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Page 454, in Table 5(a) the B-constant for Pr_5 at joint C should read 1623 instead of 1603.

Page 1632, correct the spelling of "Coulson, Benjamin LeFevre."

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CAVITATION IN HYDRAULIC STRUCTURES
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NATURE OF CAVITATION

BY JOHN K. VENNARD,[1] ASSOC. M. ASCE.

FOREWORD

As part of the program of the Hydraulic Research Committee of the Hydraulics Division, a Subcommittee on Cavitation in Hydraulic Structures was appointed in the summer of 1942. Its first objective was to summarize the facts of cavitation which are pertinent to civil engineering problems and to bring to the attention of the profession certain experiences with the phenomenon. It was decided that this objective could best be attained by the presentation of a Symposium where there would be opportunity for others, by discussion, to report their experiences and thus bring additional facts to light. In this way the store of knowledge could be increased and assembled in convenient form for civil engineering use. The Symposium developed logically into a single paper containing a summary of facts and theories, followed by three papers on practical experiences with cavitation.

After current knowledge of cavitation has been summarized by the Symposium and ensuing discussions, the committee intends to stimulate research activity on the unanswered questions and unsolved problems with the ultimate objective of obtaining, for the designer, sufficient information for cavitation-free designs to be produced with certainty.

INTRODUCTION

Cavitation first became an engineering problem about 1900 when, with the use of the steam turbine drive for ship propulsion (which resulted in higher propeller speeds), marine engineers noticed losses of efficiency and destruction of propeller materials. Shortly afterward, mechanical engineers in the hydraulic machinery field encountered the phenomenon as speeds of turbines and pumps were increased. With the design of higher hydraulic structures and with their resulting higher velocities, the civil engineer has become increasingly involved with the cavitation problem.

The generally accepted word for the destruction and subsequent erosion of materials by cavitation action is "pitting." Although the operator frequently refers to the erosion of blades, conduits, piers, liners, etc., as "cavitation," this use of the word is to be discouraged. In brief, the cavity occurs in the liquid, the pitting in the solid boundary. Although the damaging or "pitting" of structural materials by cavitation has been variously attributed to chemical action, electrochemical action (corrosion), high tension in the water, etc., present evidence indicates conclusively that the destructive action is essentially a mechanical one, caused by the impact of liquid masses on the surface of the material. The objective of all designers is to gain sufficient knowledge of the nature and occurrence of cavitation to prevent it or render it harmless in new designs.

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Notation.—The letter symbols, used in this Symposium, conform essentially to American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942. In each paper definitions are introduced where a symbol is first mentioned.

Occurrence of Cavitation

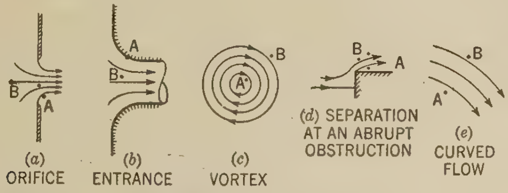
Because liquids encountered in engineering practice cannot expand and cannot support tension stress, cavities will form in them wherever the absolute pressure falls to (or close to) the vapor pressure of the liquid. Two of the factors contributing to pressure reductions may be seen, from the Bernoulli equation for a frictionless liquid—

$$\frac{p}{\gamma} + \frac{V^2}{2\,g} + Z = \text{constant} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

—to be increments of velocity, V, and of elevation, Z. (The remaining symbols in Eq. 1 are defined as follows: p is the pressure per unit area; γ is the specific weight; and g is the acceleration due to gravity.)

Considering the effect of elevation it may be concluded, for example, that (*a*) cavitation is more likely to appear at the top of a conduit than at the bottom, (*b*) higher setting of an hydraulic turbine above tailwater will increase the tendency for cavitation, and (*c*) marine propellers on surface vessels are more susceptible to cavitation than the propellers of submarines (when submerged).

The production of pressure reduction by increase of velocity is familiar to the engineer and usually results from constriction of a passage as, for example, in a venturi meter or aspirator nozzle; the tendency for cavitation to occur in regions of high velocity is obvious.



(*a*) ORIFICE (*b*) ENTRANCE (*c*) VORTEX (*d*) SEPARATION AT AN ABRUPT OBSTRUCTION (*e*) CURVED FLOW

Fig. 1.—Pressure Reductions toward the Centers of Flow Curvatures

A third, and probably the most important (yet most obscure and unpredictable), factor contributing to pressure reduction and cavitation does not appear in Eq. 1. This is flow curvature, the essential facts of which are shown in Fig. 1, points A denoting the areas of low pressure and points B the areas of high pressure. (The effects of curved flow, of course, may be included in the Bernoulli equation if one continues to use the frictionless liquid as a basis for his reasoning. Since flow curvatures are invariably caused, or accompanied, by separation, eddies, vortices, boundary layer phenomena, etc.—all of which are the result of viscous action—the assumption of a frictionless fluid is not a reasonable one.) Cavitation caused by the pressure reduction at local flow curvatures is probably more prevalent in engineering practice than any other type. These flow curvatures may exist on a boundary surface of relatively easy curvature such as a bellmouth conduit entrance, on the blades of a turbine or propeller, or at abrupt corners which produce flow separation such as gate slots, partly opened valves, or offsets in a boundary surface due to poor alinement. Furthermore, localized regions of high velocity and low pressure exist in the sharply curved flow at the centers of vortices and eddies and produce an irregular and unpredictable type of cavitation.

The relative motion of solid and liquid so necessary to the production of low pressure may be obtained by moving the solid as well as the liquid. Cavitation may be produced this way in reciprocating or vibrating devices such as plunger pumps and submarine signaling diaphragms, and in the vibratory test apparatus used by H. Peters (1).[2] This ingenious device, illustrated subsequently in Fig. 8, has proved to be an efficient means of studying cavitation under controlled conditions and producing accelerated pitting. It consists of a vertical nickel tube which is oscillated longitudinally at 6,500 cycles per sec by magnetostriction. The specimen to be tested is attached to the lower end of the rod which is immersed in a beaker of liquid. Oscillation of the rod causes cavitation in contact with the bottom of the specimen and produces pitting of the type shown subsequently in Fig. 12.

Nature of Cavitation

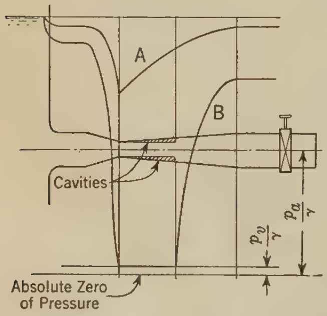


Fig. 2.—Pressure Variations in a Convergent-Divergent Passage with and without Cavitation

Cavities produced by low pressures in a flowing liquid prove to be unstable and periodically detach themselves from their point of formation. They may then be swept rapidly downstream into a region of higher pressure where they are suddenly collapsed by the surrounding liquid. As the inward rushing liquid finally fills the cavity, the momentum of the liquid is reduced to zero in an almost infinitesimal time. If the point of collapse is on a solid boundary, the pressure over an infinitesimal area becomes enormous and capable of denting or pitting the solid material within that area.

For study in its simplest form in the laboratory, cavitation may be produced in a convergent-divergent passage with transparent walls and the foregoing facts may be deduced from observations. In the apparatus of Fig. 2 the conventional hydraulic grade line A will be produced for a small valve opening and small flow rate. At a larger valve opening the higher velocity at the throat of the constricted passage produces an absolute pressure equal to the vapor pressure p_v of the liquid. Then a frothy region appears extending from the throat of the constriction into the divergent tube and the hydraulic grade line B will be obtained. Although the frothy region appears steady to the naked eye, high-speed motion pictures indicate it to be forming and reforming many times every second. The cavities form at the throat, extend themselves downstream, are torn away from the throat, and disappear near region B where the pressure gradient is high. Near B the observation window will be found after a short time to be pitted as if from delicate taps by a sharp instrument. Fig. 3 shows cavitation in a divergent passage as photographed by H. Föttinger. Fig. 4(*a*) shows schematically the creation, travel, and collapse of the cavity, and Fig. 4(*b*) shows the same series of events (cavity in black), as recorded by the high-speed camera at the Massachusetts Institute of Technology, at Cambridge, Mass.

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the "Literature Cited," which appears as the last unit of the Symposium.

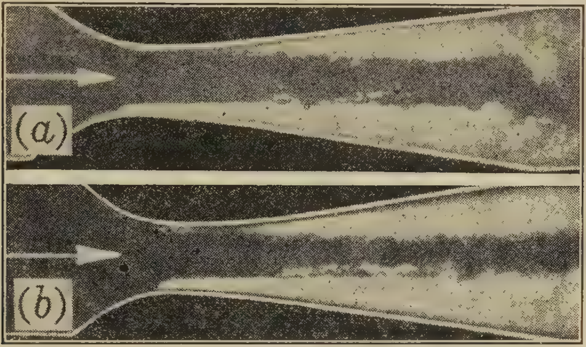
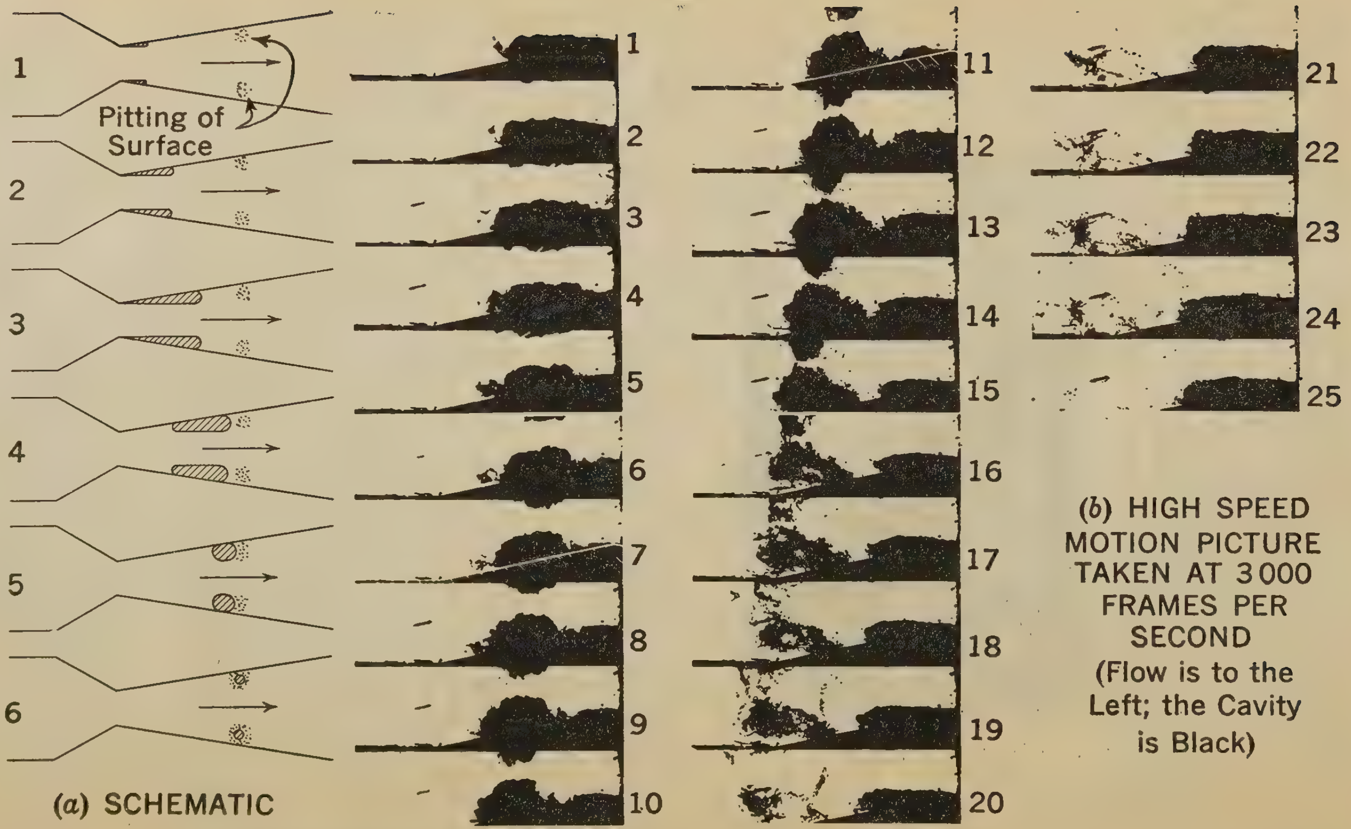


Fig. 3.—Cavitation in a Convergent-Divergent Passage

It is not implied that all cavitation assumes the form outlined in Fig. 4 but from such tests its essential nature may be deduced and the parts of the process discussed separately. Evidently the cavitation mechanism is composed of four parts: (*a*) The formation of the cavity; (*b*) its travel; (*c*) its final collapse; and (*d*) the consequences of its collapse. Since pitting of adjacent materials is one of the consequences of cavity collapse, a brief statement will be made on the pitting mechanism and on remedies that have proved successful in arresting it. In addition to the foregoing, certain luminous phenomena have been observed in the high-velocity flows on spillway aprons and in the flows downstream from the outlets of draft tubes. Since similar observations have been made on flows both with and without cavitation, it appears that these luminous phenomena are not one of the consequences of cavitation (2).

Formation of the Cavity

Because cavities are formed at regions of low pressure and because air or other gases dissolved in liquids come out of solution in these regions it would be expected that the pressures within the cavities are larger than the vapor pressure of the liquid by the partial pressure of the air within the cavity. Measurements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Pitting of Surface
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(a) SCHEMATIC

(b) HIGH SPEED MOTION PICTURE TAKEN AT 3000 FRAMES PER SECOND (Flow is to the Left; the Cavity is Black)

Fig. 4.—Complete Cycle of the Formation of Cavitation, Showing the Collapse of a Single Cavity

(3) have indicated this to be the case and have also shown that the excess of cavity pressure over vapor pressure increases with the air content of the liquid.

When cavities are formed by separation from a divergent boundary, as illustrated in Fig. 4, they have a fairly regular frequency which is approximately proportional to the velocity of flow and inversely proportional to the length of the cavity (3). The detachment of a given cavity from the boundary wall is probably caused by an instantaneous rise of pressure due to the collapse of the preceding cavity.

A cavity caused by a vortex (Fig. 5) does not form and collapse in the manner demonstrated in Fig. 4. In fact, if the flow were perfectly steady it would be difficult to see how any collapse would occur. However, the turbulent nature of the flow causes whipping of the unattached end of the vortex, which causes small portions of the major cavity to be detached and to collapse in a region of higher pressure. In Fig. 5 there is a gap between the end of the blade and the glass plate through which the pictures were taken. Upward flow occurs through the gap and produces a vortex similar to the tip vortex at the end of an airplane wing.

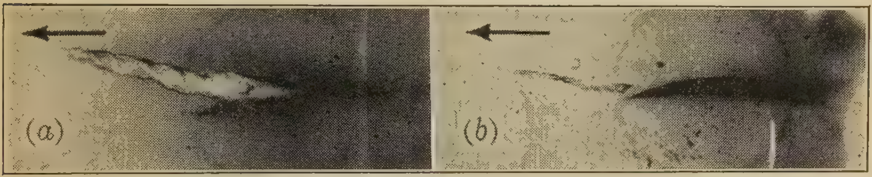

FIG. 5.—CAVITATION CAUSED BY A VORTEX (VELOCITY OF APPROACH TO BLADE, 9 FT PER SEC): (a) Gap, $\frac{3}{16}$ In.; Angle of Attack, + 2.5°; and (b) Gap, ¼ In.; Angle of Attack, − 2.5°

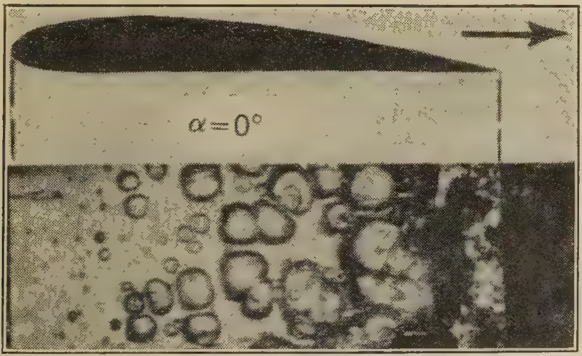

FIG. 6.—CAVITATION BUBBLES FORMING ON THE TOP OF A BLADE

When eddies that are shed from boundary irregularities, or are caused by the mixing of jets, enter a low-pressure region the pressures at their centers may be low enough to produce a cavity that will then continue to move with the flow and collapse farther downstream. Other types of cavities apparently form as small bubbles, possibly starting with a tiny air bubble as a nucleus. Such bubbles have been observed (see Fig. 6) in the flowing fluid (4), whereas others, moving very slowly, remain in contact with solid boundaries of the flow (5). Such bubbles have been observed to expand in a region of low pressure and disappear when they enter a region of higher pressure. Evidently surface tension will affect cavitation to the extent that such small bubbles play an effective rôle in the phenomenon (6).

Although the thermodynamics of cavity formation is not known quantitatively (7), a lack of equilibrium may be safely assumed except in the case of a cavity at the center of a fixed vortex. As a cavity forms and rapidly increases in size, vaporization of the liquid will occur and gases dissolved in the liquid will come out of solution into the cavity. However, due to the turbulence that always accompanies cavitation, the speed of vaporization and of release of dissolved gases is quite unpredictable. Little is known of the process except that it occurs with great rapidity.

MOTION OF THE CAVITY

It has been observed generally that the cavity proceeds from its point of formation to its point of collapse with a speed less than that of the liquid. The cavities of Fig. 4(*b*) move at about one half the velocity of the liquid at the throat of the constriction. In Fig. 6 the cavitation bubbles, approximately spherical in shape, can be seen forming and disappearing. They are moving at approximately 15 ft per sec, whereas the stream speed is approximately 22 ft per sec. However, in the case of free small bubbles and small eddies having cavities at their centers, the speed of the cavity is probably about equal to that of the liquid. On the other hand, small bubbles attached to boundary walls have been observed to move at speeds very much less than the speed of the liquid (6).

COLLAPSE OF THE CAVITY

Destruction of the cavity begins when the cavity is in motion and results from the upstream face of the cavity moving more rapidly than the downstream face. This may be observed in Fig. 4(*b*) and in the flattening of the bubbles of Fig. 6. A preliminary approach to the collapse mechanism may be obtained by considering the idealized case of collapse of a spherical cavity in a mass of liquid which is at rest. By assuming that the cavity remained spherical as it collapsed, Lord Rayleigh (8) computed the pressure within the cavity to be 68 tons per sq in. and 765 tons per sq in. when its diameter had been reduced to 1/20 and 1/100, respectively, of its original diameter. C. A. Parsons and S. S. Cook (9) also showed experimentally that enormous pressures may be developed locally when water rushes into a void space containing only water vapor. At first the bubble collapse theory was accepted as a complete explanation for the production of intense local pressures and consequent destruction of materials, but further evidence indicated it to be only a partial answer. Such evidence consisted of high-speed pictures like those of Figs. 4(*b*) and 6, destruction of materials by water jets, and the realization that it was unlikely for a spherical bubble to exist in a flow and inconceivable that it would remain spherical as it collapsed. The views in Fig. 4(*b*) show the cavity to be of irregular shape and to contain droplets of liquid as well as liquid vapor. Tests made by E. Honegger (10) and T. F. Hengstenberg (11) by moving elements of solid material through small water jets at high speed produced destruction of the material and indicated that appreciable pressures could be produced by impact of small liquid masses on solid surfaces.

From the foregoing facts it appears that the intense local pressures developed at the point of collapse are probably due to a combination of the foregoing phenomena. As collapse of the cavity occurs at a boundary, droplets may be shot at the solid surface with speeds sufficient to produce deformation and eventual destruction, or irregularities in the cavity walls may produce intense local pressures when the liquid strikes the solid material at the high speeds produced at collapse. Although the collapse of the cavity is the main cause of impact between liquid masses and solid boundaries the exact details of the impact mechanism may be quite different in successive collapses.

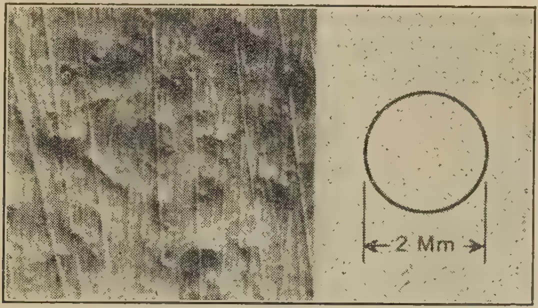

Fig. 7.—Small Pits (Dents) Produced on a Smooth Brass Plate After Five Hours of Exposure to Cavitation

Experimental measurement of the pressures produced locally at cavitation collapse has proved an exceedingly difficult problem because these pressures are not produced at exactly the same point at every collapse. Even assuming a perfect measuring instrument, it is still difficult to place the instrument at the exact point where collapse occurs. J. Ackeret (12) has approached the problem by using a piezoelectric crystal 2 mm in diameter (placed in the boundary wall) and has measured local pressures of more than 200 lb per sq in. in a region where the average pressure was only 60 lb per sq in. That local pressures at the collapse point are higher than this and that they exist on a much smaller area may be inferred from Fig. 7 by comparing the size of a dent caused by cavitation on a brass plate with the 2-mm circle. The existence of high pressures in the region of collapse was also convincingly demonstrated by some tests at Massachusetts Institute of Technology (3), in which short lead tubes, having one end closed, were filled with water and placed with their open ends in the region of cavity collapse. The tubes were stretched quickly by the intermittent high pressures and soon failed in tension. The magnitude of collapse pressures cannot be deduced from such tests, of course, but the existence of high pressure is proved beyond doubt.

The thermodynamic processes in cavity collapse consist of high-speed compression of the gases (mostly air) in the cavity and the condensation of the liquid vapor. In spite of this rapid compression and condensation, high local temperatures probably do not result because of the large water masses available to carry away heat generated by compression. Furthermore, it should be noted that the expansion and vaporization at the formation of the cavity will tend to cool the surroundings so that the heat generated during collapse will tend to restore the temperature to its original value rather than raise it appreciably above that value. Thus it appears very doubtful that thermal effects can play any appreciable part in the pitting of materials by cavitation.

Although local temperature rise appears to be of little consequence in cavity collapse, vapor pressure and air content play a significant part. Higher partial pressures of air and vapor in the collapsing cavity would be expected to offer resistance to the compression of the cavity by slowing condensation of vapor and reabsorption of air by the surrounding liquid. This in turn could be expected to produce a cushioning effect on the collapse, causing local pressures to be less intense and destruction of materials to be less rapid. Laboratory and field observations bear out the cushioning effect of increased air content but variation of vapor pressure has produced various trends. In a series of carefully controlled tests in the vibratory apparatus, using water of constant air content, H. Peters and B. G. Rightmire (13) showed that cavitation severity may increase or decrease with increments of temperature (and vapor pressure). (In Fig. 8, illustrating this apparatus, the nickel tube is oscillated longitudinally by magnetostriction.) However, J. M. Mousson, M. ASCE (14), reports swifter destruction of materials at higher temperatures in tests using a venturi type apparatus, and turbine operators have generally observed more destruction of blades and parts in the summer when temperature and vapor pressure are high and air content low than in the winter when opposite conditions exist. From the evidence available it appears that no general conclusions may be drawn as to the effect of vapor pressure on cavitation severity. Evidently the rôle that vapor pressure plays in cavity collapse is something more complicated than a cushioning effect.

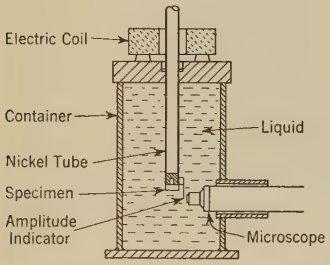

Fig. 8.—Apparatus for Producing Cavitation by the Vibratory Method

From the random paths that cavities follow before they collapse it is evident that scattering of their points of collapse over a sometimes sizable region may be expected. Since this region is three dimensional, however, it appears likely that most cavities will collapse within the free flow and out of contact with the confining boundary surface. The question naturally arises whether cavities must contact the boundary surface as they collapse in order to damage this surface. As cavities completely surrounded by liquid collapse in the flow, compression waves will be sent out in all directions and will strike the solid boundaries confining the flow. Although it appears unlikely that such pressure waves could damage the boundary material, T. C. Poulter (15) and J. Ackeret and P. deHaller (16) report the damaging of a metal surface in a liquid by the creation of high-frequency pressure waves in the liquid. If such pressure waves are caused by ordinary cavity collapse, destruction of material at a distance from the collapse point appears possible. Whether collapse of cavities in the flow can destroy boundary surfaces by the action of the pressure waves is still an open question but pressure waves may endanger a structure or machine by setting up forced vibrations. The pressure wave in traveling upstream into the low-pressure region also appears to be instrumental in detaching the succeeding cavity.

Damage from Cavitation, or Pitting

The harmful effects of cavitation are: (*a*) The pitting of the solid boundaries confining the flow; (*b*) the reduction in efficiency of machines and water passages; and (*c*) vibrations in the structure or machine caused by the periodic nature of cavity collapse. Thus, the designer of hydraulic structures may pay a three-fold penalty for allowing cavitation to occur, or he may reap a triple dividend by preventing the occurrence of cavitation. Since a discussion of vibrations and efficiency losses caused by cavitation would lead too far afield, pitting alone will be discussed in this paper. Figs. 9 to 12 illustrate the typical destruction of metals by cavitation. Fig. 9 was reported by Professor Ackeret (12), Fig. 10 by the Baldwin Southwark Company, Fig. 11 by the National Electric Light Association, and Fig. 12 by the Massachusetts Institute of Technology.

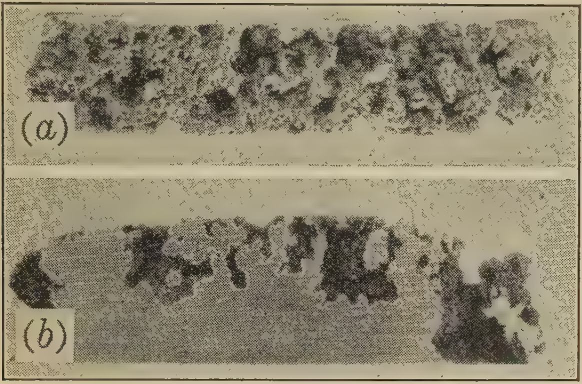

Fig. 9.—Example of Severely Pitted Cast Iron: (*a*) Surface Appearance; and (*b*) Cross Section

When a cavity collapses adjacent to a solid surface, or a droplet of liquid strikes the surface at high speed, the mechanical action is similar to striking the surface with a small ball-peen hammer. Indeed the dent that results in a malleable material appears just like the result of a hammer blow (see Fig. 7). The action of the collapsing cavity on the material is then primarily mechanical and thermodynamic or electrochemical (corrosion) effects play only a minor rôle. Although it was widely contended some time ago that pitting was primarily a corrosive action, this belief was destroyed by Professor Föttinger (18) when he obtained pitting on the glass walls of a venturi type cavitation apparatus. However, this is not to claim that corrosive action plays no part at all, because it is well known that metals fatigue more rapidly in the presence of corrosive action.

The pitting of materials by cavitation is primarily a fatiguing action in which the surface skin of the boundary is continuously hammered by millions of tiny blows until it cracks and chips off. It has been observed generally that surface finish has an effect on the speed of pitting due to cavitation and on the rapidity of destruction of metal by water jets. Rough surfaces are invariably destroyed more rapidly than surfaces of smooth finish. Thus, as the smooth surface of any material is worn away, the pitting process accelerates and very rapid destruction usually results. In some cases, however, the pitting has stopped of itself, apparently due to a water cushion covering the eroded region and preventing direct contact of collapse point and solid material.

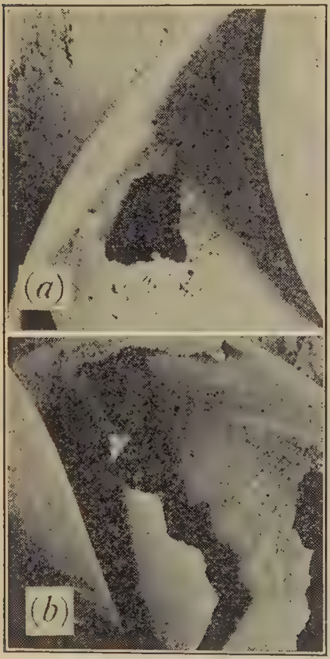

Fig. 10.—Severe Pitting of an Hydraulic Turbine Runner

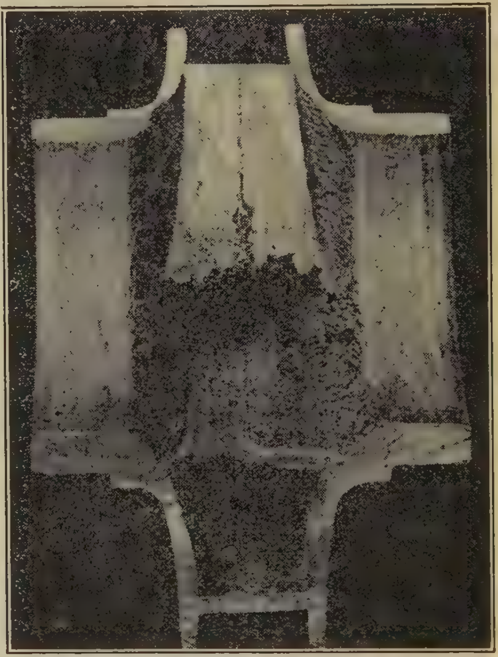

Fig. 11.—Severe Pitting of a Centrifugal Pump Impeller

The precise mechanism of the accelerated destruction of rough surfaces is not known, but higher stress concentrations on irregular surfaces undoubtedly exist and should contribute decidedly to the process. Other effects may be at work in the case of very rough or cracked surfaces, however. For example, it is possible that the high pressure nucleus of the collapse cavity (possibly a tiny air bubble) is driven into fissures in the material and explodes when the pressure drops in the surrounding region. Another possibility is that the spaces at the inner ends of the cracks in the material act as cavities themselves and collapse with destructive force when local pressures at their outer ends are increased by cavities collapsing in the flow. Still another possible explanation is the high pressure developed at the inner end of the fissure by reflection of a pressure wave entering the fissure (similar to the action of water hammer at the closed end of a pipe line). The intermolecular penetration of a high-pressure liquid into a solid has been shown by Mr. Poulter (15) to be capable of destroying the solid upon release of the pressure. Such destruction, however, is dependent upon the application of pressure over a relatively long time. Because of the almost instantaneous existence of the high collapse pressure, it does not appear likely that this effect is operative in contributing to the pitting caused by cavitation.

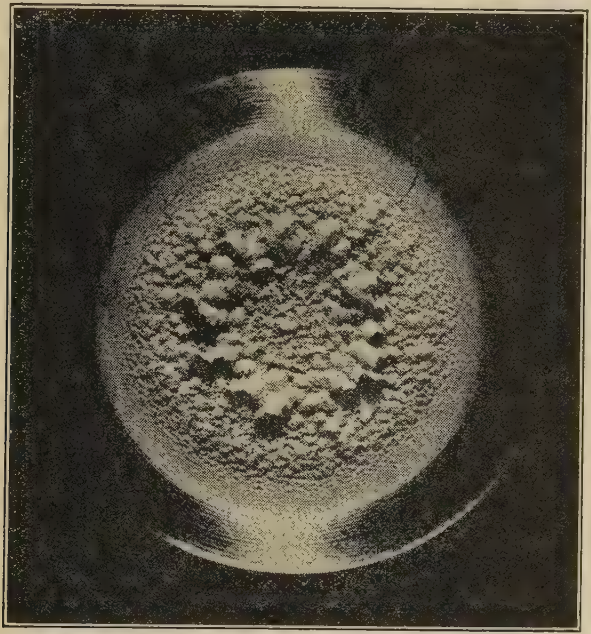

Fig. 12.—Typical Pitting Produced by the Vibratory Method

Evidence is also available that cracks in the solid boundary do not accelerate the pitting process. Mr. Mousson (19*e*) has observed that the regions in and around the check cracks in the welding of a repaired turbine runner show no more pitting than the smooth surface material. He has also reported that destruction or weakening of the material beneath the surface is possible before cracks appear in the surface, by transmission of the effects of the blows on the surface.

Pitting of solid material suspended in a liquid, in which high-frequency pressure waves are created without cavitation, has been obtained by Mr. Poulter (15) and may be explained by the induced vibrations in the solid surfaces causing destructively high local stresses. If solid boundaries are actually destroyed in cavitation by cavities collapsing out of contact with the boundaries, this phenomenon offers an explanation of such pitting.

REMEDIES FOR PITTING

The most effective means of preventing pitting in new structures is the elimination of the cavitation that causes it. However, to "design cavitation out" of structures is not possible by design office calculations only. Cavitation can be prevented only through intelligent cooperation of the design office and the hydraulic laboratory. The designer should have in mind continually the facts that sharp curvatures, abrupt corners, or any combination of circumstances which produce flow curvature, vortices, eddies, separation, or high local velocities, are all conducive to cavitation and, like the aeronautical designer, he should make every effort to "streamline" his designs with contours of easy curvature. Even with these facts at hand, the design office cannot produce designs free from the danger of cavitation (unless such designs are so conservative that their costs are prohibitive) without the assistance of the laboratory. This results from the impossibility of using the foregoing principles quantitatively for the precise prediction of hydraulic phenomena.

In completed structures where cavitation and pitting occur and drastic modifications are not possible, various methods have been used successfully to arrest the pitting process or to prevent it after repairs have been made. Replacement of eroded parts by a tougher, more resistant material (such as the use of steel liners in conduits in the region where pitting has occurred in the concrete) has proved successful. This has its counterpart in the hydraulic turbine field where it is standard practice to chip out the damaged material and replace it with a tougher substance built up by welding. In some structures the pitting process has been allowed to continue until it arrests itself, presumably by the creation of a water cushion, as mentioned previously. Although the latter method may have little appeal, it is sometimes effective where erosion does not seriously endanger the strength of a structure or its working parts.

Admission of air to the low-pressure regions has proved effective in both hydraulic structures and hydraulic machines. Higher air content of the water has been seen to cushion the cavity collapse and reduce its destructive effects, and air introduced well upstream from the cavitation region will contribute to this process. However, the effect of introducing air at the cavitation region is to "break the vacuum" there, thus raising the pressure and tending to eliminate true cavitation. The air introduced in this manner presumably forms large bubbles which are compressed downstream in the high-pressure region but are prevented from collapsing by the cushioning effect of the large quantity of air, which cannot be absorbed by the water upon compression of the bubble.

LABORATORY TESTING FOR CAVITATION

New designs may be tested for cavitation in the laboratory by suitable hydraulic measurements on a model of the new design. For cavitation testing of models there are two techniques which are commonly used: (1) The operation of the model within a vacuum tank; and (2) the operation of the model at or near atmospheric pressure conditions in the usual way. The latter method has been used successfully with water flowing in the model, and more recently with air as the fluid flowing.

The testing of models for cavitation near atmospheric conditions involves the measurement of static pressures at numerous points in the critical region so that the pressure distribution is well known. Cavitation is not produced in the model—if water is used the velocities are not high enough to cause cavitation; and, if air is used, cavitation is not possible because of the capability of the fluid to expand and fill any cavities that tend to form. However, by converting the measured model pressures to their corresponding prototype pressures by the laws of dynamic similarity, the model engineer may predict whether the prototype will have cavitation or not. The exact form of the cavitation cannot be predicted from such measurements and it is conceivable that a vortex in the flow which might produce cavitation in the prototype would be missed completely in a model that was equipped with wall piezometer openings only. In spite of this slight element of doubt, this method of testing has proved reliable in the past and it has been adapted successfully of late to the measurement of the intermittent cavitation pressures produced in highly turbulent flows, notably in the flow occurring around baffle piers. In situations like this, flow may be entirely unsteady locally, whereas the over-all flow is a perfectly steady one. At any given point, pressures may be low enough to produce cavitation at one instant and high enough to prevent cavitation at the next instant, thus producing a series of intermittent cavities or "flashes." Obviously such high-frequency pressure fluctuations could not be recorded by a piezometer column because of the inertia of the column; so the U. S. Waterways Experiment Station developed (20) an electrical recording pressure measuring device that traces the pressure fluctuations and thus may be used to predict at any point the percentage of time that cavitation pressures will occur. Assuming that all cavitation cannot be eliminated from abrupt obstructions like baffle piers (which for their very purpose must be unstreamlined), such measurements are useful in correcting designs to eliminate the cavitation pressures which exist continually over wide areas and are thus most likely to produce the regular cavitation that would be damaging to the structure.

Vacuum-tank testing of models for cavitation is much more complicated than open testing but has been shown by H. A. Thomas, M. ASCE, and E. P. Schuleen, Assoc. M. ASCE (19), to eliminate the element of doubt mentioned previously. The entire model is enclosed in a tank and the air above the model is evacuated so that the ratio of atmospheric pressure to absolute pressure imposed on the model is approximately equal to that of the model scale. By this method cavitation is produced in the model and its form and location become visible to the model operator. Photographs of expected cavitation may thus be obtained and may be supplemented by pressure measurements as well. The practical difficulties of high vacuum testing of hydraulic models are numerous but Professor Thomas has overcome these successfully and has indicated certain advantages of the vacuum-tank method. He has also developed material that will pit under the action of model cavitation and it is possible that, in the future, vacuum-tank testing will show not only the forms of the cavitation but the areas damaged by cavitation as well.

EXPERIENCES OF THE CORPS OF ENGINEERS

By John C. Harrold,[3] Assoc. M. ASCE

Synopsis

The discovery, in 1935, of severe pitting of the concrete in the conduit entrances of Madden Dam on the Chagres River in the Isthmus of Panama prompted the Corps of Engineers, U. S. Army, to undertake later that year a program of model research on the subject of cavitation. The initial purpose of this research was to develop a design of conduit entrance that would be free from cavitation. The program was later expanded to include a study of cavitation at gate slots in conduits and cavitation around baffle piers in stilling basins of high dams. The major part of this paper is devoted to a discussion of this model research and its effect on the design of structures that come under the jurisdiction of the Corps of Engineers.

The first research was conducted at Carnegie Institute of Technology, at Pittsburgh, Pa., under the direction of Harold A. Thomas, M. ASCE. Later the U. S. Waterways Experiment Station at Vicksburg, Miss., joined in the study. Since 1941 both laboratories have been engaged in cavitation research for the Corps of Engineers. At Carnegie Institute of Technology, models are tested in a vacuum tank in which the air pressure can be reduced to scale and in which actual cavitation can be made to occur. At the U. S. Waterways Experiment Station, ordinary open-air models are used and cavitation pressures are detected with especially designed electric pressure cells capable of measuring rapidly fluctuating pressures in water. These cells are attached to piezometer openings in the face of the model. Both methods have proved highly satisfactory and the results obtained at both laboratories have been correlated.

Introduction

The first high, concrete, flood control dam in the Upper Ohio River Valley—Tygart Dam—was under construction in 1935 when news reached the Pittsburgh District Office of the Corps of Engineers (under whose supervision Tygart Dam was being constructed) that severe pitting of the concrete had occurred in the entrances of the outlet conduits of Madden Dam (19*a*). Since the Tygart conduits were similar in design to the Madden conduits and were to be operated under about the same head, concern was felt lest pitting should also take place in these conduits.

The cause of the pitting in the conduits of Madden Dam appeared to be cavitation induced by the sharp entrance curve in the roof of the conduit. The curve was slightly less sharp in the case of the Tygart conduits, but it was not known whether the difference in curvature would be sufficient to prevent the formation of cavitation.

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[3] Senior Hydr. Engr., Office of Chf. of Engrs., War Dept., Washington, D. C.

The possibility of conducting model tests for studying the problem was discussed with Professor Thomas of Carnegie Institute of Technology, who had already conducted model tests of the stilling basin of Tygart Dam for the Pittsburgh District Office. Professor Thomas expressed the opinion that such tests would be feasible and agreed to undertake the tests for the Pittsburgh Office. Model tests of the conduit entrances would first be made in order to determine definitely whether cavitation had been the cause of the pitting at Madden Dam and to provide a general check on the reliability of the methods of testing used. Tests of the Tygart conduits would then be made to determine whether or not cavitation could be expected to occur in these conduits. If so, further tests would be made to determine a remedy for this condition.

These tests showed definitely that cavitation had been the cause of pitting in the Madden Dam conduits. When the results of these tests were called to the attention of the Panama Canal authorities, additional tests of the Madden Dam conduits were authorized with a view to determining a remedy for this condition. Several methods of eliminating cavitation at Madden Dam were tried—reshaping the conduit entrance, constricting the downstream end of the conduit, placing fillers in the stop-log slots, and venting with air. Reshaping of the conduit entrance was finally selected as the most practicable remedy for the condition existing at Madden Dam (19*b*).

A model of the conduit entrances at Tygart Dam was then tested and found to be free from cavitation. Apparently the degree of curvature provided in the entrances of the Tygart conduits was sufficient to prevent cavitation. No change was necessary, therefore, in the design of these entrances. Although the answer was negative in this case, the experience gained from these tests proved to be very useful in subsequent testing of this type.

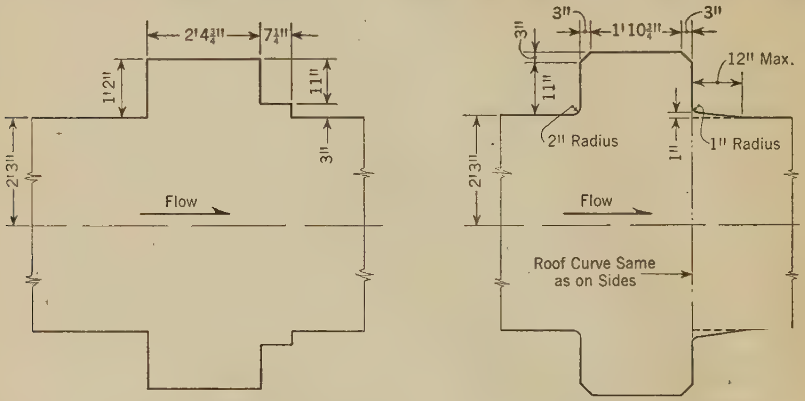
First Vacuum Apparatus

A special type of apparatus was designed by Professor Thomas and constructed for these tests—the first apparatus for testing hydraulic structures under a vacuum to be built in the United States (19*c*). It consisted of a closed, horizontal-loop, recirculation system in which the pressure could be reduced by an air exhaust pump. Water was recirculated with a centrifugal pump. A part of the system was made large enough to accommodate half-section models of conduit entrances. A glass plate was placed over the conduit on its center line so that the action of the water could be observed.

Conduit Tests

The action at the conduit entrance at Madden Dam in this apparatus has been illustrated by Professor Thomas and E. P. Schuleen, Assoc. M. ASCE (19*d*). In a similar photograph of the Tygart conduit no cavitation was visible, indicating that the conduit entrance at Tygart Dam will be free of cavitation.

A model of the conduits of another dam was tested in this apparatus in order to study cavitation phenomena in the vicinity of the gate slots. The entrance curve for this design, as a result of experience at Madden Dam, was very liberal. No cavitation was revealed either in the entrance or at the gate slots under the conditions of maximum head for which this conduit was designed. However, a slight overspeeding of the apparatus (equivalent to increasing the head) gave rise to cavitation flashes at the downstream edges of the downstream gate slot (Fig. 13(*a*)). In order to increase the factor of safety against the occurrence of cavitation in the full-sized structure, a new design of gate slot was developed (Fig. 13(*b*)) which was found to be free from cavitation under this increased head. A slight bevel and rounding was provided on the downstream edges of the gate slot in the improved design. This design has been used in several dams constructed by the Corps of Engineers.



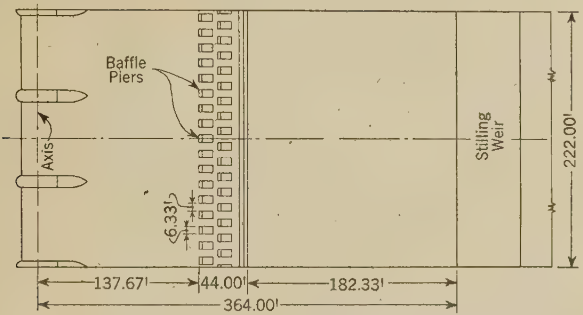
(*a*) ORIGINAL DESIGN (*b*) IMPROVED DESIGN

FIG. 13.—IMPROVEMENT IN GATE SLOT DESIGN

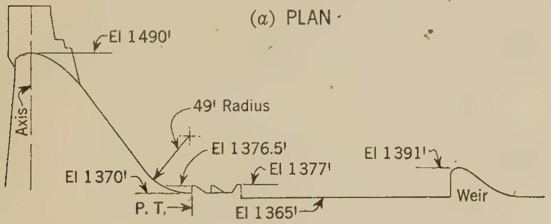
BAFFLE PIER PROBLEM

In 1939, four years after the Madden Dam incident, another incident occurred which gave further impetus to the development of the technique of cavitation model testing. A very economical stilling basin design had just been developed in the hydraulic laboratory of Carnegie Institute of Technology for use below the spillway of Bluestone Dam, one of the flood control dams in the Middle Ohio River Valley (Huntington, W. Va., District). This basin consisted of a short horizontal apron at the toe of the spillway with two rows of baffle piers and an end sill on it (see Fig. 14). Fig. 15 shows the water impinging against the baffle piers of this basin. The use of baffle piers enabled a much shallower basin to be adopted than would have been possible otherwise.

After the model study had been completed and the design accepted by the Huntington Office, it occurred to Professor Thomas that there might be cavitation present around these baffle piers. Some well-known foreign engineers who visited the laboratory at that time and saw the model in operation also voiced the same opinion. Professor Thomas, therefore, undertook an analytical study of pressures around cubical blocks in high-velocity streams to determine the likelihood that cavitation would be present. Although this investigation was entirely theoretical and had to be based on certain idealistic assumptions, the results indicated quite definitely that cavitation would be present around the Bluestone baffles. Not being satisfied with this theoretical analysis, however, Professor Thomas began to give some thought to the development of a


Baffle Piers
Axis
6.33'
Stilling Weir
222.00'
137.67'
44.00'
182.33'
364.00'

(a) PLAN


El 1490'
Axis
49' Radius
El 1376.5'
El 1377'
El 1391'
El 1370'
Weir
P. T.
El 1365'

(b) SECTION ALONG CENTER LINE

FIG. 14.—SPILLWAY AND STILLING BASIN OF BLUESTONE DAM, IN WEST VIRGINIA; SHOWING THREE BAYS

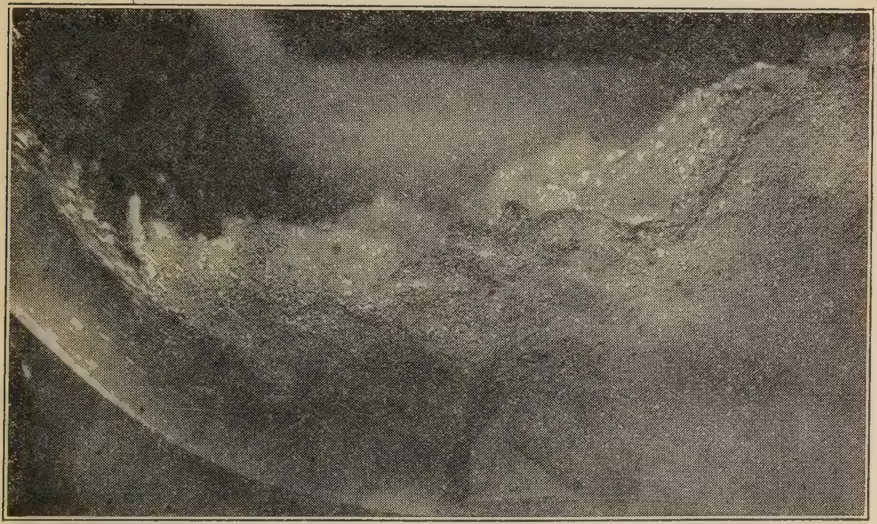


FIG. 15.—ACTION OF BLUESTONE STILLING BASIN AT MAXIMUM DISCHARGE

vacuum apparatus in which models of stilling basins could be tested. With the help of a research student he was finally able to complete the apparatus shown in Fig. 16. Fig. 16(*b*) is a view of the test chamber.

Improved Vacuum Apparatus

Professor Thomas' improved vacuum cavitation apparatus consists of a closed circulation system, as in the old apparatus; but in many other respects it differs from the old apparatus. The circulation loop is vertical instead of horizontal; and the pump is located two floors below the test chamber instead of on the same level. This latter feature eliminates the principal difficulties experienced with the previous apparatus—cavitation in the pump and air leakage in the pump packing. An ice cooling jacket was also placed on the vertical supply pipe to keep the water at a fairly constant temperature during the tests. Without the cooling jacket, the temperature of the water rises during the tests, making an additional correction in the results necessary. The test chamber is 8 ft long, 2.5 ft high, and 12 in. wide. Air is exhausted by a steam ejector located conveniently near the apparatus, the apparatus being constructed in the steam laboratory of the Institute. Windows were provided in the top and in one side of the test chamber. The top window is removable permitting the placing of models in the chamber. This window is sealed around the edges with modeling clay. Before testing, the water is recirculated for a short time under a high vacuum in order to draw as much dissolved air from the water as possible. Dissolved air has not been found to affect the results of this particular type of testing appreciably but it does produce a cloudy appearance of the water when released, which makes observation of the tests difficult. Dissolved air in large quantities has been found to affect the results of cavitation tests of hydraulic machinery.

Principle of Vacuum Apparatus

In a model study utilizing a vacuum tank apparatus, all features of the model are reproduced to scale, as in the case of an undistorted open-air model, except the atmospheric pressure surrounding the model. It would seem at first thought that the atmospheric pressure, being a unit force, should be reduced in the same proportion as the linear scale of the model. This would be correct, except for the fact that it is desired to simulate cavitation as well as correct pressures in the model. If the properties of water could be changed conveniently so that the water would cavitate, or vaporize, at, say, 1/20th or 1/50th of its natural vapor pressure depending upon the model scale used, it would be satisfactory to reduce the air pressure in direct proportion to the scale. However, it is inconvenient, if not impossible, to make water stay in a liquid state until it reaches such a low pressure. It would vaporize long before reaching this pressure. The temperature of the test water being practically the same as the prototype water, the test water will vaporize at practically the same pressure as the prototype water, not 1/20th or 1/50th of this pressure. A correction, therefore, must be added to the theoretical air pressure (atmospheric pressure divided by the model scale) to make up for the impracticability of changing the vaporizing properties of water.

A numerical example of the method of computing the atmospheric pressure to be used in a model study of this type is illustrated in Fig. 17. The spillway section of a dam has been chosen for the example but the analysis applies equally well to any hydraulic structure having a free water surface.

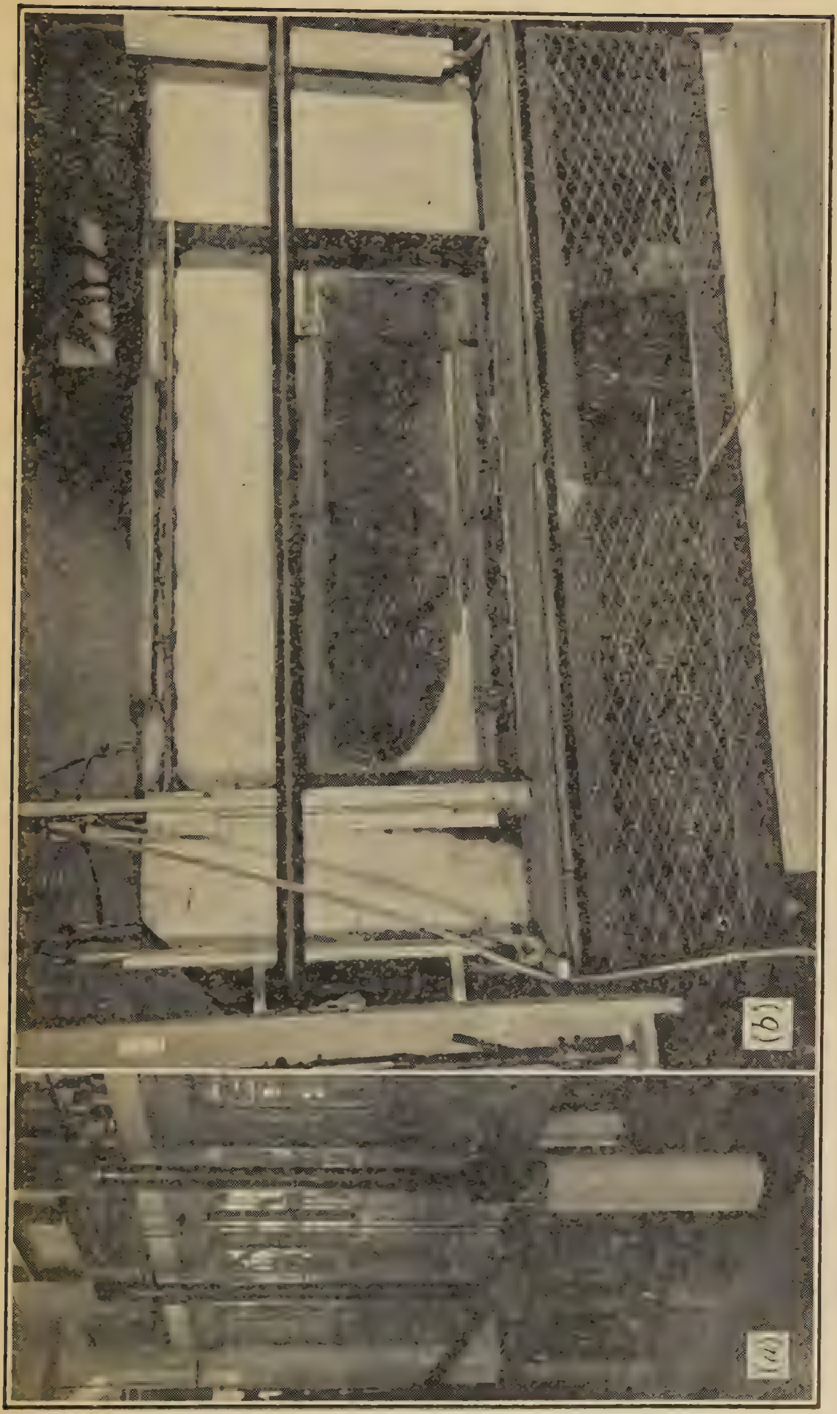


FIG. 16.—IMPROVED VACUUM APPARATUS FOR MODEL TESTS OF STILLING BASINS: (*a*) General View; and (*b*) Test Chamber

The point in question is point B on the downstream face of the spillway crest (Fig. 17(*a*)); but the analysis applies equally well to any other point on the surface of the structure, including the stilling basin. It is assumed that a model of this spillway (linear scale ratio $L_r = 1:30$) is to be installed in the vacuum tank for testing. The problem is to determine the air pressure to be maintained in the test chamber in order that cavitation action may be simulated correctly.

The example has been simplified by assuming that the water in the model will be kept at a constant temperature equal to the temperature of the prototype water. Later a correction will be developed to take care of the case in which the temperature of the test water is different from that of the prototype water. Atmospheric pressure in the prototype is assumed to be one atmosphere (33.9 ft of water, absolute).

Assume that at point B in the prototype, Fig. 17(*a*), there is a sharp break in alinement due to faulty construction and it is desired to determine the maximum head, H_p, that can be allowed on the crest without causing cavitation at point B. (The pressure on the downstream side of a spillway crest usually decreases with an increase in head on the crest, and a sharp break in alinement on this side would result in a local lowering of this pressure considerably below the normal pressure for that head.) The head, H_p, is the head that will just cause cavitation to start at point B or, in other words, just cause the local pressure at point B to be reduced to the vapor pressure of the water, p_v. Assuming the temperature of the water to be 70° F, the pressure at which the water will vaporize is $p_b = p_v = 0.838$ ft of water, absolute (obtained from a table of vapor pressures). The head, H_p, therefore, is the head which will just cause the pressure at point B to be reduced to 0.838 ft of water, absolute.

Referring to Fig. 17(*b*), the model in the vacuum tank, equations can be written for the air pressure, p_{am}, in the tank—

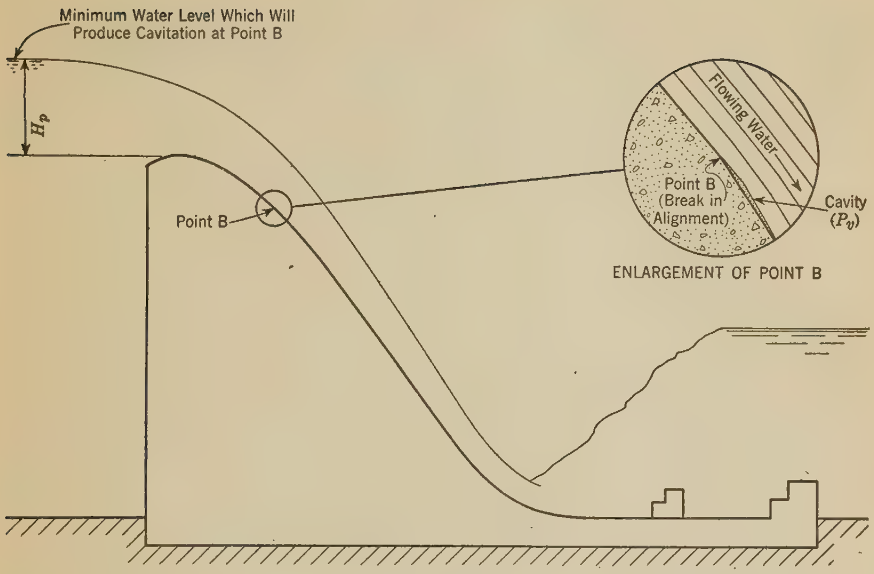
$$p_{am} = \frac{p_{ap}}{30} + \left[\frac{29}{30}\,p_v\right] \quad \text{(2a)}$$

$= 1.13 + [0.81] = 1.94$ ft of water at 70° F; and the pressure, p_{bm} at point B—

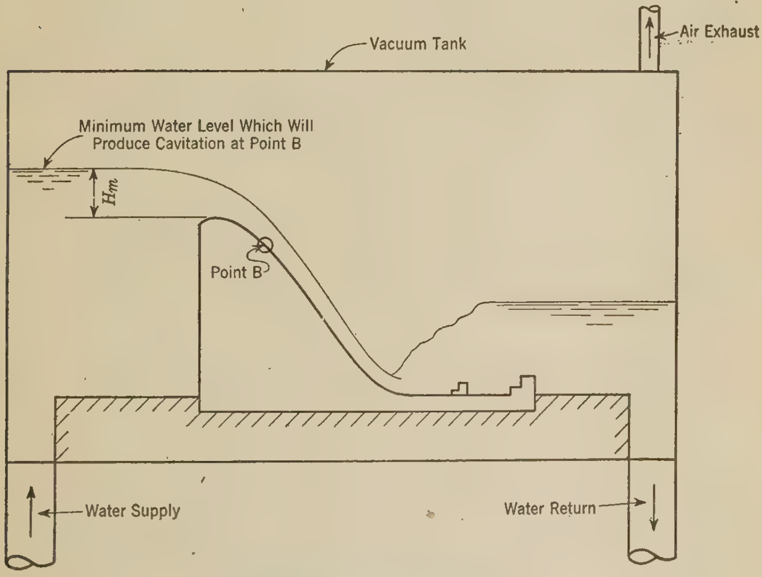
$$p_{bm} = \frac{p_v}{30} + \left[\frac{29}{30}\,p_v\right] \quad \text{(2b)}$$

$= 0.028 + [0.81] = 0.838$ ft of water at 70° F.

In Eqs. 2, the first term to the right of the equals sign is the pressure that would exist with the model head, $H_m \left(= \frac{H_p}{30} \right)$, if the air pressure in the tank is reduced by the direct linear scale ratio of the model, 1 : 30, and if the water would stay in a liquid state until a pressure as low as 0.028 ft of water, absolute, is reached. Since water at 70° F will vaporize when the higher pressure of 0.838 ft of water, absolute, is reached, a pressure as low as 0.028 ft cannot be realized. Therefore, to make the water just start to vaporize when the head, H_m, is reached, it will be necessary to raise the air pressure in the tank by the difference between 0.838 and 0.028, or 0.81 ft of water. This will raise all


Minimum Water Level Which Will Produce Cavitation at Point B
H_p
Point B
Flowing Water
Point B (Break in Alignment)
Cavity (P_v)
ENLARGEMENT OF POINT B

(*a*) PROTOTYPE


Vacuum Tank
Air Exhaust
Minimum Water Level Which Will Produce Cavitation at Point B
H_m
Point B
Water Supply
Water Return

(*b*) MODEL IN VACUUM TANK

Fig. 17.—Air Pressure in the Vacuum Apparatus for a Spillway Model (Scale, 1 : 30)

pressures on the model, including that at point B, by 0.81 ft of water. The pressure at point B will just reach 0.838 ft of water, absolute, when the head, H_m, on the crest is reached. Cavitation, therefore, will just start to form at point B when the head, H_m, is reached, which is the condition required for the correct simulation of cavitation action.

The term 0.81, or $\frac{29}{30} p_v$, has been inclosed in brackets in Eqs. 2 in order to distinguish it as the correction factor that must be added to the theoretical air pressure, $\frac{p_{ap}}{30}$, to obtain the correct air pressure p_{am} to be used in the vacuum tank. For the case in hand, it will be noted that the correct air pressure to be used in the tank is 1.94 ft of water, absolute. With this pressure in the tank, the head on the spillway crest can be increased until cavitation is observed to begin at point B, and the head at which this occurs is recorded. When this head H_m is multiplied by the reciprocal of the linear scale ratio, 1 : 30, the result will be the head H_p in the prototype, which will just cause cavitation to start at point B and which, therefore, is the head below which the dam must be operated if cavitation is to be avoided at point B.

Since the air pressure in the tank p_{am} has been increased by 0.81 ft of water over the theoretical, all absolute pressures measured in the tank will be high by this amount. Therefore, all measurements of absolute pressure will have to be reduced by this amount before being converted to prototype values. Measurements of relative pressure (differences in pressure between the air pressure in the tank and the pressure at a given point on the model) will not need to be so corrected, however, because they are independent of the pressure in the tank. The correctness of this latter statement will be seen when it is realized that in ordinary open-air models the surrounding air pressure is considerably higher than the theoretical and yet the relative pressure readings are considered reliable.

It is also desired to call attention to the fact that, in the case of a spillway model such as that in the present example, the phenomenon of separation must also be considered, since end aeration conditions in the prototype may be such that the nappe would separate from the crest before the vapor pressure at point B is reached. It was assumed that this would not take place in the above example.

The equation for the air pressure in the tank, p_{am}, given in Fig. 17(*b*), can be expressed in general terms (21), as follows:

$$p_{am} = L_r\, p_{ap} + (1 - L_r)\, p_v \quad \text{(3a)}$$

in which p_v is the vapor pressure of both the test water and the prototype water at the specified temperature. If the temperature of the test water is different from that of the prototype water, a review of the foregoing analysis on the basis of this assumption will show that the following formula applies:

$$p_{am} = L_r\, p_{ap} + p_{vm} - L_r\, p_{vp} \quad \text{(3b)}$$

in which p_{vm} is the vapor pressure of test water; and p_{vp} is the vapor pressure of prototype water. Applying Eq. 3*b* to the foregoing example, it will be seen that the correction to be added to the theoretical pressure is the difference between the vapor pressure of the test water and 1/30th the vapor pressure of the prototype water.

It will be noted in Eqs. 3 and in the accompanying example that, when the model scale L_r is small, the factor $L_r\, p_{vp}$ is extremely small. In the case of very small models, therefore, this factor may be neglected if great precision in the results is not required and the following approximate formula may be used:

$$p_{am} = L_r\, p_{ap} + p_{vm} \qquad (4)$$

The correction to be added to the theoretical pressure in Eq. 4 is only the vapor pressure of the test water.

Baffle Pier Tests

Trial runs in the vacuum tank apparatus demonstrated that this apparatus would accomplish the results desired. A preliminary model of the stilling basin of Bluestone Dam was installed in the tank and tested under the maximum discharge conditions to which the basin would be subjected. Cavitation showed up immediately around the baffle piers. This matter was then called to the attention of the Corps of Engineers and arrangements were made to have further tests of the Bluestone stilling basin conducted at both Carnegie Institute of Technology and the U. S. Waterways Experiment Station at Vicksburg, with a view to determining the severity of this cavitation condition for the complete range of discharge to which the basin would be subjected and with a view to determining a remedy for this condition, if later considered desirable and feasible. The U. S. Waterways Experiment Station, working with an open-air model, concentrated on determining a streamlined shape of baffle pier that would be relatively free from cavitation and yet be effective as a baffle pier. At Carnegie Institute of Technology tests were continued in the vacuum tank, first on a more exact model of the original basin and then on various shaped baffle piers, including the streamlined shape developed at the Experiment Station. The results obtained at the two laboratories were correlated as the tests progressed.

The tests in the vacuum tank at Carnegie Institute of Technology will be described first. A model of the original apron design with sharp-cornered baffle piers (linear scale ratio $L_r = 1/48$) was first installed in the tank and tested. Fig. 18(*a*) shows the detailed dimensions of these baffle piers. The action of this model in the vacuum tank is shown in Figs. 19(*a*) and 19(*b*). In Fig. 19(*a*), the severity of the cavitation may be judged from the fact that the upstream baffle piers (left hand in photograph) are completely enveloped in cavitation pockets. In Fig. 19(*b*) the relatively mild cavitation is revealed by a smaller pocket visible on the side of the closest upstream baffle.

Inasmuch as these tests and those at the U. S. Waterways Experiment Station (to be described subsequently) indicated that cavitation around the baffle piers of the original design would be quite severe, it was decided to investigate other shapes of baffle piers, keeping the same apron length and end sill, to see if a shape could be developed that would be relatively free from cavitation and at the same time be effective as a baffle pier and structurally rugged. Of the many shapes tested, the most practicable shape appeared to be the streamlined shape developed by the Waterways Experiment Station and shown in Fig. 18(*b*). The same width of front face, 6.33 ft, was retained in this design, the streamlining being added to the sides.

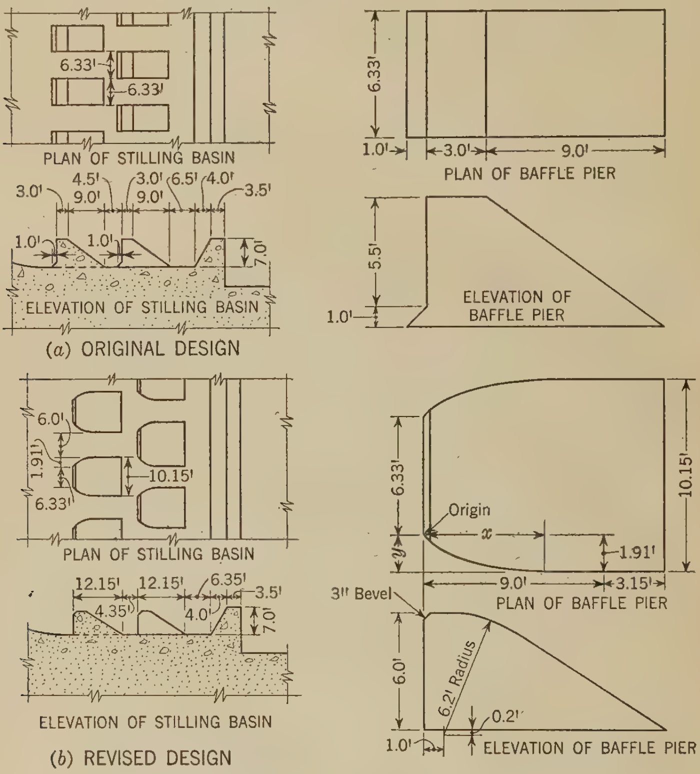

6.33'
6.33'
PLAN OF STILLING BASIN
3.0'
4.5'
9.0'
3.0'
6.5'
9.0'
4.0'
3.5'
1.0'
1.0'
7.0'
ELEVATION OF STILLING BASIN
(*a*) ORIGINAL DESIGN
6.33'
1.0'
3.0'
9.0'
PLAN OF BAFFLE PIER
5.5'
1.0'
ELEVATION OF BAFFLE PIER
6.0'
1.91'
10.15'
6.33'
PLAN OF STILLING BASIN
12.15'
12.15'
6.35'
3.5'
4.35'
4.0'
7.0'
ELEVATION OF STILLING BASIN
(*b*) REVISED DESIGN
6.33'
10.15'
Origin
x
y
1.91'
3" Bevel
9.0'
3.15'
PLAN OF BAFFLE PIER
6.0'
6.2' Radius
0.2'
1.0'
ELEVATION OF BAFFLE PIER

FIG. 18.—DIMENSIONS OF BAFFLE PIERS, BLUESTONE DAM

Figs. 19(*c*) and 19(*d*) show the action of this modified design for the same two discharge conditions as shown in Figs. 19(*a*) and 19(*b*). Cavitation on the upstream row of baffles was completely eliminated by this design. The streamer around the upstream baffle in Fig. 19(*c*) is a cavitation vortex well out in the water away from the face of the baffle pier. There is no cavitation visible in Fig. 19(*d*).

The rounding on the downstream top edges of these baffles is for the purpose of providing better ventilation of the jets of water shooting over the tops of the baffles. This tends to prevent a cavitation streamer from forming on the top of the baffle and causes it to leave the baffle if it does form. The reverse, rounding the upstream top edge, encourages the formation of a cavitation streamer and causes it to cling closer to the top of the baffle. A cavitation streamer that leaves the baffle is assumed to be harmless, whereas one that clings to the baffle is considered harmful.

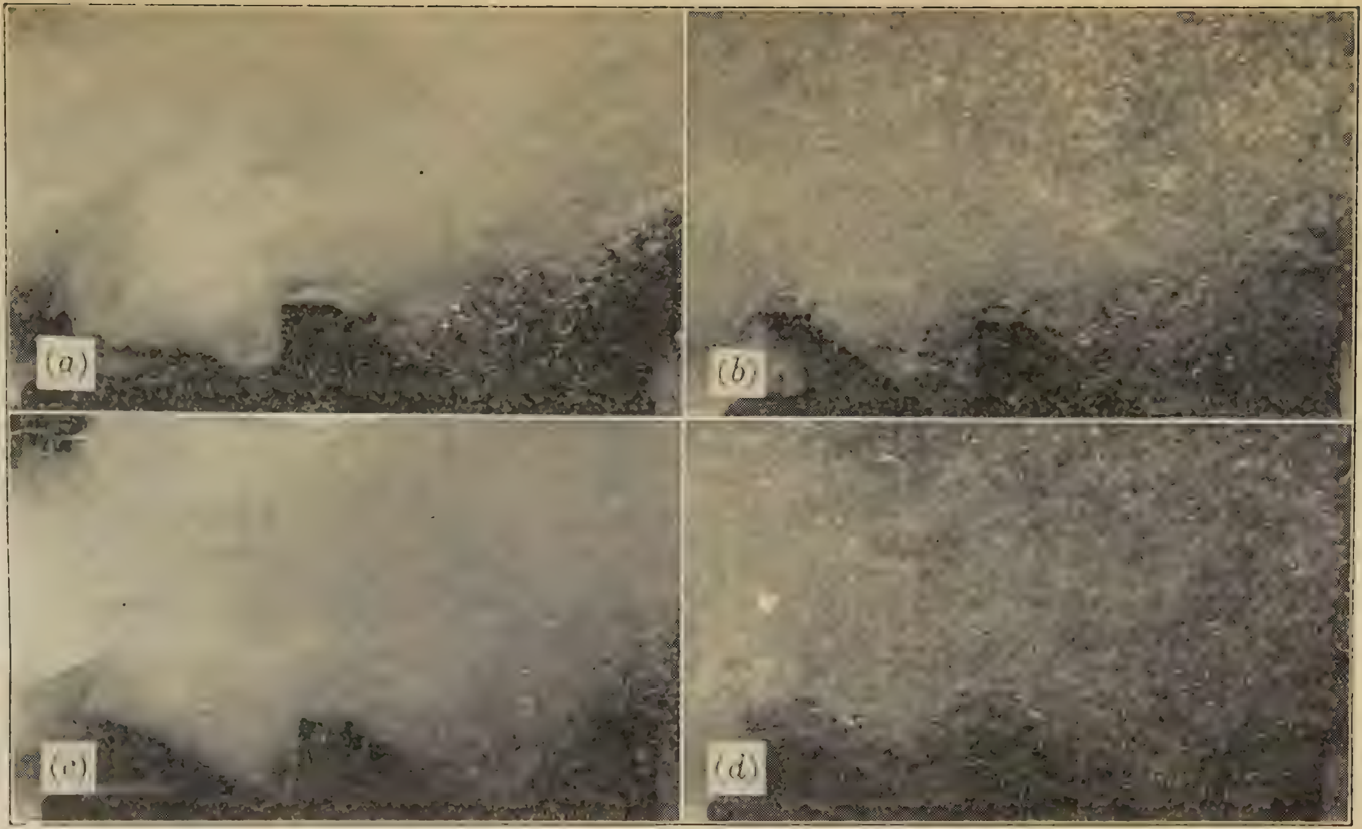


Fig 19 — Reaction of Baffle-Pier Model, Bluestone Dam, as Tested in Vacuum Apparatus (a) Maximum Discharge with Original Design, (b) Approximately One Quarter of Maximum Discharge with Original Design, (c) Maximum Discharge with Revised Design, and (d) Approximately One Quarter of Maximum Discharge with Revised Design

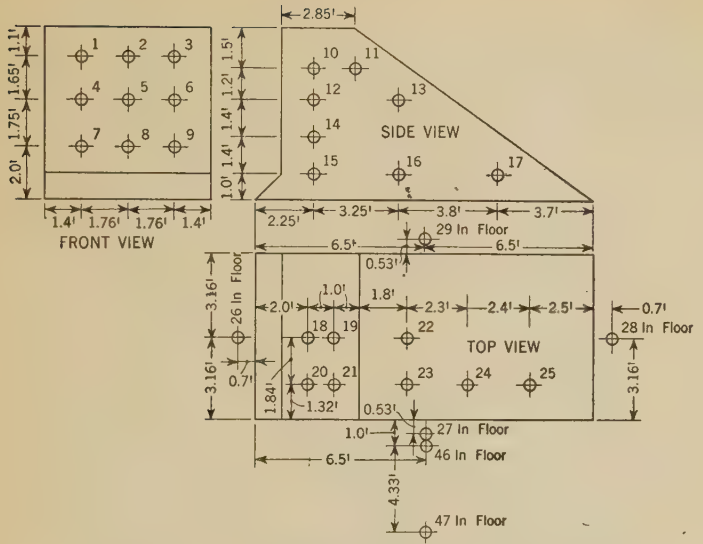
PRESSURE CELL METHOD

The tests at the U. S. Waterways Experiment Station were conducted on an open-air model for which the linear scale ratio L_r was 1/36. Two baffle piers in each row were constructed with piezometer openings in the faces as shown in Fig. 20(*a*). These piezometer openings were connected by means of copper tubing to a heavy metal pressure chamber in which was fastened an electric pressure cell, as shown in Fig. 20(*b*). (Fig. 20(*b*) shows a connection to a pitot tube but the connection to a piezometer opening is identical.) A cross section of one of the electric pressure cells is shown in Fig. 20(*c*). Several of these pressure cells can be connected to a single amplifier and oscillograph unit, and the pressure at several points recorded simultaneously.

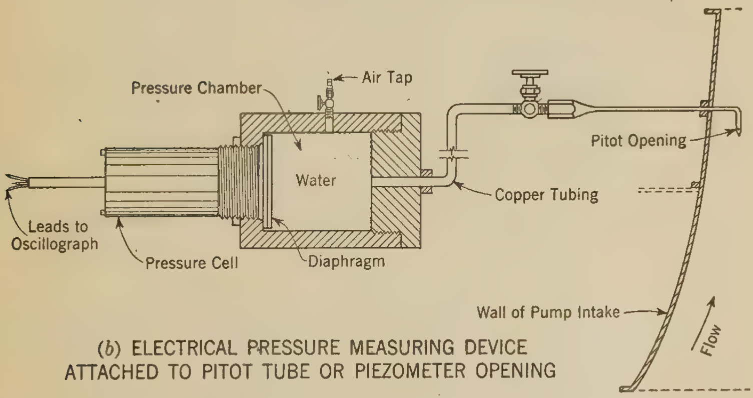
This equipment had been developed in connection with previous tests performed at the U. S. Waterways Experiment Station (20*a*) and had been found very successful in measuring rapidly fluctuating pressures in water. The cell shown in Fig. 20(*c*) utilizes the induction principle. A pressure on the diaphragm causes it to deflect and a change in the air gap causes a change in the current in the front coil. These current variations are amplified and recorded photographically by the oscillograph. Referring to Fig. 20(*b*), it was found that a considerable length of copper tubing could be used without affecting the pressure readings.

Typical oscillograph records of the pressure fluctuations on the original Bluestone baffle piers for the maximum discharge condition are shown in Fig. 21. A dashed line has been superimposed on each record representing a prototype pressure of absolute zero (33.9 ft of water below atmospheric). Pressures below the dashed line are purely imaginary but they have been used as a basis for comparing the severity of cavitation at the various points and for different discharge conditions and baffle shapes. The greater the drop below zero and the greater the percentage of time that the pressure is below zero, the more severe the cavitation is assumed to be. On this basis, the cavitation at piezometer No. 12 (see Fig. 20(*a*)) may be assumed to be the most severe. It will be noted that piezometer No. 12 is on the side of the baffle just downstream from the leading edge, the point where cavitation flashes become visible first in the vacuum tank (see Fig. 19(*b*)). The maximum pressure fluctuation recorded at this point was about 180 ft of water, prototype scale. The pressure was found to be below absolute zero (prototype scale) on the average of 82% of the time.

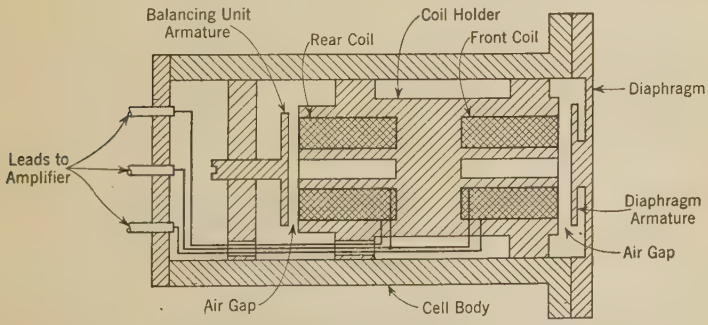
In interpreting these results, however, it should be kept in mind that in the prototype the pressure cannot drop below the vapor pressure of the liquid, or approximately absolute zero. Therefore, a fluctuation as large as the aforementioned value could not actually occur in the prototype. Some distortion of the pressure distribution in the liquid in the vicinity of a point showing cavitation may also occur due to the fact that cavitation pockets cannot form in an open-air model—a possibility that may reflect some doubt on the accuracy of pressures measured in the immediate vicinity. This is not such an important point, however, because, after it is determined that cavitation will be present in a given design, this design is usually modified until all tendencies toward


FRONT VIEW
SIDE VIEW
TOP VIEW
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26 In Floor
27 In Floor
28 In Floor
29 In Floor
46 In Floor
47 In Floor
2.85'
1.1'
1.65'
1.75'
2.0'
1.4'
1.76'
1.76'
1.4'
1.5'
1.2'
1.4'
1.4'
1.0'
2.25'
3.25'
3.8'
3.7'
6.5'
6.5'
0.53'
3.16'
3.16'
0.7'
2.0'
1.0'
1.8'
2.3'
2.4'
2.5'
0.7'
3.16'
1.84'
1.32'
0.53'
1.0'
6.5'
4.33'

(*a*) LOCATION OF PIEZOMETER OPENINGS IN BAFFLE PIERS
(Model of Bluestone Dam)


Air Tap
Pressure Chamber
Water
Pitot Opening
Copper Tubing
Leads to Oscillograph
Pressure Cell
Diaphragm
Wall of Pump Intake
Flow

(*b*) ELECTRICAL PRESSURE MEASURING DEVICE
ATTACHED TO PITOT TUBE OR PIEZOMETER OPENING


Balancing Unit Armature
Rear Coil
Coil Holder
Front Coil
Diaphragm
Leads to Amplifier
Diaphragm Armature
Air Gap
Air Gap
Cell Body

(*c*) CROSS SECTION OF ELECTRIC PRESSURE CELL

FIG. 20.—ARRANGEMENT OF APPARATUS FOR TESTS WITH ELECTRIC PRESSURE CELLS

cavitation are eliminated. When no cavitation pressures are present, all measurements are considered reliable.

Another observation worth noting at this point is that pressures on baffle piers are definitely fluctuating, thus accounting for the flashy nature of cavitation as observed in the vacuum tank. It will also be seen how futile it would be to try to determine the minimum pressure on a baffle pier by means of an ordinary manometer tube. This does not preclude the use of manometer tubes in models where the water is less turbulent, however. In such cases it is desirable to make some allowance for pressure fluctuations. It is sometimes assumed arbitrarily that the pressure as recorded by a manometer should not be allowed to drop below − 20, or − 25 ft of water (below atmospheric).

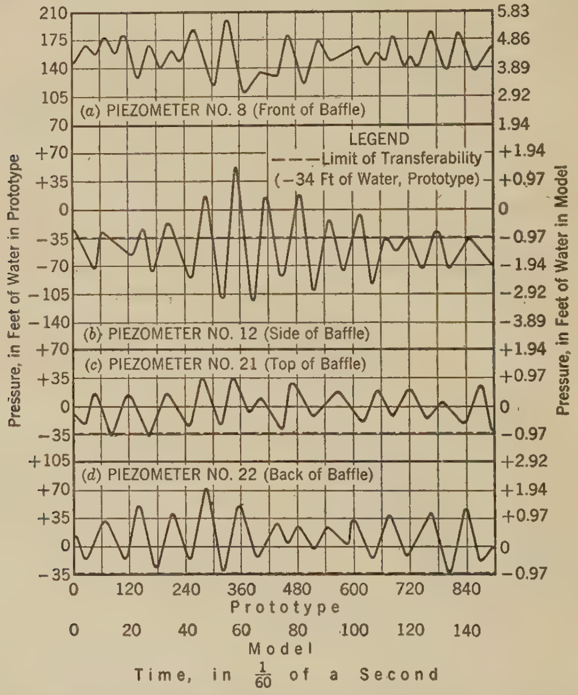


FIG. 21.—PRESSURE MEASUREMENTS ON THE ORIGINAL BAFFLE PIERS FOR BLUESTONE DAM, AT MAXIMUM DISCHARGE (PIEZOMETER LOCATIONS SHOWN IN FIG. 20(*a*))

Having definitely determined by these tests and those at Carnegie Institute of Technology that the cavitation on the original Bluestone baffle piers would be quite severe, the Waterways Experiment Station undertook to develop a streamlined baffle pier, the result of which is the design shown in Fig. 18(*b*). The curve used on the sides is the natural contraction curve of a jet from a narrow slit. As before, piezometer openings were installed in the faces of two baffle piers in each row, and tests were run for the entire discharge range. No cavitation pressures occurred on the upstream baffles and cavitation pressures were found on the second row about 25% of the time for the maximum discharge condition.

Pitting of Baffle Piers of Private Power Dam

In connection with the design of the Bluestone stilling basin, the Huntington District Office made a survey of existing dams with baffle piers. Claytor Dam, a private power dam in the Huntington District, was observed to have a dentated end sill, the sides of the dentates of which had been slightly pitted. This dam had just been constructed and was soon thereafter subjected to a large spillway discharge lasting about four days. It was thought that this experience would provide an excellent opportunity to make a model-prototype study of cavitation which would greatly aid in an interpretation of the foregoing model study results. If the percentage of time that the pressure on the sides of these dentates or baffles was below the cavitation pressure during this discharge could be determined, some idea of the significance of the 25% of time determined for the streamlined Bluestone baffles would be obtained.

Models of the stilling basin at Claytor Dam were tested at both laboratories, and cavitation was observed on the sides of the baffles during the maximum discharge that prevailed during the four-day period. Tests at the Waterways Experiment Station showed that the pressure on the sides of these baffles was below absolute zero (prototype scale) about 25% of the time at this maximum discharge, agreeing almost exactly with the degree of cavitation found on the downstream row of streamlined Bluestone baffles.

As a result of these tests on Claytor Dam, it was concluded that pitting on the downstream row of baffles at Bluestone Dam would not be excessive, particularly in view of the expected infrequency of operation of the Bluestone spillway. The streamlined baffle design shown in Fig. 18(*b*), therefore, was considered satisfactory, provided that these baffles would produce the required energy dissipation. Tests were undertaken to determine the relative effectiveness of the original square-cornered baffle piers and the newly developed streamlined design.

The two lower curves in Fig. 22 show the effectiveness of the original square-cornered baffles. At the maximum discharge, a satisfactory jump can be produced with a 12-ft lower tailwater elevation than would be required to produce a jump on a level apron without baffle piers. A curve of tailwater required to produce an equally satisfactory jump was not determined for the streamlined baffles but, at the maximum discharge, it was found that a 2-ft higher tailwater level was required. Therefore, the streamlined baffles may be considered equivalent to 10 ft of tailwater depth at the maximum discharge which is still quite effective. This 2-ft additional tailwater depth was provided in the prototype by raising the crest of the stilling weir 2 ft above that provided in the original design (see Fig. 14).

Comparison of Two Methods of Testing

The two upper curves in Fig. 22 show a comparison between the results of tests made in the vacuum tank and those made with the pressure meter. These curves represent the tailwater levels required to eliminate cavitation on the upstream row of the original Bluestone baffles. The tailwater levels were raised in each case until, in the vacuum tank, cavitation flashes disappeared and, in the open-air model, the pressure ceased to fall below absolute zero. These levels were recorded and plotted in the form of curves as shown in Fig. 22. The agreement was remarkably close, but such agreement was not obtained until some of the tests were rerun, as it was difficult to determine the exact point in the vacuum tank at which the cavitation flashes disappeared.

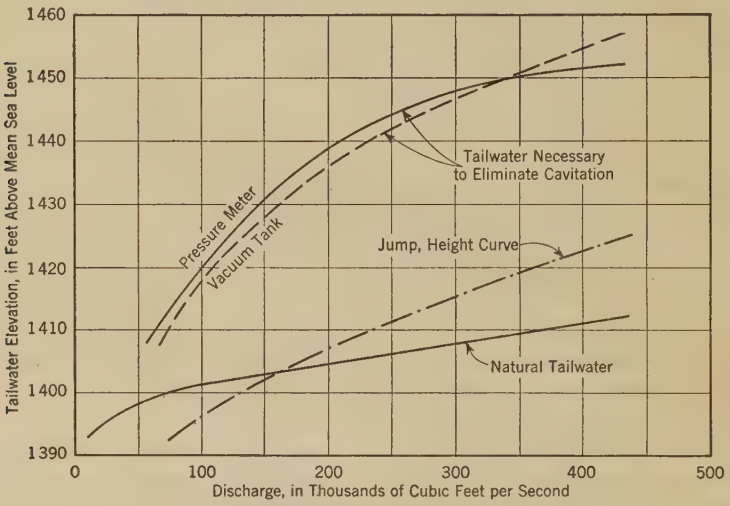


Fig. 22.—Comparison of Results Obtained by Vacuum Tank and Pressure-Meter Methods of Cavitation Testing; Original Baffle Piers, Bluestone Dam

As a result of these tests on Bluestone Dam, it was concluded that satisfactory results can be obtained by either method of testing—the vacuum-tank or pressure-meter method. Each method, however, has some advantages over the other. The main advantage of the vacuum-tank method is that the cavitation can be seen and its extent observed. The main advantage of the pressure-meter method is that it provides a quantitative measure, permitting a comparison between different degrees of cavitation and enabling the cavitation limit to be determined more accurately and with greater ease. In the pressure-meter method, however, it is necessary to know in advance of the tests where cavitation is likely to occur in order to locate the piezometer openings properly. Also, as previously stated, there is some question in the pressure-meter method as to the effect of the failure to reproduce cavitation pockets on the pressure distribution in the surrounding liquid. The two methods can be used together very nicely, however, and this is recommended whenever possible.

Additional comments on these two methods of testing have been presented elsewhere by J. B. Tiffany, Jr., M. ASCE (20*a*), and by Professor Thomas and W. J. Hopkins (22).

Actual Cases of Pitting

Several cases of actual pitting due to cavitation have been discovered in structures built under the direction of the Corps of Engineers, the most notable of which is that at Bonneville Dam on the Columbia River. An inspection of the spillway of this dam in 1939, two years after completion, revealed considerable erosion of the concrete which was thought to be due in part to cavitation (29*a*). Since this dam is a run-of-river dam, discharge over the spillway had been practically continuous. The spillway consists of eighteen 50 ft by 50 ft stoney gates on a concrete ogee section about 75 ft high. Water is discharged beneath the gates under a head of about 50 ft and is stilled on a horizontal apron 40 ft below the level of the spillway crest. The horizontal apron is 63 ft long and contains two rows of baffle piers placed at about the third points. The total fall from the upper pool to spillway apron is about 90 ft.

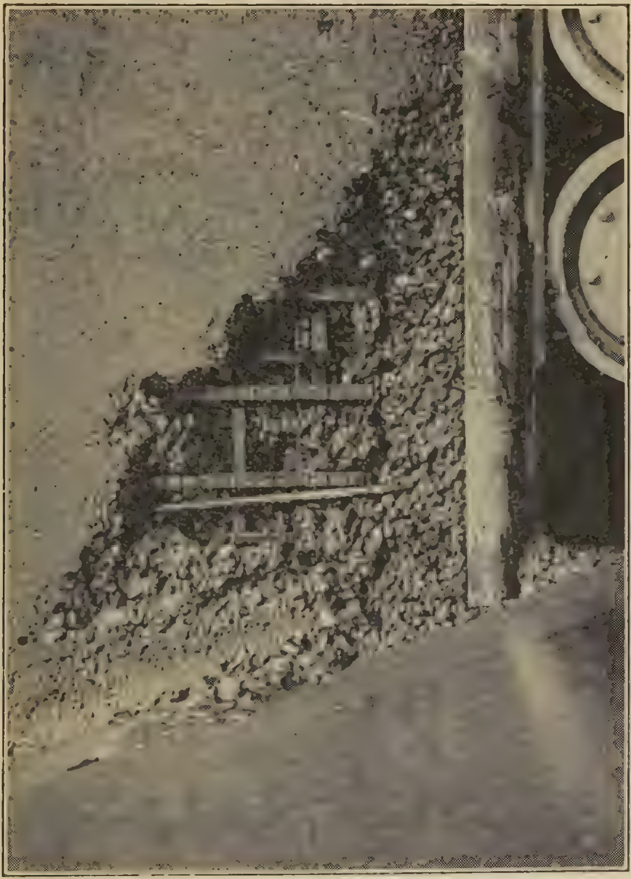


Fig. 23.—Example of Pitting; Downstream from Spillway Gate Slots at Bonneville Dam

Spillway Gate Slots

Fig. 23 shows the erosion that was discovered on the side of a spillway gate pier just downstream from the gate. It will be noted that some erosion has also occurred on the crest. This is typical of all gate piers. It was not known at first whether this erosion was due to abrasion or to cavitation. It was known, however, that the outside concrete of these piers had not attained its full strength due to cold-weather curing and the use of a pozzolanic admixture. Repair was first attempted, therefore, by refilling some of the holes with good sound gunite made with standard Portland cement, but after another year of operation the holes were found to be in as bad a condition as before. Repair was then attempted by placing ½-in. steel plates over the eroded areas, back-filled with concrete and anchored to the existing reinforcement (29*b*). Plates were placed on the crest as well as on the face of the piers. This work stood up fairly well for a while but in a few months began to fail. The lower half of the pier plate and the crest plate in some instances have been torn off and the concrete has again eroded in these areas (29*c*).

The cause of this destruction has not been determined definitely. Great pressure fluctuations on the pier face may be responsible for the ripping off of the steel plates. That cavitation is also present is indicated by the appearance of the eroded areas in the concrete. However, pitting of the steel plates has not occurred except in one instance. In this instance, the pitting could have been due to local irregularities peculiar to this particular plate. As a result of these observations, it would appear that a very mild degree of cavitation is present, one which will not pit steel but which will pit concrete. If further observations prove this to be the case, the final solution to this problem may be to replace the ½-in. plates with heavier plates and to provide better anchorage so that they will not be ripped off. However, if pitting of the plates becomes general, it may be necessary to use a cavitation-resisting alloy steel or to conduct model tests to determine a shape of gate slot that will not give rise to cavitation.

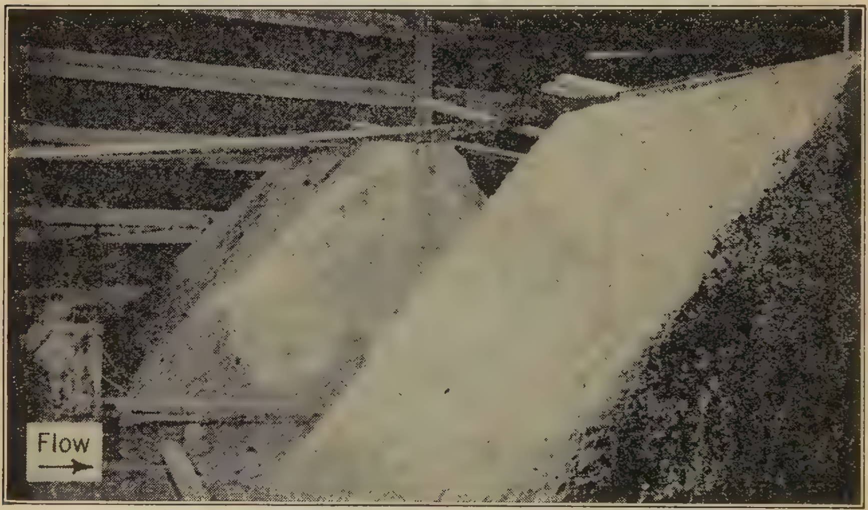
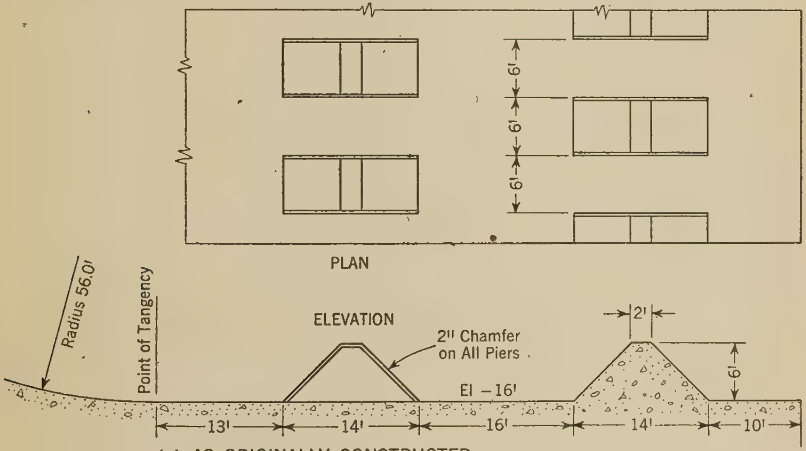


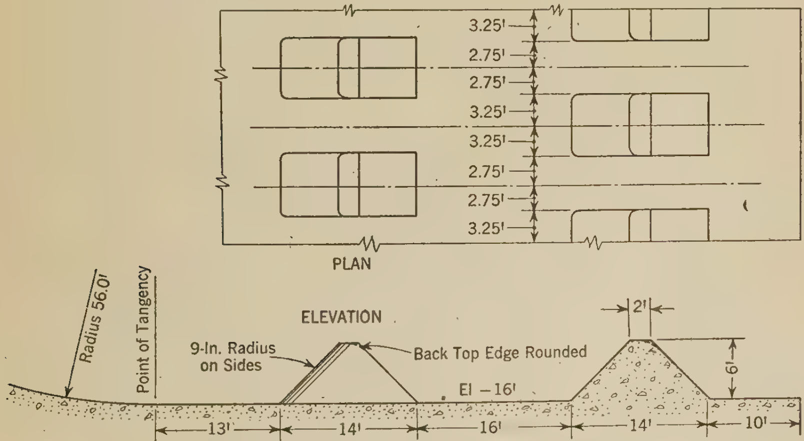
FIG. 24.—VIEW SHOWING PITTING AND EROSION OF BAFFLE PIERS ON THE SPILLWAY APRON OF BONNEVILLE DAM

Baffle Piers

The submerged spillway bucket and apron of Bonneville Dam were inspected by a diver. Considerable erosion was discovered on the floor of the apron and bucket and on all faces of the baffle piers (29*d*). The surface concrete of this part of the dam was also known to be weak due to cold-weather curing conditions. The baffle piers, however, in addition to appearing worn all over, were deeply cut on the sides just downstream from the leading edges, indicating that cavitation may have also been a factor in this erosion. Fig. 24 shows two upstream baffles in a special caisson which was later constructed to repair some of the baffles. Fig. 25(*a*) shows the layout of these baffles on the spillway apron.


6'
6'
6'
PLAN
Radius 56.0'
Point of Tangency
ELEVATION
2'' Chamfer on All Piers
2'
6'
El –16'
13'
14'
16'
14'
10'

(*a*) AS ORIGINALLY CONSTRUCTED


3.25'
2.75'
2.75'
3.25'
3.25'
2.75'
2.75'
3.25'
PLAN
Radius 56.0'
Point of Tangency
ELEVATION
9-In. Radius on Sides
Back Top Edge Rounded
2'
6'
El –16'
13'
14'
16'
14'
10'

(*b*) PROPOSED REVISION WHEN PIERS ARE REPAIRED

Fig. 25.—Dimensions of Baffle Piers, Spillway Apron, Bonneville Dam

FURTHER MODEL TESTS OF BAFFLE PIERS

In order that these baffle piers might be properly repaired, it was thought advisable to conduct model tests in the vacuum tank at Carnegie Institute of Technology for the purpose of determining whether or not cavitation had contributed to the erosion of the baffle piers. These tests were undertaken in 1941. A model of the present spillway apron ($L_r = 1/48$) was constructed and installed in the same vacuum-tank apparatus as was used for the tests on Bluestone Dam (Fig. 16). Tests were run using the operating conditions that actually had been experienced during the first two years of operation and which had caused most of the erosion. A mild degree of cavitation appeared on the sides of the baffles just downstream from the leading edges where the deep holes had scoured in the prototype, under a few of the discharge conditions tested, indicating that cavitation might have contributed to this erosion.

In view of the mild degree of cavitation observed, it was decided to try a simple rounding of the upstream side edges of the baffles to determine whether this would eliminate the cavitation. The width of the baffle piers was also increased from 6 ft to 6.5 ft to make up for the reduction in efficiency caused by rounding the upstream edges and in order to facilitate repairs since the greater thickness would permit a thicker covering of good concrete. Radii of 6 in., 9 in., and 12 in. were studied on the upstream edges. The 9-in. and 12-in. radii eliminated all tendencies toward cavitation and the 9-in. radius was selected for use in repair. The modified design is shown in Fig. 25(*b*). It will be noted that the downstream top edge has also been rounded, as in the case of the Bluestone baffles, in order to provide better ventilation for the jet of water shooting over the top edge of baffle, thus reducing the tendency for cavitation to form on the top.

Tests were also conducted on an open-air model of this spillway ($L_r = 1/40$) in order to determine the relative efficiency of the original and modified baffle piers. The original baffles were tested first and the tailwater was lowered to the point where the hydraulic jump tended to leave the apron. These tailwater elevations were recorded and plotted in curve form as shown in Fig. 26 (solid lines). The tests were then repeated with the modified baffles and the elevations recorded (dashed lines in Fig. 26). A comparison of these curves will demonstrate that the baffles have about the same efficiency with a 6-ft gate opening and that the modified baffles are slightly more efficient than the original baffles at the 12-ft and 20-ft gate openings. The lower the curve the more efficient the baffle becomes since a jump will take place with a lower tailwater level. Repair of these baffles in accordance with the modified design, therefore, will not affect adversely the efficiency of the stilling basin.

PITTING SIMULATED IN VACUUM-TANK MODEL

An attempt was made, in connection with the Bonneville tests, to simulate pitting in the model. In view of the mild nature of the cavitation that occurs in the vacuum tank on these small models, and particularly in view of the mild nature of cavitation around the Bonneville baffle piers, considerable difficulty was encountered in finding a material sufficiently weak to pit and yet strong enough to resist abrasion by the high-velocity water. Several materials were tried consisting of various mixes of cement, sand, and plaster of Paris or Opalite. It was found that a weak material with a hard outside crust was the most successful. The hard crust resisted abrasion but would fail under the pounding of cavitation. Being weak underneath, the material would then pit and form cavities such as those that occur in the prototype. In a test of one of the original square-cornered baffles, the model had to be "overspeeded," equivalent to subjecting the baffle pier to higher velocities than would exist in the prototype, in order to obtain any appreciable pitting. Some pitting occurred on the sides of the baffle just downstream from the leading edge, where pitting occurred in the prototype (see Fig. 24). These pitting tests were not entirely satisfactory, however, and further research on this subject will be necessary before any reasonable degree of success is possible with this small a model.

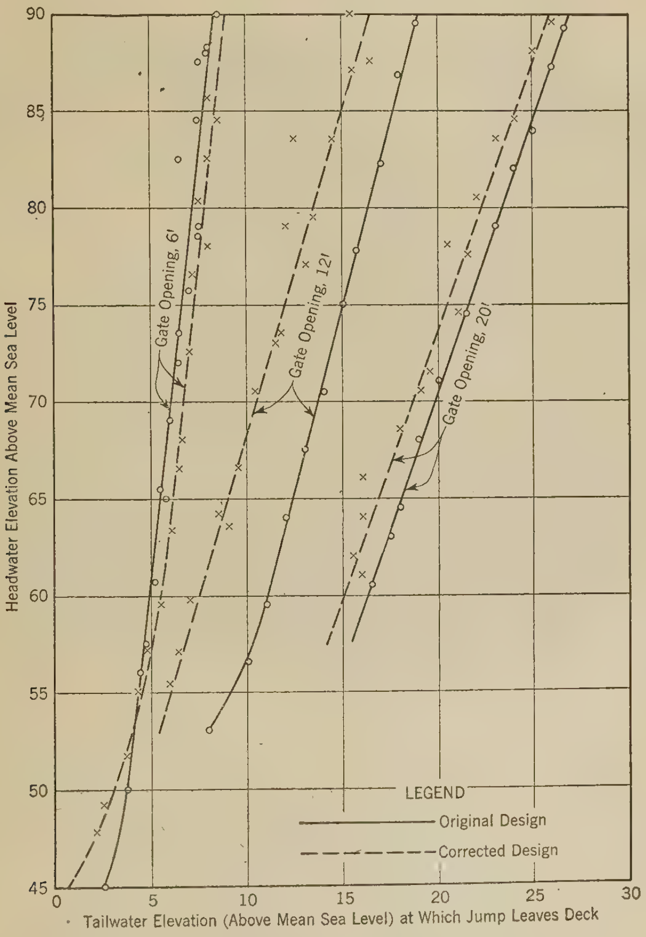


FIG. 26.—RELATIVE EFFECTIVENESS OF THE BAFFLE PIERS IN FIG. 25, AS DETERMINED BY MODEL TESTS

Another Example of Damage to Baffle Piers

Fig. 27 shows the effect of high-velocity flow on the baffle piers (height, 9 ft) in the stilling basin of Gatun Dam in the Canal Zone. The steel casting remaining in place is on the upstream face of the pier and is 3 in. thick. The side casting, which has been ripped off by the flow, was 2 in. thick. The front casting was installed with the original baffle piers. The side castings were added later after considerable erosion had occurred along the sides. The side plates were anchored into the concrete with 12-in. anchor bolts. The side plates were probably ripped off by the great pressure fluctuations on the sides such as were measured in the model of the Bluestone baffles (see Fig. 21). The original erosion of the concrete on the sides was probably caused by cavitation.

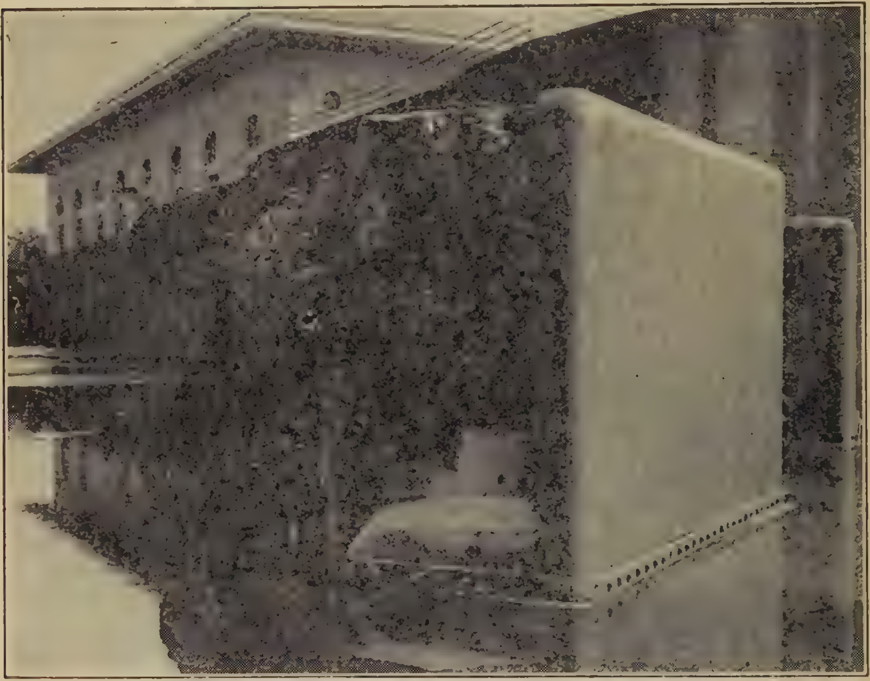


Fig. 27.—Face Plate Ripped from the Side of a Baffle Pier in the Stilling Basin of Gatun Dam

Proper Use of Baffle Piers

Certain conclusions as to the proper location of baffle piers in stilling basins of high dams can be drawn from the data presented in this paper. Square-cornered, round-cornered, and streamlined baffles were tested. It is obvious from these tests that the greater the streamlining, the higher will be the velocity in which they can be used without causing cavitation. Also, since submergence produces a positive pressure tending to neutralize the negative pressure caused by high velocities, the depth of water over the baffle piers has an important bearing on the velocity which they will stand. Since velocities are higher and depths shallower in the upstream end of a hydraulic jump basin, only streamlined baffles should be used in this location. Square-cornered baffles should be used well downstream on the apron where velocities are lower and submergence greater. The round-cornered baffles are an intermediate type. Research on this subject has not been extended far enough yet to enable one to state just what velocities and submergences each type of baffle will stand, but it is hoped that future testing will make this possible. The data presented in this paper are not intended in any way to discourage the use of baffle piers but rather to reveal some facts as to their proper use, because the use of baffle piers often results in a more economical stilling basin design and more stable hydraulic jump action.

PITTING IN SHAFT AND LOCK CULVERT

Two other instances of pitting of concrete in structures built by the Corps of Engineers are of interest. Fig. 28 is a view (looking upward) of the erosion of the concrete lining which occurred at the bottom of a vertical shaft 50 ft high at Mud Mountain Dam, in the State of Washington, through which water flowed in a downward direction for a period of about three months. Computa-

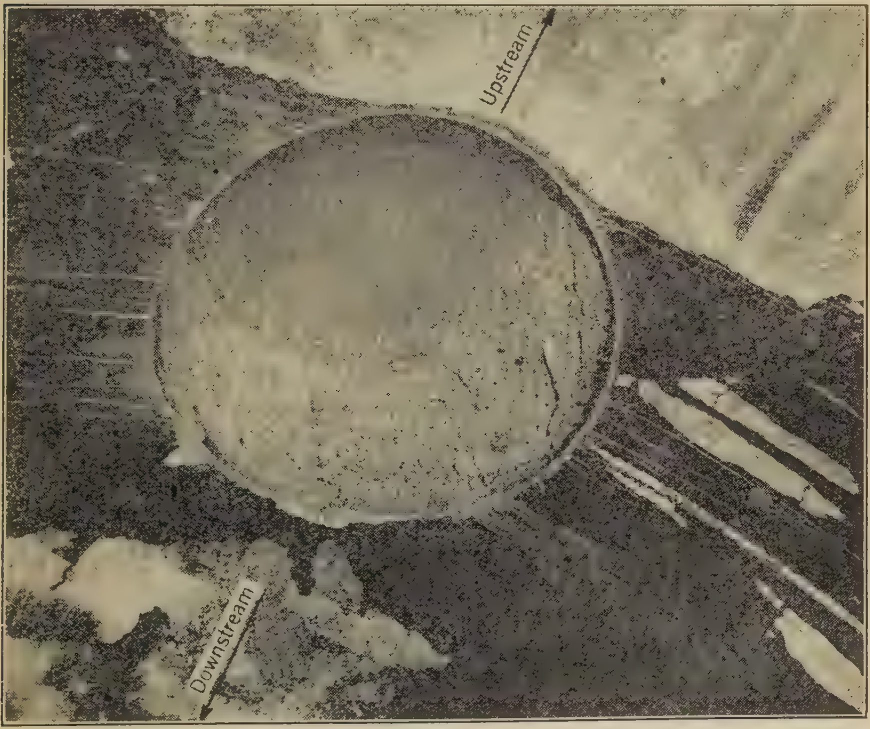


FIG. 28.—PITTING OF CONCRETE LINING AT THE BOTTOM OF A VERTICAL SHAFT (DIAMETER, 7 FT) AT MUD MOUNTAIN DAM

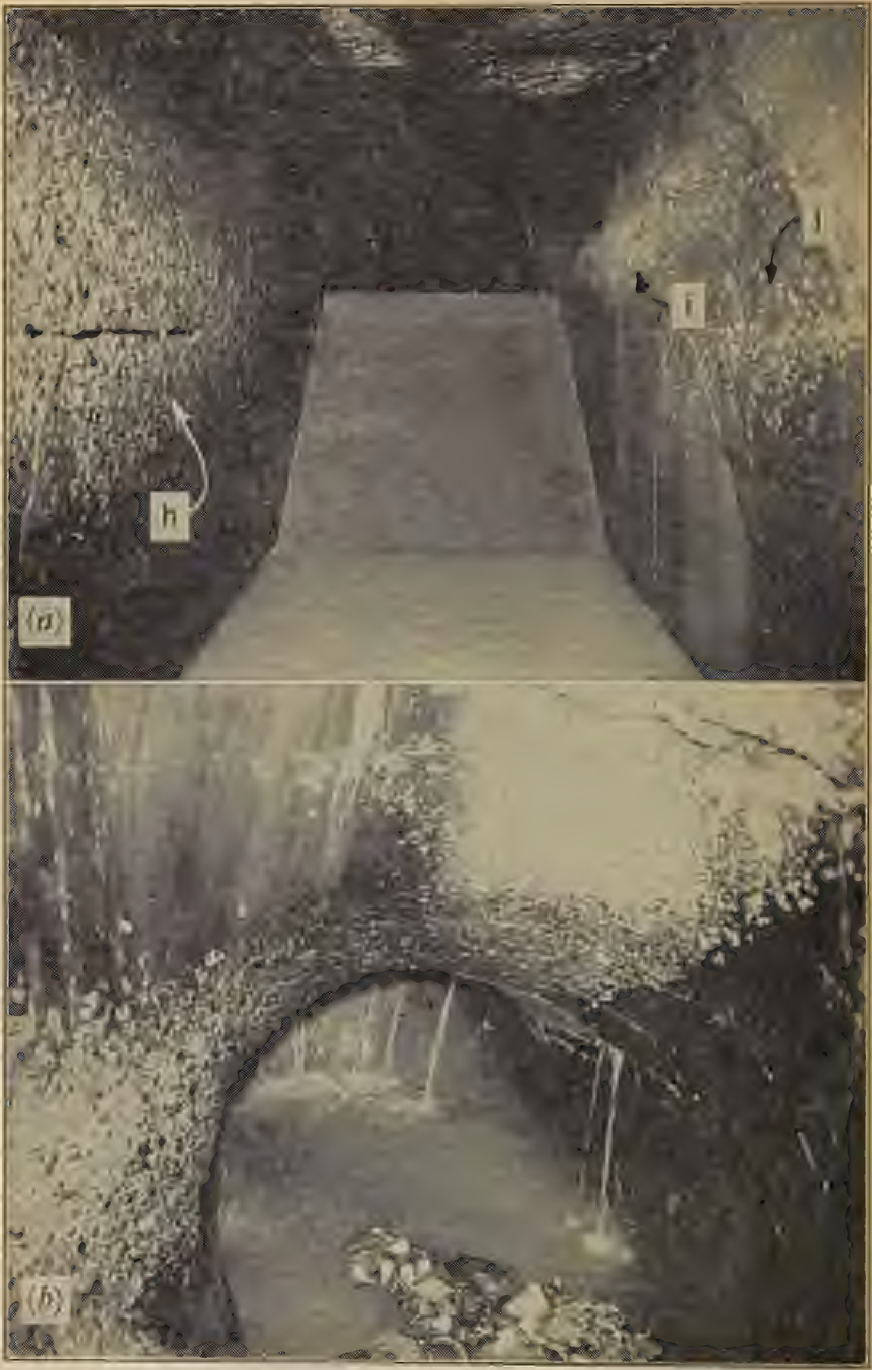


Fig. 29.—Examples of Pitting in the Lock-Filling Culvert of Wilson Dam: (a) Downstream from a Constriction in the Culvert (7 ft by 9 ft); and (b) Downstream from a Sharp Bend

tions showed that cavitation probably occurred at the top of the shaft. Just why the pitting occurred at the bottom is not too clear. Perhaps the vapor pockets did not collapse until they plunged into the pool of water at the bottom of the shaft. The pitting was about 12 in. deep in places.

Fig. 29 shows two views of the pitting that was recently discovered in the filling culvert of Wilson Lock on the Tennessee River. Fig. 29(*a*) shows the pitting that occurred downstream from a constriction in the culvert (note points h, i, and j). The purpose of this constriction is explained elsewhere (23). Fig. 29(*b*) shows the pitting on the inside of a sharp bend in this culvert near its outlet end.

INSTRUCTIVE LESSON

An interesting example of the failure to watch for cavitation in model testing appeared in the technical press in 1940 (24). An ordinary open-air model of the siphon shown in Fig. 30 was constructed and tested. The discharges with and without a flared outlet were determined. The flared outlet increased the discharge of the model 30% over the discharge without the flared outlet. When the full-sized siphon was built, it was also tested both with and without the flared outlet. The discharge in the prototype was increased only 3% by the flare. The writer of the article attributed this difference to a cavitation pocket that formed at point A in the prototype (Fig. 30(*a*)) but not in the model. The lesson is one that all designers should take seriously—to watch for pressures close to the vapor pressure in model studies. If the pressure at any point in an open-air model falls below the vapor pressure (prototype scale), the model is no longer reliable and either the design must be changed to raise all pressures above the vapor pressure, or the model must be tested in a vacuum tank where the action of cavitation can be simulated correctly.

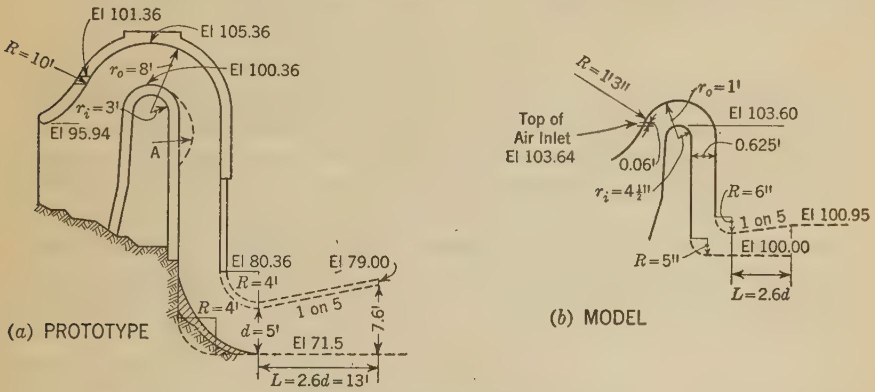


FIG. 30.—SIPHON PROTOTYPE AND MODEL IN WHICH DISCHARGES WERE FOUND TO DISAGREE BECAUSE OF CAVITATION IN THE PROTOTYPE

SUBJECTS FOR FURTHER RESEARCH

Cavitation testing has opened a broad field for research. With improved apparatus, it may be possible to test models of entire structures, under a vacuum, instead of just parts of these structures. Also, having the equipment, it may be possible to develop general laws for the occurrence of cavitation, say around baffle blocks, below gate slots, in curved conduits, siphons, and the like, which will be useful in design. As far as the writer knows there is only one vacuum tank suitable for the testing of hydraulic structures, that at Carnegie Institute of Technology, and only one set of dynamic pressure cells, those at the U. S. Waterways Experiment Station. Perhaps this Symposium will encourage others to undertake useful research along this line.

Certain full-sized tests also appear desirable. It would be interesting to know just how severe cavitation must be in order to damage concrete—that is, how large a cavitation pocket must be or what frequency of collapse is necessary to cause severe pitting. It would also be interesting to know how fast concrete will pit under various degrees of cavitation; also how close the cavitation streamer must be to the structure in order to cause pitting. A general model-prototype study, wherein identical tests are performed in model and prototype, would also help in an interpretation of model test results.

ACKNOWLEDGMENT

Grateful acknowledgment is hereby made to Professor Thomas of Carnegie Institute of Technology, to the U. S. Waterways Experiment Station, and to the various District Offices of the Corps of Engineers concerned for their assistance in assemblying material for this paper.

The writer also desires to express his appreciation to the following persons for their helpful comments on the original draft of this paper: Captain Tiffany, F. R. Brown, Assoc. M. ASCE, Mr. Schuleen, and W. J. Hopkins, and W. H. McAlpine, M. ASCE.

EXPERIENCES OF THE BUREAU OF RECLAMATION

By Jacob E. Warnock,[4] M. ASCE

Synopsis

With the trend to larger and larger hydraulic structures, cavitation and the resultant pitting have become major problems to the hydraulic designing engineer. Subatmospheric pressures in smaller structures were of little consequence, but with the increase of head in more recent structures, the approach of subatmospheric pressures to absolute zero as their limit has created previously unheard of situations. Experience in the laboratory and in the field shows that prevention of cavitation is fundamentally a function of design.

Introduction

Pitting due to cavitation is not new to the Bureau of Reclamation, U. S. Department of the Interior. Trouble was experienced as far back as 1910–1920 in outlet works, but recent examples have been more severe in extent due to the increased head. In discussing the arrangement and design of outlet works, J. M. Gaylord and J. L. Savage, Hon. M. ASCE (25), in 1923 stated that "Most of the difficulties with outlets built by the Bureau of Reclamation can be attributed to the effects of vacuum in the conduits below the regulating devices." However, it was not until recently that pitting has appeared on the surfaces of water passages normally considered to be open channels. Two examples of conduit flow are described herewith—the needle valves at Boulder Dam (Arizona-Nevada) which discharge into the atmosphere, and the balanced valve outlets at Shoshone Dam (Wyoming) which discharge into short conduits. Two examples of pitting in open channels are also shown—in the Arizona spillway tunnel at Boulder Dam and on the spillway pier faces at Parker Dam, both on the Colorado River.

The remedial measures in each case were made possible by laboratory studies. In fact, the occurrence of cavitation and pitting in the large hydraulic structures constructed in recent years would be more prevalent were it not for the availability of hydraulic laboratory facilities. A careful exploration of the pressures within a model can detect those conditions which, when transferred to a prototype structure, will cause the intermittent subatmospheric pressures producing cavitation. At one stage in the design of the spillway for Grand Coulee Dam (26), in Washington, a dentated lip at the downstream end of the apron eliminated the impingement of the high-velocity flow directly on the river bed downstream from the apron, reduced the scouring effect of the turbulent flow, and materially reduced the roughness of the water surface in the stilling pool. Minute examination of the pressures on the faces of the dentates in a

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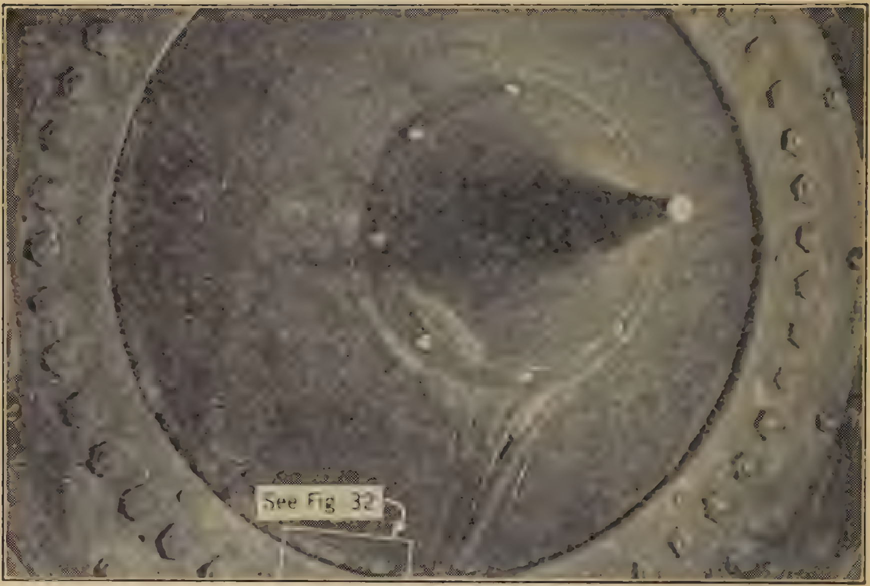


Fig. 31.—Pitting Occurs Around the Periphery of a Large Needle Valve, Immediately Below Regions of Subatmospheric Water Pressures

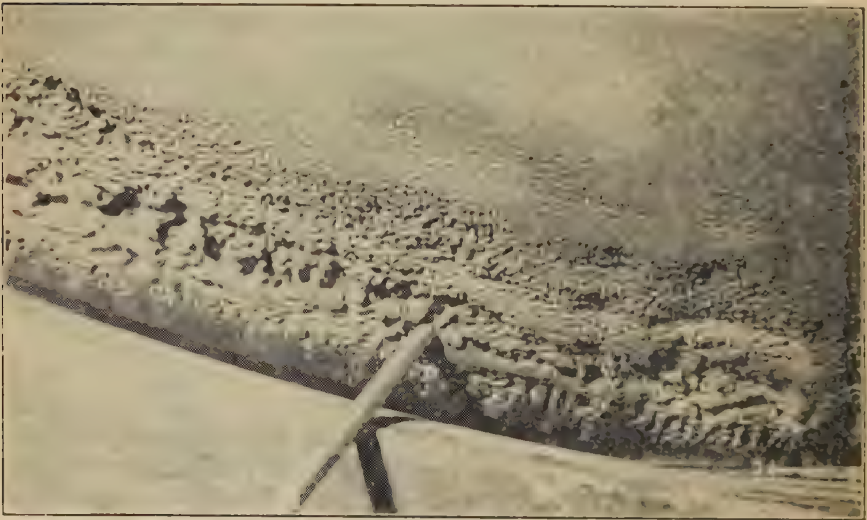


Fig. 32.—Enlarged View of Section in Fig. 31 Emphasizes the Severe Pitting on the Shoulder of the Needle Valve Which Requires Expensive Maintenance By Welding

1 : 40 model and a 1 : 15 model showed that cavitating pressures would occur in the prototype which would have destroyed the piers in a very short time. This condition was the principal factor in continuing the laboratory studies which resulted in the adoption of the bucket type of energy dissipator.

The destruction of the conduit roof in the outlets at Madden Dam, on the Chagres River in the Isthmus of Panama, by pitting from cavitation was the incentive for a complete model study of the outlets (27) for Grand Coulee Dam. Aside from the criterion of efficiency, the emphasis throughout the tests was to prevent pressures from occurring at any point in the conduit which would cause cavitation. One of the early designs of the upper and intermediate outlets showed subatmospheric pressures of such intensity that, had the design been constructed, cavitation would, without doubt, have been so severe as to hamper, if not completely prevent, successful operation. In the original design the fact was overlooked that the frictional forces in the sloping conduit were insufficient to overcome the accelerating force due to gravity, a condition which became readily apparent in the experimental studies.

There was a period prior to the Madden Dam incident when it was difficult to demonstrate that cavitation and pitting could occur in a hydraulic structure in the same manner as it has occurred in hydraulic machinery such as marine propellers, turbines, and pumps. With the experiences at Madden Dam and in certain Bureau structures, augmented by laboratory investigations, the importance of this cavitation problem is fully recognized by Bureau engineers.

The Gaylord-Savage report (25) describes outlet structures and recounts the difficulties experienced in the excessive maintenance due to damage from cavitation. Although the theory of cavitation at that time differed materially from the current conception, the adverse condition was even then associated with extreme subatmospheric pressures. It was realized that the erosion or pitting was an action accompanying subatmospheric pressures, but the cause was not completely understood. At first the pitting was believed to be a direct result of the making and breaking of the vacuum in the immediate vicinity.

As is now the case, one of the most practical remedies applied to the discharge conduits installed in early periods was the introduction of air immediately below the regulating device. Air was admitted to the discharge conduit in a number of instances during the first years of operation, but, in the light of air-requirement tests made in recent years on both model and prototype structures, it is doubtful if the air supply in most cases was either adequate or properly installed. The location of the air inlets in the conduits is often more important than the size. Thus, improper location might have been one of the main factors contributing toward failure of some of the early systems.

Streamlining the needle tips is considered the only practicable means of eliminating damage to this part of the valve; however, damage could be reduced to a minimum by restricting the valve operation to noncritical openings determined by detailed pressure measurements on the prototype or by model tests, or both.

Damage to Needle Valves

Initial operation at Boulder Dam and Alcova Dam, in Wyoming, produced severe erosion of the needle valves in the outlet structures which was expensive to maintain, since the valves were not readily accessible. The type of damage is shown on the needle in Figs. 31 and 32. In this case, the damage was produced in a relatively short time. Detailed records are not available, but the time was probably about one month at the one-half open position with a head of 145 ft.

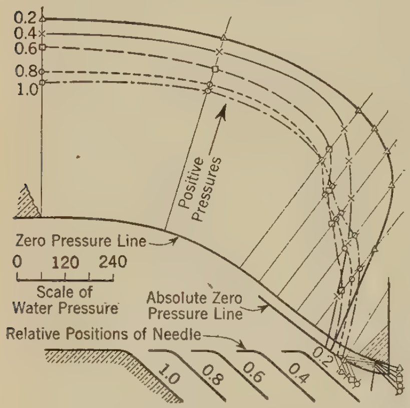

0.2
0.4
0.6
0.8
1.0
Positive Pressures
Zero Pressure Line
0 120 240
Scale of Water Pressure
Absolute Zero Pressure Line
Relative Positions of Needle
1.0
0.8
0.6
0.4
0.2

Fig. 33.—Pressure Distribution on the Nozzle of a 72-In. Needle Valve (Head, 516 Ft) Shows Regions of Low Pressure Which Correlate Perfectly with the Occurrence of Pitting

Piezometers installed in the nozzle of one of the 72-in. valves at Boulder Dam showed pressures near absolute zero in a zone (Fig. 33) immediately upstream from the region of erosion or pitting throughout the entire range of the valve. Since the installation of pressure equipment in one of the 72-in. needles would have been intricate, a homologous needle valve with an exit diameter of 5 in. was installed for testing in one of the tunnel-plug outlets at Boulder Dam. The pressure distribution on the needle and nozzle is shown in Fig. 34 for various openings of the valve under a constant head of 150 ft.

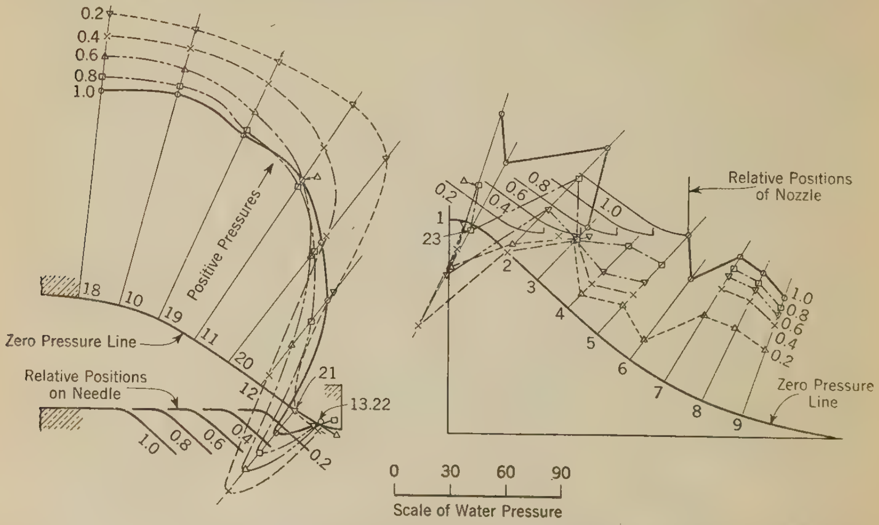

0.2
0.4
0.6
0.8
1.0
Positive Pressures
18
10
19
11
20
12
21
13.22
Zero Pressure Line
Relative Positions on Needle
1.0
0.8
0.6
0.4
0.2
Relative Positions of Nozzle
0.2
0.4
0.6
0.8
1.0
1
23
2
3
4
5
6
7
8
9
1.0
0.8
0.6
0.4
0.2
Zero Pressure Line
0 30 60 90
Scale of Water Pressure

Fig. 34.—Detail Data from Extensive Tests of a 5-In. Scale-Model Needle Valve Further Confirm the Pressure Measurements and Pitting Observed on the Prototype

In general, the pressure conditions were most critical at a valve opening of approximately 40%. The results of a wear test (Fig. 35) are shown after six days of operation under a total head on the valve of 460 ft with the valve opening of 40%—the severest condition.

A number of designs were studied in the laboratory using the same 5-in. valve and a design was developed which produced positive pressures at all valve openings and at all heads. This design, when subjected to a wear test, under the same conditions as the original design (except at an opening of 20%, the opening at which the severest conditions occurred), showed no sign of pitting on the needle after eighty-four days of operation (Fig. 36).

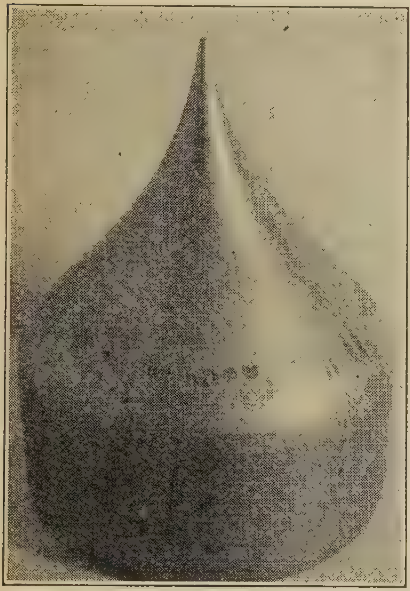


Fig. 35.—Pitting on the Needle of a 5-In. Valve, According to the Original Design, After Six Days of Operation (40% Opening and $H = 460$ Ft)

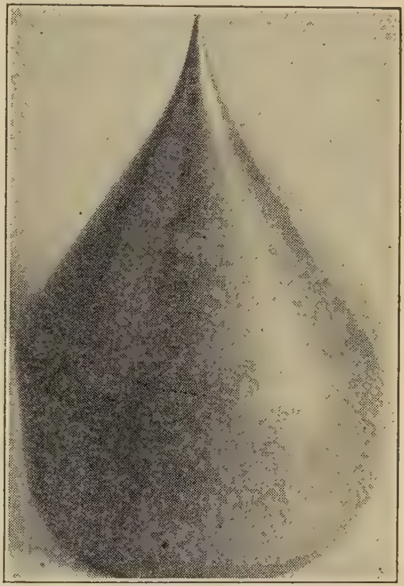


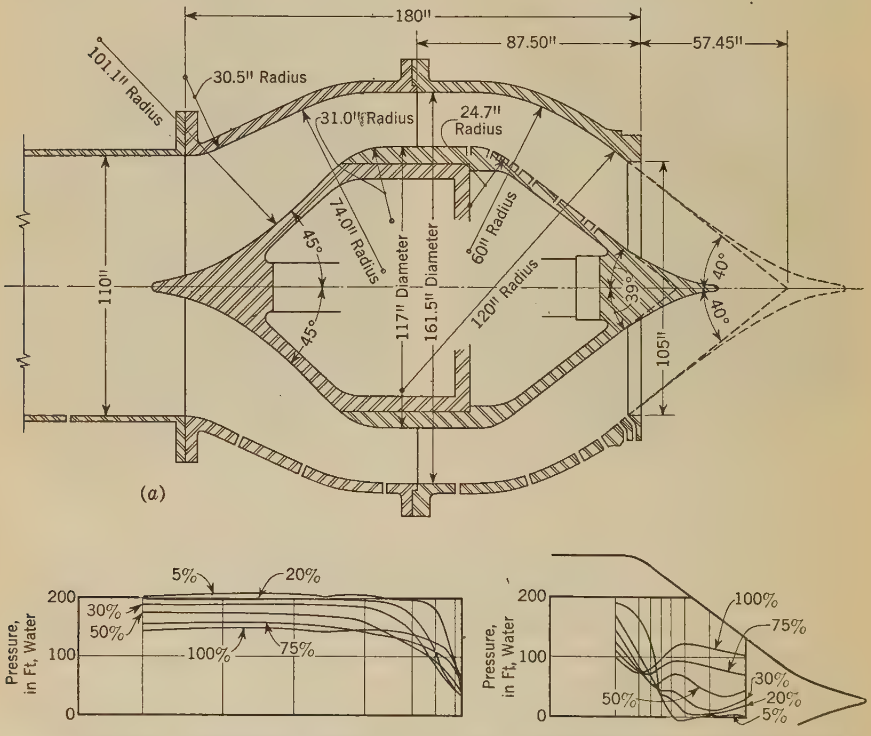
Fig. 36.—Absence of Pitting on the Needle of a 5-In. Valve, According to the Improved Design, after 84 Days of Operation (20% Opening and $H = 460$ Ft)

The original design of nozzle had an expanding water passage which tended to lower the velocity and cause a regain of velocity head. This yielded a higher discharge but created low-pressure areas in the valve. When forces in the low-pressure areas were of sufficient intensity, they produced cavitation with the accompanying pitting.

From the high-head studies of the 5-in. valve, certain specifications were developed to maintain positive pressures on the needle and nozzle of the valve at all openings. The angle between the needle and the nozzle must not be divergent in the direction of flow. The needle and nozzle profiles may be parallel and still maintain positive pressures, but a convergence of one to three degrees is preferable. The seat must be on the tangent portion of the needle; that is, the base diameter of the needle cone must be slightly larger than the outlet diameter of the nozzle. The valve nozzle should have no point of inflection; it should have a sharp edge to maintain the minimum section at the outlet of the valve nozzle and should permit free access of air to the jet at the point of emergence.

The high-head studies on the 5-in. valve gave positive proof that the 72-in. valves were pitted by cavitation and that elimination of the severe subatmospheric pressures causing the cavitation was possible by the redesign of the hydraulic passages of the valve. In eliminating the low pressure by using a sharp-edged nozzle in the improved design the discharge capacity was reduced.

Using a 6-in. valve equipped with piezometric taps throughout the profile of the needle and the nozzle, the design was altered further by increasing the outlet and equatorial diameter and the needle travel until a combination was found in which the cavitating pressures were absent over the entire range of valve opening (Fig. 37) and the discharge capacity was comparable to that of the original valve.


180"
87.50"
57.45"
101.1" Radius
30.5" Radius
31.0" Radius
24.7" Radius
74.0" Radius
45°
45°
117" Diameter
161.5" Diameter
60" Radius
120" Radius
110"
105"
39°
40°
40°
(a)
Pressure, in Ft, Water
200
100
0
5%
20%
30%
50%
100%
75%
Pressure, in Ft, Water
200
100
0
100%
75%
30%
20%
5%
50%

(*b*) PRESSURE CONDITIONS ON NOZZLE (*c*) PRESSURE CONDITIONS ON NEEDLE

FIG. 37.—COMPLETE ABSENCE OF CAVITATING PRESSURES ON THE NEEDLE AND THE NOZZLE OF A NEW DESIGN OF NEEDLE VALVE, AS DETERMINED FROM A 6-IN. MODEL

Although the new design has not been used in a field structure, the satisfactory operation of the 5-in. valve with positive pressures throughout, under a head of 460 ft for eighty-four days, indicates similar satisfactory operation in a larger valve of the new design. As designed for Friant Dam in California, the valve in Fig. 37 has an inlet diameter of 110 in. and an outlet diameter of 105 in.

Subsequently, sharp-edged nozzles were installed on certain valves at Boulder Dam and the pressure results show a marked improvement in distribution (Fig. 38). Field reports indicate a minimizing of pitting on those valves so equipped. Operation of the valves with the original nozzle shape at openings where the subatmospheric pressures were less severe, as indicated by model tests, has also tended to reduce the amount of pitting.

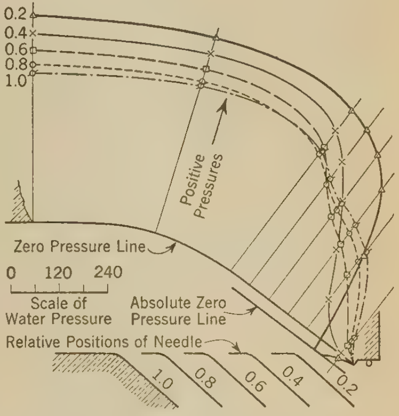


FIG. 38.—PRESSURE DISTRIBUTION ON THE NOZZLE OF THE 72-IN. NEEDLE VALVES AT BOULDER DAM, WITH REVISED PROFILE OF NOZZLE

SHOSHONE DAM BALANCED VALVES

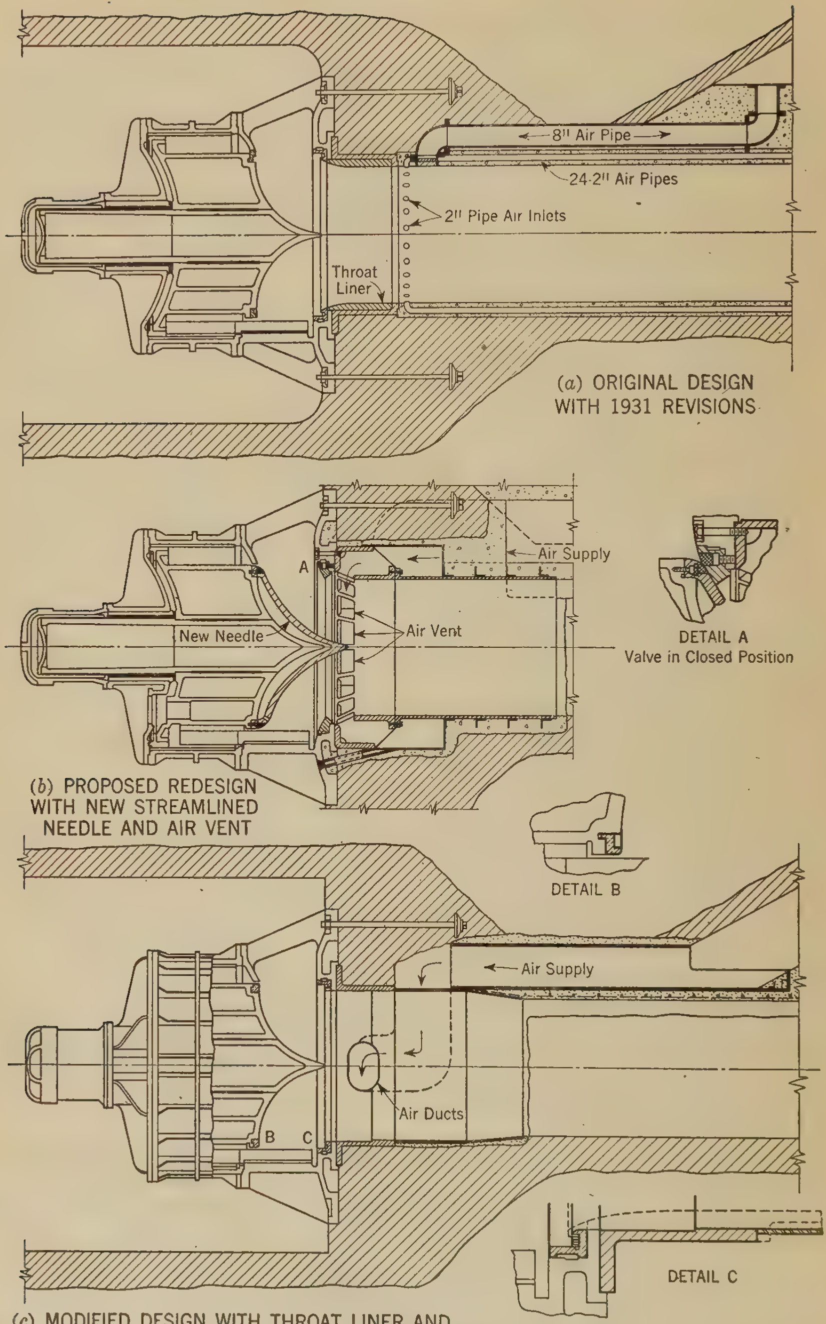
The 58-in. balanced valves in the lower outlet tunnel in the south canyon wall at Shoshone Dam, near Cody, Wyo., were installed in May, 1915. Although this type of valve had already required considerable maintenance in the installations at Roosevelt Dam, in Arizona, and Pathfinder Dam, in Wyoming, it was adopted because of the lack of a better design.

The valves were in operation only a few seasons when it became evident that seasonal maintenance similar to that at the older installations would be required. Pitting of the downstream faces of the valve needles and severe damage to the discharge conduit walls immediately below the valves occurred during extended periods of operation. Patching with various materials or filling the pitted areas by arc-welding with different metals was of no avail. With few exceptions the patches eroded more rapidly than the parent metal.

In 1930–1931, an attempt was made to relieve the Shoshone situation by installing twenty-four 2-in. pipes and an 8-in. air duct below each valve (Fig. 39(*a*)). A marked increase in the intensity of the noise accompanying the discharging water resulted, and the experiment was considered unsuccessful. Because of the failure of the vent system, resort was made to the original method of maintenance and the valves were used as little as possible. The pitting was serious and the repairs inadequate, but a more practical method of repair was not apparent.

During the season of 1942, the valves at Shoshone were operated at almost full capacity over an extended period in order to regulate flood flow and prevent crop damage downstream. Damage to the outlet structure was severe and maintenance measures became critical.

The concrete for several feet downstream from the metal lining in each conduit had been eroded severely and most of the twenty-four 2-in. pipes embedded in the conduit during the 1931 revision had been torn out in the eroded area (Fig. 40(*a*)). The "semi-steel" (high-test gray iron) conduit liner


8" Air Pipe
24-2" Air Pipes
2" Pipe Air Inlets
Throat Liner
A
Air Supply
New Needle
Air Vent
DETAIL A
Valve in Closed Position
DETAIL B
Air Supply
Air Ducts
B
C
DETAIL C

(*a*) ORIGINAL DESIGN WITH 1931 REVISIONS

(*b*) PROPOSED REDESIGN WITH NEW STREAMLINED NEEDLE AND AIR VENT

(*c*) MODIFIED DESIGN WITH THROAT LINER AND 2" PIPES REMOVED AND AIR DUCTS INSTALLED

FIG. 39.—BALANCED VALVES IN THE LOWER OUTLETS AT SHOSHONE DAM

below the valve was pitted severely and the face of the needle (Fig. 40(*b*)) had badly pitted areas (by operation in previous years) on which several kinds of metal had been tried—none satisfactorily. The areas of greatest pitting were below and above the valve guides, where only 3/4 in. of the original 2 in. of parent metal remained. The extensive welding of previous years, on the

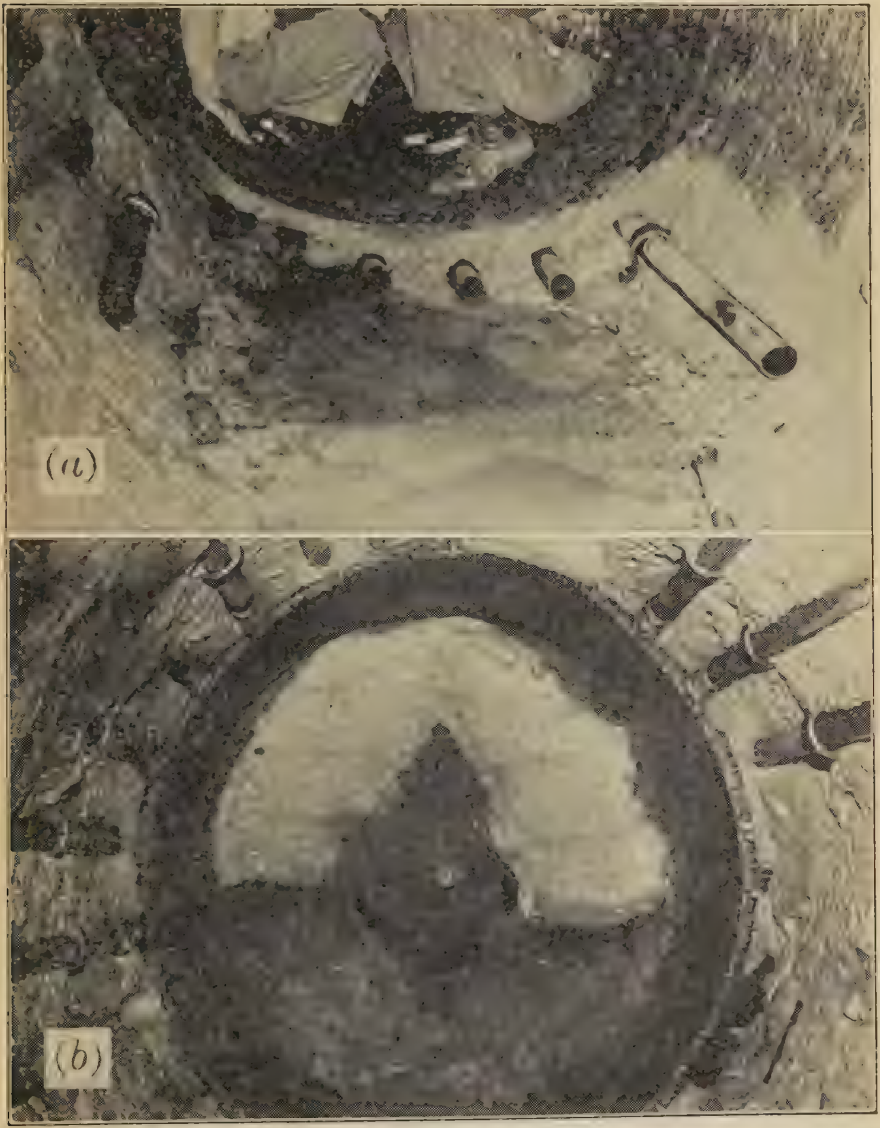


Fig. 40.—Remains of 2-In. Air Pipes, Shoshone Dam: (*a*) Vent of a 52-In. Conduit Facing Downstream from the East Valve; (*b*) East Conduit

needle face, is apparent in Fig. 40(*b*). The end of the metal liner was cut off in 1931 when the present installation was made.

Hydraulic laboratory model studies were made to evolve means of minimizing or eliminating the severe damage, to reduce the unreasonably high seasonal maintenance, and to remove the danger of a possible failure of the water-release system. This problem involved an extensive study of the pressure distribution in the valves and the discharge conduits.

Three alternatives were developed: (*a*) The range of valve opening was determined in which damage would be minimized until materials, unobtainable due to wartime restrictions, become available; (*b*) a redesign (Fig. 39(*b*)) was developed in which adverse pressure conditions were eliminated over the entire range of operation, but at some sacrifice in the discharge capacity; and (*c*) a modification of the present installation (Fig. 39(*c*)) was developed in which pressure conditions were acceptable over a range of valve opening from 25% to 100% with no reduction in the discharge capacity.

The model tests showed that the present prototype vent system is inadequate to prevent cavitation for all except a very small range of valve openings. Insufficient air is supplied between 23% and 70% openings, and some of the 2-in. vent pipes on the invert and crown become ineffective at openings above 85%, due to eddies forming immediately downstream from the V-guides. These conditions precluded safe operation of the present installation at ranges of valve opening other than 70% to 85%. Studies of the present installation indicated that the pitting on the valve needles was most severe between openings of 14% and 25%, and that damage to the conduits resulted between 23% and 70% valve opening. The damage to the conduit at these openings probably rendered the air-supply system ineffective and aggravated the destructive action for larger valve openings.

Damage by cavitation and pitting on the valve needles and discharge conduits can be eliminated entirely by a major revision (Fig. 39(*b*)) of the needle tip, the valve seat, the conduit throat, and the aeration system. This solution will reduce maintenance costs to a minimum and the valves can be operated at any opening without fear of damage due to subatmospheric pressures; but it will reduce the discharge capacity by approximately 20%, a factor to be considered in future revisions.

Minor alterations of the present structure (Fig. 39(*c*)) will involve: (*a*) Streamlining of the sealing edge of the plunger; (*b*) removal of a part of the bronze sealing ring by chipping and grinding; (*c*) removal of the throat liner; and (*d*) revamping of the air-supply system. Aeration equivalent to three 12-in. ducts would be adequate in this arrangement, but slightly more area was recommended as information on air requirements in high-velocity flow is limited. Operation of this modified design at openings smaller than 23% will have to be avoided to prevent damage to the needle. The discharge capacity is not affected noticeably by the modification.

Since materials have been unobtainable to make either the minor alterations or the major revision, the valves were operated during the 1943 irrigation season in the valve-opening range at which the subatmospheric pressures were the least severe. After thirty-five days of operation at 75% opening, the valve itself showed no evidence of additional pitting, and a very small amount of pitting had occurred in the extreme top of the discharge conduit. Forty-seven days of operation at 9% opening in 1942 had caused the damage shown in Fig. 40.

Parker Dam Spillway Piers

The spillway at Parker Dam has five 50-ft by 50-ft stoney gates to pass the flood waters. These gates were also used for passing the flow of the river during the low-water season, particularly during the early years of operation, before the power plant was completed. As a result of this early scheme of release, the gates were operated continuously over long periods, with a relatively small gate opening and a head above the spillway crest of from 40 ft to 50 ft.

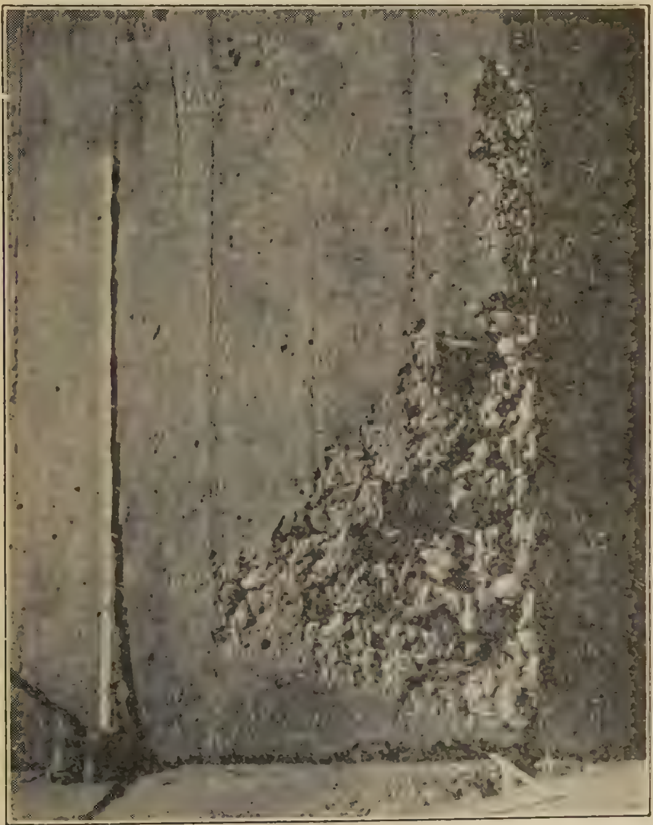


Fig. 41.—Pitted Area on the Face of a Spillway Pier Immediately Downstream from the Gate Recess at the Right, or California End, of Gate No. 5 at Parker Dam

An eroded condition, similar to that on the spillway faces at Bonneville Dam described in a previous paper, began to develop on the faces of the spillway piers and on the spillway crest immediately downstream from the gate slot

(Fig. 41). It first appeared below the gate which had the longest record of operation, but there was evidence of it downstream from the other gates. Subsequent operation of the other gates had gradually developed the same pattern on all ten of the pier faces in lesser degree of intensity. Photographic inspection at approximately yearly intervals discloses some increase in the extent of the area and depth of the erosion, but not sufficient to cause undue alarm, particularly since the power plant has been placed in operation and most of the low-water flow passes the dam through the turbines.

Model studies were undertaken to reveal the cause and means of eliminating pitting at Parker Dam and to prevent it at future installations. Incompleted studies, including all possible circumstances, have revealed several points of interest. The use of transparent models revealed cavitation under the end of the gate in the gate slot as a result of a vortex. Pitting, caused by the collapse of the low-pressure pockets breaking away from the bottom of the vortex, is the only logical explanation of the damage to the pier face.

A similar installation at Guernsey Dam, in Wyoming, showed no signs of erosion on the spillway face even though the gate has operated in the same range for a long period of time. This naturally raised the question as to why erosion occurred at Parker Dam and not at Guernsey Dam.

The gate slot at Guernsey Dam is considerably larger (Fig. 42) in horizontal cross section than the gate slot at Parker Dam. As a result, the vortex in the gate slot at Guernsey Dam was large and slow in rotation with no appreciable reduction in pressure at the core, whereas with the smaller cross section of the gate slot at Parker Dam, the angular velocity was high with a very small core and very low pressures in the core. As in the case of the cavitation zone downstream from a venturi throat, the flow condition was unstable and low-pressure pockets broke away from the bottom of the vortex. Some of the pockets collapsed against the boundary surface, resulting in the destruction of the concrete and metal.

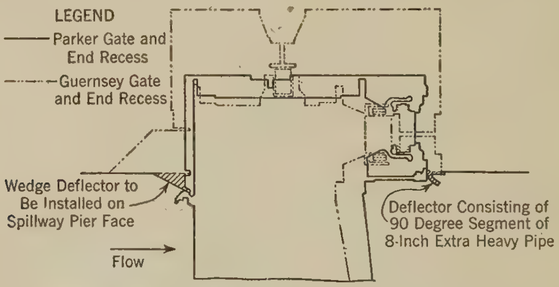

LEGEND
——— Parker Gate and End Recess
–·–·– Guernsey Gate and End Recess
Wedge Deflector to Be Installed on Spillway Pier Face
Flow
Deflector Consisting of 90 Degree Segment of 8-Inch Extra Heavy Pipe

Fig. 42.—Comparison of the Gate Recesses of Guernsey Dam and Parker Dam, Showing the Location of the Deflectors Proposed as Remedial Measures at Parker Dam

According to the present conception of the condition, the solution appears to be the elimination of the vortex. This can be done in a number of ways, none of which is universally applicable. In the case of Parker Dam, it is proposed to install a wedge-shaped deflector (Fig. 42) upstream from the gate sufficient in extent to deflect the flow of water under the gate away from the downstream corner of the gate slot, thus negating the formation of the vortex. An additional curved deflector consisting of a 90° segment of an 8-in. extra heavy pipe fastened to the metal at the downstream side of the gate slot will further deflect the flow away from the pier face and provide aeration down to the spillway crest. Another solution, practicable where the spillway crest is sufficiently far above the tailwater to provide drainage, is the extension of the end beams of the gates down into wells in the spillway crest, thus making them continuations of the gate slots. These gate-beam extensions will then serve as followers and will fill the gate slot as the gate is raised, providing continuity of the spillway pier face. In the case of a gate 50 ft high, the follower is considered structurally feasible in lengths to 6 ft. The model studies indicated that a follower length of from 2 ft to 3 ft is all that is necessary, since the occasion and duration of operation at the larger openings are infrequent and short.

Insertion of steel plates in the piers in the areas of pitting, as was done on the spillway piers at Bonneville Dam, is also a solution, but one remedying the effect rather than removing the cause.

Boulder Dam Spillway Tunnel

The channel spillway on the Arizona side at Boulder Dam was first placed in operation on August 6, 1941. On August 14, 1941, the drum gates were raised for a few hours and a hurried inspection was made of the tunnel. Little or no sign of erosion was apparent. Operation of the spillway was continued until December 1, 1941, at which time, because of the lowering of the reservoir elevation, it was necessary to start release of water through the tunnel plug outlet needle valves. During the four months of continuous operation, the average flow was approximately 13,500 cu ft per sec, except for several hours on October 28, when one of the drum gates dropped and the maximum flow was 38,000 cu ft per sec.

During a routine inspection of the spillway tunnels on December 12, 1941, an eroded area was discovered in the bottom of the curve connecting the inclined and horizontal portions of the spillway tunnel (Fig. 43(*a*)). The hole was approximately 115 ft long and 30 ft wide, with a maximum depth of 45 ft below invert grade.

The repair (Fig. 43(*b*)) of the damaged area has been described elsewhere (28). The chief concern here is an attempt to analyze the cause of the erosion. A number of theories have been advanced. In the opinion of the writer, the primary cause was misalinement of the tunnel a few feet upstream from the upper end of the eroded area. With an extremely high velocity down the inclined portion of the tunnel (at least 150 ft per sec), the stream followed the invert profile down to the hump; but, as it flowed over the hump, the water could not follow the sudden change in grade and a cavitation region formed between the sheet of flowing water and the concrete. The pressure in that region was the vapor pressure of the water, but since this condition was very unstable, the low-pressure pocket or cavity intermittently passed downstream in the region of higher pressures, collapsed and disintegrated or pitted the concrete as shown in the foreground of Fig. 44. The misalinement is defined by the position of the rope in Fig. 44.

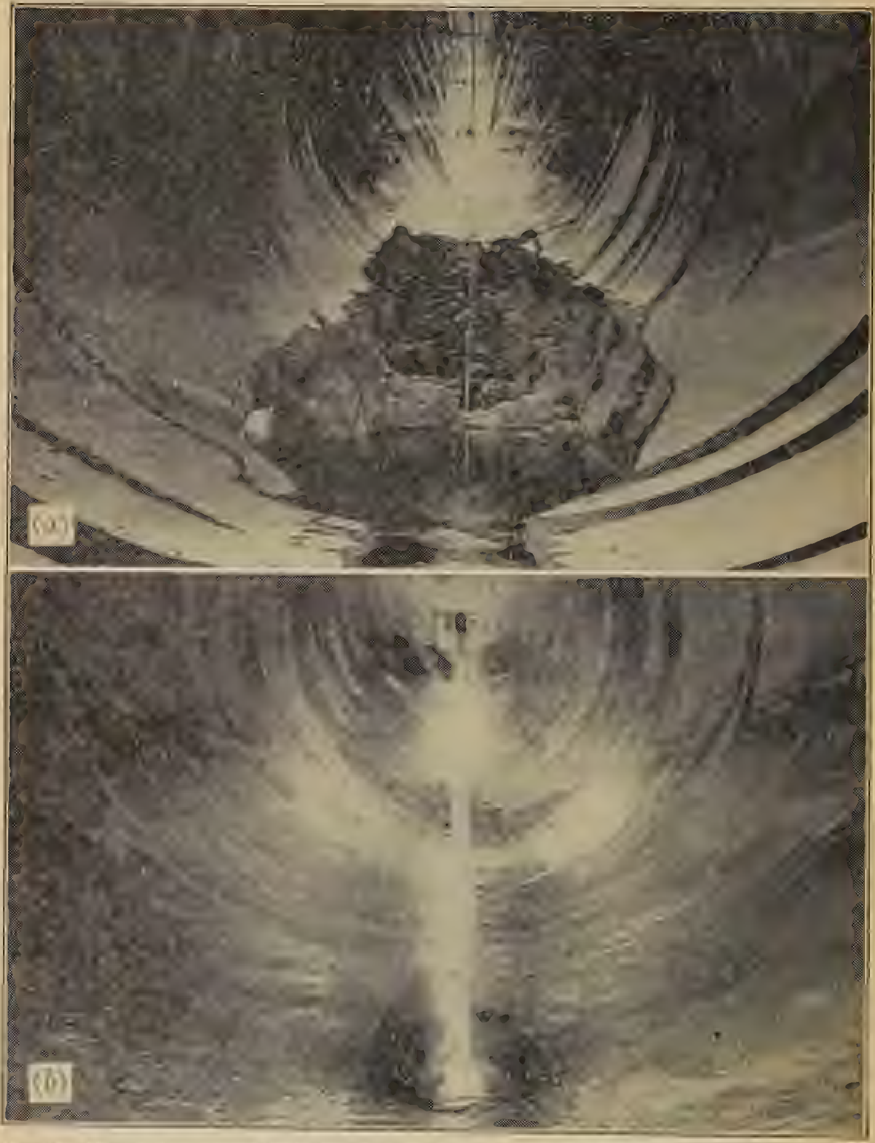


Fig. 43.—Cavitation in the Spillway Tunnel on the Arizona Side, Boulder Dam: (a) Eroded Area After Unwatering; (b) After Completion of Repairs

With the surface of the concrete broken by the pitting over a relatively small area, the high-velocity water had a grip on the concrete and destruction by impingement started. Imperfections in the concrete, such as rock pockets, cold joints, porous areas, lack of bond, etc., all made the concrete more vulnerable to this attack by impingement. Furthermore, the impingement of the high-velocity water on any exposed joints would cause the energy in the water to be converted from velocity head to pressure head. This pressure was probably transmitted through the planes of weakness in the construction joints caused by lack of proper horizontal joint cleanup prior to placement of new concrete. The concrete, being weak in tension, was dislodged in a manner similar to freezing of concrete and the resulting expansion. The concrete was probably dislodged in quite large pieces. After the concrete lining was ripped away, the shattered rock in an underlying fault was dislodged and transported away by the water. The shattered rock in the fault contributed to the extent of the erosion and not to the cause. After the surface was broken by the pitting and the joints were exposed to direct impingement, the sheet of high-velocity water down the tunnel invert acted as a mammoth hydraulic giant.

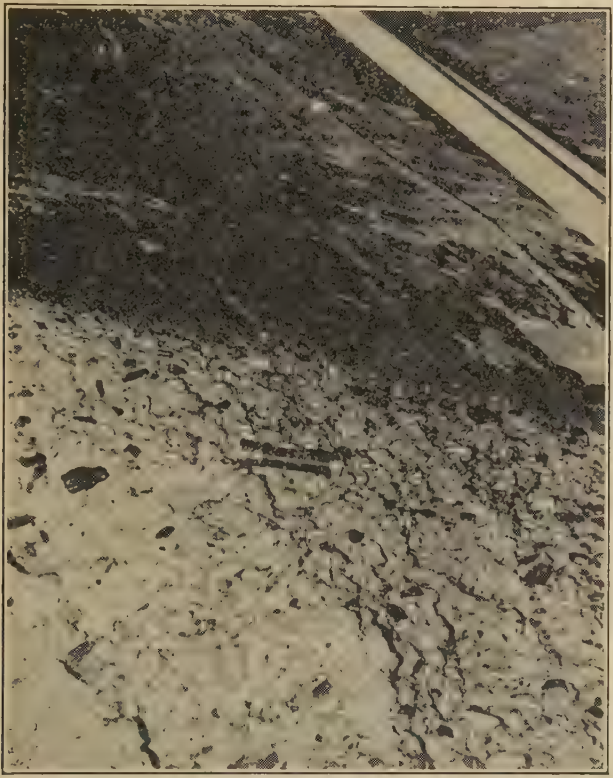


Fig. 44.—Pitted Surface Downstream from the Misalinement in the Tunnel at the Upstream End of the Eroded Area

The pitting of the concrete surface downstream from the hump is analogous to a flesh wound. Infection followed which was aggravated by the weaknesses in the concrete and the shattered condition of the underlying rock. Under the conditions of misalinement which existed at a critical position in the inclined tunnel, it is doubtful that any material could have withstood the effects of cavitation indefinitely. Of course, perfectly sound homogeneous concrete and underlying rock would have greatly reduced the extent of the erosion.

If the rock pockets, cold joints, and other porous areas in the invert are assumed to be the primary cause of failure, it is difficult to explain why the rock pockets immediately above and below the hump have not been the source of erosion, since the velocity of the water at all three points is for all practical purposes the same. Actually, the coat of black waterproofing and mineral deposit was intact in many places, showing no effect of direct scouring by the high-velocity water immediately above and below the eroded area.

In making the repairs to the tunnel, aside from providing concrete having the most suitable qualities practicable, extra effort was made to provide a smooth continuous surface with no humps or depressions. Two major humps and several minor humps in the invert above the eroded area were entirely eliminated by bushing and grinding, using a template cut to the true radius of curvature. Rock pockets were cleaned, patched, and then ground to conform to the surrounding concrete. Accumulations of grout and mineral deposits were removed. The surface of new concrete in the eroded area was finished carefully to produce a sound, continuous, uninterrupted surface. The surface was given a final grinding with a small terrazzo machine to remove board marks and objectionable offsets, leaving an extremely smooth surface. Minor bulges in the surface were removed by bushing followed by grinding, using a template cut to the correct radius of curvature. Considerable care was used in grinding the surfaces adjacent to the old concrete lining to remove all offsets and other irregularities.

Conclusion

These illustrations are typical situations which should be avoided by designing engineers. Other such examples must exist. If these could be brought to light and explained in the discussions of this Symposium, they would be a definite contribution. Since experience seems to be the principal source of knowledge, those of the profession who are intimate with the effects, even though they have attained that knowledge the hard way and in some cases the embarrassing way, should impart their experiences so that a wide variety of instances can be available to avoid repetition in the future.

EXPERIENCES OF THE TENNESSEE VALLEY AUTHORITY

By George H. Hickox,[5] M. ASCE

Synopsis

This paper describes cavitation damage experienced by the Tennessee Valley Authority (TVA) in the sluices of Norris Dam, the repairs made to the damaged areas, and the steps taken to prevent similar damage in other structures built by the Authority.

Cavitation at Sluice Entrance, Norris Dam

As of July, 1945, TVA had experienced cavitation in only one of its structures, Norris Dam, in Tennessee. The sluices through the base of Norris Dam are similar in design to those of Madden Dam, in the Isthmus of Panama, and were under construction at the time the damage to the Madden sluices was discovered. In an attempt to prevent similar damage to the Norris sluices, the entrances were bellmouthed by the addition of a half-doughnut-shaped structure on the upstream face of the dam. Fig. 45 shows vertical and horizontal sections through the Norris sluice entrance. The conduit is 5 ft 8 in. wide and 10 ft high and is lined above and below the gates with "semi-steel" (high-test gray iron). Below the liner the conduit was laid out along the trajectory of the jet so that the discharge would enter the bucket on a tangent.

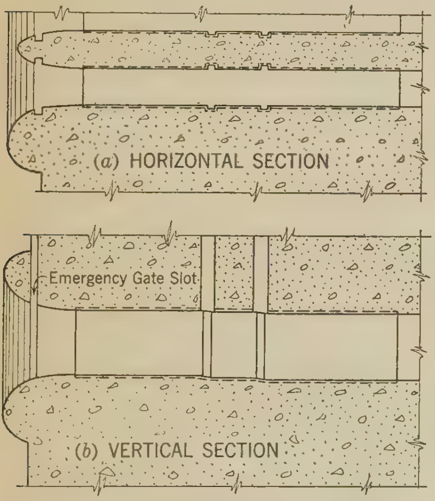

(a) HORIZONTAL SECTION
Emergency Gate Slot
(b) VERTICAL SECTION

Fig. 45.—Sections Through Sluice Entrances, Norris Dam

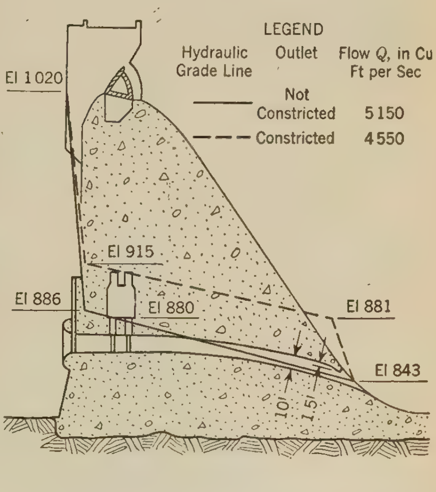

LEGEND
Hydraulic Grade Line
Outlet
Flow Q, in Cu Ft per Sec
Not Constricted 5150
Constricted 4550
El 1020
El 915
El 886
El 880
El 881
El 843
10.5'

Fig. 46.—Effect of Outlet Constriction on Hydraulic Grade Line

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[5] Senior Hydr. Engr., TVA, Hydr. Laboratory, Norris, Tenn.

Fig. 46 shows the hydraulic grade line for a discharge of 5,150 cu ft per sec and indicates that the pressure in the vicinity of the gates is relatively low, a condition favorable to cavitation. It will be noted that the low outlet contributed to the low hydraulic grade line at the entrance.

When the sluices were placed in operation with the pool at El. 1028, at the time of the release of flood water in 1937, with gates fully open, a pronounced cracking or snapping sound was heard. It seemed to originate somewhat upstream from the operating gallery in the vicinity of the sluice entrances. When the gates were closed 0.4 ft, giving a net opening of 9.6 ft, the noise stopped. It is believed that this noise was caused by cavitation occurring in the sluice entrance. An inspection made by a diver in January, 1938, showed that the sluices had not been damaged. The absence of damage, however, does not eliminate the possibility of cavitation. Accurate information on the discharge of the Norris sluices is not available but the best data at hand indicate that, with the reservoir at El. 1028, the discharges for gate openings of 10.0 ft and 9.6 ft were 5,310 and 4,920 cu ft per sec, respectively. Tests on a model of the Norris sluices, made at the TVA Hydraulic Laboratory in Norris, Tenn., showed that, with a discharge of 5,310 cu ft per sec, the prototype pressure corresponding to the model pressure immediately below the emergency gate slot at the roof of the sluice would have been 74 ft below atmospheric if such a pressure were physically possible. They also indicated that cavitation should stop when gate closure reduced the discharge to 4,850 cu ft per sec. The agreement between this value and the probable discharge of 4,920 cu ft per sec, at which the noise stopped, seems too close to be accidental. It is evident from the tests that the half-doughnut-shaped entrance structure was not satisfactory in eliminating cavitation as long as the emergency gate slot allowed the passage of water past the sharp upper corner of the entrance. Further tests showed that the low-pressure area could be eliminated by partial closure of the emergency gate slot.

TABLE 1.—EVIDENCE OF DAMAGE CAUSED BY CAVITATION, SLUICE LINERS, NORRIS DAM, IN TENNESSEE

Sluice No.	(a) OBSERVATIONS OF APRIL, 1937					(b) OBSERVATIONS OF FEBRUARY, 1938				
	Hours of Operation at the Following Gate Openings, in Feet:				Depth of pitting below gate slot (in.)	Hours of Operation at the Following Gate Openings, in Feet:				Depth of pitting below gate slot (in.)
	0 to 5.0	5.1 to 9.3	9.4 to 10.0	Total		0 to 5.0	5.1 to 9.3	9.4 to 10.0	Total	
1	280	51	219	550	Slight	793	523	297	1,613	⅛[a]
2	403	43	59	505	0	440	54	75	569	¼[b]
3	123	26	509	658	⅛[a]	674	777	1,380	2,831	½[c]
4	285	4	196	485	0	323	9	221	553	Slight
5	240	11	93	344	0	866	409	1,562	2,837	⅛
6	220	65	983	1,268	⅜[d]	659	916	1,578	3,153	¾[e]
7	424	46	35	505	0	480	60	41	581	Slight
8	15	6	380	401	Slight	210	212	1,006	1,428	Slight

[a] $\frac{1}{16}$ in. to ⅛ in. [b] At a point 3 ft from the floor. [c] ¼ in. to ½ in. [d] Near the top of the gate. [e] ½ in. to ¾ in.

Pitting Below Sluice Gates due to Cavitation

The sluices are lined above and below the gates with castings of so-called gray iron. In April, 1937, the liners below the sluice gates were inspected for evidence of damage due to cavitation. Table 1(*a*) summarizes the results of this inspection. It gives the hours of operation for gate openings of 0 ft to

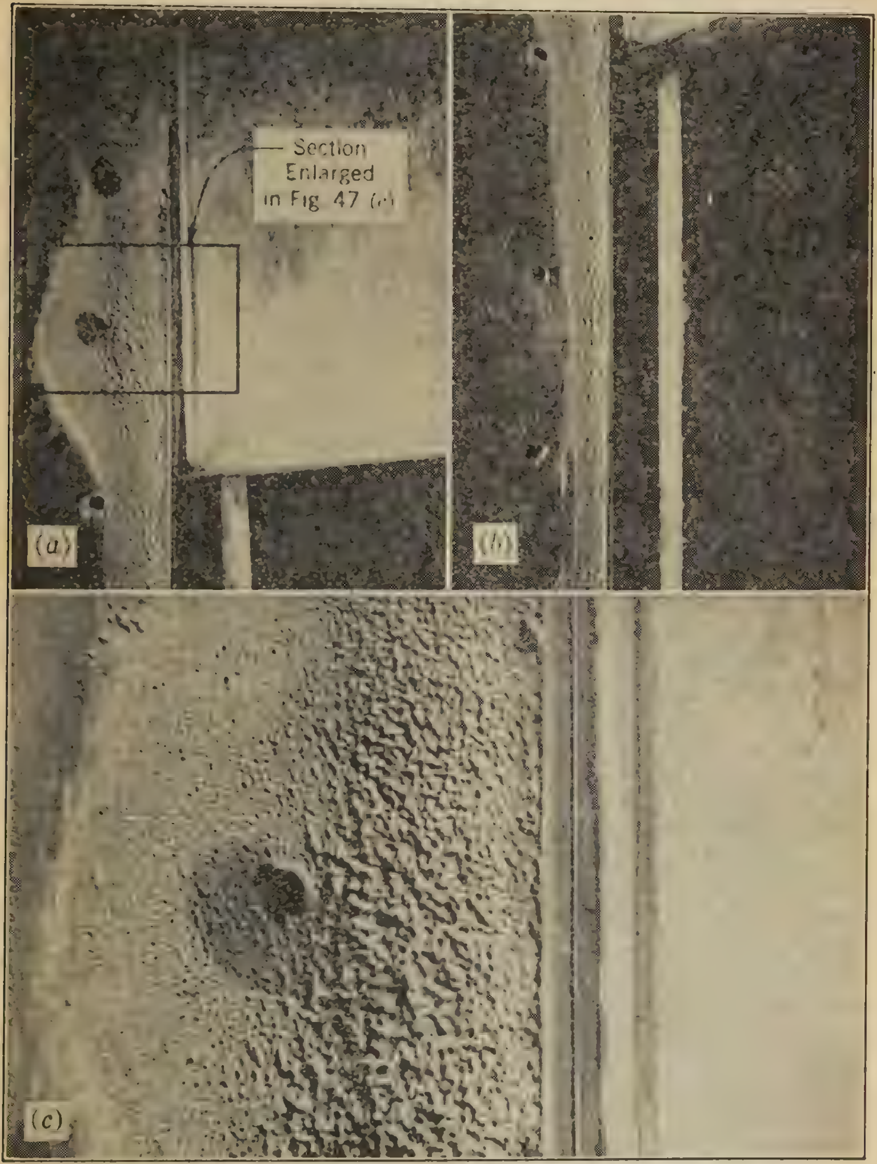


Fig. 47.—Pitting Below the Gate Slot Along the Right Side of Sluice No. 6: (*a*) Upper Part of Sluice; (*b*) Lower Part of Sluice; and (*c*) Enlarged Section (See Fig. 47(*a*))

5.0 ft, 5.1 ft to 9.3 ft, and 9.4 ft to 10.0 ft, and the depth and location of the deepest pitting below the gate slot. A similar inspection was made in February, 1938, and the results of this inspection are given in Table 1(*b*). The mean velocities during the period of operation varied between approximately 80 ft per sec and 100 ft per sec, depending on the reservoir elevation.

Table 1 demonstrates that the damage to the liner was progressive, increasing with the length of time the sluices were operating. In every case the depth of pitting was greater in 1938 than in 1937. Fig. 47 shows the pitting that occurred on the right side of sluice No. 6. Fig. 47(*c*) is a detailed view of the region indicated in Fig. 47(*a*). Similar pitting occurred on the left side of the sluice. In these illustrations it is interesting to note that, although the liner was pitted to a depth as great as 3/4 in., the bronze gate seat appeared to be undamaged.

An attempt was made to correlate the maximum depth of pitting with the time of operation. Fig. 48(*a*) shows the maximum depths of pitting plotted against the total time of operation for each sluice, as taken from Table 1. In general, there was a definite increase in the depth of pitting with the total time of operation. The correlation is not too good, however, and it was thought that the depth of pitting for the total time of operation might not be as significant as the depth resulting from operation at a particular gate opening. In Fig. 48(*b*) the maximum depth of pitting has been plotted against the time of operation for the corresponding gate opening, as defined in Table 1, in which the location of the pitting is given. Where the location of the maximum depth of pitting was not given, it was assumed that the depth was more or less uniform. The depth was plotted against the maximum number of hours for any one group of openings. The correlation does not seem to be much improved. It is probable that the breakdown of operating time is not detailed enough to be significant.

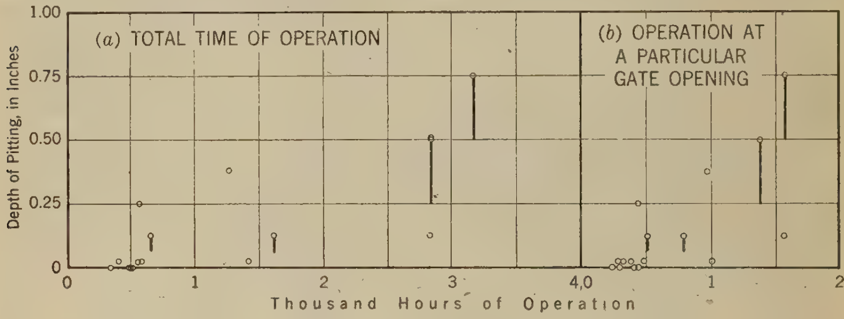
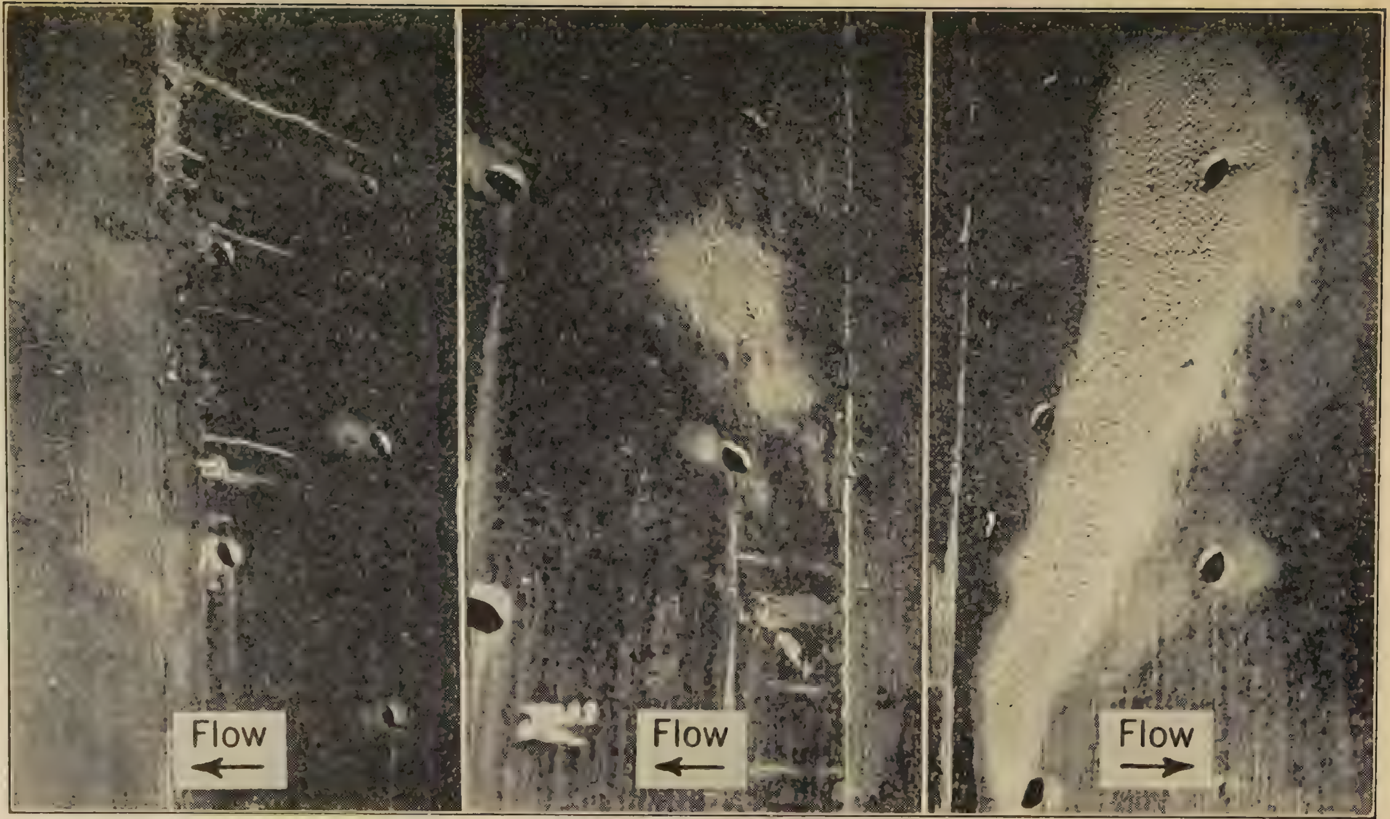


Fig. 48.—Depth of Pitting Versus Time of Operation

Pitting Below Irregularities in Liner due to Cavitation

Damage due to cavitation was noted not only below the gate slots but also downstream from the weep holes and the irregularities in the joints of the liner sections. Fig. 49(*a*) illustrates the pitting that occurred below the weep holes. Fig. 49(*b*) illustrates the effect of a slight projection into the sluices at one of



(a) Below the Weep Holes (b) Below an Obstruction at a Joint (c) Below a Mismatched Joint

Fig. 49.—Pitting Below Irregularities in the Joints of the "Semi-Steel" (High-Test Gray Iron) Liner Sections

the joints. Fig. 49(*c*) shows the damage that resulted from a slight mismatching of the liner sections at one of the joints. It is evident from these photographs that any irregularities, either depressions or projections, are sufficient to cause cavitation and pitting.

Repairs to Norris Sluices

The inspection of February, 1938, showed that some repairs to the sluice liners were necessary. The repairs were made as follows: All damaged parts that had been pitted to a depth of more than 3/16 in. were chipped out to a minimum depth of 1/4 in. The affected area was then built up to the original surface, by welding, and then ground smooth. The material used was chiefly mild steel but small areas were repaired experimentally with other pitting-resistant materials. Damaged parts that had been pitted to a depth of less than 3/16 in. were repaired by grinding the damaged part and any adjacent bumps or offsets, care being taken to leave only smoothly curved surfaces. Weep holes in the side walls were plugged by mild steel studs, driven tight, cut off flush with the surface, welded around the edges, and ground smooth.

In order to remove one of the causes of cavitation it was decided to raise the hydraulic grade line at the entrance by constricting the outlet. This was done by adding a concrete block 18 in. thick to the top of the sluices at the outlet as shown in Fig. 46. This constriction reduces the capacity of the sluices when operating at full gate opening but has no effect at smaller gate openings. The hydraulic grade line for full gate opening is shown in Fig. 46 for comparison with the original grade line. Three of the eight sluices, Nos. 1, 3, and 6, were constricted in this manner and the remainder were left in their original form.

After a sufficient period of operation, all sluices will be inspected to determine the effect of the constriction in preventing cavitation tendencies and of the repairs in preventing pitting. Sluice discharges since these repairs were made have been very infrequent and no inspection had been made as of July, 1945.

Model Tests for Prevention of Cavitation

Model tests were made of all sluices built by the TVA subsequent to the construction of Norris Dam. These included Hiwassee, Cherokee, Douglas, and Fontana dams.

Hiwassee Sluice.—The sluice through Hiwassee Dam is circular in section with a diameter of 8.5 ft. Ring follower gates were used to eliminate the possibility of cavitation at the gate slot. The hydraulic grade line was raised throughout by constricting the outlet diameter to 7 ft 10 in. The constriction was necessary because the sluices slope downward at a rate of 1 on 5. The possibility of cavitation at the entrance was investigated by means of models. The entrance and a portion of the sluice barrel were built at a scale of 1 : 15. A row of piezometers was placed along the crown of the entrance, special attention being paid to points of change of curvature. The shape of the entrance was a simple bellmouth. Three different forms were tested. The one in which the lowest pressures for the normal range of operation were nearly atmospheric was selected. Fig. 50(*a*) shows the form of the entrance as it was built. A number of piezometers were installed in the prototype structure in locations similar to those of the model. Observations on these piezometers indicate that the pressures existing on the prototype are very close to those predicted by the model. There has been no indication of any cavitation tendencies. (The dotted lines in Fig. 50(*a*) indicate the presence of a metal liner in this sluice.)

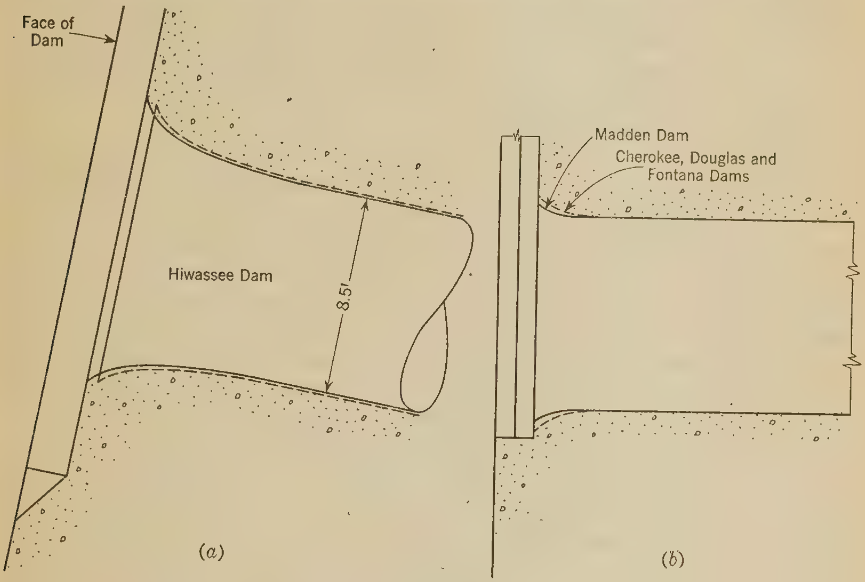


Fig. 50.—Comparison of Vertical Sections of Sluice Entrances

Cherokee, Douglas, and Fontana Dams.—The sluices through Cherokee Dam, Douglas Dam, and Fontana Dam (see Fig. 50(*b*)) have the same cross section as those at Norris Dam. The Norris gate design was used, making it unnecessary to design new gates for these dams which were built on emergency schedules. The entrances and outlets, however, were modified to avoid the difficulties experienced at Norris.

Tests were made on a 1 : 15 scale model of the entrance at the discharges to be expected in the operation of the dams. Pressures were measured by two rows of piezometers, one row along the center line of the roof and the other in the roof adjacent to the side wall. It was found that the lowest pressures occurred in the corners rather than on the center line. This is interesting in view of the fact that in the damage reported at the Madden sluices, in which the entrance shape was quite similar (see Fig. 50(*b*)), the deepest pitting was on the side walls just below the roof rather than in the roof itself. Since the cross sections of the sluices for Madden Dam and Cherokee Dam are identical, the test results may be accepted as indicative of actual pressure conditions in the Madden sluices.

To produce satisfactory operating conditions on the spillway apron, the outlets for the Cherokee and Douglas sluices curve sharply downward and the side walls are flared. The area was reduced 15% at the outlet to raise the hydraulic grade line at the entrance. A model of the outlet was built at a scale of 1 : 15 and piezometers were placed in the floor and side walls in the curved part to determine whether pressures might be low enough to induce cavitation. No low pressures were found.

Piezometers were installed in similar locations in the entrance and outlet of both the Cherokee and Douglas sluices to check the results of the model tests. Preliminary observations on the sluices in both dams indicated that a reasonably good agreement with the model tests exists. Final figures on the prototype pressures will not be available until the sluices have operated enough to establish a reliable discharge calibration.

Morning-Glory Spillway—Upper Holston Projects.—The South Holston and Watauga projects have morning-glory spillways discharging into deep vertical shafts and horizontal tunnels. The circular spillway crest was designed according to the results of tests (30) reported by Cecil S. Camp, Assoc. M. ASCE, and J. W. Howe, M. ASCE, in 1939. Piezometers were installed on the spillway face and at various locations in the shaft and tunnel. No negative pressures were found. It is planned to install piezometers at corresponding locations in the prototype structures as a check on the model tests.

Venturi Type Lock Filling Ports—Pickwick Landing and Watts Bar Dams.—Many of the locks on the Tennessee River are filled by means of culverts in the lock walls discharging through ports opening into the lock chamber. In most cases, the ports have bellmouth entrances and flaring outlets. The result is a considerable reduction in pressure at the throat section. It is possible, therefore, that cavitation conditions may exist at the throat of these ports at the beginning of the filling period when the water level in the lock chamber is low. Tests made on the lock at Pickwick Landing Dam showed that the minimum pressure at the throat of one port during filling was 9.6 ft below atmospheric. At the time of the test, the tailwater was 3.0 ft above its expected minimum elevation. The hydraulic grade line at the port throat was 26.6 ft below the water surface in the lock chamber. Model tests on the Watts Bar lock showed that pressures 22 ft below atmospheric existed in one of the ports for a short time at the beginning of the filling period when the tailwater was low. The pressures found in these two locks do not indicate that serious cavitation may be expected, since the minimum tailwater occurs only when the downstream reservoir is being drawn down in advance of floods. Cavitation may thus occur during only a very small part of the time, and it is believed that damage in these locks is negligible or nonexistent. The tests do show, however, that cavitation may occur. This possibility should always be investigated.

SUMMARY

Operation of the Norris sluices indicated that, at full-gate opening, cavitation might exist. Damage to the sluice liners actually occurred wherever the smooth surface of the liner was broken by either a projection or a depression. The depth of pitting was related to the time of operation.

Repairs were made to the metal liner by grinding and welding, using severa pitting-resistant materials. Pressures in the affected region were raised by constricting the sluice outlet.

Cavitation was prevented in the sluices of Hiwassee, Cherokee, Douglas, and Fontana dams by making model tests of the sluices and modifying the curvature of the entrance until the pressures at all points were high enough to avoid cavitation. Pressures were also raised by constricting the outlets.

Other structures of the Authority, including the spillways of the proposed Upper Holston Projects and the ports of the lock filling systems on the main river dams, were investigated for possible cavitation troubles.

LITERATURE CITED

By the Symposium Authors

The following list of references, admittedly incomplete, is offered as a representative bibliography on cavitation in so far as it comes within the scope of the Symposium:

(1) **"A New Method for Studying Cavitation Erosion of Metals,"** by W. C. Schumb, H. Peters, and L. H. Milligan, *Metals and Alloys*, Vol. 8, No. 5, May, 1937, p. 126.

(2) **"Emission of Visible Light from Cavitated Liquids,"** by L. A. Chambers, *Journal of Chemical Physics*, Vol. 5, No. 5, May, 1937, pp. 290–292.

(3) **"Cavitation Research at the Massachusetts Institute of Technology,"** by J. C. Hunsaker, *Mechanical Engineering*, Vol. 57, April, 1935, pp. 211–216.

(4) **"Slow Motion Film of a Cavitating Airfoil,"** by H. Mueller, *Hydromechanische Probleme des Schiffsantriebs*, Berlin, 1932, pp. 311, 435.

(5) **"Cavitation on Marine Propellers,"** by L. P. Smith, *Transactions*, A.S.M.E., Vol. 59, 1937, p. 409.

(6) **"Cavitation et Tension Superficielle,"** by F. K. Th. Van Iterson, *Proceedings of the Royal Academy of Science*, Amsterdam, Vol. XXXIX, Nos. 2 and 3, 1936; see also *Engineering* (London), Vol. 142, July 24, 1936, pp. 95–96.

(7) **"Theory of Stress Due to Collapse of Vapour Bubbles in a Liquid,"** by R. S. Silver, *Engineering* (London), Vol. 154, December 25, 1942, pp. 501–502.

(8) **"On the Pressure Developed in a Liquid During the Collapse of a Spherical Cavity,"** by Lord Rayleigh, *Philosophical Magazine*, Vol. 34, 1917, p. 94.

(9) **"Investigations into the Causes of Corrosion or Erosion of Propellers,"** by C. A. Parsons and S. S. Cook, *Engineering* (London), Vol. 107, April 18, 1919, p. 515.

(10) **"Tests on Erosion Caused by Jets,"** by E. Honegger, *The Brown Boveri Review*, Vol. XIV, No. 4, April, 1927, pp. 95–104; and Vol. XXI, Nos. 1 and 2, January and February, 1934, pp. 29–30.

(11) **"Rapid Erosion Tests Clarify Wear of Metals,"** by T. F. Hengstenberg, *Steel*, October 17, 1932, pp. 21–23.

(12) **"Cavitation and Erosion,"** by J. Ackeret, *Hydromechanische Probleme des Schiffsantriebs*, Berlin, 1932, pp. 227, 427.

(13) **"Cavitation Study by the Vibratory Method,"** by H. Peters and B. G. Rightmire, *Proceedings*, Fifth International Cong. for Applied Mechanics, 1938, p. 614.

(14) **"Practical Aspects of Cavitation and Pitting,"** by J. M. Mousson, *Bulletin*, Edison Elec. Inst., September, 1937, p. 373; and October, 1937, p. 423.

(15) **"The Mechanism of Cavitation Erosion,"** by T. C. Poulter, *Transactions*, A.S.M.E., *Journal of Applied Mechanics*, March, 1942, pp. A-31–A-37.

(16) **"Über Werkstoffzerstörung Durch Stosswellen in Flüssigkeiten,"** by J. Ackeret and P. deHaller, *Schweizer Archiv*, Vol. 4, No. 10, 1938.

(17) **"Cavitation Bibliography,"** Rept. R-81 of David Taylor Model Basin, Navy Dept., Washington, D. C. (unpublished).

(18) **"Hydraulische Probleme,"** by H. Föttinger, Berlin, 1926; also **"Experiments on a Few Typical Phenomena of Cavitation,"** by H. Föttinger, *Hydromechanische Probleme des Schiffsantriebs*, Berlin, 1932, pp. 241, 428.

(19) **"Cavitation in Outlet Conduits of High Dams,"** by Harold A. Thomas and Emil P. Schuleen, *Transactions*, ASCE, Vol. 107, 1942, p. 421. (*a*) Fig. 2, p. 425. (*b*) Fig. 13, p. 455. (*c*) Fig. 7, p. 438. (*d*) Fig. 10(*a*), p. 447. (*e*) p. 477.

(20) **"Recent Developments in Hydraulic Laboratory Technique,"** by J. B. Tiffany, Jr., *Proceedings*, 2d Hydraulics Conference, *Bulletin No. 27*, Univ. of Iowa, Iowa City, 1943. (*a*) p. 31.

(21) **"Sub-Atmospheric Hydraulic Models,"** by J. C. Harrold, *Civil Engineering*, June, 1941, p. 361.

(22) **"Cavitation on Baffle Piers Below Dams,"** by H. A. Thomas and W. J. Hopkins, *Proceedings*, 2d Hydraulics Conference, *Bulletin No. 27*, Univ. of Iowa, Iowa City, 1943. (*a*) p. 192.

(23) **"Manual on Lock Valves,"** *Manual of Engineering Practice No. 3*, ASCE, 1930, Appendix 10, p. 77.

(24) *Engineering* (London), November 22, 1940, p. 403.

(25) **"High-Pressure Reservoir Outlets,"** by J. M. Gaylord and J. L. Savage, Dept. of the Interior, Bureau of Reclamation, 1923, pp. 8–9.

(26) **"Experiments Aid in Design at Grand Coulee,"** by Jacob E. Warnock, *Civil Engineering*, November, 1936, p. 737.

(27) **"Distinctive Features of Grand Coulee Outlets,"** by Jacob E. Warnock, *ibid.*, December, 1940, p. 779.

(28) **"Erosion Causes Invert Break in Boulder Dam Spillway Tunnel,"** by Kenneth B. Keener, *Engineering News-Record*, November 18, 1943, p. 762.

(29) **"Bonneville Dam Concrete After Six Years,"** by C. C. Galbraith and R. R. Clark, *ibid.*, March 8, 1945. (*a*) p. 333. (*b*) Fig. 4, p. 334. (*c*) Fig. 5, p. 335. (*d*) Fig. 6, p. 335.

(30) **"Tests of Circular Weirs,"** by Cecil S. Camp and J. W. Howe, *Civil Engineering*, April, 1939, p. 247.

(31) **"Experimental and Theoretical Investigation of Cavitation,"** by J. Ackeret, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, 1930, pp. 1 and 63 (Translation No. 20, U. S. Experimental Model Basin).

(32) **"Cavitation,"** by J. Ackeret, *Handbuch der Experimental Physik*, Band IV, Leipzig, 1931 (Translation No. 22, U. S. Experimental Model Basin).

(33) **"Cavitation Laboratory Practice,"** by L. M. Davis, *Civil Engineering*, March, 1941, p. 148.

(34) **"Developments in Regulating Outlet Valves,"** by G. J. Hornsby, *Transactions*, A.S.M.E., Vol. 64, 1942, p. 85.

(35) **"Determination of the Relative Resistance to Cavitation Erosion by the Vibratory Method,"** by S. L. Kerr, *ibid.*, Vol. 59, 1937, p. 373.

(36) **"Cavitation Research—A Progress Report,"** by C. F. Merriam, presented at a meeting of Hydraulic Power Committee, New England Elec. Utilities Engrs., March 3, 1933 (on file in Engineering Societies Library).

(37) **"Progress in Cavitation Research at Princeton University,"** by L. F. Moody and A. E. Sorenson, *Transactions*, A.S.M.E., Vol. 57, 1935, p. 425.

(38) **"Pitting Resistance of Metals Under Cavitation Conditions,"** by J. M. Mousson, *ibid.*, Vol. 59, 1937, p. 399.

(39) **"Concerning the Influence of Air Content on the Development of Cavitation,"** by F. Numachi and T. Kurokawa, *Werft-Reederei-Hafen*, Vol. 20, No. 4, February 15, 1939, Berlin, pp. 45–49 (translation available at U. S. Waterways Experiment Station).

(40) **"Cavitation and Erosion Investigated as a Problem in Fluid Mechanics,"** by W. W. Pagon, A.S.M.E., 1935 (unpublished; on file in Engineering Societies Library).

(41) **"Design of Hiwassee Dam—Basic Considerations,"** by C. E. Pearce, *Civil Engineering*, June, 1940, p. 340.

(42) **"Testing for Cavitation,"** by R. E. B. Sharp, *ibid.*, May, 1941, p. 308.

(43) **"On the Behavior of Siphons,"** by J. C. Stevens, *Transactions*, ASCE, Vol. 99, 1934, p. 986.

(44) **"Cavitation at Butterfly Valves in Sluiceways,"** by G. Vogelpohl, *Zeitschrift*, Verein Deutscher Ingenieure, Vol. 79, 1935, p. 360.

(45) **"The Destructive Action of Cavitation,"** by M. Kornfeld and L. Suvorov, The Leningrad Physico-Technical Inst., Leningrad U.S.S.R.; reprinted in *Journal of Applied Physics*, June, 1944, p. 495.

(46) **"The Destruction of Material by Pitting and Cavitation,"** by J. Ackerel and P. E. DeHaller, *Schweizerische Bauzeitung*, September, 1936, p. 105.

(47) **"Failure of Metals Due to Cavitation,"** by H. N. Boetcher, *Transactions*, A.S.M.E., Vol. 58, 1936, p. 335.

(48) **"Cavitation Characteristics of Centrifugal Pumps,"** by G. F. Wislicemus, R. M. Watson, and I. J. Karassik, *ibid.*, Vol. 61, 1939, p. 17.

(49) **"Cavitation in Centrifugal Pumps,"** by A. J. Stepanoff, *ibid.*, Vol. 67, No. 7, 1945, p. 539.

(50) **"Theory of New Phase Formation Cavitation,"** by J. B. Zeldovich, *Acta Physicochimica*, U.S.S.R., Vol. XVIII, No. 1, 1943, p. 1.

(51) **"Resistance to Cavitation Erosion,"** by R. Beeching, *Engineering* (London), March 20, 1942, p. 238.

(52) **"Flow Phenomena in Hydraulic Butterfly Valves Subjected to Cavitation,"** by H. Bleuler, *Esher Wyss News*, No. 1–2, 1939.

(53) Experiments by P. DeHaller, *Schweizerische Bauzeitung*, May, 1933, p. 243.

(54) **"Cavitation and Its Effects on Turbines,"** by Giro G. Kubo, *Civil Engineering*, November, 1938, p. 768.

(55) **"Cavitation and Its Influence on Hydraulic Turbine Design,"** by Wilhelm Spannhake, Technische Hochschule, Karlsruhe, Baden, Germany.

(56) **"Cavitation Testing of Model Hydraulic Turbines and Its Bearing on Design and Operation,"** by J. D. Scoville, *Transactions*, A.S.M.E., Vol. 58, 1936, p. 323.

(57) "Cavitation," by Lewis F. Moody, Baldwin-Southwork, August, 1938, p. 14.
(58) "Cavitation Problems and Their Effect Upon the Design of Hydraulic Turbines," by J. M. Mousson, *Bulletin No. 27*, Studies in Engineering, Univ. of Iowa, Iowa City, 1943, p. 146.
(59) "Theory of Cavitation Flow in Centrifugal Pump Impellers," by C. A. Gongwer, *Transactions*, A.S.M.E., Vol. 63, 1941, p. 29.
(60) "Material Destruction by Cavitation," by H. Schroter, *Zeitschrift*, V.D.I., Vol. 78, March 17, 1934, p. 349.
(61) "Cavitation of Hydraulic Turbine Runners," by R. E. B. Sharp, *Transactions*, A.S.M.E., Vol. 62, 1940, p. 567.
(62) "A Theoretical Investigation of the Phenomenon of Cavitation in Screw Propellers," by J. Tutin, *Philosophical Magazine and Journal of Science*, July, 1927, p. 17.
(63) "Pitting in Water Turbines," by Elov Englesson, *The Engineer*, Vol. 150, October 17, 1930, p. 418.
(64) "On the Principle of Similitude for Cavitation," by A. Reinhardt *Forschungsheft*, V.D.I., No. 370, 1935, p. 1.
(65) "Cavitation in the Propagation of Sound," by R. W. Boyle, *Transactions*, Royal Soc. of Canada, Vol. 16, May, 1922, p. 157.
(66) "Cavitation in an Elastic Liquid," by E. H. Kennard, *Physical Review*, Vol. 63, March, 1943, p. 172.
(67) "On the Destructive Mechanical Effects of the Gasbubbles Liberated by the Passage of Intense Sound through a Liquid," by F. D. Smith, *Philosophical Magazine*, 7th Series, Vol. 19, 1935, p. 1147.
(68) "Supersonic Phenomena," by W. T. Richards, *Reviews of Modern Physics*, Vol. 11, January, 1939, p. 36.
(69) "Experiments to Demonstrate Cavitation Caused by Ultrasonic Waves," by K. Sollner, *Transactions*, Faraday Soc., Vol. 32, 1936, p. 1537.
(70) "On the Destructive Action of Cavitation," by M. Kornfeld and L. Suvorov, *Journal of Applied Physics*, June, 1944, p. 495.
(71) *Engineering News-Record*, August 23, 1945, p. 76.
(72) "Norfork Dam in Arkansas Nears Completion," by G. R. Schneider and H. V. Pittman, *Civil Engineering*, November, 1944, p. 455, and December, 1944, p. 499.
(73) "On The Tensile Strength of Sap," by H. H. Dixon, *Proceedings*, Royal Soc. of Dublin, Vol. NS14, 1914, p. 229.
(74) "The Effect of Temperature on the Hissing of Water When Flowing through a Constricted Tube," by S. Skinner and F. Entwistle, *Proceedings*, Royal Soc. of London, Series A, Vol. 91, 1915, p. 481.
(75) "The Formation of Bubbles," by R. B. Dean, *Journal of Applied Physics*, Vol. 15, No. 5, 1944, p. 446.
(76) "Ebulliometric Measurements," by W. Swietoslawski, Reinhold Pub. Co., New York, N. Y., 1945.
(77) "Report on Significance of Tests of Concrete and Concrete Aggregates—Wear Resistance of Concrete," by L. H. Tuthill and R. F. Blanks, A.S.T.M., 1943, p. 38.

(78) **"Model Studies of Spillways,"** *Bulletin VI–1*, Boulder Canyon Project Final Reports, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., 1938, p. 173. (*a*) p. 185. (*b*) p. 163. (*c*) p. 9. (*d*) p. 180. (*e*) p. 170. (*f*) p. 177. (*g*) p. 169.

(79) *Journal*, A.C.I., June, 1939, p. 188–1.

(80) *Ibid.*, p. 188–7.

(81) **"Technical Investigations at Boulder Dam,"** by Tom C. Mead, *Reclamation Era*, June, 1940.

(82) *Journal* (Supplement), A.C.I., November, 1945, p. 348–23.

(83) **"Performance of TVA Structures Studied,"** by George H. Hickox, *Civil Engineering*, October, 1945, p. 467.

(84) *Ibid.*, December, 1945, p. 565.

(85) *Ibid.*, April, 1946, p. 178.

DISCUSSION

C. A. Mockmore,[6] M. ASCE.—An excellent explanation of cavitation is presented by Professor Vennard. The Symposium offers some timely suggestions (which designers should bear in mind if they wish to minimize the destructive effects of cavitation)—avoidance of sharp curves and abrupt corners.

Obviously, pressures within the cavities in the flowing fluid must be low, but that does not necessitate a low pressure in the fluid a short distance from the surface of a conduit. Fig. 51(*a*) shows a cast-steel wicket gate of a water turbine operating under a head greater than 300 ft. Fig. 51(*b*) is an enlargement of one corner of this same wicket gate showing the manner in which the tough metal is worn away by cavitation. The effect of the roughness left by a milling machine during fabrication is noticeable.

Professor Vennard emphasizes the necessity that laboratory experiments should be concurrent with design to avoid the danger of cavitation. Perhaps a little extra care in fabrication would help to produce, as nearly as possible, the conditions called for in the design. Cavitation may be started by a very small irregularity in the surface of a conduit; and, once started, it may grow rapidly in destructive intensity.

Joseph N. Bradley,[7] Assoc. M. ASCE.—The Symposium is an excellent up-to-the-minute review of cavitation theory and practice. From a practical standpoint, the causes and results of cavitation and the remedies for its elimination are fairly well understood. From a more scientific viewpoint, a clearer conception of the mechanical action which produces pitting of the boundary surface during cavitation remains forthcoming.

One form of cavitation that is not uncommon is demonstrated by the following example. The profile of one of two lower outlet conduits in Ross Dam (a project of the City of Seattle, Wash.) is shown in Fig. 52. The present outlet conduit, as indicated by the solid lines, was only partly completed in January, 1946. The conduit entrance is on the upstream face of the dam; and the downstream end discharges into a large diversion tunnel. Final plans called for a vertical bend beginning at El. 1207.44, and a horizontal section of pipe with a balanced valve at the extreme downstream end of the conduit, as indicated by the dash lines in Fig. 52(*a*). Difficulties encountered in obtaining materials during the war years made it impossible to complete the downstream portion, outlined by the dash lines. War changes many things but has little influence on the flow of a river. It was necessary to pass water through the dam during this period to supply power plants located downstream. Entirely out of this necessity the partly completed lower outlet was used for this purpose, the operators relying on a slide gate at the entrance for control.

The damage resulting during operation of the partly completed conduit is explained in the notes in Fig. 52(*b*). In approximately ninety days of actual

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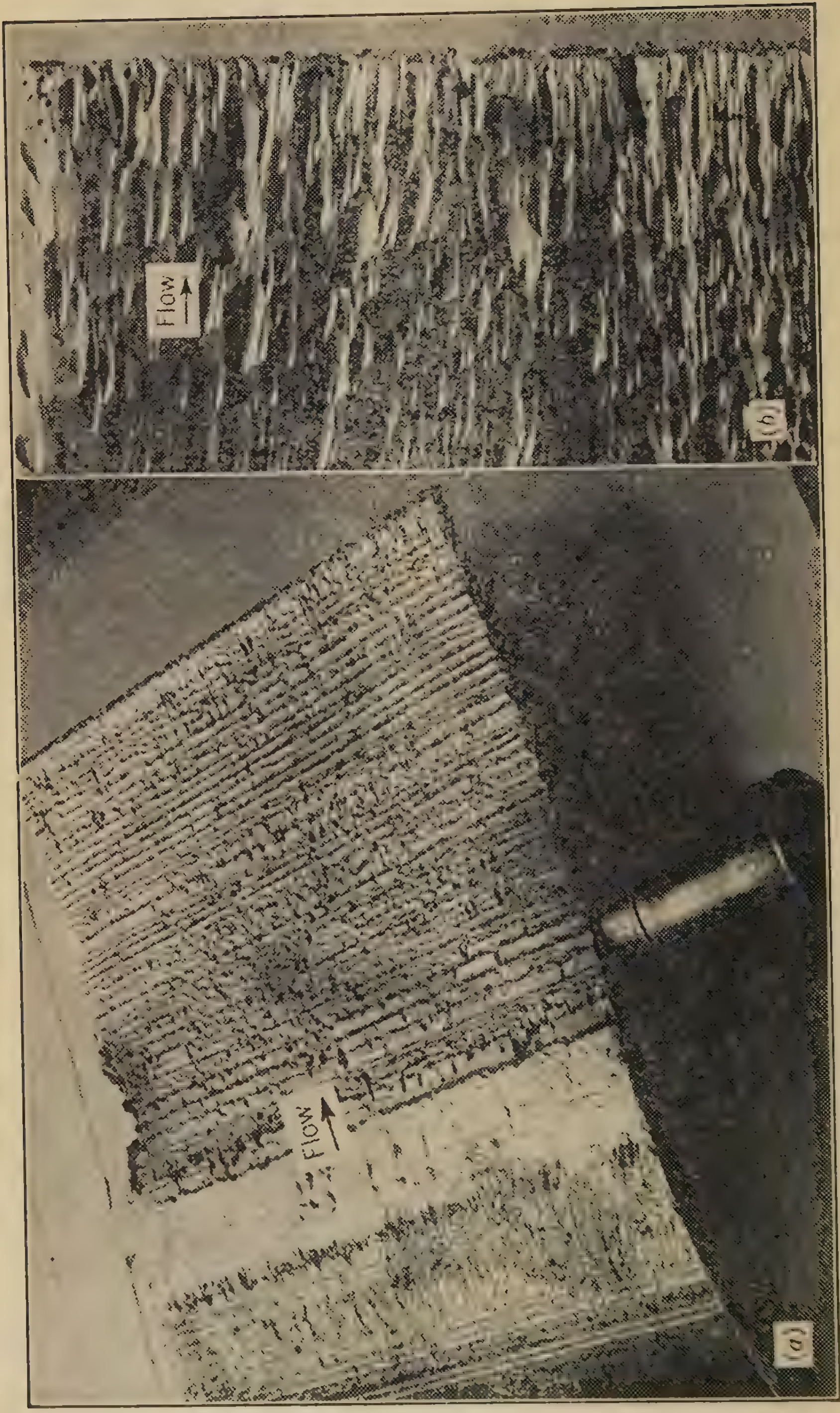

FIG. 51.—CAST-STEEL WICKET GATE FOR A WATER TURBINE, SHOWING CAVITATION EFFECTS RESULTING FROM AN IMPERFECT SURFACE FINISH

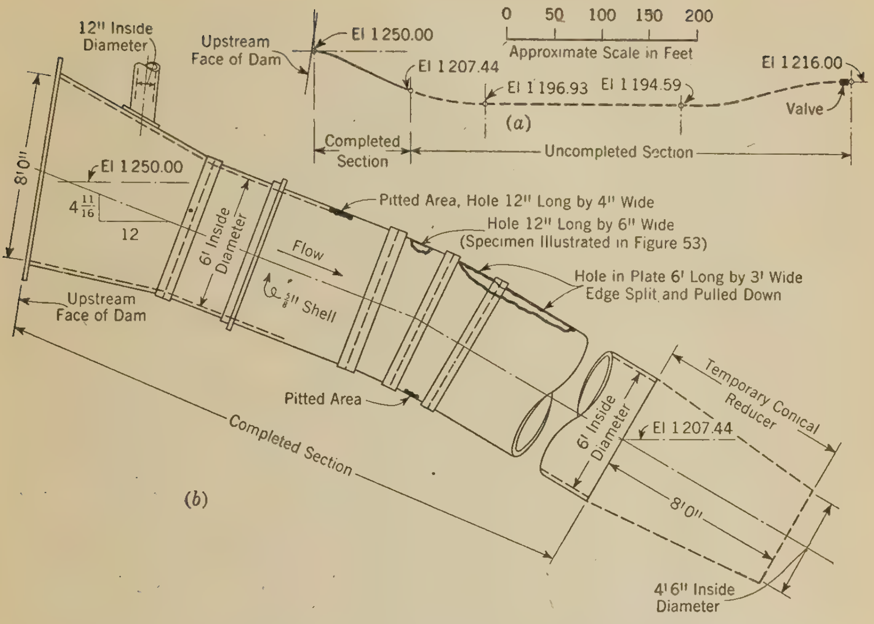
12" Inside Diameter
Upstream Face of Dam
El 1250.00
0 50 100 150 200
Approximate Scale in Feet
El 1207.44
El 1196.93
El 1194.59
El 1216.00
Valve
(a)
Completed Section
Uncompleted Section
8'0"
El 1250.00
4 11/16
12
6' Inside Diameter
Flow
5/8" Shell
Pitted Area, Hole 12" Long by 4" Wide
Hole 12" Long by 6" Wide (Specimen Illustrated in Figure 53)
Hole in Plate 6' Long by 3' Wide Edge Split and Pulled Down
Upstream Face of Dam
Pitted Area
Completed Section
6' Inside Diameter
Temporary Conical Reducer
El 1207.44
8'0"
4'6" Inside Diameter
(b)

FIG. 52.—PROFILE OF LOWER BY-PASS TUNNEL, ROSS DAM, SEATTLE, WASH.

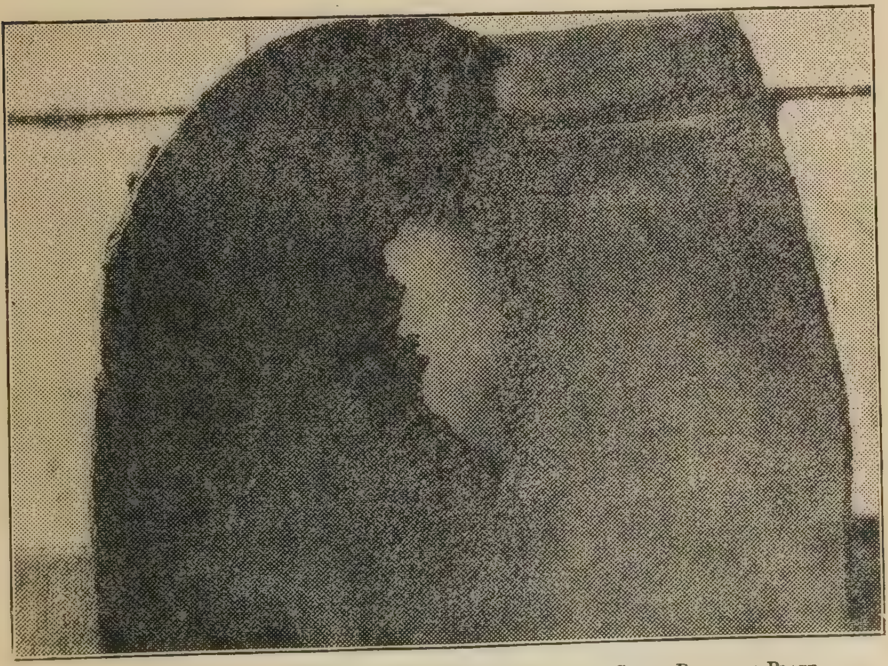

FIG. 53.—PITTING THAT HAS PENETRATED THROUGH A ⅝-IN. STEEL PENSTOCK PLATE

operation, over a period of about three years (during which the reservoir elevation did not exceed El. 1275), severe pitting occurred along the arch of the upstream tunnel section; and at some points this extended completely through the ⅝-in. steel plate liner. Fig. 53 shows a piece of an eroded section.

The cause of cavitation and the resultant pitting, in this case, is not a mystery. The difference in elevation between the entrance and the exit of the conduit in Fig. 52(*b*) is 42.6 ft. The vapor tension of water at Ross Dam approximates 32 ft of vacuum. The conduit acted in a manner similar to a siphon developing a suction head that tended to exceed the vapor pressure of water. This condition occurred principally in the upstream portion of the conduit where the water in the roof of the pipe vaporized, cavitation occurred, and pitting resulted. As the limiting value of negative pressure that can be developed is equal to the vapor pressure of water, under prevailing atmospheric conditions, this could occur only near the arch of the conduit. The pressure on the invert of the pipe was above the vapor pressure of water by an amount equivalent to the depth of water flowing in the conduit. With one exception, the damage was confined entirely to the roof of the conduit. One pitted area is indicated on the invert of the conduit, Fig. 52(*b*), immediately downstream from an abrupt change in grade. This was evidently the result of a localized, lowered pressure area produced by the sudden change of slope.

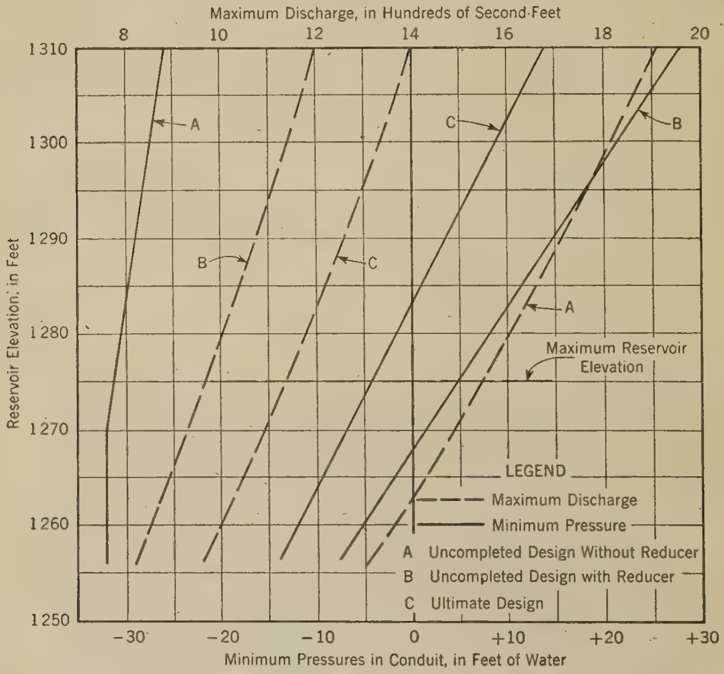

FIG. 54.—DISCHARGES AND MINIMUM PRESSURES IN LOWER BY-PASS TUNNEL

After the damage was discovered in 1943, it still remained impossible to obtain delivery of steel for the completion of the outlet conduits. Consequently, a temporary remedy for the alleviation of cavitation was administered. The severely pitted areas were removed and replaced by new plates thus making the conduit again watertight. In addition, a reducer cone from 6.0 ft to 4.5 ft in diameter was installed at the end of the pipe, as indicated by the dotted lines in Fig. 52(*b*). The cone served to reduce velocities, thus increasing pressures throughout the conduit. The outlet now (1946) operates without evidence of cavitation.

The solid lines in Fig. 54 show the minimum computed pressures in the outlet conduit for three conditions of operation. Actual field measurements are not available. Curve A, for the uncompleted tunnel, shows that negative pressures equivalent to the vapor pressure of water existed for all reservoir elevations up to El. 1270. The maximum reservoir elevation obtainable during this period was El. 1275, which is 10 ft above the top of the present dam. Curve B, Fig. 54 (which represents the uncompleted tunnel with cone installed at downstream end), indicates the minimum pressures existing in the conduit at the present time (1946). Curve C shows the minimum pressures that can be expected after final completion of the tunnel with a valve installed at the extreme downstream end. The slide gate on the face of the dam was assumed fully open for these computations.

The dashed lines in Fig. 54 show the corresponding maximum discharges to be expected for the same three conditions of operation. For the reservoir at El. 1265, the present conduit with addition of the reducer cone (curve B) will discharge 890 cu ft per sec; for the previous condition of operation without cone (curve A, Fig. 54) the discharge was 1,430 cu ft per sec. The conical reducer results in a 38% decrease in discharge at this arbitrarily selected head. As elimination of cavitation often involves a sizeable reduction in efficiency it is difficult to convince designers that avoidance of low pressure is imperative, in high-velocity design.

Acknowledgment.—The writer wishes to express his appreciation to E. R. Hoffman, superintendent of the Department of Lighting, City of Seattle, and W. B. Wolfendale, project engineer of Ross Dam, City of Seattle, for their generosity in granting permission for the presentation of the above material.

At least twenty references on cavitation ((45) to (64)) should be added to the Bibliography of the Symposium.

J. M. ROBERTSON,[8] JUN. ASCE.—The theory of cavitation can be divided roughly into two basic parts: The first is concerned with the occurrence of cavities—that is, the conditions under which cavities form; and the second is concerned with the dynamics of the cavities once they are formed. Professor Vennard's treatment of these two phases is good although somewhat elementary. However, in being elementary, it may represent correctly the state of knowledge on the subject.

Under the heading, "Occurrence of Cavitation," it is stated that "* * * liquids encountered in engineering practice cannot expand and cannot support tension stress." Although this assumption is the basis for the commonly accepted analysis used to indicate when and where cavities will form, it is not an absolute fact. According to R. W. Boyle (65), some careful static experiments have proved that liquids can be placed under tension (see also mention of this possibility by T. C. Poulter (15)). This phenomenon is similar to that of the occurrence of supersaturated solutions. If sufficient disturbing conditions are present, the phenomenon will not occur. Also, the degree of tension which a liquid may stand without parting is probably a function of the time of application of the tension. Thus, for a very short time water may withstand a high tension although for a relatively long time it may be able to withstand very little tension. This time effect is of some significance with regard to the determination of the exact location in time and space at which a cavity will form. For example, in the flow through a nozzle, if a pressure determination indicates that negative pressures occur at the throat of the nozzle, the cavities will actually appear some distance downstream, the exact point of occurrence depending upon the flow velocity. In the case of cavitation produced by vibratory methods, it is possible that, with a very high frequency of vibration, the part of the cycle during which the liquid is in tension would not be long enough for a cavity to form. The relation between the tension a liquid can stand and the time of application of the tension is also a function of the air or gas content of the liquid. Thus, in the case of the aforementioned nozzle, it is probable that, if the flowing liquid had a large dissolved gas content, the cavities would form much nearer to the point specified by elementary theory than they would if the same liquid contained little or no dissolved gas.

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[8] Asst. Prof., The Pennsylvania State College, State College, Pa.

The theory of the action of a cavity, once it is formed, is of concern mainly in connection with the manner in which cavitation produces erosion in near-by solids. In addition to the work on the pressures caused by the collapse of cavities mentioned by Professor Vennard, reference should also be made to the theoretical analysis presented by E. H. Kennard (66) and the hypothesis advanced by F. D. Smith (67). Mr. Smith's hypothesis is that the liberated cavities or gas bubbles undergo resonant vibrations producing intense local strains in the vicinity and the destructive effects are due to these strains. In addition to the mechanical effect of the collapsing or vibrating cavity in producing cavitation erosion, it is possible that other phenomena resulting from cavitation may assist in the action. W. T. Richards (68) in his summary of the state of knowledge on the effects of cavitation due to sonic or ultrasonic vibrations noted that high temperatures have been shown to occur at the edge of cavities, that the collapse of cavities has resulted in the creation of electrical potentials, and that cavitation has been shown to accelerate chemical reactions. In his studies of phenomena due to ultrasonic vibrations in nonmetallic (liquid) systems, K. Sollner (69) concludes that all the disruptive and destructive phenomena are caused by cavitation. It is possible that such secondary effects of cavitation as these may have an effect on the manner in which cavitation erosion occurs.

The vibratory form of apparatus for producing cavitation has been used extensively for the rapid comparison of the cavitation erosion resistance of various materials by S. L. Kerr (35). This type of apparatus may also be used to further understanding of the action of the cavities. With it cavities are formed at a fixed frequency in a relatively restricted and known location. By high-speed photography it is possible to observe the formation, growth, and collapse of a cavity under known pressure conditions. Research, such as that of M. Kornfeld and L. Suvorov (70), along this line should yield information on the dynamical principles surrounding the life of cavities.

Fred W. Blaisdell,[9] Assoc. M. ASCE.—The Symposium authors have shown conclusively that the possibility of cavitation in hydraulic structures must be considered, not only by the designer but also by the constructor and the inspector. Several instances are cited where the cavitation can be traced to poor design. Other instances show that cavitation is also possible in well-designed structures as a result of irregularities built into the solid boundaries during construction. In fact, Mr. Harrold uses an assumed construction irregularity in his illustration of the "Principle of Vacuum Apparatus." All the authors have shown that extensive damage, costly and continuous repairs, and operating difficulties are the price paid for neglecting to consider cavitation. A conservative office design that would eliminate all possibility of cavitation might well be uneconomical. On the other hand, although a design based on laboratory tests can work close to the cavitation limit, the possibility of cavitation damage will be greatly reduced or eliminated. This has been implied by Professor Vennard under the heading, "Remedies for Pitting."

The writer well remembers the staccato hammering in the cavitation apparatus at the Massachusetts Institute of Technology in Cambridge that made conversation in its vicinity a practical impossibility. The writer does not doubt that the cavitation in the Norris sluices could be heard, as reported by Mr. Hickox. The writer would like to ask the authors if the noise level is related to the damage caused by collapse of the vapor pocket.

At the end of the third paragraph under the heading, "Damage from Cavitation, or Pitting," Professor Vennard writes,

> "In some cases, however, the pitting has stopped of itself, apparently due to a water cushion covering the eroded region and preventing direct contact of collapse point and solid material."

The writer would like to inquire if it is desirable in some instances to design for the elimination of cavitation pitting rather than for the elimination of cavitation itself. Such a design would incorporate a "water cushion" into the structure to absorb or distribute the forces resulting from the collapse of the vapor pocket rather than streamlining to eliminate cavitation. Under the heading, "Proper Use of Baffle Piers," Mr. Harrold recommends the streamlining of baffle piers to eliminate cavitation where velocities are high and static pressures are low. Streamlining requires careful formwork and excellent workmanship. Mr. Warnock mentions the extensive damage in the Boulder Dam spillway tunnel resulting from a misalinement in the tunnel invert, and Mr. Hickox shows that cavitation pitting was caused by misalinement of joints in the sluice liners at Norris Dam. These experiences show that "streamlined" baffle piers would have to be constructed carefully if all cavitation were to be eliminated. On the other hand, if the piers were deliberately made narrower in a downstream direction, the vapor pockets would collapse in the liquid. Careful form work would not be required and pitting of the pier would presumably not occur. This is in line with Mr. Harrold's statement in the last paragraph under the heading, "Baffle Pier Tests," that "A cavitation streamer that leaves the baffle is assumed to be harmless, whereas one that clings to the baffle is considered harmful." On the other hand, the writer would like to ask if the pressure waves set up by the collapse of the vapor pockets might cause vibrations that would eventually weaken the pier. This important question has already been raised by Professor Vennard in the last paragraph under the heading, "Collapse of the Cavity." Perhaps he or Mr. Harrold will be willing to comment more fully in their closing discussions on the points raised in this paragraph.

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[9] Project Supervisor, U. S. Dept. of Agriculture, SCS, St. Anthony Falls Hydr. Laboratory, Minneapolis, Minn.

The design of "water cushions" into a baffle pier might (in the words of Professor Vennard under the heading, "Remedies for Pitting") "have little appeal," but the pier would probably be cheaper to construct than if it were streamlined. The "cushioned" piers would possibly be more effective energy dissipators than would the streamlined piers. Before this method could be applied to the design of piers, it would be necessary to know how close the cavitation streamer must be to the structure in order to cause pitting (mentioned by Mr. Harrold), the location of the cavitation streamer, and how far downstream the vapor pockets travel before collapsing. It is quite apparent that much experimental work remains to be done before adequate information on cavitation and its effects will be available.

John S. McNown,[10] Assoc. M. ASCE.—Investigations of cavitation, as indicated by the Symposium authors, have been restricted for the most part to laboratory and field observations of the factors causing cavitation and the countermeasures required for prevention. Although it is frequently possible to predict and eliminate cavitation on the basis of experience and model studies, very little is known concerning the thermodynamics of the local vaporization and the subsequent condensation of the water, the dynamics of the bubble collapse, or the manner in which boundary surfaces are pitted. Fundamental research, in time, should reveal much of this information and lead to a basic understanding of this phenomenon, but for the present it is essential that the available tools be utilized to their utmost efficiency. The writer wishes to discuss two of these tools—analytical methods, which he feels were unduly slighted in the Symposium, and the variable-pressure water tunnel.

For certain types of problems the assumption of negligible viscosity and the application of the corresponding equations of flow yield very worthwhile results, particularly since cavitation occurs in or immediately downstream from a zone of accelerated flow. Therefore, some of the statements made by Professor Vennard concerning the analytical approach are definitely misleading. The criterion for the applicability of the Bernoulli equation is not the absence of curvilinear flow, but rather the extent to which the flow can be described by the equations of irrotational motion. For example, it is obviously impossible to apply the Bernoulli equation across rectilinear laminar flow in a pipe line, whereas the familiar vortex (Fig. 1) is called the irrotational vortex because of the excellent agreement between theory and measurement for flow which is wholly curvilinear. To be sure, uncertainties in the magnitude of the pressure, which are of importance in cavitation studies, arise in the vortex core; however, these uncertainties are caused by departures from the irrotational pattern, and not by the presence of curvilinear motion.

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[10] Research Engr., Iowa Inst. of Hydr. Research, and Asst. Prof., Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

Irrotational motion is also approached very closely in regions of rapidly accelerated flow of the type indicated in Figs. 1(*a*) and 1(*b*). In fact, the two-dimensional counterpart of Fig. 1(*a*)—the horizontal slot—has been solved by the methods of classical hydrodynamics with remarkably close agreement between theory and experiment. In connection with tests in the variable-pressure water tunnel at the Iowa Institute of Hydraulic Research in Iowa City, the writer was able to predict, entirely by analytical means, the conditions for incipient cavitation on a mathematically defined boundary form. Hence, to describe all curved-flow problems (see heading, "Occurrence of Cavitation") as "most obscure and unpredictable" not only conveys a very false impression, but also completely ignores one useful avenue of approach.

Admittedly, only part of the problems encountered in engineering practice can be solved by analytical methods alone, and as a result many hydraulic structures can best be designed from model studies. Thus, consideration should be given to the methods of analysis and the experimental procedure for model studies of cavitation effects. For such a study, any dependent variable can be described by a function of viscous, gravitational, and cavitation effects, together with the appropriate description of boundary geometry. In general terms,

$$\mathbf{D} = f(\mathbf{R}, \mathbf{F}, \mathbf{K}, b/a, c/a, \cdots) \qquad (5)$$

in which **D** is any dependent variable in dimensionless form; **R** and **F** are the familiar Reynolds and Froude numbers; **K** is a cavitation parameter, the definition of which depends in part on the type of problem under consideration; and b/a, c/a, etc., are ratios of linear dimensions.

Three phases of cavitation research are of importance in engineering practice, and the general equation may be adapted to each of these. The first phase, and perhaps the most fundamental, is that of flow without a free surface such as flow past a submerged body or flow through a siphon spillway. Since viscous effects may be significant, the dependent variable can be expressed in terms of the Reynolds number, a cavitation parameter, and the boundary geometry:

$$\mathbf{D} = f_1(\mathbf{R}, \mathbf{K}, b/a, c/a, \cdots) \qquad (6)$$

The dimensionless parameter **D** may describe in relative terms a pressure drop, loss of head, length of cavitation pocket, amount of pitting which takes place, or some other pertinent quantity. The cavitation parameter is indicative of the tendency toward cavitation, and hence is determined by the normal center-line pressure p_0, the vapor pressure p_v, the undisturbed velocity v_0 and the density ρ. That is,

$$\mathbf{K} = \frac{p_0 - p_v}{\frac{1}{2}\rho v^2_0} \qquad (7)$$

and a reduction in **K**, resulting from either an increase in velocity or a reduction in pressure, increases the tendency toward cavitation. In most instances where cavitation takes place, the Reynolds numbers are large and the influence of viscosity is correspondingly slight. It must be kept in mind, however, that viscosity may play an important rôle should the model scale or velocity be too small.

The second type of problem is that of flow with a free surface for which the pattern of motion is completely described by the boundary geometry, as in flow over a spillway section. Since the Froude number cannot be varied at will once the head H and the boundary geometry are determined, **F** is not an independent variable for this type of flow; and, therefore,

$$\mathbf{D} = f_2(\mathbf{K}', a/H, b/H, \cdots) \qquad (8)$$

The simulation of prototype cavitation conditions in a model spillway requires the reduction of the atmospheric pressure p_a in the model, and the cavitation parameter for this type of flow may be written

$$\mathbf{K}' = \frac{p_a - p_v}{\rho g H} = \frac{p_a - p_v}{\gamma H} \qquad (9)$$

A third type of problem is that of flow with a free surface in which the velocity of flow can be varied independently of the depth, as in a study of cavitation effects on baffle piers. The added independent variable necessitates the inclusion of the Froude number, and the depth of approaching flow y replaces H as the linear dimension of the flow. Hence,

$$\mathbf{D} = f_3(\mathbf{F}, \mathbf{K}, a/y, b/y, \cdots) \qquad (10)$$

in which a, b, etc., are linear dimensions of the pier. The cavitation parameter may take either of the two alternative forms:

$$\mathbf{K} = \frac{p_a - p_v}{\frac{1}{2}\rho V^2} \qquad (11a)$$

or

$$\mathbf{K}' = \frac{p_a - p_v}{\rho g y} \qquad (11b)$$

which are related by the Froude number.

For any pattern of flow, such a particular value of **K** exists that for all higher values the flow is unaffected by cavitation, just as the pattern of flow past a body tends to become independent of viscous effects if **R** exceeds a characteristic magnitude. If the value for incipient cavitation is denoted by $\mathbf{K}_i$, cavitation effects commence for $\mathbf{K} = \mathbf{K}_i$ and become more and more pronounced as **K** is reduced below the critical. Furthermore, for a given value of the cavitation parameter, essentially similar cavitation phenomena will be found for various combinations of velocity and pressure on both model and prototype. It follows that Eqs. 2, 3, and 4 in Mr. Harrold's paper—which may be written in the form,

$$(p_a - p_v)_m = L_r(p_a - p_v)_p \ldots\ldots (12)$$

—are obtained if identical values of **K** are specified for model and prototype. That is, if $\mathbf{K}_m = \mathbf{K}_p$:

$$\frac{(p_a - p_v)_m}{\gamma\, H_m} = \frac{(p_a - p_v)_p}{\gamma\, H_p} \ldots\ldots (13)$$

and Eq. 12 is thereby obtained, provided the same liquid is used in both model and prototype. However, Eq. 3 does not completely determine **K** unless the Froude number depends on the boundary geometry. It is evidently not sufficiently general for use, say, on the baffle-pier study just discussed.

The problem of flow over a spillway section, discussed by Mr. Harrold, may be used further to illustrate an approach to the experimental determination of cavitation effects and to emphasize the important rôle of the cavitation parameter. If the discharge per unit width q is taken as the dependent variable, Eq. 8 takes the form,

$$\frac{q}{\sqrt{g}\, H^{3/2}} = C_d = f(\mathbf{K}', a/H, b/H, \cdots) \ldots\ldots (14)$$

For all values of **K** greater than $\mathbf{K}_i$, the variation of C_d with H, as determined by experiment, would be entirely independent of cavitation effects in this region, and would therefore be represented by the uppermost curve in Fig. 55. However, the onset of cavitation near the spillway crest would substantially reduce C_d, so that further tests at values of **K** less than $\mathbf{K}_i$ would yield a series of curves such as that shown schematically in Fig. 55. The lower the value of **K** or the greater the degree of cavitation, the greater will be the departure from cavitation-free conditions. In a similar manner, the effect of cavitation upon other dependent variables could be investigated.

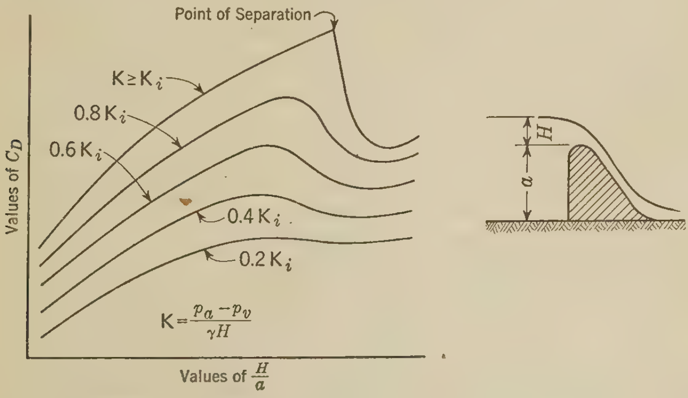

FIG. 55.—SCHEMATIC REPRESENTATION OF CAVITATION EFFECTS ON THE DISCHARGE COEFFICIENT OF A SPILLWAY

Even though it is impossible to make a model study which satisfies both the Froude and Reynolds criteria if the same fluid is used in both model and prototype, no comparable obstacle prevents the simultaneous study of cavitation phenomena and either viscous or gravitational effects if it is possible to vary the pressure on the system. As a result, much of the laboratory work on cavitation has been performed in variable-pressure water tunnels, of which more than a dozen have been built in research laboratories throughout the world. Not unlike closed-circuit wind tunnels in design, water tunnels have been used to test the cavitation characteristics of ship screws, turbine blades, and a variety of two-dimensional and three-dimensional body forms. In many respects the water tunnel is similar to the apparatus developed at Carnegie Institute of Technology in Pittsburgh, Pa., and described by Messrs. Vennard and Harrold.

Although water tunnels can be used for the variety of studies already mentioned, to date (1946) the experiments in the variable-pressure water tunnel at the Iowa Institute of Hydraulic Research have been devoted exclusively to the measurement of the effect of cavitation on the pressure distribution around underwater bodies. Nevertheless, general conclusions from these experiments concerning the effect on cavitation phenomena of viscosity and air content are applicable to all investigations of cavitation. Measurements of the pressure distribution around a given boundary form, which were taken at different Reynolds numbers and for values of **K** both above and below $\mathbf{K}_i$, indicated that viscous effects influenced the results for relatively small values of **R** and for low intensities of cavitation. This effect was found to be exaggerated for boundary curvatures of short radius, extending in one instance to Reynolds numbers of approximately 10^6.

There is usually a large amount of air dissolved in water, and the reduction in pressure that accompanies any tendency toward cavitation will release part of this dissolved air with much the same action as that involved in the formation of water vapor in zones of cavitation. Furthermore, because the pressures are low, the air released from solution may occupy a disproportionately large volume. Since it was considered likely, therefore, that the results of a study of cavitation might be affected by the quantity of air in solution, a series of tests was performed to determine the extent of this effect. The pressure was reduced steadily with a vacuum pump until an operational limit was reached below which the pump was unable to reduce the pressure in a reasonable period of time. With reference to this readily available minimum pressure, which was usually about 1.5 lb per sq in. above the vapor pressure, it was found that tests performed at pressures only slightly (0.2 lb per sq in. or less) above this operational minimum were affected by air coming from solution in such a manner as to produce results comparable with those for smaller **K**-values. For tests performed at pressures well above (0.4 lb per sq in. or more) this operational minimum, on the other hand, no effects of dissolved air were found.

For investigations directed toward the elimination of cavitation, model studies need not be conducted in variable-pressure chambers. The distribution of pressure along a boundary as determined in atmospheric-pressure models can be used to predict the tendency toward cavitation as shown by the studies at the Vicksburg (Miss.) laboratory. However, caution should be exercised in the interpretation of model results obtained in this manner, not only because atmospheric-pressure models may indicate impossibly low prototype pressures, but also because cavitation effects may be found even though the pressure on the boundary is everywhere well above the vapor pressure. Since cavitation first occurs in the fine eddies at the surface of separation, the pressure at a boundary surface a short distance away may be affected by the resulting change in the flow pattern. In one series of experiments at the Iowa Institute of Hydraulic Research on flow past an abrupt boundary transition, although the minimum pressure and the vapor pressure were equal when $K = 0.7$, cavitation effects did not vanish until the value of $K = 1.8$ was reached. In other words, even though the pressure head at the boundary was more than one velocity head above the vapor-pressure head, the pressure distribution along the boundary was affected by local cavitation a short distance away.

In conclusion the writer wishes to welcome the Symposium on Cavitation in Hydraulic Structures as a worthwhile contribution to the summarization of the available information in this relatively new field, and to concur heartily with the Symposium authors in their statements concerning the need for publication of other available information and for stimulation of further research in this field.

James W. Ball,[11] Assoc. M. ASCE.—The first Symposium paper, concerning the nature of cavitation, is an excellent contribution to the engineering profession. Although it is concise and clearly written, it would not be complete without the illustrations from the remaining Symposium papers. It is gratifying to note that the authors, by using absolute instead of gage pressure as a datum, have avoided the confusing term "negative pressure approaching, or exceeding, the vapor pressure" used by so many authors of papers on cavitation. However, it seems that the term "vapor tension" should be used instead of "vapor pressure" when cavitation is concerned. Vapor tension is defined as the maximum vapor pressure that occurs when there is an abundance of liquid present. It is a special value of vapor pressure that occurs in connection with cavitation.

Gate Grooves.—The problem of cavitation erosion below gate grooves, as described for Parker Dam on the Colorado River by Mr. Warnock, is not a new one; nor is its remedy new. In 1945, Abraham Streiff, M. ASCE, cited examples of these problems that had occurred 40 years earlier (71). The two most practicable remedial measures cited by Mr. Streiff, are almost identical to those evolved independently and 40 years later. The solutions, now as then, are mainly of two types—(1) deflectors placed upstream or downstream from the gate grooves and (2) specially-shaped piers.

Shoshone Dam Balanced Valves.—To complete the story concerning cavitation in the balanced valve outlets at Shoshone Dam near Cody, Wyo., presented by Mr. Warnock in the Symposium, it is necessary to relate what happened during the 1944 and 1945 irrigation seasons. The demand for water for irrigation and domestic use below Shoshone Dam during these two seasons was such that it has not been practicable to operate the 58-in. balanced valves in the noncritical range indicated by the model studies. As a result there has been pitting, slightly less severe, but in the same zones as before—extensive repairs being required. This is only one of many examples of the fact that scale models are invaluable instruments for determining the adequacy of proposed or existing designs of hydraulic structures.

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[11] Engr., Bureau of Reclamation, U. S. Dept. of Interior, Denver, Colo.

Boulder Dam Spillway Tunnels.—Since the Symposium was presented (before the Hydraulics Division of the Society at the Annual Meeting in January, 1944), the spillway tunnel on the Nevada bank of the Colorado River at Boulder Dam has been unwatered and inspected. Numerous discontinuities in the form of protruding pipes, humps, depressions, and "gobs" of concrete on the flow boundary were found. Cavitation erosion had occurred in the surfaces downstream from several of the pipes and "gobs" of concrete. Rough areas were noted below the other discontinuities but the appearance in these cases was such that cavitation could not be established definitely as the cause. The quantity of water which passed through this tunnel in the fall of 1941 was much smaller than that discharged by the spillway on the Arizona side. It seems fortunate that the quantity was small and that the period of operation was short, for many of the discontinuities reported would have caused cavitation that might have produced destructive pitting of the magnitude of that found in the Arizona tunnel.

Temporary Needle-Valve Outlets at Grand Coulee Dam.—The flow of the Columbia River at Grand Coulee Dam exceeds the capacity of the left powerhouse during most of the low-water season; thus, some method had to be provided for releasing the excess water downstream during the inspection, repair, and maintenance operations to the underwater part of the spillway face and bucket.

The plan developed for this purpose involved the installation of eight 84-in. needle valves (available for loan from another project) between the power penstocks and the unfinished turbine draft tubes of the right powerhouse. Steel cones and elbows connected the valves to the ends of the penstocks, and special steel transition elbows, embedded in concrete anchor blocks, completed the passages from the valves to the draft tubes from which the water discharged into the tailrace downstream (Fig. 56).

One of the outlets was operated for the first time on May 25, 1945, to check operating characteristics. That cavitation was occurring in the outlet passage downstream from the needle valve over nearly the full range of valve opening was indicated by the severe vibration and noise (crepitation) at these openings. The noise resembled the crackling of a large number of small firecrackers exploding in rapid succession, with intermittent loud popping reports. The vibration was so severe that it could be detected easily through the soles of the shoes of anyone standing on the roof or on other concrete surfaces of the powerhouse in the bay in which the valve was situated. Vibrations in the concrete anchor block, encasing the transition elbow and forming the passage between it and the lower part of the draft tube, were of such intensity as to resemble sharp blows with a small hammer on the soles of the shoes of persons standing on the anchor block. These vibrations were so severe that they were recorded at a seismograph station a short distance upstream from the dam on the shore of Roosevelt Lake.

After approximately 2 hours of operation at various valve openings (15 min at full capacity) the outlet was unwatered and inspected. Small areas of paint were missing from the inner curve of the transition elbow on both sides of the center line below the joints between the last four sections of the elbow. This condition seemed to be the result of local zones of cavitation induced by the inflection and weld seams at the joints, although no pitting of the metal could be detected. Two areas having a spongy appearance—definitely the result of cavitation—had formed in the concrete surface of the passage immediately below the areas in the elbow from which the paint had been removed. Two large areas in the original concrete of the draft tube just below the joint between the new and old construction, downstream and outward from the areas at the end of the elbow, had a pocked appearance, but it was difficult to establish this condition as cavitation erosion.

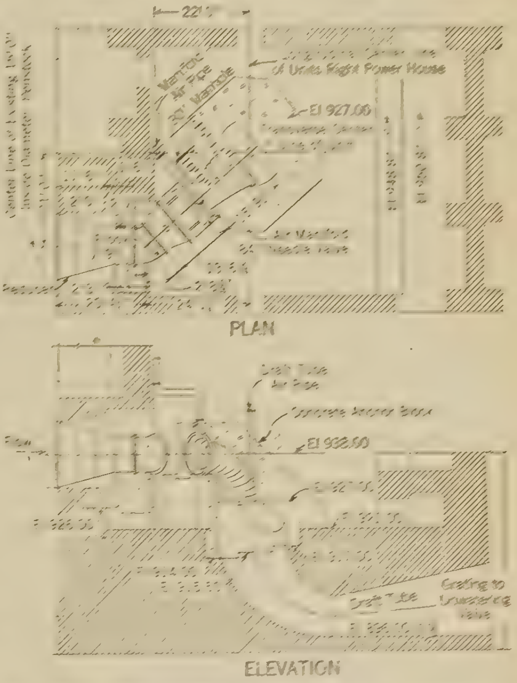


Fig. 58.—Side Elevation, Needle-Valve Outlets, Grand Coulee Dam (Needle Valve Not Shown)

In view of the indistinct markings and the undefined boundaries of the zones of erosion in the initial test, no photographs were taken. Moreover, additional operation of the outlet was desirable to establish evidence for study and analysis. Minor changes in the vent system and test equipment were made and the test was resumed, the valve being operated for 1 hour at various openings, and at full capacity for 2 hours.

The areas from which the paint had been removed in the initial test had increased in size, and their boundaries had become defined clearly. Although the bared metal surface in these areas showed no evidence of pitting, cavitation was considered to have been the cause of the removal of paint. The weld beads and the joints of the elbow combined to form a boundary of diverging surfaces, with the result that zones were formed immediately downstream from the joints, in which the pressure was reduced to the vapor tension of the water. The cavitation cavities forming in these zones collapsed to pit and removed the paint as the pressure increased in the area just above the next welded joint. The fact that this condition existed to each side of the center line on the inner curve of the elbow and not at other points was attributed to the fact that the flow is analogous to that in sharp bends (considering the axis of the bends somewhere between the horizontal and vertical planes)—and these regions are subject to low pressures regardless of discontinuities in the boundary surface.

The areas of cavitation erosion adjacent to the exit of the transition elbow in the back wall of the concrete anchor block, immediately downstream from the areas from which the paint had been removed, resulted from the cavitation originating at the last weld joint of the transition elbow.

The surface of the first-stage concrete at the back of the draft tube and directly beneath the exit of the elbow was pocked extensively (Fig. 57). It was believed that a zone of cavitation was formed adjacent to the wall immediately under the exit of the transition elbow, back of the flowing jet, and that the cavities collapsed as they reached the edges of this zone, where the pressure was greater than the vapor tension of the water, eroding the surfaces to produce the pocked appearance. The inclination of the surface of the last section of the elbow, and the receding anchor-block wall immediately below, combined to form the discontinuity producing this zone.

In addition to the pitted areas, which were more distinct than at the conclusion of the initial test, there was evidence of increased damage for which the cause was not apparent immediately. A patch 6 in. by 30 in. by 4 in. deep had been torn from the concrete floor of the draft tube (Fig. 58). Although the surface adjacent to the recess formed by the removal of the patch was rough, static pressure beneath the wedge-shaped patch, with the thick edge of the wedge at the surface, was believed to have caused the patch to "pop out." It was considered inadvisable to operate the outlet unit without patching the recess; therefore, the testing was discontinued pending the repair and alteration of parts of the flow passage below the valve.

The weld beads at the joints in the inner curve of the steel transition elbow on both sides of the center line, where cavitation pitting was indicated, were chipped and ground, and all eroded areas were smoothed and repainted.

A groove, 6 in. deep and 6 in. high at the back of the anchor block, with the bottom sloping 45° downward toward the center of the anchor dome, was chipped in the wall below the exit of the steel elbow. The groove extended approximately 30 in. beyond the extremities of the exit opening and all concrete that might protrude into the flow was removed from the wall of the anchor block. This groove is just below the exit in Fig. 59.

All the pocks in the wall of the anchor block beneath the elbow and in the curved surface of the back of the draft tube were chipped and repaired and all prominent irregularities were removed. A new patch was placed in the recess of the old wedge-shaped patch after it had been chipped to form a keyway, and five dowel rods were grouted in holes drilled several inches below the bottom surface. When the testing was resumed both 18-in. air vents were opened, and observations were made with the valve at various openings, to study the variation in vibration of parts of the powerhouse structure adjacent to the outlet and to study a phenomenon of light "flashes" reported to exist in the tailrace downstream from the outlet during its initial operation.

The vibration, although severe, was less than in the previous test when the 18-in. vent into the dome of the anchor block was closed, and no air reached the back of the outlet jet as it plunged from the transition elbow into the back of the draft tube.

"Flashes" of light resembling sheet lightning were observed in the tailrace immediately downstream from the draft-tube exit. The occurrence was irregular and varied in extent, being of a singular nature much of the time, but often having the appearance of a network of brilliant, irregular, shimmering light bands extending some distance downstream and to the sides of the outlet. When the spectacle could not be detected in darkness, it was concluded that the phenomenon was not electrical, but resulted from pressure waves that changed the refractive and reflective properties of the water as they moved outward into the tailrace. Since there were zones of cavitation in the outlet between the valve and the draft-tube exit, it was believed that the pressure waves originated from the collapse of cavitation cavities; however, it was not possible to obtain a correlation. Photographic records were unsatisfactory. The light "flashes" and crepitation accompanying the operation of the outlet were evidence that cavitation had not been eliminated. Proof of this was obtained when the outlet was unwatered after approximately 6 hours of operation.

The pitting in the transition elbow and in the back wall of the anchor-block, dome, below, and to each side of the center line of the elbow exit, was not eliminated by the grinding of the welded joints of the elbow. There was no cavitation erosion in the old concrete at the back of the draft tube or in the lower part of the anchor block. The aeration groove in the back wall of the anchor block below the exit of the transition elbow eliminated this condition.

The patch in the floor at the back of the draft tube, which had been replaced after being torn from its recess in the previous test, was missing; and there was evidence of cavitation erosion in the surrounding surface which had not been the case in previous operations (Fig. 59). The five dowels of reinforcing steel had remained in the recess, but the grout around them had been removed to a depth of from 1 in. to 2 in. below the bottom of the recess. No doubt tremendous forces were involved. Since it was desirable to establish the nature and extent of the destructive forces acting to damage the outlet, or to make it inoperable, the test was continued for 16 hours with the valve discharging at full capacity.

Examination after a total of 23 hours of operation at full capacity disclosed that, except for one area about 6 in. in diameter which had a spongy appearance, there was little change in the appearance or extent of the areas from which the paint had been removed in the sections of the steel transition elbow. Apparently the chipping and grinding of the weld beads at the joints had not reduced the discontinuities in the bounding surface sufficiently to eliminate cavitation at the inner curve of the elbow.

The pitting in the areas on the wall of the anchor block immediately below the elbow exit had deepened and increased slightly in size, and another pitted

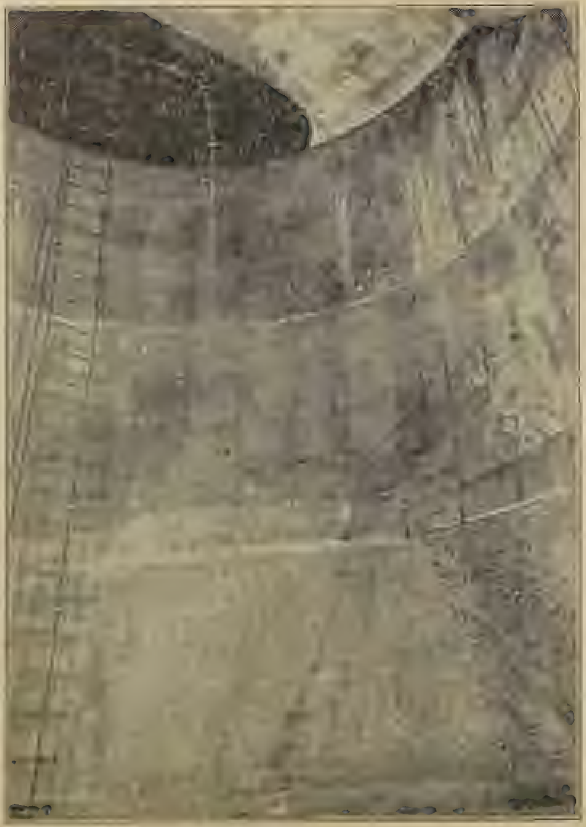


Fig. 57.—Pocked Areas in the Curved Surface at the Back of the Draft Tube

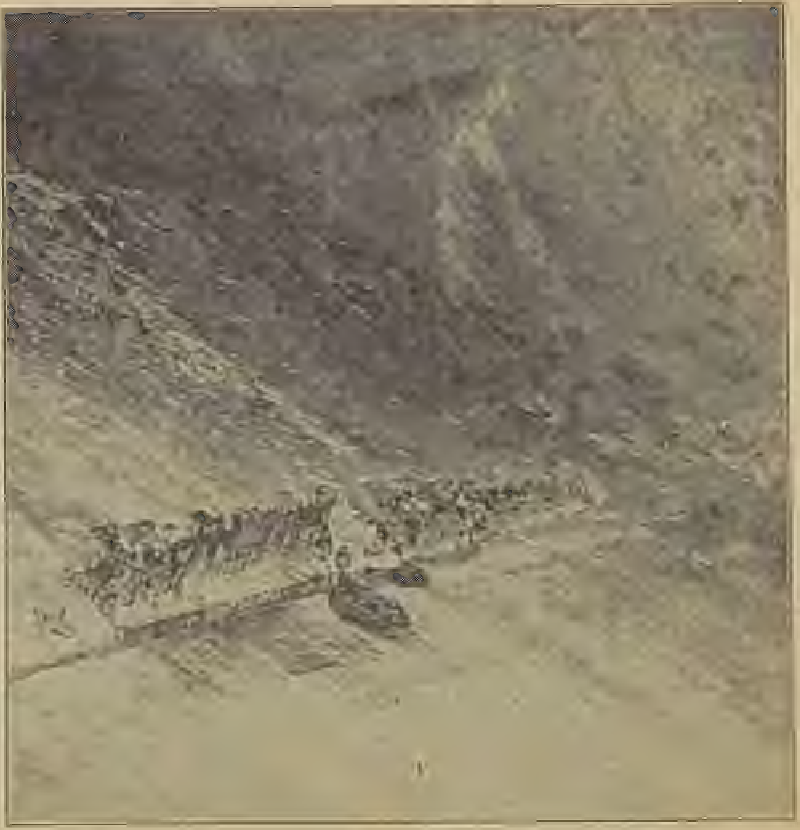


Fig. 58.—Patch in Draft-Tube Floor Torn From a Recess During the Initial Test

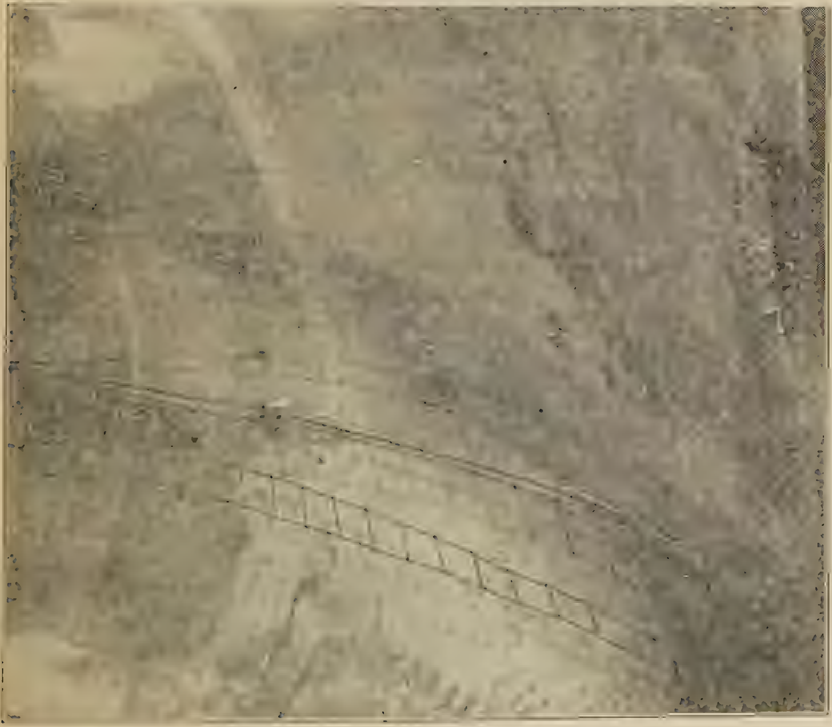


FIG. 59.—CAVITATION EROSION ADJACENT TO THE PATCH RECESS AFTER THE SECOND PATCH HAD BEEN TORN OUT

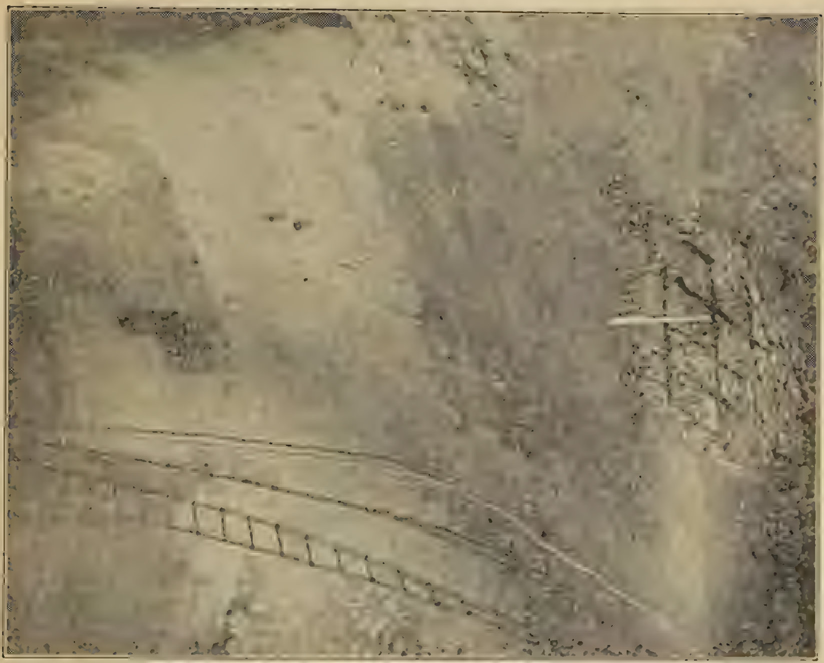


FIG. 60.—EXTENT OF CAVITATION EROSION BELOW THE ELBOW EXIT AND IN THE VICINITY OF THE PATCH RECESS AFTER OPERATING 23 HOURS AT FULL CAPACITY

area had formed directly below that which had occurred to the left of the center line and just below the exit of the elbow during the first 6 hours of operation (Fig. 60). The destructive action appeared to result from a local zone of cavitation formed when the flow near the boundary struck within the pocket just above and was deflected. This action is evidenced by the virtually undamaged surface remaining between the two eroded areas.

There was still no indication of cavitation between the back wall of the anchor block and the outlet flow, proving that the aeration groove below the elbow exit had served its purpose.

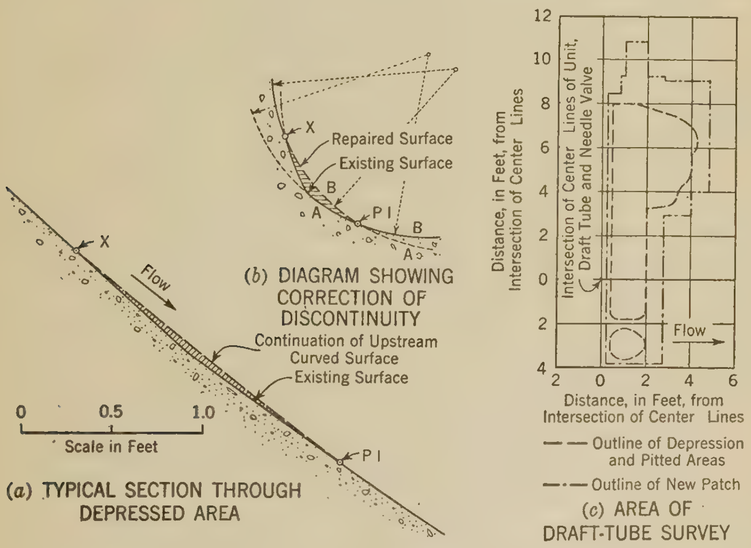
After the outlet had operated at full capacity for 23 hours, there was a band of shallow pits in the concrete surface along the top and on both sides of the recess from which the patch had been torn during the first 6 hours of operation. An area extending 5 ft below the downstream edge and approximately 18 in. to each side of the recess was pitted severely exposing several bars of reinforcing steel (Fig. 60). All the wire ties were missing from the intersections and the bars were battered severely where they had contacted one another during the test. The wear of the transverse bars occurred to the left of the natural intersection and it was not possible to force the longitudinal bars onto the worn areas by hand, indicating that tremendous forces had been involved. A short section was missing from one of the longitudinal bars, and the appearance of the remaining ends indicated a recent break due to severe vibration induced by the flowing water. The maximum depth of cavitation erosion immediately downstream from the recess was 18 in.

The source of the cavitation causing this damage was not apparent immediately, but seemed to have originated from either irregularities in the flow boundary or the tendency of the high-velocity jet from the elbow to deflect after striking the curved surface upstream. A survey of the concrete surface, adjacent to the pitted area, made several days after the test operations of August 7, 8, and 9, 1945, disclosed irregularities in the boundary surface to be the source. Longitudinal sections through the pocket and on adjacent surfaces showed a transverse discontinuity in the surface a few inches upstream from the recess (Fig. 61). The surface at this section receded suddenly to meet the concrete a short distance downstream. A maximum offset of ¾ in. was noted to the right of the patch recess. Apparently the collapse of the cavitation cavities formed in the zone just downstream from the inflection produced severe vibration with force changes that tore the patch from its recess. Once the patch was gone, a local condition, produced by the irregularity formed by this action, caused the severe damage immediately downstream.

Cavitation erosion, once started, will occur progressively where the curvature of the boundary surface is into the flow, since high-velocity flow striking the roughened surface will form small cavitation pockets, which will result in pitting the surface a short distance downstream. The two areas in the wall of the anchor block below and to the left of the exit of the transition elbow are evidence that this is the case. Later tests on the outlet indicated the same occurrence.

Some idea of the magnitude of the forces present in the draft-tube part of the outlet is indicated by the fact that two of the four cast gratings to the valves of the unwatering pumps, in the sides of the draft-tube piers (weight about 0.75 tons) were torn from their six $\frac{7}{8}$-in. anchor bolts, which were damaged severely.

After a study and discussion of the results of the 23-hr test, it was concluded that certain alterations and repairs should be made, and a test performed to determine if satisfactory operation of the 84-in. needle valve would result at partial opening. The areas on the inner curve of the transition elbow between the joints of the steel sections from which the paint had been removed by cavitation were cleaned and repainted. The holes eroded in the back wall of the concrete anchor block immediately below the exit of the transition elbow were repaired by chipping and patching. The craterlike hole below the recess from which the doweled patch had been torn, and the concrete surface adjacent to the discontinuity inducing the caviation (which damaged the surface along the upstream edge) and to each side of the recess, were chipped to form a large recess (Fig. 61). This recess was filled with a special concrete having negligible volume change and a screed used to change the form of the discontinuity to one which was not conducive to cavitation.


X
Flow
Continuation of Upstream Curved Surface
Existing Surface
P I
0
0.5
1.0
Scale in Feet
X
Repaired Surface
Existing Surface
B
A
P I
B
A
12
10
8
6
4
2
0
2
4
Distance, in Feet, from Intersection of Center Lines
Intersection of Center Lines of Unit, Draft Tube and Needle Valve
Flow
2
0
2
4
6
Distance, in Feet, from Intersection of Center Lines
—— Outline of Depression and Pitted Areas
—·— Outline of New Patch
(a) TYPICAL SECTION THROUGH DEPRESSED AREA

(b) DIAGRAM SHOWING CORRECTION OF DISCONTINUITY

(c) AREA OF DRAFT-TUBE SURVEY

Fig. 61.—Survey of Curved Surface at Back of Draft Tube After Test Operations

In general the discontinuity was changed from one receding from the flow to one encroaching upon it. The difference in the two types is illustrated in Fig. 61(*b*) by the intersecting arcs AA and BB, respectively.

Since the vibration of the outlet had been much less at smaller openings, it was planned to conduct the first operation of the next test at 40% valve opening, operating at this position for 24 hours or until damage was indicated. An inspection was then to be made, and the opening increased by increments to full capacity. The two 18-in. vents—one in the air manifold below the valve and the other in the dome of the anchor block—were to remain open. To assist in locating any damaged area in the vicinity of the new patch, a grid was painted on the floor of the draft tube.

Except for an occasional "bump" there was little vibration during the 24-hr test period. Apparently the back pressure from the tailwater damped the action, since severe pitting from cavitation action occurred on the inner curve of the elbow and in the anchor-block wall just below the elbow exit on each side of its center line. However, the damage was more extensive. Deep holes were eroded in the wall in an area extending several feet below the elbow exit. Intermittently, and in small quantities, air entered the vent in the dome of the anchor block during the test, indicating that the aeration groove at the back of the anchor and immediately below the exit was not receiving air continuously. This action caused the erosion to be more severe than that for the previous operation when the outlet discharged at full capacity for 23 hours.

In addition to the cavitation erosion below the elbow exit, there was a craterlike hole in the back curve of the draft tube just below its junction with the anchor block. Examination of this pocket disclosed that a patch, placed during the construction of the draft tube, had been torn from its recess, forming a local condition similar to that resulting when the larger patch, several feet downstream, was torn out in the previous test. Severe vibration, the result of fluctuating pressures, loosened the patch and it was then torn out by the high-velocity flow.

The repaired surface in the area of the larger patch, in which the discontinuity discovered at the end of the previous test had been made noncritical, remained intact with no damage whatever indicated. The alteration of the discontinuity to remove the offset, as shown in Fig. 61, had removed the source of cavitation and thus the cavitation erosion.

Testing, consuming a total of 9 hours, was continued at valve openings of 60%, 80%, and 100% without repairing the damaged areas. Inspections were made and photographs obtained after each operation.

The depth and extent of the cavitation erosion increased progressively in all areas except the upstream end of the craterlike hole in the back wall of the draft tube where the patch had been torn from its recess. The surface in this region remained unchanged during the test operations with the larger valve openings, indicating that the aeration groove in the anchor-block wall back of the transition elbow exit was receiving sufficient air to allow the jet to be directed farther downstream where it struck the roughened surface, deflected to form a cavitation zone, and caused erosion downstream.

From the tests described herein, it was concluded that the cavitation in the temporary needle-valve outlet at Grand Coulee Dam was induced mainly by local irregularities in the surface of the flow boundary and not by an over-all lowering of the pressure in the system. Attempts are being made to correct all local discontinuities so that satisfactory operation will result at openings between 60% and 90%. Air introduced by drilling two 36-in. diameter vent holes into the dome of the anchor block has served to reduce the vibration to a minimum, but alterations to the water passage below the valves are still indicated.

From the tests made thus far it has been observed that surface discontinuities of less than $\frac{3}{4}$ in. are sufficient to produce zones of cavitation in high-velocity flow. It would be interesting to know, for a given velocity, the maximum discontinuity that would not produce a cavitation zone.

The writer wishes to propound the theory that cavitation erosion will be progressive in instances where the boundary encroaches upon, or curves into, a stream of high velocity, until a crater of sufficient size is formed to reduce the velocity (and thus the energy) to a point where impact velocities are unable to produce a reduction in pressure that would induce cavitation. Furthermore, if the curvature is moving away from the flow boundary, it is possible that the cavitation erosion will reach a depth representing a sudden enlargement; and the collapse of cavitation cavities, if any, will occur in the fluid before the boundary surface is contacted. The depth of the erosion in this case, of course, will be much less than that for the encroaching boundary.

Fred Locher,[12] Esq.—It is customary to consider cavitation as a phenomenon created by a defect in the design of the boundary. This theory is not always true and (as has been stated by Professor Vennard, who mentions the fact briefly under the heading, "Formation of the Cavity") cavitation can also result from the mixing of two jets. A typical example of this type of cavitation occurs in a jet pump, which is shown schematically in Fig. 62. The high-velocity jet from the nozzle impinges on the center of a low-velocity jet called the "driven water." The two masses combine and in so doing produce a tremendous turbulence and a large number of rapidly whirling vortices whose speed of rotation varies with the relative velocities of the two jets. The vortices are widely dispersed throughout the mixing zone, most of them being dissipated in the fluid. However, a number of the vortices (the most rapidly rotating and dangerous ones as far as cavitation and pitting are concerned) move to the boundary along the line forming the junction between the mixing zone and the driven water. When this condition occurs in a region where the average pressure in the fluid is subatmospheric, but not necessarily sufficiently low to cause cavitation, vapor pockets form at the center of the vortices. They collapse as they move to the boundaries, and at the same time travel downstream to a region of higher pressure, causing severe pitting in the region indicated in Fig. 62.

The writer conducted a series of tests on a jet pump to determine its efficiency and to find a method for reducing cavitation in the mixing tube. It was found that cavitation could be eliminated to some extent for certain operating ranges by proper design. However, the pump was controlled by a needle in the nozzle to permit operation under widely varying conditions in the field.

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[12] Engr., U. S. Bureau of Reclamation, Denver, Colo.

Under these circumstances it is not possible to operate throughout the entire range without cavitation. Piezometers in the mixing tube of the model pump and on the downstream face of the nozzle did not indicate pressures lower than one half of an atmosphere; but even so, operation with these pressures produced severe cavitation in the mixing tube. Further study revealed that pitting occurred at, and immediately downstream from, the point where the mixing zone first came in contact with the walls of the mixing tube (Fig. 62).

When it became apparent that cavitation and pitting in the mixing tube were caused by vortex action, the problem of reducing or eliminating cavitation still remained unsolved. Aeration, as generally applied, would have reduced the efficiency of the jet pump to practically zero or would have made it inoperative. A change in the boundary of the mixing tube would have had little if any effect on the ultimate results because the change would only produce a different point at which cavitation would occur. The redeeming feature, however, was the fact that a very small volume of air, admitted at the proper place, eliminated cavitation as effectively as the admission of a large quantity to other regions. The unique nature of cavitation in the jet pump required that an air vent be placed at the downstream face of the nozzle as shown in Fig. 62. The effects of this air vent were gratifying; there was no appreciable reduction in efficiency and the pitting was eliminated in the critical operating ranges. Similar vents have since been installed on a number of jet pumps with satisfactory results.

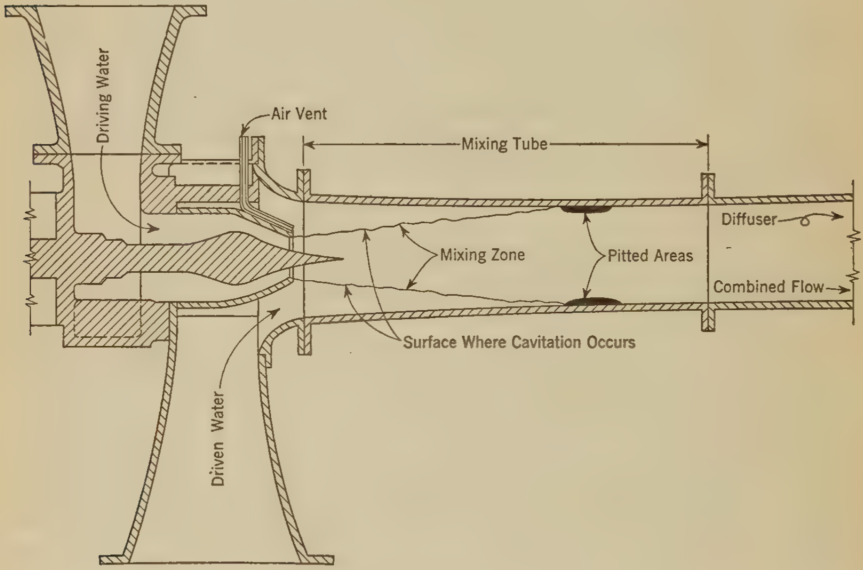
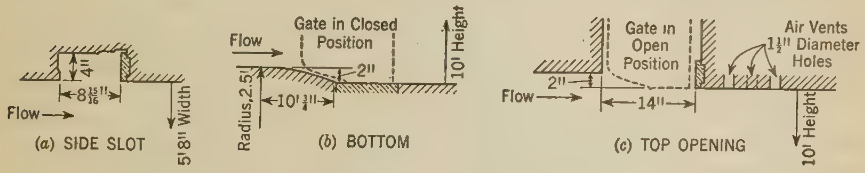


Fig. 62

Carl E. Kindsvater,[13] Jun. ASCE.—The Hydraulic Research Committee and the authors of the Symposium are to be congratulated on initiating this discussion of the cavitation problem as it affects civil engineers. The fact that most of the existing literature on cavitation is found in the journals of the mechanical engineers and physicists does not reflect the great accumulation of knowledge of the subject among civil engineers. As damage due to cavitation frequently appears to be an indication of faulty design, some engineers have apparently been reluctant to publicize their experiences. It is only by sharing different experiences, however, that a collective knowledge of the subject can be compiled.

A good share of the material contained in the several papers which compose the Symposium is concerned with design and operating experiences on spillways, stilling basins, and outlet conduits for high dams. It is the purpose of this discussion to present similar information from recent experiences of the Little Rock Engineer District, U. S. Corps of Engineers.

Outlet Conduits, Norfork Dam.—Norfork Dam (72), in Arkansas, completed by the Corps of Engineers in 1944, is a concrete gravity dam with eleven rectangular, gate-controlled outlet conduits located through the spillway monoliths. In many respects, Norfork is quite similar to Norris Dam, described by Mr. Hickox. In the period since February, 1944, when the reservoir was filled, a total of 1,214,000 acre-ft of flood storage has been passed through the conduits and 283,000 acre-ft has been passed over the spillway, for a recorded total of 1,497,000 acre-ft of water. In addition to this recorded flow since filling the reservoir, a considerable volume of water passed through the conduits and stilling basin during the construction period. Thus, it is estimated that approximately 2,000,000 acre-ft of water has been passed through the stilling basin in the life of the project. Since filling the reservoir, the eleven conduits have been operated more or less alternately, although seldom more than four at any one time. The longest aggregate period of operation of any one conduit was 60 days, and the average period of operation has been approximately 26 days per conduit. All conduits have been operated exclusively at full gate opening and largely at heads between 175 ft and 190 ft. However, during the construction period the head on the conduits was low and, due to the conduit slope of 1.5 in 10, conditions favored the development of subatmospheric pressures in the intake sections. Thorough examinations have disclosed no cavitation damage in the Norfork conduits.



(a) SIDE SLOT (b) BOTTOM (c) TOP OPENING

Fig. 63.—Norris Dam Conduit Gate Slots

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[13] Associate Prof., Georgia School of Tech., Atlanta, Ga.; formerly Associate Hydr. Engr., U. S. Engr. Office, Little Rock, Ark.

The cavitation which apparently was experienced in the Norris Dam sluices might be classified in accordance with Professor Vennard's suggestions as (1) local cavitation and (2) general cavitation. Local cavitation ordinarily occurs in regions of excessive flow curvature such as might be caused by sharp breaks in boundary lines. Thus, local cavitation must have contributed to the causes of the pitting in the conduit liner downstream from the Norris sluice gate slots. Local cavitation is often susceptible to prevention or cure by means of simple "streamlining." A comparison of Fig. 63 with Fig. 64 reflects an attempt in the design of the Norfork gate slots to avoid some of the reported difficulties at Norris Dam. The outstanding differences in the two designs are the simple streamlining measures adopted at Norfork. The 1:12 bevel provided downstream from the gate slots and air-vent header was based on the experiments by H. A. Thomas, M. ASCE, described by Mr. Harrold under the heading, "Conduit Tests," and shown in Fig. 13(*b*). Much has since been learned about this detail of design, as will be described subsequently.

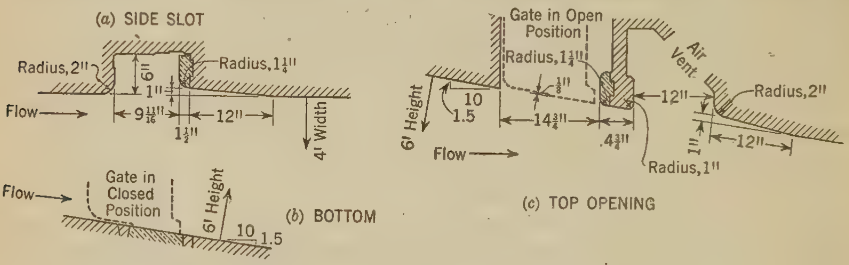


Fig. 64.—Norfork Dam Conduit Gate Slots

Damage has been reported to the upstream portions of conduits at a number of different dams. In this case, too, the damage may be attributed to local cavitation. The inlet curves at Norfork were based on flow-net studies, investigations of free jet curves, and the results of cavitation model studies for several similar structures. In view of some of the earlier experiences and model investigations, notably those described by the authors, this feature of design is no longer critical. Although the inlet portions of the Norfork conduits have not been examined, there have been no indications of difficulties such as those experienced at Norris and Madden dams.

General cavitation usually occurs when the average pressure at any cross section of flow approaches the vapor pressure of the liquid. In the case of an enclosed hydraulic turbine, general cavitation results when the wheel is set too high in relation to the tailwater level. Similarly, conditions favoring general cavitation must have occurred in the original, unconstricted Norris conduits because of their high setting with respect to the hydraulic grade line. It should be emphasized that in many instances local cavitation may or may not develop, depending on the average pressure on the entire system at the critical cross section; or, in other words, depending on the tendencies toward general cavitation at that section. At Norris, cavitation might not have occurred near the gate slots and below numerous boundary irregularities had the hydraulic grade line been higher, or had the upstream portions of the conduit been set lower in relation to the grade line. The only "post-mortem" remedy for the cavitation experienced in the Norris conduits was (as described by Mr. Hickox) the addition of outlet constrictions which had the effect of raising the grade line relative to the original setting. In this connection, it is interesting to compare typical hydraulic grade lines for Norris (Fig. 46) and Norfork (Fig. 65), for the case of free discharge with the slide gates fully open. At Norfork, with the pool at El. 561, the measured flow *Q* per conduit is 2,200 cu ft per sec and the computed average velocity *V* is 92 ft per sec. Field measurements at Norfork indicate that the grade line is approximately 15 ft above the roof of the conduit at the downstream gate slot. Of this 15 ft, nearly half is accounted for by the fact that the conduit slopes continuously downward from the inlet rather than extending horizontally to a point beyond the downstream end of the gate castings. Furthermore, the conduits at Norfork have smaller cross-sectional areas than those at Norris and, for corresponding velocities, have a greater friction gradient.

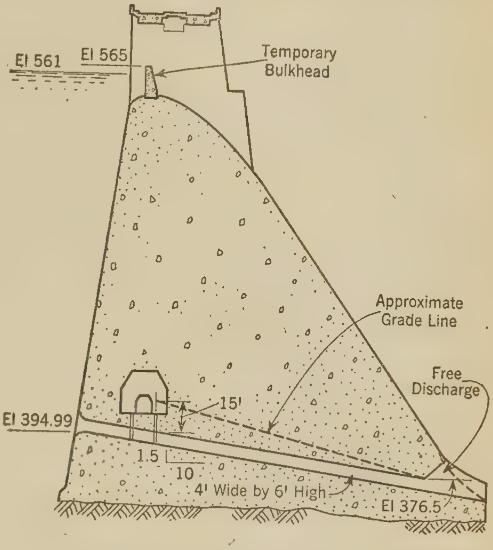
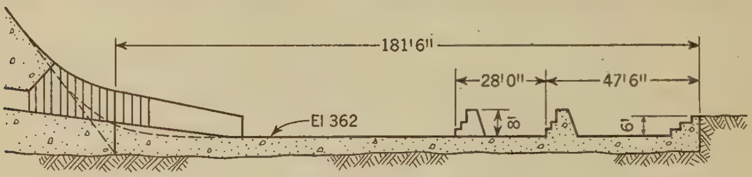


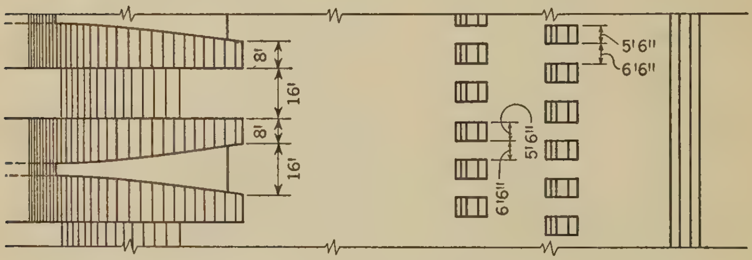
FIG. 65.—SECTION THROUGH OVERFLOW MONOLITH AND OUTLET CONDUITS, NORFORK DAM

The grade line shown in Fig. 46 for the unconstricted conduits at Norris does not correspond to that reported as the most frequent condition of operation —that is, at gate openings between 94% and 96% of full gate. For the latter condition, as for all part-gate openings, the grade line downstream from the gate will drop below that for full gate opening. Even more important, however, is the fact that, as demonstrated by field tests at Norfork, the water discharging under the gate entrains a considerable amount of air at openings as great as 90% of full gate, and for all smaller gate openings the average pressure on the roof of the conduit is reduced in inverse relation to the quantity of air supplied through roof vents (Fig. 64) or, under certain conditions, through the outlet portal.

Although it has been the usual procedure in the Little Rock District to limit continuous operation to full gate openings, it was considered important to investigate the part-gate conditions of operation at Norfork in order to determine whether the air-vent capacity provided in the design was sufficient to prevent the occurrence of excessively low pressures during the opening and closing operations. Accordingly, a pressure tap was installed in the air vent for the upstream (emergency) gate of three different conduits, and openings in the roof of the conduit at a point just below the downstream (service) gate of all eleven conduits were utilized for measuring air pressures. During these tests it appeared from observations of the outlet portals that the conduits were completely filled with water and spray when the gates were opened more than 1 ft. It was also disclosed that subatmospheric pressures occurred at the roof piezometers, downstream from the gates, for all openings up to about 5.5 ft (that is, 90% of full gate opening) and that the greatest vacuum occurred when the gates were opened about 5 ft. It appears, therefore, that extended periods of operation at 94% to 96% gate opening at Norris may have been primarily responsible for the damage to parts of the conduits downstream from the partly opened gates. It is acknowledged, of course, that this method of operation did account for a higher grade line upstream from the gate and did thereby relieve undesirable conditions which apparently existed near the inlet. The prototype tests at Norfork also revealed that, apparently, as at Norris, vents in some of the conduits, were partly obstructed, and for these conduits in particular it would be extremely hazardous to operate for any length of time at part-gate openings.



(*a*) SECTION



(*b*) PARTIAL PLAN

FIG. 66.—GENERAL LAYOUT OF STILLING BASIN, NORFORK DAM

Based on experiences at Norfork as compared to Norris, it might be concluded that the answer to cavitation difficulties in outlet conduits lies in designing for a high grade line in relation to critical portions of the conduit. Local boundary streamlining is certainly of value, but is frequently a difficult matter of compromise with the mechanical designer. The primary consideration is to avoid general cavitation tendencies due to a high setting or, as just described, due to part-gate operation. The provision of adequate air-vent capacity is important, too; but, as demonstrated both at Norris and Norfork, the vents may become obstructed. It might well be the theme of this discussion to emphasize the normal departure of completed structures from the plans produced by the design office. Too much emphasis cannot be placed on the importance of carefully prepared specifications and strict inspection in order to insure that the designer's ideas are carried over to the ultimate structure.

Baffle Piers, Norfork Stilling Basin.—The stilling basin for the Norfork Dam spillway consists of a level concrete apron with two rows of stepped baffle piers and a stepped end sill, as shown in Fig. 66. Because of the war emergency need for power when the project was completed in 1944, the customary stilling basin cleanup was delayed in order that the cofferdamming and unwatering operations would not interfere with the production of power. In November, 1945, the powerhouse was shut down for repairs to the generating unit; advantage was taken of this opportunity to clean and inspect the stilling basin. As noted previously, under the heading, "Outlet Conduits, Norfork Dam," it is estimated that approximately 2,000,000 acre-ft of water had passed through the stilling basin in the period from the initial diversion operations until the time of the inspection. When the basin was unwatered, a large quantity of debris was revealed on the apron. The debris was composed of well-worn boulders and smaller stones, steel rods, pipe of all sizes, and a miscellaneous collection of battered metal scrap. Some indiscriminately scattered baffle piers, particularly in the upstream row, had suffered considerable damage. Fig. 67(*a*) shows typical damage, and Fig. 67(*b*) shows one view of the most seriously damaged pier.

In contrast to the experiences reported in the Symposium it has been concluded that the damage to the Norfork baffle piers cannot be charged to cavitation. The photographs serve to support this conclusion. It was noted that the damaged piers were located without respect to conduit location and that the greatest damage occurred on the front or upstream faces of the piers. It is believed that the piers were eroded by the recirculated debris in the basin, and that the irregular and inconsistent nature of the damage was principally a product of chance and secondarily a result of variations in concrete quality. Again it is demonstrated that the designer's efforts are of no avail in view of practical considerations over which he has no control. This conclusion is pertinent to Mr. Harrold's discussion of streamlined baffle piers. It is acknowledged that baffle piers are advantageous in some instances, and that they may logically serve as an added safety factor in stilling-basin design. However, such vulnerable devices should be used advisedly; and the safety of the dam should not depend upon their permanence. It may be expected that damage such as that experienced at Norfork would be alleviated by the early removal of debris collected during the construction period. The recirculation of a certain quantity of stone will be unavoidable, however, unless the river channel is cleared of all loose rocks for a considerable distance below the stilling basin.

Gate-Slot Model Tests.—The Bull Shoals project, proposed for construction on the White River in Arkansas, would include a dam somewhat higher than that at Norfork. Rectangular outlet conduits similar to those at Norfork except that they are 9 ft high, would be located through the overflow monoliths. As a part of the preliminary investigations for the design of Bull Shoals Dam, conduit gate slots of the Norfork type were investigated in a model study of

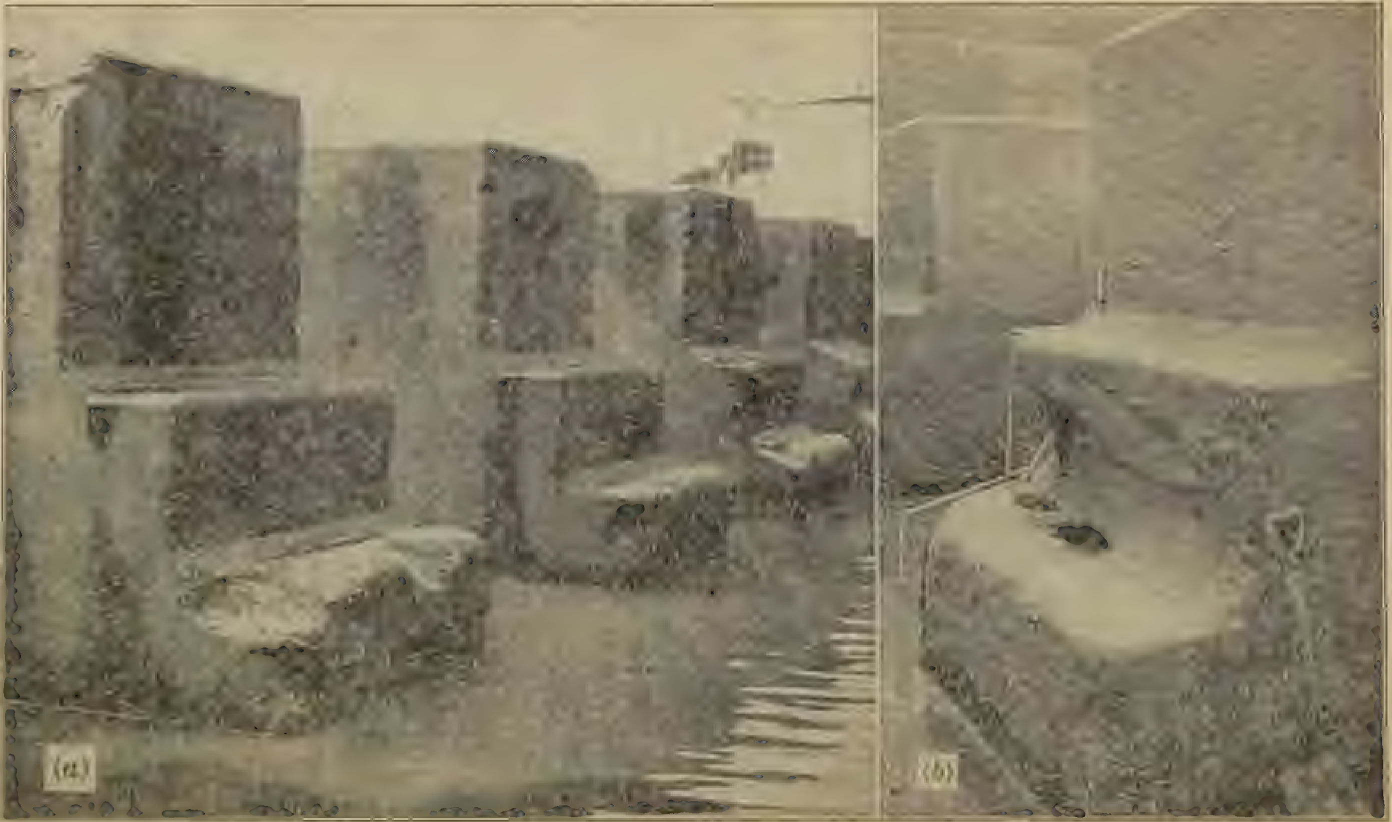


FIG. 67.—DAMAGED BAFFLE PIERS IN STILLING BASIN, WILSON DAM (ALA.): (a) TYPICAL DAMAGE; (b) MOST SERIOUSLY DAMAGED PIER

cavitation tendencies conducted at the U. S. Waterways Experiment Station, Vicksburg, Miss. The model conduit was constructed to a scale of 1:6, and two gate-slot designs were tested. The basic, nonstreamlined slot is shown in section in Fig. 68(*a*). The Norfork-type slot is shown in Fig. 68(*b*). Each model was equipped with a large number of small piezometers, and for each a series of tests were run for a wide range of velocities. The data consisted largely of open manometer readings of average pressures at each piezometer. The pressure cell-oscillograph device described by Mr. Harrold gave inconsistent results, and these data were discarded.

The principal results of the gate-slot model tests are summarized in Fig. 68, which shows dimensionless pressure coefficients, C_d, for piezometers on the boundary of the gate-slot models. Values of C_d were computed from the relation,

$$C_d = H_d \div \frac{V^2}{2\,g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which H_d was taken as the measured difference in head between the various gate-slot piezometers and an arbitrary upstream control piezometer. The plotted points in Fig. 68 represent test velocities ranging from 42 ft per sec to 94 ft per sec. Curves drawn through these points describe the normal pressure variations in the vicinity of the slots.

As an example in the use of Fig. 68, let it be required to determine the maximum pressure drop in the vicinity of both nonstreamlined and streamlined gate slots, referred to the normal friction grade line. In the first case, the maximum departure of the boundary pressure coefficient from the friction coefficient occurs near the downstream corner of the gate slot and has a value (Fig. 68(*a*)) of approximately 0.127. It follows from Eq. 15, therefore, that the maximum pressure drop at the boundary, referred to the normal grade line, would be $0.127\left(\frac{V^2}{2\,g}\right)$, where $\frac{V^2}{2\,g}$ is the computed average velocity head. For the estimated maximum discharge, V is 94 ft per sec or $\frac{V^2}{2\,g}$ is 137 ft, and the resulting maximum pressure drop for the case of the nonstreamlined slot is 17.4 ft. Similarly, for the case of the streamlined slot, the maximum pressure drop is only 8.2 ft. It should be apparent from these examples that the elevation of the normal grade line (that is, the magnitude of the average pressure in the conduit at the critical cross sections) is the primary index of whether or not a local pressure drop will result in cavitation. Thus, even in the first of the foregoing examples, the 17-ft pressure drop would not be expected to result in cavitation unless the grade line was extremely low. Combination of a low grade line with such a marked local pressure drop must have been the case in the Norris sluices. It should be noted, of course, that the data included in Fig. 68 are based on average pressures rather than on instantaneous pressures such as it was hoped could be obtained with the pressure-cell device. The diagrams, therefore, tell only a part of the story, for it is recognized that momentary pressure fluctuations far exceed the damped variations shown by the manometer readings. In order to have some assurance that cavitation damage will not occur as a result of the most extreme pressure fluctuations, the manometric pressure data from the model must be compared to similar data on designs

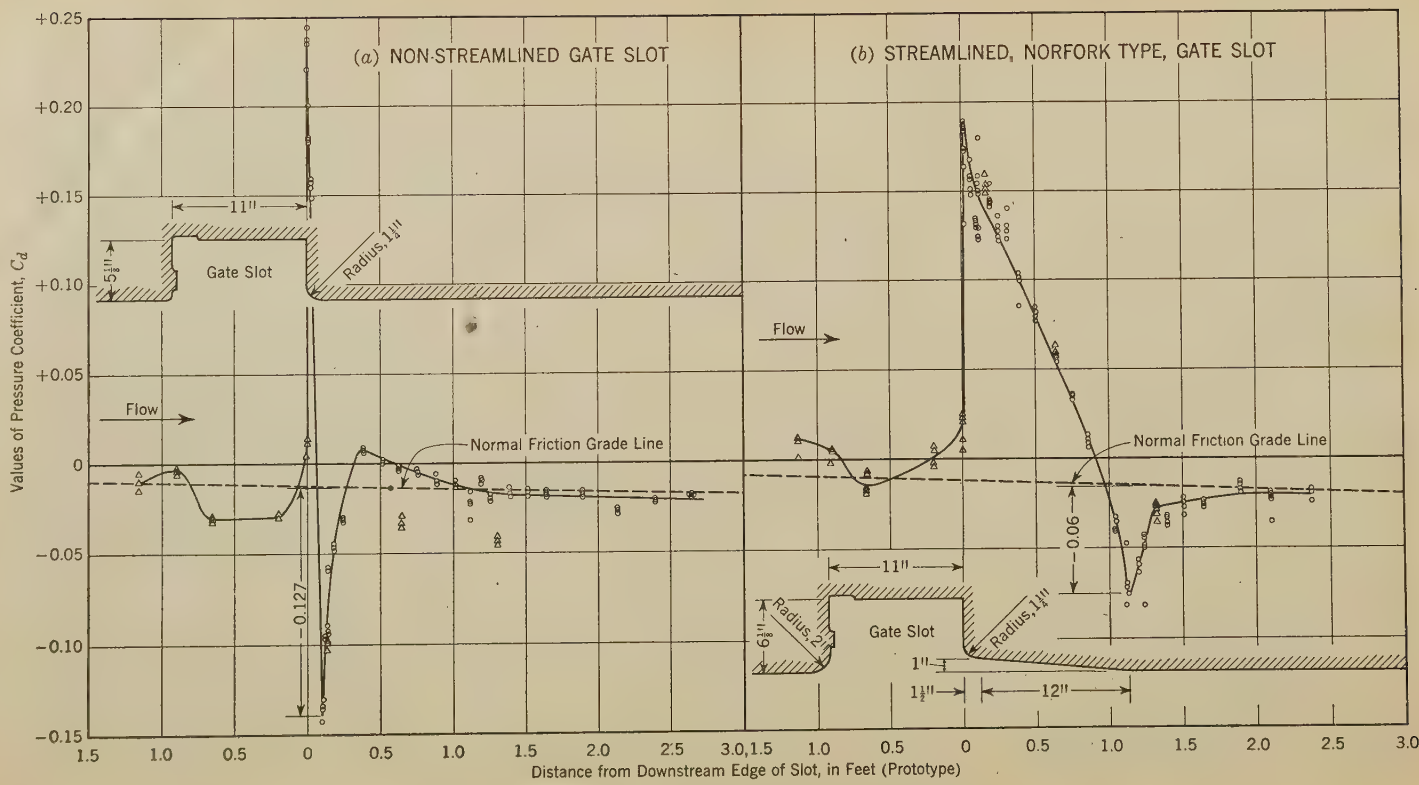


FIG. 68.—PRESSURE VARIATION IN THE VICINITY OF GATE SLOTS

which have been checked in the prototype. This is believed to be a valid and quite informative alternate procedure as compared with the infrequent opportunity to contrast a variety of designs in the prototype. A similar procedure has long been used as the most positive means of establishing the cavitation characteristics of Francis turbines.

Fig. 68(*b*), compared with Fig. 68(*a*), not only shows a marked benefit from the 1:12 bevel on the downstream edge of the gate slot but also suggests a need for further streamlining near the downstream end of the bevel. Practical considerations make it undesirable to specify a modified curve at this point, but in the case of the gate castings at Bull Shoals Dam the intersection of the bevel with the parallel side walls will be "rounded off" by grinding.

It seems reasonable to expect that the pitting of spillway gate slots described by Messrs. Harrold and Warnock would be susceptible to treatment such as that developed for the Norfork conduit gate slots. As shown in Fig. 68, local pressure variations are functions of the velocity head, and comparatively high velocities may be attained in regions near the bottom of spillway gate slots. It would appear that appropriate streamlining measures, rather than deflectors designed to eliminate the slot vortex, are the logical solution for the difficulties described. Mr. Warnock's discussion of experiences with needle-valve design aptly demonstrates that adequate streamlining cannot always be achieved by estimate.

An interesting sidelight to the tests on the gate-slot models was an accidental indication of the flow pattern in the vicinity of the slot, shown in Fig. 69. Quite unintentionally on one occasion when the model was being assembled, a small particle of heavy yellow grease was left on the top of the conduit upstream from the transparent cover plate over the test section. Several minutes after the model was adjusted to a steady velocity of approximately 38 ft per sec (model), observers spotted the grease particle as it "unrolled" in an unwavering, needle-sharp path (much smoother than the freehand lines in Fig. 69). Only once did the path divide, and in this case it joined a less sharply defined pattern being evolved in a similar manner within the slot itself. Obviously, the path lines shown in Fig. 69 cannot be typical for the region in the center of the conduit and removed from the boundary and corner effects. It is interesting, however, to have this unusual indication of the flow pattern in the vicinity of the slot. Comparison of Fig. 69 with Fig. 68(*b*) shows that, as would be expected, the extremely high pressure measured on the downstream corner of the gate slot is directly opposite a point of sharp convex curvature in the indicated streamline.

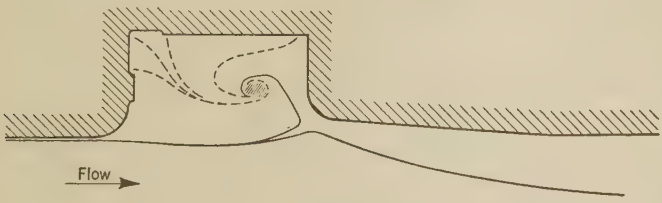


Fig. 69.—Indicated Flow Pattern in the Vicinity of a Gate

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Stilling Basin Studies.—In preliminary design studies for the Bull Shoals project it was assumed that a stilling basin similar to that at Norfork could be used. When a model of the proposed basin was investigated at the U. S. Waterways Experiment Station, it was found to operate in a very satisfactory manner for all spillway flows. However, the same setting, with the apron founded on the estimated firm rock line, did not provide sufficient tailwater depth to produce satisfactory stilling when a relatively few conduits were opened alone; that is, with no release from the powerhouse or spillway. As the tests proceeded, it appeared that the baffle-pier type of basin would have to be lowered about 10 ft below the firm rock line in order to insure satisfactory performance for all conditions of conduit operation. An alternate type of basin, a stepped apron similar to that developed by the Tennessee Valley Authority for Cherokee Dam, was then investigated. This basin involved an average maximum lowering of 7.5 ft below the firm rock line near the toe of the dam. The new design eliminated the baffle piers entirely, thereby resulting in a considerable saving in steel and concrete. As finally developed, the stepped apron proved to be satisfactory for all conditions of operation. It is believed to be a thoroughly practical design, well suited to the tailwater conditions that prevail at Bull Shoals. A comparison of the two types is shown in Fig. 70.

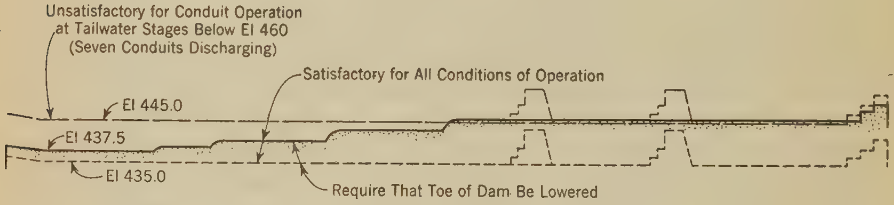
Unsatisfactory for Conduit Operation at Tailwater Stages Below El 460 (Seven Conduits Discharging)
Satisfactory for All Conditions of Operation
El 445.0
El 437.5
El 435.0
Require That Toe of Dam Be Lowered

FIG. 70.—STILLING BASIN STUDIES, BULL SHOALS DAM

Since the steps of the Bull Shoals apron design, like the baffle piers at Bluestone Dam described by Mr. Harrold, are near the toe of the dam where they receive a minimum benefit from tailwater cover, cavitation pressures may be expected to develop below the top corner of each step unless they are streamlined. Although pitting tendencies near the sharp edges of the steps might be expected to be self-healing, it was found that an elliptical step such as that shown in Fig. 70 would eliminate zones of negative pressure which had been observed below sharp-edged steps. Mr. Harrold points out that, in the case of the Bluestone tests, the streamlined baffle piers were less efficient as stilling devices than square-edged baffles. Similarly, the streamlined steps for Bull Shoals were found to sacrifice some quality of performance in comparison with preliminary tests on square-edged steps.

The writer is indebted to G. R. Schneider, Assoc. M. ASCE, and W. M. Mulholland of the U. S. Engineer Office, Little Rock, Ark., and to Fred R. Brown, M. ASCE, and associates of the U. S. Waterways Experiment Station, Lower Mississippi Valley Division, Vicksburg, for their contributions to the information from which this discussion was drawn.

E. R. VAN DRIEST,[14] ASSOC. M. ASCE.—The theoretical and practical aspects of this Symposium have stimulated, in the writer, a sharp interest in the phenomenon of cavitation.

In his desire to see the actual cavitation in action, the writer made some simple tests on the flow of water through a glass venturi tube, with the following results.

A glass tube was heated and drawn into a venturi tube; after the tests it was found that the diameter of the exit and throat were 0.3967 in. and 0.1114 in., respectively. This tube was then connected by a rubber hose to the cold-water faucet in a sink, and the flow was increased until cavitation appeared strongly. After an attempt to destroy the glass by five days of such action, the writer found no destruction but, instead, a discoloration of the diffuser of the tube, which discoloration was readily washed out by an acid. The first result, therefore, was that chemicals in solution in the water entering the cavitated region can come out of solution and be deposited in concentrated form on the solid boundaries adjacent to the region. The effect could be due to supersaturation of the chemicals upon vaporization of the liquid. This result may have some consequence in the damage of surfaces; in other words, a corrosive effect may be encouraged under such conditions in addition to the well-known erosive effect.

The next result observed is by far the more interesting because it shows that tension can be sustained in a flowing fluid. Pure tension has been observed under static conditions by various experimenters. H. H. Dixon (73) obtained a tension of nearly 160 atmospheres; and the writer, after making his own observations, found that S. Skinner and F. Entwistle (74) had concluded in 1915 that tension can occur at the throat of a constricted tube although the discharges and dimensions of the tube were not given in their paper. Continuing with the writer's experiments, the succession of events leading to cavitation must be described: Water was forced through the tube very gradually until suddenly the flow would burst into terrific cavitation. Then the discharge could be diminished appreciably before the cavitation disappeared. Such cavitation always occurred about ½ in. downstream from the minimum constriction. These circumstances naturally led the writer to suspect the existence of tension in the liquid at the throat. However, it was possible to obtain cavitation at the throat when the inlet valve was adjusted a certain way, so that apparently extremely small bubbles of air were created (which could not be seen, since the water looked perfectly clear) that formed nuclei for cavitation when the pressure was brought low enough.

The appearance of the cavities downstream from the throat can be ascribed to the sudden rupture of the fluid due to the presence of free vortices (75). The zone of separation of the fluid from the wall is an opportune place for the occurrence of such vortices as there they are not too restricted by the wall. The free vortex indeed is the one simple type of motion that is easily generated and has centrifugal forces sufficient to rupture the fluid. Vorticity at the throat of the tube cannot readily fly into free vortices due to the stabilizing effect of the wall.

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[14] Asst. Prof., Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

Since tension in the fluid cannot be measured with a manometer, it was necessary to compute the average tensile stress. To use the energy equation for computation, the head-loss coefficient (K_L) had to be known. Therefore, a measurement of pressure was made at the throat at the threshold of cavitation—when cavitation could be made to appear at the throat. At this threshold, the absolute pressure was found to be 30 mm Hg with a discharge of 13.8 lb per min. Such a pressure was expected since a pressure as low as vapor pressure (11 mm Hg) would not necessarily have to be reached to allow the small bubble nuclei to expand and form cavities. In the formula—

$$h_L = K_L \frac{(V_1 - V_2)^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

—the constant K_L was found to be 0.335.

Application of this value of K_L to the maximum flow of 16.5 lb per min just before the fluid ruptured gave 770 lb per sq ft in tension; and application to the minimum flow of 14.5 lb per min when cavitation downstream just disappeared yielded a value of 118 lb per sq ft in tension.

The foregoing demonstration proves that a fluid does not necessarily cavitate when the pressure is reduced to vapor pressure or even below—not to mention tension. Furthermore, one should be cautious of the notion that water always boils at 100° C. Water can be superheated well over 100° C without boiling; in fact, the temperature of ordinary boiling water is always greater than the true boiling temperature (75)(76). Also, water can be saturated with a gas at a pressure of 100 atmospheres and then reduced to 1 atmosphere without producing bubbles (75). Carbonated water will not effervesce unless agitated. All this leads to the conclusion that either the fluid is absolutely pure or gas is occluded in the liquid in such infinitely small bubbles that the surface tension forces are so great as to hold the fluid together. When cavitation occurs, either the pressure difference across the bubble wall must become sufficiently great to expand the bubble, or a free vortex with its tremendous centrifugal action must appear to produce bubble expansion or rupture of the fluid.

It seems worthwhile at this point to investigate the mechanics of the bubble in more detail. Fig. 71 shows the bubble and the forces holding it in equilibrium. The pressures p_v and p_a are the partial pressures of the vapor and air, respectively, within the bubble; p_o is the pressure in the fluid outside the bubble; σ is the surface tension of the liquid, and r is the radius of the bubble. For equilibrium,

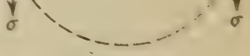

Fig. 71

$$(p_v + p_a) - p_o = \frac{2\sigma}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Hence, the bubble will expand when $(p_v + p_a) > \frac{2\sigma}{r} + p_o$. Since superheating of the fluid is coincidental with boiling (certainly near the walls of the vessel), it follows that the additional energy must be necessary to overcome the surface tension of the fluid. High superheat is required at the boundaries where the bubbles grow, whereas low superheat can maintain the size of the bubbles once they begin to rise and become larger, when p_a and σ are not effective.

Because surface tension, diameter of bubble, and viscosity have an effect on the growth of bubbles or rupture of the fluid, they should be included in a thorough dimensional analysis of any cavitation problem. For example, in the case of Fig. 17:

$$H = f(p_o, p_v, \sigma, r, \mu, \rho, g, b, c, \cdots) \ldots\ldots\ldots\ldots\ldots (18)$$

in which b, c, $\cdots$, are linear dimensions. In Eq. 18 p_a is not included because it can be determined from Eq. 17. Dimensional analysis then yields:

$$\frac{H}{b} = \phi\left(\frac{p_v}{\rho\, g\, b}, \frac{p_v}{p_o}, \frac{\sigma}{b\, p_v}, \frac{\rho\, p_v\, b^2}{\mu^2}, \frac{r}{b}, \frac{b}{c}, \cdots\right) \ldots\ldots\ldots\ldots (19a)$$

If it is now assumed that vortex action is the cause of the formation of bubbles and that the bubbles when formed are of appreciable size, then the products $\frac{\sigma}{b\, p_v}$ and $\frac{r}{b}$ will disappear from the function. Furthermore, the viscosity parameter $\frac{\rho\, p_v\, b^2}{\mu^2}$ will probably cover a large range during the production of strong vortices so that it too may sensibly not affect the function. This leaves only the difference between the vapor pressure and the pressure outside the bubble as effective in further enlargement of the bubble into a cavity. Hence,

$$\frac{H}{b} = \phi\left(\frac{p_o - p_v}{\rho\, g\, b}, \frac{b}{c}, \cdots\right) \ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

The parameter $\frac{p_o - p_v}{\rho\, g\, b}$ in Eq. 19b leads immediately to Eq. 2a or 3a since

$$\left(\frac{p_o - p_v}{\rho\, g\, b}\right)_m = \left(\frac{p_o - p_v}{\rho\, g\, b}\right)_p \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

whence (see Eq. 3a): $p_{om} = p_v + L_r\, p_{op} - L_r\, p_v = L_r\, p_{op} + (1 - L_r) p_v$. This procedure is essentially the same as that for the pressure cell method in which the experimenter predicts what could happen to the bubbles if he allowed a low enough pressure to exist.

In conclusion, the writer hopes that these remarks will help to clarify the physics of the cavitation phenomenon. Certainly with the development of higher dams and high-velocity structures demanding precise design, the hydraulic engineer will be stimulated to further research in this interesting field.

Duff A. Abrams,[15] M. ASCE.—The third Symposium paper, by Mr. Warnock, consists mostly of a recital of some of the minor ailments that have been encountered by the Bureau of Reclamation, U. S. Department of the Interior (USBR), since about 1910, in the operation and maintenance of some of its hydraulic structures. The treatment of a "Boulder Dam Spillway Tunnel" could have been expanded to advantage, in proportion to its importance.

Boulder Dam Spillway Tunnel.—The considerable damage to a Boulder Dam spillway tunnel, discovered on December 12, 1941, is attributed to "cavitation" by Mr. Warnock, although it is apparent that this explanation is highly speculative. The same damage has been attributed by earlier USBR writers to "erosion" (28) and to "concrete wear" (77). The only evidence given to support this claim for "cavitation" is: "The misalinement is defined by the position of the rope in Fig. 44." In the writer's opinion, a photograph of a rope is not very satisfying as scientific evidence; especially in view of certain other considerations:

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[15] Cons. Engr., New York, N. Y.

1. The "misalinement" shown may have been due to any one of a number of causes that had nothing to do with the damage to the concrete lining.
2. The "misalinement" may have occurred after the damage.
3. An official report of the USBR published in 1938 stated that there was no misalinement in the Boulder Dam spillway tunnels (78) (78*a*):

> "* * * The Boulder Dam spillway system is designed with the expectation of obtaining practically streamline flow in the lower tunnel sections where maximum velocities occur. Comprehensive specification provisions for accuracy of alinement and rigid control of concrete manufacture and placement help to insure this.
>
> * * *
>
> "Since considerable care was taken in designing the shapes of the spillway transitions to eliminate negative pressures or vacuums, it is not expected that erosion will occur due to cavitation."

4. The damage was in exact accord with the teachings of model tests of concrete erosion, published by the USBR prior to damage.
5. Apparently previous inspections during seven years after completion had revealed no misalinement in the tunnel lining.
6. Even if one wished to accept the hypothesis of "cavitation," a statement by Mr. Warnock makes it impossible to do so:

> "Actually, the coat of black waterproofing and mineral deposit was intact in many places, showing no effect of direct scouring by the high-velocity water immediately above and below the eroded area."

Mr. Warnock continuously keeps before the reader the merits of many different types of model tests. The word "model" ("laboratory studies," "laboratory investigations," etc.) is used twenty times; but in all his comment on model testing he omits one of the most significant "model-prototype" relations to be found in the literature. By whatever name it is designated, the performance of the Boulder Dam spillway was exactly in accord with the teachings of model tests made by the USBR ten years earlier.

Prior to the construction of the dam, two types of model tests were conducted: (*a*) Erosion of concrete blocks by a jet of water at high velocity; and (*b*) hydraulic model tests of various spillway features.

Before discussing the model tests, it may be well to review what happened. Boulder Dam is a concrete arch-gravity structure, 727 ft high, completed by USBR in 1935. Two spillways were built. Water discharges into a side-channel spillway and then into a concrete-lined tunnel (50-ft inside diameter), inclined to the horizontal at 50°. Water storage in Lake Mead (above Boulder Dam) began in 1935; but it was six years before an opportunity arose to compare the performance of the "prototype" with the models. During four months in 1941, about 13,500 cu ft per sec was discharged by the Arizona spillway. This spillway was designed for a capacity of 200,000 cu ft per sec. The quantity discharged was about 7% of the rated capacity, and was considered so insignificant that no hydraulic model tests were made with less than equivalent discharges of about 40,000 cu ft per sec. Nevertheless, the discharge for a short time of 7% of rated capacity produced the following rather startling results:

(a) Ripped a hole 115 ft long and 33 ft wide at the bottom of the drop in the 50-ft circular tunnel;

(b) Washed away a seven-year-old concrete lining to depths of as much as 25 ft;

(c) Excavated a hole 25 ft deep in the native rock; and

(d) Repairs required seventeen months, and cost $250,000.

The principal feature of the repairs was the placing of 1,460 cu yd of concrete. It may seem strange that the concrete lining was as much as 25 ft thick; but the bottom of the drop was in a "tunnel plug" built to seal off the upstream part of the original 50-ft diversion tunnel.

Model Tests of Concrete Erosion.—In 1931, Arthur Reuttgers and B. W. Steele, Assoc. M. Am. Soc. C. E., wrote "Tests of Concrete Blocks Subjected to High Water Jet Velocities." These tests were made two years before the first concrete was placed in Boulder Dam. The complete report was reprinted in 1938 (78*b*). The original purpose of the tests was stated as follows (78*c*):

> "Experiments were made to determine the resistance of concrete to erosive action of clear water flowing at extremely high velocities. Although the experiments were not part of the laboratory tests of hydraulic properties of spillways, they were definitely related to the practical performance of the spillway structures. Since no previously existent structures had been known to carry water at velocities of nearly 180 feet per second the investigations were of considerable importance."

Attention is directed particularly to the statements made seven years after the tests and three years after the completion of Boulder Dam to the effect that:

(a) The tests "were made to determine the resistance of concrete to erosive action of clear water,"

(b) They "were definitely related to the practical performance of the spillway structures,"

(c) "* * * the investigations were of considerable importance."

In 1938 the USBR announced the titles of forty-one volumes of reports that were in process of publication, giving technical details of all features of the design and construction of Boulder Dam and appurtenant works. Seven of these books are devoted to the $500,000 research program on cement and concrete. Perhaps it was some measure of the relative importance of the subject that the report on "Model Studies of Spillways" (78) was the first to appear of those having to do with concrete.

Sixteen small blocks of concrete, 18 in. square and 6 in. thick, were made in Denver, Colo. The mix was 1:2:4 using sand and gravel graded to $1\frac{1}{2}$ in.; water-cement ratio, 0.63, by weight; and slump from 3 in. to 5 in. The blocks were cast on smooth, plane steel plates. The type of cement and the strength of concrete were not reported.

At ages of from 40 days to 138 days the blocks were subjected to a stream of water at velocities of from 100 ft per sec to 175 ft per sec. The stream was applied in a downward direction by means of a fixed nozzle that tapered from 4-in. inside diameter to 1 in. in 20 in. The test block was bolted at the required angle, so that the stream impinged on the surface 15 in. from the nozzle. Generally the stream was applied at an angle of 90° to the bottom surface of the blocks, although angles of 5°, 30°, and 45° were also used. On some of the blocks, other methods were used, such as playing the stream in a circular slot cast in the concrete, or against an offset cast in the concrete. The treatment was continued, from a few hours to two days—in one instance for five weeks.

Typical Results of Model Tests of Concrete Erosion.—Block A–2 was subjected to water at a nozzle velocity of 175 ft per sec, at an angle of 90° for five days. It has been stated (78*d*) that this block was:

> "Unintentionally tested on upper, trowelled surface (as cast). The mortar surface was worn off in the shape of an annular ring."

The sections of Boulder Dam spillways that are subjected to the highest velocities have unformed surfaces; hence the foregoing test, made contrary to plan, gave the only information in the entire report that is directly applicable to the conditions at Boulder Dam. The remaining tests on block A–2 were made on the "formed" surface, but gave more tangible information on the dimensions of the "annular ring." At the end of a three-week test:

> "Erosion was confined to strip 1-inch wide on outside of a circle of approximately 4-inch diameter. Pits less than ½-inch deep" (78*d*).

After block A–2 had been under test at 175 ft per sec at 90° for five weeks: "Largest hole in block 1-inch deep and 12-inches square at the bottom. Rest of erosion less than ½-inch deep" (78*d*). It is not clear how a nozzle that discharged a scattered spray 4 in. in diameter could form a hole in the concrete "12-inches square at the bottom." This question would not ordinarily be important, but in this instance the information appears in an official USBR report and concerns data on which Boulder Dam was designed and built.

The tests on block A-2 were the most extended in the program. They demonstrated fully the fundamental error of applying a scattered spray, rather than a concentrated jet of water. At the same time, they showed the inability of this concrete to withstand even this mild type of erosion. In testing block F–1, the stream from the nozzle entered tangentially a "12-inch semicircular groove" cast in the concrete. After one day at 175 ft per sec: "Maximum depth of cavitation approximately ⅛ inch."

TABLE 2.—TESTS ON CONCRETE BLOCKS SUBJECTED TO HIGH WATER JET VELOCITIES

Block	Velocity[a]	Days	Observed data
A–1..	125	1	No appreciable erosion
A–1..	150	1	No appreciable erosion
A–1..	175	9.6	Small rough spot, some enlargement of surface pits
B–1..	175	2	Very little erosion
C–1..	175	2	Very little erosion

[a] Feet per second.

Because of entrainment of air, scattering of water, rebound, etc., the tests made with this setup were entirely inadequate when applied to Boulder Dam spillways. When the stream was applied at small angles the treatment was still less significant. Notes from the report on three blocks tested with the nozzle set at 5° to the test surface are presented in Table 2. Of course, "very little erosion" resulted from this scattered spray, which did not remotely resemble operating conditions at Boulder Dam. These notes are typical of the observed data from the model tests of concrete erosion; but the destructive effects of these innocuous treatments were overlooked entirely in the design and construction of Boulder Dam.

Comparison of Model and Prototype.—Although it is apparent that the model tests of concrete erosion were inadequate to represent conditions at Boulder Dam, it is extremely interesting to note the almost perfect agreement between phenomena for model and "prototype":

Concrete model tests, 1931	Prototype tests, 1941
Under jet velocity of 175 ft per sec; cavitation of $\frac{1}{8}$ in. in one day; wore an annular ring in five days; a hole "1-inch deep and 12-inches square" in five weeks.	At a velocity estimated by Mr. Warnock of "at least 150 ft per sec," and at 7% of design capacity, 3,000 tons of rock and concrete was washed away in four months.

The superior quality of the spillway concrete, as a result of its age of seven years, was more than counterbalanced by the longer period of test. Ten years before the "prototype" was used, the model tests showed that concrete could not withstand even this mild treatment. The model tests predicted exactly the nature but not the full extent of the damage that was to occur in the spillway tunnel.

Concrete of Controlled Quality.—The two quotations (78) (78*a*) given near the beginning of this discussion, published three years after the completion of Boulder Dam, assured the profession that: "Comprehensive specification provisions for accuracy of alinement and rigid control of concrete manufacture and placement help to insure this [streamline flow at critical sections]"; and that: "* * * it is not expected that erosion will occur due to cavitation." Mr. Warnock now claims that the spillway failure was due to the very causes that were renounced years earlier:

> "Imperfections in the concrete, such as rock pockets, cold joints, porous areas, lack of bond, etc., all made the concrete more vulnerable to this attack by impingement. Furthermore, the impingement of the high-velocity water on any exposed joints would cause the energy in the water to be converted from velocity head to pressure head. This pressure was probably transmitted through the planes of weakness in the construction joints caused by lack of proper horizontal joint cleanup prior to placement of new concrete."

Horizontal Construction Joints.—The damage described by Mr. Warnock is attributed to the horizontal construction joints. Nearly three years prior to the discovery of the damage, the writer called attention to the defects of the water-air-gun method of joint cleanup that was used at Boulder Dam, and pointed out that the method (79): (*a*) Was dangerous, (*b*) should never have been specified, and (*c*) should be strictly prohibited. In response to this criticism, R. F. Blanks, M. ASCE (80), made a vigorous defense of this method, to which Mr. Warnock now attributes the severe damage of the spillway tunnel.

That reservoir water is finding its way freely into or through the horizontal construction joints at Boulder Dam is amply attested by the following:

(1) In 1940 Tom C. Mead (81) reported that:

> "Knowledge of the composition of the seepage water should help also in understanding the reason for the deposition of calcium carbonate in the foundation drains."

The mechanism here is quite plain. Cold reservoir water, about 60° F, enters the drains where they intercept the horizontal joints. In passing through the joints, the cold water became saturated with calcium hydroxide from the cement. Water in the drains is heated to about 100° F by the warm rock. Warm water cannot hold in solution as much calcium hydroxide as cold water; hence the excess hydroxide is precipitated in the drain holes, and gradually becomes carbonated from combination with the carbon dioxide in the water. It seems extremely likely that these drains will soon become entirely inoperative as a result of this deposition of carbonate.

(2) In 1945 Clarence Rawhouser, Assoc. M. Am. Soc. C. E. (82), called attention to "drilling of additional drain holes" under the middle of Boulder Dam. This was made necessary, no doubt, by the development of excessive hydrostatic uplift, due to water from the lake passing freely into the horizontal joints.

(3) In the same discussion Mr. Rawhouser gave data on one thermometer embedded in the concrete near the middle of the bottom of the dam which showed a most rapid rise in temperature for the first two years after artificial cooling was discontinued; then there was a much smaller rise for three years, followed by a drop of about 10° F during the last five years. The "unexplained" behavior of this thermometer was probably due to the gradual cooling of the hot concrete by the cold reservoir water passing freely through the joints.

(4) Mr. Warnock concludes that: "The concrete was probably dislodged in quite large pieces." If so, this was facilitated, no doubt, by hydrostatic uplift from lake water in the joints.

(5) In two places in his paper Mr. Warnock mentions the "mineral deposits" encountered in the area of the failure of the tunnel lining. These deposits were undoubtedly due to lime from the cement carried to the surface by lake water passing through the horizontal construction joints.

The Durability of Boulder Dam Concrete.—One of the most disturbing features of the damage in the Boulder Dam spillway tunnel was a report issued by representatives of the USBR. At the same time that the USBR was spending $250,000 to repair the damaged tunnel, it was informing the profession that (77):

> "Bureau of Reclamation tests indicated that concrete of ordinary proportions and controlled quality will withstand any erosive action likely to be produced in Boulder Dam spillways."

Both the concrete tests and the spillway had previously demonstrated that "concrete of ordinary proportions and controlled quality" would not do what is here claimed. The foregoing statement was published more than a year after it was known that 3,000 tons of concrete and rock had been washed out of one of the Boulder Dam spillways the first time it had been required to discharge 7% of its design capacity.

Summary.—The concrete model testing done for the design of the Boulder Dam spillways affords one of the most remarkable opportunities for "model-prototype" comparisons in engineering literature. The results of the concrete block tests in 1931 were completely corroborated ten years later by the first trial run of a Boulder Dam spillway. The model showed decided pitting at one day, and serious erosion after five weeks of exposure to a scattered spray of water at 175 ft per sec. In the spillway, 3,000 tons of concrete and rock were washed away, in four months, by water equivalent to 7% of the rated capacity.

The damage to the spillway tunnel was primarily due to erosion of the concrete (as had been shown by the model tests) and secondarily to hydrostatic uplift from reservoir water that found its way through the horizontal construction joints. This effect under a discharge of 7% of rated capacity, has a significance much more important than merely an interesting example of concrete erosion. It demonstrates that: (*a*) Concrete cannot withstand the erosive action of water flowing in these spillways; and (*b*) this spillway, having failed to withstand a discharge of 13,500 cu ft per sec, is entirely inadequate to discharge the 200,000 cu ft per sec for which it was designed.

In spite of the plain failure of the concrete in the model tests of erosion, and more than a year after the experience at the spillway tunnel representatives of the USBR issued a misleading statement on the capability of this concrete to withstand the service conditions to which it is exposed. This statement is almost certain to lure other engineers into the same types of costly and dangerous decisions that have occurred at Boulder Dam.

John K. Vennard,[16] Assoc. M. ASCE.—It is gratifying that that Symposium has stimulated so many civil engineers to place their cavitation experiences before the profession as a whole. The main purpose of the Symposium was to provide a convenient summary of theories, experiences, and references on cavitation. It is felt that the excellent discussions submitted have contributed heavily toward attaining this objective.

Professor Mockmore rightly emphasizes the necessity for care in surface finish to reduce cavitation and pitting. The importance of careful finishing of surfaces has been recognized in the propeller and turbine field for some time; on the other hand, a few tests described in the literature seem to indicate that surface roughness is quite irrelevant. This question is another ramification of the problem that has never been decided conclusively.

Professor Robertson's comment on the "elementary" treatment of cavitation theory is not taken to be a criticism since an elementary treatment was the intention of the paper. Professor Robertson raises the question of the effect on cavitation of internal tensions in a liquid, and thus encounters another unsolved problem of cavitation. Pure, air-free, unagitated water has been shown capable of withstanding enormous tensions for relatively long periods, but obviously such conditions would never be satisfied in engineering occurrences of cavitation. Nevertheless, it is possible that, even in "engineering water," tensions may exist for a sufficiently long time to contribute significantly to the rapid formation of the cavity. Future research may answer this question.

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Thanks are due Professor Robertson for his neat summary of hypotheses and phenomena connected with cavity collapse which were not included in the original paper. This summary has served to strengthen the effectiveness of the Symposium.

The question asked by Mr. Blaisdell cannot be answered at the present time. Whether pitting may result from the collapse of cavities out of contact with the pitted area is another question for the research laboratory. Eventually, if this question is answered in the negative (assuming other advances in the knowledge of cavitation), the designer will have a valuable tool for rendering cavitation harmless.

Professor McNown indicates the possibility of dealing with some problems of cavitation caused by flow curvature by assuming a perfect fluid and applying the methods of classical hydrodynamics. Situations where this procedure can be followed, however, are a very small minority of those encountered in engineering practice; it still appears to the writer that, in the large percentage of problems in which cavitation results mainly from flow curvature, pressure reduction is quantitatively unpredictable and its causes are obscured by the complexities of three-dimensional flow. Professor McNown's remarks on a dimensionless approach to the cavitation problem, on the effect of air content, and on the use of the water tunnel have contributed materially to the value of the Symposium and are much appreciated.

Professor Van Driest's observation on the contribution of corrosive action is pertinent. Originally it was felt by numerous engineers that corrosion was the mechanism by which pitting occurred. All evidence now seems to indicate that cavity collapse is the main destructive effect, with corrosion playing a small, usually negligible, part; however, this is not to deny that there may be an effect of corrosion since metals are known to fatigue more rapidly in the presence of corrosive action.

The writer is skeptical of the validity of Professor Van Driest's conclusion concerning tension in a column of flowing water although he cannot offer conclusive refutation. The evidence that there was tension in the liquid at the throat of the constriction is certainly circumstantial; a moving liquid column (presumably containing dissolved air), sustaining tension for the time implied by Professor Van Driest, is inconceivable to the writer. Furthermore, the numerical calculations offered by Professor Van Driest do not appear convincing. The writer suspects that a change in the flow picture has produced a change in the value of K_L which Professor Van Driest has assumed to be constant; if this is the case, the calculations leading to tensions of 770 lb per sq ft and 118 lb per sq ft would be in error.

John C. Harrold,[17] Assoc. M. ASCE—The writer was pleased with the many original ideas, practical suggestions, and additional cavitation experiences presented in the discussions. They should add greatly to the value of the Symposium.

Professor Mockmore's suggestion that care in fabrication is just as important as proper design has also been learned by the Corps of Engineers through experience at Norfolk Dam (Arkansas) where, in constructing the sluice gate slots described by Professor Kindsvater (Fig. 64(*a*)), the castings did not match within $\frac{1}{2}$ in. in some cases and had to be ground flush after the dam was put in operation.

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Messrs. Bradley and Kindsvater brought out the necessity for distinguishing between the cavitation due to a general lowering of the pressure gradient in a conduit and that due to a localized reduction caused by a discontinuity or sharp curvature in the boundary. Mr. Bradley's example (Fig. 52) showing that, in a straight uniform conduit, cavitation can occur only in the roof of the conduit because the bottom is always under higher pressure than the roof was very interesting.

The remedy described by Mr. Bradley for the low-pressure gradient in the Ross Dam (Washington) by-pass tunnel is essentially the same as that used in the Norris Dam (Tennessee) sluices (Fig. 46). In each case, the downstream end was constricted in order to raise the general pressure gradient in the conduit. Constrictions of the Norris type have been built into the sluices of some of the dams recently constructed by the Corps of Engineers.

Use of the beveled gate slot, described by Professor Kindsvater (Fig. 64(*a*)), has also become common practice with the Corps of Engineers. A general series of model tests is under way (1946) at the U. S. Waterways Experiment Station in Vicksburg, Miss., to attempt to improve the streamlining of gate slots. General tests are also under way at the Experiment Station which have for their purpose improvement in the design of the bottom edge of rectangular slide gates and the determination of the cavitation characteristics of elliptical entrance curves for rectangular sluices.

In reply to Mr. Blaisdell's question about the practicability of baffle piers with the sides cut away to form a water cushion, a model test of just such a baffle pier conducted in the vacuum tank at Carnegie Institute of Technology in Pittsburgh, Pa., in connection with the Bluestone Dam (West Virginia) tests (Fig. 18) will be described. The baffle piers for this test were 6 ft high and 5 ft wide, with a vertical upstream face and sloping downstream face as in the other tests; but the upstream face in this test was flared to an 8-ft width providing a $1\frac{1}{2}$-ft overhang on each side. The upstream face was made convex in plan with a 6-ft radius and the transition from the 8-ft width to the 5-ft width was made with a concave surface, also with a 6-ft radius. The piers were spaced on 8-ft centers and were staggered in two rows as in the other tests. The following is quoted from the report on these tests:

> "The former [above] design was based on the assumption that the destructive effects of cavitation on the sides of the baffle could be eliminated by providing an area, or water pocket, for the formation of eddies in the region where cavitation would likely occur. The front of the baffle was designed in such a way as to direct the jet away from its sides, these being depressed to allow the water to feed in from behind and provide an area of positive pressure against them due to eddy formation. The behavior of this design was as expected, no pockets occurring adjacent to the pier. The jet was deflected away from the sides of the baffle, and the resulting low pressure areas were confined to the water in between the baffles. However, the effectiveness of this 'side-pocket' type of pier in energy dissipation was not as good as for the other types tested for Bluestone Dam."

Although in this particular case the "water cushion" baffles did not prove to be the best solution to the problem, the idea is basically sound and might prove more advantageous in another case. There is one practical objection to these baffles, however—they are structurally less rugged than the streamlined type. The upstream side edges being sharp and relatively thin would be subject to damage by impact from heavy drift or ice. The streamlined baffles have no projecting corners and would be less subject to damage. On the other hand, as stated by Mr. Blaisdell, the streamlined baffles also have a practical disadvantage in that the curved surfaces must be constructed more accurately.

Concerning the remaining two questions raised by Mr. Blaisdell: The writer does not know how the noise level of collapsing cavities is related to the damage caused by them; nor does he know of any instance where the structure, as a whole, failed because of the forces resulting from collapsing cavities. As stated in the writer's paper, sections of armor plating on the sides of the Bonneville (Oregon) gate piers and on the sides of the Gatun (Panama Canal) baffle piers were ripped off due to fluctuating pressures possibly involving the action of cavitation. However, the structures in these instances are of such rugged construction that no tendency toward failure of the structures, as a whole, has been noticed. Perhaps, in time, some such tendency may be observed. However, it is possible that the great width of these piers and the heavy steel reinforcement used in them may cause unit stresses resulting from the fluctuating pressures to be quite low. It is also possible that the forces themselves may not be great because the area exposed to extreme high and low pressures may not be great.

The writer appreciated the more complete theoretical analyses of the cavitation phenomenon and the model simulation thereof given by Professors McNown and Van Driest, and he was grateful that these analyses checked Eqs. 3. Their discussions also brought to light many interesting physical characteristics of the cavitation phenomenon which those not well versed in physics would not realize existed and which should be useful to practicing hydraulic engineers in interpreting cavitation experiences.

The additional cavitation experiences cited by Messrs. Ball, Locher, and Kindsvater should be helpful in future hydraulic designs.

There is one more comment on Professor Kindsvater's discussion which appears desirable. In writing about the reliability of electric pressure cells for measuring pressure fluctuations in the model of the Bull Shoals (Arkansas) sluice gate slots, Professor Kindsvater states that the cells "* * * gave inconsistent results, and these data were discarded." This may appear contrary to the statement in the writer's paper that satisfactory results can be obtained by this method of measurement. However, the statements will not seem contradictory when it is explained that the writer's statement applies to great fluctuations in pressure such as were measured on the sides of the baffle piers in the model of Bluestone Dam whereas Professor Kindsvater's statement applies to small pressure fluctuations downstream from the gate slots in the model of the Bull Shoals sluices. The pressure cells have been found unreliable in the lower range when measuring rapidly fluctuating pressures; and, as a result, further experimentation is under way (1946) at the U. S. Waterways Experiment Station to determine the causes of this trouble and the remedies for it, if possible. It is suspected that the presence of minute air bubbles in the copper tubing leading from the piezometer openings to the cell may be the principal causes of the difficulty. Apparently, the error is inappreciable with great fluctuations in pressure; but, with small fluctuations, the results are quite inconsistent as Professor Kindsvater states. This leaves hydraulicians at present (1946) with no reliable method of measuring small fluctuations and with the necessity of applying an arbitrary factor of safety to measurements made with an ordinary water manometer.

Since the writing of the original papers of this Symposium (1944), severe pitting, due to cavitation was discovered in the outlet works control towers of Fort Peck Dam in Montana. The outlet works consist of three circular tunnels 24 ft 8 in. in diameter and about a mile long with vertical control shafts near the center. The discharge of each conduit is controlled by a cylinder gate, 27 ft 7 in. in diameter and 8 ft 2 in. high, 55 ft above the invert of the tunnel in the control shaft. The water rises vertically through an annular ring 6 ft 6 in. wide concentric with, and outside of, the 28-ft 1-in. circular shaft. It then flows laterally through six rectangular openings 8 ft 2 in. high and 7 ft 4 in. wide into the shaft; thence downward into a 24-ft 8-in. elbow which deflects the water horizontally into the 24-ft 8-in. diameter tunnel. The cylinder gate controls the six lateral openings and seats on the bottom edge of these openings. The sides and top of these openings are plane surfaces. The sides are radial with respect to the center line of the shaft and the top slopes downward in the direction of flow on a slope of 30° with the horizontal. The bottom of the opening is 6 ft 2 in. thick in the direction of flow and is shaped like the crest of an overflow dam. The sides, top, and bottom of the openings and the inside surfaces of the shaft in the vicinity of the openings are lined with semisteel armor plating. Severe pitting of these curved crests and the shaft lining in the vicinity of the openings has occurred to a maximum depth of $1\frac{1}{2}$ in. The cylinder gates have been operated partly open and wide open for extended periods under heads up to 200 ft (pool to invert of tunnel). In addition, the cylinder gates themselves have been damaged by vibration and a section of the steel liner plating in the 24-ft 8-in. elbow has been torn out. Some of this damage may have been caused by the closing of the air vents in the control shaft, which was found necessary during winter operation to prevent freezing of the spray in the shaft. Very little winter operation is contemplated in the future. If the foregoing conditions continue to exist, it may be necessary to work out a change in design to remedy them. Thus far, repairs have been made in accordance with the original design.

The Corps of Engineers is constructing (1946) a large vacuum tank at the U. S. Waterways Experiment Station, similar to the tank at Carnegie Institute of Technology for the cavitation testing of various parts of flood control structures. The inside dimensions of the test chamber will be 15 ft long by 5 ft high by 5 ft wide. The test chamber has been designed for testing models with or without a free water surface.

JACOB E. WARNOCK,[18] M. ASCE.—Although the Symposium papers have stimulated a worthwhile budget of cavitation pitfalls to guide the efforts of future designers, the writer is somewhat loath to see the discussion close. He is certain that, if every organization dealing with hydraulic structures and machinery would open its files, as the authors of the Symposium papers have done for their respective organizations, a more comprehensive list of examples of cavitation damage could be contributed. If this were done, the purpose of the Symposium (namely, that of allowing the profession at large to benefit by the experience of its individual members) would be more fully realized.

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Of the many discussions submitted only one is in need of direct reply and comment. The discussion by Mr. Abrams covers the writer's portion of the Symposium very thoroughly, and it is unfortunate that much of his discussion is based on misconceptions on his part. He does not concede that cavitation can damage a concrete tunnel lining. Some of Mr. Abrams' comments are concerned with the quality and placing of concrete and are not discussed herein because they have no place in this Symposium; and, also, the subject has been fully treated in other publications. Those dealing with the subject at hand are fully discussed herein.

In his discussion of the Boulder Dam spillway tunnel, Mr. Abrams states, "* * * a photograph of a rope is not very satisfying as scientific evidence * * *" of tunnel misalinement. Actually, the misalinement was carefully surveyed. The photograph, however, was submitted because it was a more striking evidence than a contour drawing. Mr. Abrams' statements regarding the misalinement of the tunnels have no basis if the facts are considered. Certainly, the entire lining could not shift after the damage occurred without leaving evidence of such movement. Also, the misalinement indicated by the rope could not have been caused by erosion, since the black waterproofing was still on the surface of the concrete in many places after the damage had occurred.

Mr. Abrams then states, "An official report of the USBR published in 1938 stated that there was no misalinement in the Boulder Dam spillway tunnels." Careful reading of the excerpt chosen by Mr. Abrams (78*a*) will show this to be a misstatement:

> "* * * The Boulder Dam spillway system is designed with the expectation of obtaining practically streamline flow in the lower tunnel sections where maximum velocities occur. Comprehensive specification provisions for accuracy of alinement and rigid control of concrete manufacture and placement help to insure this.
>
> * * *
>
> "Since considerable care was taken in designing the shapes of the spillway transitions to eliminate negative pressures or vacuums, it is not expected that erosion will occur due to cavitation."

Nowhere in these excerpts is it stated that there was no misalinement of the tunnels. It is stated (78*a*) that, "specification provisions for accuracy of alinement" were made and that "care was taken" in design. These procedures were followed, but in spite of the precautions, the misalinement still occurred.

Mr. Abrams' understanding of "cavitation" does not agree with the accepted definition. He quotes the writer's statement:

> "Actually, the coat of black waterproofing and mineral deposit was intact in many places, showing no effect of direct scouring by the high-velocity water immediately above and below the eroded area."

and interprets it to mean that cavitation could not have occurred. Actually, this is positive evidence that cavitation did occur. Pitting or erosion due to cavitation begins a short distance downstream from a source of low pressure—in this case, the misalinement at the point shown by the rope. No measurable erosion occurred above the misalinement; and this fact constitutes strong evidence that, even with high-velocity flow, if changes in contour are not present, there is no tendency for the high-velocity water to erode the concrete. Apparently, the shearing action of the fluid against the boundary in the absence of abrasive material did not wear the concrete.

Considerable space is devoted by Mr. Abrams to a discussion of erosion of concrete test blocks, using quotations from a USBR report (78) to illustrate his points. The tests selected by Mr. Abrams to illustrate excessive concrete erosion are impingement tests. The jet of high-velocity water was pointed directly at, or at a sharp angle to the test block surface. These tests (78*e*)(78*f*) were run (quoting directly from the report)—

> "* * * in order to determine the erosion produced by such extremely severe conditions * * * tests at a jet angle of 45 degrees and 90 degrees were intentionally extreme or accelerative and have no direct application to the spillways. These tests are enlightening, however, in revealing the resistance of the concrete to severe or abusive treatment."

Nowhere in the Boulder Dam tunnels is the concrete subjected to direct exposure to a jet of water traveling 175 ft per sec and impinging at a 45° or 90° angle. Flow in the Boulder tunnels is such that the streamlines are parallel, or nearly so, to the concrete surfaces. Quoting again from the report (78*g*), and referring to the test blocks,

> "There was practically no evidence of wear or erosion of the concrete on any plane or jointed surface subjected to the water jet at any velocity when the angle between the jet and the block was small."

Mr. Abrams refers to the test jet as "a scattered spray" and then declares, "It is not clear how a nozzle that discharged a scattered spray 4 in. in diameter could form a hole in the concrete * * *." Actually, the "scattered spray" referred to by Mr. Abrams was a concentrated jet of water emanating from a nozzle that tapered from 4 in. in inside diameter to 1 in. in a length of 20 in. The jet impinged on the sample 15 in. from the nozzle with a velocity of 175 ft per sec; yet Mr. Abrams calls this a "mild type of erosion."

In two places Mr. Abrams calls attention to the results of tests on block A-2 and refers to a "hole 1-inch deep and 12 square inches at the bottom." Here he is quoting a typographical error (78*d*) meant to read $\frac{1}{2}$ sq. in. This should have been clear, regardless of the typesetter's omission of a diagonal line, since a photograph of the test block, included in the report, shows the area of the hole to be considerably less than 1 sq in. at the bottom.

In discussing the results of tests on block F-1, Mr. Abrams completely disregards the text of the report which states (78):

> "In blocks having a cylindrical hole or semicircular groove, the apparent wear was inappreciable, except near the point of water exit from the block, see Figures 129 and 130. At those points, substantial cavitation occurred under jet velocity of 105 to 175 feet per second. The cavitation is attributed principally to outlet disturbances accompanying the sudden release of the stream from its constricting channel."

These statements are substantiated by a photograph of the block taken after testing was completed.

In connection with the foregoing discussion of cavitation and erosion, it is interesting to note the relative location of the erosion that occurred on the test blocks. Referring to tests with jet angles of 90° the report states (78*e*), "Areas affected had the shape of an annular ring approximately 1-inch wide around a 2-inch inside diameter circle * * *." Contrary to Mr. Abrams' description, the jet was a solid stream of water about 1 in. in diameter where it impinged on the test block surface; yet, there was no erosion within the area of impingement or within a concentric circle about 2 in. in diameter. All the measurable erosion occurred outside the 2-in. circle. A satisfactory explanation of this phenomenon is not readily apparent when one considers the known facts concerning either erosion or cavitation. Similar erosion has been experienced by others and no explanatory conclusions have been reached. Further experience and knowledge are needed to fully understand this type of surface destruction.

The comparison of model and prototype given by Mr. Abrams is not valid because of his misinterpretation of the model data. Furthermore, the nature of the damage to the prototype cannot be predicted from the model tests since cavitation did not occur in the model. Cavitation in the prototype was due to a local condition which did not exist in the model and was not expected to exist in the prototype. At velocities of about 175 ft per sec, under conditions approaching those in the prototype tunnel, there was no measurable damage to the model test blocks. At velocities of about 150 ft per sec in the prototype there was no damage to the tunnel except immediately below the misalinement.

Mr. Abrams concludes, "* * * this spillway, having failed to withstand a discharge of 13,500 cu ft per sec, is entirely inadequate to discharge the 200,000 cu ft per sec for which it was designed." This statement is certainly a pessimistic forecast which is not substantiated by the facts. Model tests showed that the tunnel was indeed capable of discharging the design flood. Cavitation is the result of high-velocity flow and the maximum velocity for 200,000 cu ft per sec will not be greater, proportionately, than that for 13,500 cu ft per sec, since the height of fall for both discharges is approximately the same. Thus, the tunnel has already been subjected to velocities which approach the maximum, and the effects of high-velocity flow in the tunnel where no imperfections exist have already been determined. Also, with increased discharges the possibility of cavitation occurring in the elbow is reduced in two ways: (1) The resulting greater depth increases the pressure at the boundary; and (2) the greater centrifugal force exerted by the greater mass of water in passing around the bend further increases the pressure at the boundary.

If the writer's paper serves no other purpose than to encourage a more wary attitude toward the performance of any hydraulic feature with respect to possible cavitation damage, he will feel that a worthy purpose has been served. It is to be hoped that in the future various individuals will feel encouraged by the candor of the Symposium authors to contribute their cavitation experiences, especially when they are novel or unexpected.

Acknowledgment is made to Messrs. Bradley, Ball, and Locher for contributing additional examples of cavitation erosion as experienced by the Bureau of Reclamation.

GEORGE H. HICKOX,[19] M. ASCE.—The response to the Symposium on cavitation experiences has been very gratifying. It is clear that the difficulties caused by cavitation have been rather widespread, and are to be expected wherever high-head structures are in operation.

The experiences reported in the incompleted conduits of Ross Dam (Seattle, Wash.) by Mr. Bradley and in the temporary elbows at Grand Coulee Dam (Washington) by Mr. Ball are good examples of the way in which cavitation can be prevented, both by raising the general pressure level and by surface treatment to correct local reductions of pressure. These experiences are similar to those cited in the case of the Norris Dam (Tennessee) sluices.

Mr. Ball's notes on the beginning of cavitation damage in the Nevada tunnel of Boulder Dam (Arizona-Nevada) support the supposition that cavitation was the cause of the extensive damage in the Arizona tunnel. His description of the progressive nature of cavitation damage, supported as it is by direct evidence, is especially valuable and shows how an otherwise insignificant irregularity of surface may become a menace to the safety of an entire structure. If a smooth continuous surface, such as a tunnel, sluice, or face of a dam, contains an irregularity, either depression or projection, of sufficient size, cavitation will occur whenever the velocity is high enough and the general pressure level is low enough. If, then, the cavitation results in damage, the resulting depression may be deep enough to cause still further damage; and from that point on the effects are progressive and cumulative. For this reason, the writer wishes to endorse, heartily, Professor Kindsvater's statement that structures such as baffle piers that are subject to cavitation damage must not be depended on to insure the safety of dams.

Mr. Blaisdell asks if the noise level is related to the cavitation damage. In the case of the Norris sluices, there appeared to be a relationship between the duration of cavitation and the damage sustained, but there are no data to indicate that the rate of damage is related to the noise level. However, if a higher noise level indicates an increased intensity of cavitation, it would seem reasonable to expect that it would also be accompanied by an increased rate of damage.

Mr. Ball's observation of flashes of light in the tailrace is of interest as such flashes have also been observed on spillway aprons during the operation of sluices discharging at high velocity (83). His hypothesis that these flashes are caused by differences in the refractive and reflective properties of the water due to compression waves is probably correct. This hypothesis has been advanced by Professor Kindsvater (84). A similar explanation for the appearance of rapidly moving light bands accompanying the eruption of Paricutin volcano has been given by O. H. Gish (85). The flashes are not necessarily the result of compression waves set up by cavitation within the structure. In the case of the sluices discharging on the apron at Cherokee and Hiwassee dams, the flashes appeared to originate in the boundary of the jet from the sluice. It seems probable that cavitation existed in the vortices set up at the sides of the jet. It is believed that the sluices themselves were free from cavitation as they had been constructed after extensive model tests, and measurements on the prototype structure disclosed no regions of low pressure. The existence of cavitation at the boundaries of a jet has been described by Mr. Locher in his discussion of the jet pump.

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Professor McNown advances certain analytical methods of studying cavitation which undoubtedly have merit. It is suggested that such methods be used with considerable caution, however, since mathematical predictions based on the assumption of irrotational motion may be quite misleading. An example may be found in the case he cites in which cavitation actually occurred at a value of $K = 1.8$, although the vapor pressure of the water was not reached until $K = 0.7$.

Suggestions for further research are found in the various discussions. Professors Robertson and Van Driest suggest that water may be capable of supporting considerable tension while in motion, and that the formation of cavities may thereby be affected materially. Professor Robertson's assumption that the tension is a function of time is an interesting hypothesis but is subject to verification. Cavities that appear downstream from the point of lowest pressure may be caused by the formation of vortices in a region of expanding flow. This subject is certainly open to further investigation. Professor Van Driest's observations appear to indicate considerable tension in the liquid at times when no cavitation was observed. The existence of tension is possible, of course, as it is known to exist, and can be measured, in liquids at rest. His analysis does not show any definite relationship between tension and the occurrence of cavitation. It may be of interest to note, however, that, within the limits referred to by Professor McNown, most experimenters have found a good agreement between the occurrence of cavitation and the approach of boundary pressures to the vapor pressure of the liquid. It should be noted in this connection, as pointed out by Professor Kindsvater, that measurements of fluctuating pressure by a manometer are subject to considerable damping because of the relatively high frequency of the fluctuations and the large mass of the manometer fluid.

The bibliography included with the Symposium was not intended to be complete. A complete bibliography of papers on cavitation runs through many hundreds of titles.

Much investigation and research will need to be done before the mechanism of cavitation is understood. The same is true of the manner in which cavitation causes damage. The subcommittee sincerely hopes that discussers of the Symposium will be able to devote some effort to the solution of these problems. In the meantime, it is hoped that the Symposium has served to indicate the dangers that may exist when the possibility of cavitation is neglected.

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TRANSACTIONS

Paper No. 2296

THE SAFETY OF STRUCTURES

By Alfred M. Freudenthal,[1] Assoc. M. ASCE

With Discussion by Messrs. F. H. Frankland, Elliott B. Roberts, A. G. Pugsley, Lynn Perry, Nomer Gray, I. M. Nelidov, M. Hirschthal, and Alfred M. Freudenthal.

Synopsis

The purpose of this paper is to analyze the safety factor in engineering structures in order to establish a rational method of evaluating its magnitude. The discrepancy between the highly refined procedure of modern design and the rather arbitrary manner of choosing the safety factor is seriously hampering the development of more effective design methods based upon a perfect balance of safety and economy.

The true character of the safety factor is disclosed by the introduction of a statistical concept of physical qualities, according to which the individual properties composing the structural phenomena of strain and resistance are represented by frequency distributions instead of by individual (minimum or maximum) values. The safety factor, correlating the strain induced with the resistance of the structure, may be derived from observable and measurable physical properties and phenomena.

The correlation of strain and resistance requires a careful analysis of all features of structural design, both from the points of view of the basic assumptions and of statistical interpretation. For the purpose of this analysis the available experimental and observational evidence is rather inadequate; the results obtainable at present, therefore, should be considered suggestive rather than conclusive. It is the insight gained into the essence of the safety factor that is of immediate practical interest.

The word "strain" in this paper is not to be confused with its conventional use to denote a linear measure, or proportion of stretch. It has a meaning roughly equivalent to what is commonly called "stress," but is more general as it is intended to denote the mechanical effect of load, or external conditions on the structure, the structural member, or the section. The word "resistance"

Note.—Published in October, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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is a general term indicating "strength" or capacity to resist failure. It can be expressed in any units appropriate to the matter under discussion.

PART 1.—GENERAL PRINCIPLES

INTRODUCTION

Structural design is generally understood to be composed of two operations—(1) The determination of the strain induced in the structure by loads and secondary effects, and (2) the comparison of the maximum strain that occurs with a rather fictitious resistance characteristic termed the "permissible stress," which is a fraction of the strength of the respective structural material. Although attempts have been made to modify this latter concept,[2,3] which is theoretically inadequate for the rational solution of any but the most simple structural problems, such new ways have not succeeded in affecting the practical procedure of design.

Structural design, like technical problems in general, must not be considered solely from the point of view of theoretical accuracy detached from the aspect of practical application. The conventional methods have proved their expediency under widely differing conditions. Although the underlying concepts have lost their original significance as a consequence of the better understanding of the behavior of structures and structural materials, it appears reasonable to retain the well-tested traditional form and to adapt it to the present improved knowledge of the problem, by redefining the basic concepts in accordance with the results of advanced research.

The fundamental, conventional concept of "allowable stress" involves a comparison between a computed maximum strain and the strength of the material, and implies the existence of a margin between the two. The justification of this margin has never been contested. As its conventional name "margin of safety" suggests, it reveals the subjective striving on the part of the designer for an adequate measure of safety as well as a consciousness of the limitations of his knowledge and the arbitrariness of his assumptions. Its real character has remained obscure, however, and its magnitude is generally estimated on the basis of subjective judgment rather than objective fact. Nothing has been done to establish a criterion for its determination on a more rational basis than the experience and judgment of the designer. The most refined design is thus deprived of its merits, the designer being free to choose the fundamental assumptions of his design largely on the basis of subjective arguments, without being compelled to ascertain their validity by the identification of the objective conditions. Research in the sphere of new materials cannot be expected to bear its full weight upon the economy of structures if the safety factor can be fixed rather arbitrarily "between 1.25 and 4.00 or even higher."[4] Therefore, the analysis of the safety factor to identify its true character and to determine its objective values has become of increasing urgency.

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[2] *Final Report*, 2d Cong. of the International Assn. for Bridge and Structural Eng., 1936, Pt. I, Ductility of Steel, Wilhelm Ernst & Sohn, Berlin, 1939, p. 27.

[3] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, ASCE, Vol. 105, 1940, pp. 638–730.

[4] "Structural Application of Steel and Light-Weight Alloys," A Symposium, *ibid.*, Vol. 102, 1937, pp. 1207–1208.

The Factor of Safety

The principle underlying the concept of the safety factor can be understood best by reviewing the fundamental difficulty of structural design. The computed structural characteristic ("strain") cannot be equated to the characteristic derived from observable and measurable physical properties ("resistance") because the results of an intellectual process—design—cannot be equated to the result of material perceptions—resistance. The limitations of human observation are such that no observable quality can be measured "exactly." The designer is able to assert only that a value is larger than a lower limit and smaller than an upper limit—not that it is "equal" to another value. The manifest lack of correspondence between the conceived action and the performed action in any sphere of human activity explains why no real identity may be expected in a series of actions or events which were planned to be identical. Consequently, intellectual concepts, contrived to reproduce material phenomena with a certain grade of perfection, may be correlated with material observations regarding those phenomena only by a relation of "inequality," expressed in terms of a probable correlation range. This range itself will be a function of the degree of perfection in the concept. It must therefore provide for: (1) The imperfection of human observations and actions (uncertainty); and (2) the imperfections of intellectual concepts devised to reproduce physical phenomena (ignorance). This range represents the objective minimum value of the safety margin, which is thus identified as a function of objective uncertainty as well as subjective ignorance.

With increasing perfection of design methods the element of "ignorance" can be largely eliminated; but the element of "uncertainty" is caused by circumstances that can be changed, to a certain extent, but can never be removed. Hence, the safety factor is a measure of uncertainty rather than of ignorance. The trend toward reducing its numerical value is not so much the result of improved design methods as it is the result of modified objective circumstances; that is, standardizing engineering materials by introducing quality control in production, applying standard acceptance tests by the users of such materials, and introducing stringent regulations for the control of workmanship.

The laws of structural design are derived from the principles of classical mechanics, and are based on the existence of a causal relationship between the antecedent and the consequent events. They are mostly expressed in the form of differential equations, the solution of which enables the engineer to determine all the consequences following one or a number of given antecedent events. Within the range for which the initial assumptions are valid, therefore, the designer should be able to go confidently from cause to effect, all phenomena concerned being strictly predictable. A certain number of parameters of these equations represent observable and measurable physical properties or phenomena. The application of the differential equations to structural design requires the introduction of the real values of such properties under all conceivable conditions of practical importance. Some of these values must be predicted or estimated since their observation and measurement under all relevant conditions are impracticable. Such prediction is entirely different from that based upon differential equations. It can only refer to past experience; and the effect of chance, inherent in any activity or perception, becomes essential. The concept of deterministic causality is superseded by a new concept, in which every unknown cause is termed a "chance" cause. Systems of chance causes produce events in accordance with the law of large numbers and thus give rise to statistical laws represented by frequency distributions. Prediction based on a statistical causality, therefore, may be expressed only in terms of the probability that a certain event will occur within definite limits.

With the introduction of statistical causality the conventional conceptions of physical qualities undergo a considerable change. Every physical property is a statistical distribution so that the constancy of physical qualities is found to be of a purely statistical nature. A given quality approaches a constant value only in the sense that it may be represented by a frequency distribution within specified limits. The laws of structural design, therefore, must be considered a combination of functional and statistical relationships—functional so far as the laws of the theory of structures are concerned and statistical to the extent that real physical properties appear as parameters of the functional relations.

There are cases in which apparently functional relations are intrinsically statistic. Relationships that are important in a practical sense frequently cannot be derived from differential equations either because such equations are not known for the given case or because their solution is too difficult. The laws derived in such cases by statistical interpretation of past experience are empiric and do not represent real functional relationships. They represent the trend revealed by the correlation of assumedly relevant variables; the pertinent frequency distributions form an integral part of the information.

The interrelation between buckling resistance s_b and the slenderness ratio $\frac{l}{r}$ of steel columns is an example of the simultaneous existence of functional, empirical, and statistical laws in one and the same problem. Fig. 1 affords an illustration of the relation—

$$s_b = f\left(\frac{l}{r}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Region c defines Euler's hyperbola as derived from the differential equation of elastic buckling; region b represents the empirical laws of inelastic buckling; and region a shows the range of pure compression in which the frequency distribution of the resistance is the effect of a system of genuine chance causes only.

The value of the safety factor ζ may be derived from the condition that the maximum strain s_a induced in the structure by actual service conditions must never cause such damage as to impede its fitness for service, even were this maximum strain to coincide with the lowest value of the structure's resistance s_r

$$\zeta = \frac{s_r}{s_a} > 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If $\pm\ \Delta s_a$ denotes the maximum range of fluctuation of actual strain about the expected value s_{ao} and $\pm\ \Delta s_r$ the maximum range of fluctuation of the structure's resistance about its expected value s_{ro} the maximum strain to be expected will be $(s_{ao} + \Delta s_a)$ and the minimum resistance will never be less than $s_{ro} - \Delta s_{ro}$. According to Eq. 2 failure is just prevented if s_r (minimum) $= s_a$ (maximum) or

$$s_{ao} + \Delta s_a = s_{ro} - \Delta s_r \quad \text{(3)}$$

Eq. 3 leads to the condition of minimum safety based on the correlation of expected values of strain and resistance; thus:

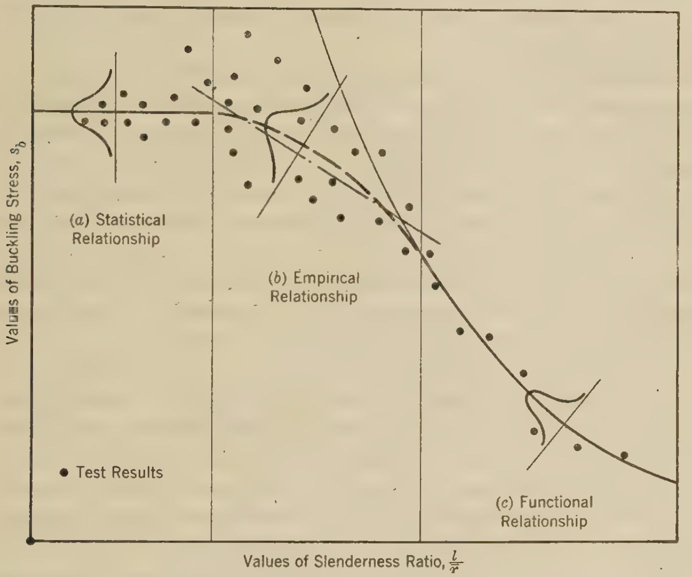

Fig. 1.—Buckling Stress $s_b = f(l/r)$ of Steel Columns

$$s_{ao} = \frac{1 - \dfrac{\Delta s_r}{s_{ro}}}{1 + \dfrac{\Delta s_a}{s_{ao}}} s_{ro} = \frac{1}{\zeta} s_{ro} \quad \text{(4)}$$

in which,

$$\zeta = \frac{1 + \dfrac{\Delta s_a}{s_{ao}}}{1 - \dfrac{\Delta s_r}{s_{ro}}} \quad \text{(5)}$$

denotes the safety factor. For convenience, the correlation of strain and resistance may be expressed as a correlation of load and carrying capacity.

Absolute Versus Relative, or "Economic," Safety

The value of the safety factor derived from the condition of absolute safety expressed by Eq. 2 depends upon the variability of the design parameters (dimensions, weights, elasticity, strength, temperature) and of the initial and boundary conditions. The individual ranges of fluctuation determine the ultimate ranges of variation of strain and resistance or of load and carrying capacity, in accordance with the law of statistical superposition. An increase in the individual ranges as well as in the number of parameters increases the resulting factor of safety. In order to keep this factor as low as is reasonably possible it is necessary to reduce the range of dispersion of the individual parameter by: (*a*) Modifying the objective circumstances and conditions under which it operates until the range of variability of the parameters of the design is that of genuine chance fluctuations; and (*b*) introducing the economic aspect of the problem of safety.

The requirement that the fluctuation of the individual values of the relevant material properties must be confined to the range of genuine chance fluctuation, produced by a system of large numbers of chance causes in which any one cause does not produce an effect greater than the resultant effect of all the others, expresses the purpose of control in modern industrial production and represents its standard definition.[5] A quality or a phenomenon is said to be controlled when, through the use of past experience, one can predict, within limits, how this quality or phenomenon may be expected to vary in the future. Prediction within limits means that one can state the probability that an individual value will fall within given limits.

By enforcing a régime of maximum control upon all material properties and phenomena that affect the design, eliminating all assignable causes of fluctuation, the objective minimum value of the factor of "absolute" safety is obtained. Such "absolute" safety is economically impracticable and theoretically meaningless. Failure cannot be prevented with certainty but only with a high degree of probability, which may approach certainty (a probability of 1) very closely but can never attain it. The ranges of fluctuation (which tend to become infinite to insure certainty) are thus reduced to a reasonable magnitude. A factor of safety may be based on the probability that individual values of either strain or resistance will fall beyond the ranges of either

$$s_a = s_{ao} + j\,\sigma_s \qquad \text{(6a)}$$

or

$$s_r = s_{ro} - j\,\sigma_r \qquad \text{(6b)}$$

in which σ_s and σ_r denote the standard deviation of the universes of s_a and s_r, respectively, and j is an arbitrary constant. Failure can be made highly improbable if this probability is small enough, but cannot be prevented absolutely.

The degree of safety will depend on the selection of the values of the coefficient j, resulting from a reasonable compromise between the requirements of safety and those of economy. It will be comparatively easy to reach such a compromise if the entire process of construction, beginning with the manufacture of the structural material, has been under rigorous control. If a certain level of economy is to be maintained, deficiency of control must be compensated by an increase in the risk faced by the designer.

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[5] "Economic Control of Quality of Manufactured Product," by W. A. Shewart, D. Van Nostrand Co., Inc., New York, N. Y., 1931, p. 151.

PREDICTION FROM PAST EXPERIENCE

A structure is designed by predicting its future behavior on the strength of knowledge gained by past experience and the analysis of factual data. The computation of its safety factor requires an analysis of the variability of all influences bearing upon its resistance and the strain imposed. The quantitative information available is inadequate; where data have actually been collected they are usually presented inadequately and are thus deprived of much of their value.

Past experience concerning physical properties is generally recorded as a series of observations of the properties themselves, or of quality characteristics expressing a known relationship with the property considered. This latter relationship may be either functional (as in observing the decreasing amplitude of vibrations to determine the coefficient of damping of a material) or empirical (as in deriving the tensile strength of steel by observing the hardness of its surface).

When, in the case of some complex phenomena, research has not yet resulted in the establishment of such relationships as would be required for the resolution of the phenomenon into its fundamental components, past experience must be trusted to a correspondingly greater degree. Then past experience may be expressed either by series of observations or measurements of a characteristic manifestation of the phenomenon, or, if such evidence is inconclusive, by the ultimate limits within which all values of the characteristic may be expected to lie. Such limits will frequently be deducible from general considerations, as, for example, in delimiting the possible extreme fluctuations in the bending resistance of sections of mild steel between the resistance of an entirely plastic section with a rectangular stress distribution and that of a maximum elastic section with a triangular stress distribution.[2] Sometimes the frequency distribution itself (and thus the probability of the occurrence) of an individual event can be predicted from the knowledge of its a priori probability, which is the objective probability that an event will occur in an infinite number of trials. This probability can generally be derived by inference from collateral information at the disposal of the designer.

When dealing with material properties such as strength, elasticity, or linear dimensions—all of which affect the resistance of the structure—it will be necessary to predict their most probable values and the probable ranges of fluctuations of individual values. Observations of only a comparatively small number of samples, supplemented by relevant knowledge based on experience with the manufacture and the procedure of control and selection will be available for this purpose. The character of that process and the extent and reliability of current knowledge concerning it will determine the character and shape of the distribution function of the property considered, and therefore the accuracy of the prediction.

When dealing with external influences, such as service loads or wind pressure, that affect the strain induced in the structure the designer will have to assess the probability that certain extremes will occur; or he will be required to estimate the most probable conditions and probable ranges of fluctuations from objective a priori probabilities based, generally, not on direct statistical evidence (which is mostly not available) but on circumstantial evidence. The form of the distribution curve of the probabilities of occurrence of certain loading conditions may be determined by the law of large numbers. For example, the probability that a certain load will occur m times in n cases is given by the binomial, which is the mth successive term of the expression $(P_p + P_q)^n$, in which P_p denotes the a priori or mathematical probability of the occurrence of the particular load and $P_q = (1 - P_p)$ the probability that it will not occur.

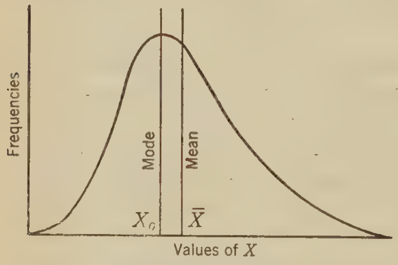

FIG. 2.—FREQUENCY DISTRIBUTION OF CHARACTERISTIC X

In order that data and statistical information may be used effectively, they will have to be condensed and presented in the form of a frequency distribution. Subsequently, the frequency distribution should be approximated by an algebraic distribution function $f(X, \bar{X}, \sigma, k)$ of the respective quality characteristic X (Fig. 2), containing the three principal statistics of the frequency distribution computed from the n observations X_n of the characteristic X; namely, its expected or mean value

$$\bar{X} = \frac{\Sigma X_n}{n} \qquad (7a)$$

its standard deviation

$$\sigma = \sqrt{\frac{\Sigma(X_n - \bar{X})^2}{n}} \qquad (7b)$$

and its skewness

$$k = \frac{\Sigma(X_n - \bar{X})^3}{n\,\sigma^3} \qquad (7c)$$

If the parameters of the distribution function are chosen so that

$$\int_{-\infty}^{+\infty} f(X, \bar{X}, \sigma, k)\, dX = 1 \qquad (8)$$

the integral $\int_a^b f(X, \bar{X}, \sigma, k)\, dX$ is a measure of the probability that an individual value X_n will occur within the limits $X = a$ and $X = b$.

The shape of the frequency distribution of a material property is an indication of the conditions under which this property has been, or is being, produced. In manufacturing processes an effort is generally made to attain a definite value of the characteristic property. This objective can be expressed either by stipulating a minimum value of the property or by imposing a limit on the fluctuations about its most probable value. In either case the objective cannot be achieved in any degree without imposing controls on the manufacturing process, in order to insure the systematic elimination of assignable causes of fluctuation, thus reducing the range of dispersion of the individual values. The visible effect of such control is to produce a unimodal frequency distribution of the characteristic bell shape, which, in the ideal case of maximum control, may be expressed by the normal or Gaussian law:

$$f(X, \bar{X}, \sigma) = \Phi_o(z) = \frac{1}{2 \cdot \pi} e^{-0.5z^2} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which e is the base of Napierian logarithms and

$$z = \frac{X - \bar{X}}{\sigma} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The distribution functions of real properties generally deviate more or less from the ideal form expressed by Eq. 9, even if the degree of production control imposed is high. They are mostly asymmetric and the most probable or peak value of the distribution—its modal value X_o—differs from the mean value $\bar{X}$ (see Fig. 2). The first two terms of the Gram-Charlier series[6] containing the skewness k:

$$f(X, \bar{X}, \sigma, k) = \Phi_o(z) - \frac{k}{6} \Phi'''_o(z) = \Phi_o(z) \left[1 - \frac{k}{6} (3\,z - z^3) \right] \ldots . (11)$$

(in which $\Phi_o(z)$ is given by Eq. 10 and $\Phi'''_o(z)$ denotes its third derivative) will be found to furnish fair approximations of real frequency distributions of controlled physical properties and phenomena (Fig. 3).

Eqs. 10 and 11 can also be used to advantage if the probability of occurrence is based, not on experience expressed by series of direct observations, but on the knowledge or assumption of an a priori probability P_p, since they represent good approximations of the binomial law $(P_p + P_q)^n$. This law is unsuitable for practical computation if the result desired is the probability that an individual occurrence will exceed, rather than equal, $P_p + j\,\sigma$. If P_p is known, the standard deviation σ and the skewness k are given by the expressions

$$\sigma = \sqrt{P_p\,P_q\,n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

and

$$k = \frac{1 - 2\,P_p}{\sigma} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12b)$$

If a physical property is manufactured without control, or if the designer has no conclusive evidence or knowledge of its being controlled, the best he can do toward determining the probability that individual values will fall within specified intervals is to use the theorem[7] proposed by P. L. Tchebycheff. According to this theorem, the probability that a value will fall outside the limits $\bar{X} \pm j\,\sigma$ is always smaller than $\frac{1}{j^2}$, regardless of the shape of the distribution function. The effect of quality control on the safety of structures is evident from a comparison of the accuracy of predictions based on the

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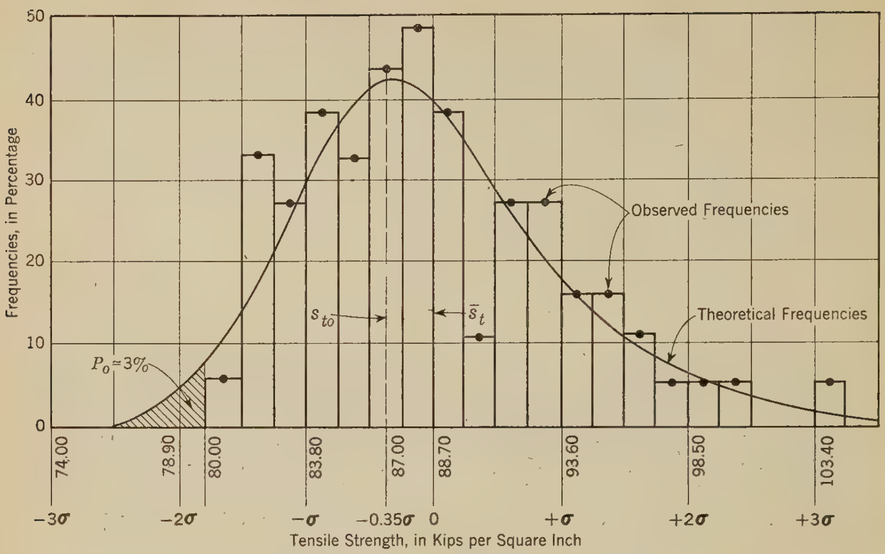
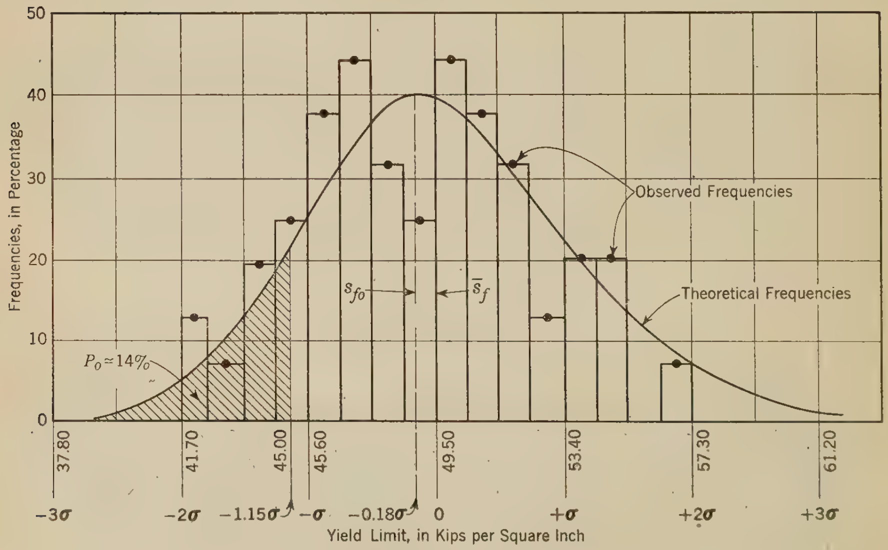
[6] "Probability and Its Engineering Uses," by Thornton C. Fry, D. Van Nostrand Co., Inc., New York, N. Y., 1928, p. 261.

[7] "A Treatise on Probability," by J. M. Keynes, Macmillan Co., New York, N. Y., 1929, p. 355.



(*a*) TENSILE STRENGTH: $\bar{s}_t = 88.70$ Kips per Square Inch; $\sigma = \pm 4.9$ Kips per Square Inch; and, $k = +0.72$



(*b*) ELASTIC LIMIT: $\bar{s}_f = 49.5$ Kips per Square Inch; $\sigma = \pm 3.9$ Kips per Square Inch; and, $k = +0.27$

FIG. 3.—COMPARISON OF OBSERVED AND THEORETICAL FREQUENCY DISTRIBUTIONS

Tchebycheff theorem with those derivable, under controlled conditions, by Eq. 10 or 11.

When imposing a standard of control on the manufacturing process of a given physical object the question necessarily arises as to whether the value aimed at should be stipulated as minimum limit or whether the inevitable dispersion of values should be controlled about an expected average. It can easily be shown that the setting of minimum limits, although still the prevalent conventional procedure, is considerably less effective in safeguarding such a limit and less reliable than the control of the fluctuations about an expected mean or most probable value. The current opinion that adequate safety can be insured only by specifying minimum limits is unfounded.

As an illustration consider the published results[8] of acceptance tests of structural shapes of silicon steel for the towers of the Golden Gate Bridge in California. The statistics have been computed, and frequency distributions of tensile strength and yield point developed from the published data, and the latter have been expressed in the form of Eq. 11 (see Fig. 3). The specifications for this bridge required minimum values of X_m = 80,000 lb per sq in. and 45,000 lb per sq in. for the tensile strength and the yield point, respectively. The integral

$$\int_0^{X_m} f(X, \bar{X}, \sigma, k)\, dX = p \qquad (13)$$

represents the "defective fraction" or the probability that an individual value will fall below the minimum limit stipulated. If two samples are taken from each melt and if the melt is accepted only when both samples prove satisfactory,[9] the probability that a given melt will be rejected will be $p^2 + 2\,p\,(1 - p) = 2\,p - p^2$ according to the binomial law. Hence, it will be more probable that a melt will be rejected than that it will be accepted only if $(2\,p - p^2) > 0.5$ or if $p > 0.3$; that is, if the defective fraction exceeds 30%, a possibility that is conceivable but under conditions of most inadequate control. Under the stipulation of a minimum limit, therefore, the chance of eliminating defective melts by the usual procedure of sampling is real only if the manufacture of the product is subjected to a practically inconceivable degree of control. Even under adequate conditions of control the occurrence of values lower than the stipulated minimum is still possible; this probability is computable by a statistical interpretation of the tests. The only possibility of reducing this probability practically to zero is by controlling the dispersion of values about an expected mean value, thus defining the minimum limit in terms of the probable extreme range of fluctuations below this mean value. The distribution functions reproducing the results of the check tests determine the range of fluctuation $\pm\, j\,\sigma$ about the mean, or the most probable value of X, in such a manner as to enable the designer to make the probability of an individual value smaller than $\bar{X} - j\,\sigma$ lower than any arbitrarily chosen figure. For a reasonably high level of control the introduction of $j = 3.5$ with regard to fluctuations about the mean value usually reduces this probability to less than 0.0001, as can be shown by evaluating the integral of Eq. 9. Fig. 3 shows that by controlling the dispersion of values around the mean, or the mode, and introducing the probable maximum range of fluctuations $\bar{X} - 2.5\,\sigma$ or $X_o - 2.15\,\sigma$ for tensile strength and $\bar{X} - 2.75\,\sigma$ or $X_o - 2.57\,\sigma$ for elastic limit, the minimum limit is safeguarded more effectively than by stipulating it in the conventional manner. This procedure should be adopted in engineering practice, and experience should be represented by the mean value $\bar{X}$ of the observations, or test results, together with its standard deviation σ and its skewness k. Only in cases in which the available data are unsuitable for statistical interpretation owing to deficiencies in scope and character will it be necessary to stipulate extreme limits for all possible values of the phenomenon or the characteristic considered. Only then will it be necessary to limit the maximum range of fluctuation by assumptions based on collateral information, and to introduce the midpoint between the extremes as a tentative approximation of the most probable value.

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[8] "Structural Applications of Steel and Light-Weight Alloys," A Symposium, *Transactions*, ASCE, Vol. 102, 1937, pp. 1328–1329.

[9] Standard Specification for Structural Steel, A.S.T.M., A-7-42, Philadelphia, Pa., 1943.

According to Eq. 3, the safety factor is derived from the ranges of probable maximum deviation of strain and resistance from their expected (that is, their most probable) values, which are to be considered the actual design values. Since the real distribution functions will usually be asymmetric and since, therefore, the mean value will not be identical with the modal value, it will be necessary to determine the modal value of X_o as the maximum of the function $f(X, \bar{X}, \sigma, k)$ by solving the equation

$$\frac{d}{dX}[f(X, \bar{X}, \sigma, k)] = 0 \ldots\ldots\ldots\ldots (14)$$

and introducing it as the design value and as the reference value for determining the relevant range of fluctuation.

CLASSIFICATION OF INFLUENCE BEARING UPON THE SAFETY OF STRUCTURES

The factor of safety is affected by two groups of influences:

(1) Influences that control the strain induced in the structure or the load that produces such strain; and

(2) Influences that control the resistance of the structure or its carrying capacity.

In both groups the existence of assignable causes as well as of chance causes may be presumed. Conditions for controlling all relevant properties and phenomena leading to the complete elimination of assignable causes and to the reduction of chance causes to a constant system of equivalent effects, which have been successfully imposed on highly organized industrial mass production, will necessarily be less perfect in the more individual, less standardized organization for the production of such units as civil engineering structures. In the latter case, economic considerations prevent the rigorous enforcement of conditions of maximum control, and they define a certain state of control as a limit beyond which assignable causes are to be considered chance causes. Beyond such a specified limit any further elimination of assignable causes would not be economically justified, since the control measures, when applied beyond a reasonable scope, are not only expensive, but may become a serious obstacle, impeding the rate of progress of construction so considerably as to outweigh all economic advantages. It is reasonable, therefore, not to lay too much stress on the discrimination between assignable and chance causes, but to regard the difference as a difference of grade rather than of principle, and thus to extend the concept of "chance causes" to cover also assignable causes left unidentified, completely or beyond a certain limit, because their identification would not be worth while.

For some purposes it will even be convenient arbitrarily to present genuine assignable causes as chance causes, particularly in connection with problems of design. Extensive and intricate computations, such as are sometimes required for the exact design of complex but relatively unimportant structures, are disproportionately accurate from an economic point of view. For the most part, the maximum range that embraces all possible values of the several characteristics of design, however, can be evaluated by a short-cut method; or such a range is known by experience. By introducing the midpoint value between the extremes as the "design value," a range of artificial "chance fluctuations" is created, arbitrarily, which will affect the safety factor by expressing it as a function of the accuracy of the design. In a highly refined design, in which the conditions of strain and resistance are reproduced perfectly, genuine chance fluctuations, mostly controlled, will affect the safety factor within comparatively narrow limits. A crude design, requiring only a rapid computation of approximate values (and therefore marked by a high degree of uncertainty represented by a wide range of "artificial chance fluctuations") will lead to a considerable increase in the safety factor required.

SUPERPOSITION OF INDIVIDUAL INFLUENCES

In order to superimpose the fluctuations of the individual constituent qualities of strain and resistance, respectively, the method of statistical superposition must be introduced. In the functional relationship

$$X = F(x_1, x_2, x_3, \cdots, x_n) \quad \text{(15a)}$$

the characteristics $x_1, x_2, x_3, \cdots, x_n$ are more or less controlled around the corresponding "design values" $x_{1o}, x_{2o}, x_{3o}, \cdots, x_{no}$ (which may be the means or the modal values) and the value of the principal characteristic X is subject to fluctuations $\pm j\sigma$ about its "design value":

$$X_o = F(x_{1o}, x_{2o}, x_{3o}, \cdots, x_{no}) \quad \text{(15b)}$$

The range of fluctuations $\pm j\sigma$ can be determined by applying the following rule for the evaluation of the standard deviation of the resultant characteristic from the standard deviation of the constituents:[10]

$$\sigma = \sqrt{a^2_1\sigma^2_1 + a^2_2\sigma^2_2 + a^2_3\sigma^2_3 + \cdots a^2_n\sigma^2_n} \quad \text{(16)}$$

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[10] "Economic Control of Quality of Manufactured Product," by W. A. Shewart, D. Van Nostrand Co., Inc., New York, N. Y., 1931, p. 393.

in which

$$a_n = \left(\frac{\partial F}{\partial x_n}\right)_{x_n = \bar{x}_n} \qquad (17)$$

Eq. 16 shows that owing to the relatively slight probability that all the component fluctuations will occur simultaneously, and in full intensity, the fluctuations of the resultant characteristic will be included within considerably narrower limits than might be inferred from the algebraic addition of the component fluctuations.

For the large number of material properties that have a bearing on the design, the level of control maintained will scarcely be uniform. The conditions of production vary considerably, not only with regard to the different structural materials but with regard to the individual stages of the construction process as well. Conditions of maximum control of certain properties may exist simultaneously with moderate control of other phenomena. In order to superimpose the fluctuations of the individual constituents of strain and resistance in spite of the different shapes of their distribution curves, the factor j (which relates the probable maximum range of fluctuations with the standard deviation of the frequency distribution) must be chosen individually for each component quality with regard to the particular degree of production control maintained. This must be done in such a way as to insure that the probability of occurrence of values beyond the range of variation considered shall be equal for each and every constituent. In other words, for strain all integrals $\int_{X_0+j\sigma}^{+\infty} f(X, \bar{X}, \sigma, k)\, dX$ of the distribution functions of the component qualities must be equal; and, for resistance, all integrals $\int_{-\infty}^{X_0-j\sigma} f(X, \bar{X}, \sigma, k)\, dX$ must be equal.

Assuming that different levels of control exist in different parts or in different stages of a construction process, distinguished by the factors j_k, j_l, j_m, etc., and leading to equal probabilities of defective individual values of the respective qualities, and considering absolute ranges of variability Δ_n of certain properties or phenomena, the range of fluctuation of the resultant characteristic X may be written in the form

$$\Delta X = \sqrt{j^2_k\, \Sigma(a_k\, \sigma_k)^2 + j^2_l\, \Sigma(a_l\, \sigma_l)^2 + j^2_m\, \Sigma(a_m\, \sigma_m)^2 \cdots + \Sigma(\Delta_n)^2}. \qquad (18)$$

SAFETY FACTOR AND PROBABILITY OF FAILURE

The safety factor, as expressed by Eqs. 2 and 5, is derived from the condition that it should prevent failure even if a highly unfavorable and very improbable upward fluctuation of strain or load were to coincide with a particularly unfavorable and improbable downward fluctuation of resistance or carrying capacity. If P_s denotes the probability of occurrence of an extreme strain $(s_{ao} + \Delta s_a)$ and P_r is the probability of occurrence of the minimum resistance $(s_{ro} - \Delta s_r)$, the probability of coincidence of both extremes—that is, the probability of failure P_f—will be, according to statistical principles

$$P_f = P_s\, P_r \qquad (19)$$

Denoting by n the expected number of load repetitions during the presumed period of service of the structure, $n\, P_f$ represents the number of load repetitions during this period which might actually lead to failure. In order to prevent even a single occurrence of this critical state of the structure it is necessary to stipulate that $n\, P_f < 1$, or that $P_f \doteq P_r\, P_s < \frac{1}{n}$.

Since, for known distribution functions of strain and resistance the choice of P_s and P_r determines the respective ranges of fluctuation, the foregoing condition expresses the necessary relation between these ranges for every probability associated with the prevention of failure. Drawing the distribution function of the resistance as abscissas and that of strain as ordinates and

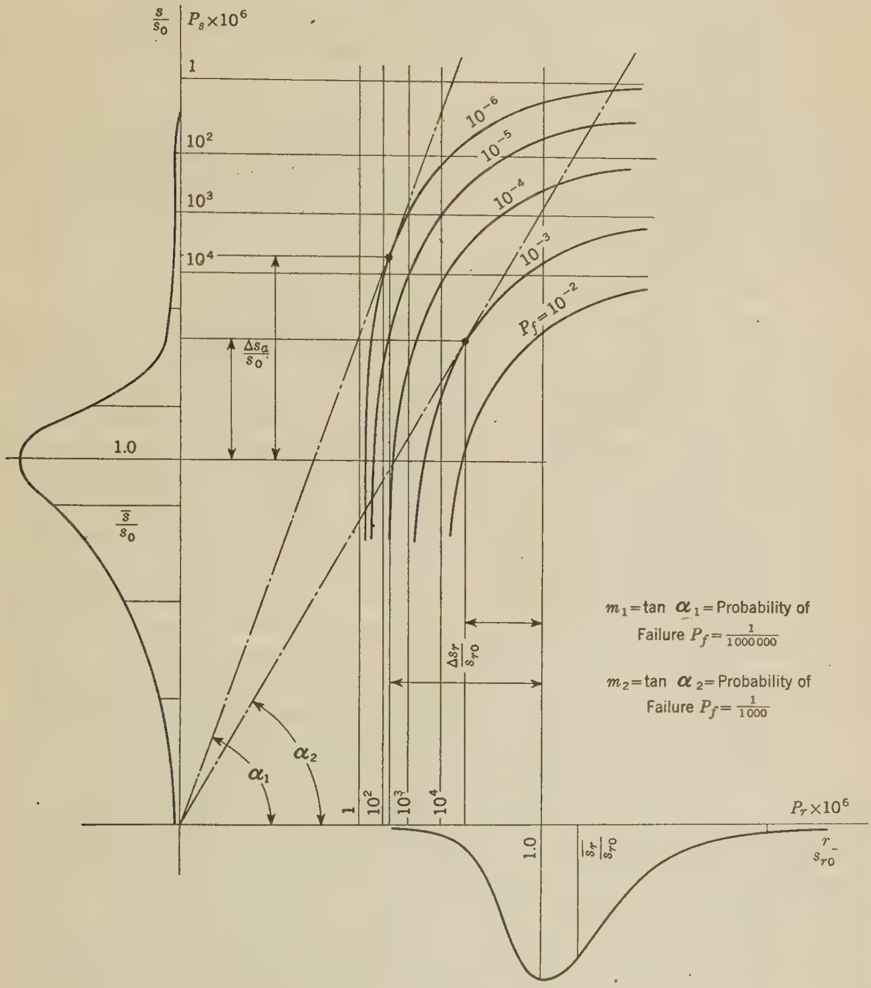

FIG. 4.—CURVES OF EQUAL PROBABILITY OF FAILURE (SEE EQ. 21)

indicating on each specific range of fluctuations $\frac{j\,\sigma_s}{s_{ao}}$ and $\frac{j\,\sigma_r}{s_{ro}}$, respectively, the pertinent probability of values falling outside this range, it is possible to trace the family of curves of equal probability of failure (see Fig. 4)

$$P_f = P_s\,P_r = \text{constant} \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The safety factor for each point on those curves is represented by the slope of the line connecting the respective point with the origin because

$$\tan\alpha = \frac{1 + \dfrac{\Delta s_a}{s_{ao}}}{1 - \dfrac{\Delta s_r}{s_{ro}}} = \zeta \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Since the highest safety factor pertaining to a certain probability P_f is represented by the slope of the tangent to the respective curve through the origin (which does not intersect any curve of higher probability) this slope expresses the safety factor which, when used in design, will prevent failure with at least the selected degree of probability.

Selection of Relevant Influences

Causes that affect the range of dispersion of the strain and the resistance values of a structure are almost infinite in number. A single component quality, such as the strength of concrete in compression, for instance, itself depends upon a large number of factors—the quality of the cement, quality of the aggregates, quality of the water, quantities of cement, aggregate, and water in the mix, influences of workmanship (mixing, transporting, placing, and curing), and uncontrollable external influences, such as changes of temperature and humidity of the air. Most of these individual factors, in their turn, are subject to further influences.

Thus, the compressive strength of concrete is actually a function of several thousands of independent factors. However, only a restricted number of influences will be assignable and relevant during a certain stage in the transition from the raw material to the completed structure. The designer, being concerned with a certain stage of this process, may regard the assignable causes of fluctuations relevant to previous stages as genuine chance fluctuations, if he is satisfied that those stages have been subject to a state of control consistent with the manufacture of a uniform product which is expected to comply with certain standard requirements. Therefore, the number of influences relevant in a certain stage of the construction process is only a small fraction of the actual number of influences.

The influences that affect the factor of safety will be divided into two groups bearing respectively upon strain and resistance or (whenever such correlation appears more expedient) upon load and carrying capacity. In addition to these principal groups, an intermediate group, embodying influences of the method and procedure of computation of strain, must be introduced.

The fundamental types of influences are enumerated in the following outline, separated into classes and groups in conformity with the aforementioned divisions.

Group A. Causes of Fluctuation in Strain.
- I. Uncertainty and variability of loading conditions.
  - a. Dead load.
  - b. Live load (including dynamic effects).
- II. Uncertainty and variability of external conditions that are independent of the load.
  - a. Change of temperature.
  - b. Wind forces.
  - c. Uncertainty of behavior of the subsoil.

Intermediate Group; Causes of Uncertainty of Strain Computations.
- III. Variation of rigidity.
- IV. Imperfection of methods and shortcomings of assumptions.
  - a. Accuracy of method and tolerances of numerical computation.
  - b. Inadequacy of assumptions concerning initial and boundary conditions, stress concentration, and secondary strain.

Group B. Causes of Fluctuation of Resistance.
- V. Uncertainty and inaccuracy of the assumed mechanism of resistance.
  - a. Inaccuracy or inadequacy of conceived mechanism.
  - b. Variability of resistance limits of materials.
- VI. Variation of structural dimensions.

In order to illustrate the determination of the appropriate design values of the individual component qualities and characteristics, and the establishment of ranges of dispersion about these values, some of the typical influences enumerated herein will be analyzed. Since neither the scope nor the character of the data that are assumed to represent past experience concerning these influences is adequate to the purpose of the present investigation, the following examination of the individual causes is tentative and suggestive rather than conclusive. Its main object is the indication of methods and of procedure to be adopted, deficiencies in the data to be supplied by future research. Consequently, the numerical values presented will have to be considered with some reservation, the main emphasis being laid upon the rational insight obtained into the true conditions of safety in engineering structures.

PART 2.—ANALYSIS OF PARTICULAR INFLUENCES

Dead Load

The fluctuations of the dead-load values are caused by the variability of the specific weight of the materials concerned and of the dimensions of the component parts of the structure. Therefore, the combined consideration of the tolerances of weight and dimensions will lead to a reasonably accurate estimate of the variation of the dead load.

Moreover, a discrepancy may generally be assumed to exist between the design load (mode of an assumed distribution curve) and the mean value due partly to the inaccuracy of the weights estimated (which are more frequently underrated than overrated), and partly to slight alterations of design during construction; also (except in very simple structures) the actual distribution of specifications for traffic loads. A certain degree of differentiation is justified, however, with regard to the volume of traffic, which will affect the number of load repetitions, but not the load intensity.

(4) One factor that exerts considerable influence upon the load intensity is the distance between motor vehicles moving along the same traffic lane. Evaluating investigations reported by the U. S. Bureau of Public Roads,[12] the relation between the center to center spacing l of vehicles and their traveling velocity v can be approximated by the formula

$$l = \frac{L}{1 - 0.15\, v/V}\,(1 \pm 0.20) \qquad (23)$$

valid for $0 \leqq v \leqq 60$ miles per hr, in which $V = 10$ miles per hr. For trucks, $L = 40$ ft; for passenger cars, $L = 30$ ft. A speed of 10 miles per hr denotes extremely congested traffic with all vehicles moving at equal speed; speeds exceeding 40 miles per hr are possible only if the rate of movement of each vehicle is independent, distances between adjacent vehicles being such that the equivalent loads exerted are very small.

In formulating specification for the design loads of railroad bridges, the frequency of occurrence of different types of load units is irrelevant. The design load should represent the heaviest train expected to run on the given track within the assumed period of service.

Dynamic Effects of Live Loads.—The action of the service load is generally considered as static in the design of buildings and as dynamic in the design of bridges. Actually, however, only a limited number of structures is subject to genuinely static load action (dams, aqueducts, etc.). The loads acting on most structures are variable and transitory. Whether the effect of such loads is static or dynamic is thus a question of the rapidity of application of the strain.

The dynamic behavior of the load intensifies the strain induced in the structure and thus affects its safety. The "dynamic increment" I, indicating the excess of the dynamic over the static load, is conventionally considered the dynamic characteristic of the design. This concept, however expedient it may be, hardly conveys a picture of the real conditions in the structure under dynamic load action. The complex process of transfer of potential (strain) energy into kinetic (vibration) energy cannot be expressed adequately by such a simple device, based on the unjustified assumption of proportionality between the effect of a static load and that of a load moving at a certain speed. This fact, although generally recognized,[13] so far has had little effect on experimental research, which has mainly been directed at the determination of definite numerical values of the dynamic increment. It may be anticipated, therefore, that the results of such research will appear widely scattered and that final conclusions of general validity cannot be formulated.

The main reason for the inconclusiveness of impact research is the fact that only one part of the strain in a structure subject to traveling loads is functionally dependent on major assignable causes such as masses, weights, velocities, dimensions, and elastic constants. The other part depends on more or less accidental circumstances and effects, such as the damping capacity of the structure, its state, the conditions of track or roadway, the wear of the wheels of the vehicles, etc. If, for the lack of a more expedient dynamic characteristic, the dynamic increment I is to be retained, and only the underlying concept is to be modified so far as to make it consistent with experience, it will be necessary to distinguish between assignable influences on this increment and chance influences, these being understood to cover genuine chance causes as well as unidentified assignable causes.

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[12] "Preliminary Results of Highway Capacity Studies," by O. K. Normann, *Public Roads*, February, 1939, Fig. 5, p. 227.

[13] "European Developments in the Study of Impact and Fatigue," by F. H. Frankland, *Civil Engineering*, April, 1940, p. 208.

The functional relation between the average value of the increment and the load impulse has been found to be very nearly linear.[14] The range of dispersion about this "line of best fit" must be expected to be greatest for structures of medium span for which vibrations represent a substantial source of strain. In such structures the range of magnitude of the frequency of natural vibrations compares with the frequency of the real load impulses; the intensity and number of such impulses are sufficient to overcome the inertia of the structure within the period that the load is acting. For these reasons, conditions approaching resonance may be reproduced. Short-span structures protected by their rigidity and the resultant high frequencies of natural vibration, and long-span structures protected by their great masses from being excited to considerable vibrations by the short action of comparatively light service loads, may be expected to show a relatively narrow range of dispersion of values of the dynamic increment. The range of fluctuation about the mean value $\bar{I}$ thus tends to decrease with spans increasing beyond, as well as dropping below, the critical medium-span range. Practically, however, only the tendency to decrease with increasing span is manifest. For short spans the growing influence of local irregularities of the load action, and of the path of the load upon the intensity of individual impact impulses, is strong enough to reverse the opposite trend.

The genuine chance effects are of a considerably narrower range of variation in railroad bridges than they are in highway bridges. In the latter, the intensity, as well as the frequency and number of consecutive load impulses, varies within wide limits owing to the complexity of traffic conditions and the fact that the local condition of the roadway may become deteriorated to an extent that would be practically unimaginable on a railroad track in service. A similar explanation accounts for the wear of rolling surfaces of vehicle wheels —another factor that intensifies the load impact.

The mean or "line of best fit" of observations[15] pertaining to a single motor vehicle traveling at the speed v may be represented by:

$$I_o = \frac{1.50}{1 + \dfrac{V}{3\,v}} \quad \text{.........................} \quad (24)$$

in which V = 10 miles per hr. The increment pertaining to n consecutive specifications for traffic loads. A certain degree of differentiation is justified, however, with regard to the volume of traffic, which will affect the number of load repetitions, but not the load intensity.

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[14] "Vibration Problems in Engineering," by S. Timoshenko, 2d Ed., D. Van Nostrand Co., Inc., New York, N. Y., 1937, p. 363.

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in which V = 10 miles per hr. The increment pertaining to n consecutive vehicles may be derived by assuming that the probability of the simultaneous occurrence of the dynamic increment I in n vehicles is governed by chance. Hence the most probable increment in an observation of n vehicles, I_{no}, is obtained by applying Eq. 16 to Eq. 24, thus

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[14] "Vibration Problems in Engineering," by S. Timoshenko, 2d Ed., D. Van Nostrand Co., Inc., New York, N. Y., 1937, p. 363.

[15] "Impact in Highway Bridges," by A. H. Fuller, *Final Report*, 2d Cong. for Bridge and Structural Eng., Springer, Berlin, 1929, p. 56.

$$I_{no} = \frac{1}{\sqrt{n}} I_o = \frac{1.50}{(1 + V/3\,v)\sqrt{n}} \qquad (25)$$

The influences of the structure, and of the irregularities of the roadway and the tires, are represented by the range of fluctuations about the foregoing average. For single vehicles acting on stringers of very short span, the range may be as great as ± 100% of the average if the possibility of trucks with hard-worn tires traveling on rather worn roadways is to be provided for. The dynamic increment for n vehicles of weight W may be written in the form

$$w_l I_n = \frac{1.5\,w_l}{(1 + V/3\,v)\sqrt{n}}\left(1 \pm \frac{1.00}{\sqrt{n}}\right) \qquad (26)$$

in which $w_l = \frac{\Sigma W}{n\,l}$ denotes the load intensity in pounds per linear foot of traffic lane. If the structure is long enough to accommodate more than a single load unit per lane, the load intensity w_l becomes a function of the traveling speed of the load, because the spacing between the consecutive load units increases with increasing speed. A critical speed for which the product of load intensity and dynamic effect $(1 + I)\,w_l$ attains a maximum value, and which represents the traveling speed of the design load, is obtained by a differentiation of this product. With the resulting $v/V = 1.15$, the "critical" velocity of the design load is shown to be not more than 11.5 miles per hr. Only for comparatively short spans does one single load unit per lane, traveling at maximum speed, represent the design load.

Any presentation of the dynamic effect of the load is incomplete without a knowledge of the velocity of strain, the maximum number of consecutive load impulses, and the total number of impulses on the structure during its entire expected period of service.

In short-span steel bridges for railroads, for which the share of the dead-load strain in the total strain is almost negligible, the velocity of specific impact strain may attain a magnitude of 0.8% per sec for mild steel, the assumption being that the ultimate strain may almost attain the yield-point limit if the service load is acting under the most unfavorable conditions. With increasing spans the velocity of strain due to vibrations becomes dominant, which is in the range of magnitude of 0.2% to 0.3% per sec for mild steel. In large-span structures the maximum rate of specific strain, due to transitory load movement, will generally remain below 0.05% per sec. The rates of strain application in highway bridges are considerably lower than those in railroad bridges because the "critical" speed of the design load is less than one fifth of the traveling speed of fast trains. The strain rates in structures of low-alloy steel are necessarily higher than those for mild steel; and the values for reinforced concrete structures are considerably lower owing to the high dead-load ratio.

Load repetitions on single-track railroad bridges seldom attain 1,000 per day even for very dense traffic of about 80 to 100 trains daily, since the critical impulses in short-span structures are exerted by the locomotive driving axles only. The number of repetitions of the load of railroad bridges may thus exceed ten million for short-span structural parts if a 30-yr period of service is assumed. This number decreases rapidly with increasing spans.

In order to estimate the number of load repetitions as well as the number of consecutive load impulses on highway bridges a certain amount of evidence concerning the volume and distribution of traffic is necessary. The evaluation of traffic records[16] suggests that dense traffic is concentrated over eight consecutive hours of the day only, the maximum hourly volume reaching 6% of the daily total. For short-span structures and structural parts, the number of repetitions of the design load is determined by the number of trucks passing. This number during a 30-yr period of service will probably reach ten million on main city thoroughfares with 4,500 vehicles per day per lane. For medium and large spans, however, the design load consists of a congested, slowly moving row of vehicles extending over the entire span or the critical length of the influence line. Its maximum frequency (although extremely improbable) would be reached if all the traffic during this period were distributed exclusively in groups representing the design load. The total number of design-load repetitions during the expected period of service would thus decrease rapidly with increasing spans. The numbers estimated for single lanes are considerably larger than those for multiple-lane bridges owing to the small probability that full design loads will occur at one time on different lanes.

CHANGES OF TEMPERATURE

Temperature changes in a structure involve chance fluctuations. The probability that extreme temperatures will occur is comparatively small. Changes of temperature within a restricted range, however, will occur frequently enough to justify their inclusion in any study concerning service conditions. Therefore, temperature changes should be divided, according to frequency of occurrence, into changes affecting normal conditions of service and changes over an extreme and infrequent range affecting the safety factor.

WIND FORCES

The magnitude and distribution of wind forces are chance events. Within a certain range of wind pressures the frequency is comparatively high and, consequently, the structure can be expected to withstand wind pressure permanently and with a degree of safety equal to that required for the principal service loads. Extreme values of wind forces, such as those that occur during severe storms, need be expected on rare occasions only. Hence, wind forces should be introduced into the actual design only as far as moderate and frequently occurring values are concerned, whereas maximum effects should be anticipated in the factor of safety. Long-time records of wind velocities will furnish the material for the preparation of distribution curves from which design values and fluctuations can be derived.

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[16] "An Automatic Recorder for Counting Highway Traffic," by R. E. Craig, *Public Roads*, May, 1938, p. 48.

UNCERTAINTY IN THE BEHAVIOR OF THE SUBSOIL

The foundation is an integral part of the structure, and its behavior, mainly depending upon the behavior of the subsoil, exerts considerable influence on the state of strain in statically indeterminate structures. The strain resulting from such movement in the structure is either entirely neglected in the conventional design (being based mostly on the assumption that the supports always remain fixed or that they settle uniformly, by equal amounts) or it is regarded as of secondary importance. The movement of the subsoil being a reality, this attitude is not justified.

Probably maximum values of settlement differences should be estimated with the aid of selected methods defined by soil mechanics, and the range of fluctuation of the dead-load strain, such as might actually occur when these differences are most unfavorable, should be established. Such fluctuations considered as chance events, varying about the design values of the dead-load stress, will necessarily increase the dispersion of the individual stress values and, consequently, will increase the required factor of safety. Owing to the high degree of rigidity and the almost perfect elasticity of the subsoil under the action of transient loads, the mutual effect of subsoil and structure is restricted to dead-load stress only.

VARIATION OF STRUCTURAL RIGIDITY

The rigidity of a structural part is expressed as the product of a sectional value, such as the moment of inertia or the area, and the respective modulus of elasticity of the material, divided by the length of the member. The degree of rigidity will vary, therefore, with changes of either the modulus of elasticity or the sectional value, or both, fluctuations in length being insignificant.

It is evident that the rigidity of all parts or members of one and the same structure will not necessarily be subject to uniform variation. Allowance must be made for the fact that the rigidity of individual parts or members will differ among themselves. The effect of such variation upon the strain of redundant structures may be determined by a method which has been called "analytical experiment" by Hardy Cross,[17] M. ASCE. This method consists of the actual computation of certain characteristics of strain in a structure subject to any possible combination of variation of rigidity in its individual parts or members.

In metallic structures subject to air temperatures only, no change of the initial rigidity need be expected unless the cross sections are reduced by deterioration. In concrete and reinforced concrete structures, however, the additional deformation (creep) that occurs under action of sustained compressive loads makes the initial modulus of elasticity appear to decrease. This apparent decrease affects the state of strain, and thus the safety, of plain and reinforced concrete in which sustained compression is a dominant phenomenon, such as redundant large-span arches and pre-stressed structures.[18]

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[17] "Dependability of the Theory of Concrete Arches," by Hardy Cross, *Bulletin No. 203*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[18] "Theory of Wide-Span Arches in Concrete and Reinforced Concrete," by A. M. Freudenthal, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 4, Zurich, 1936, pp. 249–264.

Accuracy of Methods and Tolerances of Numerical Computation

Methods of computing strain characteristics are generally based upon the assumption of perfect elasticity. This assumption is justified for metallic structures under air temperatures, and it is correct even for concrete structures strained within the range of the conventional working stresses. If the strict methods of the theory of elasticity are applied to the design, and if the respective boundary conditions are selected so as to reproduce the actual behavior, the resulting strain values are within the range of accuracy of the assumed boundary conditions.

If short-cut methods are applied as substitutes for a strict theoretical analysis, the range of error must be compensated by an appropriate increase in the general factor of safety. The functional dependence of the safety factor on the relative crudeness of the design method is thus established. The maximum range of possible error must be ascertained reliably and subsequently introduced as a constituent chance fluctuation.

Boundary Conditions, Secondary Strain and Stress Concentration

Boundary conditions are expressed in terms of displacement and the angular deflection of end sections. The boundary conditions for the supports may be derived from observations of the movement of the foundations and the subsoil. There are cases, however, in which boundary conditions are chosen rather arbitrarily or, to simplify the mathematics, without satisfactory evidence that the choice is rational.

The manner in which the boundary conditions for end sections are assumed will considerably influence the actual safety of members subject to bending or buckling. It is doubtful that any assumed degree of fixity at the ends will ever be realized precisely in the actual structure. Therefore, minimum and maximum values of the probable angular deflection at the ends should be estimated, and the mean of these values should be selected as the design value, the range of variation about the mean being considered a constituent part of the safety factor.

An important group of influences upon the safety of structures is represented by the effects of secondary strain and stress concentrations, mostly occurring at or near structural connections—thermal stresses in welds, stress concentrations around rivets, residual stresses due to rolling or cold-working, shrinkage stresses in reinforced concrete, and stress concentrations produced by surface conditions due to corrosion. The magnitude and effect of all these influences depend on a large number of chance causes and vary considerably, often in a rather erratic manner. Their effect upon the strain should be reflected in the design by reducing the resistance values rather than by increasing the intensity of the strain. In this paper, therefore, they will be treated in connection with resistance problems.

Uncertainty and Inaccuracy of the Mechanism of Resistance

The resistance of engineering structures is affected by the character of the load, by external conditions, and by the characteristic properties of structural materials. In order to reproduce the real phenomenon in a form suitable for design, abstractions and assumptions must be introduced.

Structural resistance is defined, generally, with reference to the state of strain which delimits the fitness for service of a structure. It is principally a function of two variables, a sectional characteristic and the resistance (strength) of the material. For practically every structural material this resistance is influenced by the rate at which strain is applied—the same material that appears permanently deformable and tough under low strain rates will appear perfectly elastic and brittle under impact strain and will show increased strength.

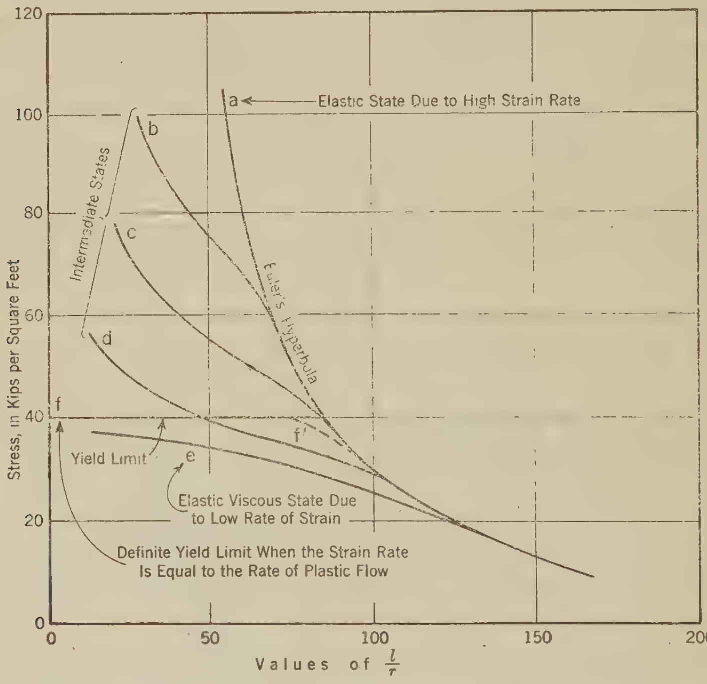


Fig. 5.—Buckling Stress s_b as Function of Slenderness Ratio l/r and the Strain Rate Applied

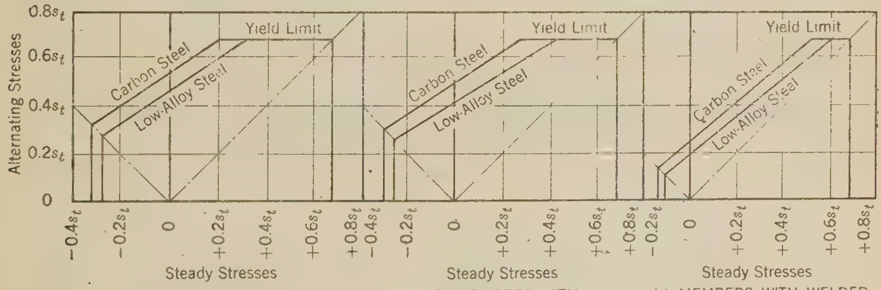
In certain cases resistance is not a function of sectional strength alone but also of the structure's stability, as in members subject to axial compression. The "buckling stress" embodies both sectional strength and geometrical criteria. Fig. 5, which presents the buckling stress as a function of the slenderness ratio l/r and of the strain rate applied, illustrates the velocity sensitiveness of the buckling phenomenon. This aspect of the buckling problem is important in the design of compressed members under conditions of impact, under which the Euler hyperbola applies over a wider range than is conventionally considered elastic, and under conditions of creep under which the elastic range is considerably diminished.

The real resistance mechanism is normally rather complex, and it is hardly possible to conceive such a mechanism that will effectively reproduce the actual phenomenon and which is at the same time simple enough to be suitable for practical design. Every devised mechanism, therefore, is fictitious to a certain degree; but it will embody the relevant physical qualities that are determinable by simple standardized tests. The efficiency of the mechanism is ascertained by statistically interpretable experiments, and its reliability and the range of dispersion of individual values about the average trend established. Its suitability is to be judged by the simplicity of the concept, by the closeness with which experimental results are reproduced, and by the narrowness of the range of dispersion of such results about the "theoretical" course.

The range of uncertainty characterizing a certain resistance mechanism will be affected by the state defined as undesired with regard to the safety of the structure. Satisfactory correlation of the structural resistance limits which determine this state, with the material strength limits which can be observed on reproducible test specimen, has so far been attained for the most simple conditions only.

UNCERTAINTY AND VARIATION OF RESISTANCE LIMITS OF STRUCTURAL MATERIALS

The mechanical resistance of engineering materials is affected by the strain field, the rate and duration of strain, the amplitude of strain cycles, and their number. The influence of the strain field is generally expressed by the conceived mechanism of resistance. Effects of local concentrations of strain due to notches, welds, or rivets are not embodied in the general mechanism of resistance but are considered by empirically reducing the resistance (strength) of the material in the undistorted field of strain. Since the resistance of a material relevant for purposes of structural design is its resistance at, and in, the vicinity of the heaviest concentration of strain which occurs within the areas of the structural connections, the specific resistance of the material at, or near, connections of standard shape must be determined experimentally.



(*a*) PLAIN MEMBERS (with Mill Scale)

(*b*) MEMBERS WITH RIVETED CONNECTIONS

(*c*) MEMBERS WITH WELDED CONNECTIONS (Fillet-Welds)

FIG. 6.—ENDURANCE LIMIT IN TERMS OF ULTIMATE TENSILE STRENGTH s_t AS FUNCTION OF AMPLITUDE OF STRESS OSCILLATIONS ABOUT STEADY STRESS

The resistance-reducing effect of structural connections subject to load cycles of various amplitudes has been investigated extensively for steel.[19,20]

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[19] "On the Fatigue Strength of Riveted and Welded Joints and the Design of Dynamically Stressed Structural Members Based on Conclusions Drawn from Fatigue Tests," by K. Schaechterle, *Publications*, International Assn. for Bridge and Structural Eng., Zurich, Vol. 2, 1933–1934, pp. 312–379.

[20] "On the Fatigue Strength of Riveted and Welded Joints Made of Steel St. 52," by E. H. Schulz and H. Buchholtz, *ibid.*, pp. 380–399.

Fig. 6 shows the results of the investigations in a concise form, relating the endurance limit for various amplitudes of stress cycles repeated two million times, and the tensile strength s_t. Owing to the different character of the connections different relationships are obtained for riveted and for welded connections. Different limits are also obtained for different numbers of stress cycles (Fig. 7). The results presented in Figs. 7 and 8, however, have been derived from arbitrarily defined minimum values of the endurance limit; no statistical interpretation has been attempted and no adequate information is

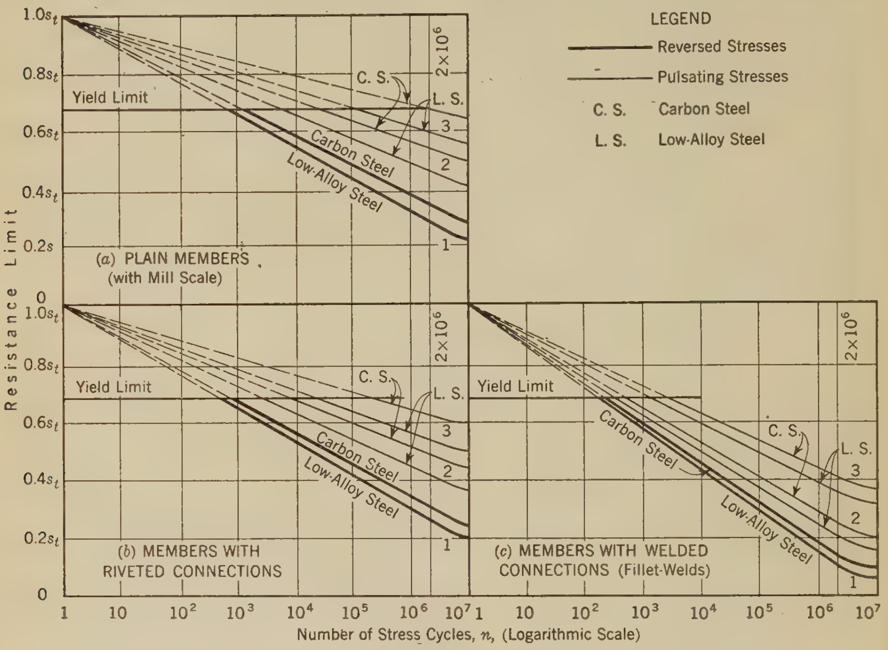

FIG. 7.—ENDURANCE LIMIT IN TERMS OF ULTIMATE TENSILE STRENGTH s_t AS FUNCTION OF NUMBER OF STRESS CYCLES ("1" FULL REVERSAL OF STRESS, "2" STRESS PULSATING BETWEEN ZERO AND LIMIT, "3" STRESS PULSATING BETWEEN 0.2 s_t AND LIMIT)

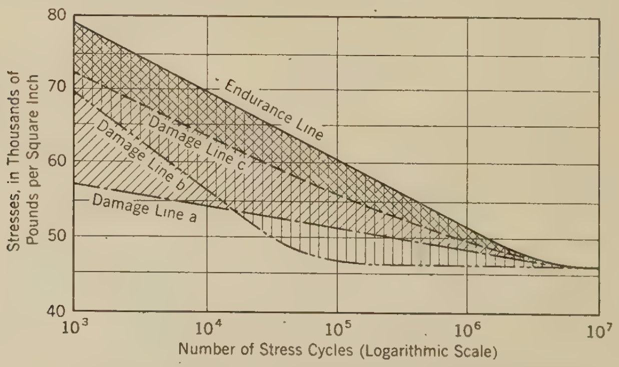

FIG. 8.—CHARACTERISTIC DAMAGE LINES AND DAMAGE RANGES

available concerning the range of fluctuations of the dynamical resistance limits in general. This range will certainly exceed that pertaining to static resistance limits.

In order to ascertain the range of validity of the diagrams of Fig. 7, two different sets of conditions of load application should be considered—(1) loads inducing strain rates that exceed the rate of permanent deformation (rate of yield) of the material and (2) loads inducing lower strain rates. The fact that, in the latter case, the yield point cannot be exceeded furnishes the criterion which defines a critical number of load cycles below which the resistance under conditions of static strain is lower than that under conditions of rapid load repetition.

The picture of the dynamical resistance of metals would be incomplete without the inclusion of the concept of "damage" by overstrain which has considerable bearing on the safety of structures. The "damage range"[21,22] expresses the extent of damage with regard to the endurance limit which is caused by incidental overstrain. The "damage line" delimits this range by relating the amplitude of overstrain that can be sustained by the material without the final endurance limit being affected, to the maximum permissible number of repetitions of this overstrain cycle (Fig. 8). Information about this phenomenon is scarce and refers to conditions of total load reversal only.

The capacity of a structural material to sustain occasional overstrain without damage to its ultimate endurance limit is very important from a design point of view. According to the previously stated stipulation, the design live load should provide for service conditions of reasonably high frequency of occurrence, whereas less probable, and therefore less frequent, extreme loading conditions should be covered by the factor of safety. If the material is able to carry this occasional excess of extreme loads over the design load by its capacity to sustain occasional overstrain without damage to its ultimate endurance limit, this limit may be directly correlated with the design load without introducing any margin of safety that would otherwise be necessary to cover the fluctuations of the load intensity above the design value specified.

Resistance to "damage" by overstrain of structural materials should be made an urgent object of research. The results of such research will most certainly have some influence upon the manufacture of such materials, since it will reveal the relation of "damage sensitivity" and the general mechanical properties. Structural metals can have "damage lines" of different shapes but still have similar standard properties, such as static strength, elongation, and endurance limit. In Fig. 8 line a characterizes a material with a capacity to sustain frequent overstrains of moderate intensity, line b a variety of the same material with a capacity to sustain considerable overstrain if its frequency of occurrence is low, line c a variety with a remarkably high resistance against "damage" by frequent and high overstrain. The endurance limit as a function of the number of load repetitions is the same for all three varieties; but their performance will be different and will depend on the frequency of overstrain

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[21] "Damage and Overstress in the Fatigue of Ferrous Materials," by H. W. Russel and W. A. Welcker, *Proceedings*, A.S.T.M., 1936, Pt. II, pp. 1118–1135.

[22] "Effect of Overstressing and Understressing," by J. B. Kommers, *ibid.*, 1938, Pt. II, pp. 249–268.

and its amplitudes. Every one of the three varieties of the material is suited to a particular character of load fluctuation.

Like all physical properties the resistance values of structural materials are constants in a statistical sense only; the level of control enforced in all stages of the manufacturing process determines the character of the distribution function. In Fig. 3 the distribution functions for the yield point and the tensile strength of the silicon steel manufactured for the Golden Gate Bridge have been drawn. The practical range of chance fluctuations reaches about 15% below the mean or about 12% below the modal value. Concrete is less uniform than steel; investigations[23,24] have shown maximum ranges of fluctuations of individual values of compressive strength of concrete manufactured on the site under normal control to be approximately 35% about the mean; this range is reduced to one half by control under laboratory conditions.

Fluctuations and Uncertainty in the Dimensions

The ranges of variation of the sectional resistance values can be derived from the tolerances of the linear dimensions. Since these tolerances do not generally exceed 2% to 3% the fluctuations of areas and section moduli are within the range of 4% to 6% about the mean.

Design as a process of prediction, however, is not concerned only with the initial state of the structure but also with its change in the future owing to deterioration and corrosion. These influences must be considered, therefore, in the computation of the initial safety factor.

A more detailed treatment of all influences is included in the original manuscript which has been placed on file for reference in the Engineering Societies Library in New York, N. Y.

Application of Methods Developed

The method of evaluating the appropriate safety factor will be illustrated and its practical application demonstrated by computing the working stresses to be used in the design of stringers and floor beams of highway bridges of low-alloy (silicon) steel.

In order to derive working stresses or safety factors it is necessary to establish a uniform basis of reference since both concepts have a definite meaning only in connection with a specified service load. The evaluation of this load must be the first step in any analysis of this kind. It requires the computation of the probabilities of occurrence of different groupings of pertinent load units with reference to the definition of an adequate design live load.

Under the assumption that one fourth of all vehicles are trucks or units of similar load intensity, and that only the heaviest types circulating at present are considered (see conclusion 2 under the heading "Live Load: Intensity and Distribution of Live Loads for Bridges"), the frequency of occurrence of trucks among a total number $n = t\,m$ of vehicles (in which t denotes the number of

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[23] Discussion by C. E. Arnold of "Manufacturing Concrete of Uniform Quality," by William M. Hall, *Transactions*, ASCE, Vol. 96, 1932, pp. 1367–1371.

[24] "Betondruckfestigkeit," by D. Bendel, Rept. of Cong. for Reinforced Concrete, Liege, Belgium, 1930, p. 238.

traffic lanes and m the number of vehicles per lane) can be derived by the binomial law and approximated by the Gram-Charlier series. The frequency distributions for $n = 6$, 12, 24, and 96 vehicles, respectively, have been presented in Fig. 9. With the aid of Eqs. 23 and 25 and introducing $v = 11.5$ miles per hr as the "critical" speed of the design load, the design load intensities —including the dynamic effect $w_{lo}\ (1 + I_o)$ pertaining to the respective modes of the distribution functions as well as the ranges of fluctuation of those functions pertaining to the probability $P_s < 0.001$ of values to fall outside this range—can be established as shown in Table 1.

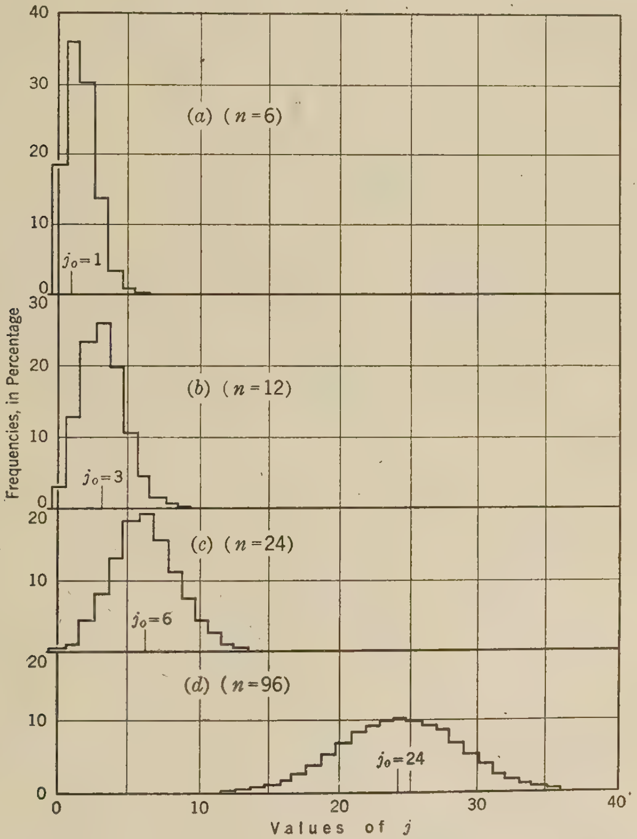

FIG. 9.—FREQUENCY DISTRIBUTION OF THE NUMBER OF TRUCKS COUNTED IN A TOTAL NUMBER n OF VEHICLES

For short spans the passing of a single truck per lane traveling at maximum speed may produce higher design load intensities than those given in Table 1. A direct comparison of load intensities would be misleading, however, because the difference in the ranges of fluctuation for the two types of loading results in different values of the safety factor to be applied.

Introducing the bending moment as a strain characteristic

$$s_a = M = C_1 \, w_d \, l^2 + C_2 \, (1 + I) \, w_l \, l^2 \ldots\ldots\ldots\ldots\ldots (27)$$

and as resistance characteristic the product of section modulus S and strength of the steel at failure s_f

$$s_r = S \, s_f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

TABLE 1.—DESIGN LIVE LOADS PER LANE AND RANGES OF FLUCTUATIONS[a]

Number of vehicles, n	NUMBER OF TRUCKS		DESIGN VALUES			Maximum w_l (lb per ft)	MAXIMUM FLUCTUATION (%)			LOADED LENGTH (FT)			
	Modal	Maximum	w_{lo} (lb per ft)	$1 + I_{no}$	Col. 4 × Col. 5 (lb per ft)		$+\Delta w_{lo}$	$+\Delta w_l$	$+\Delta(1 + I)$	$t = 1$	$t = 2$	$t = 4$	$t = 6$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
6	2	5	585	1.48	865	925	25	58	28	240	120	60	40
12	3	8	510	1.33	680	830	25	63	22	480	240	120	80
24	6	14	510	1.24	630	770	25	51	17	940	470	235	117
96	24	36	505	1.12	565	745	25	47	10	3,800	1,900	950	475

[a] Ranges of fluctuation pertain to probability $P_s < 0.001$.

—and applying Eqs. 17 and 18:

$$\left.\begin{aligned} a_1 &= \frac{\partial M}{\partial w_d} = C_1 \, l^2 \\ a_2 &= \frac{\partial M}{\partial w_l} = C_2 \, (1 + I_o) \, l^2 \\ a_3 &= \frac{\partial M}{\partial (1 + I)} = C_2 \, w_{lo} \, l^2 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (29)$$

and

$$\left.\begin{aligned} (a_1)' &= \frac{\partial s_r}{\partial S} = s_{fo} \\ (a_2)' &= \frac{\partial s_r}{\partial s_f} = S_o \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (30)$$

Hence

$$s_a = s_{ao} \left(1 + \frac{\Delta s_a}{s_{ao}} \right) = [C_1 \, w_{do} \, l^2 + C_2 \, (1 + I_o) \, w_{lo} \, l^2]$$

$$\times \left[1 + \frac{1}{1 + \dfrac{C_1}{C_2} \dfrac{w_{do}}{w_{lo} \, (1 + I_o)}} \right]$$

$$\times \sqrt{\left(\frac{C_1}{C_2}\right)^2 \left(\frac{w_{do}}{w_{lo}}\right)^2 \left(\frac{\Delta w_d}{w_{do}}\right)^2 + \left(\frac{\Delta w_l}{w_{lo}}\right)^2 + \left[\frac{\Delta(1 + I)}{(1 + I_o)}\right]^2} \ldots\ldots (31)$$

and

$$s_r = s_{ro} \left(1 - \frac{\Delta s_r}{s_{ro}} \right) = S_o \, s_{fo} \left[1 - \sqrt{\left(\frac{\Delta S}{S_o}\right)^2 + \left(\frac{\Delta s_f}{s_{fo}}\right)^2} \right] \ldots\ldots (32)$$

For the freely supported short-span structural members considered (stringers and floor beams) the respective design load intensities are produced by one truck per lane traveling at a maximum speed of 50 miles per hr. The ratios $\frac{w_{do}}{w_{lo}}$ may be assumed to vary between 0.1 (for stringers) and 0.4 (for heavy floor beams). With such comparatively small effects of the dead load, the strain may be assumed to fluctuate between zero and a maximum value. The total number of strain cycles generally will reach, or even exceed, the two-million mark for bridges carrying main highways, the rate of strain induced by the rapidly moving load being in the vicinity of 0.3% per sec in structures of low-alloy steel. Since this rate exceeds considerably the intrinsic plastic flow rate of steel of about 0.05% per sec, failure will be brittle, and the structure's resistance will be a function of the ultimate tensile strength s_t of the material. For a load oscillating between zero and a maximum value, numbering a total of more than two million cycles, the actual strength of riveted structural parts (floor beams) is $s_{fo} = 0.4\,s_{to}$ and of plain members (stringers) is $s_{fo} = 0.45\,s_{to}$ (see Fig. 7).

Let: $\frac{\Delta w_l}{w_{lo}} = 0$ for one-lane and two-lane highways on which the design load is equal to the maximum load; $\frac{\Delta w_l}{w_{lo}} = 0.58$ for six-lane highways (see Table 1); $\frac{\Delta w_{do}}{w_{do}} = 0.1$ (in which, see Eq. 22, $w_{do} = 1.1\,\bar{w}_{do}$); $\frac{\Delta(1 + I)}{1 + I_o} = \frac{1.00}{(1 + I_o)\sqrt{n}}$ (see Eq. 26); $\frac{\Delta S_a}{S_a} = 0.06$; and $\frac{\Delta s_t}{s_{to}} = 0.12$ (see Fig. 3). For free supports $C_1 = C_2$. Inserting the foregoing values in Eqs. 5, 31, and 32, the ranges Δs_a and Δs_r, the safety factor ζ, and the allowable stresses $\frac{1}{\zeta} s_{fo}$ are found to be:

For stringers ($t = 1$)—

$\Delta s_a = +\,0.40\,s_{to}$; $\Delta s_r = -\,0.14\,s_{ro}$; $\zeta = 1.64$; allowable stress $s = 0.275\,s_{to}$

for floor beams ($t = 2$)—

$\Delta s_a = +\,0.32\,s_{to}$; $\Delta s_r = -\,0.14\,s_{ro}$; $\zeta = 1.54$; allowable stress $s = 0.260\,s_{to}$

for floor beams ($t = 6$)—

$\Delta s_a = +\,0.51\,s_{to}$; $\Delta s_r = -\,0.14\,s_{ro}$; $\zeta = 1.75$; allowable stress $s = 0.229\,s_{to}$.

The comparatively high value of the safety factor for six traffic lanes is due to the fact that for six vehicles the design load intensity is no longer represented by the maximum load intensity (see Fig. 9 for $n = 6$).

The data utilized in this example are not elaborate enough to enable one to evaluate the probability of failure pertaining to the foregoing safety factors. A rough estimate becomes possible, however, if the distribution functions of the constituents w_l and s_t are assumed to be representative of the frequency distributions of the resultant characteristics s_a and s_r. For the six-lane floor beam $P_s < 6\,(\frac{1}{4})^5\,(\frac{3}{4}) = 0.005$ and $P_r < 0.0001$. Therefore, the probability of failure $P_f < 1/2{,}000{,}000$ as required for an expected number of load cycles which slightly exceeds 2,000,000.

In order to compare the foregoing with values specified in actual design a common base of reference is found in the "reduced" permissible stress s', which is the stress permissible under the action of dead load $\bar{w}_{do}$ and the static weight of the maximum live load w_l (max), if the specified safety of the design is to be maintained. Hence, for a uniformly distributed live load and freely supported span

$$s' = s \frac{\bar{w}_{do} + w_l\,(\max)}{w_{do} + (1 + I_o)\,w_{lo}} \qquad (33)$$

In the design of the Golden Gate Bridge for which the frequency distribution of the tensile strength s_t of silicon steel was established (Fig. 3), and the permissible stress specified for stringers and floor beams was 22,000 lb per sq in. to be applied with an impact increment of 50% of the live load for stringers and 25% for floor beams. For the latter, a reduction of 25% of the live load was intended to allow for the large number of traffic lanes ($n = 6$).[25] These stipulations lead to the following values for the reduced permissible stress: For stringers—$s' = 0.69 \times 22{,}000 = 15{,}000$ lb per sq in.; and for floor beams—$s' = 1.04 \times 22{,}000 = 23{,}000$ lb per sq in. The reduced permissible stresses resulting from the present analysis of the safety factor can be computed by introducing the mode of the frequency distribution (Fig. 3), $s_{to} = 87{,}000$ lb per sq in. Hence: For stringers—$s' = 0.44 \times 0.275 \times 87{,}000 = 10{,}500$ lb per sq in.; and for floor beams—$s' = 1.35 \times 0.229 \times 87{,}000 = 27{,}000$ lb per sq in.

Comparison of the foregoing values shows that even a complex design (such as that of the Golden Gate Bridge, which empirically provides for the different probabilities of occurrence of maximum load intensity in different structural parts) will not yield adequately balanced stresses and dimensions as long as the safety factor is being "selected" instead of computed by rational and objective methods. It appears that the permissible stress specified is too high for the stringers on Golden Gate Bridge, and unduly low for the comparatively long-span floor beams.

PART 3.—CONCLUSIONS

The safety factor is an objective value of correlation between actual strain and potential resistance that can be ascertained by the rational methods suggested in the paper. Loads, external conditions, and pertinent physical properties and phenomena are analyzed under the common aspect of their relation to the magnitude of the safety factor. This factor is intended to provide for relatively infrequent loading, and other conditions, whereas the design itself considers conditions of relatively frequent occurrence. The computation of the objective minimum safety factor and the consequent permissible stresses is in a balanced design of no less importance than the determination of the principal static characteristics.

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[25] "The Golden Gate Bridge," Rept. of the Chf. Engr. to the Board of Directors of the Golden Gate Bridge and Highway Dist., Schwabacher-Frey, San Francisco, Calif., 1937, p. 81.

Since the safety factor, expressing the excess of possible, or potential, resistance over the computed strain to be provided in order to prevent failure with a reasonably high probability, derives its existence from the imperfection inherent in the performance of any human action and the uncertainty marking the results, as well as from the acknowledged ignorance about certain phenomena, and since it relates computed strain to actual resistance, it can refer to the design as an entity only and not to different features of it. Therefore, it can have only one value for a certain design. Individual safety factors for different influences, such as for dead load or for live load, separately, being inconsistent with the basic concept, can have no real meaning.

The current specifications and standards applying to design loads and secondary influences, as well as strength and permissible stresses, should be analyzed as to their correspondence to reality, and the objective conditions should be ascertained as a base for new and more adequate specifications. Conventional specifications seem too lenient as far as short-span structural members are concerned, and unnecessarily severe for structural members and structures of medium and long span. The use of adequately balanced permissible stresses in the different elements of a structure will solve, automatically, the problem of the occasional passage of exceptionally heavy loads, raised in connection with the transport of military equipment with very heavy axle concentrations.[26]

In spite of extensive experimental research concerning a great number of structural qualities, lack of adequate information about certain basic aspects of design calls for urgent attention and an effort to make up the deficiencies. Some of the most important effects, such as that of the rate of the induced strain, or the capacity of the material to sustain occasional overstress, have been almost neglected. The evidence applying to other phenomena and qualities which have actually been investigated has generally not been collected with regard to subsequent statistical interpretation, and therefore needs amplification.

By adopting the principle that neither design loads nor safety factors and permissible stresses should be specified arbitrarily, it will be possible not only to eliminate inadequate design, but frequently to achieve considerable economy. It will be possible, moreover, to determine correct safety factors and permissible stresses for unconventional structures or new structural forms and materials.

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[26] *Civil Engineering*, February, 1941, p. 116.

DISCUSSION

F. H. Frankland,[27] M. ASCE.—The structural designer is concerned with the solution of probability problems by the consideration of the various factors governing his design, such as safety, use characteristics, economy, future suitability, and durability. Although, in recent times, structural engineering has undergone changes at an accelerated pace, structural engineers have not generally kept pace with the rapid advance made in some other branches of engineering.

The author states, in the "Synopsis," that: "The word 'resistance' is a general term indicating 'strength' or capacity to resist failure." It is important that, in discussing safety factors, this use of the term "resistance" should receive general acceptance and understanding.

The safety of a design is not determined by the use of an average factor of safety but is governed by the safety factor at the weakest part. However, in most structural systems there is often a redistribution of stress, under load, throughout the system which makes the resulting safety factor greater than that of any one part:[28]

> "In practically all cases a structure designed by the use of conventional methods, with a high safety factor, will be less safe than one designed by the use of more refined methods and using a smaller safety factor. The conventional safety factor idea is that the safety factor for a particular design under given loading conditions is a constant coefficient; but actually the safety factor varies."

The skill of the designer is indicated, in part, by his understanding of the degree to which elasticity and plasticity influence the behavior of a structure because of geometrical displacements under load in the various parts and the consequent possible creation of multiaxial and other secondary stresses.

The author states (under the heading, "The Factor of Safety"):

> "With increasing perfection of design methods the element of 'ignorance' can be largely eliminated; but the element of 'uncertainty' is caused by circumstances that can be changed, to a certain extent, but can never be removed."

Nevertheless, little real betterment of design methods can be achieved by "refinements" in the application of mathematical theories unless it is accompanied by an intelligent understanding of the behavior of structures in use. Such refinements mean little when basic assumptions are wide of reality. It is of prime importance that engineers realize that there is but little excuse for the element of so-called ignorance and that the safety factor is a matter governed mainly by the degree of uncertainty. Generally accepted design methods too often neglect stress concentrations and the effects of fatigue and consider that materials possess perfectly elastic properties. It should be remembered that, for high-alloy structural steels, such as 18–8 stainless and nonferrous structural alloys like aluminum and magnesium, the elastic limit is indefinite and the stress-strain curve is continuous. The general and exclusive use of the elastic theory in the design of structures is partly responsible for the lack of understanding among many engineers of the actual behavior of structural materials. The "aspect of practical application" which the author refers to in "Part 1.—General Principles: Introduction" should be clear in the mind of the designer at all times. As a consequence of the better understanding of the behavior of structures and structural materials, referred to by the author, some of the generally accepted underlying concepts of structural design are inadequate.

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[27] Cons. Engr., New York, N. Y.

[28] "Modern Stress Theories," by A. V. Karpov, *Transactions*, ASCE, Vol. 102, 1937, p. 1207.

Recent investigations of stress distribution in structures under load clearly justify dissatisfaction with design methods which have hitherto been accepted as adequate. In this connection it would be well to study the paper "Theory of Elastic Stability Applied to Structural Design."[29]

An understanding of the theory of limit design will explain the capacity of a structural material to sustain occasional overstrain without damage to the ultimate endurance limit from the design point of view. The term "endurance limit" should not be confused with "fatigue strength." The endurance limit may be defined as the fatigue stress under complete stress reversal below which metal can withstand an infinitely large number of stress reversals without failure.

Any loaded structure will behave in a manner inconsistent with the elastic theory in that various parts and connections may be locally overstressed, sometimes beyond the local yield point of the material, thus causing plastic flow and a consequent redistribution of stress at the affected parts. The elastic theory is based on the assumption that the behavior of all parts of the structure is entirely elastic at all times when not loaded beyond the safe allowable design stresses. This assumption is obviously untrue as it ignores the element of uncertainty involved in differentials of stress in various parts of riveted, bolted, or welded construction, as well as conditions of locked-up stresses and local variations in the yield point of the material. These variations and constraints are "ironed out" by the application of service loads, and only then may a structure behave in accordance with the concepts of the elastic theory. However, should a structure be further loaded beyond this point of stress redistribution, the limit theory must be resorted to in order to care for actual conditions. In this connection consideration must be given to what Raymond J. Roark calls "damaging stress,"[30] which is:

> "* * * the least unit stress, of a given kind and for a given material and condition of service, that will render a member unfit for service before the end of its normal life. It may do this by producing excessive set, or by causing creep to occur at an excessive rate, or by causing fatigue cracking, excessive strain hardening, or rupture."

The designer should also recognize what may be called the factor of utilization, which is the ratio of the allowable stress to the ultimate strength. Where the stress is proportional to the load, this factor is the reciprocal of the safety factor. Designers in the aeronautical field use the term "margin of safety" to express the percentage by which the ultimate strength exceeds the design load.

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[29] "Theory of Elastic Stability Applied to Structural Design," by Leon S. Moisseiff and Frederick Lienhard, *Transactions*, ASCE, Vol. 106, 1941, p. 1052.

[30] "Formulas for Stress and Strain," by Raymond J. Roark, McGraw-Hill Book Co., Inc., New York and London, 2d Ed., 1943, p. 5.

Plasticity is a most valuable characteristic in any structural material when the material also possesses elasticity. If structural members did not have the ability to accommodate themselves to the redistribution of stress through this characteristic of plasticity, many bridges and buildings would not stand up no matter what numerical safety factor was used in their design by the elastic theory.

There is a distinct trend among structural engineers toward the evaluation of adjustments necessary to bring the idealized conditions embraced within the elastic theory into agreement with the realism of actual conditions as comprehended in the limit theory of design.

These comments are not offered in a spirit of criticism, but with the purpose of calling the attention of engineers to the desirability of understanding what is the true strength of a structure, and, above all, to the desirability of adopting a realistic approach to design problems.

ELLIOTT B. ROBERTS,[31] M. ASCE.—Although the validity of this paper is not questioned, it is nevertheless regrettable that the author failed to introduce a discussion of earthquake factors.

Earthquakes are frequent in certain well-recognized zones. What is not, perhaps, so generally recognized is that they may, and do, occur almost anywhere. No one could safely state that any place is perfectly free from the danger of a destructive earthquake. Evaluation of the risk in a specific location may introduce a delicate problem; it should never be ignored completely.

The subject of earthquake factors has been treated adequately by the American Standards Association,[32] and a more detailed treatment is contained in "Uniform Building Code."[33] The latter publication, of course, is the result of extensive experience gained by constructors in the most seismic areas of the United States. Experience in these areas is supported by extensive field observations and analysis of ground and building vibrations made by the United States Coast and Geodetic Survey.

A. G. PUGSLEY,[34] ASSOC. M. ASCE.—The general question of structural safety is treated from the statistical standpoint by Mr. Freudenthal, and, by so doing, the author contributes to a cause that the writer has much at heart. The cause is the rationalization of current ideas and habits regarding factors of safety. In the aeronautical field, the very large numbers of aircraft involved in World War II have presented a unique opportunity to explore and apply statistical methods of approach. In this latter field it has been possible to collect much of the basic information required for such an approach—the magnitudes and variations of the loads experienced in war on a number of aeroplane structures of a given type during their life histories; the actual strengths of these structures as measured by mechanical tests representing conditions approximating the observed flight conditions; and the variation of these strengths among structures built to the same plans. Admittedly, in many respects, even the data obtained in this relatively favored field have fallen short of what one would wish for, but a stage was reached in which the "ideal" situation visualized by the statistical approach—the correlation of loads and strengths on the one hand with recorded structural accident rates on the other—could at least be discussed in rough numerical terms.

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[31] Lt. Comdr.; U. S. Coast and Geodetic Survey, Washington, D. C.

[32] "American Standard Building Code Requirements for Minimum Design Loads in Buildings and Other Structures," ASA—A58.1—1945, pp. 11 and 12.

[33] "Uniform Building Code," Pacific Building Officials Conference, Los Angeles, Calif., 1937.

[34] Prof. of Civ. Eng., Univ. of Bristol, Bristol, England.

In 1942 the writer made a very simple statement of this problem in statistical terms, intended to introduce to aircraft designers what was then a new and unfamiliar approach to the rationalization of strength factors.[35] The mathematical background of this approach, and the limitations of this simplified statement, were subsequently discussed by D. J. Bishop;[36] and the position has been admirably summed up in popular terms by W. Tye in 1944.[37]

In reading the Freudenthal paper, the writer appreciated the careful distinction made between "absolute" and "economic" safety, which lies at the root of any statistical approach. Whether all will agree that such a distinction is wise in relation to all types of civil engineering structures is open to doubt. Thus, although a statistical "philosophy" of strength was ultimately widely accepted in relation to the structures of military aircraft, it is already clear that a wider philosophy may be required to meet the needs of designers of civil aircraft. Operational and maintenance considerations in the design of civil aircraft, clothed perhaps in some economic garb, loom large compared with accident rates which, for the peace of the public mind, must be very small indeed. The simple basic aim that the writer proposed in 1942,[35] however well it fitted the military situation, may require some modification or amplification in other applications:

> "Let us start with the simple assumption that, in deciding upon the strength of an aeroplane structure, we are aiming primarily to ensure that the aeroplane shall be able to perform its duties efficiently without breaking. In practice, because of the importance of minimizing structure weight, an aeroplane cannot be made so strong as to be unbreakable, and structural failures occasionally occur under extreme conditions, such as may arise if a pilot attempts to manoeuver a large bomber too violently or if he makes a very bad landing. Structural accidents may also sometimes occur due to errors of design or workmanship or to the use of faulty material. It is natural to consider all such accidents in relation to the hours flown, and to express the history of structural failure among aeroplanes by the number of structural accidents arising in a given number of flying hours, i. e. by a structural accident 'rate.' Expressed in more precise terms, then, our basic aim is to learn how to choose aeroplane strength factors so that structural accident rates shall be as small as possible consistent with efficient production and operation."

Whatever changes may be required, however, every effort should be made to advance the process of rationalizing the engineer's position regarding the safety of structures of all types. The writer, therefore, heartily endorses the sentiments in the concluding paragraph of this paper:

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[35] "A Philosophy of Aeroplane Strength Factors," by A. G. Pugsley, *Report and Memorandum No. 1906*, British Aeronautical Research Committee, 1942.

[36] "Renewal of Aircraft," by D. J. Bishop, *Report and Memorandum No. 1907*, British Aeronautical Research Committee, 1942.

[37] "Factors of Safety—or of Habit?" by W. Tye, *Journal of the Royal Aeronautical Society*, Vol. XLVIII, No. 407, November, 1944, p. 487.

> "By adopting the principle that neither design loads nor safety factors and permissible stresses should be specified arbitrarily, it will be possible not only to eliminate inadequate design, but frequently to achieve considerable economy. It will be possible, moreover, to determine correct safety factors and permissible stresses for unconventional structures or new structural forms and materials."

LYNN PERRY,[38] M. ASCE.—To those who have been using the word "strain" to signify the deformation and "stress" as the load per unit area, the rather nebulous new meanings that the author has used for these words is confusing. A great deal of time is required to grasp the proper understanding of the theory he has advanced. This would be unnecessary if the standard terminology had been rigidly adhered to.

The subject of "Factor of Safety" is always timely. Since it involves economy, each case should be studied and the factor fixed by personnel with sufficient vision and experience to afford the proper balance—giving weight to both the physical and the economic considerations in the particular structure.

In many countries including all Europe, the cost of materials is so high and the labor rates are so low, compared with those prevailing in the Americas, that the economic feature involved in fixing a factor of safety is entirely different. An additional feature, at least up to the end of World War II, is the remarkably stable society of even the most progressive European countries. Old structures have been used for the same purpose, with little change in the loading, for generations. Old locomotives that resemble museum pieces are still being used because it is more economical to maintain them and keep them in service than to meet the requirements that would be necessary for larger and heavier rolling equipment. Some highway bridges were built in Europe during the interim between World War I and World War II with a much more slender factor of safety than most American engineers would feel warranted in adopting. Because of this fundamental economic phase, engineers in the Americas are accustomed to using somewhat more material than engineers abroad. For economy, also, standard sections are used—frequently heavier than necessary but cheaper than an exact section, rolled to specified dimensions.

What the author has termed "uncertainty" in contradistinction to "ignorance" is always in the mind of the experienced designing engineer. There have been so many failures of well-designed structures due to overloading that the designer is prone to imagine the probability of heavier locomotives, trucks, and trailers, larger impact on grandstands, heavier floor loadings, and more violent wind storms than has been provided for in past design. The writer has calibrated and posted loadings for many old highway bridges. Most, if not all, of these were well designed with an adequate factor of safety for any conceivable load that could possibly cross the bridge—that is, from the viewpoint of the designer in 1890. When a loaded 10-ton truck crashes into an end post or fascia girder of such a bridge, it is simply "too bad" and the computation of a factor of safety based on the probable maximum load degenerates to an academic discussion.

With Eqs. 6*a* and 6*b*, the author computes the maximum strain and the lowest value of the structure's resistance using equations containing arbitrary constants. In principle this is no different from current practice.

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[38] Designing Engr., Dept. of Water and Sewers, Miami, Fla.

In Eq. 22, the author suggests that the computed dead load be increased 10%. This may be a constructive suggestion. Concrete and curtain walls seldom weigh 10% more than the estimate. The main steel members do not exceed the estimate by any such amount. In some cases, a series of complex details might be heavier than the preliminary estimate but a 10% leeway would certainly be adequate. Failures inspected by the writer have not been caused by the weakness of details.

Under the heading, "Part 2.—Analysis of Particular Influences: Live Load," the author seems to have missed the primary purpose of the specifications: To make every person concerned with the design thoroughly understand that the final design will be safe under the loadings specified. Such specified loadings do take into consideration the immediate requirements and the estimated future requirements in so far as it is possible to estimate. Steel is sold by the ton and concrete by the cubic yard. It is obvious that money can be saved in either case by decreasing the load or by increasing the allowable unit stresses. After considering all the factors involved, the owner specifies the loadings and allowable unit stresses and makes an effort toward economy by securing the best grade of material and workmanship obtainable at the time and place.

Considerable information is available about axle concentration. Highway bridges in the United States are designed for safe peacetime traffic. When bridges designed for such traffic become subjected to military loadings, the factors of safety disappear. Military traffic, however, can be rigidly regulated: Speed can be reduced to a creep; traffic can be reduced to one way; only one vehicle can be permitted on a span; and unbelievably heavy loads can be moved over a bridge without failure. It is the firm conviction of the present generation of designers that structures will be safe for any load to which they are likely to be subjected during the expected and estimated useful life of the structure. This was also the conviction of the designers of the mauve decade; but any person who imagines that a very old firm will not purchase larger elevator cable and move the largest and heaviest safe from the basement to an upper story and roll it across a floor without any attempt to spread the load is naive indeed. The effect of this procedure might be included as an element of "uncertainty." Police powers can prevent the overloading of structures in a thoroughly regimented military or bureaucratic society, and the uncertain and evasive human element can be eliminated. Under such control, the factor of safety could be materially reduced, with resulting economies.

The change in temperature affecting a structure cannot be greatly in error. A feature that has been noted recently, and on which investigations are still in progress, is the thermal coefficient of expansion of concrete. Undetermined qualities inherent in the coarse aggregate appear to influence this constant as much as ± 50% from the time-worn value of 0.0000065. Further research in this field will be of definite value in the construction of highways and curbs and may even influence specifications for structures.

Structures, designed in many places for a wind load of 30 lb per sq in., have withstood storms for years and have given satisfactory service. For the higher buildings in Miami, Fla., this load has been increased 50%. Observations over a longer period of time will be necessary before a reasonably safe estimate for this loading can be made. Unfortunately, maximum wind velocities are reported as the maximum recorded values, sustained continuously over a period of 5 min. Wind comes in gusts and the dynamic effect of the highest velocity observed would lead to a more justified design. In some of the most severe storms since accurate anemometers have been available, the instruments either were not calibrated sufficiently high to register the maximum velocities or were destroyed together with the structures in which they were installed. In the Aleutian Islands, where high wind velocities prevail, the wind is frequently accompanied by fine snow which seals the orifices of the pitot-type anemometer or clogs the vanes of the revolving type, thus invalidating or impairing the accuracy and reliability of the readings. Nevertheless, a few reliable records of wind velocities of more than 150 miles per hr are available, and the factor of safety under such exposures has not been regarded as wasteful.

Steel has been referred to as "the only elastic structural material." For many years, steel developing a strength, to the yield point, of 30,000 lb per sq in. in tension was standard. Almost uniformly, engineers used a working stress of 16,000 lb per sq in. in tension for such material—a factor of safety of 1.87. Some years ago, it was learned that a very small additional amount of manganese would raise the elastic limit to 40,000 lb per sq in. or more without materially affecting other physical values. Thus, the new product would carry 33% more load safely, and the cost of the steel for a structure would be reduced correspondingly. However, it was years before the profession as a whole was willing to raise the value of the working stress to 18,000 lb per sq in. Later this value was recognized and increased to 20,000 lb per sq in. In some current specifications the use of 22,000 lb per sq in. is allowed, indicating a factor of safety of 1.82; and, as the tests actually show from 41,000 lb per sq in. to 42,000 lb per sq in., the factor of safety may be even higher. The author has computed factors of safety under an assumed set of conditions and has reached values of from 1.54 to 1.75, resulting in a fair correlation.

It is doubtful whether the manufacturers of building materials have obtained final perfection in their effort to produce a homogeneous structural material economically. Structural steel test specimens cut from the top of the ingot, after rolling, usually develop about 4,000 lb per sq in. more stress in tension than similar specimens cut from the bottom of the ingot. Further research may eliminate this differential entirely; but, for the time being, the lower value will have to be used for design. Too frequently, no value is given. It is usual to specify that the test specimens shall be cut with their longest dimension in the direction of the rolling for uniformity and because specimens, in tension, usually develop a higher unit strength when pulled at right angles to the direction of rolling. However, steel details are designed with little reference to the direction of rolling and any excess strength inherent in the material increases the factor of safety.

A factor of safety, to be adequate, must take care of the inherent lack of homogeneity in the material, some items on which more accurate measurements will be available in the future, local consideration, and the human element.

"High rigidity" and "almost perfect elasticity" of subsoil should be the subjects for other discussions.

Nomer Gray,[39] Assoc. M. ASCE.—It is inevitable that the attention of structural engineers will be directed with increasing frequency to a reexamination of the entire subject of factors or margins of safety. When consideration is given to the profound part played by this factor in the design of structures, the wonder is that it has not been continuously the subject of discussion. Certainly the device of using "working stresses," although perfectly justifiable on the ground of expediency, has diverted attention from this most important element of design. However, the true reason is the inherent complexity of the problem—a complexity which becomes more evident from a study of this paper.

Studies leading to refinements in the methods of stress calculation have persisted at a high level of activity for years, and devising new variations on old and tried methods of stress calculation can be classed as an occupational ailment. Although it leaves much to be desired, current knowledge of materials has been enhanced greatly since the 1920's, thus materially reducing this element of "uncertainty" in design. These two statements do not necessarily sponsor higher working stresses, but rather question the validity of a uniform factor of safety for a great variety of members in a structure regardless of function.

An extreme example may be found in the selection of the required sectional area for the main cables of suspension bridges. The three "uncertainties" requiring the use of a safety factor may be simply stated as: (1) Accuracy of method of stress calculation; (2) uniformity of material; and (3) present and future validity of the design loading.

In regard to uncertainty (1), few members in the entire field of structures inspire as little doubt as to the correctness of the computed stress. With respect to uncertainty (2), the material used in the cables of large bridges is remarkably uniform. Taking the wire used in the George Washington Bridge cables as an example, the variation in yield point is recorded[40] as from 170 kips per sq in. to 189 kips per sq in.—a spread of 10%; and the variation in ultimate strength is from 225 kips per sq in. to 240 kips per sq in.—only 6.4%. These are extremes. Actually, the separate values cluster closely around the mean (the standard deviation is small).

It is in respect to uncertainty (3), however, that this member (the cable) is almost unique. For long-span highway bridges, the ratio of dead to live load is frequently 5 to 1. There is little uncertainty about the design dead load and an excess of 10% may be regarded as extreme if the weight is calculated with moderate care. Any major increase in the dead load arising from alterations made during the life of the structure is certain to occur only after an analysis, and the justification of the increase can be decided at that time. There is very little uncertainty regarding the dead load.

An increase of 100% in the live load will only produce an approximate 17% increase in the cable stress; and such an increase could not take place without adequate warning, for the stiffening system would be seriously overstressed (and perhaps fail) before this large increase in live load could occur.

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[39] Civ. Engr., Dept. of Public Works, City of New York, New York, N. Y.

[40] "George Washington Bridge: Materials and Fabrication of Steel Structure," by Herbert J. Baker, *Transactions*, ASCE, Vol. 97, 1933, p. 366, Table 9.

In spite of the uniquely certain character of material and stress in the cables, however, the working stress is commonly set at 80 kips per sq in. This gives a factor of safety of approximately 2.1 based upon the least value of yield point and 2.8 if based upon ultimate strength; and, as the cable wire does not have a pronounced yield point, the latter value is more significant. In the light of the foregoing, some decrease in the factor of safety appears entirely justifiable.

In view of the firmly established and completely accepted meaning associated with the word "strain," its use in a different sense by the author is unfortunate.

Throughout the paper the use of the technique and nomenclature of the mathematics of statistics is evident. The statistical approach alone cannot give the answer, for the determination of a factor of safety involves questions of a philosophical nature. Statistics can present the facts upon which a judgment may be made, however. The author is to be congratulated for having so fully developed the statistical approach to this inherently complex and difficult problem.

I. M. Nelidov,[41] M. ASCE.—A very erudite and comprehensive treatise on the safety of structures and on the factor of safety in particular has been presented by the author. His definition of this factor is based on the theory of chance and implies a statistical meaning of the safety factor.

In Eq. 2 the factor of safety is defined as the ratio of the minimum resistance to the maximum strain. The terms maximum strain and minimum resistance represent average value plus or minus the maximum range of fluctuations. Although the validity of this statement can scarcely be disputed, another aspect seems to affect the definition of the factor of safety.

Suppose that the resistance is known to such a degree that the probability of its correct value is unity; suppose also that the same assumption is true of the applied load. Then, taking the simplest case of a cable composed of 100 straight wires and subject to tensile force, by definition, the factor of safety $\zeta = \frac{s_{r0}}{s_{a0}}$.

Since each wire is taut to the same degree, the factor of safety of a single wire will be also the factor of safety for the entire cable. On the other hand, consider the same cable under bending. Then the stress in each individual wire will vary according to a linear, or some other, law.

If both the resistance and the strain are known as accurately as in the preceding case, the now variable factor of safety of an individual wire will be that previously expressed. However, the factor of safety of the entire cable will have some other value. Since the summation of all the individual factors of safety does not have any practical application, the next usable value will be the average of the individual factors of safety taken for the entire cross section of the cable. This condition will exist in a majority of cases of structural or mechanical engineering and will be especially true for structures of large dimensions with wide variations in stresses. Good examples of the latter case are gravity dams in which the stresses of any one of the sections vary not only in magnitude but also in sign.

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[41] Civ. and Structural Engr., El Cerrito, Calif.

Therefore, the definition of the factor of safety should also include the degree of variation, from the average, of strains within the structure. There are also other qualifying factors pertaining to the factor of safety: (1) The necessity of analogy between the type of the strain and that of the resistance—pure shear, pure tension, combined stress, availability of lateral support, etc.; and (2) the statement of the limit against which the factor of safety is judged.

Thus, for a certain structure, elastic limit may be a criterion since plastic deformations may be not permissible; or, in another case, the ultimate strength may be a criterion, if plastic deformations are permissible. In still other cases, the deflection limit may or may not be a criterion, depending on the functional purpose of the structures. In addition, stability (such as sliding or overturning) may become a criterion.

In all these and numerous other cases, not noted herein, there will be individual and average over-all factors of safety. It is the opinion of the writer that an average over-all factor of safety should be used for cases of variable stresses in which the values of both resistance and strains are known to such a degree that they may be taken as correct values, with deviations being considered below the general precision of calculations. In concluding, the writer wishes to congratulate the author on presenting such a scientific treatment of this widely interpreted subject.

M. Hirschthal,[42] M. ASCE.—Publication of this paper is a laudable attempt to effect economies in design by reducing the "uncertainties," thus effecting a saving in the use of materials of construction by designing for the minimum required for safety. The author's thesis is the reduction of uncertainties, both in the strength of materials and external forces, by the application of the theory of probabilities to each of the factors entering into the design of a structure, both external and internal. The profession is certainly indebted to him for the thorough treatment of the subject and the painstaking analysis of the factors involved.

All engineers are agreed that the method now being used to obtain safety with a "factor" applied to the ultimate strength or elastic limit of a material is wasteful to some extent; but they have not yet found a satisfactory substitute, and the writer fears that the present paper will not provide the "perfect answer" even with complete quality control of all materials of construction. It is to be feared that a "crystal ball" would be required to look into the future for assurance that designers were making proper provision for future conditions, such as loadings and action of materials under long time loading and reversal of stress, despite the fact that marked progress has been made in the latter field.

Consider, for example, the case of railroad structures: At the turn of the century bridges carrying railroad traffic were being designed for class E40, or class E45, Cooper engine loads, and in a few cases for class E50 loads; yet in 1935 the standard American Railway Engineering Association design loading was set at E72. These loadings are now being exceeded on some of the railroads of the United States. In addition, the increase in locomotive tender loadings has been even more marked, so that 63,000-lb axle loads for tenders as compared with the 26,000-lb axle load in the class E40 loading are not at all unusual. Cars of 40-ton capacity of 40 years ago, are supplanted by cars up to 100 tons (and more) capacity.

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[42] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

Inasmuch as the life of a steel structure is set (by the Interstate Commerce Commission) at 60 years, and concrete structures have been accorded a life of 100 years, it is apparent that well within the life of the structure there has been an increase of 80% from E40 to E72 in loads on the driving axles and of 142% (from 26,000 lb to 63,000 lb) in engine tender loads over those of the original design. Therefore, it seems advisable to proceed slowly in making radical changes in the method of design based on safety factors since, had the structures of 1900 been designed by the proposed method, they would have been recommended for retirement before this.

Although it is not recommended that engineers design for such a contingency, there is still a feeling of satisfaction that the impact of the bomber that struck the tower of the Empire State Building in New York, N. Y., did not result in its demolition. Moreover, one never knows when a "tremor" or more severe earthquake will strike a locality never before struck, as has been declared possible by experts.

Various other methods of applying different factors of safety to dead loads and live loads have been proposed on the basis of the fact that there is very little likelihood of a major variation in the dead load of a structure (both A. J. Boase and Shortridge Hardesty,[43,44] Members, ASCE, made such proposals in connection with wartime conservation of materials), but as a matter of fact railroad bridges were designed under this theory and the practice was discarded. (Open-floor bridges have been provided with solid ballasted decks within their serviceable life.) Another factor in this connection is the variation of the ratio of live load to dead load within the same structure. Consider the case of slab beam-girder construction: The live load is a far greater proportion of the total load of the slab than is the load on the beam; the girder has a still smaller proportionate live load than the beam; and so on to the column or pier. A similar condition obtains in the case of an arch bridge with open spandrel construction where the live load on the arch ring is a much smaller proportion of the total load than is the load on the spandrel arches (or floor slab) or the load of the piers or columns that transmit the loads to the arch ring.

The theory of probabilities will give satisfactory results when determining maximum design factors for a condition of known loads or a combination of loads. However, in the case of railroad structures particularly, the factors of safety must also include allowances for an anticipated increase of loads; and the foregoing example is not encouraging.

Another complication arises in considering resistance or internal stresses. Recent research on the subject has revealed that "fatigue" influences a riveted or welded connection to a different extent than it does the main member, and that it varies with different types of connections. Nevertheless, designers are not yet in a position to assign quantitative results to these tests.

Attempts have been made to take account of plastic flow in concrete for long-time application of loads, but these attempts have been sporadic and

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[43] "Saving Reinforcing Steel By Rationalizing Safety Factors," by A. J. Boase, *Engineering News-Record*, May 7, 1942, p. 81.

[44] "Steel Bridge Design to Conserve Material," by Shortridge Hardesty, *Civil Engineering*, August, 1942, p. 431.

without general acceptance. In addition, reinforced concrete is looked upon as two distinct materials of construction: One with the ultimate strength and elastic limit at the same point of the stress-strain relationship, the other with a distinct elastic limit somewhere between one half to two thirds of the ultimate strength. Moreover, the modulus of elasticity of concrete varies with age. Nevertheless, in observations of tests of reinforced concrete members the action is somewhere between that of each component material. Deflections of reinforced concrete beams rarely reach the values obtained theoretically from the conditions of loading assumed. Refinement in the analysis of redundant (or rigid frame) structures will not necessarily result in a more accurate solution when the assumptions contain inaccuracies to degrees far from minute. All theories, after all, are based on assumptions, whether applying to flexure, elasticity, applications to reinforced concrete, or to suspension bridges.

The answer to all this is research, and more research, to provide a firm basis for the theoretical formula that will eventually solve the problem of safety, combined with economy.

Alfred M. Freudenthal,[45] Assoc. M. ASCE.—Discussers of this paper have been handicapped by the brevity of the published version, which is a wartime, paper-saving condensation of a considerably longer manuscript, that has been filed for reference in the Engineering Societies Library in New York, N. Y. Part 2, dealing with " Analysis of Particular Influences" has been cut most severely, whereas not much condensation was possible in Part 1, the general and mathematical treatment. The result has been a slight distortion of perspective by the emphasis on the statistical and mathematical aspects of the problem rather than on the engineering aspects.

The impression appears to have been created that the writer had tried to put forward the claim that the theory of probability and statistical methods could be expected to provide the "perfect answer" to a problem as complex as that of the safety of structures. Actually, the purpose of the paper was to draw attention to the fact that, by the application of the theory of probability (probably the most efficient tool of modern scientific thought) the concept of safety can be rationalized, and to develop a method for the evaluation of the minimum safety factor on the basis of the available objective evidence instead of the usual procedure of arbitrary selection. Not only is it evident that the successful application of the method depends on the extent and reliability of the evidence, but that a certain "residue" of unknown and subjective factors will always be present, requiring appraisal on a level beyond that of statistical inference and prediction from the objective analysis of recorded past experience. That rationally unpredictable "residue" can be isolated by a rational analysis of all statistically predictable influences. Reliable data for this analysis, however, can only be obtained by following Mr. Hirschthal's call for "research and more research."

In his thoughtful comments, Mr. Frankland has raised the important point of the effect on the safety of structures of the distribution of elastic stresses through inelastic behavior. There is no doubt that "some of the generally accepted underlying concepts of structural design are inadequate." It is really paradoxical that one's ability to analyze structures should depend on the assumption that structures are perfectly elastic, whereas the only real safeguard insuring satisfactory performance in service is the extent to which structures are inelastic. This inelastic behavior, by producing a certain redistribution of stresses, necessarily affects the safety. However, by devising an appropriate "mechanism of resistance" and establishing its range of uncertainty, that effect can easily be considered in the evaluation of the safety factor.

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It is too frequently forgotten, however, that the resistance mechanism essentially depends on the rate of application of stress. This is shown by the variation of the stress-deformation diagrams of concrete (Fig. 10(*a*)) and of steel (Fig. 10(*b*)). The structural member or connection in a steel frame which accommodates itself to slowly applied overstress by plastic redistribution of elastic peak stresses may fail without redistribution by cracking in a brittle manner if the same stresses are applied rapidly. This fact alone would contradict Mr. Frankland's assumption that "* * * the theory of limit design will explain the capacity of a structural material to sustain occasional overstrain without damage to the ultimate endurance limit * * *." This theory which, under the name of "Classical Theory of Plasticity"[46] has been studied very extensively in prewar Europe since the late 1920's, is certainly not applicable to conditions of repeated, rapidly moving loads which are implied in the term "endurance limit"; it has been applied, in general, to structures subjected to steady or to slowly moving loads only. However, even for this type of load values of the resistance or the carrying capacity are obtained which might be considered theoretical extremes never attainable in practice. The real values should be somewhere between that extreme and the minimum value, defined by the state of elastic stress immediately preceding the occurrence of the first plastic strain in the most highly stressed section. Such a concept is borne out by the results of experimental investigations between 1930 and 1936[47] which have led to the replacement of the rather crude "classical" concept of plastic redistribution of stress by the so-called "New Theory of Plasticity."[48]

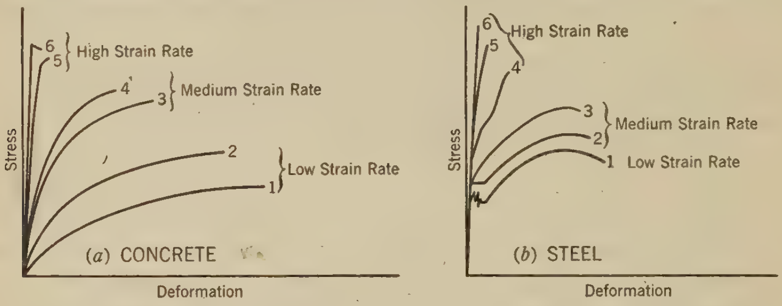

FIG. 10.—INFLUENCE OF THE RATE OF DEFORMATION ON DEFORMABILITY AND STRENGTH

According to the classical theory of plasticity, the upper resistance limit of a steel section in simple bending is defined by a fully plastic rectangular stress distribution. The lower limit is reached when the extreme fiber stress of the triangular (elastic) stress distribution attains the yield limit s_y. Since the scarcity of the available experimental evidence does not justify a statistical interpretation of the results, this is an example of the midpoint value between known extremes, being the most reasonable tentative assumption for the design value; and this value is subject to fluctuations within the range delimited by the extremes. Let S denote the elastic section modulus; T, the plastic section modulus (which is the sum of the static moments of the section elements about the neutral axis); the design value of the resisting moment can be expressed as:

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[46] *Preliminary and Final Reports*, 2d Cong. of the International Assn. for Bridge and Structural Eng., Berlin, 1936, question I.

[47] "Test Results, Their Interpretation and Application," by H. Meier-Leibnitz, *Preliminary Report*, 2d Cong. of the International Assn. for Bridge and Structural Eng., Berlin, 1936, pp. 103–137.

[48] "Fundamental Principles of the Theory of Plasticity," by I. Fritsche, *ibid.*, pp. 15–41; and "Suggestions," question I, pp. 933–934.

$$\bar{M}_R = \tfrac{1}{2} S s_y (1 + T/S) = \tfrac{1}{2} \bar{M}_{RO} (1 + T/S) \qquad (34)$$

Eq. 34 is subject to fluctuations within a range of ± $[(T - S)/(T + S)]$.

As the resisting moment of an individual section is subject to chance fluctuations within the foregoing range, the range of fluctuations of the carrying capacity of an n-fold statically indeterminate girder, being a function of the resisting moments of its $n + 1$ critical sections, increases with an increasing number of such sections.

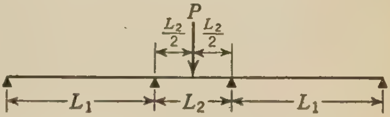

Fig. 11

As an example, consider the continuous beam shown in Fig. 11 under the loading conditions indicated. The capacity load, P_{El}, under the conventional assumptions of the elastic theory can be expressed by

$$P_{El} = \frac{4\, S_m\, s_y}{L_2 (1 - \alpha)} \qquad (35)$$

in which S_m denotes the section modulus at the point of load application and

$$\alpha = \frac{3\, L_2}{4\, L_1 + 6\, L_2} \qquad (36)$$

For a freely supported central span, $L_1 = \infty$ and $\alpha = 0$ and the capacity load,

$$P_o = \frac{4\, S_m\, s_y}{L_2} \qquad (37)$$

The ratio—

$$\frac{P_{El}}{P_o} = \frac{1}{1 - \alpha} \qquad (38)$$

—varies between 1.0 for the freely supported central span and 2.0 for a central span with perfectly fixed ends defined by $L_1 = 0$ and $\alpha = 0.5$.

According to the theory of limit design, the capacity load, P_{LD}, of the structure is reached with the formation of three (that is, $n + 1$) plastic hinges over the supports and at the midspan of the central span. Therefore, it is independent of α and can be expressed by

$$P_{LD} = \frac{4\, (T_m + T_s)\, s_y}{L_2} \qquad (39)$$

in which T_m and T_s denote the plastic section modulus at the midspan of the central span and over the supports, respectively. For a girder with constant sections, $T_m = T_s = T$:

$$P_{LD} = \frac{8\, s_y\, T}{L_2} \qquad (40a)$$

and

$$\frac{P_{LD}}{P_o} = \frac{2\,T}{S} \qquad (40b)$$

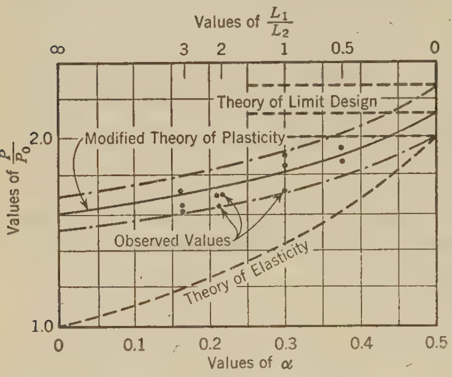

Fig. 12

Under the assumption that the most probable value $\bar{P}$ of the capacity load is the mean of the elastic and the limit-design values:

$$\bar{P} = \frac{2\,\bar{M}_R}{L_2} \times \frac{3 - 2\,\alpha}{1 - \alpha} \qquad (41a)$$

and

$$\frac{\bar{P}}{P_o} = \frac{1}{4}\left(1 + \frac{T}{S}\right)\frac{3 - 2\,\alpha}{1 - \alpha} \qquad (41b)$$

The mechanism of resistance of the girder defined by the "design" load $\bar{P}$ is subject to fluctuations between the extremes within a specific range of $\pm\frac{1 - 2\,\alpha}{3 - 2\,\alpha}$, as well as to fluctuations due to the uncertainty in the individual resistance mechanism of every one of the three critical sections. Hence, the capacity load is:

$$P = \bar{P}\left[1 \pm \sqrt{\left(\frac{1 - 2\,\alpha}{3 - 2\,\alpha}\right)^2 + 3\left(\frac{T - S}{T + S}\right)^2}\;\right] \qquad (42)$$

Comparison of Eq. 42 with the results of tests performed by F. Stuessi and G. F. Kollbrunner[49] and H. Meier-Leibnitz[50] has been presented in Fig. 12; the experimental values are scattered within a comparatively narrow range about the expected or "design" value $\bar{P}$ computed with the aid of Eq. 41*a*, the actual range of variation being much narrower than the range between the extremes as expressed by Eq. 42.

The foregoing example illustrates that a design based on the assumption of perfect elasticity is frequently inadequate; it also shows that a design based on a somewhat crude concept of plastic behavior, as expressed by the classical theory of limit design, may be less adequate. However, the real, rather complex behavior can be fairly well approximated by a simple assumption. This serves to support the writer's assertion (see heading, "Part II. Analysis of Particular Influences: Uncertainty and Inaccuracy of the Mechanism of Resistance") that

> "* * * it is hardly possible to conceive such a mechanism [of resistance] that will effectively reproduce the actual phenomenon and which is at the same time simple enough to be suitable for practical design. Every devised mechanism, therefore, is fictitious to a certain degree * * * Its suitability is to be judged by the simplicity of the concept, by the closeness with which

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[49] "Beitrag zum Traglastverfahren," by F. Stuessi and G. F. Kollbrunner, *Bautechnik*, Berlin, 1935, Vol. 13, p. 264.

[50] "Relations in Girders Continuous Over Three Spans," by H. Meier-Leibnitz, *Final Report*, 2d Cong. of the International Assn. for Bridge and Structural Eng., Berlin, 1936, p. 70.

experimental results are reproduced, and by the narrowness of the range of dispersion of such results about the 'theoretical' course."

The writer regrets (certainly not less than Lt.-Comdr. Elliott B. Roberts) the lack of a discussion of earthquake factors, and he agrees fully with the assertion that although "Evaluation of the [earthquake] risk in a specific location may introduce a delicate problem; it should never be ignored completely." A short comment on the principal aspects of this problem is included in the complete manuscript.

Although it is true that "no one could safely state that any place is perfectly free from the danger of destructive earthquakes," the frequency of occurrence of seismic forces at one location is hardly of an order of magnitude comparable to that of the occurrence of primary design loads (service loads or impact) and of secondary effects (wind or temperature changes). Even in the principal seismic areas of the world, earthquake shocks occur rather less frequently than maximum loads or extreme wind velocities. Therefore, their effects should reasonably be considered on the same level as those extreme conditions—that is, as a source of stress fluctuation of appropriate range about the stresses produced in the structure by the design values of primary loads and secondary effects. Because of the relative infrequency of earthquakes of any intensity there is hardly any justification for introducing even a small seismic acceleration force as "design value." There is even less justification for considering the stresses produced in the structure by acceleration forces which result from severe shocks (from 0.10 g to 0.15 g) similar in character to the stresses produced by the design service loads, and to compare them with the conventional values of permissible stresses.

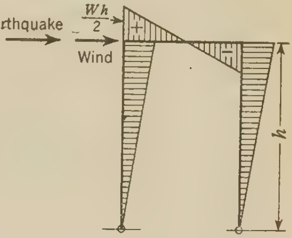

FIG. 13

A procedure recommended for use in the design for seismic forces can be illustrated by reference to the portal frame shown in Fig. 13, which represents the pier of a viaduct. Both the wind forces on the structure and the seismic acceleration force acting at its center of gravity are transmitted into the frame at its top corner. If it is assumed that (a) the extreme wind force P_w exceeds the "design" force W_o by 100%, (b) the maximum seismic force P_E is of the order of magnitude of the design wind force, and (c) the maximum range of fluctuations in the resistance value s_r is $\Delta s_r = 0.2\, s_{ro}$, then the corners of the portal bracing strut should be proportioned for the bending moment $M = \frac{1}{2} W_o h$ with a safety factor (see Eqs. 5 and 18):

$$\zeta = \frac{1 + \sqrt{\left(\frac{\Delta W}{W_o}\right)^2 + \left(\frac{P_E}{W_o}\right)^2}}{1 - \frac{\Delta s_r}{s_{ro}}} = \frac{1 + \sqrt{1 + 1}}{0.8} \cong 3 \ldots\ldots (43)$$

Eq. 43 provides for the stresses occasionally produced by seismic forces of the assumed magnitude. If earthquake effects were not considered in the design, the strut would be proportioned for the same bending moment but with a safety factor:

$$\zeta = \frac{1 + \frac{\Delta W}{W_o}}{1 - \frac{\Delta s_r}{s_{ro}}} \qquad (44)$$

From which $\zeta = \frac{1+1}{0.8} = 2.5$. For the strut under consideration a 17% reduction in the permissible stress, therefore, would be sufficient to provide for the effects of an earthquake force of the magnitude of the design-wind force. In this example, the absence of vertical loads tends to intensify the effect of the seismic forces.

The conventional procedure of earthquake design—in which the effects of horizontal acceleration forces ($= 0.1\,g$) are simply added to those of vertical loads and of secondary influences—undoubtedly provides an excessive safety reserve. Nevertheless, the acceleration to be used in the proposed procedure may possibly have to exceed $0.10\,g$ since it should reproduce extreme earthquake effects. Only very careful and extensive statistically interpretable observations, and analysis of seismic vibrations both of the ground and of structures, can provide a reply to this last question.

Designers of aircraft structures operate with the aid of a considerably more advanced concept of safety than is generally applied in civil engineering. This fact is shown clearly by Professor Pugsley's stimulating comments; and it is not surprising, since in aircraft design any reduction in the weight of structural material which may result from an advanced approach to the problem of safety will improve the performance of the aircraft and effect a direct saving in the initial cost.

Efficiency and economy are of such immediate concern to the air transport industry as to justify unusually large appropriations for research, particularly in wartime. The principal object of such research must be to evaluate the cumulative fatigue effects of repeated stress cycles of varying amplitude, in relation to the capacity of the material to sustain "damage by overstrain" (see heading, "Part 2. Analysis of Particular Influences: Uncertainty and Variation of Resistance Limits of Structural Materials"). In a number of excellent papers Professor Pugsley[51,52] and others have analyzed this difficult problem thoroughly and have presented important results. It appears, however, that even in the aeronautical field the results from experiment and observation are rather erratic, the correlation between the principal factors being far from satisfactory. What is probably needed is not only research on a large scale, but an entirely new approach to the problem of fatigue. The writer has attempted to provide such an approach[53] by establishing the statistical character of fatigue as an expression, on a macroscopic scale, of the progressive destruction of cohesive bonds. Fatigue is the result of the repetitive action of an external load and has the typical features of a mass phenomenon, both the cohesive bonds and the load repetitions being collectives in a statistical

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[51] "Modern Experimental Work on Aeroplane Structures," by A. G. Pugsley, *Journal*, Royal Aeronautical Soc., London, September, 1945.

[52] "Specification of Loading Conditions for Strength Tests on Aeroplane Structures," by A. G. Pugsley, *Aircraft Engineering*, London, December, 1945.

[53] "The Statistical Aspect of Fatigue," by A. M. Freudenthal, *Proceedings*, Royal Soc. of London (publication pending).

sense. By combining the probabilities of bond destruction for various amplitudes and numbers of stress cycles the cumulative damage effect can be evaluated and predicted.

The distinction between "absolute" and "economic" safety admittedly presents a very delicate problem; and it is evident that the approach to this problem will be affected by the character of the structure and the seriousness of the consequences of failure. For military aircraft Professor Pugsley's recommendation that strength factors should be chosen so that "structural accident rates shall be as small as possible consistent with efficient production and operation" is probably adequate. Civil engineering structures, on the other hand, must be designed sufficiently safe to make the chance of one single occurrence of a critical or of an undesirable condition negligible. It should be recognized, however, that "absolute" safety is unattainable and that the admission of a finite (however small) probability of failure is inherent in the statistical approach.

The writer very much regrets that by using the word "strain" in a sense which differs from the established meaning of "specific deformation" the intelligibility of the paper should have suffered—a remark made by both Mr. Perry and Mr. Gray. In favor of such use the writer would like to point out that, in contrast with other languages (French or German), the English technical language lacks a term expressing the general effect produced in a structure by loads and other influences (French: *Effort and sollicitation;* German: *Beanspruchung*). The common use of the word "stress" in this general sense is still more confusing than the writer's use of "strain"; "stress" is an abstract concept implying the action of a mechanical force per unit area, whereas "strain," even in the established meaning, is the observable effect on the structure, or part of it, of influences which are not necessarily forces. Direct correlation between strain and failure has been established reliably. Although fracture does not occur unless some strain is present, the action of a force is not essential. Furthermore, the meaning of the word "strain" in nontechnical language represents, very well, the meaning proposed by the writer.

Mr. Perry's comments on the fundamental difference between the economic aspect of structural safety prevailing in the United States and in prewar Europe are very much to the point. This difference is the result not only of the substantial difference in the ratio of the cost of material and labor but also of the relative importance of standardization of production and organization of work in a country the size of the United States. The implication is unjustified, however, that because of the comparatively low cost of materials there is no need to rationalize the fundamentals of design and that the "experienced designing engineer" can be relied upon to provide all the answers by pure intuition. Such an approach to engineering denies the value of research as an instrument of progress and leads logically to "trial and error" as the only method of making engineering progress, and "apprenticeship" as the only appropriate system of engineering education.

Although it may be perfectly obvious that "money can be saved by decreasing the load or by increasing the allowable unit stress" this statement contributes nothing toward the rational appraisal of the effect of load reduction or of a unit stress increase on the real value of the safety factor. Nine out of ten "experienced designing engineers" will not be able correctly to evaluate this effect; nor will they be able to justify, by rational argument, a definite numerical value for the safety factor. The engineer's client is generally the least qualified person to specify the loading and the allowable stresses. To specify the actual loads to be carried by the structure at present and expected in the future is the best he can be expected to do; but he generally lacks the knowledge necessary to translate these data into a workable loading specification and to select the rational safety factor pertaining to this specification.

Failures of well-designed structures have almost never been caused by simple overloading except in cases of gross neglect. They have mostly resulted from too much reliance on the past experience of the designer, and on his failure to recognize the emergence of new factors in the design.[54]

The crash of a truck into an end post of a bridge, or of a ship's mast into a truss chord of a bridge, should be prevented by the specification of adequate width of curbs and of ample clearances; it has nothing to do with the factor of structural safety, not even in "academic discussion."

Although it should provide an adequate leeway for the guidance of a very careful designer, the writer's proposal to provide for a 10% fluctuation of dead load is tentative, and does not appear to be as ample as Mr. Perry assumes. It is significant in this connection that the dead load of the original Quebec Bridge, computed after its collapse, exceeded the design load by some 20% to 30%.[55] An investigation conducted during the erection of a 48-story building[56] on eight of its main columns furnished the following ratios between the observed dead load stresses and those computed on the basis of the completed drawings: 1.20, 1.31, 1.23, 1.29, 1.27, 1.26, 1.22, and 1.30—an average of 1.26.

The fact that structures designed for a wind load of 30 lb per sq ft have given satisfactory service is hardly surprising, since such a wind load represents the pressure exerted by a storm of the infrequent velocity of about 85 miles per hr. This value is derived from the known formula,

$$p = \frac{c\,q\,v^2}{2\,g} = 0.00256\,c\,V^2 \qquad (45)$$

in which V denotes the wind velocity in miles per hour and c is an empirical factor of shape, which for structural shapes varies between 1.3 and 1.9.[57]

Wind velocities fluctuate within a wide range; wind records taken at a number of stations at New York, N. Y.,[58] reporting maximum velocities and covering several years, show that not more than one observation out of every ten recorded velocities in excess of 46 miles per hr at about 500 ft above ground level and 65 miles per hr at 1,250 ft above ground level. The maximum velocities at these heights were 71 miles per hr and 102 miles per hr, respectively. A design load of 20 lb per sq ft corresponding to a wind velocity of 60 miles per hr should be more than adequate under normal climatic conditions. An occasional increase to as much as 40 lb per sq ft ($V = 100$ miles per hr) would have to be covered by the safety factor. Such reduction of the design wind load would not controvert the fact that a number of serious accidents have occurred as a result of wind action (Tacoma-Narrows Bridge in the State of Washington, and Chester Bridge in Chester, Ill.). These accidents were not caused by excessive wind load but by the failure to provide adequately for the accompanying dynamic effects, and for uplift.

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[54] "Dynamics and Bridge Design," by A. M. Freudenthal, *Engineering News-Record*, Vol. 129, 1942, pp. 540–541.

[55] "Structural Engineering," by G. F. Swain, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1927.

[56] "Dead Load Stresses in the Columns of Tall Buildings," *Bulletin No. 40*, Eng. Experimental Station, Ohio State Univ., Columbus, Ohio.

[57] "Wind Pressure on Structures," by G. E. Howe, *Civil Engineering*, March, 1940, pp. 149–152.

[58] "Wind Forces on a Tall Building," by J. Charles Rathbun, *Transactions*, ASCE, Vol. 105, 1940, pp. 8–11.

The lack of uniformity in the properties of structural materials is an important source of fluctuation of structural resistance. For steel, the range of fluctuation inherent in the individual manufacturing process is further increased in the structure by the fact that structural shapes in one framework may originate not only from the top or the bottom of ingots of the same heat or from ingots of different heats in the same manufacturing process, but from entirely different processes and mills. Moreover, the properties are affected by the rolling, and they differ for different shapes and thicknesses. The frequency distributions drawn in Fig. 3 represent the yield limit and tensile strength of an alloy steel in which all those sources of nonhomogeneity have been present. Nevertheless, the fluctuations are enclosed within a range of from 15% to 20% above and below the mean value. Even for concrete produced on the site, the manufacturing process of which is considerably less controlled than that of steel, the range of fluctuation of the compressive strength about its modal value seldom exceeds ± 35%. This is shown in Fig. 14 which represents the results of an investigation involving more than 1,000 samples.[59] It is interesting to compare the foregoing value (± 35%) with the range of fluctuation of the compressive strength of concrete manufactured under laboratory conditions which, normally, does not exceed ± 15%.

The writer heartily endorses Mr. Gray's comments. The lack of interest in fundamentals among engineers is really surprising. The number of discussers of a paper concerned with methods of elastic stress calculation, for instance, is generally a multiple of those taking part in the discussion of paper concerned with basic principles.

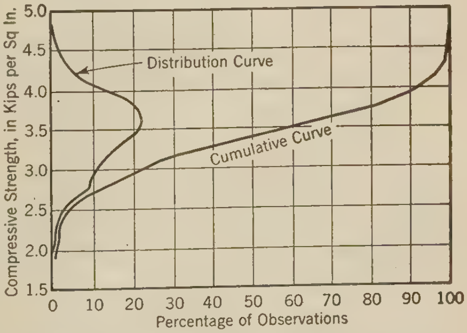

Fig. 14.—Frequency Distribution of the Cylinder Strength of Concrete (Modal Strength Approximately 3,600 Lb per Sq In.)

With regard to the safety of main cables of suspension bridges there can be no doubt that here is a structural element which probably requires an exceptionally low safety factor. If the frequency distribution of the strength of individual wires is narrow, that of the cable is still narrower, since the standard deviation of the strength of a cable consisting of m wires would be $\frac{\sigma}{\sqrt{m}}$ if σ denotes the standard deviation for the individual wire.

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[59] *Ibid.*, Vol. 96, 1932, pp. 1367–1371.

As correctly stated by Mr. Gray, the uncertainty regarding the loading is also exceptionally small. However, the writer is not so sure about the reliability of the computed stress, particularly with regard to the uniformity of the distribution of stress over the individual wires. Since the capacity of the hard wire to secure inelastic relief of local overstress is rather small, the initial uniformity of stress distribution appears to be the principal question that needs careful study before designers can undertake the probably justified substantial reduction of the safety factor for suspension bridge cables.

Mr. Nelidov comments on the difference in the treatment of homogeneous and nonhomogeneous states of stress. This is a rather complex problem which, in most engineering applications, is best solved by the introduction of an adequate "mechanism of resistance"; this mechanism is an empiric function of the geometrical dimensions and the observable elementary physical properties. The variability of the resistance which determines the safety factor can be either directly evaluated from test results or computed by applying Eqs. 16 or 18 to the results of tests concerned with the observation of the constituent properties.

In a more scientific approach to this problem the probability of failure at every point of the stress field could be expressed as a function of the coordinates, considering both the variable stress intensity and the frequency distribution of the strength of an element. This approach leads to rather involved statistical computations; but it must be applied if no reasonable "resistance mechanism" can be devised a priori.

The essential difference between both approaches can be illustrated by considering the case of simple bending of a section composed of wires. The "resistance mechanism" of the section expresses the resisting moment in terms of the geometrical dimensions and the tensile strength of the material, the assumption being that failure is always initiated in the extreme fiber of the section. In contrast to this assumption, in the strict statistical approach, a definite probability of failure in relation to the distance from the neutral axis and to the fluctuation of strength is attributed to every wire of the section. Fracture may thus start in any individual wire, whatever its stress, if its actual strength is low enough. The resulting frequency distribution of sectional resistance is then used to evaluate the safety factor of the entire section.

The importance of a breadth of vision in predicting future developments, particularly with regard to the weight of railroad traffic is rightly stressed by Mr. Hirschthal. There is no other type of structure for the economic design of which the adequate appraisal of such development is as essential as for the railroad bridge. Here this aspect tends to overshadow all other aspects of the design; therefore, rational analysis and evaluation of the initial safety factor loses much of its practical value.

The difficulties of design for repeated stress cycles, to which Mr. Hirschthal refers, have been dealt with in the paper and the available quantitative results presented in Figs. 6 and 7. Admittedly, these results do not as yet fully account even for the most elementary conditions encountered in the design of structures subject to dynamic loads.

The writer greatly appreciates the stimulating and constructive comments of those who have discussed the paper and wishes to convey to them his thanks for their contributions.

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Paper No. 2297

TECHNICAL SURVEY—BROOKLYN BRIDGE AFTER SIXTY YEARS

A SYMPOSIUM

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Note.—Published in January, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

Foreword

The Brooklyn Bridge is a suspension structure that crosses the East River between the Borough of Manhattan and the Borough of Brooklyn, in New York, N. Y. In 1943, when an examination of its physical condition and load-bearing capacity became desirable, it was 60 years old.

This investigation, which is described in the Symposium, was conducted by the Department of Public Works, City of New York, of which Irving V. A. Huie, M. ASCE, was then commissioner. Homer R. Seely, M. ASCE (author of the first paper), had general charge of the investigation for the Department, which was made under the guidance of a Board of Consultants comprising in addition to O. H. Ammann, M. ASCE (author of the second paper), the late Holton D. Robinson and the late Leon S. Moisseiff, Members, ASCE. The so-called technical survey was implemented by members of the engineering staff of the Department under the direction of Isidore Delson, M. ASCE, with Nomer Gray, Assoc. M. ASCE (author of the third paper), in charge of the work in the field. The tests described in the fourth and last paper were made under the direction of Harold E. Wessman, M. ASCE (author of the fourth paper).

HISTORICAL FEATURES OF CONSTRUCTION AND OPERATION

By Homer R. Seely,[1] M. ASCE

Preliminary Studies

While describing the anchorages of the Brooklyn Bridge across the East River between Brooklyn and New York, N. Y., in his progress report of 1876, Col. Washington A. Roebling made the following statement:

> "In European suspension bridges, the chains are usually carried in tunnels open to access, which are confidingly left to the care of future generations, and this trust is not betrayed. This, however, is contrary to the genius of the American people, with whom everything has to look out for and take care of itself; hence, in the arrangement of this anchorage, the chains are inaccessibly preserved and are not intrusted to the neglect of posterity."

In May, 1945, the Brooklyn Bridge had withstood the ravages of time and use for 62 years. How well the bridge has survived the "neglect of posterity" and how well it can serve the traffic of the future is the subject of this Symposium. This paper is devoted to a brief history of the bridge and a review of some of the more interesting features of its construction and operation.

The Brooklyn Bridge (or New York and Brooklyn Bridge as it was officially called in those days), like all undertakings of great magnitude, had been agitated, on occasion, since the early years of the nineteenth century. However, it remained for the action of three men—Henry C. Murphy, then a New York State Senator, William C. Kingsley, F. ASCE, and Alexander McCue, who met on December 21, 1866, in the City of Brooklyn—to reach an agreement resulting in the passage of an act by the New York State Legislature on April 16, 1867, incorporating the New York Bridge Company for the purpose of building and maintaining such a bridge. This action was undoubtedly aided by the unusually severe weather that winter, which caused the East River to freeze. As a result, it was possible to travel from Albany to New York by railroad in less time than it took the business man living in Brooklyn to reach his office in downtown Manhattan.

The corporators of the New York Bridge Company first met on May 13, 1867, and on May 23 retained John A. Roebling, M. ASCE, as their chief engineer. Mr. Roebling was admirably fitted for this assignment—ten years previously he had accomplished the proclaimed "impossible" by constructing a railway suspension bridge over the Niagara gorge at Niagara Falls, N. Y., and earlier that same year (1867) had completed the Cincinnati (Ohio)-Covington (Ky.) Bridge over the Ohio River with a record span of 1,057 ft.

Mr. Roebling immediately proceeded with the necessary surveys, preliminary plans, and estimates of cost and submitted his initial report on September 1, 1867. He investigated three locations, recommending the south or City Hall Park location which was ultimately adopted (see Fig. 1). The river crossing required a clear span of approximately 1,600 ft. In his report, Mr. Roebling[2] stated:

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[1] Deputy Commr., Dept. of Public Works, New York, N. Y.

> "Any span inside of 3,000 feet is practicable. With the best quality of steel wire, even greater spans may be made secure for all kinds of traffic; but, of course, the cost of such works increases with the length of span."

His implicit faith in the safety of a bridge of this magnitude, however, was not shared by a large number of the citizens. Therefore, he requested and received the counsel of a Board of Consulting Engineers (Horatio Allen and Julius W. Adams, Past-Presidents and Hon. Members, ASCE; James P. Kirkwood, Past-President, ASCE; Benjamin H. Latrobe; William J. McAlpin; John J. Serrell; and J. Dutton Steele), who, after several months of intensive review and study, unanimously agreed in May, 1869:[3]

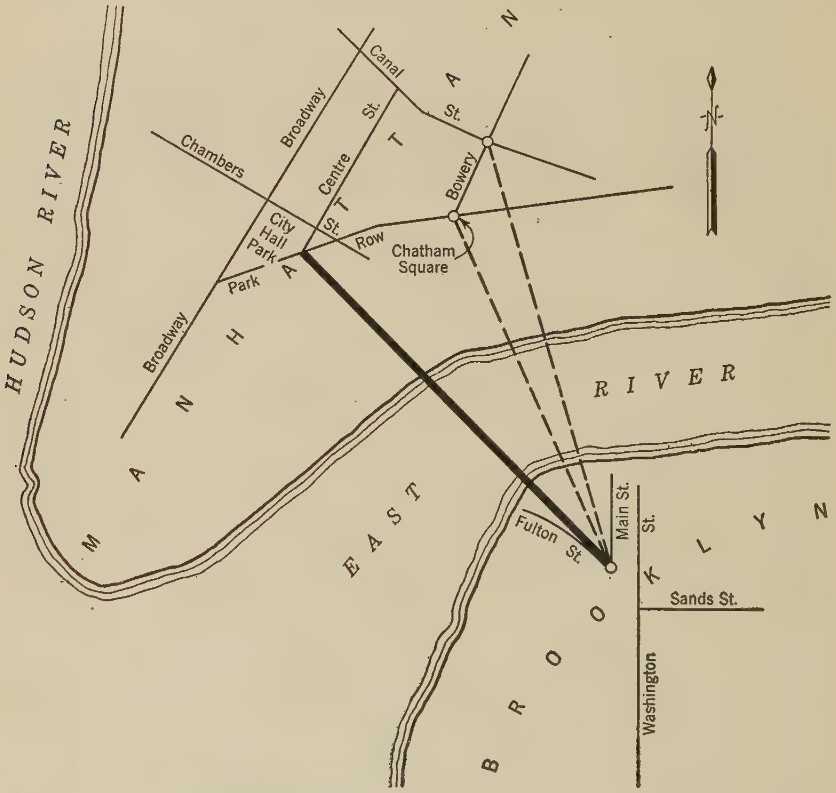
HUDSON RIVER
Canal St.
Broadway
Centre St.
Chambers St.
City Hall Park
Row
Bowery
Chatham Square
Park
Broadway
MANHATTAN
EAST RIVER
Fulton St.
Main St.
Washington St.
Sands St.
BROOKLYN
N

FIG. 1.—ALTERNATE AND ADOPTED LOCATIONS OF THE BROOKLYN BRIDGE

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[2] "Report of John A. Roebling, C. E., to the President and Directors of the New York Bridge Company on the Proposed East River Bridge," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1867, p. 8.

[3] "Brooklyn Bridge 1883–1933," Dept. of Plant and Structures, New York, N. Y., 1933, p. 8.

> "It is beyond doubt entirely practical to erect a steel wire suspension bridge of 1,600 feet span, 135 feet elevation, across the East River, in accordance with the plans of Mr. Roebling, and that such a structure will have all the strength and durability that should attend the permanent connection by a bridge of the Cities of New York and Brooklyn."

An Act of Congress (Public No. 53), approved on March 3, 1869, legalized the bridge as a post road for the conveyance of mails of the United States provided that it was constructed so as to not "obstruct, impair or injuriously modify the navigation of the river," and, to secure compliance with these conditions, the act stipulated that the plans of the bridge be submitted for the approval of the Secretary of War. A commission of three U. S. Army Engineers (Lt.-Col. H. G. Wright and Lt.-Col. John Newton, Hon. Members, ASCE, and Capt. W. R. King) was designated to report on the structure. After an exhaustive investigation, it was recommended that the bridge (with an underclearance of 130 ft) be approved, as planned. The recommendation was also endorsed by Brig.-Gen. A. A. Humphreys, Hon. M. ASCE, then Chief of Engineers, U. S. Army. However, in granting his approval on June 21, 1869, John A. Rawlins, then Secretary of War, acceded to the navigation interests and stipulated that the clear height of the bridge at the center of the main span be not less than 135 ft above "mean high water of the spring tides." It was recognized that even at this height it would be necessary for some of the larger merchant vessels, in order to sail under the bridge, to "send down or house their royals, and in some cases, their top gallant masts." Even so, for many years after the bridge was completed, the annual reports state that several ships were deprived of their topmasts each year.

Final surveys for constructing the bridge were started in June, 1869, but fate decreed that its master was not to build the bridge, as, on June 28, John A. Roebling, while standing on a fender rack, had his right foot crushed by a ferryboat approaching its slip. Tetanus developed and he died on July 22. Fortunately, associated with him on the work was his son, Col. Washington A. Roebling. In 1865, when only twenty-eight Colonel Roebling had resigned his commission in the Union Army to assist his father in completing the Cincinnati Bridge, taking practically complete charge of the construction. Obviously, in spite of his age, he was the logical successor to the position of chief engineer, and he was so appointed on August 3, 1869.

The trials and tribulations which confronted Colonel Roebling and his associates in the construction of the bridge were enough to exhaust the patience of ordinary men. The problems which arose in the sinking of the pneumatic caisson for the Brooklyn tower alone would furnish sufficient interesting material for a paper.

Foundation Problems

Actual construction started in January, 1870, at the site of the Brooklyn tower. The site selected for the tower fell upon a glacial deposit which proved to be a mass of boulders cemented together with decomposed fragments of green serpentine rock, making them almost impossible to dislodge. These boulders ranged in size from 1 cu ft to larger than 300 cu ft. A collection of the various kinds of boulders encountered during the sinking of the caisson included all the varieties of rock found for a hundred miles north and northeast of Brooklyn; 90% of the boulders were of traprock, most likely originating in the Palisades.

Of course, it was necessary to break up the larger boulders in order to remove them from the working chamber. At first, this was done by plug and feather, but some more expedient method soon became necessary. Blasting had been considered from the beginning, but it was feared that, in addition to rupturing the eardrums of the workmen, a violent explosion in the compressed air of the working chamber would cause disastrous results to the caisson. After experimenting at first with a pistol, using successively heavier charges and then still larger charges fired by fuse, it was found that blasting caused no harmful effects. The powder smoke was quite a nuisance, however, as it would fill the working chamber for a half hour or more and obscure the lights. The sulfur odor did not matter as the sense of smell practically disappears in compressed air.

Illumination of the working chambers was a difficult problem. The general illumination was furnished by calcium lights. Care had to be taken, however, to prevent explosions caused by leaking gas or carelessness in turning off the burners. One explosion did take place, which, according to Colonel Roebling was "sufficient to singe off whiskers and create some alarm." Candles were used for special work where extra light was necessary. Incomplete combustion of the candles resulted in an intolerable amount of smoke in spite of all precautions, such as reducing the size of wick as well as soaking the wick in vinegar and mixing alum with the tallow. The smoke was very injurious to the lungs of the workmen; six months after the caisson work was completed, the workmen still coughed up sputa streaked with carbon black.

In the Brooklyn caisson the men worked in 8-hr tours, including 1 hour for meals, which they usually ate in the working chamber. Their wages were $2.00 a day, which was increased to $2.25 when the caisson had reached a depth of 28 ft. On the New York side, because of the greater depth and correspondingly greater pressure, the daily working hours were gradually reduced from two 4-hr tours to two 2-hr tours. The wage scale was also increased to $2.75 a day.

As one might expect, caisson disease, or the "bends," presented an acute problem, particularly on the New York side. According to Colonel Roebling, "scarcely any man escaped without being somewhat affected by intense pain in his limbs or bones or by a temporary paralysis of arms or legs." Full advantage was taken of all prior experience and knowledge of the subject—Colonel Roebling having previously made a special trip to Europe to learn of new developments in the use of pneumatic caissons. In addition, Andrew H. Smith, M.D., was retained to attend those afflicted and to conduct research. In his report Dr. Smith stated that, during his employment on the work, there were 110 cases that required treatment, of which three proved fatal. Colonel Roebling, himself, was a victim, being permanently crippled and confined to his home, from which point he continued to direct the work through his loyal and capable assistants. Some question was raised as to his ability to continue in charge of the work. A resolution introduced by Seth Low, then Mayor of

Brooklyn, to the Board of Trustees proposed that he be appointed as consulting engineer because:

> "In the judgment of this Board the absence of the Chief Engineer from the post of active supervision is necessarily, in many ways, a source of delay * * *."

Colonel Roebling positively refused to consider such an appointment and requested: "* * * that the vote of the board be taken simply as to whether or not I am to remain chief engineer." After a prolonged and heated debate, the resolution failed to be adopted by a vote of 10 to 7. In his defense Colonel Roebling also prepared a paper which Mrs. Roebling read before a meeting of the Society. There is no record, however, that the Society took any official action in his behalf. The ire of some of the Trustees was undoubtedly aroused by Colonel Roebling's inability to comply with the Board's request to attend one of its meetings when he telegraphed curtly: "Cannot meet the Trustees today." To their repeated requests, he wrote:

> "I am not well enough to attend the meetings of the Board as I can talk for only a few moments at a time and cannot listen to conversation if it is continued very long. * * * I did not telegraph you before the last meeting that I was sick and could not come, because everyone knows I am sick and they must be as tired as I am of hearing my health discussed in the newspapers."

Colonel Roebling resigned from the position of chief engineer on June 30, 1883, shortly after the bridge was opened to traffic.

The caissons were constructed of yellow pine with 2-ft-square oak sills resting on a semicircular casting, forming the cutting edge. The caissons were 102 ft wide and approximately 170 ft long—by far the largest pneumatic caissons ever constructed to that time.

The manways and supply shafts to the working chambers were equipped with air locks (see Fig. 2). The relative position of the air locks on the manway shafts became the subject of a suit for infringement of patent by Capt. James B. Eads, M. and F. ASCE, builder of the well-known Eads Bridge at St. Louis, Mo. In writing about this subject in the technical press,[4] Captain Eads stated:

> "I trust I shall not be understood as finding fault with Colonel Roebling for copying my plans in his New York caisson. * * * Colonel Roebling's failure for the past three years to credit me with the plans appropriated by him was not deemed of sufficient moment to cause me to trouble the public with the matter but having frequent occasion myself for them when designing foundations of other works, his omission being now coupled with an effort to deprive me of all merit of originality in them, compels me most reluctantly to correct his statements and show my right to use my own property."

In rebuttal, Colonel Roebling wrote:[5]

> "Captain Eads virtually makes the broad claim that any device which has been used in a pneumatic cylinder can be made the subject of a new

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[4] *Engineering* (London), Vol. 15, May 16, 1873, p. 337.

[5] *Ibid.*, June 27, 1873, p. 458.

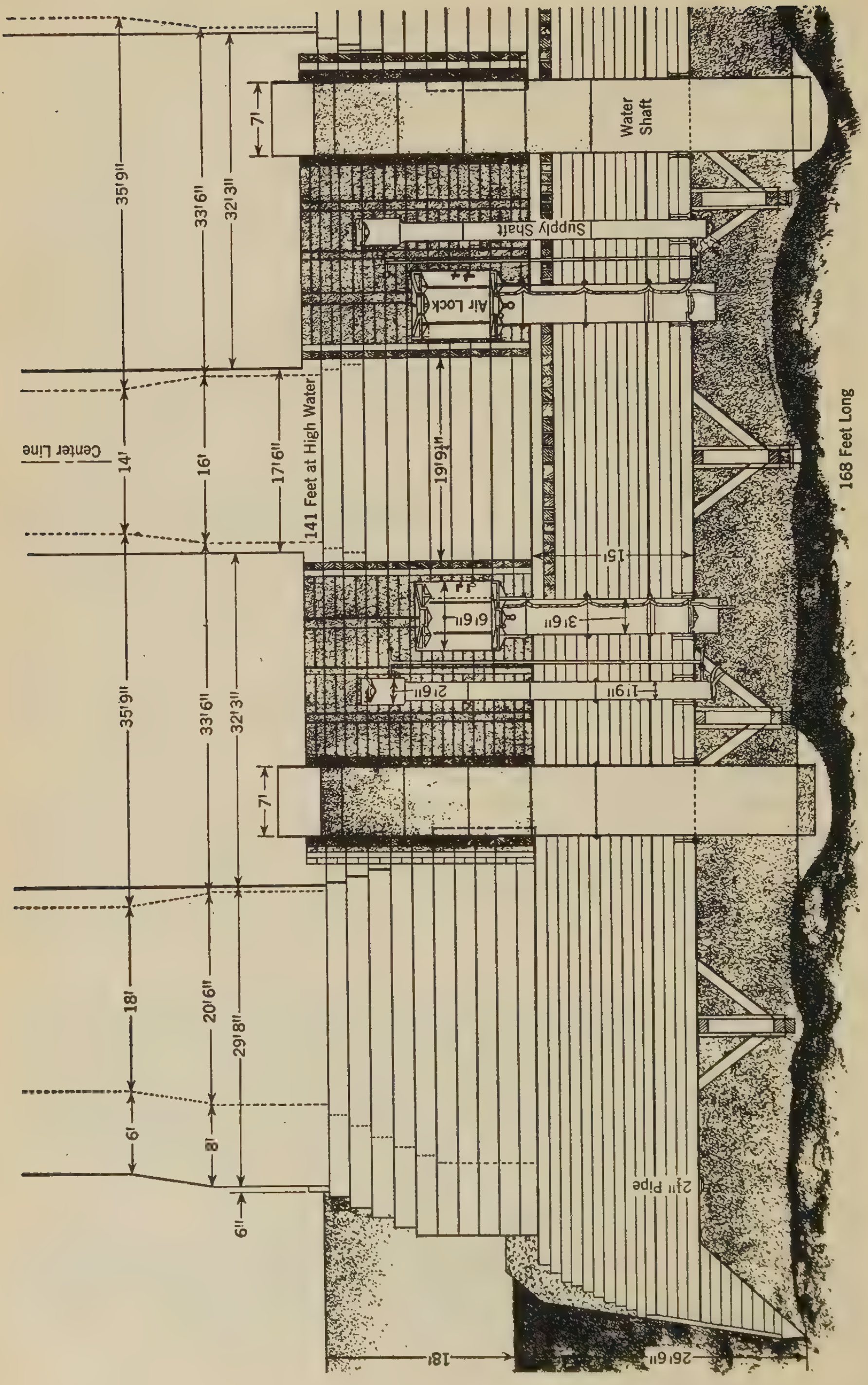
35'9"
33'6"
32'3"
7'
Water Shaft
Supply Shaft
Air Lock
Center Line
14'
16'
17'6"
141 Feet at High Water
19'9½"
15'
3'6"
6'6"
2'6"
1'9"
35'9"
33'6"
32'3"
7'
18'
20'6"
29'8"
6'
8'
6"
2½" Pipe
18'
26'6"
168 Feet Long

FIG. 2.—LONGITUDINAL SECTION, BROOKLYN TOWER CAISSON

> patent when applied to a 'masonry caisson' and in that spirit has had several patents granted. I choose to differ with him on that point and before paying the round sum with which he proposes to tax the engineering world for the next fifteen years, I have preferred to leave the matter to the decision of the court at law where it is now being tried."

A resolution adopted in 1876 by the Board of Trustees of the bridge indicates that the suit was settled in favor of Captain Eads.

"Water shafts" (see Fig. 3) were used for removing the excavated material. There were two shafts, entirely open from top to bottom, with the lower ends submerged in pools of water. The air pressure in the working chamber was balanced by columns of water extending up the shafts. The men in the working chambers excavated the material and deposited it in the pools of water where it was picked up and hoisted up the shafts by a clamshell bucket, which was apparently a patented device in those days and called a "Grapnel" bucket.

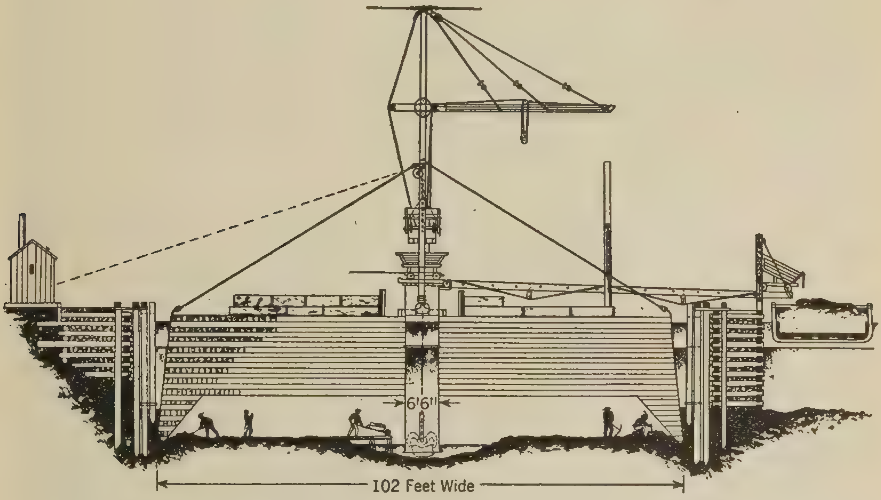

Fig. 3.—Arrangement of Excavating Machinery, Brooklyn Tower Caisson

Occasionally, a stone would prevent the bucket from closing properly, and the men would dive into the pool and clear the obstruction. With compressed air in their lungs, they were able to stay under water 3 min or 4 min with little difficulty. Contrary to expectations, it was found that the buckets alone were not able to excavate enough material under the shafts to maintain the pools so that, periodically, it was necessary to discontinue operations, pump out the pools, and excavate the material by hand. To maintain the air pressure, a cover was bolted on the shaft, loaded with rock to balance the air pressure, and the water in the shaft was released by admitting compressed air into the top of the shaft. One or two days were lost each time this operation became necessary.

During the sinking of the Brooklyn caisson, an incident occurred which Colonel Roebling refused to admit was an accident; but, rather,

> "* * * simply the legitimate result of carelessness brought about by an over-confidence in supposing that matters would take care of themselves."

Early one Sunday morning, one of the earth dams retaining the water in the pools under the water shafts was washed away, allowing the air to escape through the shaft. The incident was recorded in Col. Washington Roebling's report as follows:[6]

> "Eye-witnesses outside state that a dense column of water, fog, mud and stones was thrown up five hundred feet into the air, accompanied by a terrific roar and a shower of falling fragments, covering the houses for squares around. This column was seen a mile off. The noise was so frightful that the whole neighborhood was stampeded and made a rush up Fulton Street. Even the toll-collectors at the ferry abandoned their tills. There were three men on the caisson at the time, including the watchman. He reports that the current of air rushing toward the blowing water shaft was so strong as to knock him down; while down, he was hit on the back by a stone, and further than that he does not remember. One of the other men jumped into the river and the third buried himself in a coal pile. It was all over in a minute. Both doors of the air-lock fell open. The dry bottom was visible through the air and water shaft; not a particle of water had entered under the shoe into the air chamber, and for the first and only time the caisson could dispense with artificial illumination."

Fortunately, very little, if any, real damage was done to the caisson and air pressure was quickly restored.

Another incident occurred later, however, which nearly proved disastrous. With the cutting edges within 2 ft of final elevation (– 45 ft) one of the workmen, while retrieving his hidden dinner pail, inadvertently held a candle against the roof of the working chamber, igniting the timber. The leakage of air, acting as a draft, carried the fire zigzagging up through the caisson roof. All efforts to extinguish the fire failed and the working chamber had to be flooded. Colonel Roebling remained in the caisson all night directing the fight against the fire, which resulted in his first attack of the "bends"—an illness destined to cripple him for life. The fire spread laterally to points some 50 ft apart and upward through several courses of timber. Cement grout was injected into the burnt cavities through boreholes to prevent the leakage of air. It was hoped that this might also serve to repair the damage; but, since further inspection indicated the presence of considerable quantities of charred wood, it was necessary to cut out the burned timbers progressively and to replace them or to scrape away the charred portions and pack the cavities with cement mortar. The repair work was particularly disagreeable:

> "* * * the men having to lie for hours in confined spots without room to turn, and breathing a foul mixture of candle smoke and cement dust, combined with powdered charcoal * * *."

The New York tower caisson, although founded at nearly twice the depth of the Brooklyn tower (– 78 ft) was sunk in 20% less time. This was largely because the material excavated was principally sand, which was blown to the surface through pipes with the compressed air in the working chamber. Some difficulty was encountered in finding a material that would deflect the sand at the top—otherwise the sand would have shot some 400 ft into the air. Wrought-iron and cast-iron elbows, used at first, would last only 1 hour or 2 hours. The elbows were finally discarded and heavy blocks of granite were hung over the pipes. The pebbles discharged with the sand, however, proved quite a hazard—a boatman on the river had a finger shot off and one of the laborers working on the caisson was shot through the arm. Comparatively little material was excavated through the "water shafts"; in fact, they were capped and not used at all during the last 10 ft of excavation.

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[6] "Report of the Chief Engineer and General Superintendent to the Board of Directors of the New York Bridge Company," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1871, p. 20.

Cable Spinning

The masonry towers and anchorages were completed sufficiently to start cable construction in August, 1876. One of the traveling or "hauling" ropes and a "carrier" rope were erected by first laying them on the bed of the river and then hoisting them to the top of the towers. The remaining ropes were erected by hauling them across the river supported on hangers running on the carrier rope. Only one footbridge, 3.5 ft wide (see Fig. 4), was used to give access to the different parts of the work. At three points in the main span and the midpoint of each land span, "cradles" or crosswalks were erected to give access from the footbridge to the wire adjusting platforms. The cable strands were spun approximately 60 ft above their final position at the center of the main span to insure the wire being pulled straight under the correspondingly higher tension. Also, the greater underclearance of the temporary ropes

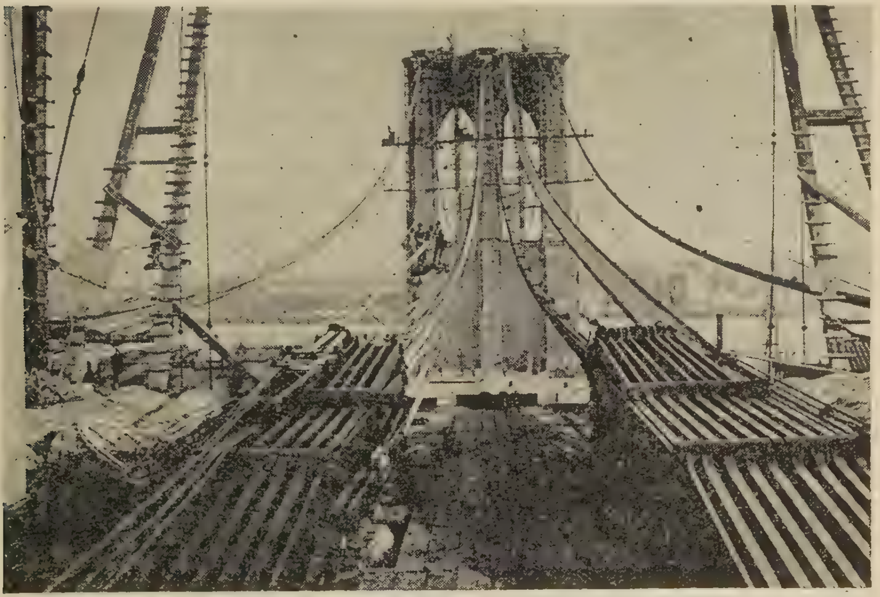

Fig. 4.—Main Cables Under Construction

offered less interference to navigation for a considerable added length of time. Hanging platforms were erected for adjusting the strands which were reached by ladders from the wire adjusting platforms.

The cable wire was spun basically the same as for modern bridges. However, only one strand for each cable was spun at a time, so that the spinning wheels returned empty each trip. Eighteen months were required to spin the cables, the last wires being placed on October 5, 1878. Based on his experience at Cincinnati, Colonel Roebling had estimated that it would take at least 2.5 years to 3 years to spin the cables. Only one major mishap occurred on this phase of the work. While one of the strands was being slacked off to adjust it to the proper sag, the strand broke loose at the New York anchorage and slipped over the tower until it came to rest on the bed of the river. Fortunately, no boat happened to be under the bridge or it might have been cut in half; but, unfortunately, the strand shoe did strike and kill two of the men working on the anchorage.

Difficulties of Material Inspection

Galvanized steel wire was used for the main cables instead of iron wire which had been used for all previous parallel wire bridge cables. The specifications established limitations as to unit weight, strength, elastic limit, and modulus of elasticity. However, no restrictions were included on the process of manufacturing the steel or the wire even though at the time it was generally believed that open hearth or Bessemer steel would not produce acceptable wire. All parties expecting to submit a bid were requested to furnish samples of the wire they proposed to supply. Colonel Roebling realized that this was no assurance that all the wire would conform to the sample, but (he indicated) it would serve to acquaint each bidder with all the problems involved in producing the wire and convince him that there were no unjust or arbitrary conditions in the specifications, that would be impossible to fulfil. The samples submitted by prospective bidders soon indicated:[7]

> "* * * that both Open Hearth and Bessemer steels could be so manipulated as to meet the requirements in every respect, giving *uniformity* equal to and even superior to any brand of cast-steel * * *."

John A. Roebling's Sons Company submitted alternative bids for wire of crucible cast steel and of Bessemer steel. The Executive Committee recommended the award of the contract to the Roebling Company on the basis of their low bid for Bessemer steel wire. Nevertheless, a strong vote of the Board of Trustees in favor of crucible cast-steel wire caused the contract finally to be awarded to the firm submitting the low bid for such wire. The contract price was 8.7¢ a lb—nearly 2¢ a lb more than the Roebling bid. The bids were taken at so much per pound, gold, to eliminate all questions of probable changes in the value of currency during the life of the contract.

The successful bidder was a local Brooklyn company and everything apparently went well until inspectors noticed that the stock of rejected wire, instead of growing larger, was gradually growing smaller. It was determined that the contractor was substituting rejected wire for all approved wire which happened to remain at the plant overnight and then, of course, on the following morning, carted it to the bridge without further inspection. When this skulduggery was discovered, the contractor conceived the idea of having each wagonload of wire driven to a building en route where the approved wire was replaced with an equal weight of rejected wire. The approved wire was then returned to the plant and resubmitted to the inspector as new wire. Colonel Roebling, upon learning this, assigned a man on horseback to accompany each wagonload, and in reporting the matter to the president of the Board stated (letter from the chief engineer to the president of Board of Trustees, New York and Brooklyn Bridge, dated July 9, 1878):

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[7] "Report of the Chief Engineer of the New York and Brooklyn Bridge, January 1, 1877," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1877, p. 21.

> "I owe it to myself to protect my reputation in this matter and in case a want of strength shall in the future be found in the cables, I wish the responsibility to rest where it belongs, with the Board of Trustees."

When asked to explain this statement, Colonel Roebling replied that the old Board of Trustees must be held responsible because,

> "They awarded so important a contract as the cable wire to a man who had no standing commercially or otherwise, and the same responsibility must be assumed by the present Board if they fail to at once put an end to [his] * * * contract, now that he has been detected in fraud. * * * I further recommend that you call for new bids for the wrapping wire as [his] * * * bid is below the cost of production and he must be intending to furnish something else than the material called for."

Colonel Roebling's bitterness was probably aggravated by a previous action of the Board of Trustees in voting to prohibit acceptance of bids from any firm or company in which any trustee, officer, or engineer of the bridge was interested, thus forcing him to sever all connection with the John A. Roebling's Sons Company. Although the Trustees did not cancel the cable wire contract, as urged by Colonel Roebling, they did reaward the contract for the wrapping wire.

It was estimated that about 221 tons of rejected wire had been spun into the cables. To compensate, 14.25 tons of wire, or, roughly, 1.7% was added to each cable. To have fully compensated for the estimated loss in strength would have required 22 tons of wire per cable, which would have made the cables too large to accommodate the cable bands.

Suspended Structure

The floor structure, as originally planned, consisted of six stiffening trusses of uniform depth when the floor beams framed at the neutral axis of the trusses. Between the outer pairs of trusses, provision was made for a sidewalk and a roadway for one lane of vehicles. An elevated promenade for the use of people of leisure and invalids was included between the inner pair of trusses in the plane of the top chords. The two intermediate spaces were designated for railway cars. To provide the additional underclearance required by the Secretary of War, the stiffening trusses were raised in relation to the floor beams, thus making it unnecessary to change the profile of the roadway. The sidewalks were also eliminated, and the bridge was widened to provide roadways for two lanes of vehicles—a most fortunate decision, considering the needs of present-day motor traffic. At the same time, the inner trusses were made deeper to serve as railings for the promenade. The floor structure was to be constructed mainly of wrought iron. The design remained in this status until May, 1878, when, in spite of repeated objections by Colonel Roebling, the Board of Trustees voted that the bridge should be constructed to accommodate standard-gage "palace" or sleeping cars in order to make possible continuous transit between the two cities without transfer to bridge cars. Colonel Roebling had previously determined that provision should be made for a lighter weight passenger car with not less than 6-ft gages to insure steadiness in high winds.

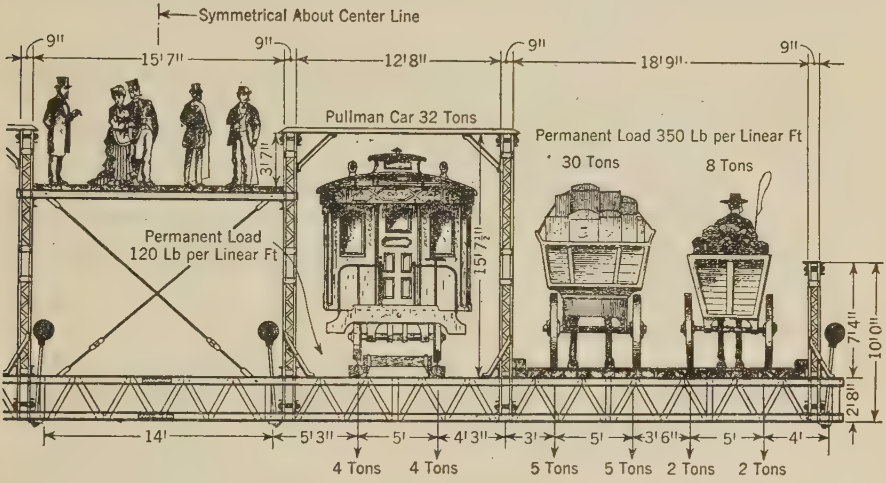
The Board of Trustees believed its action would not cause a serious change in the plans (which had been completed); and, accordingly, bids for furnishing the ironwork were taken in June, 1878. The president was authorized by the Board to award the contract; but the cash on hand at the time was only $2,582.06, and, with liabilities of more than $157,000, no award was made. In the meantime, Colonel Roebling investigated the possibility of using steel and reported that (letter from the chief engineer to the president of Board of Trustees, New York and Brooklyn Bridge, dated April 2, 1879):

> "There will be nothing experimental, therefore, now on our part in the use of steel, as manufacturers have proved satisfactorily that a mild grade of steel in every way suited to bridge construction can be produced."

In April, 1879, the Board ordered that revised plans and specifications be prepared on the basis of using principally steel in place of wrought iron. Bids on this basis were to have been received on May 1 but were delayed until May 27, pending the outcome of an investigation of a charge against the engineers that they released advance information of the steel specifications for a consideration of $10,000. A committee of Trustees, after investigating, stated: "It is unnecessary to say that there is not the slightest evidence to sustain the charge."

The contract was finally consummated on August 6, 1879, with the Edge Moor Iron Company, calling for the delivery of not more than 500 tons of steel during the remainder of that year. For financial reasons, no deliveries were to be made between January 1 and May 1, 1880, and the Board reserved the right, until May 1, to order the remainder of the steel called for in the contract. Progress was slow. Revised drawings were not completed until January, 1880. The unprecedented size of the channel sections made it necessary for the mill to design and construct new shears to cut the blooms and new rolls to form the channels. The engineers had also assumed that it would be possible to fabricate the eyebar stiffening-truss diagonals by the same procedure that had been used to make iron eyebars; but, when made in this way, the diagonals would not meet the specifications. Four months of constant and costly experimentation were required to develop a satisfactory method. It was not surprising, therefore, that the contract price increased from 4.35¢ a lb to 8.50¢ a lb for subsequent orders of steel not covered by the original contract. The first shipment was made in November, 1880, and field erection was begun in January, 1881.

The original contract called for approximately 5,500 tons of steel. When the change from iron to steel was made, it was estimated that there would be a saving in tonnage of from 12% to 15%. As the work progressed, it was discovered that some additional 1,100 tons would be required. This was due partly to an increase in the depth of the four inner trusses (see Fig. 5(*a*)) and other strengthening necessary to accommodate the Pullman cars, but largely because the steel could not be rolled as thin as iron. Considerable concern was expressed by the Trustees regarding the resulting margin of safety of the bridge, to which Colonel Roebling replied: "The margin of safety in this bridge is still over four, which I consider safe." He added, however, that perhaps an error of judgment had been committed in using a design stress of 15,000 lb per sq in. in place of 20,000 lb per sq in., although, even at the lower unit stress, some of the sections were deliberately made thicker to compensate for corrosion, which he probably visualized would result from "neglect of posterity."


Symmetrical About Center Line
9"
15'7"
9"
12'8"
9"
18'9"
9"
Pullman Car 32 Tons
Permanent Load 350 Lb per Linear Ft
30 Tons
8 Tons
3'7"
Permanent Load
120 Lb per Linear Ft
15'7½"
7'4"
10'0"
2'8"
14'
5'3"
5'
4'3"
3'
5'
3'6"
5'
4'
4 Tons
4 Tons
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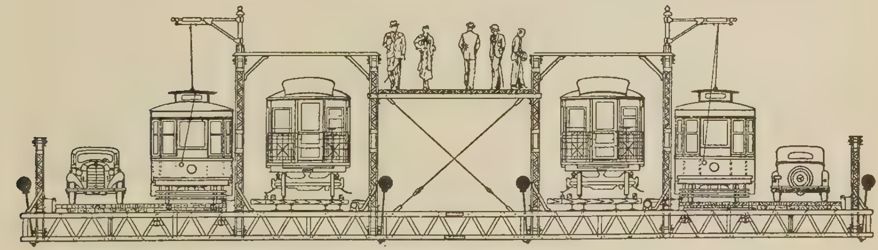
5 Tons
2 Tons
2 Tons

(*a*) 1883–1897



(*b*) After 1897 (Trolley Cars Rerouted over Railway Tracks, December 15, 1944)

Fig. 5.—Cross Section at Middle of River Span

Financial and Political Difficulties

Engineering and construction problems were not the only difficulties that confronted Colonel Roebling and his associates. In the preparation of the original design of the bridge, his father had estimated the cost of the structure at $7,000,000. This estimate, intentionally, did not make any allowance for real estate. John Roebling visualized approach structures supported on small masonry pillars and iron columns in such a way that the property beneath could be utilized for commercial purposes such as a market hall to supersede the "old Fulton Market." He stated:[2]

> "It is desirable that the construction of the Bridge and the improvements underneath should be carried on simultaneously, and upon one and the same plan. The Bridge floor will thus form the roof of the houses underneath and must be made fire and water-proof. My estimate includes the necessary pillars for the support of the Bridge floor, but no more. The balance of the work must be charged to the improvements of the ground, which will pay for themselves. For this same reason have I not included an estimate of real estate, because its value will not be destroyed, but on the contrary it will, in connection with the contemplated improvements, be considerably enhanced."

Furthermore, he reported:

> "It so happens that this part of New York has been very much neglected. The blocks are densely crowded by the poorest class of buildings, the removal of which will be desired by every citizen who feels an interest in the general improvement of the city."

Real estate costs, however, eventually totaled about $3,800,000. As construction advanced, it soon became evident that the cost was exceeding the original estimate. Frequently, the funds were almost entirely depleted, and upon such occasions Colonel Roebling was asked for an accounting and also an estimate of cost for completing the bridge. On at least two occasions, all work had to be halted and the engineers placed on half pay until such time as additional funds could be obtained. Once this required entering a suit of mandamus against John Kelly, then Comptroller of New York City, which was carried to the Court of Appeals for final decision. The cost of the bridge when opened to traffic was approximately $15,500,000.

It was not surprising, therefore, that suspicions were aroused as to the proper expenditure of the funds, and the records of the day are filled with insinuations and accusations which quite often led to investigations.

As organized by the Act of 1867 (Chapter 399), the New York Bridge Company was a private corporation with an authorized capital stock of $5,000,000. Of this sum, $3,000,000 was subscribed by the City of Brooklyn, $1,500,000 by the City of New York, and $500,000 by individual investors. The act was amended in 1869 (Chapter 26) to provide for representation of the cities on the Board of Directors; but, as these directors had no vote in the election of others, the control of the Board remained entirely in the hands of the private stockholders. Decisions on all business matters were made by an Executive Committee, it being ordered by the president of the company at the very beginning that: "* * * the Chief Engineer should have nothing to do with the making of any contracts or purchase of any supplies." The action of this committee in appointing the majority stockholder as "general superintendent" on a 15% fee basis (including the cost of real estate) for supervising the construction of the tower foundations to high-water level, was, for a time, the subject of much bitter debate and an investigation which resulted in a considerable downward adjustment of the fee finally paid.

In June, 1874, the New York State Legislature again amended the Act of 1867 to provide for the purchase of the stock held by the private investors and to transfer the completion of the bridge to the direction of the mayors and comptrollers of each city, together with a Board of Directors appointed by them. The reorganization on this basis was modified by still another Act of Legislature passed in May, 1875 (Chapter 300), which provided for the dissolution of the New York Bridge Company and the establishment of the bridge as a public work of the two cities. The first meeting of the Board of Trustees appointed under authority of the act was held on June 9, 1875. This change seemed only to add to the suspicions of the general public. In 1876 an application for an injunction to halt further construction was made to the Circuit Court of the United States; and again, in 1879, upon petition of D. Willis James and others stating that the bridge would be unsafe as well as an obstruction to traffic, that it would reduce property values, and, furthermore, that it would cost in excess of the cost fixed by law, a complete investigation of the project from the very beginning was conducted by the Committee on Commerce and Navigation of the New York State Assembly. In the fall of 1882, another investigation of the affairs of the bridge was initiated by the Board of Trustees, following charges made by a daily newspaper, the *New York World*, accusing the Trustees of malfeasance in office, neglect of duty, conspiracy, and misappropriation of funds. All that this investigation disclosed, however, after more than a year of search, was an overpayment of $9,578.67 to some of the material contractors because of clerical errors.

Then, there were problems, such as the case of widow Delaney, whose husband had the misfortune to fall off the bridge. The Trustees paid the funeral expenses, amounting to $51.12 and already donated the sum of $25 to Mrs. Delaney; but she sought further assistance. It was decided that, although she had no legal claim on the Trustees and any donation granted her would be simply an act of charity, a sum not to exceed $100 was to be paid to her in instalments, at the discretion of the vice-president.

Opening Ceremonies

The bridge was finally completed and opened to the public on May 24, 1883. The opening ceremonies were of national importance, being attended by the President of the United States, the governors of several states, and the mayors of nearly all the cities in the vicinity.

It was a gala holiday and excursions were run by the railroads from the neighboring cities and towns. As President Chester A. Arthur, Governor Grover Cleveland, and Mayor Franklin Edson of New York City, walked over the bridge, escorted by regiments of the National Guard, salutes were fired from the Navy Yard and from the forts in the harbor, joined by five naval ships anchored below the bridge. Whistles were blown and the chimes of Trinity Church in New York City rang out.

After the exercises, which were held on the Brooklyn side, the President, the Governor, and the Trustees were driven to the near-by home of Colonel Roebling to offer him their felicitations as he was unable even to attend the ceremonies. In the evening, the bridge was cleared for an elaborate display of fireworks.

The joys of opening day, however, were dampened a week later on Memorial Day when a skeptic yelling that the bridge was falling created a panic and caused the death of thirteen people on the promenade stairway at the New York anchorage.

TRAFFIC

After considerable discussion and voting, the following toll rates were established by the Board of Trustees:

Traffic	Cents
Pedestrians	1
Railroad fare	5
One horse or horse and man	5
One horse vehicle	10
Two horse vehicle	20
Additional horses, each	5
Sheep and hogs, each	2
Neat cattle, each	5

In later years these rates were modified from time to time and, except for the railroad fare, were finally abolished in 1911.

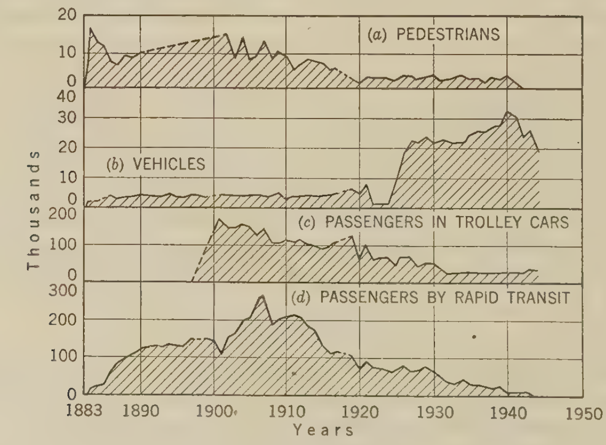


FIG. 6.—RECORD OF DAILY TRAFFIC

Pedestrian traffic was particularly heavy at first and on Sundays and holidays one of the roadways was usually closed to vehicle traffic and made available for pedestrians to Brooklyn, the promenade being restricted to those crossing to New York. In the first year pedestrian traffic averaged about 16,600 daily (see Fig. 6) with morning and evening hourly peaks of more than

2,000. The fad of walking over the bridge apparently ended about 1910, because since then traffic has gradually decreased to about 3,000 people a day. By order of the Navy Department, the bridge was closed to pedestrian traffic on July 27, 1942, for the duration of World War II.

Railway traffic over the bridge began on September 24, 1883. This facility probably received more thought and planning than any other feature. John A. Roebling visualized multiple-car, rope-driven trains running alternately back and forth over each track. In 1878 Colonel Roebling still recommended this system, making the following pertinent comment in regard to the circulating system (letter from the chief engineer to the president of Board of Trustees, New York and Brooklyn Bridge, dated February 25, 1878):

> "Imagine for a moment that at 6 o'clock P.M., a single car on the circulating system with a carrying capacity of fifty passengers has arrived. A crowd of two hundred is waiting to get in! Who will enter it? Why the strong men and active boys. In a minute-and-a-half or probably two minutes, another car arrives. By this time a fresh relay of strong men and active boys is at hand, and the performance is repeated. It may thus happen that ladies and timid persons will have to wait a whole hour before they can cross."

It was finally suggested, however, to use circulating multiple-car trains stopping at elevated platforms on each end of the bridge and, on this basis, Colonel Roebling agreed that the circulating system was preferable and recommended its adoption. He also proposed the installation of tracks on the roadways for horsecar transportation, but this facility was later abandoned by action of the Board of Trustees.

Traffic using the rapid transit trains increased from approximately 25,000 passengers a day in 1883 to a maximum of about 266,000 a day in 1907, with a peak hour count of more than 46,000—nearly 82% more than the most optimistic forecast of such traffic made in 1879. Early in 1908, the first subway to Brooklyn was placed in operation and two years later the Manhattan Bridge was opened to traffic. Passenger traffic over the Brooklyn Bridge, accordingly, went on the decline and on March 5, 1944, when the service was discontinued, there were only about 7,800 passengers using the trains each day.

Single cars were used at the beginning, increasing in number as the traffic demanded. Steam locomotives were used to switch the cars at either end and to haul them over the bridge during the slack hours. On foggy days, flagmen were stationed at frequent intervals along the tracks to guard against rear-end collisions, until a satisfactory automatic block signal system became available in 1908. Oil lamps in the cars were replaced in 1895 with electric lights served by overhead trolleys. The locomotives were replaced by electric motor-driven cars in 1897, but use of the rope or cable power continued until 1908. The trains increased in length until six-car trains operating on a 1-min headway were used during the peak hours. Through trains from Brooklyn started running over the bridge in 1898 (the year that New York and Brooklyn were consolidated and the bridge was placed under the jurisdiction of the Department of Bridges of the City of New York).

By 1901 traffic facilities at the Manhattan terminal had become so overtaxed that the condition was brought to the attention of the New York County

Grand Jury by the District Attorney. During the next decade continual effort was made to alleviate the congestion. Legislation was passed and numerous plans were projected for a new station. Elevated lines to disperse the traffic to several stations were proposed and lines connecting with the Williamsburg and Manhattan bridges, then under construction, were also urged; but, except for acquiring property for a new station, nothing was ever accomplished because of the stolid inaction on the part of the city officials in the face of certain injunctions to any construction which might have been attempted. However, after repeated applications, approval was granted to the extension of the head house first to the west side of Park Row and later to the west side of Centre Street (see Fig. 1), thus providing for six-car trains starting in January, 1908. This construction was regarded as temporary until a new station could be built but it remained in place until 1935.

The property acquired for the station was finally used as a site for the Municipal Building. In one last effort to connect the rapid transit tracks with those of the other two bridges, connections were constructed from the bridge approach passing under Park Row to a subway station beneath the Municipal Building. At this point the tracks were intended to connect with those of the Centre Street subway which was opened to traffic in 1913. The construction of the connections, except for the installation of the tracks, was completed in 1914, but these connections were never used because the operating company refused to enter into an agreement with the city.

Tracks for trolley cars were installed on the bridge roadways in 1897 (see Fig. 5(*b*)) and within a few years the trolleys were transporting approximately 169,000 passengers a day. This traffic has also decreased to about 38,000 passengers a day. Work has been completed which on December 15, 1944, enabled the rerouting of the trolley cars over the tracks formerly used by the trains. The stations, which had long since become an eyesore as well as a maintenance headache, have also been demolished.

Vehicular traffic over the bridge increased, roughly, from 1,700 vehicles a day in 1883 to approximately 30,000 a day in 1940. This seems to be about the limit of capacity, however, when compared with the traffic trends of the other East River bridges.

Pneumatic mail-tube service over the bridge was initiated in August, 1898, reducing the time of transporting mail between Manhattan and Brooklyn from 25 min to 3 min.

INTERRUPTIONS OF TRAFFIC

During its existence, the Brooklyn Bridge has not always operated without interruption. On July 29, 1898, a horse died on the south roadway near the Brooklyn tower, completely blocking all Brooklyn-bound vehicular and trolley traffic. The entire river span was soon jammed, causing a shifting of the cable saddles riverward and a readjustment of stresses in the overfloor stays. Some of these stays, being attached directly to the tower tops rather than to the saddles, picked up more than their share of the load, thus causing a severe lateral distortion of the stiffening trusses described in the records as a "buckling" of the bottom chords. No mention was recorded of what repairs, if any, were necessary; nor is there any physical evidence of any having been made. The condition partly corrected itself with the dispersion of the live load and the saddles slowly returned to their former position.

Again, on July 24, 1901, nine consecutive suspenders at the center of the river span along the north cable were found broken, requiring the cessation of all rail traffic for 36 hours. These suspenders are short, solid, round rods connected to the floor system through trunnions to permit longitudinal movement. The discovery attracted the attention of Eugene Philbin, then District Attorney for New York County who retained Edwin Duryea, Jr., and Joseph Mayer, Members, ASCE, consulting engineers, to

> "* * * make an examination of the bridge * * * with a view of ascertaining whether or not the capacity of the bridge is overtaxed * * * and as to whether or not the bridge has been allowed to deteriorate because of improper supervision, inspection and repair."

These engineers submitted a lengthy report severely criticizing the men entrusted with the care of the bridge and also certain features of the design of the structure. A careful review of the report made by Richard S. Buck, M. ASCE, chief engineer and Henry A. LaChicotte, assistant engineer of the Manhattan and Queensborough bridges at the request of Charles C. Martin, M. ASCE, chief engineer of the Brooklyn Bridge, not only refuted all accusations of lax supervision, inspection, and maintenance, but also showed basic fallacies in the Duryea-Mayer analysis of the design. Wilhelm Hildenbrand M. ASCE, who, with Mr. Martin, had been associated with Colonel Roebling on the design and construction of the bridge also refuted the Duryea-Mayer report. No official action was taken by the District Attorney.

Motor vehicles were barred from the bridge on July 6, 1922, following a riverward shifting of the northerly two cable saddles over the Manhattan tower. From a study of saddle movements, it appears that these movements were simply the readjustment of cable stresses consistent with the condition existing in the other two cables. The ban against motor vehicles was lifted to passenger cars on May 12, 1925.

The Eighth Wonder of the World

The completion of the Brooklyn Bridge was without question a milestone in the science of suspension bridge construction. Its span held the record for 20 years and was not materially exceeded until 1925, when the Philadelphia (Pa.)-Camden (N. J.) Bridge across the Delaware River was opened to traffic. It also marked an advance of 51% in length of span which was not surpassed until the completion of the George Washington Bridge across the Hudson River between New York City and Fort Lee, N. J., in 1931, which increased span length 89%.

Considering the manifold unprecedented problems which confronted its builders, and the limited means with which to solve them, the Brooklyn Bridge truly stands as, so frequently described, "The Eighth Wonder of the World."

References

Many references were consulted in the preparation of the Symposium, some of them probably available only in the files of the Department of Public Works of the City of New York. An exceedingly comprehensive bibliography on the Brooklyn Bridge may be found in **"A History of Suspension Bridges in Bibliographical Form,"** by A. A. Jakkula, U. S. Public Roads Administration and the Agricultural and Mechanical College of Texas, College Station, 1941, pp. 198–236. The following references proved to be the most productive of useful information:

"Report of John A. Roebling, C. E., to the President and Directors of the New York Bridge Company on the Proposed East River Bridge," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1867.

"Pneumatic Tower Foundations of the East River Suspension Bridge," by W. A. Roebling, Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1872.

"Notes on the Masonry of the East River Bridge," by Francis Collingwood, *Transactions*, ASCE, Vol. VI, 1877, p. 7.

"Report of the Chief Engineer of the New York and Brooklyn Bridge, January 1, 1877," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1877.

"Cable-Making for Suspension Bridges," by Wilhelm Hildenbrand, D. Van Nostrand Co., New York, N. Y., 1877.

"Report of the Chief Engineer on the Strength of the Cables and Suspended Superstructure of the Bridge to the Board of Trustees, January 9, 1882," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1882.

"New York and Brooklyn Bridge—Proceedings 1867–1884," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1884.

"New York and Brooklyn Bridge: Report of the Board of Experts on the Feasibility and Practicability of Permitting Cars of Surface Railroads and Trains of Elevated Railroads or Either of Them to Operate on the Bridge," report by V. G. Bogue, G. H. Thomson, and L. L. Buck, February 8, 1897 to the Board of Trustees, Brooklyn, N. Y., *The Railroad Gazette*, Vol. 30, 1898, pp. 564, 824, 841, and 874.

"The Condition of the Brooklyn Bridge," a report to E. A. Philbin, District Attorney, New York, by Edwin Duryea, Jr., and Joseph Mayer, *Engineering News*, Vol. 46, No. 15, October 10, 1901, pp. 250 to 262.

"Report of the Commissioner of Bridges of the City of New York," January 1, 1901, to November 30, 1901, Dept. of Bridges, New York, N. Y., pp. 54 to 81.

"Mr. Hildenbrand's Reply to the Report of Messrs. Duryea and Mayer on the New York and Brooklyn Bridge," *Engineering News*, Vol. 47, No. 3, January 16, 1902, pp. 54–60.

"Precise Levelling in New York City," by Frederick W. Koop, City of New York, Board of Estimate and Apportionment, Office of the Chf. Engr., New York, N. Y., 1915.

"Brooklyn Bridge—1883 to 1933," Dept. of Plant and Structures, New York, N. Y., 1933.

DESIGN AND STRESS CONDITION

By O. H. Ammann,[8] M. ASCE

Introduction

The Brooklyn Bridge is a structure of momentous interest in the history of bridge building. The bridge furnishes a wealth of information on the safety and useful life of a structure of its kind and on the merits and shortcomings of its design, as viewed from present-day experience and advanced knowledge.

Its construction was the culmination of a series of splendid records in suspension bridge building in the nineteenth century. Remarkable progress in suspension bridge building began with the achievements of English and French engineers in the early part of that century. In 1826 Thomas Telford completed the famous Menai Bridge in Wales with the then unprecedented span of 576 ft; and, in 1834 the French engineer, M. Chaley, built an 870-ft span at Fribourg, Switzerland.

In the second half of the nineteenth century attention was focused on the United States because of new achievements in the field of suspension bridges and advances in the art of building wire cables to new records of span. John A. Roebling, who planned and initiated the building of the Brooklyn Bridge, himself had created a number of outstanding bridges of this type, among them the unique Railway Suspension Bridge across the Niagara River in 1855 and the bridge across the Ohio River at Cincinnati in 1867. With its span of 1,057 ft, the Ohio Bridge was at that time the longest suspension bridge and served as a close model for the Brooklyn Bridge.

The latter, however, outranked its predecessor because of a 50% longer span (1,595 ft), exceptionally long side spans (933 ft), and more than 100% greater carrying capacity. The Brooklyn Bridge is also distinguished by the first introduction of steel wire in place of the wrought-iron wire previously used for cables. Cable wire was increased in strength from 90 kips per sq in. to 160 kips per sq in., and thus much longer and heavier spans were possible. For the first time, also, the cable wire received a galvanized zinc coating, instead of a film of grease, which contributed to increased life.

Together with the Eads Bridge in St. Louis, which was under construction at the same time, the Brooklyn Bridge has the distinction of being one of the first large bridges in which pneumatic caissons were used in building great foundations.

Conception of the Original Design

Although the design of the Brooklyn Bridge reflects general practice in the nineteenth century, in many respects it reflects the individual conceptions of John A. Roebling. The most conspicuous features are the massive masonry towers, which were common in early suspension bridges (Fig. 7). Mr. Roebling properly remarked that "in a work of such magnitude, located between two great cities, good architectural proportions should be observed." He stated:[9]

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[8] Cons. Engr., New York, N. Y.

> "The great towers will serve as landmarks to the adjoining cities, and they will be entitled to be ranked as national monuments. As a great work of art, and as a successful specimen of advanced bridge engineering, the Brooklyn Bridge will forever testify to the energy, enterprise and wealth of that community which shall secure its erection."

The graceful cable curve with its small sag (one thirteenth of the span, which is characteristic of older suspension bridges) also contributes to good appearance. The relatively shallow parallel-chord stiffening trusses along the floor, although considerably deeper than those originally conceived by Mr. Roebling, do not detract from the elegance of the structure. The inclined stay ropes add to the unique appearance.

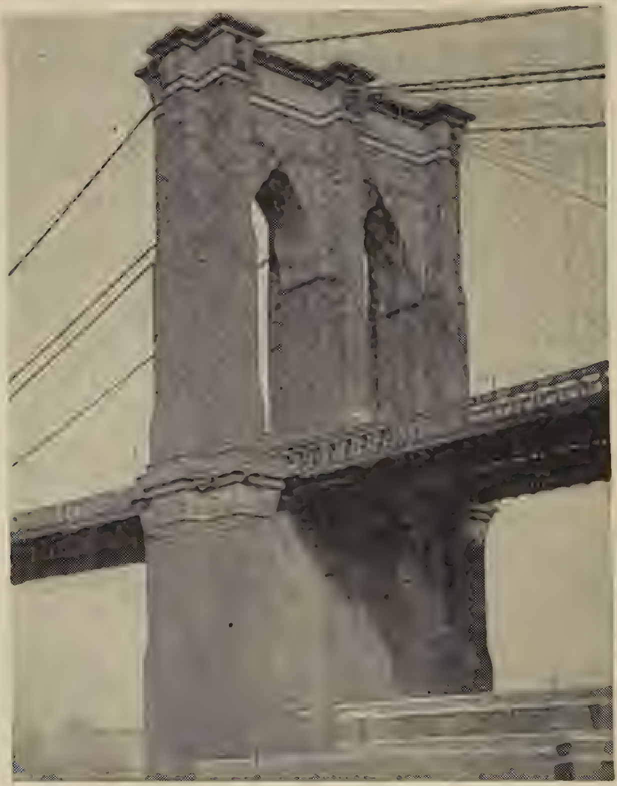

Fig. 7.—"The Most Conspicuous Features Are the Massive Masonry Towers"

At the time of the conception of the Brooklyn Bridge, suspension bridge building had already been highly developed as an art, but proportioning had not yet reached the stage of refined modern science. The theory of the suspended cable was well known, as was also the common theory of elastic structures; but the designers of the Brooklyn Bridge were not familiar with more refined methods which would have aided in the analysis of the highly indeterminate structure. The so-called deflection method of calculating stresses, which is now commonly applied to suspension bridges, was developed only in the early years of the twentieth century. Whatever records remain of the stress calculations used in designing the Brooklyn Bridge indicate a very crude conception and analysis of the stress action in the stiffening system.

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[9] "Report of John A. Roebling, C. E., to the President and Directors of the New York Bridge Company on the Proposed East River Bridge," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1867, p. 3.

Nevertheless, Mr. Roebling had a keen practical understanding of the suspension system and the functions of the component parts. He appreciated the merits inherent in the simple unstiffened cable suspended from the towers. Experience had amply demonstrated that the cable had to be stiffened against objectionable or destructive oscillatory motions incited by moving loads or wind; but Mr. Roebling realized that such stiffening must be kept to a rational limit. Contrary to this wise view, in the early part of the twentieth century, there was a fallacious tendency to make a virtue of rigidity, which resulted in many excessively rigid, economically wasteful, and clumsy looking suspension bridges.

Previous developments had led to two essentially different methods of stiffening, one by a system of inclined stay ropes and the other by stiffening girders or trusses along the floor. Mr. Roebling believed that he could obtain greatest effectiveness by a combination of the two. He assigned to the trusses the principal task of stiffening the cables vertically in the center of the center span and in the anchorage portion of the side spans, where the stays cannot be effective. He also considered the trusses essential in resisting dynamic wind action, for he stated:[10]

> "* * * to guard against vertical and horizontal oscillations, and to insure that degree of stiffness in the flooring which is absolutely necessary to meet the effects of violent gales in such an exposed situation, I have provided six lines of iron trusses, which run the whole length of the suspended floor from anchor wall to anchor wall * * *. Those parts of the trusses which extend below the floorbeams, afford an excellent means for lateral trussing and bracing."

According to Mr. Roebling's original plans, all six trusses were to be 12 ft deep, or only one one hundred and thirtieth of the center span. For the traffic then contemplated these flexible trusses would have been adequate, but (see Mr. Seely's paper) it appears that, during construction of the bridge, Col. Washington A. Roebling (who became chief engineer after his father's death) was induced by the directors of the Bridge Company to provide for the possibility that heavy Pullman cars might eventually be moved over the bridge. This called for stiffer trusses near the tracks and thus the four inner trusses were increased in height to 17 ft.

John A. Roebling assigned to the stay system the principal task of reducing the motions of the cable saddles under unbalanced loads on center and side spans. The stays, in combination with the stiffening trusses, fixed at the towers, form a cantilever system that relieves the cables of a substantial part of the vertical load. This stay system is so strong (John A. Roebling stated), that:[11]

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[10] "Report of John A. Roebling. C. E., to the President and Directors of the New York Bridge Company on the Proposed East River Bridge," Eagle Book and Job Printing Dept., Brooklyn. N. Y., 1867, p. 17.

> "The supporting power of the stays alone will be ample to hold up the floor. If the cables were removed, the Bridge would sink in the center, but would not fall."

Col. Washington A. Roebling disclosed that he was aware that the stays do not act in perfect harmony with the cables and that for a single span he would not advocate their use; yet in this case, on account of interaction between center span and the long side spans, he considered them indispensable and he maintained that the "want of harmony between stays and cables is less in reality than in theory."

It is not clear from the records why unusually long side spans were adopted—very likely to reduce the height of the anchorages and thus to minimize their cost. John A. Roebling realized the resulting difficulties, especially in maintaining "the balance of spans," as he called it. Originally, he considered anchoring down the cables of the side spans halfway between the towers and the anchorages—an idea which was advanced in subsequent years by several engineers who reviewed the design. Mr. Roebling abandoned this feature, however, as not being very effective, and he decided that the motions of the cable saddles were most effectively checked by the stay system.

The bridge was originally built to accommodate (on two tracks) passenger trains composed of cars attached to an endless rope propelled by a stationary engine. Two 2-lane roadways were to serve horse-drawn vehicles, and a pedestrian promenade was provided on a central elevated platform (see Fig. 5).

The live load corresponding to this traffic capacity was estimated at 1,800 lb per ft of bridge and, with an estimated dead load of 8,750 lb, brought the total load to be carried by the bridge to 10,550 lb.

In course of time a trolley track was placed on each roadway, the inner tracks were used by heavier electric trains, and certain additions were made to the floor structure, so that for a period of time the bridge carried as much as 40% more load than was originally intended.

REVIEW OF DESIGN IN LIGHT OF PRESENT KNOWLEDGE

In 1943, on the eve of transforming the Brooklyn Bridge from a structure principally for passenger traffic in electric trains and trolley cars into a structure for modern highway traffic, it appeared advisable to review its design with a complete calculation of the stresses and to undertake an examination of its physical condition.

The completed investigation aimed first at determining the limitations of capacity without major alterations and second at exploring the possibilities of major reconstruction to create an up-to-date link in the highway system.

The Main Carrying System: Cables, Stiffening Trusses, and Stays.—Study of the original design and changes made in course of time reveal a stress condition that is far more complex, statically indeterminate, and uncertain than would appear from a superficial examination of the geometry of the system.

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[11] "Report of John A. Roebling. C. E., to the President and Directors of the New York Bridge Company on the Proposed East River Bridge," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1867, pp. 18 and 19.

Fortunately, however, the uncertainty of stress action does not apply to any great extent to the carrying members upon which the safety of the structure depends—the cables, towers, and anchorages. Uncertainty as to stress action is confined to the vertical and lateral stiffening systems, damage to which does not necessarily endanger the safety of the structure.

As in most suspension bridges the four cables are continuous from anchorage to anchorage and form natural catenaries under the loads transmitted to them by the suspenders (Fig. 8). On top of the towers the cables rest on cast-iron saddles and these in turn bear on nests of rollers designed to permit longitudinal motions of the cables under temperature changes and unbalanced loads. These rollers are very small (only 3.5 in. in diameter); and in course of time collection of dirt and rust, and in some cases dislocation of the rollers, increased the friction to such an extent that under most ordinary conditions no motion took place. Only on a few rare occasions of severe loadings was a "slipping" of the saddles observed. For future highway traffic, the existing roller nests may be considered frozen, and any unequal cable pull can be assumed transmitted to the rigid towers.

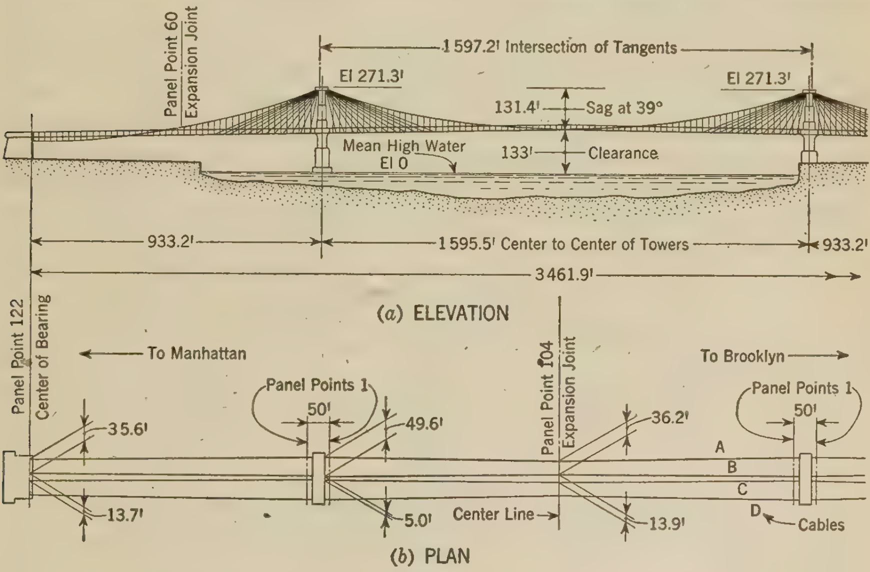

Panel Point 60
Expansion Joint
1 597.2' Intersection of Tangents
El 271.3'
El 271.3'
131.4'
Sag at 39°
Mean High Water
El 0
133'
Clearance
933.2'
1 595.5' Center to Center of Towers
933.2'
3 461.9'
(a) ELEVATION
Panel Point 122
Center of Bearing
To Manhattan
Panel Points 1
50'
35.6'
49.6'
Panel Point 104
Expansion Joint
36.2'
To Brooklyn
Panel Points 1
50'
A
B
C
D
Cables
13.7'
5.0'
Center Line
13.9'
(b) PLAN

Fig. 8.—General Plan and Elevation, the Brooklyn Bridge

The bridge thus acts as three independent suspension spans, and this condition was assumed for the calculation of stresses in the first phase of conversion, in which the cable saddles would be left as they were. This realistic assumption simplified the stress calculations. On the other hand, the inaction of the roller nests has introduced uncertain stresses because it is not known under what temperature and live loads the rollers did become frozen. The floor is suspended from the cables by the customary wire-rope suspenders at the floor beams.

Six stiffening trusses are placed along the floor (see Fig. 5); four of them are close to the four cables and the two intermediate trusses are placed between the outer and inner cables. The four inner trusses were made 17 ft deep, whereas the outer trusses are only 8.75 ft deep. This cross-sectional arrangement is one source of indeterminateness, although not a major one since the continuous and closely spaced floor beams are sufficiently rigid to assure a fairly even deformation of the six trusses. Furthermore, the moment of inertia of the two outer trusses is only about one eighth of that of the four inner ones, so that their effect could be neglected.

Differing from the common stiffening trusses of modern suspension bridges, which are hinged and free to move longitudinally at the towers, those of the Brooklyn Bridge are practically fixed at the towers. At the center of the center span, an expansion joint is provided which, although it acts as a hinge can transmit shears from unsymmetrical loadings. In each side span, a cantilever arm extends from the tower to an expansion joint acting as a hinge at about midspan. From there to the anchorage the trusses act by themselves as simple spans (Fig. 9).

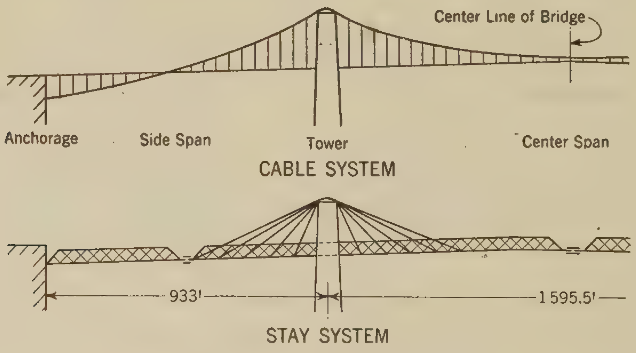


Fig. 9.—Cable and Stay Systems, Brooklyn Bridge

The four stiffening trusses near the cables are supplemented by a system of diagonal stays which run from the tops of the towers to the bottom chords of the trusses (Fig. 8). Because the trusses are fixed longitudinally at the towers they form (together with the stays) a complete cantilever system superimposed upon the cable system. Thus, there are two distinct, but interacting, vertical carrying systems, which form a highly indeterminate compound system. Although the interaction between the two systems for temperature, live load, and any additional dead load can be calculated fairly reliably on the basis of the elastic properties of the component members, the stresses under the original dead load are by no means determinable with any degree of accuracy.

The lower chords of the stiffening trusses must resist the horizontal components of the stay tensions in addition to the flexural stresses caused by the bending of the trusses. These direct stresses become cumulative and very large toward the towers and have been the reason for overstress in the bottom chords.

The web system of the four deep stiffening trusses is formed by vertical posts at the floor beams and a quadruple system of diagonals composed of forged bars (Fig. 10). Half the diagonals (the so-called "counters") are adjustable in length. In course of time many of the counters have been tightened far beyond a nominal initial stress, thus inducing appreciable uncertain stresses.

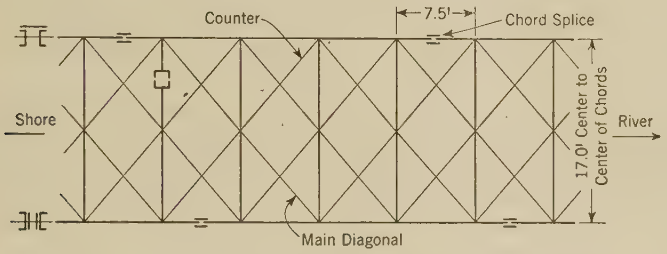

Counter
7.5'
Chord Splice
Shore
17.0' Center to Center of Chords
River
Main Diagonal

Fig. 10.—Inner and Intermediate (High) Stiffening Trusses

The top chords of the trusses were provided with a number of construction joints, three in each half of the center span, which created zero moments at those points. These joints were later bolted tightly; but, since there is no record of the stress condition existing at the time of the bolting or of the final riveting of other truss connections, another uncertainty is introduced with respect to the stresses from the original dead load.

In addition to these complications and uncertainties affecting the primary or axial stresses, secondary stresses are caused by faulty or crude design details or lack of proper maintenance. In fact, such defects have led to a number of local structural failures in the past.

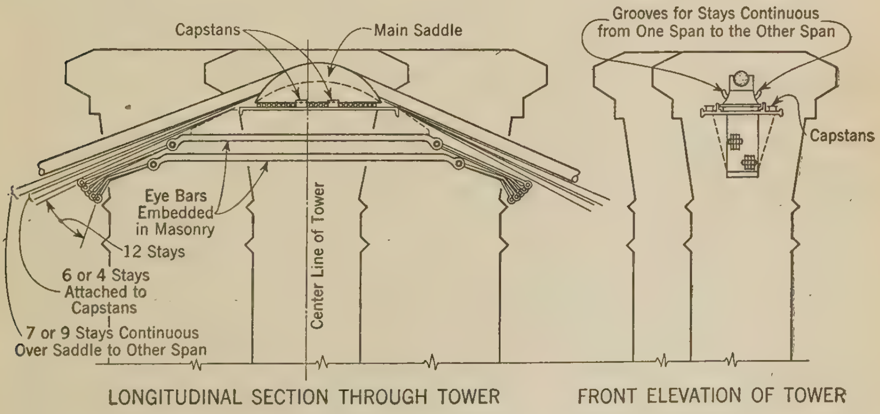

Capstans
Main Saddle
Grooves for Stays Continuous from One Span to the Other Span
Capstans
Eye Bars Embedded in Masonry
12 Stays
6 or 4 Stays Attached to Capstans
7 or 9 Stays Continuous Over Saddle to Other Span
Center Line of Tower
LONGITUDINAL SECTION THROUGH TOWER
FRONT ELEVATION OF TOWER

Fig. 11.—Arrangement of Overfloor Stays at the Tower Tops

One of the most incongruous details is the attachment of the stay ropes at the tops of the towers (Fig. 11). The stay ropes nearest the towers are connected to wrought-iron bars anchored in the tower masonry. Others are fixed to the saddle bed plates and some pass over the saddles. As a result, the latter group moved longitudinally with the saddles, while the others remained fixed.

Such motions must at times have caused appreciable redistribution of the stresses between the various stays and between the stays and the main cables. It would seem that on one occasion excessive strain put on certain stays caused excessive compression, with resulting bending or initial buckling, in the bottom chords of the stiffening trusses to which the stays are attached.

Under these conditions an accurate stress calculation throughout the structure is impossible. However, by the approximate analysis adopted, the stress condition has been determined with a sufficient degree of reliability to assure safety of the structure under actual load conditions.

Determination of Loads.—The present dead load was determined from the available records, supplemented by field measurements. The field survey showed a fairly uniform present dead load of 9,200 lb per lin ft or 800 lb per lin ft more than the weight when the bridge was completed. This increase is due largely to changes made in the deck structure.

When an estimate of the live load was being made, it was known that the elevated trains would be abandoned. They have since been removed, permitting the use of the tracks between the inner stiffening trusses by the trolley cars and leaving the two 17-ft roadways free for vehicular traffic.

Preliminary calculations indicated that a live load corresponding to unrestricted vehicular traffic on the two roadways would cause excessive stresses; and for this reason, as well as on account of the narrowness of the roadways, it was concluded that the vehicular traffic should be restricted to passenger automobiles as, in fact, it has been since 1925. On this assumption, the live load used for the calculations of the stresses in the existing structure was estimated at 2,400 lb per ft of bridge.

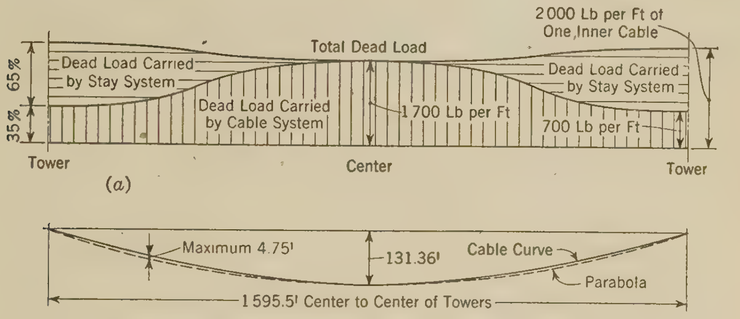
When applying this live load in the design of the stiffening system, it was realized that, imposing maximum load on partial stretches, with no load on the remainder of the span, would cause an excessively severe condition. Therefore, a light load of 600 lb per ft was assumed on the intervening stretches. The live load was combined with a maximum variation of temperature of ± 50° F.

Method of Stress Analysis in Main Carrying System.—The stresses in the vertical carrying system were determined by a combination of analytical approximations and measurement of certain stresses and deflections on the bridge. This procedure was ably devised by the staff of the Department of Public Works under the general guidance of Mr. Seely (author of the first Symposium paper). Mr. Delson developed and directed the theoretical analysis; and Mr. Gray (author of the third Symposium paper), the field operations.

It was essential to establish the exact cable curve under dead load and at known temperature, since it deviates considerably from the parabola due to the effect of the stay system. This was done by measuring the cable ordinates with reference to a carefully surveyed profile of the bridge floor. In the central section, outside the stay attachments to the floor, the cable curve was found to be a parabola, as expected; but it flattens considerably toward the towers, indicating the increasing absorption of the floor panel loads by the stay system. The maximum deviation from the parabola under dead load at 39° F is 4.75 ft in the center span and 5.3 ft in the side span (Fig. 12).

It was also essential to determine directly the dead-load tensions in the stay ropes. This was done by measuring the sag of certain selected stays and calculating the tension which produces that sag—proper adjustment being made for temperature differences.

From the dead-load concentrations and the stay tensions thus found, it was possible to determine the distribution of the dead load between cables and stays, to check the form of the cable curve, and to calculate the dead-load stresses in the cables.


2000 Lb per Ft of One Inner Cable
Total Dead Load
Dead Load Carried by Stay System
Dead Load Carried by Stay System
Dead Load Carried by Cable System
1700 Lb per Ft
700 Lb per Ft
65%
35%
Tower
Center
Tower
(a)
Maximum 4.75'
131.36'
Cable Curve
Parabola
1595.5' Center to Center of Towers

(*b*) CABLE CURVE AT 39° F

Fig. 12.—Distribution of Dead Load in the Center Span

Since there was no record of the erection procedure with respect to the stiffening trusses, it appeared reasonable to assume that, as is done in modern practice, the bolted construction joints in the top chords remained untightened and free to move and also that the other truss connections remained unriveted, until at least the greater part of the dead weight was in place. Under this assumption the trusses would be practically free from the original dead-load stresses. The stresses from subsequently added dead load were calculated by the method developed for the live load.

The stresses from live load and temperature were calculated first by a so-called elastic theory—that is, the relieving effect of the deflections was ignored. Since there are fifty stays in the center span, with twenty five near each end, in addition to the unknown cable pull, for any unsymmetrical live load, the elastic system would be statically indeterminate in the fifty-first degree. To simplify the laborious and unnecessarily refined procedure involved, the stays were assumed to act in groups of several adjacent stays of equal tension and each group was replaced by one resultant. Thus, the center-span system was simplified to a system statically indeterminate in the seventh degree for unsymmetrical loads. For symmetrical loads in the center span and for each side span the system became indeterminate in the fourth degree, requiring the solution, respectively, of seven and four elastic equations for each separate loading.

A further simplification was introduced by dividing the spans into approximately equal individual live-load lengths and assuming only full-load lengths for any load position. The moments and shears were calculated separately for each load length and the results were combined for maximum values (Fig.13). The chord stresses so found were then reduced to allow for the effect of the deflections caused by the live load. The reduction was determined by measuring the deflections and certain chord stresses under an experimental elevated train placed on the bridge and then comparing the stresses with those calculated by the elastic theory for that loading. This procedure led to stress reductions of from 15% to 25% in the side span and from zero to 20% in the center span. These reductions compare reasonably with those calculated for a number of other suspension bridges, considering the contribution of the stays to the stiffness of the Brooklyn Bridge. Finally, the readily determinable direct compression stresses in the bottom chords were added to the flexural stresses, so reduced, as horizontal components of the stay-rope tensions.

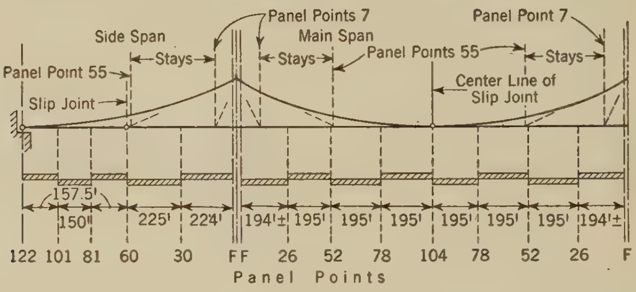


FIG. 13.—ASSUMED LOAD LENGTHS

The Make-Up and Strength of the Principal Members and Their Safety Under the Assumed Loads.—The stress investigations were supplemented by a program of tests to determine the present strength and other physical characteristics of the more important members, cables, anchorage chains, stay ropes, chords, and web members of the stiffening trusses. These tests are described more fully in the third and fourth papers of this Symposium.

Each of the four cables has a diameter of 15.75 in. and is composed of nineteen strands of wires which vary considerably in size, some being No. 7 gage and some No. 8 gage. The aggregate net area has been estimated at 144.8 sq in. Specimen tests on sixty wires cut from the cables show an ultimate strength ranging from 146,000 lb per sq in. to 192,000 lb per sq in., with an average of 162,600 lb per sq in. Other properties, although indicating a fairly hard steel, give assurance of an acceptable condition of the wire, with no apparent deterioration since it was erected.

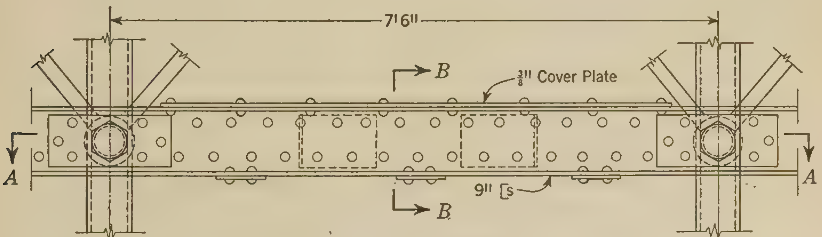
The maximum axial stress in the cable, under the live load of 2,400 lb per sq in., is calculated at 44,600 lb per sq in.—less than 30% of the average strength of the wire. The physical qualities of the wire would justify a cable stress of 60,000 lb and therefore permit an increase in cable stress of as much as 33% in case of an eventual reconstruction of the floor structure. In such reconstruction, if the stay system is eliminated, thus throwing the entire load on the cables, the margin for additional load would be only about 18%.

The stays are steel wire ropes 1.75 in., 1.875 in., and 2 in. in diameter.

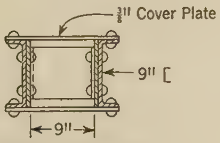
A full-size test of a rope disclosed an ultimate strength of 147,000 lb per sq in. The calculated live-load stress of 23,000 lb added to the dead-load stress of 21,000 lb derived from the sag measurements gives a maximum stress of 44,000 lb per sq in., which is only 30% of the ultimate strength of the rope. Therefore, the stays are also capable of sustaining a considerable increase in live load.

The stress calculations and measurements disclosed that the chords and web members of the stiffening trusses were the weakest parts of the structure. This is corroborated by the many web members that have broken and the chords that have been bent under severe load conditions.

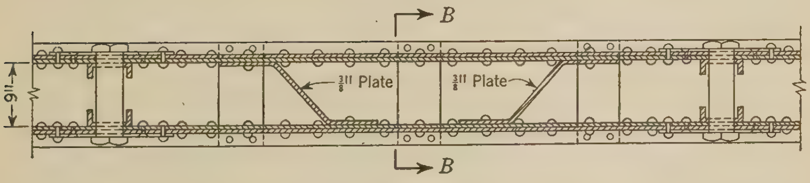
A typical chord is composed of two 9-in. rolled channels, reinforced in the heavier sections by web plates. The top chords were originally also provided with a cover plate, and in later years cover plates were added to many bottom chords (Fig. 14). In place of efficient latticing, the two channels are connected (in each 7.5-ft panel only) by two bent plate diaphragms. This very inefficient detail gave reason to expect that the chords have a relatively low buckling strength and it was deemed advisable to make a full-size model test. The model chord, built without cover plate and of ordinary structural steel, actually developed an ultimate strength of only 78% of the average yield point of the material.



(*a*) ELEVATION



(*b*) SECTION *B-B*



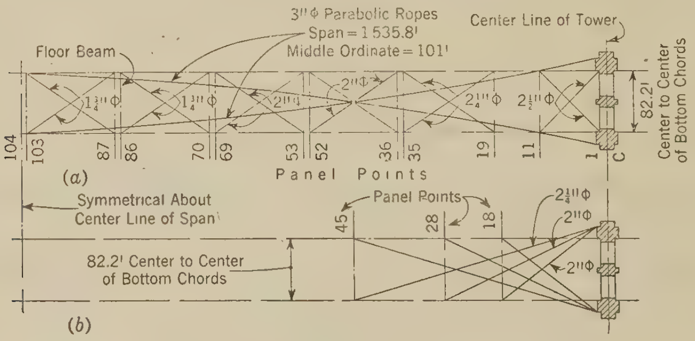
(*c*) SECTION *A-A*

Fig. 14.—Typical Panel, Bottom Chord

The actual chords are made of high carbon steel. Specimen tests cut from the chords revealed an average yield point of 48,700 lb per sq in.—as high as that for present-day silicon steel. Application of the 78% ratio to this yield point gave an estimated buckling strength of 38,800 lb per sq in. for the chord members without cover plates. With allowance for the cover plates, where added in later years, the strength was estimated at between 40,000 lb per sq in. and 42,000 lb per sq in. of the original section.

Compared to these strength figures, the calculations indicate compression stresses in some bottom chord members of more than 40,000 lb per sq in. and in a large number of more than 30,000 lb per sq in. In view of these large calculated stresses, it is not surprising that, in the region of the most highly stressed chord members, the latter sustained permanent deformations. On the other hand, although the chords were never straightened, no more serious consequences resulted thereafter, demonstrating that relatively high working stresses in the stiffening trusses of suspension bridges can be permitted. Nevertheless, it is advisable to reinforce the overstressed chord members in the Brooklyn Bridge.

All truss diagonals and counters are forged steel eyebars. A number of full-size bars, which were removed from the bridge and tested, revealed a yield point of from 43,000 lb per sq in. to 46,000 lb per sq in. and an ultimate strength of between 69,000 lb per sq in. and 76,000 lb per sq in. The initial dead-load stresses in the diagonals should be only nominal. Actually, because of evidently promiscuous tightening of the counters, these stresses in the counters, and consequently also in the main diagonals, were greatly increased. Stress measurements revealed actual dead-load stresses of more than 20,000 lb per sq in. That such stresses are easily produced is indicated by the fact that a single 360° turn of the adjusting sleeve causes a stress of 30,000 lb per sq in.


3" Φ Parabolic Ropes
Span = 1 535.8'
Middle Ordinate = 101'
Center Line of Tower
Floor Beam
1¾" Φ
1¾" Φ
2" Φ
2" Φ
2¼" Φ
2½" Φ
82.2'
Center to Center of Bottom Chords
104
103
87
86
70
69
53
52
36
35
19
11
1
C
(a)
Panel Points
Symmetrical About Center Line of Span
Panel Points
45
28
18
2¼" Φ
2" Φ
2" Φ
82.2' Center to Center of Bottom Chords
(b)

Note: (a) and (b) Represent Ropes in Same Plane

FIG. 15.—PLAN OF UNDERFLOOR STAYS

The sum of the calculated live load and temperature stresses, which in some members are also nearly 20,000 lb, explains why under former severe load conditions not infrequent breakage of the diagonals occurred.

For the proposed restricted vehicular traffic it appears advisable to adjust all counters so as to reduce the dead-load stresses to a nominal minimum. The diagonals will then have enough spare strength for such traffic.

The Lateral System.—The system designed to resist wind and other lateral forces is also one of great complexity (Fig. 15). It is composed in part of a lateral truss whose chords are the lower chords of the outer stiffening trusses. The web system is formed by the floor beams and double wire-rope diagonals. Additional diagonal ropes are connected directly to the towers. Thus, in each half of the center span, and between the tower and the hinge in the side span this lateral truss acts as a cantilever arm fixed at the towers.

The truss system is supplemented by a pair of parabolic wire-rope cables, 3 in. in diameter, which, in the center span, stretch from tower to tower with a central sag somewhat greater than the width of the floor. In spite of the relatively wide floor (one nineteenth of the main span), the lateral truss is very flexible on account of the extensibility of the wire-rope diagonals. The very flexible parabolic wind cables likewise offer little resistance to lateral deflection. Consequently, the floor deflects easily and transmits a substantial part of the wind force to the main cables, which thus become the third important element resisting the lateral forces. Fig. 16 shows the calculated distribution of the wind load between lateral truss, wind cables, and main cables. Area ABC in Fig. 16 represents the wind load transferred from the deck to the cables.

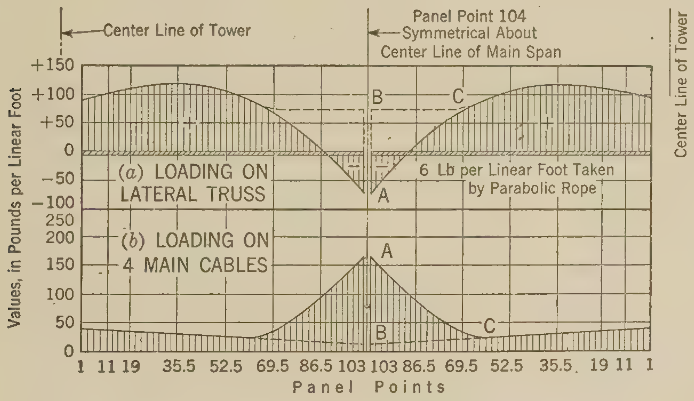


Fig. 16.—Wind-Load Distribution, Main Span

To enhance lateral resistance, the main cables were placed in inclined planes as in many earlier bridges. Today this cradling of the cables is recognized to be of negligible effect. The prevailing practice of proportioning the lateral system for a static wind force of 30 lb per sq ft of exposed surface (which corresponds to a wind velocity of about 100 miles per hr) would require the Brooklyn Bridge to be adequate for a wind force of about 950 lb per ft of bridge. This force would deflect the bridge laterally at the center 6.5 ft, or by one two hundred and fiftieths of the span, and would cause wind stresses of about 30,000 lb per sq in. in the truss chords. Such stresses in possible combination with the high live-load stresses must be considered excessive.

Such a condition, however, appeared unrealistic. Observations showed maximum deflections, under a quartering wind of 35 miles per hr, of only 2 in.—or a fraction of the theoretical deflection corresponding to this wind velocity. This great difference between actual and theoretical deflections must be due in large part to the fact that the static wind pressures acting on large suspension bridges are far lower than the theoretical ones assumed in modern design. This is confirmed by observations on a number of other large suspension bridges.

On the basis of observed deflections it was concluded that under severest wind velocity the Brooklyn Bridge would probably not deflect more than 1 ft at the center. This assumption was made in computing the wind stresses, which were found to be very moderate, both in the trusses and in the parabolic wind cables. The latter are probably rarely under stress.

In view of the fact that the lateral flexibility of the Brooklyn Bridge is within the range of that of several longer and much narrower suspension bridges which have experienced oscillations in wind, it seems pertinent to inquire what resistance the Brooklyn Bridge offers to such motions, which were disastrous to the Tacoma Narrows Bridge in the State of Washington.

Contrary to widespread belief, evidence shows that within practical limits lateral flexibility has little or no influence on oscillations in wind. The behavior of many suspension bridges during the 120 years since about 1825 furnishes unmistakable evidence that rigidity of the carrying system in the vertical planes of the cables is the predominant factor in the resistance against aerodynamic motions. Based on this empirical information, expressed in rational analytical form, it is possible to evaluate existing bridges and to design new ones for optimum safety with respect to other forces. On the basis of such analysis, the Brooklyn Bridge is shown to have ample resistance against aerodynamic oscillations—during its life it has never experienced such motions or any ill effects under wind action.

The Floor System.—The floor system of the Brooklyn Bridge is very simple (Fig. 5). It consists essentially of trussed floor beams spaced at 7.5 ft. Originally wooden stringers supported the rails and the roadway planks rested directly on the floor beams. The wooden beams have since been replaced by shallow steel stringers. The floor beams are continuous on four elastic supports formed by the four cables. In addition to transmitting the floor loads to the cables, the floor beams must distribute the proper share in resisting bending to the intermediate stiffening trusses.

Under former severe traffic conditions, coupled with high secondary stresses due to faulty details, the floor beams must have been subjected to high stresses, and consequently suffered local fractures of the chords on several occasions. Under restricted vehicular traffic, even allowing for an occasional 10-ton truck, the stresses are well within permissible limits, so that, except for local corrections, strengthening of the floor beams is not necessary.

The Towers.—The towers below the floor are massive, but partly hollow masonry pillars, mostly of granite (Fig. 17). Above the floor they consist of three masonry shafts joined at the top by Gothic arches. The towers are 271 ft above high water, 140 ft long, and 59 ft wide at the water line. Through a spreading base, mostly of limestone, each tower rests on a concrete-filled timber caisson about 170 ft long and 102 ft wide. As previously mentioned, these caissons were sunk by the pneumatic process. They were launched in 1870 and 1871 and were then of unprecedented size.

The New York caisson rests, at a depth of 78 ft, on a shallow layer of hard compact material overlying the bedrock of gneiss. The Brooklyn caisson is founded on a compact formation of sand, gravel, and boulders at a depth of 45 ft.

Because of the immovability of the cable saddles, any unbalanced horizontal cable pull must be resisted by the towers in bending. The calculations indicate resulting compression stresses in the masonry, at the floor level, of more than 700 lb per sq in.—about 50% of which is due to bending. In view of the excellent condition of the masonry, these stresses may be considered permissible, but in any major reconstruction they should not be allowed to increase. Such an increase can be prevented by reconstructing the cable saddles to assure longitudinal motion and thus avoid bending the towers.

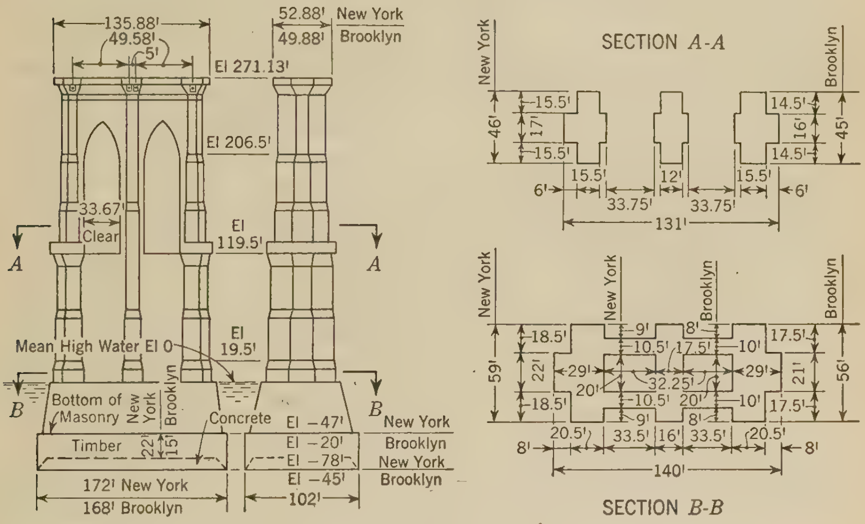

135.88'
49.58'
5'
52.88' New York
49.88' Brooklyn
El 271.13'
El 206.5'
33.67' Clear
El 119.5'
A
A
Mean High Water El 0
El 19.5'
B
B
Bottom of Masonry
New York
Brooklyn
Concrete
Timber
22'
15'
El −47'
El −20'
El −78'
El −45'
New York
Brooklyn
New York
Brooklyn
172' New York
168' Brooklyn
102'
SECTION A-A
New York
Brooklyn
15.5'
17'
15.5'
46'
14.5'
16'
14.5'
45'
15.5'
12'
15.5'
6'
6'
33.75'
33.75'
131'
New York
New York
Brooklyn
Brooklyn
18.5'
22'
18.5'
59'
9'
8'
10.5'
17.5'
10'
29'
29'
20'
32.25'
10.5'
20'
10'
9'
8'
17.5'
21'
17.5'
56'
20.5'
33.5'
16'
33.5'
20.5'
8'
8'
140'
SECTION B-B

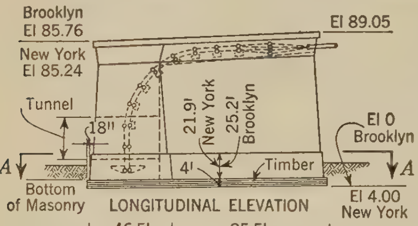
Fig. 17.—The Towers

The pressure at the bottom of the New York tower foundation, under balanced cable pull, is calculated at 5.5 tons per sq ft; and the maximum edge pressure under unbalanced cable pull, at 9 tons. The pressures under the Brooklyn tower are somewhat less. There is no evidence that any settlements have occurred; but, like the stresses in the masonry, the foundation pressures should not be increased.

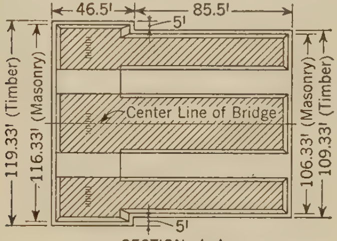
The Anchorages.—The anchorages form massive limestone masonry blocks about 90 ft high (Fig. 18). They rest on shallow timber grillages about 130 ft long and 120 ft wide, originally placed entirely below the ground-water level. The anchor chains, from their attachment to the cables at the top of the anchorage, pass through the anchor block with a right-angle arc. At the bottom they are attached to cast-iron anchor plates. The anchor chains are composed of forged wrought-iron bars and, except for a short piece at the upper end, are completely embedded in the masonry. The cable stress becomes gradually dissipated by friction in the masonry and the partly exposed upper links of the chain may be considered the most critical ones.

The anchorage bars had been made for a specified strength of 50,000 lb per sq in., and a test coupon cut from a similar forged erection bar which had been left in place showed a strength of 48,000 lb per sq in., whereas the maximum stress in the anchor bars under a live load of 2,400 lb per ft is only 12,000 lb per sq in. A piece exposed by removing the surrounding concrete revealed the bar in an excellent state of preservation. The anchor chains, like the cables proper, would therefore be able to resist, safely, a considerable increase in load.

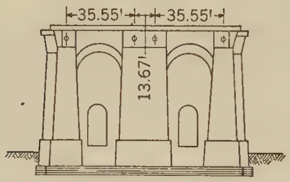
The maximum toe pressure of the anchor block on the underlying compact sand formation is estimated at about 6.5 tons per sq ft. This appears somewhat high. In fact, slight settlements are indicated, but there is evidence that they occurred mostly during or soon after construction. Nevertheless, it was thought that the settlement might be due partly to a deterioration of the timber platform on which the anchor blocks rest, and in 1943 it was decided to excavate pits to the bottom of the base to permit examination of the timber. Although the ground water was lowered, so that on the New York side the timber platform is partly, and on the Brooklyn side entirely, above the present water level, the condition of the timber was perfectly sound. The masonry likewise is in excellent condition so that there is sufficient cause for confidence that even a moderate increase in pressure would not result in further settlements or in any way endanger the stability of the anchorages.



LONGITUDINAL ELEVATION



SECTION *A-A*



TRANSVERSE ELEVATION

Fig. 18.—The Anchorages

Possibilities for Reconstruction to Serve Six-Lane Highway Traffic

A simple plan of alteration has been developed by which the bridge can accommodate two 3-lane roadways for passenger automobiles and buses when the surface car tracks are removed (Fig. 19). The scheme involves the removal of the top chord and web members of the two intermediate trusses and their re-erection in place of the two shallow outer trusses. The top chords of each remaining pair of trusses are to be connected by knee-braced struts. The trolley tracks and the present wooden roadway flooring would be replaced by a light open-grid steel floor over the full width of each 3-lane roadway.

The cost involved in this alteration is very moderate and the changed condition is expected to be adequate for many years, especially since the present approaches and street connections make continued exclusion of truck traffic advisable. It is estimated that the present capacity volume of vehicular traffic would be more than doubled.

A tentative study also indicates that it will be feasible to reconstruct the bridge for 6-lane unrestricted highway traffic, corresponding to a live load of 3,000 lb per ft of bridge. By the use of a light floor, the increased load would be resisted safely by the cables, towers, and anchorages, provided the cable saddles on top of the towers are rebuilt to assure longitudinal motion under live load and temperature changes. It is questionable whether this can be done at the same time making the necessary floor alterations economically while the cables remain in place. It may be found more economical to rebuild the cables completely. Whether the diagonal stays would be retained is largely a question of possible sentiment in favor of preserving the characteristic appearance of the bridge.

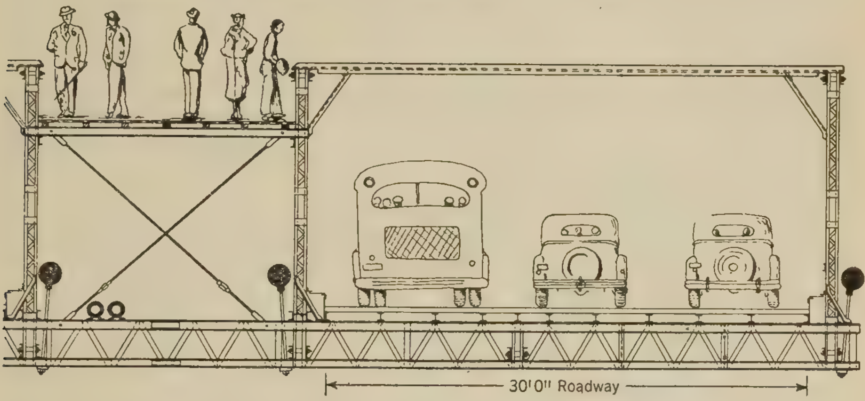


Fig. 19

On account of excessive stresses in the stiffening trusses and floor system, complete reconstruction of the suspended structure would be necessary.

Conclusion

Despite the crudeness of the theories employed in its design, the incongruities of some of the design features, primitive structural details, and consequent defects which developed during the life of the Brooklyn Bridge, this structure has given greater service than was originally anticipated.

Its behavior is a testimony to the fundamental soundness and safety of its design and that of the suspension type of bridge in general. The bridge as it exists today cannot be considered serviceable for unrestricted modern highway traffic, with two tracks for electric surface cars; but with moderate structural changes it can be reconditioned to serve a greatly increased volume of automobile traffic for many years.

A complete reconstruction of the suspended structure, with retention of the towers and anchorages, and possibly even the cables, to transform it into a modern 6-lane highway bridge for unrestricted traffic is feasible and can be accomplished without altering the general appearance of this magnificent structure, which has become world renowned and a cherished landmark of the City of New York.

PHYSICAL EXAMINATION AND FIELD OBSERVATIONS

By Nomer Gray,[12] Assoc. M. ASCE

Introduction

The need for a physical examination in an investigation of this kind is self-evident. Parts of the Brooklyn Bridge are more than seventy years old. For a great part of the life of the bridge, the loads were considerably in excess of those originally contemplated. The location is an unusually exposed one. At the time of construction, steel was just beginning to be used in large bridges and there was no general agreement on the best specifications governing its manufacture. In the light of these facts, the present physical condition of the bridge is of general interest.

In a structure as old as Brooklyn Bridge, the original adjustments are frequently disturbed as a result of settlement, added dead load, or maintenance operations. Such changes can affect the stress condition materially, and stress calculations become dependent, to some extent, upon field observations. It is the purpose of this paper to present, briefly, some of the methods and results of the physical examination and field observations made in connection with this study of Brooklyn Bridge.

The datum originally used in the construction of the bridge was mean high water at the site, as established by tide gage observations made by the bridge engineers. It was considered advisable to refer the 1943 surveys to this original datum. Unfortunately, no records of old bench marks were found, and many discrepancies arose in attempting to reconcile old plan elevations with the 1943 level runs. This problem was solved satisfactorily by reference to a very comprehensive set of precise levels[13] made about 1910 by the City of New York for the purpose of establishing one datum for all city departments. Library records of this excellent work provided ties with the datum planes used by the borough governments. The recorded relationship between the datum planes of Brooklyn Bridge and those of the Borough of Brooklyn and of the Borough of Manhattan controlled the level surveys. The first levels taken were for anchorage settlement.

Settlement Observations

The two anchorages are of limestone masonry, laid with joints of Rosendale cement mortar. Their construction is identical, except that the base of the masonry of the Manhattan anchorage is 4 ft lower than the base of the masonry of the Brooklyn anchorage. Both are supported on timber grillages 4 ft thick which, in turn, rest upon well-compacted sand. Close examination of the masonry does not reveal any cracks or spalls, and the original mortar of the exterior joints is in excellent condition after 70 years. A level check of a bed joint near the ground surface at the Brooklyn anchorage showed an extreme variation in elevation of 3 in. and an average settlement, from plan elevation, of 5 in.

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[12] Civ. Engr., Dept. of Public Works, City of New York, New York, N. Y.

[13] "Precise Levelling in New York City," by Frederick W. Koop, City of New York Board of Estimate and Apportionment, Office of the Chf. Engr., New York, N. Y., 1915.

In the absence of exact information regarding allowances made, during construction, for anticipated initial settlement, a comparison between lower-course elevations and plan elevations can be misleading. A level check with the top course is more dependable, since corrections in the estimated settlement during construction will have been made to bring the upper courses closer to plan grade. The average settlement of the Brooklyn anchorage based upon top-course elevations was 3 in. From this evidence it appears that the anchorage settled about 2 in. during construction. The observed settlement and tilt of the Brooklyn anchorage, coupled with the knowledge that the bridge rested on a timber grillage, aroused suspicion of the condition of the timber. Old construction records showed the ground-water level to be just below the top of the grillage. It is well known that the ground-water level has been lowered materially in Brooklyn. This was confirmed by a test boring which showed a drop of 13 ft, placing the present ground-water level 11 ft below the grillage and, incidentally, 11 ft below mean high water. Thus, it became necessary to examine the grillage.

A test pit 26 ft deep was excavated at the face of this anchorage, at the point of greatest settlement. When the concrete encasement was stripped off, the yellow pine 12-in. by 12-in. timbers were found to be in an excellent state of preservation, except for the first inch penetrated. It had been intended to take horizontal cores from the timber grillage, using a standard test boring rig placed in the bottom of the test pit, but great difficulty was encountered because of numerous drift bolts in the timber. Hand-operated auger borings, 5 ft deep, were made, without revealing any change in the quality of the chips. The auger holes may be seen in the exposed timbers in Fig. 20.

Conditions at the New York anchorage were better. The settlement (5 in.) was uniform judging by the lower-course elevations, and only 1.5 in. as measured by the top-course elevations. The present ground-water level is about 8 in. below the top of the grillage, which (as nearly as could be observed) is kept completely wet by capillary action. Auger borings showed sound wood in the accessible upper courses of the grillage. There is additional evidence that most of the settlement in both anchorages occurred during construction or shortly thereafter.

Physical Condition of Structural Members

The exposed ends of the wrought-iron eyebars in the anchorage galleries showed no sign of corrosion. At the point where the eyebars enter the wall, concrete was removed for a depth of several inches. The original coating of red-lead paint on the bars was found to be intact. The massiveness of the masonry towers (Fig. 7) contrasts strongly with the relative slenderness and flexibility of modern steel towers. Some conception of the size of the towers may be gained from the fact that the respective weights are 93,000 tons and 108,000 tons, each being considerably heavier than the 60,000-ton anchorages.

The combined weight is about twelve times that of the suspended superstructure.

The tower masonry is almost entirely of granite. Although subjected in places to appreciable stresses, there is no apparent sign of distress in the masonry. With the exception of one or two places where the stone has been exposed to an unusual amount of wetting and drying, the mortar of the original joints is in excellent condition.

As in the case of the anchorages, the two towers rest upon heavy timber grillages 15 ft and 22 ft thick, respectively. Their great thickness is explained by the fact that they originally formed the tops of the compressed air caissons used in construction, and as such were designed as slabs to bear tremendous loads. Immediately below the grillage there is a mass of concrete, from 6 ft to 7 ft thick, filling what was formerly the working chamber of the caisson.

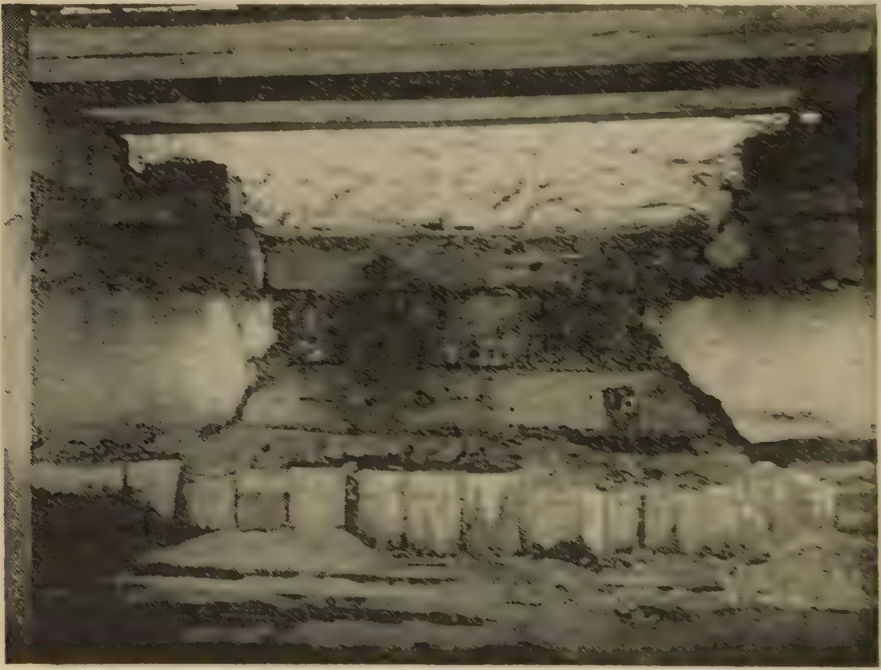

Fig. 20.—Edge of a Timber Crib Under the Brooklyn Anchorage

At the site of the Brooklyn tower, rock lies more than 90 ft deep. The difficulty of excavating and the fact that the material encountered gave evidence of a bearing capacity greater than 20 tons per sq ft led Washington A. Roebling to found the Brooklyn tower on compact sand, gravel, and boulders at El. − 45.

The Manhattan tower is generally regarded as resting on rock. Old reports indicate that the rock surface under the tower varies from El. − 75 to El. − 90. The caisson cutting edge is at El. − 78 and the concrete fill bears upon rock projections and hardpan, filling the depressions in the rock. The direct bearing pressure of about 7 tons is conservative for these materials.

The situation at the Manhattan anchorage with respect to the tower grillages differs materially from that at the Brooklyn anchorage. The tops of the grillages are, respectively, 20.0 ft and 46.5 ft below mean high water. Soundings indicate that the river bottom adjacent to the towers is 20 ft and 34 ft above the respective grillages. At these depths, the timber, which is completely encased in concrete, should be entirely safe from attack by marine organisms.

Surveys to determine plumbness and settlement of the towers confirm this judgment of the state of the grillages. No settlement or tilting was detected in either tower. The towers, which are higher than 300 ft (Fig. 17), are plumb within 0.5 in.

MOVEMENT OF CABLE SADDLES

The cable saddles at the tower tops each rest upon about forty cylindrical rollers, 3.5 in. in diameter. This design had been adopted principally to provide for saddle movement during erection. The base plates were placed 12 in. shoreward and the saddles were originally an additional 6 in. shoreward from the transverse center line of tower. The positions of the saddles at various times are given in Table 1.

TABLE 1.—RIVERWARD MOVEMENT OF CABLE SADDLES A, B, C, AND D, FIG. 8, FROM THEIR ORIGINAL POSITIONS[a]

Date	Brooklyn Tower				Manhattan Tower				Averages[b]	
	A	B	C	D	A	B	C	D	Total	Operation
May 21, 1883[c]......	3–11	3–9	3–10	3–6	3–5	3–12	3–12	3–4	3–9	0
May 26, 1883......	4–9	4–10	4–11	4–10	3–8	3–15	3–15	3–5	4–2	0–9
May 2, 1898[d].......	5–4	5–6	5–2	4–14	3–12	5–0	5–2	4–10	4–14	1–5
July 30, 1898[e]......	6–8	7–10	7–8	8–2	3–12	5–14	6–6	6–12	6–9	3–0
June, 1902.........	6–0	6–10	6–8	7–1	4–2	5–6	5–7	5–1	5–12	2–3
June, 1915.........	6–0	6–13	6–10	7–2	4–8	5–6	5–6	5–2	5–14	2–5
June, 1943.........	7–12	7–8	8–8	8–0	5–14	6–12	6–12	6–10	7–3	3–10

[a] The original position of each saddle was 18 in. shoreward from the transverse center line of the tower. Dimensions are in inches and sixteenths—for example, "3–11" denotes 3 $\frac{11}{16}$ in. [b] The column of totals gives the average total movements from the beginning of construction to the date indicated. The column marked "operation" gives the average total movements during the time the bridge has been in operation. [c] Two days before the bridge was opened. [d] The first year that trolley cars were operated on the roadways. [e] Immediately after the bottom chords buckled.

Col. Washington A. Roebling stated[14] that the sensitiveness of the saddles under unequal loading, during the erection of the superstructure was quite marked but that the saddle movement in the completed structure, under ordinary traffic, would be very small indeed. This proved to be the case, although under increasing dead loads and live loads the saddles moved very gradually toward the river.

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[14] "Report of the Chief Engineer on the Strength of the Cables and Suspended Superstructure of the Bridge to the Board of Trustees, January 9, 1882," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1882.

During erection of the cables and superstructure, the saddles moved an average of $3\frac{9}{16}$ in. toward the river. After the bridge was opened their movement, to June, 1943, has amounted to an additional $3\frac{5}{8}$ in. Until 1887 movements of the saddles under seasonal temperature changes were observable and amounted to from $\frac{3}{8}$ in. to 0, indicating a variation in rolling friction for the various saddles.

Since 1902 the saddle movement has been small and very gradual except in 1922 when two saddles moved suddenly under heavy vehicular traffic and high temperature. Although records of the exact movements since 1915 are lacking, an examination of the saddles indicates that they have not moved for years. The likelihood of movement is dependent, of course, upon the magnitude of unbalanced side and main span cable pulls. The friction is now very great as a result of a skewing of some of the rollers and an accumulation of rust and dirt, the rollers being practically inaccessible. No movement occurred under a fairly heavy unbalanced test load.

Live loads and temperature changes tend to produce unbalanced cable pulls which, in modern flexible steel towers, are compensated for by movements of the tower tops. This is not so, however, in the masonry towers of the Brooklyn Bridge. As a result of saddle friction and failure of the massive towers to yield easily, the unbalanced cable pulls induce appreciable bending stresses in the towers. The determination of the magnitude of these stresses depends to some extent upon the movement of the tower tops. Analytically, this is a complex problem. It was a relatively simple matter, however, to measure the movement by observations on a full-scale model which, in effect, is what the investigators had in this case. A transit was set at the base of the tower on the prolongation of the transverse center line. Empty elevated trains weighing a total of 600 tons, centrally placed on one span at a time, were used. Movements of the Brooklyn tower top of approximately $1\frac{1}{4}$ in. and $1\frac{1}{2}$ in., respectively, were observed for the load on the main and side spans. Further transit observations revealed that the elastic line of the tower, with the top so deflected, was practically a straight line—a significant phenomenon, as it indicates that the tower rotates principally as a rigid body, about the base. This angular yielding of the base reduces the bending stresses in the masonry.

The towers, generally, were found to be in excellent condition, testifying to the soundness of their construction and the durability of good stone masonry.

The Main Cables

The main cables are the principal supporting members and, as such, merit careful examination. These cables, four in number, are $15\frac{5}{8}$ in. in diameter under the wire wrapping. The exact net area of the cables is not known; but, taking the best estimate as 144.8 sq in., the percentage of voids is 24%. This may be compared with from 19% to 21% of voids in modern cable construction.

To examine the cables in detail, it was necessary to remove the wire wrapping at a number of places. On this bridge, the cable bands were placed over the wrapping, which is continuous from anchorage to anchorage, except through the towers. The wrapping, which originally had been placed by hand, was found to be tight and generally well painted. The watertightness of the painted wrapping is indicated by the fact that the interior of the cables is dry at the low points. In the worst case after several days of rain, a slight moisture was detected in one cable at the midpoint of the main span.

The wrapping wire was removed from the cables for lengths of from 7 ft to 10 ft at twenty-four points; twelve cable wires were removed at each point—or 0.2% of the cross section of the cable. In each case, a length of new wire was spliced in to replace the wire removed for test. It was possible to remove and splice in surface wires within 1 ft of the end of the wrapping. An effort was made to achieve approximately the original tension in each spliced wire by screwing up the threaded sleeve splice. The method of obtaining the correct tension (admittedly not exact) was to approximate the force required to deflect the wire from its straight alinement.

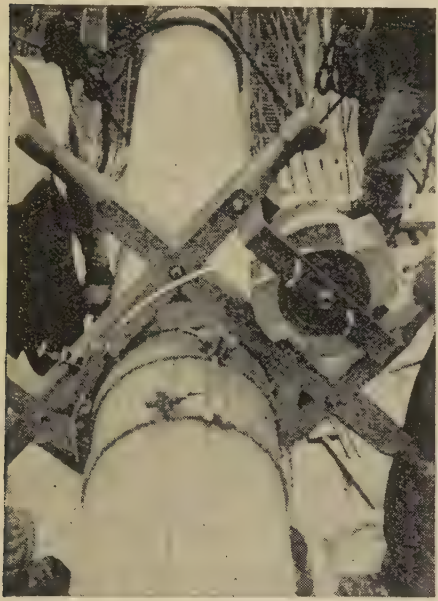


Fig. 21.—Device for Rewrapping the Cable by Hand

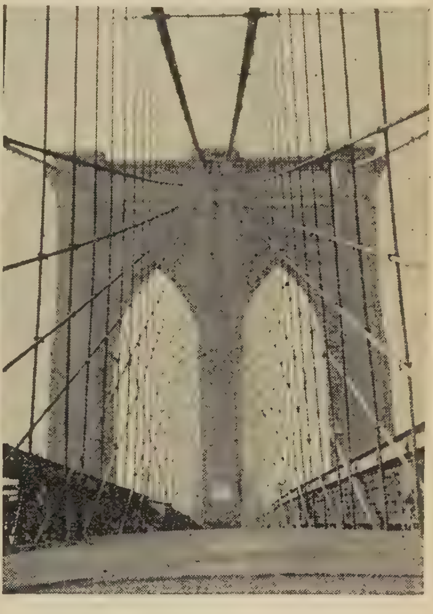


Fig. 22.—Long Overfloor Stay Ropes from the Center Line of the Bridge

Under the wrapping wire, a loose, dry, chalky substance (which was probably the white lead of the paint applied to the wrapping) appeared between the surface cable wires. When this was brushed away, the cable wires were found to be almost entirely free from rust. At a few points, notably near the center of the main span, some of the surface wires were rusted just under the cable band. This condition rarely extended to a depth, into the cable, of more than two wires. The writer attributes the rusting at these points to two factors: (1) The very short suspender rods, at these points of maximum truss expansion, cause some "working" of the cable bands which tends to abrade the surface wires; and (2) canvas fillers under the cable bands at these points undoubtedly contribute to the retention of moisture and consequent promotion of rust.

For some distance out from the anchorages the deck is supported on top of the cables by short posts. Because of the greater rigidity of this connection between cable and deck, the "working" of the cable bands has caused the wrapping wires to wear ridges into the top surface wires of the main cables and in some cases to break a few of them. The cables have been inspected regularly at these points and the broken wires repaired.

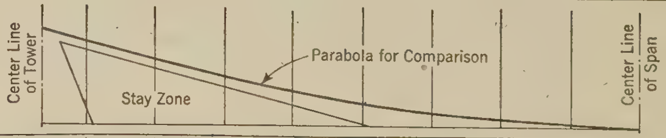
The cables were wedged open to a depth of about 4 in. to permit visual examination of the interior wires at many points. In every instance the interior wires appeared as clean as, and generally cleaner than, the surface wires.

The hand-wrapping frame for replacing the wrapping wire is shown in Fig. 21. Its operation will be self-evident from the photograph. For close contact between adjacent wires the practice, of course, was to wrap uphill—a fact which is not apparent in the photograph because of the angle from which it was taken. The clamps around the cable were necessary at points where the cable had been wedged open, as the wedging usually increased the cable diameter slightly. The wrapping frame was used wherever sufficient clearance permitted its use.

The physical characteristics of the cable wire, as determined from many tests, are the subject of the fourth and final paper in the Symposium.

The shape of the cable curve was a matter of interest, differing as it does from the conventional form. In most modern suspension bridges, the approximately uniformly distributed suspended weight results in an equilibrium polygon very close to a true parabola. Actually, in the stress computations for a truss it is commonly treated as a parabola. However, in Brooklyn Bridge, the effect of the overfloor stays is to reduce, appreciably, the weight suspended from the main cables in the quarter span adjacent to the towers. This effect of nonuniform distribution of cable load was so marked that a survey of the cable curves became necessary.

TABLE 2.—ORDINATE MEASUREMENTS TO THE MAIN SPAN CABLE, IN FEET

Description	Panel Points									
	T	6	18	30	42	54	66	78	90	104
										
Brooklyn side	131.33	113.01	89.65	68.25	48.79	32.41	19.43	9.25	2.71	0
Manhattan side	131.39	113.05	89.39	67.89	48.42	32.25	19.23	9.13	2.62	0
Variation from parabola	0	1.60	3.70	4.52	3.99	3.30	2.56	1.33	0.39	0

Two factors tended to complicate the cable-curve survey: (*a*) The curve changes from minute to minute under passing train loads; and (*b*) there is the usual uncertainty regarding the true cable temperature during the daylight hours. By making the survey at night, both these difficulties could be overcome, because the train service could be discontinued. However, the difficulty of measuring cable ordinates accurately, in total darkness, will be readily appreciated. The solution of the problem was to make a profile survey of the more accessible top chord of the truss between midnight and dawn, with the train service stopped. With the profile of the top chord as a reference line, it was an easy matter to measure the vertical distances from the cable center line to the top chord at any convenient time, even with trains running. The resulting cable curve, of course, would be that which existed at the time the chord profile survey was made. It may be of interest to record that passing trains produced a deflection at the center of the main span in excess of 1 ft. Their observed effect in elongating suspenders did not exceed $\frac{3}{16}$ in.

Referring to Table 2, the effect of the stays in distorting the curve from parabolic form is clearly discernible. The parabola used for comparison is that which would contain the points of intersection of the tangents at the towers and the low point of the midspan. The measured ordinates were used in the truss stress analysis.

STIFFENING EFFECT OF STAY ROPES

One of the distinguishing features of Brooklyn Bridge is the use of overfloor stays to stiffen the suspended structure. These are wire ropes, from 1.75 in. to 2 in. in diameter, which radiate from the tower tops to connect with the trusses at two-panel intervals in the quarter span adjacent to the towers. A view of the stays looking along the center line of bridge is given in Fig. 22. The stays are clipped to the suspenders at every intersection. The flatness of the catenaries of the long stays emphasizes more clearly than figures the high tensions of the stays.

The present condition of the ropes is fair considering their long service in an extremely exposed location and the constantly varying tensions under passing loads. Care has been taken to prevent loss of section by the use of metal sleeves or leather spacers at points of contact with other surfaces. A tensile test was made on a single $1\frac{7}{8}$-in. stay rope, removed from the bridge for this purpose. This stay rope, which had been in service for 60 years, developed an ultimate strength of 147,000 lb per sq in. The modulus of elasticity was approximately 20,000,000 lb per sq in.

The tensions in the stays were set at from 15 tons to 17 tons each in the original design. Knowledge of the present stay tensions was necessary to determine the thrust induced in the bottom chords of the trusses to which they are attached. The cumulative effect of these stay tensions is surprisingly large, being of the order of 500,000 lb in a single bottom chord.

Observations were made to determine the tension in thirty two of the one hundred stays which lie under one cable. As will be appreciated by any one who has ever used a surveyor's tape, the tension in a rope is a function of the sag. The stay tensions were found by measuring the vertical sags. The method, as indicated in Fig. 23, is as follows:

A transit is set up adjacent to the lower end of the stay, the horizontal axis being on a level with a point on the stay. The telescope is directed along the closing chord, by sighting at the upper end of the stay; and then it is clamped, fixing the vertical angle, which is recorded. Next, the transit is placed, by trial, in a new position so that the line of sight is tangent to the stay, while maintaining the original vertical angle. The method of calculating the desired vertical sag will be evident from Fig. 23. In this manner, it is possible to make all the observations without leaving the bridge floor. The entire operation was frequently completed in 20 min, using one observer and one assistant.

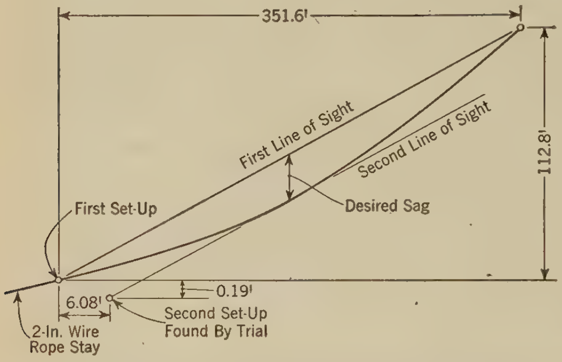


Fig. 23.—Method of Measuring Stay-Rope Stresses (Measured Sag 2.14 Ft and Computed Stress 50.8 Kips Per Sq In.)

The observations were necessarily made at temperatures which differed considerably from the basic temperature of 39° F used in the stress calculations. Therefore, corrections had to be applied to the stay tensions as calculated from the observed sags. When temperatures change, several factors influence the stay tension, as follows:

(1) The stay rope itself tends to change length;
(2) The curve of the main cable changes as the cable length changes; and
(3) The truss shortens or lengthens, thus moving the stay connection horizontally and influencing the stay tension.

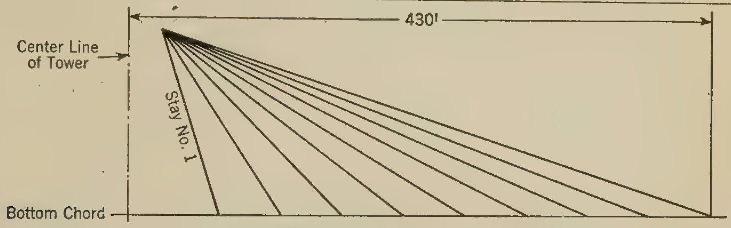
The effect of factor (2) is to raise or lower the truss to which the lower end of the stay is connected, thus influencing the tension. Factors (1) and (3) can be treated analytically, but factor (2) offers great difficulty analytically because of the interaction of a number of elements. It proved fairly simple to observe the elevation of the truss at the two temperatures—namely, at 39° F and the temperature prevailing when the sag observation was made. For the long stays, the difference in elevation amounted to as much as 5 in. for a 50° F change in temperature. The increment in stay tension was as great as 23% for a 50° F rise in temperature.

A comparison of the design tensions and the measured tensions is given in Table 3. In maintenance operations on the stays, through the years, the practice has been to turn the adjusting nuts back to their original positions, which procedure, if faithfully followed, should have preserved the initial tensions. The slightly higher present tensions are probably caused by increases in the dead load since the time of the original adjustment.

Stiffening Trusses

The stiffening trusses have tension web members, with adjustable counter diagonals pin-connected to the chords. The small size of the members, the nature of the details, and the absence of traveling platforms combine to make painting costly. However, inspection indicates that, through most of its long life, the bridge has been well painted. There is no marked reduction in sectional area through corrosion, except at the towers. At this point, curiously enough, the chords lie partly in chases, in the granite, making access for painting impossible. Fortunately, the sectional area is greatest here and the same loss of area is not as significant as it would be elsewhere. Nevertheless, it will be necessary to strengthen the corroded sections of the bottom chords at the towers.

TABLE 3.—TENSIONS MEASURED IN THE STAY ROPES, IN KIPS



Item	Description	Stay No.:							
		4	7	10	13	16	19	22	25
1	Panel point	13	19	25	31	37	43	49	55
2	Diameter, in inches	1.75	1.75	1.875	1.875	1.875	1.875	2	2
3	Original tension, probably at 90° F.	33.3	33.3	30	30	30	30	30	30
	Present Dead-Load Tension, at 39° F:								
4	Brooklyn end; side span	21.7	27.4	28.4	21.8	23.2	30.1	32.9	31.2
5	Brooklyn end; main span	26.3	22.7	25.3	24.8	24.9	32.9	37.8	41.4
6	Manhattan end; side span	19.4	20.5	29.6	27.1	27.0	29.7	37.1	33.3
7	Manhattan end; main span	30.2	30.8	22.6	22.4	19.8	28.4	33.9	28.8

In years past, the deflections under live load, common to suspension bridges, have resulted in considerable pin wear. During those years, the pins have been inspected and replaced where necessary as a regular maintenance practice and, in consequence, their present condition is not bad.

During the first decade of the twentieth century, the bridge was subjected to very much heavier live loads than at present. At that time the diagonals frequently failed and the top flanges of the floor beams cracked. Since 1922 no heavy trucks have been permitted on the roadways and the elevated train service has decreased progressively until, in March, 1944, train service was discontinued. Under the prevailing light live loads, the aforementioned difficulties have practically disappeared.

Analysis showed that the chord stresses were high, and it was decided to test the physical characteristics of the chord materials. Since it was impracticable to remove a full-chord section, coupons were cut from the flanges of the channels making up the chord section at sixteen well-distributed points. They were cut out with hack saw and chisel, sufficient excess being taken to permit machining away the material disturbed in the cutting. The results of these tests are reported in the last paper of this Symposium.

As stated, the cumulative tensions of the overfloor stays, applied to the bottom chords of the stiffening trusses, result in a large compressive force near the towers. At a point about 250 ft from the towers, the bottom chord has a low sectional area (13.72 sq in.). The compressive unit stress from the stays alone reaches a high value at this point. It was realized that other longitudinal members must participate in resisting this compressive force. The best approach to the determination of the extent of the relief afforded by other members appeared to be a direct field measurement of the bottom-chord stress by strain gage. To obtain a zero stress gage reading, auxiliary channels were riveted and welded to the top and bottom of the existing bottom chord and zero stress readings were taken on the new channels after they were in place. This arrangement with the new channels in place is shown in Fig. 24. Six gage lines were used on each new channel, completely traversing the section in order to obtain a fair total stress.

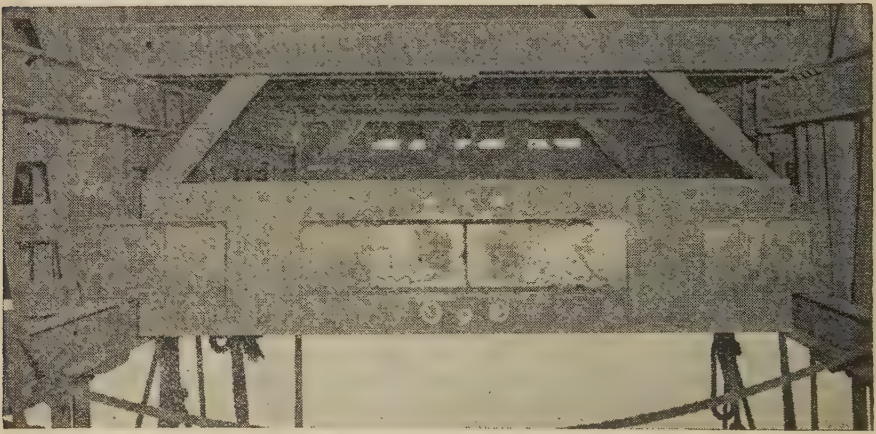


Fig. 24.—Bottom Chord of Stiffening Truss with Auxiliary Channels in Place

The original chord section was then burned through at the middle of the panel, transferring the compressive stress into the new channels. In manipulating the torch care was taken to remove metal concentrically and progressively, in order to avoid high temporary eccentric stresses. The strain-gage measurements then taken on the new channels gave a unit stress of 17,000 lb per sq in. for the original chord section. This value is about 30% less than it should have been for the then existing loading condition, which indicates that other members participate in the bottom-chord stress to that extent. In this manner, it was possible to obtain a more exact knowledge of the actual bottom-chord stresses, confirming an analytical treatment of participation.

On July 30, 1898 (see Table 1), a traffic block on one roadway caused a large overload and a partial failure of the bottom chords of the stiffening trusses, resulting in considerable discussion in the technical journals of that period. The bottom chords of four main-span trusses were bent out of line, because of the extremely high compressive stress caused by the stays. In time, the same bending was produced in the side spans. The bending referred to is clearly visible in the single plank walk shown in Fig. 25. Although the misalinement was never corrected, the chords were reinforced with cover plates and bottom battens about 1908.

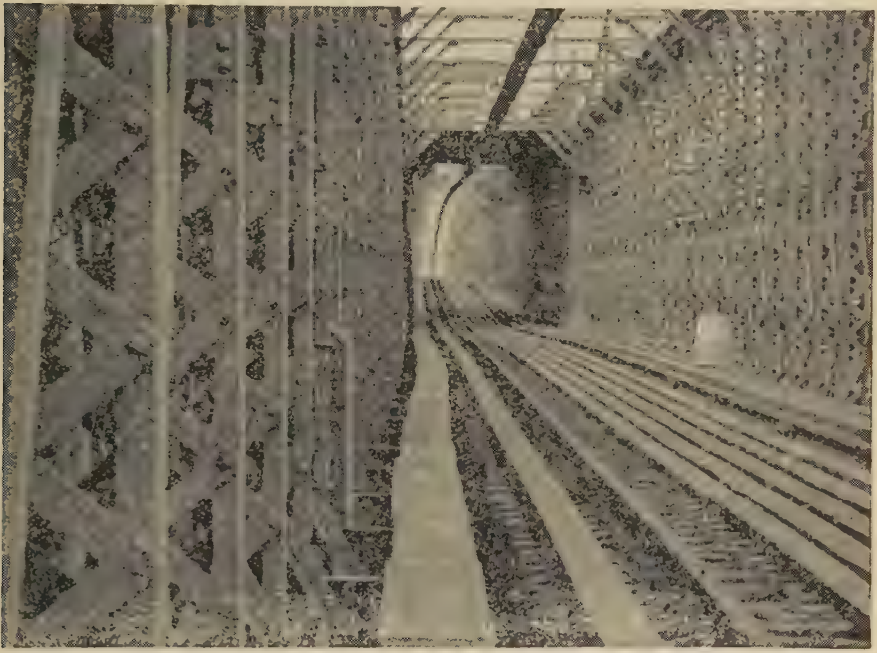


Fig. 25.—View Along the Track, Showing the Bend in the Floor Structure After Chord Members had Buckled

TABLE 4.—Tensile Tests on Eyebar Diagonals

Item	Description	Bar 1	Bar 2	Bar 3	Bar 4
	Nominal Section, in Inches:				
1	Width	3	2	2	2
2	Thickness	0.625	0.5	0.5	0.5
3	Measured areas, in square inches	1.93	1.12	1.12	1.14
4	Yield point, in kips per square inch	44.6	42.7	44.9	46.1
5	Ultimate strength, in kips per square inch	75.8	69.2	71.8	71.3
6	Ultimate percentage elongation in 22.5 ft	13.7	9.28	13.5	14.0
7	Percentage reduction of area	46	41	46	53

In order to learn something of the physical characteristics of the web members, four eyebar diagonals were removed from the high trusses for full-size tensile tests. The results of these tests are presented in Table 4. None of the eyebars broke in the head, a considerable excess sectional area having been provided in their manufacture. This excess is made up partly by additional thickness in the head, since there is insufficient clearance within the chords for widening the head. The additional thickness is fortunate, since corrosion is greatest in the eyebar head, which is inaccessible for painting.

Decision to remove the eyebars for test provided an opportunity to check their stresses by strain gage. Accordingly, gage points were drilled in the members to be removed, and readings were taken at a moment of extremely light live load. Theoretically, there should be only nominal dead-load stresses in a properly adjusted truss at mean temperature, and the analysis indicated small live-load stresses. Surprisingly, on measuring the no-load gage lengths after removal from the truss, the stresses in the diagonals were found to be approximately 24,000 lb per sq in.

The four eyebars were removed from points well spaced along the main span. The discovery of this high shear could not be explained satisfactorily by the general truss analysis. However, it was in agreement with the records of failures of many diagonals in past years. After some study, it became apparent that these stresses could be caused by high tensions in the adjustable counterdiagonals.

To test this hypothesis, gage holes were drilled into the main diagonals and counters of three adjacent panels at two widely separated points. The gage distances in the counters were measured before and after slacking off the adjusting sleeves to zero tension. As had been supposed, the countertensions were found to be unnecessarily high—from 12,000 lb per sq in. to 17,000 lb per sq in. Simultaneous readings on the adjacent main diagonals showed smaller changes in their tensions—suggesting that the horizontal shear produced by the over-tightened adjustable counters is resisted over long lengths by groups of diagonals rather than by the immediately adjacent main diagonal. Thus, efforts to correct this condition will require the scheduling of the adjustments and a large amount of checking and rechecking by the strain gage.

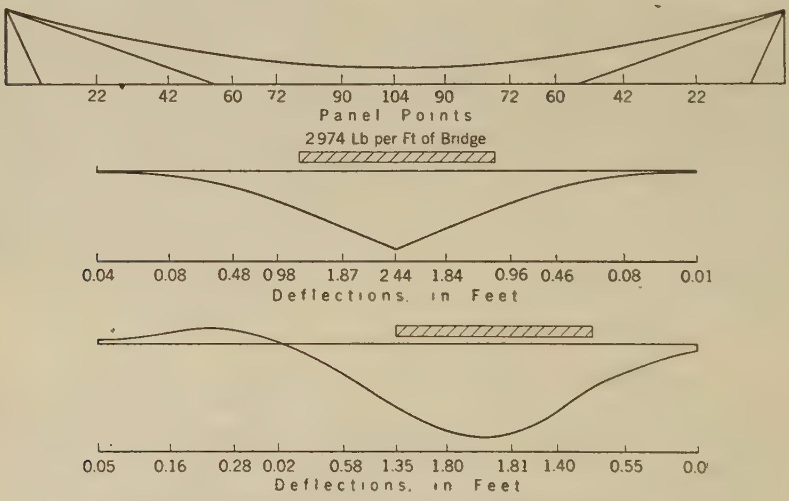


Fig. 26.—Deflections of the Main Span Under Live Load
(One Eight-Car Train on Each Track; Total Length, 394 Ft and Total Load, 586 Tons)

LIVE-LOAD DEFLECTIONS

In the course of the analysis of truss stresses, by an adaptation of the so-called elastic theory, the need arose for a check on the deflections under live loads. Arrangements were made to place trains in predetermined positions on the main span and side spans. Profile surveys of the top chord were made before and while the trains were in position. The resulting deflections are shown in Fig. 26 and Fig. 27.

Historically, the introduction of the Melan deflection theory into the United States is associated with Brooklyn Bridge. The late Mr. Moisseiff (who is generally credited with the first application of this theory to an actual structure) has stated that it was his study of the failure of the suspender rods of this bridge in 1901, which first drew his attention to the effect of the distributed dead load in resisting live-load deflections.

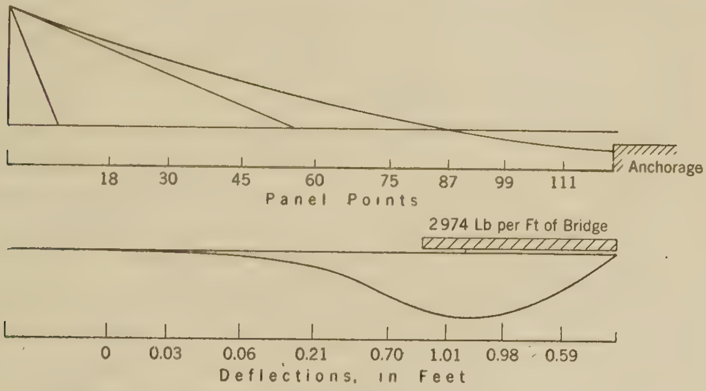


FIG. 27.—DEFLECTIONS OF A SIDE SPAN UNDER LIVE LOAD
(One Six-Car Train on Each Track; Total Length, 295 Ft and Total Load, 439 Tons)

Further checks on the live-load chord stresses were made with the strain gage, using the train loads. The results are too detailed for enumeration in this paper. It may suffice to state that the strain gage showed stresses lower than those computed by the elastic theory and the measurements aided in the determination of stress reduction percentages.

THE FLOOR SYSTEM

The floor beams are continuous over the four cable supports. The solution of the floor-beam stresses is complicated by consideration of the amount these four supports yield under load. The exact calculation of the deflections of the six trusses is an extremely difficult, if not impossible, problem if approached from a purely analytical angle because of the interaction of the main cables, six trusses, the overfloor stays, and the floor beams. A more direct approach is to select that floor beam which will deflect the most, because of differential deflections of the supporting cables, and measure the deflection under a known load. This was done, using the same train loads as shown in Fig. 26. At midspan the inner cables and trusses deflected 2.5 in. more than the outer trusses, under the centrally placed load. After the yielding of the floor-beam supports under a known load had been measured, it was possible to calculate the floor-beam stresses for the working loads used in the study.

CONCLUSION

The Brooklyn Bridge has had a long and extremely useful life; and, although it is customary to associate old age with decrepitude, there is nothing in the results of the physical examination to suggest that advancing age has seriously weakened any major element in the structure. Stone masonry plays a larger part in this bridge than in modern long-span suspension bridges, and the durability of such construction, when properly executed, is well known. The lasting quality of steel in a well-designed bridge is almost entirely a matter of protection from the elements. The examination has shown the bridge to have been well painted during the major part of its life.

The bridge is statically indeterminate to a high degree. The analytical approach to its stress condition is very complex. It was logical that direct measurements should be taken whenever possible, as an aid to stress determination. Seldom does a bridge designer have a full-scale model before him on which to test the accuracy of his simplifying assumptions. Much could still be learned from a judicious application of the strain gage.

In closing, the writer feels impelled to call attention to the great value of detailed construction reports on large projects. To know all about a structure, an engineer must look further than the final plans and specifications. The progress reports of the chief engineer and his several assistants on the construction of Brooklyn Bridge are veritable mines of information which are still available in technical libraries. In the course of this investigation, they helped resolve many questions; and they can be read today, with profit, by the practicing engineer as well as by the student of bridge engineering.

TESTS OF METALS REMOVED FROM CABLES AND STIFFENING TRUSSES

BY HAROLD E. WESSMAN,[15] M. ASCE

INTRODUCTION

The tests reported in this paper were made on wire specimens cut from the cables and on chord specimens cut from the stiffening trusses of the Brooklyn Bridge. As stated in the other papers of this Symposium, this historic structure was opened to traffic in 1883; and, consequently, the test specimens have been in service for at least 60 years.

CABLE WIRE

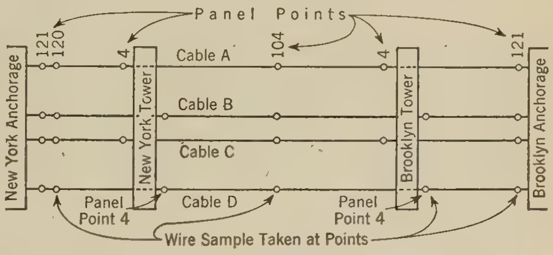


FIG. 28

Wire samples were cut from the top, side, and bottom of each of the four cables at six different points on each cable as indicated in Fig. 28. Of the total, sixty samples were subjected to tensile tests to failure. The ultimate strength, elongation, and reduction in area were determined for all the specimens. In addition, strain measurements over an 8-in. gage length were taken on twenty four of the sixty samples and stress-strain diagrams plotted to determine the proportional limit, yield strength, and modulus of elasticity. The wire samples when cut from the main cables tended to assume immediately the curvature of the coils in which they were delivered to the bridge site years ago. Hence, after being placed in the grips of the testing machine, the wire was subjected to an initial load of 250 lb to straighten it before the extensometer was attached.

Simultaneous load and strain measurements were also taken on twelve long samples using a gage length of 84 in. to obtain a more accurate value for the modulus of elasticity. In these tests, the maximum load was approximately that corresponding to the proportional limit of the material.

Values for the physical properties of the cable wire are summarized in Table 5, Cols. 2 to 5. The values in Cols. 4, 7, and 10, Table 5, are arithmetical averages. Maximum and minimum values are also tabulated in order to indicate the range of values encountered among the samples tested.

The average "ultimate tensile strength" for sixty samples is 162,600 lb per sq in. The original specifications,[16] phrased in terms of the small-diameter wires initially contemplated for the bridge cables, specified that a single wire should sustain an ultimate load of 3,400 lb before breaking. Inasmuch as the approximate diameter of the small wires after galvanizing is 0.165 in., the specified breaking load is equivalent to a unit stress of 159,000 lb per sq in. Only a small quantity of the small-diameter wire had been used, when a change was made to a larger wire with an approximate diameter, after galvanizing, of 0.184 in.

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[16] "Specifications for Steel Cable Wire for East River Suspension Bridge," in "Report of the Chief Engineer of the New York and Brooklyn Bridge, January 1, 1877," Eagle Book and Job Printing Dept., Brooklyn, N. Y., 1877, p. 71.

The average unit tensile strength of the wires tested is a little greater than the unit strength indicated in the original specification; thirty-two specimens had strengths greater than 159,000 lb per sq in. whereas twenty-eight specimens had strengths less than this value. It is of some interest to note that, in general, the bottom wires of the cables exhibited higher strengths than the side and top wires.

TABLE 5.—PHYSICAL PROPERTIES OF SPECIMENS TESTED IN TENSION

Property[a]	BROOKLYN BRIDGE							GEORGE WASHINGTON BRIDGE (CABLE WIRE)[c]		
	Cable Wire				Stiffening Truss[b]					
	No.[d]	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Ultimate tensile strength	60	140.0	162.6	195.0	69.8	79.0	89.5	220.0	234.0	259.0
Yield strength	24	128.2	143.8	165.5	43.2	48.7	53.8	150.0	184.0	202.0
Proportional limit	24	90.2	99.9	111.6	....	....	....	....	....	....
Modulus of Elasticity:										
8-in. gage length	24	27,400	29,010	32,300	....	....	....	....	....	....
84-in. gage length	11	28,200	28,760	29,500	....	....	....	....	....	....
Percentage elongation	60	0.8	2.88	4.8	18.8	23.2	25.4	....	6.0	....
Reduction in area (%)	60	3.0	15.6	28.8	45.5	49.6	52.4	....	....	....
Diameter, in inches	54	0.1793	0.1841	0.1910	....	....	....	....	0.1965	....
	6[d]	....	0.1653	....	....	....	....	....	....	....

[a] Units are in kips per square inch except where specially noted. [b] Tensile tests of chord coupons. [c] *Transactions*, ASCE, Vol. 97, 1933, Table 8, p. 364. [d] The number of samples tested is given in Col. 2. The six reported in the last line were smaller wires used in a few bottom strands of the cables.

The average yield strength of twenty-four samples (Col. 4, Table 5) is 143,800 lb per sq in. This value is 88.4% of the average ultimate strength. The yield strength in these tests is defined as the unit stress at that point on the stress-strain diagram at which the elongation is 0.7%—in other words, the unit strain is 0.007.

The average value for the proportional limit of twenty-four samples is 99,900 lb per sq in. The proportional limit is defined as the unit stress at the point at which there is an observable deviation from the straight part of the stress-strain diagram. This point could be determined more accurately, however, from a study of the variation in increments of strain noted in the tabulated data for each test in combination with a scrutiny of the stress-strain diagram.

The average values for the proportional limit and the yield strength cannot be compared directly with specification requirements. The original specifications state that the elastic limit (of the small-diameter wire) should be 1,600 lb. This is equivalent to a unit stress of 75,000 lb per sq in. What was meant at that time by the elastic limit is not clearly defined in the specifications.

The modulus of elasticity was found to have an average value of 28,760,000 lb per sq in. This value is based on stress-strain diagrams for eleven of the twelve tests of long samples using an 84-in. gage length. One test was discarded because the wire was badly corroded. The original specifications required that values for the modulus of elasticity should fall between 27,000,000 lb per sq in. and 29,000,000 lb per sq in. Values for E were also determined from twenty-four tests of short samples, using a gage length of 8 in. The average for this series is 29,010,000 lb per sq in. However, in one of the tests averaged, the modulus was determined as 32,300,000 lb per sq in., which was considerably "out of line" with the values for the remaining twenty-three tests. Omitting this value, the average was reduced to 28,870,000 lb per sq in. The average values for the short-gage tests and the long-gage tests show very good agreement.

The average value for ultimate elongation in 10 in., based on sixty tests, is 2.88%; twenty-five samples had elongations in the interval between 2.0% and 3.0%; and twenty four were in the interval between 3.0% and 4.0%. The original specifications (for the small-diameter wire) called for a minimum elongation of 2% in a 50-ft gage length or 3.5% in a 5-ft gage length.

A direct comparison of test values for elongation with original specified values is not possible because of the difference in gage lengths. However, the wire seems somewhat deficient in ductility. It is of some interest to compare it in this respect to modern cold-drawn suspension bridge wire. The wire for the George Washington Bridge (Cols. 9 to 11, Table 5) was required to have a minimum elongation of 4% in 10 in., but the average value of 26,274 tests[17] was 6.0%. Moreover, the average tensile strength of the cable wires in the George Washington Bridge was 234,000 lb per sq in., which is considerably higher than the average tensile strength for wire in the Brooklyn Bridge. It is important to remember that, in general, ductility decreases as ultimate strength increases.

The average value for reduction in area at the point of fracture is 15.6%. The original specifications for the small-diameter wire stated that necking at the break should be such as to give a diameter less than 0.15 in. Since the original diameter of this wire was approximately 0.165 in., this statement is equivalent to specifying a minimum reduction in area of 17.5%. The significance of "reduction in area" is a subject for considerable debate. There is no definite correlation evident in these tests between reduction in area and elongation or tensile strength.

A comparison of ultimate tensile strengths with carbon content, determined from chemical analyses of twelve samples indicates that, in general, strength increases as the carbon content increases. The carbon ranged from 0.55% to

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[17] *Transactions*, ASCE, Vol. 97, 1933, p. 364.

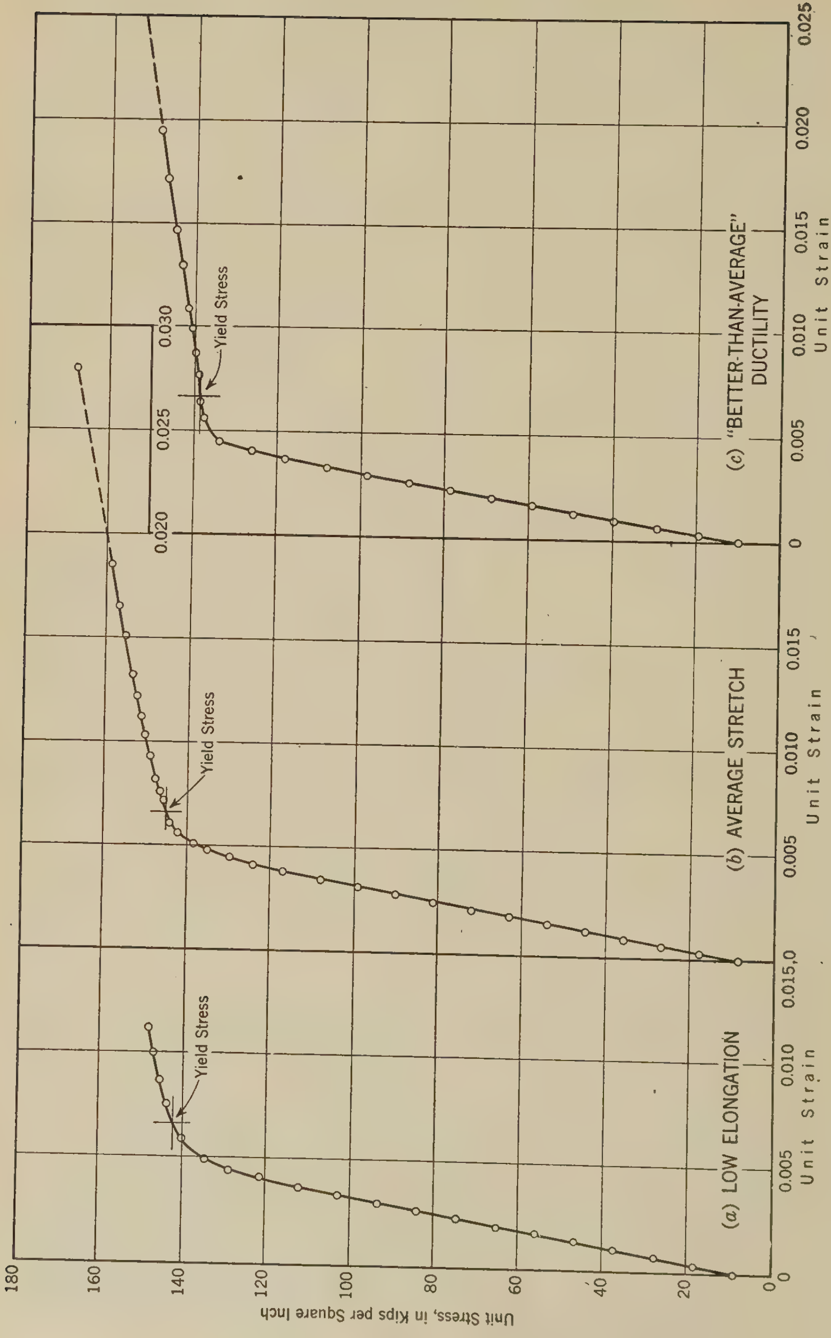

Fig. 29.—Typical Stress-Strain Diagrams for Cable Wire (See Table 6)

0.89% whereas the tensile strength for the twelve samples ranged from 143,600 lb per sq in. to 184,000 lb per sq in., the detailed comparison being as follows:

Ultimate strength (kips per sq in.)	Carbon (%)
143.6	0.63
144.6	0.55
151.0	0.68
154.0	0.63
157.8 (Fig. 29(*c*))	0.67
162.2	0.71
166.7	0.74
167.4 (Fig. 29(*b*))	0.76
172.0	0.70
174.0	0.82
178.5	0.89
184.0	0.84

Typical stress-strain diagrams for the tensile tests are shown in Fig. 29. The physical properties accompanying these diagrams are listed in Table 6.

TABLE 6.—CHARACTERISTICS OF SPECIMENS PLOTTED IN FIG. 29

Characteristic	Fig. 29(*a*)	Fig. 29(*b*)	Fig. 29(*c*)
Ultimate tensile strength (lb per sq in.)	148,500	167,400	157,800
Yield strength (lb per sq in.)	142,500	144,800	138,000
Proportional limit (lb per sq in.)	102,900	99,000	98,000
Modulus of elasticity (lb per sq in.)	28,700,000	28,800,000	29,500,000
Percentage elongation in 10 in.	1.1	2.8	3.2
Percentage reduction in area	0	17.4	13.9

Fig. 29(*a*) pictures the data for a specimen having low elongation; Fig. 29(*b*), for one with average stretch, and Fig. 29(*c*), for one with "better-than-average" ductility. All the stress-strain curves show a flat slope above the yield point. In general, the reserve of strength between the yield stress and the ultimate strength is small. The diagrams also exhibit a sharp increase in curvature or "knee" in the transition zone from elastic to plastic behavior. Although the yield strength is a value rather arbitrarily defined in cable-wire tests, nevertheless, in these tests, it seems to coincide rather closely with what might be termed a point or region where yield begins to be pronounced.

Bend tests on samples of the wire from which the galvanizing had been removed were conducted by the New York City Department of Public Works in its own laboratory. The detailed results of those tests are not included in this paper. The results indicated that the wire was deficient in flexibility. More than one half of the samples tested could not be coiled continuously (at least one 360° turn) around a cylindrical mandrel less than 1 in. in diameter. There was practically no correlation between the results of these tests and the elongation obtained in the tensile tests. On the other hand, some correlation was evident between the bend data and the values for reduction in area. In general, the wires that broke on the large-diameter mandrels (1 in. and 1.5 in.) gave small reductions in area on the tensile tests.

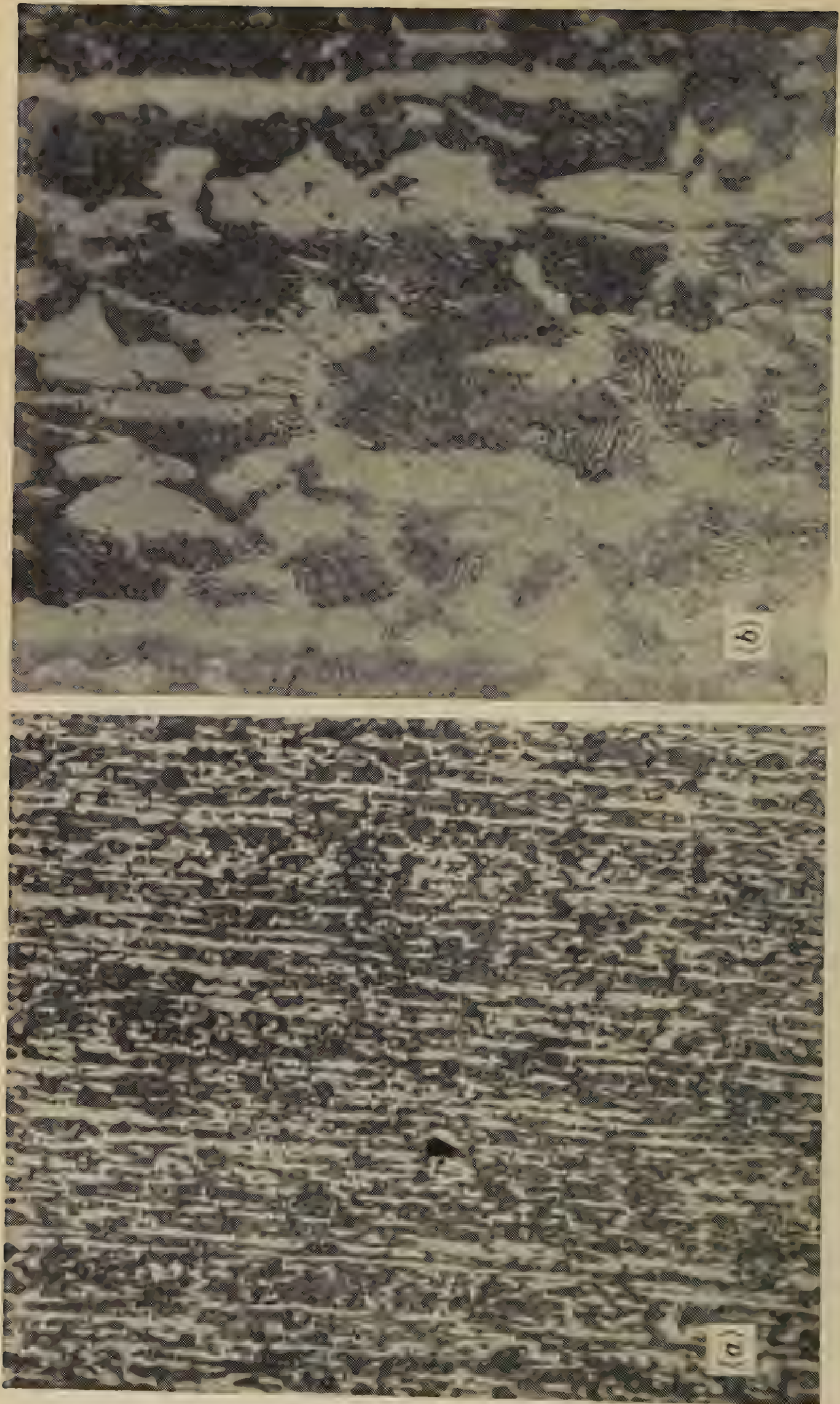

FIG. 30.—WIRE SAMPLE WITH BANDED STRUCTURE
(*a*) MAGNIFICATION 100 ×
(*b*) MAGNIFICATION 500 ×

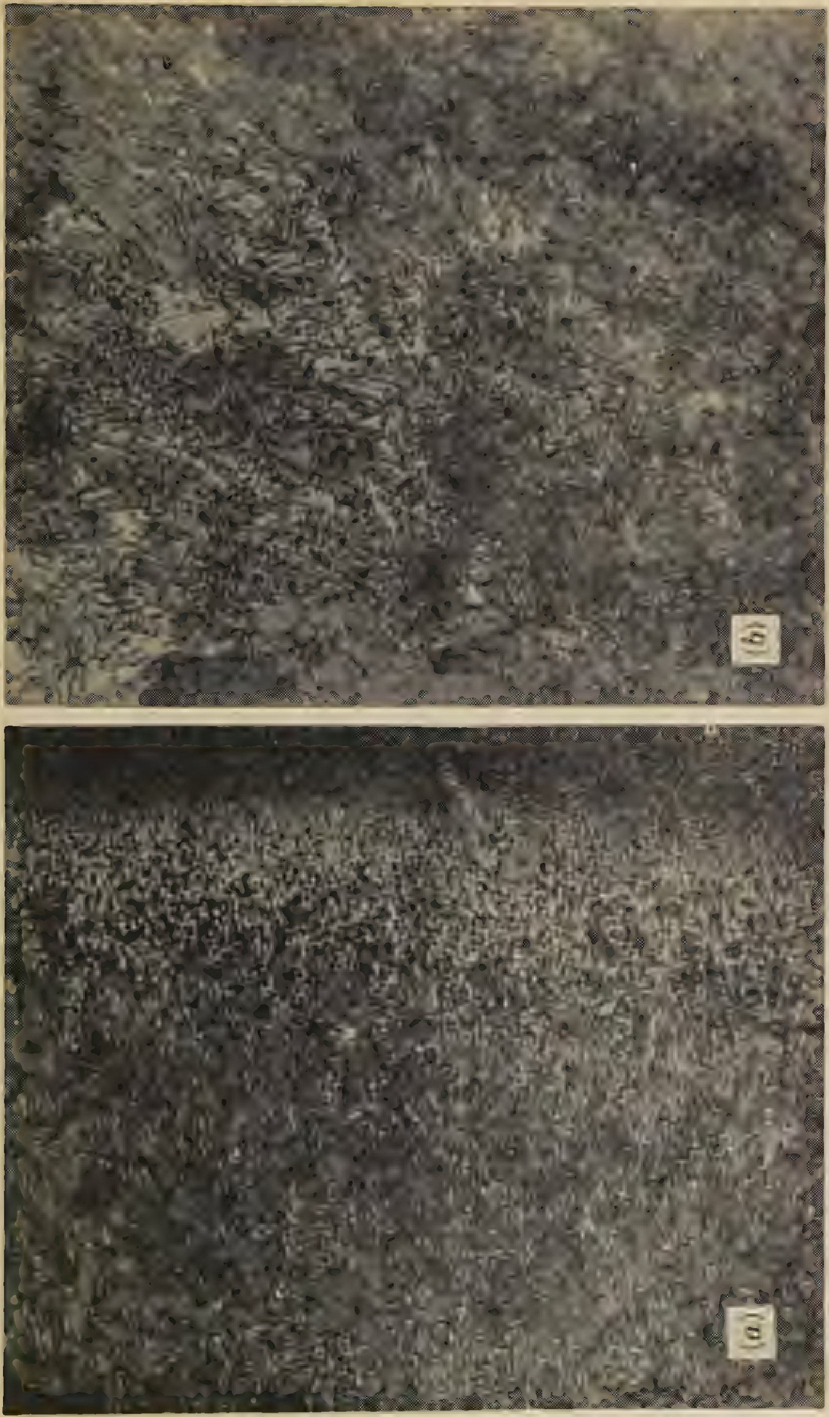

FIG. 31.—PHOTOMICROGRAPHS SHOWING PERLITE STRUCTURE
(*a*) MAGNIFICATION 100 ×
(*b*) MAGNIFICATION 600 ×

"Scleroscope" hardness tests were made on twenty-four samples. The hardness ranged from 55 to 66 except for one very soft spot on one sample. In general, as hardness increased, tensile strength increased.

The metallographic examination of twenty-four samples of cable wire showed variations and irregularities that would not be encountered in wires made more recently, under rigid, modern standards of inspection. It must be remembered that these wires were made at a time when it was the exception rather than the rule to make analyses of steels. Very few steel plants at the time were equipped with chemical laboratories, and metallographic examination was still unknown.

Most of the wires seem to be a good grade of steel, of eutectoid (0.83% C) composition, approximately. However, two wires were of hypereutectoid structure, five were decidedly hypoeutectoid, and three showed a banded structure like that illustrated, in Fig. 30. The same sample is shown in both Figs. 30(*a*) and 30(*b*), but Fig. 30(*a*) was originally taken at a magnification of 100 diameters; and Fig. 30(*b*), at 500 diameters. This banded structure is generally considered indicative of poor quality and low strength. The white streaks are ferrite, practically pure iron, and the dark areas are perlite, an iron-iron carbide mixture.

Phosphorus, silicon, and other elements present in steel sometimes remain segregated, even after rolling. They become elongated in the direction of hot working. Carbon is less soluble in these areas than in the regions of lower impurity content and the result is the banded structure in Fig. 30. Banding gives directional properties to steel.

The photomicrograph in Fig. 31(*a*) shows a sample of better wire typical of most of the wires, under a magnification of 100 diameters. Fig. 31(*b*) is the same sample magnified 600 diameters. This particular sample is a little higher in carbon than most of those tested but, nevertheless, illustrates the eutectoid composition. The structure is almost entirely of perlite, as evidenced by the predominance of dark areas. Fig. 31(*a*) shows, however, appreciable decarburization along the outer surface of the wire. A short sliver of steel should be noticeable at the left center extending into the zinc coating (black).

In one respect—namely, in the structure of the perlite—all the wires examined were alike and of high quality. The individual grains were small, and the laminations within the grains were fine. Both these characteristics tend to improve the strength of the wire—other factors being the same, the smaller the grain size, the greater the tensile strength. Otherwise, every feature that could be observed under the microscope varied from wire to wire.

Although some of the wires were as free of nonmetallic inclusions as wire produced today, many of the samples had far too many slag and oxide inclusions. The banded structure shown by several of the wires indicated not only hot rolling but also too high a percentage of undesirable constituents such as phosphorus, sulfur, and slag.

Several of the samples were nonuniform in the distribution of the carbon. This variation appeared in two independent forms. In one case, isolated spots of relatively carbon-free ferrite of appreciable size were scattered throughout a matrix of essentially 0.8% carbon. In the other case, which is more serious, the wires show a widely varying amount of decarburization along the surface. This decarburization will give the wire a lower tensile strength than it would otherwise have had.

The technical press has recorded[18,19,20] that the original small-diameter wire was cold-drawn through two dies from rolled crucible rod steel, oval in section, with diameters of approximately 7/32 in. by 15/64 in. The first drawing reduced the wire to No. 7 gage and the second drawing to No. 8 gage, the latter being approximately 0.165 in. in diameter. Only a relatively little of the smaller wire had been used when the change to a diameter of about 0.184 in. was made, which was slightly less than that of No. 6 wire. Available literature offers no evidence that the larger wire was still given two drawings and that the size of the basic crucible rod steel was increased.

From the microscopic examination of the samples it would appear that only a few of the wires were given a real wire-drawing operation. This lack of wire drawing is indicated in two independent ways: (1) The longitudinal section of cold-drawn material shows the grains, especially near the surface, to be drawn out axially; and (2) wire drawing produces a smooth surface. Most of the samples showed no axial tendency at all; and, in cases where it would be observed, such a tendency was restricted to a very superficial depth. If the hole in the die is not perfectly circular, ridges extending along the wire may be produced; but, even so, the cross-sectional shape does not vary from place to place. Many of the wires examined were extremely rough, the high and low spots occurring in random order. That this was the condition of the wire at the time of manufacture is shown by the way in which the zinc coating has filled in these irregularities.

The greatest irregularities were noted in the zinc coating. Some samples showed a heavy, uniform coating; others, a very thin uniform coating; and in some the coating was extremely irregular. The irregularity may have been due, in part at least, to the flow of the zinc occurring under pressure when the individual strands were pressed into the form of the cable and then wrapped.

TABLE 7.—CHEMICAL ANALYSES OF CABLE WIRE (PERCENTAGES)

Description	BROOKLYN BRIDGE					GEORGE WASHINGTON BRIDGE[a]				
	Carbon	Manganese	Phosphorus	Sulfur	Silicon	Carbon	Manganese	Phosphorus	Sulfur	Silicon
Minimum..	0.55	0.30	0.076	0.038	0.16	0.72	0.44	0.021	0.022	0.07
Average...	0.75	0.38	0.099	0.052	0.19	0.81	0.63	0.029	0.034	0.19
Maximum .	0.91	0.40	0.128	0.067	0.23	0.93	0.77	0.042	0.046	0.34

[a] *Transactions*, ASCE, Vol. 97, 1933, Table 7, p. 362.

The results of the chemical analyses of the cable wire are compared with similar analyses for George Washington Bridge[21] in Table 7. Carbon,

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[18] "East River Bridge Wire," *The Iron Age*, January 4, 1877, p. 5.

[19] "Making Wire for the Brooklyn Bridge," *ibid.*, March 1, 1877, p. 1.

[20] "Wire for the East River Bridge," *Scientific American*, March 3, 1877, pp. 127 and 130.

[21] *Transactions*, ASCE, Vol. 97, 1933, Table 7, p. 362.

phosphorus, and sulfur contents were determined in twenty-four samples, whereas manganese and silicon contents were found for four samples. The wide range of carbon content is apparent. In the cable of the George Washington Bridge, the range for 958 melts of steel was from 0.72 to 0.93, with an average of 0.81. The phosphorus and sulfur contents are considerably higher than those in modern bridge wire which generally average less than 0.04%.

It is not within the scope of this paper to recommend a working stress to be used as a basis in evaluating the Brooklyn Bridge cables under modern loading conditions. Nevertheless, it is advisable to call attention to a number of factors evident from the physical, chemical, and metallographic studies which have a bearing on this question. The stress-strain diagrams show a rather flat slope above the yield strength. There is not much difference between the yield strength and the ultimate strength. The elongation is low. Even though the ultimate strength of this wire is about 70% of that for modern bridge wire, the energy capacity is less than 50% of the energy capacity obtained today.

The carbon content exhibits a wide range. The metallographic studies corroborate this conclusion and also indicate nonuniformity in other characteristics. The galvanizing coat is quite variable in thickness. In view of all these factors, it seems wise to be conservative in establishing a limiting working stress for any revision in loading conditions.

This statement is not to be construed as indicating a belief by the writer that the wire is definitely inferior in quality. Such is not the case. It must be remembered that this wire was manufactured more than 60 years ago, when standards for insuring uniformity were not as definitely established as they are today. The cable wire has stood the test of many years of satisfactory service and its quality today is high enough to warrant the expectation of many years of added service.

Chord Samples from Stiffening Trusses

Sixteen flat coupons were machined from steel samples cut from the chords of the stiffening trusses of the Brooklyn Bridge. The results of the physical tests made on these specimens are summarized in Table 5, Cols. 6 to 8.

The average value for the ultimate strength is 79,000 lb per sq in. The average value for the yield point is 48,700 lb per sq in. These values are somewhat higher than those generally encountered today in low-carbon structural steel as defined by Specification A7–42 of the American Society for Testing Materials for bridges and buildings.

The average value for ultimate elongation for all sixteen tests is 23.2%. The average for the eight samples with 8-in. gage is 23.1% whereas the average for the eight samples with 7-in. gage length is 23.2%. These values are practically the same as values obtained today in tests of low-carbon steel.

A metallographic study and a chemical analysis were made for two of the samples, the one with lowest strength (69,800 lb per sq in.) and the one with highest strength (89,500 lb per sq in.). The low-strength sample had a carbon content of 0.17% and a manganese content of 0.68%. The high-strength specimen had a carbon content of 0.26% and a manganese content of 1.43%.

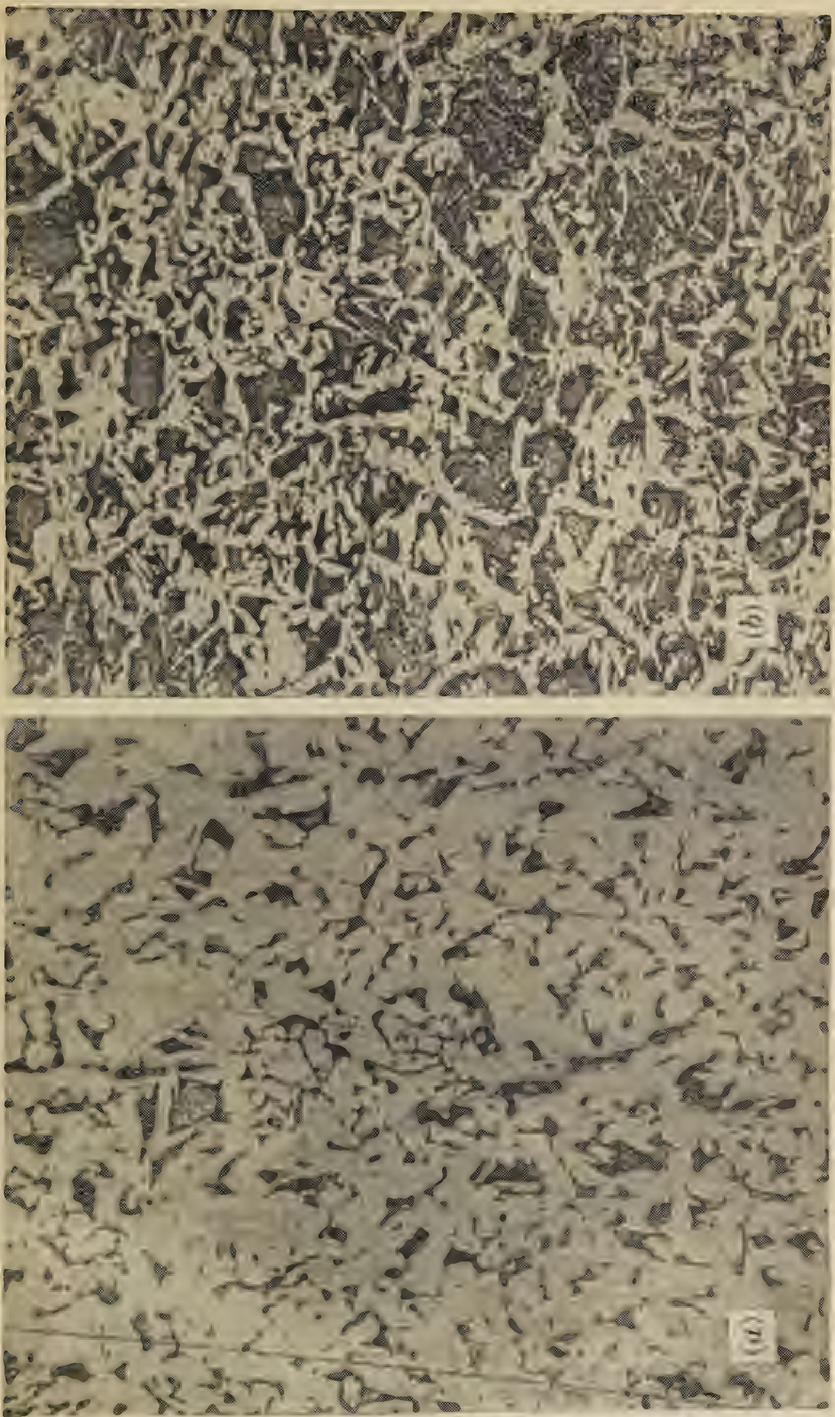

Fig. 32.—Photomicrographs of Chord Specimens
(a) Low-Strength Sample at 100 ×
(b) High-Strength Sample at 100 ×

The difference in the quantities of these elements undoubtedly was one of the factors accounting for the difference in ultimate strengths of the two samples.

Fig. 32(*a*) is a photomicrograph of the low-strength sample with a magnification of 100 diameters. Fig. 32(*b*) shows the high-strength specimen. When compared with Figs. 30(*a*) and 31(*a*), which are also magnified 100 diameters, these views show clearly the larger grain structure associated with lower-strength steels.

As is to be expected, Fig. 32(*b*) shows a higher proportion of perlite (dark area) than does Fig. 32(*a*). This is due, not only to the higher carbon content, but also to the higher manganese content which places this steel, from a metallographic standpoint, outside the low-carbon steel class. Without a chemical analysis of such a steel, an estimate of carbon content based on the area of perlite might be considerably in error.

From the sixteen tests of steel coupons, it seems safe to conclude that the structural steel placed in the Brooklyn Bridge trusses more than 60 years ago is of unusually good quality. Whether the sections have the most desirable shape and sufficient area for modern loadings, however, is another matter—one which is beyond the scope of this paper.

ACKNOWLEDGMENT

The tests described in this paper were made at the laboratories of the College of Engineering, New York University. The metallographic studies (including photomicrographs) and the galvanizing tests were made by F. C. Fair, assistant professor, Department of Chemical Engineering, New York University; and the chemical analyses were made by H. B. Hope, professor, Department of Chemical Engineering, The Cooper Union—under the general supervision of H. J. Masson, chairman, Department of Chemical Engineering, New York University.

DISCUSSION

Isidore Delson,[22] M. ASCE.—In presenting the several papers on the Brooklyn Bridge, the authors have done a fine service to the engineering profession.

Mr. Ammann's authoritative paper leaves little for discussion to one who, like the writer, has had an intimate part in the investigation of the bridge.

The writer's preoccupation with the Brooklyn Bridge dates back to 1939, when he was required to make a study of a memorandum on the reconstruction of the bridge submitted by the late Leon S. Moisseiff, M. ASCE.

As far back as 1903, Mr. Moisseiff had collaborated in a plan for reconstruction advocated by the late Gustav Lindenthal, Hon. M. ASCE, then commissioner of the old Department of Bridges of the City of New York. Ever since then Mr. Moisseiff had nurtured the hope of some day participating in such a reconstruction. In his memorandum Mr. Moisseiff stated that:

> "Thorough physical examination and surveys of the material and the structure must be made. An examination of the state of preservation of the existing cables should be made and a substantial number of wires from the cables should be tested for strength, behavior and duration."

The investigation which began four years later led to the recommendation, as stated by Mr. Ammann, that, with motor trucks excluded from the roadways, the two so-called intermediate trusses may be removed, thus providing 30-ft roadways in place of the present narrow roadways which are less than 17 ft wide. Under this plan, as noted by Mr. Ammann, the shallow outer trusses will be rebuilt to the full height of the inner trusses.

As indicated in Fig. 5 (Mr. Seely's paper), four of the six stiffening trusses are directly suspended from the cables. The two intermediate trusses are supported on the floor beams about 14.5 ft from the point of suspension of the inner trusses.

The interaction between the floor beams and the intermediate trusses is extremely indeterminate. Entirely neglecting any support given to the floor beams by these trusses, it was found that under a local loading of a 35-ton trolley car, and a 10-ton truck, the maximum stresses in any floor beam were moderate, especially in view of the high strength of the steel.

Inasmuch as the proposed removal of the intermediate trusses is predicated on the carrying capacity of the floor beams, it may not be amiss to discuss the fractures to which Mr. Ammann has referred.

Virtually all these fractures occurred in the top chords of the floor beams close to the point of attachment of the inner suspenders. It was rather baffling to find that all but one of thirty-two fractures recorded between 1929 and 1936 occurred in the side spans, nearer to the anchorages.

A reasonable explanation was suggested to the writer by the very competent bridge foreman at the time the fractures had occurred; namely, that they could have been induced by the braking of the trains approaching the terminals on the downgrades.

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[22] Cons. Engr., Staten Island, N. Y.

Mr. Ammann's paper states the simplifying assumptions made for the analysis of the stiffening trusses. As regards the effect of the diagonal stays, actually each group of twenty-five stays was divided into three zones, in each of which the upward vertical pulls of the stays (the vertical components of the stay tensions) were assumed to be uniform, their resultant acting at the center of the zone. The stays may be conceived as furnishing points of yielding supports to the stiffening trusses.

As regards the very high unit stresses of the bottom chords found in parts of the span in the region of the stays, they were due, to a large extent, to the longitudinal components of the stay tensions, the so-called direct stresses. The original designers had made the rather naive assumption that these direct stresses would be resisted equally by all the longitudinal material on the bridge, both steel and timber. Mr. Gray's paper describes the method used for measuring, in one of the most highly stressed chords, the direct stress due the dead-load stay tension, upon the basis of which participation by the floor system was fixed at 25%. For the direct stresses due to live load, the same ratio was used, in addition to which, participation by the top chords through the web system was determined by analysis.

The revelation of these high unit stresses, in some members exceeding their yield point, was somewhat startling. The figures, of course, were the result of laborious computations based on a theoretical analysis which was set up by the aid of reasonable assumptions for simplifying the complicated structure into a calculable pattern.

Mr. Gray has described the deflection tests under predetermined train loads. At the time that these tests were made on the Brooklyn side span, Mr. Gray also took strain-gage readings on a top chord, which disclosed a stress, 75% of that computed for the particular loading. This compared closely with the result of the computation for stress reduction due to the deflection at that point. It was reassuring to find that measure of agreement between the calculated and the measured stress.

T. Kennard Thomson,[23] M. ASCE.—Under the heading, "The Main Cables," Mr. Gray states that "passing trains produced a deflection at the center of the main span in excess of 1 ft." In other words, the vertical deflection from live loads is given as about 1 ft, but of course the vertical variation caused by the expansion and contraction of the cable is much greater. For the Brooklyn Bridge the total possible deflection is 18 ft, 9 ft above and 9 ft below the mean level. For the Williamsburg Bridge the total possible variation is about 6 ft 9 in.

The writer obtained these values from the Bridge Department of New York City, N. Y., for an article that appeared in 1909.[24]

Theodore Belzner,[25] Affiliate, ASCE.—In addition to its real technical importance, this Symposium is of unusual interest and is a very valuable and helpful contribution to engineering literature. Brooklyn Bridge, the oldest long-span suspension bridge, has had 62 years of extremely useful service. For years it carried much heavier loads than were originally contemplated without undergoing any major reconstruction.

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[23] Cons. Engr., New York, N. Y.

[24] "New York City Bridges," by T. K. Thomson, *The Engineering Magazine*, September and October, 1909.

[25] Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Public Works, City of New York, New York, N. Y.

The more that is known about existing long-span bridge structures, after many years of useful service, the more it is realized that they demand careful and searching study to be designed efficiently and that computation alone is no measure of efficiency. Mathematical analysis and reasoning are mental operations, and there is usually enough uncertainty about the distribution of steel in a structure (particularly a structure of long service) to make strain-gage measurements indispensable for a satisfactory knowledge of service effects. Stress-gage measurements will demonstrate the agreement between engineering theory and the actual behavior of the structure.

Very true and timely are Mr. Gray's statements (see "Conclusion") that:

> "Seldom does a bridge designer have a full-scale model before him on which to test the accuracy of his simplifying assumptions. Much could still be learned from a judicious application of the strain gage."

Also applicable is the old saying: "Anything worth doing at all is worth doing well." This applies to investigations of engineering structures, as well as to many other activities.

The time has come when large bridges, with long service (particularly those of the magnitude of the East River bridges), should be investigated for the purpose of comparing computed stresses with those measured by the strain gage.

Admitting that field stress measurements are more or less expensive, time consuming, and laborious and that they require a high order of instrumental and technical skill to be of value, the fact remains that there are many situations in practice where the results of these stress investigations would give the engineer the answer to many otherwise indeterminate stress problems.

An important instance may be mentioned in connection with a difficult erection problem, some years ago on the Williamsburg Suspension Bridge across the East River in New York, N. Y. To the writer's knowledge, the first time in the history of the New York City Department of Bridges (later the Department of Plant and Structures and still later the Department of Public Works) that extensometer investigations were ever conducted on one of its bridges, and particularly on a structure of such magnitude, was in connection with this bridge. Extensometer investigations were made during 1913 and 1914 under the direction of the late Austin Lord Bowman, M. ASCE, chief engineer, in cooperation with the National Bureau of Standards, under the auspices of the late James E. Howard, engineer physicist, in connection with strengthening the end spans of that bridge.

The writer was assigned to the extensometer investigation of the important truss members of the main towers and of the legs of the intermediate towers at the Brooklyn end. He recalls distinctly not only the difficult problems involved but also the value of the extensometer measurements. One of the wedging operations,[26] particularly, solved a critical and perplexing erection problem, not only because of the close agreement of the computed stresses but also because of the fact that, after the removal of the pins, there were no apparent changes in the stresses of the members.

Also of the utmost importance to this or any other great bridge is its potential life as it affects capacity, maintenance, and repairs. With proper design, construction, protection, inspection, and maintenance, the life of a steel structure, under reasonable conditions, may be practically unlimited. In other words, there is no danger or deterioration from working stresses if properly provided for, and none from corrosion, if the steel is properly used and protected.

In conclusion, it can be stated without fear of contradiction that the Brooklyn Bridge, long regarded as "The Eighth Wonder of the World," is still the most famous and esthetic span in existence.

C. H. Gronquist,[27] M. ASCE.—The history of the major undertakings of the past is always engrossing, in engineering as in other fields. In that part of the Symposium on the Brooklyn Bridge concerning the practical difficulties of the construction job of the eighties, it will be seen from Mr. Seely's account that the problems of today are the problems of the past. Because of the ingenuity and capacity of the engineering pioneers of the past, the solution of that day now has become routine.

The analysis of the stresses in the stiffening trusses of the Brooklyn Bridge is complicated by the unusual conditions of fixity at the towers and hinged connections at the centers of the three spans, combined with the action of the overfloor stays in stiffening the suspended structure. The solution has been handled in a practical manner that is undoubtedly sufficiently accurate. The deflection theory correction in the simple span part of the side span can be estimated closely by the use of buckling loads,[28] but the correction in the main span and in the cantilever part of the side span is most difficult to estimate and is not constant. These corrections were approximated by the investigators with the help of the results of the test loading of the structure.

The overfloor stays are considered to add much to the esthetic appeal of the Brooklyn Bridge, and in addition they contribute greatly to the stiffness of the structure. The long side span, with the center hinge eliminated, still would be very flexible if it were without the stays. This fact should require careful consideration in the complete rebuilding of the structure.

The relative effectiveness of the outer trusses as measured by their stiffness is much less than is indicated by comparison of moment of inertia, since deflection is proportional to the total buckling load—the sum of the buckling load of the truss and the value of the cable tension.[28] The proposed replacement of the shallow outer trusses by the deep intermediate trusses, now not directly connected to the cables, is fully justified; it will decrease the total stiffness of the bridge by an entirely inappreciable percentage.

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[26] Discussion by Isidore Delson of the paper, "Stress Measurements on the Hell Gate Arch Bridge," by D. B. Steinman, *Transactions*, ASCE, Vol. LXXXII, 1918, pp. 1087–1089.

[27] Associate Engr., Robinson & Steinman, New York, N. Y.

[28] Discussion by C. H. Gronquist of "Rigidity and Aerodynamic Stability of Suspension Bridges," by D. B. Steinman, *Transactions*, ASCE, Vol. 110, 1945, p. 490.

The report of the physical examination of New York's first suspension bridge more than sixty years after its completion attests to the thoroughness of the workmanship employed in its construction, and the results of the tests on the cable wire and truss metal are on the whole reassuring. The authors and their associates are to be complimented on the thoroughness and effectiveness with which they attacked the interesting problems attendant to making a technical survey of the condition of the historic Brooklyn Bridge.

Blair Birdsall,[29] Assoc. M. ASCE.—The papers composing this Symposium represent not only an ingenious and competent study of a very complicated engineering problem but also a fitting tribute by present day engineers to the lasting genius of the creators of the Brooklyn Bridge. The subject has been well covered by the four papers but the following comments may prove of interest as footnotes to some of the statements in the Symposium.

No printed matter could appear to be more dry and uninteresting than the publication entitled, "Testimony Taken by the Committee on Commerce and Navigation of the Assembly in Relation to the New York and Brooklyn Bridge Pursuant to Resolution of the Assembly, Passed February 5, 1879."[30] However, to one who has an interest in the history of the Brooklyn Bridge or of the cities of New York and Brooklyn, this book makes reading which is exceedingly interesting if not exciting. It includes the testimony taken pursuant to the petition of D. Willis James and others mentioned by Mr. Seely under the heading, "Financial and Political Difficulties." The following excerpts from this testimony will be of considerable interest to those who have read the Symposium. It is in startling contrast with the 60 years' service record of the Brooklyn Bridge.

Edward W. Serrell, Jr., was a witness for the opposition. Incidentally, he was a nephew of the John J. Serrell who was a member of the Board of Engineers called in by the late John A. Roebling, M. ASCE, in 1869. As a part of his testimony purporting to show the insufficiency of the design of the bridge, he made the following comments:

> "* * * for 25 years past there has been discussion as to the deterioration of metals under strain and what that deterioration is has not, so far as I know, been accurately determined even for wrought iron which is the most familiar that we have now in bridge construction; now, I understand that the maximum strength in any steel wires used in these bridge cables is 160,000 lb. per sq. in.; now, I am willing to place anything I have to say in the matter on the assumption that every wire there now is capable of carrying that amount."
>
> "But that metal, such a metal as that, is a new metal; it is a metal, to be sure, we have used I don't know how long; but steel wires of such a strength as that, so far as I have been informed, have not been subjected to the test of time in such a way as to determine whether the factor of safety for a number of years can be properly found by calculating the present strength of that steel."

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[29] Asst. Chf. Engr., Bridge Div., John A. Roebling's Sons Co., Trenton, N. J.

[30] "Testimony Taken by the Committee on Commerce and Navigation of the Assembly in Relation to the New York and Brooklyn Bridge Pursuant to Resolution of the Assembly, Passed February 5, 1879," Weed, Parsons and Co., Albany N. Y., 1879.

Statements submitted as evidence by William H. Webb, president of the Council of Political Reform (whose particular interest was tax reduction) included the following section headings which were supposed to be descriptive of the bridge:

> "An obstruction to commerce * * *. Depreciates dock property * * *. Impracticable * * *. Inconvenient * * *. Will not accommodate passengers * * *. Illegally unsafe."

In summing up the case William H. Arnoux, who appears to have been the counsel for the opposition, makes the following statements:

> "It seems to me this committee ought to report to the Legislature that this work should stop until a competent board of engineers should be appointed who should examine into this work and determine whether it is safe for the public. All other questions sink into insignificance compared with the loss of human life which would result, if, upon some occasion, the bridge should be crowded and should give way under the strain. It therefore becomes a very serious problem. And in connection with this matter, as I did not feel competent myself to discuss the engineering project in view of the testimony Mr. Serrell gave to us, I have invited him and the Council of Political Reform have asked him to come here and give to you his views upon that question of the bridge."
>
> "Is it not preposterous, in view of all the matters that are going on day by day, to say that anybody shall construct a bridge that shall provide not only for the uses of today but for the additional uses of a period 75 years hence?"

In connection with the traffic volume mentioned by Mr. Seely under the heading, "Opening Ceremonies," it might be interesting to show the forecasts of this traffic made by both the opposition and the defendants during the investigation of 1879. Data quoted from the forecasts of 1879, in Table 8, are based on one direction of travel only. This is probably more nearly comparative to the traffic recorded for the maximum hour in 1907 as that was undoubtedly a rush hour during which most traffic would be in one direction. In any case it is very apparent that even the most sanguine hopes of those who favored the bridge in 1879 fell far below the actual potentialities of the bridge.

TABLE 8.—PREDICTED AND ACTUAL NUMBER OF PEOPLE CROSSING BROOKLYN BRIDGE (MAXIMUM HOURLY CAPACITY)

Item	Type of traffic	Trolley cars	Elevated cars	Vehicles	Total passengers	Pedestrians	Grand total
	1879 Forecast:						
1	By the opposition........	....	6,000	Few	6,000 +	8,000	14,000 +
2	By the defendants........	....	25,350	4,650	30,000	36,000	66,000
3	1907 actual...............	25,000	46,256	474	71,730	2,270	74,000

NOMER GRAY,[31] M. ASCE.—In commenting on the statement in the writer's paper that "passing trains produced a deflection at the center of the main span in excess of 1 ft," Mr. Thomson states that "the total possible deflection is 18 ft, 9 ft above and 9 ft below the mean level." Even as the most extreme hypothetical deflection at midspan that one could rationalize, 18 ft is much too large for Brooklyn Bridge. The maximum range of main-span deflection is about 6 ft—3 ft above and 3 ft below a mean level. This estimate is based on a temperature range of from 120° F to − 10° F, and a live load such as prevailed in the first decade of the twentieth century, when the bridge was subjected to the heaviest traffic. Elevated railway service was discontinued on March 5, 1944. With the present live load, consisting of trolley cars and passenger automobiles, the corresponding maximum range of deflection is not likely to exceed 4 ft.

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[31] With Ammann and Whitney, Cons. Engrs., New York, N. Y.

The writer takes this opportunity of expressing the appreciation of the authors of the four papers comprising the Symposium for the contributions made by several discussers.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2298

ANALYSIS OF UNSYMMETRICAL BEAMS BY THE METHOD OF SEGMENTS

By Sol Lifsitz,[1] Esq.

With Discussion by Messrs. William A. Conwell, Ralph W. Hutchinson, Thomas P. Revelise, Victor R. Bergman, A. A. Eremin, and Sol Lifsitz.

Synopsis

A considerable number of tables and diagrams are available from which fixed-end moments, stiffness, and carry-over factors may be obtained for beams with variable moments of inertia. However, most of these tables and diagrams were prepared for symmetrical beams or for beams haunched at one end only. Since the combinations of shapes for unsymmetrical beams are limitless, it is obviously impracticable to tabulate data for but a limited range of shapes. The method proposed herein enables the designer to utilize the data available for beams haunched at one end only for the purpose of determining, quickly and accurately, the end moments, the stiffness, and the carry-over factors for unsymmetrical beams of any combination of shapes. This method should prove to be shorter than any other heretofore published for this purpose, with an accuracy equal to that obtained from the so-called "exact methods."

Nomenclature

The letter symbols used in this paper are defined where they first appear, in the text or by diagrams, and are assembled alphabetically, for convenience of reference, in the Appendix. Discussers are requested to make their nomenclature conform to these symbols. In order to avoid confusion as to signs, all terms are expressed in absolute values and the directions of end moments are indicated by arrows.

Fixed-End Moments

Divide beam AB, Fig. 1, into two segments, AO and OB, by introducing a temporary support at point O, the point of minimum thickness. Values of the fixed-end moment M_F, stiffness ratio K, and carry-over factor C, for each segment, may be obtained from available data.

Note.—Published in March, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Civ. Engr., Board of County Road Commissioners, Wayne County, Detroit, Mich.

Assume a vertical settlement, y_o, at point O. Compute the fixed-end moments m_F due to y_o and determine the final end moments at all the joints by the moment-distribution method.[2] Also compute F_y, the reaction at point O which is equal to, but of opposite sign to, the force acting on beam AB at point O (see Fig. 2). The downward force is simply the sum of the reactions at point O from the beams AO and OB resulting from the final end moments in the two beams.

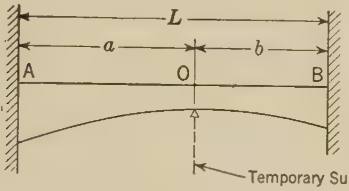

FIG. 1

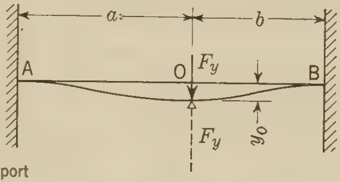

FIG. 2

Fig. 3(a) shows a beam AO, fixed at point O and hinged at end A. Apply a partial moment m'_{AO} at end A, rotating joint A through the angle θ (since θ is small, $\theta = \tan\theta = \frac{y_o}{a}$). This in turn will induce a moment at the fixed joint O which is equal to m'_{AO} times the carry-over factor C_{AO}. Fig. 3(b) shows the position of the beam after this rotation. The end moments at end A and end O caused by this rotation are:

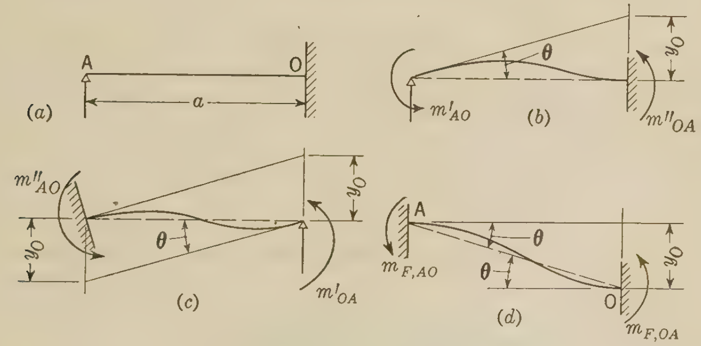

FIG. 3.—DERIVATION OF EQS. 1

$$\curvearrowleft m'_{AO} = K_{AO}\,\theta = K_{AO}\,\frac{y_o}{a} \quad\text{.................. (1a)}$$

and

$$\curvearrowleft m''_{OA} = m'_{AO}\,C_{AO} = K_{AO}\,C_{AO}\,\frac{y_o}{a} \quad\text{............... (1b)}$$

By the principles of moment distribution:

$$K_{AO}\,C_{AO} = K_{OA}\,C_{OA} \quad\text{...................... (2)}$$

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[2] *Transactions*, Vol. 96, 1932, p. 1.

Therefore,

$$m''_{OA} = K_{OA}\,C_{OA}\,\frac{y_o}{a} \tag{3}$$

Lock joint A in its new position and apply a moment m'_{OA} at end O, rotating joint O through the same angle θ as shown in Fig. 3(*c*). The end moments from this second rotation are:

$$\curvearrowleft m'_{OA} = K_{OA}\,\theta = K_{OA}\,\frac{y_o}{a} \tag{4a}$$

and

$$\curvearrowleft m''_{AO} = m'_{OA}\,C_{OA} = K_{OA}\,C_{OA}\,\frac{y_o}{a} = K_{AO}\,C_{AO}\,\frac{y_o}{a} \tag{4b}$$

The final position of the beam is shown in Fig. 3(*d*). The final end moment at each joint is the algebraic sum of the moments at the respective joint. Therefore,

$$\begin{aligned} \curvearrowleft m_{F,AO} &= \curvearrowleft m'_{AO} + \curvearrowleft m''_{AO} = K_{AO}\,\frac{y_o}{a} + K_{AO}\,C_{AO}\,\frac{y_o}{a} \\ &= K_{AO}\,\frac{y_o}{a}\,(1 + C_{AO}) \end{aligned} \tag{5a}$$

$$\begin{aligned} \curvearrowleft m_{F,OA} &= \curvearrowleft m'_{OA} + \curvearrowleft m''_{OA} = K_{OA}\,\frac{y_o}{a} + K_{OA}\,C_{OA}\,\frac{y_o}{a} \\ &= K_{OA}\,\frac{y_o}{a}\,(1 + C_{OA}) \end{aligned} \tag{5b}$$

Corresponding formulas for the right-hand end of beam AB may be similarly derived:

$$\curvearrowright m_{F,OB} = K_{OB}\,\frac{y_o}{b}\,(1 + C_{OB}) \tag{5c}$$

and

$$\curvearrowright m_{F,BO} = K_{BO}\,\frac{y_o}{b}\,(1 + C_{BO}) \tag{5d}$$

Values of *K* and *C* are taken from standard tables, which are readily available. Final end moments caused by a settlement y_o are:

$$\curvearrowleft m_{AO} = m_{F,AO} + (m_{F,OB} - m_{F,OA})\,d_{OA}\,C_{OA} \tag{6a}$$

$$\curvearrowleft m_{OA} = m_{F,OA} + (m_{F,OB} - m_{F,OA})\,d_{OA} \tag{6b}$$

$$\curvearrowright m_{OB} = -\,m_{OA} \tag{6c}$$

and

$$\curvearrowright m_{BO} = m_{F,BO} + (m_{F,OA} - m_{F,OB})\,d_{OB}\,C_{OB} \tag{6d}$$

The reaction at point O necessary to restore the settlement y_o is:

$$F_y = \frac{m_{AO} + m_{OA}}{a} + \frac{m_{BO} + m_{OB}}{b} \tag{7}$$

If a load *P* is applied at any point along span AO, partial end moments M' will be produced at all the joints. These are computed by distributing the fixed-end moments at end A and at end O to all the joints. The reaction at point O is obtained from statics (see Fig. 4).

Partial end moments caused by load P acting in span AO are:

$$\curvearrowleft M'_{AO} = M_{F,AO} + M_{F,OA}\, d_{OA}\, C_{OA} \quad \text{(8a)}$$

$$\curvearrowright M'_{OA} = M_{F,OA}\, d_{OB} \quad \text{(8b)}$$

$$\curvearrowleft M'_{OB} = -M'_{OA} \quad \text{(8c)}$$

and

$$\curvearrowleft M'_{BO} = M_{F,OA}\, d_{OB}\, C_{OB} \quad \text{(8d)}$$

Values of M_F are taken from standard tables.

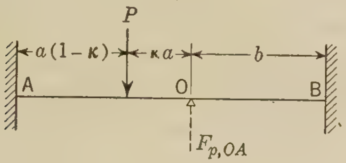

FIG. 4

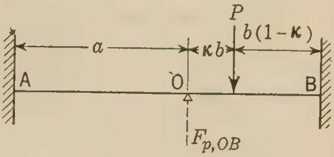

FIG. 5

The reaction at point O to resist load P in span AO is

$$F_{P,OA} = \frac{P\,a\,(1-\kappa) + M'_{OA} - M'_{AO}}{a} + \frac{M'_{OB} + M'_{BO}}{b} \quad \text{(9)}$$

If the temporary support is removed, point O will deflect and additional end moments will be produced at all the joints similar to those caused by the displacement y_o. Since the force required to restore point O to its original position is evidently equal to $F_{p,OA}$, it is clear that an equal but opposite force is acting downward on the beam at point O after the removal of the support. Therefore, the additional end moments created by this deflection can be obtained by direct proportion from the end moments caused by y_o. The final end moments at each joint are equal to the algebraic sum of the moments at the respective joints; or:

$$\curvearrowleft M_{AB} = \frac{F_{p,OA}}{F_y}\, m_{AO} + M'_{AO} \quad \text{(10a)}$$

and

$$\curvearrowright M_{BA} = \frac{F_{p,OA}}{F_y}\, m_{BO} - M'_{BO} \quad \text{(10b)}$$

If a load P is applied along span OB, formulas similar to Eqs. 10 are derived in the same manner (see Fig. 5). Thus, partial end moments because of load P in span OB (compare Eqs. 8) are:

$$\curvearrowright M'_{AO} = M_{F,OB}\, d_{OA}\, C_{OA} \quad \text{(11a)}$$

$$\curvearrowright M'_{OA} = M_{F,OB}\, d_{OA} \quad \text{(11b)}$$

$$\curvearrowleft M'_{OB} = -M'_{OA} \quad \text{(11c)}$$

and

$$\curvearrowright M'_{BO} = M_{F,BO} + M_{F,OB}\, d_{OB}\, C_{OB} \quad \text{(11d)}$$

The reaction at point O to offset load P in span OB (compare Eq. 9) is:

$$F_{p,OB} = \frac{P\,b\,(1-\kappa) + M'_{OB} - M'_{BO}}{b} + \frac{M'_{AO} + M'_{OA}}{a} \quad \ldots\ldots (12)$$

Final end moments corresponding to load P in span OB (compare Eqs. 10) are:

$$\curvearrowleft M_{AB} = \frac{F_{p,OB}}{F_y}\, m_{AO} - M'_{AO} \quad \ldots\ldots (13a)$$

and

$$\curvearrowright M_{BA} = \frac{F_{p,OB}}{F_y}\, m_{BO} + M'_{BO} \quad \ldots\ldots (13b)$$

Stiffness and Carry-Over Factors

To find the K-values and C-values for beam AB, the first step is to find the elastic area (A), the centroidal distance of this area ($\bar{x}$), and the moment of inertia I' about the centroid of each of the two segments of the beam (see Fig. 6). This is easily accomplished by the column analogy method[3] from the known values of K and C for the two beam segments; thus:

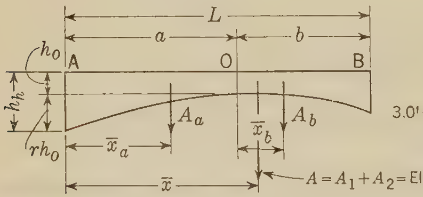

Fig. 6

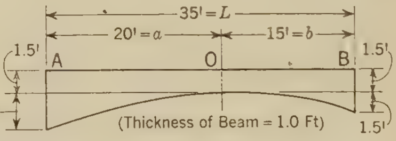

Fig. 7

$$K_{AO} = \frac{1}{A_a} + \frac{\bar{x}^2_a}{I'_a} \quad \ldots\ldots (14a)$$

and

$$K_{OA} = \frac{1}{A_a} + \frac{(a - \bar{x}_a)^2}{I'_a} \quad \ldots\ldots (14b)$$

in which A_a is the elastic area of span AO; $\bar{x}_a$ is the centroidal distance from end A to elastic center of gravity; and I'_a is the moment of inertia of the elastic area about its center of gravity. Furthermore:

$$C_{AO} = \left[-\frac{1}{A_a} + \frac{\bar{x}_a\,(a - \bar{x}_a)}{I'_a}\right] \div \left[\frac{1}{A_a} + \frac{\bar{x}^2_a}{I'_a}\right] \quad \ldots\ldots (15a)$$

and

$$C_{OA} = \left[-\frac{1}{A_a} + \bar{x}_a\,\frac{(a - \bar{x}_a)}{I'_a}\right] \div \left[\frac{1}{A_a} + \frac{(a - \bar{x}_a)^2}{I'_a}\right] \quad \ldots\ldots (15b)$$

The signs were changed in the numerator of Eqs. 15 in order to obtain positive values for the carry-over factors. Solving Eqs. 14 and 15 simultaneously:

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[3] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Univ. of Ill. Experiment Station, Urbana, 1930.

$$\bar{x}_a = \frac{K_{AO}(1 + C_{AO})\,a}{K_{AO}(1 + C_{AO}) + K_{OA}(1 + C_{OA})} \qquad \text{(16a)}$$

$$I'_a = \frac{a\,\bar{x}_a}{K_{AO}(1 + C_{AO})} \qquad \text{(16b)}$$

and

$$A_a = \frac{1}{K_{AO} - \frac{\bar{x}_a}{a} K_{AO}(1 + C_{AO})} \qquad \text{(16c)}$$

In a similar manner the following expressions for segment OB are derived:

$$\bar{x}_b = \frac{K_{OB}(1 + C_{OB})\,b}{K_{OB}(1 + C_{OB}) + K_{BO}(1 + C_{BO})} \qquad \text{(17a)}$$

$$I'_b = \frac{b\,\bar{x}_b}{K_{OB}(1 + C_{OB})} \qquad \text{(17b)}$$

and

$$A_b = \frac{1}{K_{OB} - \frac{\bar{x}_b}{b} K_{OB}(1 + C_{OB})} \qquad \text{(17c)}$$

Let:

$$A = A_a + A_b \qquad \text{(18a)}$$

$$\bar{x} = \frac{A_a\,\bar{x}_a + A_b(a + \bar{x}_b)}{A} \qquad \text{(18b)}$$

and

$$I' = I'_a + A_a(\bar{x} - \bar{x}_a)^2 + I'_b + A_b(a + \bar{x}_b - \bar{x})^2 \qquad \text{(18c)}$$

Therefore,

$$K_{AB} = \frac{1}{A} + \frac{\bar{x}^2}{I'} \qquad \text{(19a)}$$

$$C_{AB} = \frac{-\frac{1}{A} + \frac{\bar{x}(L - \bar{x})}{I'}}{K_{AB}} \qquad \text{(19b)}$$

$$K_{BA} = \frac{1}{A} + \frac{(L - \bar{x})^2}{I'} \qquad \text{(19c)}$$

and

$$C_{BA} = \frac{-\frac{1}{A} + \frac{\bar{x}(L - \bar{x})}{I'}}{K_{BA}} \qquad \text{(19d)}$$

TYPICAL EXAMPLE

It is required to find K_{AB}, K_{BA}, C_{AB} and C_{BA} for the unsymmetrical beam AB shown in Fig. 7; and to plot the influence lines for the fixed-end moments at end A and at end B. Data for segments AO and OA may be taken from the curves in Fig. 8 and Fig. 9. In Fig. 6, let *r* be a ratio of minimum beam

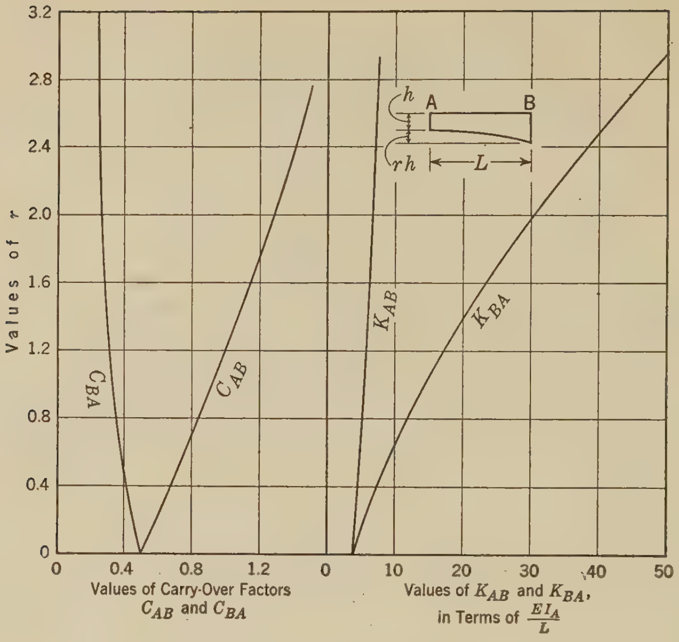

Fig. 8.—Curves for the Selection of Stiffness Ratios K and Carry-Over Factors C

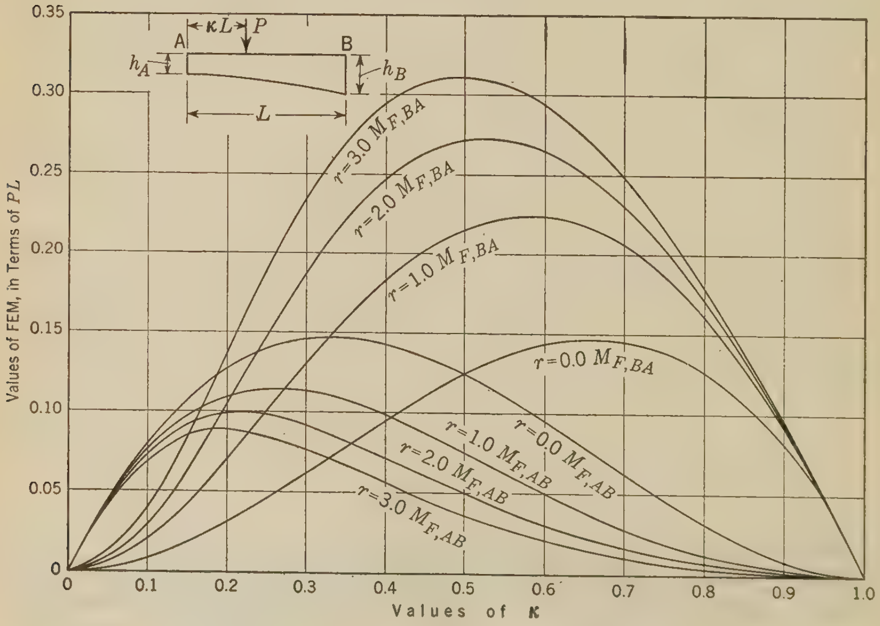

Fig. 9.—Curves for Selection of Fixed-End Moments

depth h_o to haunch depth h_h such that $r = \frac{h_h}{h_o} - 1$. Then (see Fig. 7): $r_A = \frac{3.0}{1.5} = 2.0$; $r_B = \frac{1.5}{1.5} = 1.0$; and $I_o = 1.0 \times 1.5^3 \times \frac{1}{12} = 0.281$ ft⁴. Values for stiffness and carry-over factors at the ends of each beam segment are taken from Fig. 8:

$$K_{AO} = \frac{30.5\,E\,I_o}{20} = 0.43\,E; \qquad C_{AB} = 0.27$$

$$K_{OA} = \frac{6.5\,E\,I_o}{20} = 0.091\,E; \qquad C_{OA} = 1.27$$

$$K_{OB} = \frac{5.35\,E\,I_o}{15} = 0.10\,E; \qquad C_{OB} = 0.91$$

$$K_{BO} = \frac{14.5\,E\,I_o}{15} = 0.272\,E; \qquad C_{BO} = 0.33$$

With the foregoing values substituted in Eqs. 16:

$$\bar{x}_a = \frac{0.43\,E \times 1.27 \times 20}{0.43\,E \times 1.27 + 0.091\,E \times 2.27} = 14.51$$

$$I'_a = \frac{20 \times 14.51}{0.43\,E \times 1.27} = \frac{532}{E}$$

$$A_a = \frac{1}{0.43\,E - \frac{14.51}{20} \times 0.43\,E \times 1.27} = \frac{29.4}{E}$$

—in Eqs. 17:

$$\bar{x}_b = \frac{0.10\,E \times 1.91 \times 15}{0.10\,E \times 1.91 + 0.272\,E \times 1.33} = 5.18$$

$$I'_b = \frac{15 \times 5.18}{0.10\,E \times 1.91} = \frac{407}{E}$$

$$A_b = \frac{1}{0.10\,E - \frac{5.18}{15} \times 0.10\,E \times 1.91} = \frac{29.4}{E}$$

—in Eqs. 18:

$$A = A_a + A_b = \frac{29.4}{E} + \frac{29.4}{E} = \frac{58.8}{E}$$

$$\bar{x} = \left[\frac{29.4}{E} \times 14.51 + \frac{29.4}{E} \times (20 + 5.18)\right] \times \frac{E}{58.8} = 19.85$$

$$I' = \frac{532}{E} + \frac{29.4}{E} \times 5.34^2 + \frac{407}{E} + \frac{29.4}{E} \times 5.33^2 = \frac{2{,}612}{E}$$

—and in Eqs. 19:

$$
\begin{aligned}
K_{AB} &= \frac{E}{58.8} + \frac{19.85^2\,E}{2{,}612} &&= 0.168\,E \\
C_{AB} &= \left(\frac{-E}{58.8} + \frac{19.85 \times 15.15\,E}{2{,}612} \right) \times \frac{1}{0.168\,E} &&= 0.583 \\
K_{BA} &= \frac{E}{58.8} + \frac{15.15^2\,E}{2{,}612} &&= 0.105\,E \\
C_{BA} &= \left(\frac{-E}{58.8} + \frac{19.85 \times 15.15\,E}{2{,}612} \right) \times \frac{1}{0.105\,E} &&= 0.933
\end{aligned}
$$

Assuming a displacement y_o of 1,000 units at point O, Eqs. 5 yield:

$$
\begin{aligned}
\curvearrowleft m_{F,AO} &= 0.43\,E \times \frac{1{,}000}{20}(1 + 0.27) &&= 27.3\,E \\
\curvearrowleft m_{F,OA} &= 0.091\,E \times \frac{1{,}000}{20}(1 + 1.27) &&= 10.33\,E \\
\curvearrowright m_{F,OB} &= 0.10\,E \times \frac{1{,}000}{15}(1 + 0.91) &&= 12.73\,E \\
\curvearrowright m_{F,BO} &= 0.272\,E \times \frac{1{,}000}{15}(1 + 0.33) &&= 24.12\,E
\end{aligned}
$$

By definition:

$$
\begin{aligned}
d_{OA} &= \frac{0.091}{0.091 + 0.10} &&= 0.475 \\
d_{OB} &= 1 - d_{OA} &&= 0.525
\end{aligned}
$$

—by Eqs. 6:

$$
\begin{aligned}
\curvearrowleft m_{AO} &= 27.3\,E + (12.72 - 10.33)\,E \times 0.475 \times 1.27 &&= 28.75\,E \\
\curvearrowleft m_{OA} &= 10.33\,E + (12.73 - 10.33)\,E \times 0.475 &&= 11.47\,E \\
\curvearrowright m_{OB} &= -\,m_{OA} &&= 11.47\,E \\
\curvearrowright m_{BO} &= 24.12\,E + (10.33 - 12.73)\,E \times 0.525 \times 0.91 &&= 22.97\,E
\end{aligned}
$$

—and by Eq. 7:

$$
F_y = (28.75 + 11.47)\frac{E}{20} + (22.97 + 11.47)\frac{E}{15} = 4.30\,E
$$

For load P in span AO equal to 1 lb, Eqs. 8 yield:

$$
\begin{aligned}
\curvearrowleft M'_{AO} &= M_{F,AO} + M_{F,OA} \times 0.475 \times 1.27 &&= M_{F,AO} + 0.603\,M_{F,OA} \ldots \text{(20}a\text{)} \\
\curvearrowright M'_{OA} &= M_{F,OA} \times 0.525 &&= 0.525\,M_{F,OA} \ldots\ldots\ldots\ldots \text{(20}b\text{)} \\
\curvearrowleft M'_{OB} &= -\,M'_{OA} &&= 0.525\,M_{F,OA} \ldots\ldots\ldots\ldots \text{(20}c\text{)} \\
\curvearrowleft M'_{BO} &= M_{F,OA} \times 0.525 \times 0.91 &&= 0.478\,M_{F,OA} \ldots\ldots\ldots\ldots \text{(20}d\text{)}
\end{aligned}
$$

Values of M_F for use in Eqs. 20 are selected from Fig. 9 and entered in Cols. 2 and 3, Table 1. The resulting values of partial end moments are entered in Cols. 4 to 7, and the remaining solution leading to influence line ordinates (see Fig. 10) appear in Cols. 14 and 15, Table 1.

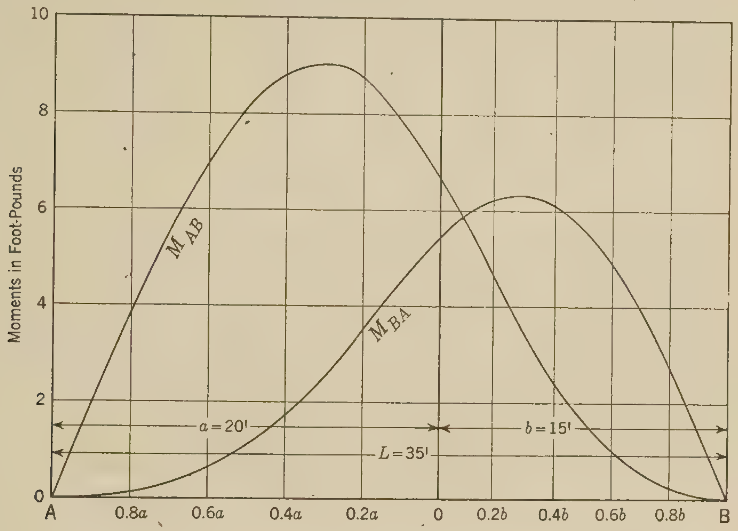

FIG. 10.—INFLUENCE LINES FOR M_{AB} AND M_{BA}

Similarly, for a load P equal to 1 lb in the span OB, Eqs. 11 yield:

$$\curvearrowright M'_{AO} = M_{F,OB} \times 0.475 \times 1.27 = 0.603\, M_{F,OB} \qquad (21a)$$

$$\curvearrowright M'_{OA} = M_{F,OB} \times 0.475 = 0.475\, M_{F,OB} \qquad (21b)$$

$$\curvearrowleft M'_{OB} = -\, M'_{OA} = 0.475\, M_{F,OB} \qquad (21c)$$

and

$$\curvearrowright M'_{BO} = M_{F,BO} + M_{F,OB} \times 0.525 \times 0.91 = M_{F,BO} + 0.478\, M_{F,OB} \qquad (21d)$$

TABLE 1.—INFLUENCE OF ORDINATES FOR END MOMENTS, WITH A LOAD P OF ONE POUND IN SPAN AO

κ	FIXED-END MOMENTS (FIG. 9)		PARTIAL END MOMENTS (EQS. 8)			
	$M_{F,AO}$	$M_{F,OA}$	M'_{AO}	M'_{OA}	M'_{OB}	M'_{BO}
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	0	0	0	0	0	0
0.2	0.103 × 20	0.10 × 20	3.27	1.05	1.05	0.96
0.4	0.245 × 20	0.072 × 20	5.77	0.76	0.76	0.69
0.6	0.267 × 20	0.032 × 20	5.73	0.34	0.34	0.31
0.8	0.173 × 20	0.009 × 20	3.57	0.09	0.09	0.09

TABLE 1.—(*Continued*)

κ	$a(1-\kappa) = 20(1-\kappa)$	$\frac{1}{20}$ (Col. 5 + Col. 8 − Col. 4)	$\frac{1}{15}$ (Col. 6 + Col. 7)	$F_{p,OA}$ = Col. 9 + Col. 10 or Eq. 9	$\frac{F_{p,OA}}{F_v} m_{AO} = 6.69\,F_{p,OA}$	$\frac{F_{p,OA}}{F_v} m_{BO} = 5.34\,F_{p,OA}$	EQUATIONS 10	
							$\curvearrowleft M_{AB}$ (Col. 12 + Col. 4)	$\curvearrowright M_{BA}$ (Col. 13 − Col. 7)
(1)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
0	20	1.0	0	1.0	6.69	5.34	6.69	5.34
0.2	16	0.69	0.134	0.824	5.51	4.40	8.78	3.44
0.4	12	0.35	0.096	0.446	2.98	2.38	8.75	1.69
0.6	8	0.13	0.043	0.173	1.16	0.92	6.89	0.61
0.8	4	0.026	0.012	0.038	0.25	0.20	3.82	0.11

The influence ordinates can then be computed for the case of a load P in span OB = 1 lb (Table 2), the same as demonstrated in Table 1.

TABLE 2.—INFLUENCE ORDINATES FOR END MOMENTS, WITH A LOAD P OF ONE POUND IN SPAN OB

κ	FIXED-END MOMENTS (FIG. 9)		PARTIAL END MOMENTS (EQS. 11)			
	$M_{F,OB}$	$M_{F,BO}$	M'_{AO}	M'_{OA}	M'_{OB}	M'_{BO}
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.2	0.11 ×15	0.07 ×15	1.00	0.78	0.78	1.84
0.4	0.10 ×15	0.182 ×15	0.90	0.71	0.71	3.45
0.6	0.05 ×15	0.225 ×15	0.45	0.36	0.36	3.74
0.8	0.012 ×15	0.162 ×15	0.11	0.085	0.085	2.52

TABLE 2.—(*Continued*)

κ	$b(1-\kappa) = 15(1-\kappa)$	$\frac{1}{15}$ (Col. 6 + Col. 8 − Col. 7)	$\frac{1}{20}$ (Col. 4 + Col. 5)	$F_{p,OB}$ = Col. 9 + Col. 10 or Eq. 12	$\frac{F_{p,OB}}{F_v} m_{AO} = 6.69\,F_{p,OB}$	$\frac{F_{p,OB}}{F_v} m_{BO} = 5.34\,F_{p,OB}$	EQUATIONS 13	
							$\curvearrowleft M_{AB}$ (Col. 12 − Col. 4)	$\curvearrowright M_{BA}$ (Col. 13 + Col. 7)
(1)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
0.2	12.0	0.73	0.09	0.82	5.48	4.38	4.48	6.22
0.4	9.0	0.42	0.08	0.50	3.35	2.67	2.45	6.12
0.6	6.0	0.175	0.04	0.215	1.44	1.15	0.99	4.89
0.8	3.0	0.038	0.01	0.048	0.32	0.26	0.21	2.78

SUMMARY

The advantages claimed for this method are clearly demonstrated in the preceding example.

By readily available curves similar to those in Fig. 9, together with a few simple equations, influence-line data for fixed-end moments for any unsymmetrical beam may be quickly tabulated. The K-values and C-values for the same beam are just as easily determined by curves similar to those in Fig. 8 plus a few more simple equations.

As to the accuracy of the results obtained by this method, it is apparent that it is no less than the accuracy of the curves from which the data for the two segments are taken.

In striking contrast with the tedious computations required by the exact methods, the comparative ease with which accurate results are obtained with this method should be apparent to anyone familiar with the analysis of unsymmetrical beams.

APPENDIX. NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10*a*—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932: Subscripts *AB*, *OA*, etc. denote "in span AB," "in span OA"; or (depending on the context) "end A of Span AB," "end O of span OA," etc. Subscript *F* denotes "fixed-end"; *y* denotes "caused by settlement"; and subscript *p* denotes "caused by a concentrated load *P*."

A = elastic area of a beam = $\sum \frac{\Delta x}{E\, I_x}$;

a = left-hand segment of span L;

b = right-hand segment of span L;

C = carry-over factor;

d = distribution factor, $\frac{K}{\Sigma K}$;

E = modulus of elasticity;

F = force or reaction:

- F_y = reaction at point O of a beam necessary to restore the settlement y_o;
- F_p = reaction at point O of a beam necessary to correct the deflection caused by load P;

h = depth of a beam, with appropriate subscript to denote the section;

I = rectangular moment of inertia of the cross section of a beam = $\frac{t\, h^3}{12}$;

I_o = value of I at a point O;

I' = moment of inertia of elastic area = $\sum \frac{x^2\, \Delta x}{E\, I_x}$;

- I'_x = value of I' for variable segments x of a beam;

K = stiffness of a beam = moment required to rotate the simply supported end of a beam through a unit angle when the other end is fixed;

L = span length = $a + b$;

M = end moment caused by external loads P on a beam:

  M_F = fixed-end moment;

  M' = partial end moment;

m = end moment caused by a displacement y_o:

  m_F = fixed-end moment;

  m' = partial moment, hinged end (see Fig. 3);

  m'' = partial moment, fixed end (see Fig. 3);

P = an externally applied concentrated load;

r = a ratio of the minimum beam depth h_o to the haunch depth h_h (Fig. 6) such that $r = \frac{h_h}{h_o} - 1$;

t = thickness, or width, of the cross section of a beam;

x = variable distance from the left end of a designated span; $\bar{x}$ = distance $\bar{x}$ to the mass center of an elastic area $= \frac{1}{A} \sum \frac{x \, \Delta x}{E \, I_x}$;

y = deflection; y_o = vertical displacement imposed on a point O;

θ = angle of rotation = tan θ;

κ = a percentage of span length (see Figs. 4 and 5) depending on the position of load P.

DISCUSSION

William A. Conwell,[4] M. ASCE.—The treatment given by Mr. Lifsitz to this important subject has the virtue of maintaining close contact with fundamentals and thus eliminating elaborate tables. Thus, it appeals to the engineer who prefers to visualize structural action while making his calculations rather than to read results from tables and then interpret them. A favorable comparison between the results of the method of segments and the so-called "exact" methods referred to by the author in the "Synopsis" and "Summary" is not dependent upon any approximation inherent in the method. The only element entering the procedure and likely to affect the accuracy is the degree to which the shape of the beam upon which tables or charts (such as those of Figs. 8 and 9) are based conforms to that of the beam being analyzed.

Despite the fact that the material of the paper is not unduly complicated, it is not easily read. The reason for this difficulty may be traced to two items—organization and sacrifice of clarity and fullness for brevity.

Unlike the "Typical Example" which is superlatively organized, the preceding theory does not proceed logically to a solution. If the theory were to parallel the "Typical Example" there would first be a treatment of "Stiffness and Carry-Over Factors" and then, under "Fixed-End Moments," a sharp division of the steps of the procedure as follows:

1. Bending moments in segmental spans AO and OB resulting from load *P*;
2. Bending moments in beam AB resulting from a vertical load at point O; and
3. Fixed-end moments in beam AB resulting from load *P*.

A clear statement of the author's intention at the beginning of each division would aid the reader immeasurably.

An example of brevity at the expense of clarity is evident in the second paragraph under "Fixed-End Moments." The first sentence—"Assume a vertical settlement, y_o, at point O"—immediately raises a question. "Settlement" is usually associated with the relaxation of action or removal of a support of a loaded beam. In the instance under consideration, removal of the temporary support at point O would cause no settlement because the beam is not loaded. If, then, it is desired to produce a deflection at point O, it must be done by positive action—that is, by placing loads on the beam. Only one condition of loading—namely, a vertical load at point O—fits the requirements of the problem, however. It is suggested, therefore, that this step in the process of determining fixed-end moments for span AB be considered merely the computation of the bending moments at points A and B resulting from the application of a vertical load at point O. Thus, y_o is considered a means rather than an end and is relegated to the background where it really belongs.

In omitting the derivation of Eq. 2 brevity is achieved at the expense of fullness and the introduction of rich background material. Although this relation has appeared elsewhere,[5] it would seem more in keeping with the author's practice of dealing in fundamentals if a more detailed reference or a demonstration were included. The relation goes much deeper than the "principles" of moment distribution which really furnish only the definitions of the terms. The full implication of the equation is apparent only upon return to the principle that the amount of work done by forces acting upon a body is independent of the order of application of the forces. A demonstration similar to the following would seem apropos:

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[5] "Design Constants for Beams with Nonsymmetrical Straight Haunches," by August L. Ahlf, *Transactions*, Vol. 110, 1945, p. 1019.

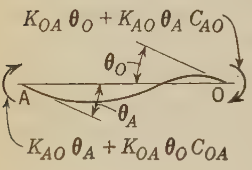
$K_{OA}\,\theta_O + K_{AO}\,\theta_A\,C_{AO}$
θ_O
A
O
θ_A
$K_{AO}\,\theta_A + K_{OA}\,\theta_O\,C_{OA}$

Fig. 11

If a moment $K_{AO}\,\theta_A$ is applied at end A of a beam AO while point O is restrained against rotation, it will produce a rotation θ_A at end A and induce a bending moment, $K_{AO}\,\theta_A\,C_{AO}$, at point O. A similar application of a moment $K_{OA}\,\theta_O$ at point O while end A is restrained will produce a rotation of θ_O at point O and induce a bending moment, $K_{OA}\,\theta_O\,C_{OA}$, at end A. The beam AO under the simultaneous action of these moments is shown in Fig. 11. The work (W) by the application of end moments in the order $K_{AO}\,\theta_A$, $K_{OA}\,\theta_O$ is

$$W = \tfrac{1}{2}\,K_{AO}\,\theta^2_A + \tfrac{1}{2}\,K_{OA}\,\theta^2_O + K_{AO}\,\theta_A\,C_{AO}\,\theta_O \quad\ldots\ldots\ldots(20a)$$

Similarly, application in the order $K_{OA}\,\theta_O$, $K_{AO}\,\theta_A$ gives for the amount of work,

$$W = \tfrac{1}{2}\,K_{OA}\,\theta^2_O + \tfrac{1}{2}\,K_{AO}\,\theta^2_A + K_{OA}\,\theta_O\,C_{OA}\,\theta_A \quad\ldots\ldots\ldots(20b)$$

Equating these expressions gives: $\tfrac{1}{2}\,K_{AO}\,\theta^2_A + \tfrac{1}{2}\,K_{OA}\,\theta^2_O + K_{AO}\,\theta_A\,C_{AO}\,\theta_O = \tfrac{1}{2}\,K_{OA}\,\theta^2_O + \tfrac{1}{2}\,K_{AO}\,\theta^2_A + K_{OA}\,\theta_O\,C_{OA}\,\theta_A$; and

$$(K_{AO}\,\theta_A\,C_{AO})\,\theta_O = (K_{OA}\,\theta_O\,C_{OA})\,\theta_A \quad\ldots\ldots\ldots\ldots\ldots(21)$$

In general terms, Eq. 21 states that the bending moment at point O, induced by a rotation θ_A, multiplied by the rotation θ_O, equals the bending moment at end A, induced by a rotation θ_O, multiplied by the rotation θ_A. In particular, if $\theta_A = \theta_O = 1$,

$$K_{AO}\,C_{AO} = K_{OA}\,C_{OA} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

and a statement of this relation becomes simply: The bending moment at point O induced by a unit rotation at end A equals the bending moment at end A induced by a unit rotation at point O. The principles involved are evidently those of the Maxwell reciprocal theorem.

Clarity again appears to be sacrificed in the twelfth paragraph under "Fixed-End Moments" in the sentence:

> "Since the force required to restore point O to its original position is evidently equal to $F_{p,OA}$, it is clear that an equal but opposite force is acting downward on the beam at point O after removal of the support."

It is suggested that this step, after computation of the reaction $F_{p,OA}$, be considered as follows:

1. Replace the reaction, $F_{p,OA}$, with an equal, upward-acting load $F_{p,OA}$. Of course, this action will leave the state of stress and deflection in the beam

precisely as it was with the reaction in place; but it may now be treated as a single span AB.

2. By the method outlined in the discussion of the second paragraph under "Fixed-End Moments," compute the end moments resulting from a vertical downward load at point O equal to $F_{p,OA}$ and acting alone on beam AB.

3. Superimpose the loading and bending conditions of the second paragraph on those of the first paragraph. The equal and opposite loads at point O cancel each other and the bending moments are those resulting from P only.

Although Figs. 8 and 9 could have been omitted, their inclusion is commendable in that they round out the paper and make it a fairly complete tool. Values more accurate than those that can be read from the diagrams may often be desired but they are readily available in standard tables, as the author states. A description of the shape of beam upon which Figs. 8 and 9 were based would be of value. The application of tables for straight haunches[5] to the beam of the "Typical Example" yields the values $K_{AB} = 0.294\,E$, $C_{AB} = 0.541$, $K_{BA} = 0.159\,E$, and $C_{BA} = 0.997$. When these values are compared with those of the "Typical Example" ($K_{AB} = 0.168\,E$, $C_{AB} = 0.583$, $K_{BA} = 0.105\,E$, and $C_{BA} = 0.933$), some idea is obtained of the extreme variations in these characteristics resulting from differences in shape only.

In view of the foregoing it may be concluded that, despite reading made difficult by easily-removable obstacles, this paper has a more-than-ordinary value and that the presentation of the "Typical Example" is far superior to that of the remainder of the paper.

Ralph W. Hutchinson,[6] Assoc. M. ASCE.—The method of segments is similar to a method developed by engineers of the writer's organization for obtaining the properties of beams with intermediate hinges from those of the solid beam, or the same beam without the hinge. The properties of the solid beam can usually be obtained from published charts whereas available data on beams with hinges are meager.

As in the method of segments the elastic area is first obtained, together with the moment of inertia about its centroid and the location of the centroid. Because of the similarity between a beam with an intermediate hinge and one with a hinge at one end, the formulas used were based on the properties of a beam hinged at one end. By this method a formula for the elastic area A may be expressed in terms of properties of the beam without first solving for the location of the centroid. For comparison with the formula used by the author, this formula may be written in terms of a beam fixed at both ends by including the factor $(1 - C_{AB} \times C_{BA})$:

$$\frac{A}{(1 - C_{AB} \times C_{BA})} = \frac{1 - C_{AB}}{K_{BA}} + \frac{1 - C_{BA}}{K_{AB}} \qquad (23)$$

The properties of the hinged beam are computed by first finding the moment of inertia of the elastic area about the hinge. In the following formulas, the properties of the hinged beam are indicated by adding the letter H to the letter symbols used for these same properties in the solid beam: $c = \bar{x} - a$; $I_H = I + A\,c^2$; $K_{ABH} = \frac{a^2}{I_H}$; $K_{BAH} = \frac{b^2}{I_H}$; $C_{ABH} = \frac{b}{a}$; $C_{BAH} = \frac{a}{b}$; and $d = \frac{I_H}{C\,A}$.

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[6] Associate Bridge Engr., State of California, Sacramento, Calif.

Applying the foregoing equations to the solid beam in the Lifsitz paper: $A = 58.8$; $I = 2{,}612$; $\bar{x} = 19.85$; $a = 7$ ft; $b = 28$ ft; $c = 19.85 - 7.00 = 12.85$ ft; $I_H = 2{,}612 + 58.8 \times 12.85^2 = 12{,}332$; $K_{ABH} = \frac{7 \times 7}{12{,}332} = 0.00397$; $K_{BAH} = \frac{28 \times 28}{12{,}332} = 0.0636$; $C_{ABH} = \frac{28}{7} = 4$; $C_{BAH} = \frac{7}{28} = 0.25$; and $d = \frac{12{,}320}{12.85 \times 58.8} = 16.3$ ft. From the tables in Mr. Lifsitz' paper $(\text{FEM})_A = 8.82$; and $(\text{FEM})_B = 1.69$. For graphical solution the analyst can plot a simple beam moment diagram and draw a closing line through the fixed-end moments for the solid beam. The closing line for the hinged condition is drawn through the simple beam moment curve at the hinge and through the fixed-end moment line at the conjugate point.

After the properties of the beam have been obtained, fixed-end moments may be obtained by either graphical or analytical means. An illustrative example of the graphical method is given in which a hinge is inserted at the fifth-point in the beam used by the author to illustrate the method of segments. The fixed-end moments are computed for a load of $P = 1$ at 12 ft from end A (Fig. 12).

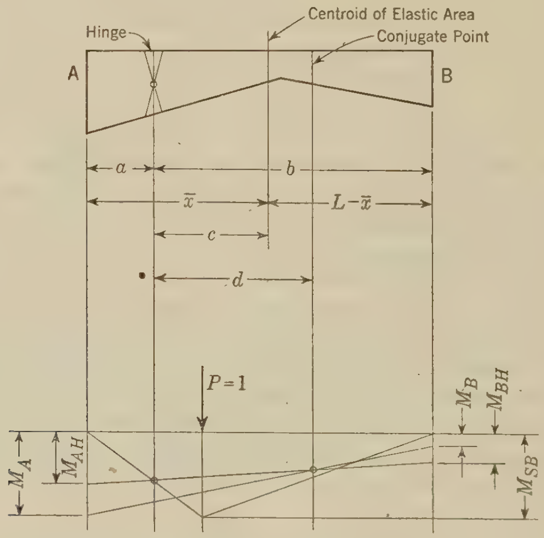

FIG. 12

Time often does not permit an exact determination of the properties of beams of unusual shape, and every engineer has had to rely on approximations extrapolated from working tables. Even though these approximations are usually close enough for the purpose there is a real need for a quick method of obtaining results with a precision equal to data picked from charts for regular beams. The method of segments gives promise of meeting this need, but the work required for the computation of fixed-end moments is disappointing. Some engineers may find it economical to use the method of segments to determine the elastic properties of the beam and then compute fixed-end moments by a more familiar method.

Before deciding for or against the adaptation of the method of segments as a working tool, one should compare the work involved in a complete solution with the work required by a method of "multiplying factors." Such a method was developed by Carl Wagner, Jr., Jun. ASCE, from contributions by George E. Large[7] and A. W. Earl,[8] Members, ASCE.

Thomas P. Revelise,[9] Esq.—The technique of computing design constants for unsymmetrical beams has been advanced perceptibly by the publication of this paper. However, the writer does not consider the method as general in scope as the author implies (see "Summary"):

> "By readily available curves similar to those in Fig. 9, together with a few simple equations, influence-line data for fixed-end moments for any unsymmetrical beam may be quickly tabulated."

Curves cannot be constructed either for use alone or in conjunction with supplementary methods for computing beam design constants if the shape or cross section of the member is sufficiently discontinuous. Several important types occur in this category. A familiar example is the continuous plate girder, with multiple cover plates of varying lengths, placed unsymmetrically over supports and elsewhere in the spans. In this case, the moments of inertia at short successive sections are both variable and discontinuous, and there is no short cut to a rational solution. The problem is also encountered occasionally in the design of reinforced concrete beams whose cross section at successive points is discontinuous.

The few exceptions do not detract from the merit of Mr. Lifsitz's method and other short-cut methods, which are applicable to most cases encountered in routine structural practice; but it is well to remember that these methods are necessarily based on certain assumptions of geometric continuity, which, if not fulfilled, make recourse to a so-called "exact" analysis the only remaining alternative.

In view of the considerable interest in this subject, evidenced by various published papers, the writer would like to say a word about its most neglected phase—the exact procedure. Most writers seem agreed that an exact procedure is too involved and unwieldy for practical use. The reader is invited to accept this premise without question, since, as a rule, no comparative demonstration is given. Although most textbooks on structural analysis present the theoretical fundamentals, few writers give consideration to a well-coordinated exact procedure arranged to yield, with a minimum of computation, the answers in which the average designer is primarily interested—namely, the stiffness and carry-over constants at both ends of a given beam and the (FEM)-coefficients at both ends for a unit load at the tenth points (or the equivalent, unit influence lines for (FEM) at both ends).

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[7] *Transactions*, ASCE, Vol. 96, 1932, p. 101.

[8] *Ibid.*, p. 112.

[9] Highway Bridge Engr., U. S. Public Roads Administration, Atlanta, Ga.

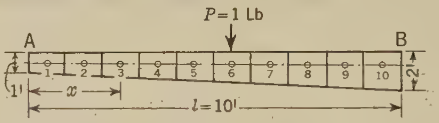
If properly organized, an exact procedure for the determination of beam design constants is much less formidable than is generally appreciated, and its applicability to all cases makes it a valuable tool for anyone engaged in this phase of structural design. Practically any variant of the elastic theory may be used in deriving the necessary expressions. Writers of today show a preference for the method of column analogy. Other variants, such as moment areas, least work, or slope deflection, can be used with fully comparable results.

TABLE 3.—COMPUTATION OF CONSTANTS FOR BEAMS WITH VARIABLE MOMENT OF INERTIA (UNIT LOADS SUCCESSIVELY AT POINTS x; WIDTH OF BEAM, 1 FT)

$Z_A = AC - B^2 = 1.42$

$K_A = \frac{C}{Z_A l} = 0.058$

$C_A = 1 - \frac{B}{C} = -0.83$



$M_f = f P l$

$K E$ = absolute stiffness[a]

$K_B = \frac{E}{Z_A l} = 0.161$

$C_B = 1 - \frac{D}{E} = -0.30$

x (point No.) (1)	I (2)	$\frac{x}{l}$ (3)	$\frac{0.1}{I}$ (4)	Col. 3 times Col. 4 (5)	Col. 3 times Col. 5 (6)	Σ Col. 4 minus Col. 4_x (7)	Σ Col. 7 minus Col. 7_{x-1} (8)	Σ Col. 5 minus Col. 5_x (9)	Σ Col. 9 minus Col. 9_{x-1} (10)	Col. 10 times B (11)	Col. 8 times C (12)
1	0.097	0.05	1.03	0.05	0	3.44	12.76	1.45	7.38	11.07	10.46
2	0.127	0.15	0.79	0.12	0.02	2.65	9.32	1.33	5.93	8.90	7.64
3	0.163	0.25	0.61	0.15	0.04	2.04	6.67	1.18	4.60	6.90	5.47
4	0.205	0.35	0.49	0.17	0.06	1.55	4.63	1.01	3.42	5.13	3.80
5	0.254	0.45	0.39	0.18	0.08	1.16	3.08	0.83	2.41	3.62	2.53
6	0.311	0.55	0.32	0.18	0.10	0.84	1.92	0.65	1.58	2.37	1.57
7	0.375	0.65	0.27	0.18	0.12	0.57	1.08	0.47	0.93	1.40	0.89
8	0.447	0.75	0.22	0.16	0.12	0.35	0.51	0.31	0.46	0.69	0.42
9	0.528	0.85	0.19	0.16	0.14	0.16	0.16	0.15	0.15	0.23	0.13
10	0.619	0.95	0.16	0.15	0.14	0	0	0	0	0	0
Σ	....	....	4.47	1.50	0.82	12.76	....	7.38	....	....	....
Design constants		....	A	B	C	....	....	....	....	....	....

TABLE 3.—(*Continued*)

x (point No.) (1)	Col. 11 minus Col. 12 (13)	Col. 13 times $\frac{1}{10Z_A}$ (14)	Col. 14 times A (15)	Col. 15 plus Col. 8/10 (16)	Col. 16 times $1/B$ (17)	1 minus Col. 3 (18)	Col. 17 minus Col. 18 (19)	Col. 14 minus Col. 19 (20)	Col. 18 times Col. 4 (21)	Col. 18 times Col. 21 (22)
1	0.61	0.043	0.192	1.469	0.980	0.95	+0.030	0.013	0.98	0.93
2	1.26	0.089	0.398	1.330	0.887	0.85	+0.037	0.052	0.67	0.57
3	1.43	0.01	0.451	1.118	0.746	0.75	−0.004	0.105	0.46	0.35
4	1.33	0.094	0.420	0.883	0.589	0.65	−0.061	0.155	0.32	0.21
5	1.09	0.077	0.344	0.652	0.435	0.55	−0.115	0.192	0.21	0.12
6	0.80	0.056	0.250	0.442	0.295	0.45	−0.155	0.211	0.14	0.06
7	0.51	0.036	0.161	0.269	0.179	0.35	−0.171	0.207	0.09	0.03
8	0.27	0.019	0.085	0.136	0.091	0.25	−0.159	0.178	0.06	0.02
9	0.10	0.007	0.031	0.047	0.031	0.15	−0.119	0.126	0.03	0
10	0	0	0	0	0	0.05	−0.050	0.050	0.01	0
Σ	....	....	....	....	....	....	....	....	2.97	2.29
Design constants		f_A	....	....	....	....	....	f_B	D	E

[a] $K \times$ modulus of elasticity equals absolute stiffness.

Several years ago, the writer arranged a tabular form for the analysis of unusual and borderline cases, which is offered as an example of what can be prepared along this line.

The basic expressions were obtained by developing, algebraically, the fundamental elastic equation—

$$d\phi = \int \frac{M\,ds}{E\,I} \qquad \text{(24)}$$

—and using physical summations instead of integrals. If approached in this manner, the most advanced mathematical operation is found to be the solution (for this problem) of two simple simultaneous equations. The resultant expressions were then separated into component parts and rearranged in orderly fashion in a tabular form which is nonmathematical in character and can be executed without reference to the theoretical derivations.

The member to be analyzed is first drawn to a convenient scale and divided into ten sections of equal horizontal length as shown in Table 3. The depth, d, at the center of each section is scaled, and values of $I = \frac{b\,d^3}{12}$ are computed and entered in Col. 2, Table 3. Successive operations are indicated under each column heading; and, upon completion of the form, all necessary design constants, including fixed-end moments at both ends for unit concentrations at the tenth points are obtained. The meaning of the headings for Cols. 7 to 10 may be further clarified by reference to point $x = 3$, as follows:

$$\text{Col. } 7 = \Sigma \text{ Col. } 4 - \text{Col. } 4_x = 4.47 - (1.03 + 0.79 + 0.61) = 2.04$$
$$\text{Col. } 8 = \Sigma \text{ Col. } 7 - \text{Col. } 7_{x-1} = 12.76 - (3.44 + 2.65) = 6.67$$
$$\text{Col. } 9 = \Sigma \text{ Col. } 5 - \text{Col. } 5_x = 1.50 - (0.05 + 0.12 + 0.15) = 1.18$$
$$\text{Col. } 10 = \Sigma \text{ Col. } 9 - \text{Col. } 9_{x-1} = 7.38 - (1.45 + 1.33) = 4.60$$

The example used to illustrate the procedure was chosen to afford a comparison between computed values and those obtained from available curves. The procedure remains the same, regardless of what form the dissymmetry or discontinuity takes.

Forms of the type of Table 3 can be derived independently by other methods, or the form illustrated can be altered or modified to suit the needs or preferences of the user.

VICTOR R. BERGMAN,[10] ASSOC. M. ASCE.—In the analysis of beams of variable moments of inertia by the moment-distribution method, the work preliminary to the actual distribution frequently requires a major part of the total time involved. The determination of fixed-end moments, stiffness factors, and carry-over factors can be a very tedious and time-consuming task, unless facilitated by the use of tables and diagrams of adequate scope. Therefore, the author's presentation of a fairly simple method for extending the range of applicability of readily available data is to be commended. Incidentally, Eqs. 5 are not peculiar to the problem under discussion; they have been developed elsewhere in connection with other phases of structural analysis.[11]

Several years ago, the writer had occasion to develop a "Method of Sections" in which the procedure for the calculation of fixed-end moments is practically identical with that presented by Mr. Lifsitz, but in which the method of computing stiffness factors and carry-over factors is somewhat simpler.

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[10] Vice-Pres., Godwin Constr. Co., New York, N. Y.

[11] "One-Story Concrete Frames Analyzed by Moment Distribution," *Concrete Information Bulletin No. ST42*, Portland Cement Assn.

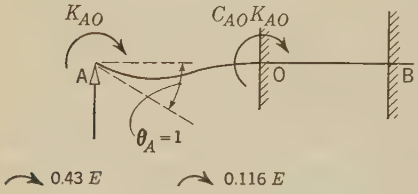

K_{AO}
$C_{AO}K_{AO}$
A
O
B
$\theta_A=1$
0.43 E
0.116 E

FIG. 13.—STEP 1

Consider the unsymmetrically haunched beam shown in Fig. 7 for which the author, by use of Fig. 8, determined values of K and C for each end of both segments. The writer's procedure for determining values of K_{AB} and C_{AB} involves four simple steps, as follows:

Step 1.—Assume the beam to be both supported and fixed against rotation at point O, and hinged at point A. Apply to end A (Fig. 13) a clockwise moment equal to K_{AO} ($= 0.43\ E$). This will produce on end O of the segment AO a clockwise moment equal to $C_{AO}\ K_{AO}$ ($= 0.116\ E$).

Step 2.—At end A, the applied moment will have produced (by the definition of stiffness factor) a unit rotation. Lock end A in this rotated position and unlock joint O, distributing moments as shown in Fig. 14.

Step 3.—Determine the reaction provided by the temporary support at point O, as follows:

$$F'_O = \frac{0.360\ E + 0.061\ E}{20} + \frac{0.061\ E + 0.056\ E}{15} = 0.0289\ E \ldots\ldots (25)$$

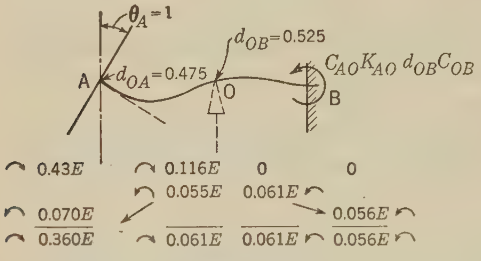

$\theta_A=1$
$d_{OB}=0.525$
$C_{AO}K_{AO}\ d_{OB}C_{OB}$
A
$d_{OA}=0.475$
O
B
0.43E
0.116E
0
0
0.055E
0.061E
0.070E
0.056E
0.360E
0.061E
0.061E
0.056E

FIG. 14

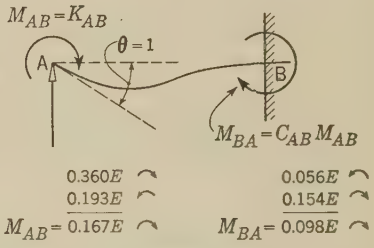

$M_{AB}=K_{AB}$
$\theta=1$
A
B
$M_{BA}=C_{AB}M_{AB}$
0.360E
0.193E
$M_{AB}=$ 0.167E
0.056E
0.154E
$M_{BA}=$ 0.098E

FIG. 15

Next remove the temporary support at point O, and replace it with an equal but opposite load. In the computation preceding Eqs. 20, the author found that a load of 4.30 E at point O corresponds to moments at ends A and B equal, respectively, to:

$$m_{AO} = 28.75\ E;\ \text{and}\ m_{BO} = 22.97\ E \ldots\ldots\ldots\ldots\ldots (26)$$

By proportion, for a load of 0.0289 E at point O,

$$m_{AO} = \frac{0.0289\ E}{4.30\ E} \times 28.75\ E = 0.193\ E \ldots\ldots\ldots\ldots (27a)$$

and

$$m_{BO} = \frac{0.0289\ E}{4.30\ E} \times 22.97\ E = 0.154\ E \ldots\ldots\ldots\ldots (27b)$$

Step 4.—Combine the end moments calculated in step 3 with those determined in step 2, to arrive at the final end moments at A and B (Fig. 15). After receiving a unit rotation, end A has been maintained in that position;

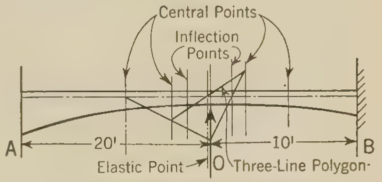
it is evident, therefore, that the final moment ($=0.167\ E$) is equal to K_{AB}, the required stiffness factor, in accordance with the definition of the term. Furthermore, by the definition of carry-over factor, $C_{AB} = \frac{M_{BA}}{M_{AB}} = \frac{0.098\ E}{0.167\ E} = 0.586.$

The values thus found are seen to check, very closely, those calculated by the author. The stiffness and carry-over factors at end B, of course, can be determined in a manner similar to that followed at end A. The foregoing method involves only basic moment-distribution concepts, and, therefore, it has some slight advantage in ease and simplicity over that proposed by the author.

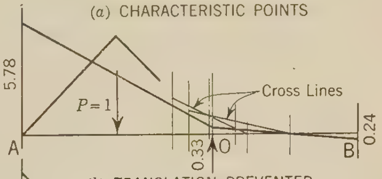
A. A. Eremin,[12] Assoc. M. ASCE.—A temporary support, inserted hypothetically, in an unsymmetrical member simplifies the computation of moments, stiffness, and carry-over factors. The algebraic procedures used by Mr. Lifsitz for this purpose can be clarified by the use of graphics. For illustration, the beam in Fig. 1 is analyzed graphically in Fig. 16. The member is sustaining a unit load at the midpoint of distance AO in Fig. 16(*b*).

The characteristic points in the member with a temporary support at point O are shown by the three-line polygon, Fig. 16(*a*). The moments in the member with the temporary support, fixed against linear displacement, are determined by constructing cross lines as shown in Fig. 16(*b*). The effect of a unit displacement at the temporary support is shown in Fig. 16(*c*). The final moments in the member are obtained by the summation of moments in Fig. 16(*b*) and Fig. 16(*c*), multiplied by the factor of elastic displacement of the support at point O. The resulting moments check the values in Fig. 10 very closely. Elsewhere[13] the writer has demonstrated a similar graphical construction for the distribution of moments in a continuous beam with a suspension hinge.

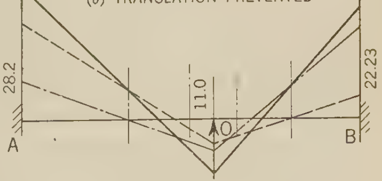
The author's algebraic formulas for moments, stiffness, and carry-over factors serve as the preliminary steps in the distribution of moments in continuous beams. The graphical principles in Fig. 16 may be applied directly to the distribution of moments in any continuous beam or frame containing unsymmetrical members.



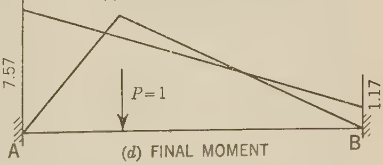
(*a*) CHARACTERISTIC POINTS



(*b*) TRANSLATION PREVENTED



(*c*) SETTLEMENT AT O, Δ=1



(*d*) FINAL MOMENT

Fig. 16.—Graphic Construction of Moments in an Unsymmetrical Beam

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[12] Associate Bridge Engr., California State Highways Dept., Sacramento, Calif.

[13] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, Sacramento, Calif., 1943.

Unsymmetrical members are often used in modern engineering structures; therefore, the paper constitutes a valuable contribution in the analytical interpretation of the temporary support method.

Sol Lifsitz,[14] Esq.—By clarifying some of the "hard-to-read" paragraphs of the paper Mr. Conwell has supplemented the paper considerably. As to the omission of the derivation for Eq. 2 which Mr. Conwell mentions, the writer was of the opinion that its inclusion would serve no useful purpose. The demonstration given by Mr. Conwell, based on the principle of work, is only one of several proofs available in the literature on moment distribution. As a matter of fact, the proof of Eq. 2 is inherent in Eqs. 19, where it is seen by inspection that $C_{AB}\ K_{AB} = C_{BA}\ K_{BA}$.

Mr. Hutchinson finds the work involved in the computation of fixed-end moments by the method of segments disappointing, citing the method of "multiplying factors" developed by Carl Wagner, Jr., Jun. ASCE, as an alternative for comparison. The criterion for deciding on either of two methods, both yielding the same degree of accuracy, should be the ease of computation and the time required to obtain results. After comparing the two methods, the writer finds that with the same accuracy of results the time required by his own method is considerably less than that required for the suggested alternative.

Mr. Revelise is quite right in stating that the application of the method is limited to members in which the variation in moment of inertia is a continuous function, or at most contains but a single discontinuity in each segment of the beam. The efficient use of the method is confined to cases for which tables or curves of the segmental beam constants are available, and to the writer's knowledge such data for beams of highly discontinuous outline do not exist. For cases in which it is necessary to derive the beam constants from first principles, there is no economy of time over other methods, of which Mr. Revelise has given one very good example.

Mr. Bergman's method for finding stiffness and carry-over factors appears to be very simple and bears evidence that his approach to the problem in his method of sections closely parallels that of the writer's method of segments.

In conclusion, the writer wishes to thank the discussers for their interest in the paper and their kind and helpful criticism, which he found very stimulating.

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[14] Civ. Engr., Board of County Road Commissioners, Wayne County, Detroit, Mich.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2299

NEW PROJECT FOR STABILIZING AND DEEPENING LOWER MISSISSIPPI RIVER

BY CHARLES SENOUR,[1] M. ASCE

WITH DISCUSSION BY MESSRS. CHARLES W. OKEY, F. NEWHOUSE, GERARD H. MATTHES, AND CHARLES SENOUR.

SYNOPSIS

The project authorized by the 1944 Act for stabilizing and deepening the Lower Mississippi River between Cairo, Ill., and Baton Rouge, La., included in flood control bill H. R. 4485 (approved December 22, 1944) and Public Law No. 534 (78th Congress, Chapter 665—second session), is the necessary sequel to the flood control and navigation improvement hitherto accomplished, which it is designed to supplement and protect.

INTRODUCTION

There is presently maintained for navigation between Cairo and Baton Rouge (head of deep-water navigation) a channel not less than 300 ft wide and 9 ft deep at lowest water. This condition or better obtains naturally throughout much of the year; but, at low stages, some of the so-called crossing bars, which constitute broad-crested sand weirs between the deep water in successive bends, require dredging. The river consists, in general, of alternating pools and crossings. The pools, lying along the concave banks of the bend, tend to scour in high water and fill somewhat on declining stages but always are from 40 ft deep to more than 100 ft deep. The crossings, however, build up many feet by deposition during high-river stages and some of them do not scour off rapidly enough during the decline to maintain the 9-ft depth. A fleet of nine dredges is operated to correct deficiencies as rapidly as they occur and to prevent their occurrence by premaintenance dredging where trouble can be foreseen. The capacity of the river to carry commerce is practically unlimited as long as the crossings can be maintained at proper depth, since there are no locks or other bottlenecks of any sort. The tonnage now being moved is limited only by the availability of carriers; between 1935 and 1944 it increased from 5½ million tons per yr to

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NOTE.—Published in February, 1946, *Proceedings*. Positions and titles given are those in effect when the paper was received for publication.

[1] Chf. Engr., Mississippi River Comm., Vicksburg, Miss.

13½ million tons per yr. Data on commerce from the mouth of the Ohio River to, but not including, Baton Rouge for the years 1935–1944 are as follows:

Year	Total tons
1935	5,595,991
1936	5,786,129
1937	6,574,573
1938	6,915,034
1939	7,009,130
1940	7,055,675
1941	9,605,602
1942	9,474,591
1943	10,891,691
1944	13,115,914

History

Flood control of the Lower Mississippi began with the earliest settlements along its banks. It was, and is, absolutely essential to dependable agriculture on the flood plain. From time to time the proposition is advanced that flood protection has been a mistake and that the soil should have been rejuvenated by natural overflows. This plan has no advocates among the 3,000,000 people who live behind levees in the lower alluvial valley, for floods were early found to be too long sustained, and too uncertain as to season, to permit satisfactory correlation between overflows and farming operations.

Fortunately, the topography of the region is ideally suited to flood protection by levees. In general terms, the 20,000,000 odd acres subject to overflow comprise a flood plain of low relief about 600 miles long and from 30 miles to 110 miles wide, divided by major tributaries into seven main basins ranging in size from 956 sq miles to 6,648 sq miles.

The terrain slopes downward away from the river instead of toward it; and the natural watercourses that drain the slopes roughly parallel the river and discharge into the tributary which forms the lower boundary of the basin. Thus, some very long levee lines can be used without drainage complications of any consequence. The Yazoo Basin, for example, extending from Memphis, Tenn., to Vicksburg, Miss., embraces an area about the size of Connecticut and Rhode Island combined. It is protected by a levee 272 miles long. The Tensas Basin, about the size of North Ireland, is protected by a continuous levee 380 miles long, extending from Pine Bluff almost to Old River, in Louisiana. The levee lines do not cross the tributaries which separate the basins, so the lower ends of the basins are subject to backwater.

Proposed Plan of Flood Control

Fig. 1 shows the general features of the present plan of flood control. Briefly, this plan embraces (1) the confinement of the project flood by levees to the main channel of the river from the Commerce Hills to Cairo, and between New Madrid, Mo., and the mouth of Red River; (2) an overbank floodway about 5 miles wide supplementing the leveed river channel between Cairo and New Madrid when stage exceeds 55 ft on the Cairo gage; (3) an equal division of project flood flow in the vicinity of the mouth of Red River, half continuing down the main river between levees, and half down the Atchafalaya Basin. Of the flow to be carried by the Atchafalaya Basin, part will pass down the Atchafalaya River (which is leveed for some 40 miles below its head) and part will flow through leveed overbank floodways paralleling the Atchafalaya River on the east and west. The plan also includes a controlled outlet into Lake Pontchartrain at Bonnet Carré some 20 miles above New Orleans, La., which is to abstract up to 250,000 cu ft per sec and thereby keep New Orleans stages from exceeding 20 ft on the Carrollton gage. Supplemental features are the protection of the St. Francis Basin against headwater floods on the St. Francis River by reservoir construction, channel improvement, and levees; the protection of the Yazoo Basin by similar methods against headwater floods of the Yazoo and of the backwaters of the Mississippi; the protection by levees of parts of the Red River backwater area, and of the White River backwater area; and the improvement of Bayou Rapides, Bayou Boeuf, and Bayou Cocodrie in Louisiana.

The project flood is the maximum flood which the

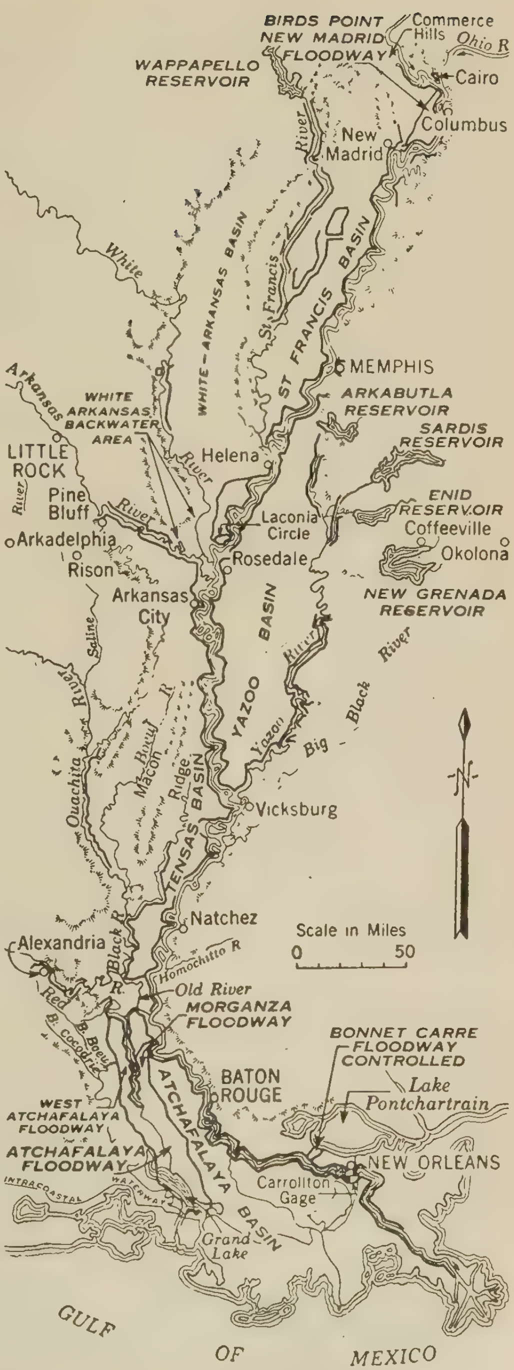
BIRDS POINT
NEW MADRID
FLOODWAY
Commerce
Hills
Ohio R
Cairo
Columbus
WAPPAPELLO
RESERVOIR
River
New
Madrid
White
WHITE – ARKANSAS BASIN
St. Francis
ST FRANCIS BASIN
MEMPHIS
ARKABUTLA
RESERVOIR
SARDIS
RESERVOIR
Arkansas
WHITE
ARKANSAS
BACKWATER
AREA
LITTLE
ROCK
River
Helena
River
Pine
Bluff
River
Laconia
Circle
ENID
RESERVOIR
Coffeeville
Okolona
Arkadelphia
Rison
Rosedale
NEW GRENADA
RESERVOIR
Arkansas
City
Saline
YAZOO BASIN
River
River
Black
Yazoo
River
Big
River
Ouachita
Bayou
Macon
Ridge
TENSAS BASIN
Vicksburg
Natchez
Scale in Miles
0
50
Alexandria
Black
R.
Homochitto R
Red
Old River
MORGANZA
FLOODWAY
B. Cocodrie
B. Boeuf
BONNET CARRE
FLOODWAY
CONTROLLED
BATON
ROUGE
Lake
Pontchartrain
WEST
ATCHAFALAYA
FLOODWAY
ATCHAFALAYA BASIN
ATCHAFALAYA
FLOODWAY
NEW ORLEANS
INTRACOASTAL WATERWAY
Carrollton
Gage
Grand
Lake
GULF
OF
MEXICO

FIG. 1.—FLOOD CONTROL PLAN FOR THE ALLUVIAL VALLEY OF THE MISSISSIPPI RIVER

War Department's hydrologists and the meteorologists of the Weather Bureau consider reasonable to expect. Estimated discharge at Cairo is 2,450,000 cu ft per sec (reduced from a probable value of 2,600,000 cu ft per sec by headwater reservoirs). At the latitude of Arkansas City, Ark., discharge is estimated at 3,065,000 cu ft per sec; and at Red River, at 3,000,000 cu ft per sec.

As adopted in 1928, the plan for flood control did not include the aforementioned supplemental features; nor did it contemplate confinement of project flood flow to the leveed river channel between the Arkansas and Red rivers. Calculations had indicated that such confinement would require unreasonable increases in the height of existing levees. In the vicinity of Arkansas City, for example, the grade would have had to be raised about 15 ft.

Since the levees were already about 22 ft high on the average, and occasionally attained heights of from 30 ft to 40 ft, the resultant structures would have been so large that their cost would have appeared prohibitive had it been considered practical to make them dependably safe with the materials and foundations available.

Therefore, part of the flow of extreme floods between the Arkansas River and the Red River was to be carried by an overbank floodway leveed from the Arkansas River to the head of the Red River backwater area. The floodway was to follow the valley of the Boeuf River, west of Macon Ridge. Local interests opposed its construction; so it was shifted to the east of Macon Ridge and became known as the Eudora Floodway Project—where it was no less obdurately opposed by the people in that part of the basin.

In the meantime, however, great progress had been made in lowering the flood flow line by channel rectification through cutoffs and corrective dredging. The cutoffs were made by dredging through the narrow necks or peninsulas created by the river's meander. The distances across the necks varied from 1.4 miles to 4.8 miles, and the distances around the bends were from 7.3 miles to 17.2 miles. The fall between opposite sides of the necks ranged from 2.2 ft to 5.4 ft. Fifteen artificial cutoffs have been made and one occurred naturally in 1929. In conjunction with the cutoffs, certain chutes which fitted into the new river alinement better than did the main channel were developed by dredging. These operations have shortened the former low-water river distance of 545 miles between Memphis and Old River (mouth of Red River) by about 170 miles or 28%. By 1941, it had become apparent to everyone that the flood plane had been reduced more than 12 ft at Arkansas City and more than 6 ft at Vicksburg. Thus, the levee raising necessary to confine the project flood in the river channel was reduced to moderate and practical dimensions; and in August, 1929, Congress authorized abandonment of the Boeuf and Eudora floodway projects and the substitution of moderately enlarged levees capable of carrying the project flood from the mouth of the Arkansas River to Old River.

The recurrent necessity through the years of enlarging and re-enlarging the levees has resulted in raising them to imposing heights. When brought to the 1941 grade the levees of the Yazoo Basin, for example, will average about 30 ft high and will be from 270 ft to 290 ft wide at the base plus whatever berms may be required for foundational stability. As the size of levees has increased, the cost per mile has increased disproportionately because of the considerably greater length of haul, and because of the present necessity for expensive foundation treatment over part of the line that was stable at the former height.

This condition, together with several other factors, has been emphasizing more and more one of the most troublesome of the river's characteristics—namely, its tendency to meander by caving its banks. No matter how wide and high the levee, no matter how impregnable against the assault of floods, it is an easy and frequent victim of bank caving. In earlier years, retreat from caving banks by the simple process of setting the levee farther landward wherever it was threatened was an easy and fairly inexpensive expedient—but that is no longer true. Levees are so massive that construction on a more retired position is very costly; and, since the ground normally drops away to the landward, the new location is almost invariably on lower ground. This physical phenomenon, in addition to requiring a higher levee, also frequently means that the new foundation is less stable than the old; and two or more years may be required for construction to avoid overloading the base and causing resultant failure.

This much time is not always available. The bank sometimes caves very rapidly. For example, at Homochitto, near Mayersville, Miss., while the engineers struggled (none too calmly) to make an extremely high setback levee stand up on a weak foundation, the bank receded 1,200 ft in three months. In 1943, caving banks required the construction of 18½ miles of levee setbacks costing approximately $4,800,000. During the ten years since 1935, caving banks have necessitated the construction of 135 miles of setbacks at approximate cost of $13,000,000. Most of this expenditure represents a total loss, since setting the levee back in no way advances the program. Moving the levee is pure appeasement of the river's insatiable appetite for land—an appetite which largely precludes the industrial or even agricultural development of riparian property, causes shoaling of channel and harbors, and, over a period of years, engulfs a surprising acreage. Accretion on one bank keeps pace with erosion on the other, but accretion is in the form of a sand bar which will not become useful for generations, while erosion claims the best and highest lands.

Considered as a geologic process, the lateral movement of the river is a matter of give and take; and, except perhaps for occasional horseshoe lakes and various scars and swales, the flood plain abandoned by the stream, in time, becomes as good as the one which it invades. However, this theory does not hold true where only periods commensurate with the short span of human life are considered and when the practical aspects of present-day economy govern. Within the memory of those who live in the valley, river meander has meant loss. The levees are almost always moved back. Even when an occasional accretion achieves a usable elevation and fertility it is not practical as a rule from the economic standpoint to include it within the levee system, since the cost per acre of protecting such isolated areas greatly exceeds the return. The levee of the lower Yazoo Levee District extending from about 25 miles above Rosedale, Miss., to Vicksburg, is 178 miles long, but not one mile stands today upon its original location. Indeed, since 1880, the length of levee yielded to the river by setbacks in this particular district has aggregated 305 miles. The average distance between the controlling levee lines on opposite banks of the river between Memphis and Vicksburg is almost a mile greater than it was 40 years ago.

A continuation of this process of attrition must inevitably make serious inroads on the resources of the region. For example, if the channel and banks of a river are sufficiently stable, industries can load their products directly into barges; and they can import raw materials and fuel most economically by water. One effect of bank and channel instability is to discourage the establishment of such river ports and terminals.

In describing the flood control plan which was later adopted by the Act of May 15, 1928, paragraph 131 of Document No. 90, Seventieth Congress, calls attention to the menace of bank caving to the levee system and its deleterious effect upon the navigation channel in the following language:

> "131. *Channel Stabilization.* Since the levees within the limits of this project are to be greatly enlarged, they will be much more expensive than heretofore, so something must be done to avoid the frequent moving of them from the proximity of caving banks. In addition, the river cannot be regulated for low water navigation until the banks are made stable, this both to keep the channel in one place and to stop the enormous dumping of earth into the river by bank caving. A general bank protection scheme must be carried out * * *."

This statement was made in 1928. The events of succeeding years have not lessened the need. On the contrary, they have rendered it more or less imperative, for, it will be remembered, the finally adopted plan for flood control between the Arkansas River and the Red River by confinement of flow to the leveed channel is based upon the reduction in flood heights caused by shortening the river; and a return toward former length by continued bank caving would mean a corresponding return toward former flood heights and the necessity for overbank floodways—with consequent loss of the large investment made in channel rectification.

Methods of preventing bank caving are few and costly. Temporary relief may sometimes be had by dredging. The situation at Homochitto previously referred to has been corrected for the time being by this means. The circumstances are seldom such that this method is suitable, however, and at best the remedy is usually not permanent. Deflecting dikes of various types have been tried but they also are usable at very few places. The main reliance has always been upon bank revetment, consisting of mats of various types such as woven willow, lumber, asphalt, and articulated concrete, laid upon the river bed from water surface to the toe of slope which may be anywhere from 40 ft to 140 ft below low water and from 120 ft to 350 ft or more offshore. The bank above the water line may be paved with asphalt, or with riprap or articulated concrete laid on a gravel base.

Partly because of scarcity of lumber, willows, and man power, the most widely employed type of subaqueous revetment currently is the articulated concrete mattress, composed of concrete slabs 14 in. wide, 4 ft long in the upstream-and-downstream direction and 3 in. thick, spaced 1 in. The slabs are cast in groups of twenty and are reinforced and connected by wire mesh, fabricated of noncorrosive No. 8 gage wire in sheets 4 ft by 25 ft, to form a twenty-slab unit of those dimensions. Units are laid side by side on the inclined ways of a launching barge, and $\frac{5}{16}$-in. or $\frac{7}{16}$-in. launching cables occupy the $1\frac{3}{4}$-in. spaces left between each pair of adjacent units and pass over drums on the launching barge. Loops of the reinforcing mesh projecting from the ends of the slabs of adjacent units are fastened to the launching cables by clips or twist wires to form a mattress 25 ft wide measured normal to the shore and 140 ft measured parallel to the shore. The mattress is anchored ashore by noncorrosive wires leading from the loops of reinforcing mesh projecting from the shoreward slabs to screw anchors spaced from 4 ft to 8 ft apart. It is lauched partly by withdrawing the barge riverward, concurrently paying off the launching cables. As soon as the ways are sufficiently cleared, a second series of units is placed upon them, made fast to the launching cables, and attached to the riverward ends of the preceding series by means of projecting loops of the reinforcing mesh. The procedure of launching and of adding units is repeated until the mattress has been carried as a continuous sheet a short distance riverward of the toe of the underwater slope. Then, the launching cables are cut at the barge, the outer end of the mattress released, and the barge moved in to the bank ready to assemble the next mattress immediately upstream, with an overlap of from 5 ft to 10 ft on the one previously placed.

Because of the large investment in plant required, and the short working season, revetment has heretofore been placed by hired labor. It costs from about $650,000 to $700,000 a mile under 1946 conditions as to labor and materials. An intensive research program is underway to develop a less expensive product and one that can be made more attractive to contractors from the plant angle; there is reason to believe that progress is being made.

Halfway measures cannot be adopted in an effective bank revetment program. The river's unrevetted bends are not static, but tend to move down the valley, resulting in a constant migration of the caving banks and resultant bars. A revetment put in at great expense to meet an emergency one year may be masked by a sand bar a few years later; or the current may be progressively directed against the bank farther and farther beyond it and thus require repeated extensions. The procedure of protecting the banks only where immediate disaster impends is wasteful and ineffective. However, it has been the only course possible with the funds made available and the other urgent demands upon them. The ideal method would be to stabilize all the bends at once. That is obviously impossible. The next best course would be to work downstream in orderly sequence from certain points where the stream is fixed in location by the presence of bluffs.

The force of events appeared to have made a general stabilization of the river's banks the next and essential step in the improvement. However, this would involve large sums of money and the modification of a fundamental aspect of regimen. It seemed advisable, therefore, to approach such stabilization with caution and, before recommending that it be undertaken, to make certain, if possible, (*a*) that the need was in fact as real as it seemed; (*b*) that there was not some geologic or other cause that would arrest meander or restrict its ultimate scope to limits that could be permitted; (*c*) whether the river's past history indicated that if let alone it would in fact regain the length eliminated by cutoffs as the rapid caving since their completion seemed to indicate; (*d*) whether restriction of meander by artificial means would set up conditions adverse to other aspects of regimen; and (*e*) to what extent revetment of banks would be required to effect satisfactory stabilization.

The Commission had been concerned, of course, with problems of meander for many years; but the river is so large, its phenomena so complex and interrelated, its geology so complicated, and its cycles of change so deliberate, that several fundamental questions had either no answer or too many conflicting answers.

Certain facilities have become available for investigation that were not formerly extant. The making of literally thousands of deep borings in the alluvial valley by the petroleum industry, and coverage of the valley by aerial photographs in connection with the crop control program of the Department of Agriculture provide opportunities, far beyond anything ever approached before, for studying the geologic background of the river. The significance of the oil-well logs is too apparent to require comment. The value of aerial photographs is that ancient river courses and other phenomena not discernible to an observer on the ground are easily detectable in an aerial photograph, and once located in this manner may be examined on the ground to the extent desired.

Another recently developed method of investigation is model experimentation. In the model the numerous variables which combine in nature to obscure one another's effects can be isolated and controlled; cycles of floods and low waters that would consume years in nature can be reproduced in the model in a matter of hours. It seemed possible that exploration of the river's geology, and model experimentation might resolve some of the questions that the usual methods of observation had not entirely clarified.

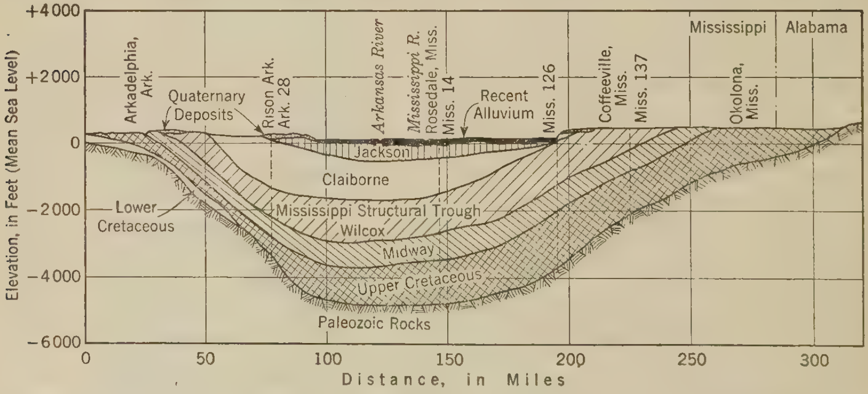
+4000
+2000
0
−2000
−4000
−6000
Elevation, in Feet (Mean Sea Level)
Arkadelphia, Ark.
Quaternary Deposits
Rison Ark. Ark. 28
Arkansas River
Mississippi R.
Rosedale, Miss.
Miss. 14
Recent Alluvium
Miss. 126
Coffeeville, Miss.
Miss. 137
Mississippi
Alabama
Okolona, Miss.
Jackson
Claiborne
Lower Cretaceous
Mississippi Structural Trough
Wilcox
Midway
Upper Cretaceous
Paleozoic Rocks
0
50
100
150
200
250
300
Distance, in Miles

Fig. 2.—East-West Cross Section at the Southern Limit of the Mississippi Embayment, Along Latitude 34° 00′ (see Fig. 1)

The geologic investigation was undertaken by H. N. Fisk of Louisiana State University at Baton Rouge as consulting geologist to the Commission, from whose data Figs. 2 to 5 have been reproduced. The borings made in connection with exploration for oil were supplemented as necessary to develop geologic cross sections of the valley; and the study of aerial photographs was followed by field exploration. The results were interesting and useful.

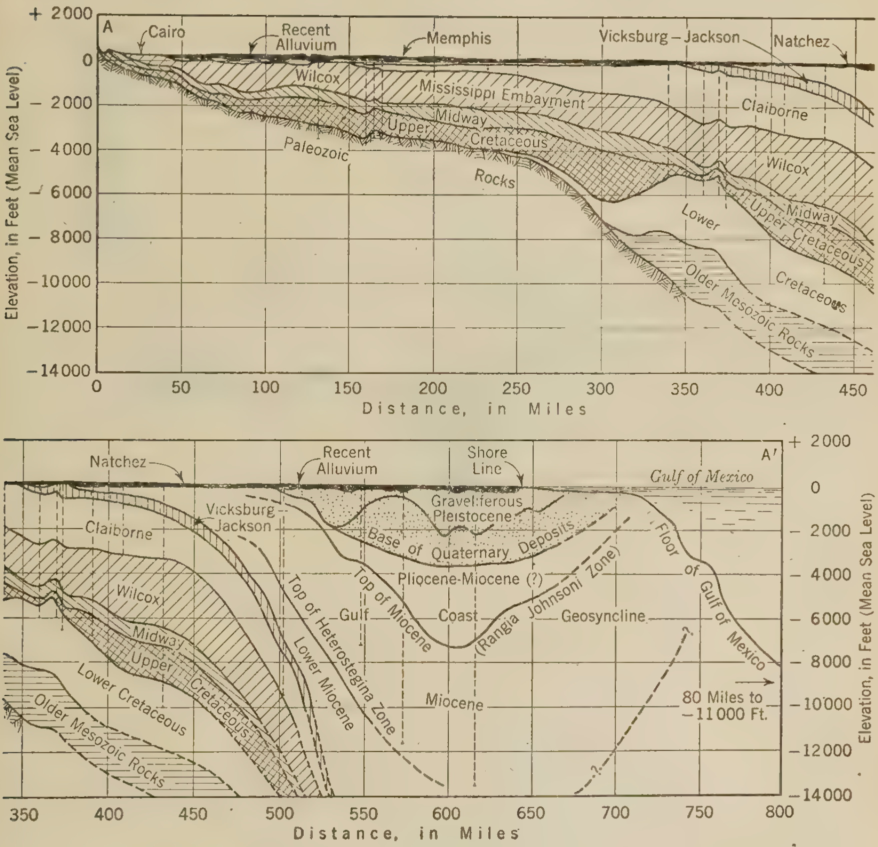
A
Cairo
Recent Alluvium
Memphis
Vicksburg – Jackson
Natchez
Wilcox
Mississippi Embayment
Claiborne
Midway
Upper
Cretaceous
Paleozoic
Rocks
Wilcox
Lower
Upper
Midway
Cretaceous
Older Mesozoic Rocks
Cretaceous
+ 2000
0
– 2000
– 4000
– 6000
– 8000
–10000
–12000
–14000
Elevation, in Feet (Mean Sea Level)
0
50
100
150
200
250
300
350
400
450
Distance, in Miles
Natchez
Recent Alluvium
Shore Line
A′
Gulf of Mexico
Vicksburg
Jackson
Claiborne
Graveliferous Pleistocene
Base of Quaternary Deposits
Floor of Gulf of Mexico
Pliocene-Miocene (?)
Wilcox
Top of Miocene
Gulf
Coast
(Rangia Johnsoni Zone)
Geosyncline
Midway
Upper
Top of Heterostegina Zone
Lower Miocene
Lower Cretaceous
Cretaceous
Older Mesozoic Rocks
Miocene
80 Miles to –11000 Ft.
?
?
+ 2000
0
– 2000
– 4000
– 6000
– 8000
–10000
–12000
–14000
Elevation, in Feet (Mean Sea Level)
350
400
450
500
550
600
650
700
750
800
Distance, in Miles

Fig. 3.—North-South Cross Section, Central Gulf Coastal Plain (see Fig. 1)

The Paleozoic floor of the lower alluvial valley apparently dips down to form a great north-south structural trough roughly following the course of the modern river and an east-west geosyncline along the Gulf Coast. Fig. 2 is a west-east cross section through the structural trough at about the latitude of Rosedale, Miss. Fig. 3 is a profile along the trough and through the geosyncline. The floor of the north-south trough is about 15,000 ft below sea level at the latitude of Natchez, Miss., 10,000 ft below sea level at Lake Providence, La., 5,000 ft at Rosedale, and 3,500 ft at Memphis. The bottom of the Gulf Coast geosyncline is about 20,000 ft below sea level. These depressions are thought to have resulted from the weight of the sediments which fill them and which were brought down by tributary streams from the Appalachians and the Ouachita-Ozark Mountains for the 180,000,000 years to 200,000,000 years comprised in the Mesozoic era and the Tertiary period of Cenozoic time. No stream of great size seems to have entered the valley from the north until the ice cap of the first glacial stage of the Pleistocene epoch interrupted northward flowing drainage and ponded it, and the drainage overflowed the divide into the lower alluvial valley, about 1,000,000 years ago. Several periods of glaciation and retreat of the ice cap followed—the former always resulting in a lowering of sea level and the deep entrenchment of streams in the alluvial plain; the latter, in rising ocean levels and the refilling of the valley and the

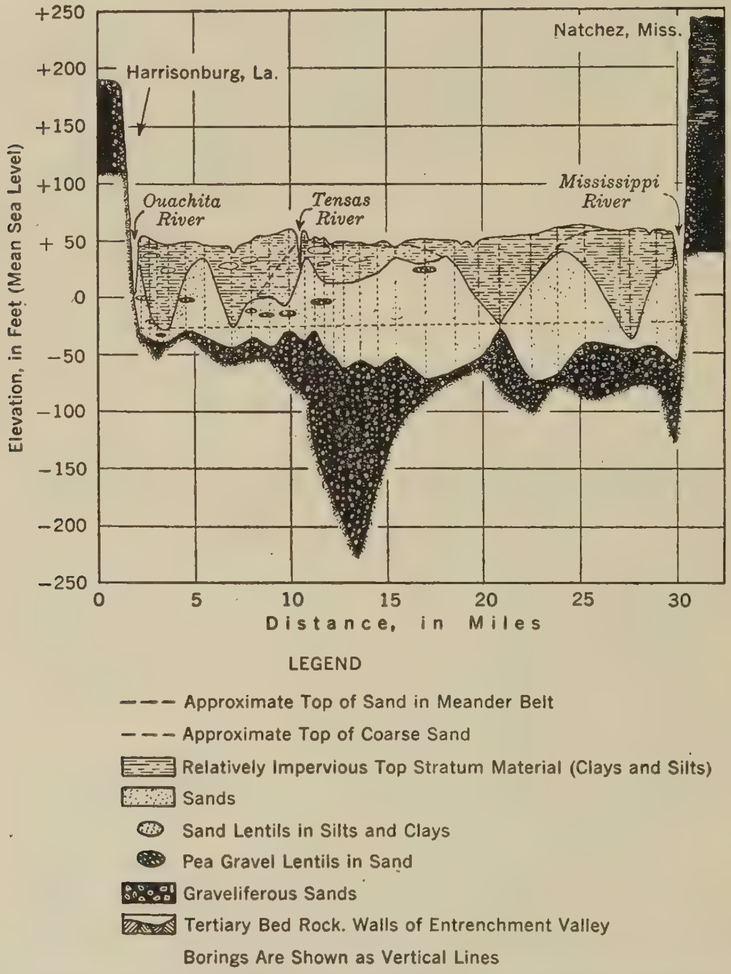
Natchez, Miss.
Harrisonburg, La.
Ouachita River
Tensas River
Mississippi River
Elevation, in Feet (Mean Sea Level)
+250
+200
+150
+100
+ 50
0
−50
−100
−150
−200
−250
0
5
10
15
20
25
30
Distance, in Miles
LEGEND
Approximate Top of Sand in Meander Belt
Approximate Top of Coarse Sand
Relatively Impervious Top Stratum Material (Clays and Silts)
Sands
Sand Lentils in Silts and Clays
Pea Gravel Lentils in Sand
Graveliferous Sands
Tertiary Bed Rock. Walls of Entrenchment Valley
Borings Are Shown as Vertical Lines

Fig. 4.—West-East Cross Section from Harrisburg, La. to Natchez, Miss., Showing the Generalized Nature of the Alluvial Filling of the Valley Cut During the Last Glacial Stage (see Fig. 1)

basins. The retreat of the last glacier (late Wisconsin stage) began about 30,000 years ago. At that time sea level was about 350 ft below present elevation, and the slope of the entrenched valley, gradually steepening as it approached the sea, was about twice that of the present valley.

The filling of the valley to approximate present levels following final retreat of the glacier consumed from about 20,000 years to 24,000 years, so the present plain is about 6,000 years to 10,000 years old. Fig. 4 is a cross section through the narrowest part of the valley—at the latitude of Natchez—extending to the Tertiary level, and it shows that the Pleistocene, and recent sediments to depths far below present river bed, consist of easily erodible deposits comprised of coarse gravels grading finer upward into sands and silts. The Mississippi River carried gravel into the Gulf of Mexico until sea level had risen to El. −200.

Fig. 5 records the history, as developed by use of aerial photographs, of a typical section of the present 2,000-year-old meander belt. The reach shown extends from the Arkansas River to Fitler, Miss. The meander belt has covered a width of from 15 miles to 20 miles. The study did not disclose any basis for believing that future behavior will differ materially from past behavior. It seems well established that, unless its meanders are artificially checked, in course of time the river will destroy everything within many miles of its present course.

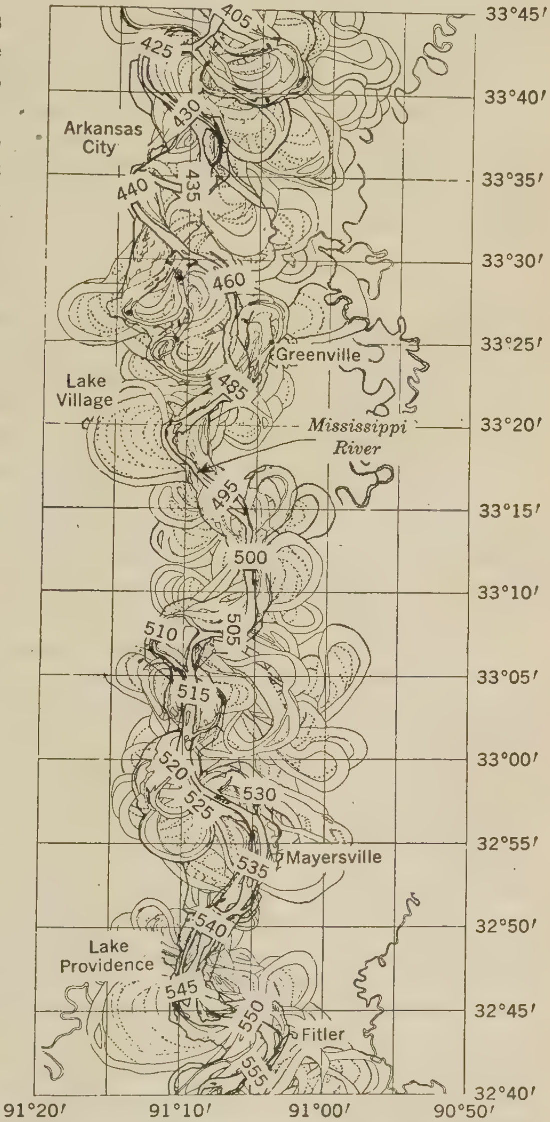
33°45′
33°40′
33°35′
33°30′
33°25′
33°20′
33°15′
33°10′
33°05′
33°00′
32°55′
32°50′
32°45′
32°40′
91°20′
91°10′
91°00′
90°50′
Arkansas City
Lake Village
Greenville
Mississippi River
Mayersville
Lake Providence
Fitler
405
425
430
440
435
460
485
495
500
505
510
515
520
525
530
535
540
545
550
555

Fig. 5.—Mississippi River Meander Belt

The study of ancient meanders has developed another interesting fact—namely, that the elevations of the banks along the river's various courses and the depths of its previous channels are practically the same as the elevations of its present banks and its present thalweg depths at correlative locations. This is convincing evidence that the river is not an aggrading stream in the ordinary sense of that term, and confirms the indication of data procured by surveys, that the Lower Mississippi River is now a "poised" stream—neither aggrading nor degrading.

A study of the length of the lower river in the sixteen reconstructed courses it has previously occupied in its present meander belt shows that it has remained 1,200 miles long with a variation of not more than 10%. Shortening has occurred when it has forsaken a part of its course to take a more advantageous route, but this has always been followed by lengthening. Thus, the river will probably regain the length it had prior to the cutoff program if it is not artificially prevented from so doing.

Model Investigation

Model investigation of the meander phenomenon proved unexpectedly slow. Much time and energy were expended in trying to find a material that would produce a stream which, like the Mississippi River, would neither aggrade nor degrade.

The model rivers behaved either better or worse than the Mississippi. They formed meanders with pools and crossing bars and point bars and became very natural-looking streams. However, those molded in granular, cohesionless materials soon began to deteriorate by widening and shoaling and lost depth and flow-carrying capacity; and those which were molded in difficultly erodible materials caved their bends slowly, showed much less tendency to build bars, and gradually deepened and entrenched themselves with increases in bankfull flow-carrying capacity. This inability to produce a poised condition, although it precluded the creation of a model stream that would behave exactly like the Mississippi River, did not interfere with the study of the general phenomenon of meander.

From observation, checked by the use of dyed sand, the movement of bed load, upon which the whole of the meander phenomenon seems principally to depend, normally appeared to be a very local movement. The load was provided by caving of the bank in the bend; and, where the bends were uniform, most of it came to rest on the convex bar immediately below—a part depositing in the crossing, and a small part moving diagonally across to deposit on the tail of the bar on the opposite bank. When the bends were not uniform some of the material eroded in the longer bends would be carried on through an adjoining short bend to deposit on the second or third convex bar beyond the point of caving.

The model rivers repeatedly showed that a meandering stream picks up the bulk of its bed load from the river bottom. If the banks are high and friable, the removal of material at their base causes them to slough off into the stream, thereby increasing the width and decreasing the depth with resultant steepening of the water-surface slope. If the stream still receives more bed load than it can handle, even with its increase in slope, the process continues until at length, if the surplus of load persists, it becomes a shallow and sometimes a "braided" stream whose low-water flow is carried in a series of small interconnected channels wandering across the surface of its sediment-filled trough.

When the banks are tough, they do not slough off so readily when load is picked up from the bed; and so, instead of being surfeited with bed load, the stream is starved and continues to remove material from the bed with consequent deepening and flattening of slope and with small tendency toward building up crossing bars between successive pools. This type of stream is illustrated in nature by the deep river below Baton Rouge where the banks and bed cutting across the clay deposits of a Pleistocene delta are fairly resistant to erosion. There as elsewhere on the Lower Mississippi River, however, a balance seems to have been struck between the forces at work, so that there is not any evidence of continuing entrenchment.

When the caving banks of the model rivers were protected by revetment, they could no longer cave into the stream; and, in consequence, the quantity of bed load was reduced. In general, the bends deepened, and the flow-carrying capacity either ceased deteriorating or positively improved—usually the latter—and sometimes to a remarkable extent, when the revetment extended throughout each bend. No good reason is apparent why revetting the banks of the Mississippi River should not reduce the bed load in that stream, nor why such reduction should not similarly affect flow capacity.

Revetment is placed in caving bends. Hence, its direct effects are upon the bends rather than upon the crossings. However, since the crossing bars are formed by the deposition of bed load, reducing the quantity of bed load by restricting its procurement to the bed of the river should tend to discourage the growth of the crossing bars, which interfere with navigation.

Most of the crossings do not offer any difficulties to 9-ft navigation under present conditions. For example, a recent typical channel report at very low water for the river between Cairo and the mouth of the Arkansas River, where most of the dredging of crossings is required, lists a total of seventy-four crossings. Of this number, five were 9 ft deep, thirty nine were 12 ft deep or more, and thirty were from 10 ft to 11½ ft deep. Where trouble does occur, it is usually due to a dissipation of the flow as a thin sheet over a wide expanse of sand bar or as several shallow divided channels across the bar. Its concentration into a single channel of moderate width would correct the deficiency. Revetment alone probably cannot accomplish this. However, if the migrating bends are revetted, so that their positions and curvature do not change from year to year, it should be practical in many cases to confine the extreme low-stage flows over the crossing bars to courses predetermined by low dikes which would not in any way interfere with flood flows but would only come into play to prevent the formation of multiple or too wide low-stage bar channels.

Because of the reduction in quantity of bed load and stabilization of channel that revetting the banks would entail, the amount of dredging now performed should insure the maintenance of a channel at least 12 ft deep after the banks have been stabilized.

SUMMARY

Concurrently with the model studies, analysis was conducted to develop the probable length of revetment required to stabilize the river upon an acceptable alinement. The study developed that, in the 737 miles between Cairo and Baton Rouge, 97½ miles of effective bank revetment were already in place, and indicated that, to stabilize the banks between the two points, about 230 additional miles would be required—representing an aggregate of 44½% of the total river length and a cost of about $165,000,000. Allowing $35,000,000 for possible improvement dredging and the construction of low dikes, the cost of the total requirement was estimated at about $200,000,000.

The whole question is succinctly summarized in the following statement from the report of the Mississippi River Commission (Document No. 509, Seventy-eighth Congress, second session):

> "The Commission is of the opinion that stabilization of the river is necessary in order to retain reduction in flood heights obtained by channel rectification, and is advisable for the purpose of safeguarding the main Mississippi River levees and protecting the investment which they represent. The Commission is of the further opinion that such stabilization may materially increase the flood carrying capacity of the river channel and together with the maintenance dredging already authorized, will provide a minimum depth of 12 feet at low water for navigation.
>
> "The great investments made by the Government and the local people in the levee system, the constantly increasing population and property values protected by the levees, and the benefits accruing to the country as a whole from the navigation of the river in both war and peace justify the further expenditures required."

DISCUSSION

CHARLES W. OKEY,[2] M. ASCE—In describing the new project for stabilizing and deepening the Mississippi River, and recording a condensed history of the development of the present flood control plans, Mr. Senour has performed a service of great value. This paper is a most valuable summary of the results of geologic investigations by the Corps of Engineers, U. S. Army, for the past several years on the Mississippi River below Cairo, Ill.

This new project is gigantic, as compared to anything tried elsewhere, both in the cost of the proposed work, and the size of the river and volume of flows involved. The field observations and the model investigations give good reason to believe that the principal objects of the project will be attained in the years to come. The principal uncertainty seems to be: What will be the progressive development of the crossing bars between the bends? If the supply of bed load from the upstream caving banks were reduced by revetment, then dredging might be reduced. As Mr. Senour states, if the migrating bends have their position and curvature fixed by revetment, low dikes would be a possible solution for a channel across those bars where the required depth is not maintained by the river itself. Without such low dikes to confine the low-water flow to a predetermined course, the elevation of the crossing bar might be reduced by less bed-load supply, but with a corresponding reduction in the elevation of the water across the bar, resulting in no substantial increase in navigable depth.

The prevention of the loss of levees on caving banks and the prevention of a gradual increase in the length of the channel, so beneficially reduced by cutoffs in the past few years, are objectives of great importance. They are much to be desired even though a great increase in the navigable depth over the crossing bars might not be automatically attained and continuously preserved at low-water stages.

The writer has been interested in this matter since March, 1911, when he read a manuscript copy of a paper by the late Maj. T. G. Dabney, for many years the chief engineer of the Yazoo-Mississippi Levee District. That paper (which was not published) had been originally written as a plea to the Mississippi River Commission to do what is now proposed by the project authorized in the 1944 Act. In reading Mr. Senour's paper, the writer's mind traveled along a familiar path which seemed to agree with Major Dabney's plea to the Mississippi River Commission. This plea of Major Dabney's if found in the records of the Mississippi River Commission or in the levee board office at Clarksdale, Miss., would constitute a most interesting commentary on Mr. Senour's paper.

F. NEWHOUSE,[3] ESQ.—The practice of building bigger and better levees to control a meandering river has proved ineffective on the Lower Mississippi, as is revealed in this valuable and interesting paper. The experiments and research described in the paper appear to have led the authorities to decide to deal with this engineering problem by adopting a method whose necessity can be deduced from basic theory.

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[2] Head Hydr. Engr., TVA, Knoxville, Tenn.

[3] Twickenham, Middlesex, England.

Any river carves its course through its own alluvial plane by the erosive power of its current. This erosive power depends on the velocity of the water, and the degree of erosion depends on the resistance of the soil. A river, therefore, achieves stability when its velocity is such that it cannot erode its channel. Velocity, of course, depends on slope. This fact leads to the basic theory that the shape of the channel of a river depends on two things only—(1) the slope and (2) the soil through which the river flows. The slope in turn depends on two quantities—(*a*) the fall from the source to the sink and (*b*) the length of the course.

For practical purposes, the fall must be accepted as fixed over any river as a whole, since it can only be changed to a significant extent by earth movements. On individual reaches the slope could be altered by building weirs and falls, but such construction would be attended by economic and engineering difficulties, probably prohibitive because of their extent on large rivers.

The other element in slope, length of course, is altered by the forces of nature in two ways: The river eats into one bank or the other forming a curve and carries the eroded material onward, in a series of repeated depositions and erosions, until it reaches the sea. Shoals are built up and the land is gradually advanced, by the deposition of the eroded material and of the silt brought down from the catchment area by normal (or abnormal) denudation. At some river mouths this formation is very conspicuous, as, for instance, at the Rosetta and Damietta mouths of the Nile River. Any map, even a small-scale map, shows the capes formed in the smooth coast line of Egypt.

The combined effects of the curves formed by erosion and of the deposition at the mouth lengthen the course of a river and therefore reduce the slope and hence the velocity. Finally, a state of equilibrium is established in the flood bed of the river, when the velocity is so reduced that it can no longer produce scour. Such a condition is easily upset by slight local variations in velocity whose effect is to set the deep channel of the river swinging from side to side. The channel then becomes a series of curves, full of shifting shoals, and tends constantly to erode one bank or the other. Everything possible has been done by nature in the way of reducing slope and velocity to fit the soil, without producing a satisfactory river channel for man's purposes. Therefore, action must be taken on the second factor that affects the course of a river—namely, the soil through which the river flows.

Nature changes soil by denudation but not in a manner designed to offer greater resistance to erosion. Excessive denudation of the catchment area causes floods to carry a heavier charge of silt, and produces sandbanks, inequalities in current, erosion, and general instability; but it does not improve a river—just the reverse! In fact, soil cannot be improved by nature, from the point of view of resistance to erosion, and the indisputable conclusion is that it must be protected by the art of the engineer—by the use of stone. Thus theory has led to the conclusion that revetment alone can stabilize the course of a river.

Cutting off a bend on a river must increase velocities locally, and start again the aforementioned erosion and other ill effects. If every cutoff is not revetted as it is built, the "Works" should be given the code name of "Operation Sisyphus."

It has been decided to revet cutoffs on the Mississippi, but the extent of the revetment of the cross section has not been stated in the paper. If the banks alone are revetted, there will be erosion of the bed and the revetment on the slopes will be underscoured, and will collapse, with consequences easy to foresee. It would be of interest to know the steps being taken to meet this danger and, eventually, how they succeed.

GERARD H. MATTHES,[4] HON. M. ASCE—The author is to be commended for the conciseness with which he has presented the historical and physical sequences of flood control and channel regulation in the Lower Mississippi Valley; also, for setting forth so clearly the stable characteristics of the river as regards its being neither an aggrading nor a degrading stream. This truly remarkable stability, despite its ceaseless meandering, has been fundamental to the success of much of the engineering work executed. Broadly viewed, it is the basic justification for the large expenditures for flood control and river-channel improvement that have been made in the past, and for the expenditures yet to be made.

Had the physical conditions in the Lower Mississippi Valley paralleled those in Mesopotamia (Iraq), its history would have been differently written. Mesopotamia, an alluvial valley of comparable size, has been, and still is, outstanding for the difficulties that beset its large irrigation and flood control works. The sediment deposited by two large rivers, the Tigris and the Euphrates, raises river beds and flood plain at a rapid rate. Great floods, repeatedly, have buried, under silt, the extensive irrigation systems, levees, and even entire cities whose ruins have been found at successive levels underground. The alluvium, furthermore, has built out into the Persian Gulf 170 miles in 5,500 years or at the average rate of 160 ft a year. In marked contrast are the extremely slow advance of the mouths of the Mississippi into the Gulf of Mexico, and the absence of bed aggrading in the 1,000 miles of Lower Mississippi River.

The river has suffered aggradation only to the extent of a few feet at its junction with the Ohio due to local conditions downstream created by the hand of man. In other places, the bed has become lowered as much as 5 ft where cutoffs or dredging operations have shortened the channel. Aside from these local changes (which resulted from channel improvement), the low-water profile has shown but little change. An interesting checkup became available in 1936 when the low-water profile of that year was compared with that of 1901, both representing extreme low-water conditions with fairly comparable discharges. It was found that the elevation above sea level, of the water surfaces at New Madrid, Mo., Memphis, Tenn., Vicksburg, Miss., and Angola, La., on these two occasions were in close agreement, differences ranging from 1 ft to almost zero. The important point brought out by this comparison was that, wherever the river had not become affected by engineering operations, its low-water profile had not been altered by natural causes over a period of 35 years.

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[4] New York N. Y.

These stable characteristics of the Lower Mississippi are referred to by the author when he terms it a "poised river." As this is an unfamiliar term as applied to rivers, a word of explanation is in order. The word "poised" was first used in 1941[5] to denote a status in which a stream shows no apparent natural tendency toward over-all raising or lowering of its bed. It is merely a convenient label for such a status in connection with the construction and maintenance of engineering works, the useful life of which commonly does not exceed half a century. The word implies no permanency, however; rather, it implies the possibility that the status may change at any time, or may be in the process of changing, as measured over the long course of geologic time. A river may be "poised" regardless of whether, concurrently, the land surface of its flood plain is being raised or eroded. Such, in substance, was the prevailing status of the Lower Mississippi up to 1932 when major channel improvements were undertaken.

The subject of the river's stability is of interest to engineers for another reason; namely, the widespread misunderstandings that are entertained concerning it. Statements have emanated from supposedly well-informed sources[6,7,8,9] to the effect that the Lower Mississippi is "forever" building up its bed and banks to higher levels, and that, consequently, the levees built for protection against overflow will have to be raised "indefinitely" as flood crests reach higher elevations. In a geological sense, the river is actually an aggrading river, because were it not for the levees it would continue to raise its flood plain by depositing silt. Deposits of this kind still take place on overflowed areas which have no levee protection, as evidenced by the gradual filling of lakes and sloughs in backwater areas. On the whole, however, the silt layers deposited by the Lower Mississippi, as described by the author, have been much thinner than commonly imagined; otherwise they would have obliterated the early channel courses of the river of 2,000 years ago.[10]

CHARLES SENOUR,[11] M. ASCE.—The writer is grateful for the several discussions of his paper and for the consideration and thoughtfulness which they exemplify.

With regard to Mr. Okey's arresting reference to a paper by the late Maj. T. G. Dabney (to whose memory a handsome granite marker has been erected at the head of the Yazoo Basin levee), it may be stated that, although this particular paper has not come to light either at Vicksburg (Miss.) or Clarksdale (Miss.), the desirability and eventual necessity for more or less complete stabilization of the banks of the lower Mississippi have been voiced by various writings of Major Dabney both prior and subsequent to 1911, and also, from time to time, by others. The very considerable sum of money and the large plant investment involved in such an undertaking, the necessity of accomplishing the work at a fairly rapid tempo, and the apparently more urgent need for other types of work have, in the past, inhibited anything beyond academic consideration of such a project. Authorization of the present undertaking was made possible largely through the efforts of Maj.-Gen. Max C. Tyler, M. ASCE, who was president of the Mississippi River Commission between 1939 and 1945.

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[5] "Basic Aspects of Stream Meanders," by Gerard H. Matthes, *Transactions*, Am. Geophysical Union, 1941, p. 632.

[6] *Compton's Pictured Encyclopedia*, 1940 Ed., Vol. 9, p. 204; and Vol. 12, p. 110.

[7] *World's Work*, May, 1913, p. 23.

[8] *National Geographic Magazine*, July, 1926, p. 118.

[9] *Harpers Monthly Magazine*, September, 1936, p. 376.

[10] Report on "Geological Investigation of the Alluvial Valley of the Mississippi River," by Harold N. Fisk, Mississippi River Comm., Vicksburg, Miss., 1945.

[11] Chf. Engr., Mississippi River Comm., Vicksburg, Miss.

With reference to the question of the direct effect of bank stabilization upon actual depths over the crossing bars without low dikes to direct low-water flow, it may be remarked that Mr. Okey's observations are borne out by the model studies made at the Waterways Experiment Station at Vicksburg. In all cases the model river bed lowered when completely stabilized. In some cases, the resultant actual low-water depths over the crossing bars increased; in others the bed and water surface were lowered in practically equal measure. It seems certain that at many points dredging must continue even after stabilization, but, it is hoped, with greatly improved results in navigable depths for equal effort.

As Mr. Newhouse states in the preamble to his interesting analysis, the necessity for bank stabilization can be deduced from basic theory. It is not believed that—for many years past, at least—engineers have cherished the hope that the construction of levees could solve the meander problem. Mr. Newhouse mentions that the shape of a river channel depends upon two factors only—the slope, and the character of the soil through which the river flows—and that slope depends only upon the fall from source to mouth and the length of the stream's course between those points. So inextricably interrelated are the factors that govern river regimen, that, although the shape of the channel does indeed depend upon slope and soil characteristics of bed and banks, in a large degree, the slope, itself (at least with respect to quite long reaches of the stream), is determined by the soil characteristics of bed and banks, tough materials apparently conducing to flat, and sandy materials to steeper, slopes.

Mr. Newhouse also stresses a very important point in his comment as to the extent of the cross section revetted upon the Mississippi River, and the possibilities of trouble inherent in revetment of the banks alone. It is true that, with only the banks protected, the river bed will generally scour at the outer edge of the revetment. On the Mississippi River the revetment is normally built to the toe of the bank's slope and beyond the toe a distance sufficient to permit it to follow down as deepening occurs. The assumption is made, for purposes of design, that the ultimate depth attained will approximate the maximum depth observed elsewhere in the general vicinity where bank recession is not in progress. The revetment is articulated and so is flexible; it has considerable strength in tension. By virtue of these characteristics it is able, in some cases, to adjust itself to a moderate amount of gradual deepening without apparent distress. In other cases, repairs are required. The process of adjustment takes place a great distance beneath the surface of the muddy stream and its mechanics are not observable. It seems probable that, when the adjustment takes place by a gradual flowing out of sand from beneath the mattress, the latter accommodates itself to the change without much damage. However, when the character of the bank is such that a considerable deepening occurs in advance of bank movement, and for this or some other reason the movement partakes of the character of a slough of sizable proportions, the outer parts of the mattress, despite its considerable strength, are ruptured and replacement of the riverward sections is indicated. Plans have been under consideration for some time for a large-scale experiment with clear water to continue to investigate the mechanics of failure, but the problem poses certain practical difficulties. The matter is also being studied from another angle—the possibility of using low abatis dikes or similar permeable current-retarding structures sunk and anchored at the toe of the revetment to prevent erosion at that immediate point and to force the locus of deepening farther channelward. Some experimental work along this line has been performed on model rivers, and it is possible that an installation will be made in the prototype during 1947.

Mr. Matthes' scholarly observations anent the contrasting characteristics of the Mississippi and Mesopotamian alluvial valleys are most interesting, as is his discussion of the implications and limitations of the term "poised" as used to connote the river's lack of apparent tendency to aggrade or degrade. As to aggradation of the flood plain, the deposition of silt from overbank flows took place for centuries along the present meander belt of the river before its floods were confined by levees which, in a geological sense, are parvenu indeed. The heavy deposits, however, consisting of the coarser sediments, took place on the immediate banks, which, accordingly, in course of time, became natural levees sloping off landward into the lowlands upon which the silt deposits were less generous. Since the river's meander was constantly caving its immediate banks, it was constantly tending to work its way into lower and lower ground. Thus, while through the medium of its flood deposits it was always tending to increase the general elevation of its banks, it was at the same time tending to decrease bank elevation by caving the highest ground into the river, thereby forcing the bank line back into lower terrain—which in turn built up and in turn caved in, and so ad infinitum. The net result of all this "give and take" appears to be a present natural levee at about the same height as those of previous meander belts. It can scarcely be doubted that the general elevation of the lowlands too remote from the stream to be attacked by it must have increased through the years, but, as stated by Mr. Matthes, the deposits, except at the immediate banks, appear to have been comparatively light and—over most of the territory—have been stopped entirely by the construction of levees. Some building of the batture (the land lying between the levee and the river) is in progress, particularly below Baton Rouge, La., where in many places the levee looks appreciably higher when viewed from the land side than it does when viewed from the river side. Hydrographic surveys, repeated at intervals over most of the period of levee building, have revealed no general tendency toward any aggradation of river bed. The necessity for upward revision of levee grades from time to time has been brought about entirely by the increasingly more complete confinement of flood flows warranted by the continuing economic development of the alluvial plain.

W. E. Elam, M. ASCE, chief engineer, Board of Mississippi Levee Commissioners, has advised by letter that the assertion that none of the levees of the lower Yazoo Levee District remains upon its original alinement is in error—that according to a map of the district made in 1867 by its then chief engineer, Minor Meriwether, he has located (in small stretches scattered here and there through the almost 180 miles of line) a little more than 27 miles that the river has overlooked. The assertion referred to has been current for some years in this vicinity. It apparently represents an erroneous conclusion drawn from a statement in the 1943 report of the levee board to the effect that the river had claimed 305 miles of its levees since 1880. The writer is at a loss to explain such crass inefficiency in a stream as generally thorough in such matters as is the Lower Mississippi, but is glad to make the correction.

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Paper No. 2300

SHIPWAYS WITH CELLULAR WALLS ON A MARL FOUNDATION

By M. M. Fitz Hugh,[1] Esq., J. S. Miller,[2] Esq.,
and Karl Terzaghi,[3] M. ASCE

Synopsis

Two full-length submerged shipways, whose walls consist of cellular cofferdams, are described in this paper. The cells of the cofferdams are filled with marl and founded on marl. The term "marl" is applied to a rather dense, greenish, calcareous sediment which has the grain-size characteristics of a fine, silty sand but, in an undistrubed state, the consistency of a stiff clay. The shipways are separated from each other by a partition wall which also consists of a cellular cofferdam. All the cells are of the diaphragm type. The hydrostatic uplift on the floors of the shipways is relieved not only by weep holes in the concrete mat but also, wherever necessary, by sand wells that drain the subsoil at a considerable depth below the floors.

The theoretical principles for the design of cellular cofferdams have been described elsewhere.[4] In connection with the design of the new shipways the local hydraulic and geologic conditions precluded an accurate forecast of the deflection of the walls and of the force that tends to lift the shipway floors. In this paper it will be shown that the risk due to such inevitable initial uncertainties can be eliminated completely by appropriate observations during construction, combined with modifications in the details of the design as soon as the required data become available.

General Layout and Soil Conditions

The general layout and two typical cross sections through the shipways are shown in Fig. 1. In a paper published in 1943, C. B. Jansen, M. ASCE,

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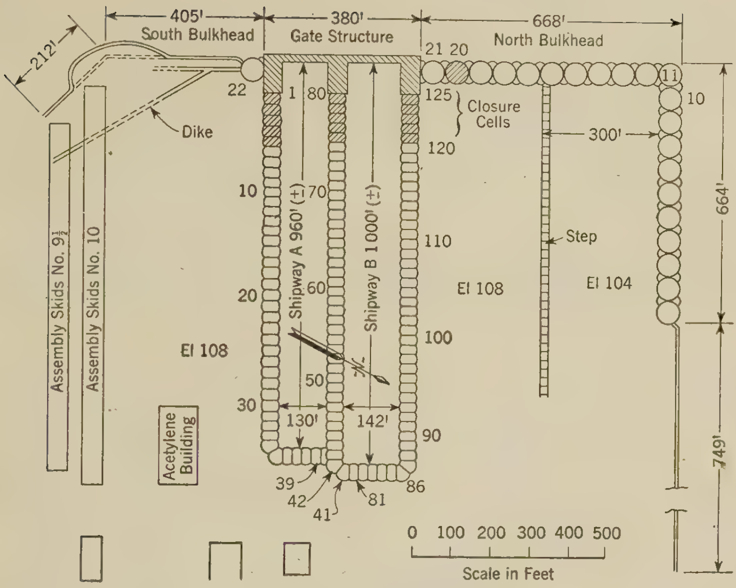
Note.—Published in November, 1945, *Proceedings*. Positions and titles given are those in effect when the paper was received for publication.

[1] Plant Engr., Newport News Dry Dock and Shipbuilding Co., Newport News, Va.

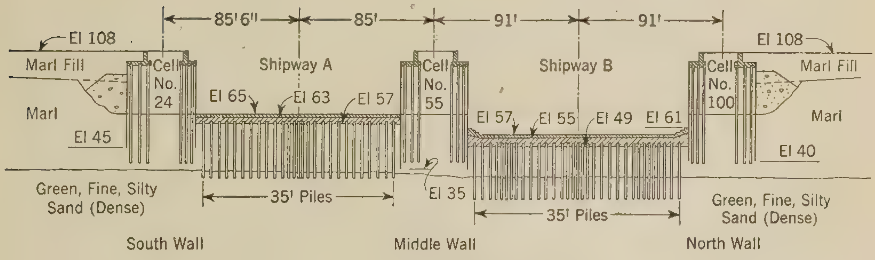
[2] Senior Vice-Pres., Dravo Corp., Pittsburgh, Pa.

[3] Cons. Engr. and Lecturer, Graduate School of Eng., Harvard Univ., Cambridge, Mass.

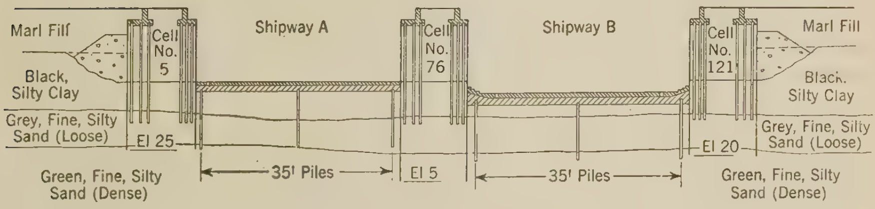
[4] "Stability and Stiffness of Cellular Cofferdams," by Karl Terzaghi, *Transactions*, ASCE, Vol. 110, 1945, p. 1112.


405'
South Bulkhead
380'
Gate Structure
668'
North Bulkhead
212'
21 20
22
Dike
1
80
125
Closure Cells
120
300'
10
11
10
Shipway A 960' (±)
70
Shipway B 1 000' (±)
110
Step
664'
Assembly Skids No. 9½
Assembly Skids No. 10
20
60
El 108
El 104
El 108
100
50
30
130'
142'
90
Acetylene Building
39
42
41
81
86
749'
0 100 200 300 400 500
Scale in Feet

(a) GENERAL LAYOUT


85'6"
85'
91'
91'
El 108
El 108
Marl Fill
Cell No. 24
Shipway A
Cell No. 55
Shipway B
Cell No. 100
Marl Fill
Marl
El 65
El 63
El 57
El 57
El 55
El 49
El 61
Marl
El 45
El 40
Green, Fine, Silty Sand (Dense)
35' Piles
El 35
35' Piles
Green, Fine, Silty Sand (Dense)
South Wall
Middle Wall
North Wall

(b) SECTION THROUGH EASTERN PART


Marl Fill
Cell No. 5
Shipway A
Cell No. 76
Shipway B
Cell No. 121
Marl Fill
Black, Silty Clay
Black, Silty Clay
Grey, Fine, Silty Sand (Loose)
El 25
El 20
Grey, Fine, Silty Sand (Loose)
35' Piles
El 5
35' Piles
Green, Fine, Silty Sand (Dense)
Green, Fine, Silty Sand (Dense)

(c) SECTION THROUGH WESTERN PART

FIG. 1.—PLAN AND TYPICAL SECTIONS OF SHIPWAYS

included a view of shipway B after completion.[5] While both shipways are empty the outside walls are acted upon by earth pressure whereas the middle wall carries no horizontal load. However, from time to time, one of the two shipways is flooded and pumped out again. Hence the middle wall must occasionally sustain full water pressure from one side. Since the crane-rail girders are located on top of the longitudinal sections of the walls it was highly desirable that the horizontal deflection of the crest of the walls after construction should not change by more than a few inches, or 1% of the height of the wall.

The soil conditions are indicated in Figs. 1(*b*) and 1(*c*). Prior to construction the site of the shipways was occupied by a shallow body of water whose depth increased from about 5 ft at the shore (eastern) side of the shipway to about 15 ft at the water front (west side). The site of the shipways and the adjoining areas were explored by eighteen 2-in. Shelby-tube borings. Before taking each sample the bottom of the hole was cleaned with a soil spoon whose diameter was slightly smaller than that of the casing. The samples were obtained in 2.5-ft sections and, in exceptional cases, in 5-ft sections. The sampler was forced into the clay by static pressure. After recovery the two ends of the Shelby tube were cleaned and sealed with a metal disk and paraffin. The samples were shipped in crates to the Soils Laboratory at Harvard University in Cambridge, Mass., where they remained in the Shelby tubes until they were tested. The routine tests included determination of the natural water content of every 6-in. section of every sample, and of the Atterberg limits and the unconfined compressive strengths of representative samples.

The tests showed that the uppermost stratum consisted of dark, silty, highly organic clay whose liquid limit ranges between 70 and 110. The natural water content is close to the liquid limit and the average unconfined compressive strength is about 0.2 kg per sq cm. North of the area occupied by the shipways the dark clay contains lenses of peat which suggest that this material was deposited during a low-water stage. The dark clay rests on a stratum of marl whose surface descends from about El. 90 at the east end of the shipways to about El. 25 or 30 at the west end. Within the area covered by the shipways the base of the marl stratum is fairly horizontal. Its elevation is between 15 and 30 ft. Hence, the thickness of the marl stratum decreases from about 60 ft or 70 ft at the east end to zero near the west end. Along its western rim the marl stratum is separated from the soft black top layer by a bed of fine, loose, silty sand. The marl rests on a thick bed of fine, dense, greenish sand with a variable admixture of silt.

The marl is a stiff, greenish soil which contains many sea-shell fragments. The grain-size curve of the marl is that of a fine silty sand. The coefficient of permeability is about 2×10^{-6} cm per sec which is that of a fine, silty sand with an effective size of about 0.02 mm. Slow triaxial compression tests furnished a value of 32° as the angle of internal friction of the marl, which is also the angle of friction for a silty sand. However, the Atterberg limits are those of a silty clay. The plastic limit is about 20 and the liquid limit varies between 30 and 40—in exceptional cases reaching 45. The natural water content ranges between 22 and 34. The apparent contradiction between the results of the mechanical analysis and the Atterberg limit tests suggests that the marl is completely flocculated. The clay and the fine silt particles are firmly united in hard clusters which behave like sand grains during the mechanical analysis. However, when the marl is remolded for the purpose of performing the Atterberg limit tests, the flocs are destroyed and, as a consequence, the character of the marl changes from that of a sand to that of a clay. This assumption is supported by the fact that remolding at an unaltered water content reduces the coefficient of permeability of the marl to about one thirtieth of its initial value. Remolding also reduces the shrinkage limit from about 20 to about 15 and it increases the compression index from about 0.05×10^{-5} to about 5×10^{-5}.

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[5] "Constructing the Shipways," by C. B. Jansen, *Civil Engineering*, July, 1943, p. 313.

DESIGN OF SHIPWAYS

The walls of the shipways were designed on the assumption that the cells were to be filled with sand and gravel. However, immediately before beginning construction, the owners and the contractor agreed to fill the cells with marl because local experience had proved that artificial fills consisting of marl gradually become very stiff and stable. Since the base of the eastern and middle part of the shipway floors is located between 35 ft and 45 ft below the original surface of the marl, all the marl required for filling the cells became available while making the excavation for the shipways. Hence, the substitution of marl for sand and gravel involved a very considerable saving. On the other hand, it introduced an element of uncertainty into the design, which had to be compensated by careful observations in the field during construction and by occasional modifications of the design in accordance with the results of the observations.

The most vital decision in connection with the design of the shipway walls was in choosing the depth of penetration of the sheet piles. In a previous paper the writer has shown[4] that a cellular cofferdam on a soil foundation is not safe unless the bearing capacity of the sheet piles, per unit of width, is at least equal to 1.5 times the friction between the sheet piles and the fill of the cells, per unit of width of the sheet piles. When the shipways were designed, early in 1940, this rule was not yet known, and in the published theories for the design of cellular cofferdams, the depth of penetration received no consideration. Nevertheless, the contractor decided to drive all the sheet piles to firm bearing, regardless of the depth at which sufficient resistance is encountered. The sheet piles for some of the cells were driven to a depth of more than 50 ft. This precaution is largely responsible for the success of the job and it discloses remarkable practical judgment. To make sure that the sheet piles were well anchored and seated the sheet piles were driven at least to the depth at which the driving of test sheet piles required about twenty blows of a single-acting steam hammer (5,000-lb ram and 15,000 ft-lb per blow) per foot of penetration. In this connection it should be mentioned that a steel sheet pile which is interlocked with an adjacent pile offers a greater resistance against driving than a single one.

The design of the shipway floor was based on the observation that the floors of old dry docks in the vicinity of the new shipway did not heave. The floor of the deepest of these docks is at El. + 65, or 35 ft below mean tide level. These floors consist of lagging spiked to the caps of pile bents driven to support the ships. The floors of these dry docks have never shown any signs of heaving, which indicates that the upward hydrostatic pressure on the base of the marl stratum at about El. 30 (Fig. 1) at the site of these floors is smaller than the weight of the marl. If the floors of the old dry docks were covered, not with lagging, but with concrete, they could not be lifted except by water accumulating between the marl and the floors. Such an accident can be prevented easily by weep holes in the concrete floor. Hence, the designer duplicated the old floors, except that he replaced the lagging by a plain concrete mat punctured by weep holes consisting of 4-in. relief pipes. Prior to pouring the underwater concrete, the pipes were driven into the marl. These pipes are spaced 12 ft on centers on lines 4 ft out from each wall, and 22 ft on centers on lines about 48 ft apart across the shipways.[6] The water that comes out of the weep holes flows into covered drains and discharges into small pump sumps near the gate structure at the western end of the shipways.

Consideration of the geological situation at the site of the shipyards indicates that the conditions determining the danger of heave at the site of the new shipways are by no means necessarily identical with those at the site of the old dry docks. First, the distance between the center line of the middle wall of the new shipways and that of the deepest of the old dry docks is about 2,350 ft. Over such a distance the elevation of the base of the marl may vary considerably. Second, for a given thickness of the marl the hydrostatic uplift on the base of the marl stratum depends on the ratio between the permeability of the fine green sand and the marl. This ratio also varies from place to place within unknown limits. Third, part of the hydrostatic uplift on the base of the marl at the site of the old floors may be carried by shearing stresses along vertical planes through the outer boundry of the floors. Since the floor of shipway A is very much wider than that of either one of the floors of the older shipways, the perimeter shearing resistance per unit of area of the floor is smaller. This difference has an important influence on the hydrostatic pressure required to produce heave. Since the factor of safety of the old floors is unknown, all these circumstances had to be considered as potential sources of danger.

In connection with shipway B, the design of the floor on the basis of local precedent involved an additional risk because, during the period from the unwatering of the shipway to the placing of concrete to the finished floor level, the surface of the concrete floor was located at El. 55, or 10 ft below the level of the old floors. This situation was without any precedent at the site of the shipway floors. To make sure that the floor would not heave while the shipway was unwatered for the first time, the piezometric pressure in the pore water was measured at different points of the base of the marl stratum beneath the floor of shipway B at different stages of unwatering. The results of the observations are used to estimate in advance the hydrostatic pressure condition for complete drawdown. If the pressure ever threatens to become excessive, it can be relieved by means of sand wells before the shipway is completely emptied.

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[6] "Constructing the Shipways," by C. B. Jansen, *Civil Engineering*, July, 1943, p. 310.

In both shipways the concrete floor is supported by 35-ft green pine piles (or longer). The tops of all the piles were notched and encased in the concrete over a length of at least 2 ft. The layout of the piles was determined by the loads to be carried by the piles while ships are being built or repaired in the shipways. Each pile is assigned a maximum load of 20 tons.

The shipways are served by cranes that travel on tracks parallel to the center line of the shipways.[7] Each crane rail rests on the crest of a concrete wall which in turn is supported by piles.

The specifications required that the space behind the north and south walls of the shipways be filled to a distance of about 300 ft from each wall up to El. 108, and beyond 300 ft to El. 104. The western face of the backfill north of shipway B is supported by a cofferdam consisting of cylindrical cells with a diameter of 51 ft. This is known as the north bulkhead (Fig. 1). This bulkhead is remarkable inasmuch as it was constructed under exceptionally unfavorable soil conditions and no attempt was made to consolidate the marl in any of the cells except the closure cell. The results of the observations on the bulkhead have been published elsewhere.[4]

INCIDENTS DURING CONSTRUCTION

The process of constructing the cofferdam and the construction equipment have been described by Mr. Jansen.[6] Within the area to be occupied by the shipway floors the marl was excavated by a heavy-duty dredging clamshell bucket operated from a derrick boat. It was either deposited directly in the cells or rehandled by barge and again by clamshell. In some instances the marl dropped out of the bucket in one big chunk, but usually it broke up into small lumps. During the first weeks after a cell was filled, the fill consisted of stiff chunks embedded in a matrix which was so soft that it offered very little resistance to the penetration of a man's finger. The sides, even of shallow test pits, had to be braced. The soil for filling the space behind the south wall was excavated by a suction dredge in a shallow borrow pit north of the shipway area and then was transported to the site through a pipe line about 2,000 ft long. In its natural state the material resembled the dark, silty clay which covered the marl at the site of the shipways. The filling operations started at the east end of the south wall. The fill consisted of clay balls with a diameter of about 4 in. embedded in a dark colored, almost liquid matrix. This material was so soft that it was not possible to walk on its surface.

During the construction of the cofferdams the deflection of the crest of the length walls (east-west walls) was ascertained about once every two weeks. While the highest part of the surface of the hydraulic fill was still 4 ft below final grade, the crest of the eastern part of the south wall adjoining the fill moved about 3 in. inward. During the following weeks, with the elevation of the top of the fill remaining unchanged, the greatest deflection of this wall increased to more than 5 in., as shown by the deflection curve a, Fig. 2. Subsequent driving of some of the piles for supporting the track for the gantry cranes south of the south wall further increased the greatest deflection to more than 7 in., as shown in curve b, Fig. 2, and the deflection continued to increase at constant horizontal pressure on the wall at a considerable rate; yet the inside of the south wall was still acted upon by full water pressure. Since the removal of this pressure would triple the overturning moment it became increasingly doubtful whether the wall would survive unwatering the adjoining shipway.

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[7] "Constructing the Shipways," by C. B. Jansen, *Civil Engineering*, July, 1943, Fig. 2, p. 311.

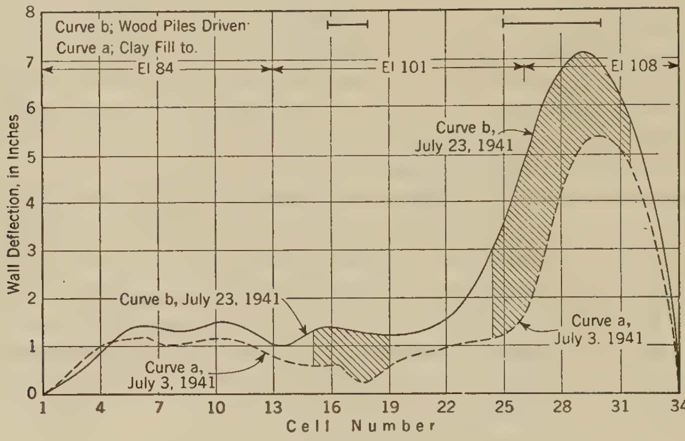

Fig. 2

The excessive deflection of the wall was obviously due to two independent causes—inadequate stiffness of the fill in the cells and the absence of internal friction in the hydraulic fill. To remedy the situation it was decided to increase the stiffness of the marl fill in the cells by compaction and drainage and to replace the hydraulic fill with gravel.

Replacement of the hydraulic fill by gravel offered an opportunity for a further reduction of the unbalanced pressure on the cofferdams by unwatering the gravel. Fig. 3, which is a vertical section through the gravel fill, shows that the gravel rests on marl and is covered with marl whose permeability is very low. After the east end of the gravel pocket also had been covered with marl, it was possible to maintain the water level in the gravel pocket adjoining the south wall, with a total length of almost 1,000 ft, at 20 ft below mean tide level by pumping not more than 0.5 gal per sec. This quantity is so small that it was later decided to divert the water from the gravel pockets, by pipes through the cellular walls, into the shipways and to pump it out of the sumps that collected the leakage from the weep holes and gate seats.

The effects of the partial replacement of the clay backfill by a gravel fill, and of draining the gravel pockets on the unbalanced horizontal pressure and on the overturning moment, are illustrated by Table 1. Line 1, Table 1, contains the values corresponding to the original conditions, and lines 2 and 3 contain those corresponding to the modified design.

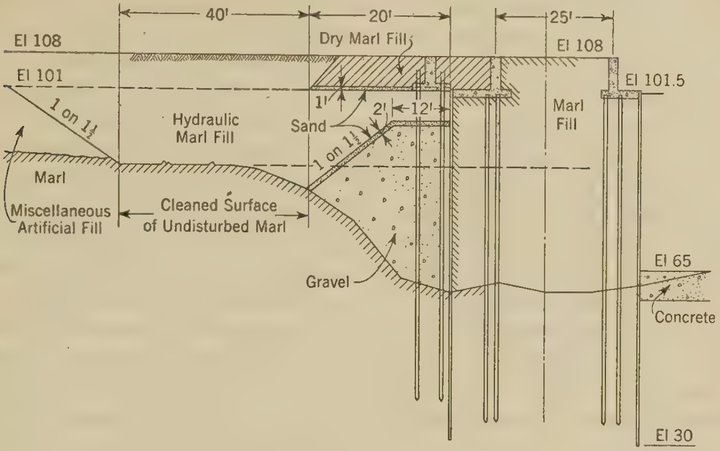
40'
20'
25'
El 108
Dry Marl Fill
El 108
El 101
El 101.5
1 on 1½
Hydraulic Marl Fill
1'
Sand
2'
12'
Marl Fill
1 on 1½
Marl
Miscellaneous Artificial Fill
Cleaned Surface of Undisturbed Marl
Gravel
El 65
Concrete
El 30

FIG. 3.—SECTION AT STATION 4 + 50 (±)

TABLE 1.—EFFECT OF DRAINAGE AND REPLACEMENT OF CLAY BACKFILL BY GRAVEL

Line	Description	HORIZONTAL PRESSURE (KIPS PER FOOT)		OVERTURNING MOMENT (KIP-FEET PER FOOT)	
		South wall	North wall	South wall	North wall
1	Wall entirely backfilled with black clay......	112	....	1,605	....
	Modified Design:				
2	Water in the gravel pocket at El. 80.......	28.6	36.1	325	441
3	Gravel pocket flooded..................	53.7	65.8	669	905

The estimate of the values contained in Table 1 was based on the following data which were determined in the laboratory:

Material	Pounds per cubic foot
Brackish water........................	63.4
Saturated dark clay backfill.............	110
Submerged dark clay backfill...........	48
Saturated marl fill.....................	120
Submerged marl fill....................	57
Moist gravel..........................	112
Submerged gravel.....................	67

The active earth pressure p_A on the wall was computed by means of Rankine's equation for cohesionless soils,

$$p_A = p \tan^2\left(45 - \frac{\phi}{2}\right) = Np \qquad \text{(1)}$$

in which: ϕ is the angle of internal friction; p is the vertical unit pressure in the soil at any depth; and $N = 1.00$ for the black clay backfill (original condition), and $N = 0.20$ for the gravel. To simplify the computation it was assumed that the top surface of the gravel pockets is located at El. 100 and not at El. 94, as shown in Fig. 3. Since the space between El. 94 and El. 100 was occupied by a marl fill subject to rapid consolidation between two horizontal, permeable layers, the error due to this simplifying assumption is unimportant. The marl backfill between El. 100 and El. 108 acts on the lower strata like a uniformly distributed surcharge of 960 lb per sq ft. The horizontal pressure exerted by the marl itself, above El. 100, has been disregarded because it is almost equal to zero. The active earth pressure of the marl in situ north of the north wall below El. 65 was disregarded because, in a natural state, the marl is very stiff.

Shipway A was unwatered in August, 1941, according to program. At that time the south wall was not yet backfilled and, as a consequence, it was acted upon by water pressure only. While the shipway was empty the deflection of the western part of the middle wall continued to increase very considerably at constant horizontal pressure on the wall. To stop the movement a temporary support was constructed as described subsequently herein.

While the water was being pumped out of shipway B, in January, 1942, the pressure-gage observations indicated exceptionally high piezometric levels at the base of the marl beneath the eastern part of the shipway. This pressure was relieved temporarily by installing shallow sand wells (lower end at about El. 20). At a later date, after the shipway had been in operation for more than a year, the provisional sand wells were replaced by permanent ones whose lower end is located at about El. 0.

Since January, 1942, observations have shown that the average movement of the crest of the middle wall due to the application of full water pressure alternately from the north and the south remains within a range of less than 3 in. which, from a practical point of view, is negligible. This result indicates that the attempt to consolidate the fill in the cells was successful.

Observations During and After Construction

The deflections of the crest of the walls were determined about once every two weeks by measuring the offset of each cell from a base line. One base line was established for every wall. Observations on the short north-south walls were begun but were soon discontinued because it became evident that the deflection of these walls is very small compared with that of the long east-west walls, for reasons discussed elsewhere.[4]

The deflection observations were supplemented by the measurement of the length of the upper edge of all the cross-walls of all the cells immediately after filling the cells with marl, after driving the wooden piles, and after driving the sand piles that were used for consolidating the fill in the cells. After shipway A was unwatered, the deformation of the inner walls of several cells of the adjoining cofferdams was surveyed carefully. The survey was repeated on a few cells of the middle wall after shipway B had also been pumped out.

Several months before shipway B was unwatered, seventeen pressure gages of the Bourdon type were installed in shipway A to obtain some information regarding the distribution of the pore-water pressure through the subsoil of the shipway; thirteen of these gages were connected with observation points located at a shallow depth below the shipway floor and four were connected with points located beneath the base of the marl stratum, at about El. 25. A short time before shipway B was unwatered, six gages were installed to measure the hydrostatic pressure at the base of the marl beneath shipway B and, in addition, four 6-in. observation wells were drilled from the crest of the north and the middle wall through the fill in the cells and the marl stratum into the fine, green sand. All the pressure gages were of the Bourdon type.

After construction was finished the deflection observations on the middle wall were repeated—(*a*) before one of the two adjoining shipways was flooded, (*b*) several times while it was flooded, and (*c*) several times after it was unwatered. The temporary pressure-gage installation was replaced by a permanent one, of thirty-one pressure gages, which furnishes fairly complete information on the pressure conditions at the base of the shipway floors. The well observations were discontinued, because the results cannot be relied upon unless the hydrostatic pressure conditions change very slowly. All the other observations will be continued until there is ample evidence that the elastic properties of the walls and the permeability of their base have become practically constant.

SAND-PILE COMPACTION OF FILL IN CELLS

Since compaction of a saturated soil requires the expulsion of pore water, the efficacy of any attempt to compact a mass of saturated soil by mechanical means in a short time depends on the permeability of the soil. One of the most expedient means of compacting saturated relatively permeable soil consists in ramming sand or gravel into it. This method has been practiced for centuries in marsh lands. If the sand is rammed into holes made by driving piles or tubes into the ground, the columns of sand thus obtained are known as sand piles. The process of ramming increases, temporarily, the pressure in the soil surrounding the seat of the successive impacts and the excess water can escape in a horizontal direction into the voids of the sand piles. By bailing or pumping from the sand piles for several weeks after their installation, simple gravity drainage is added to the effect of the drainage produced by driving the sand piles.

If the permeability of a soil is as low as that of a fine silt or clay, driving sand piles into it merely causes the soil to rise between the sand piles with almost no change in water content and the process of subsequent gravity drainage is too slow to be of practical value. A failure to consolidate a clay fill in cells by gravity drainage was experienced on the cofferdam used for raising the *Maine*. The fill consisted of sandy to heavy clay excavated by means of a hydraulic dredge. In some cells the water was pumped from 3-ft by 3-ft shafts and in others from 8-in. wells with perforated walls;[8,9] yet the effect of pumping was disappointing. In the process of drainage toward wells

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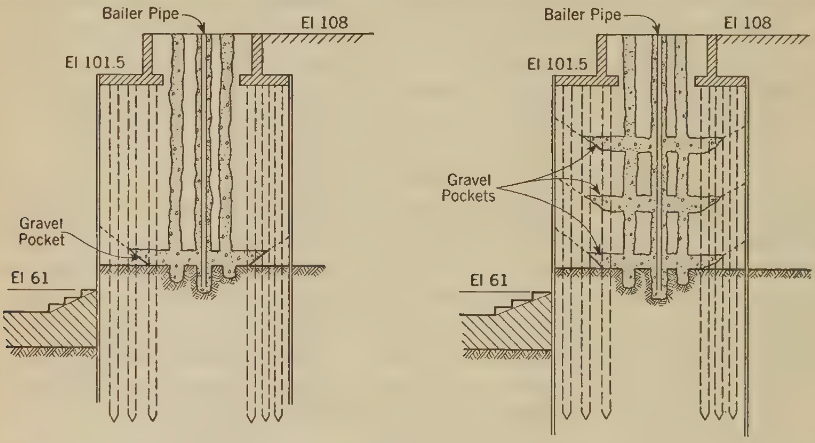
[8] *Engineering Record*, Vol. 63, 1911, pp. 548–549.

[9] Discussion by D. A. Watt of "Unusual Coffer-Dam for 1,000-Foot Pier, New York City," by Charles W. Staniford, *Transactions*, ASCE, Vol. LXXXI, December, 1917, p. 544.

the hydraulic gradient is maintained very much longer than during the process of driving sand piles into the soil. Hence, if bailing from filter wells fails to produce noticeable consolidation, it is practically certain that the driving of sand piles would also be ineffective.

The fill in the cells of the shipways consisted of stiff chunks of marl embedded in a soft matrix. The sole purpose of compaction was to weld the stiff chunks together by driving the excess water out of the matrix. Owing to the peculiar properties of the marl, the matrix seemed permeable enough to permit fairly rapid drainage. Therefore, it was decided to try the sand-pile procedure, and experience showed that the decision was justified.

Every sand pile is inevitably surrounded by a mantle of remolded marl whose permeability is very much smaller than that for the remainder of the fill. In order to establish, within the fill, a large unobstructed area of contact between fill and drain in addition to the partly obstructed area of contact with the sand piles the design illustrated in Fig. 4 was finally adopted. Fig. 4 (*a*) is a vertical section through the cells of the middle part of the north wall and Fig. 4(*b*) is a section through the closure cells at the west end of this wall. Each cell contains a central bailing well and one (Fig. 4(*a*)) or three (Fig. 4(*b*)) gravel pockets. Prior to placing the gravel for the pockets, marl was dumped into the cell along the inner and outer sheet-pile wall to intercept free communication between the gravel pockets and the open seams in the locks of the sheet-pile walls. During the filling operations the site for the bailing well was occupied by a 12-in. casing whose lower end rested on the surface of the lowest layer of gravel. After the cell had been filled with marl, a 4-in. bailer pipe was lowered into the 12-in. casing; the space between the two casings was filled with gravel; and the outer casing was pulled. The next step consisted in driving the sand piles, as described elsewhere by Mr. Jansen.[6] The driving of additional sand piles was discontinued when the total number of blows required to drive the shells for the piles (12-in. tubes with capped lower end) be-

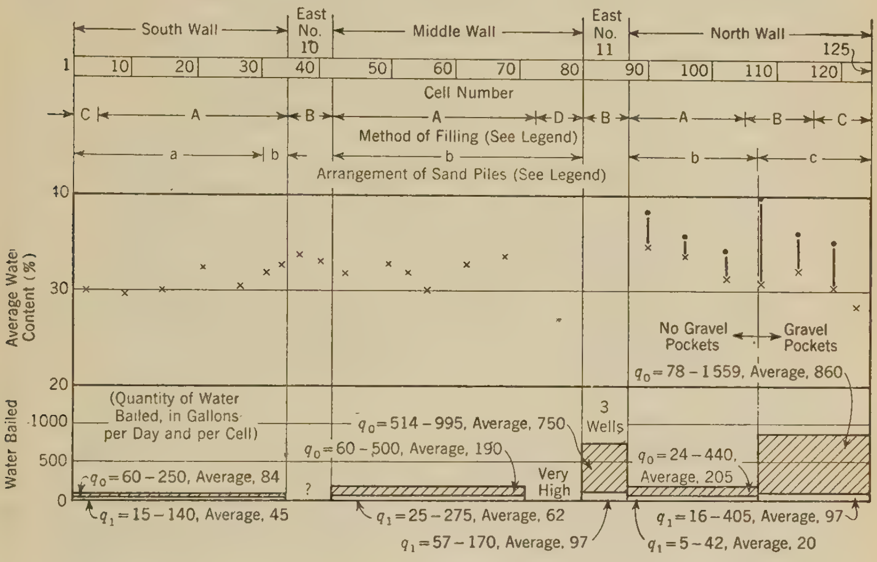


(*a*) CELLS 108 TO 121 (*b*) CLOSURE CELLS 122 TO 125

FIG. 4

came equal to 5,000. For the north wall, whose cells contained a softer fill than the other walls, 6,000 blows were specified.

Immediately after a sand pile was installed the voids in the sand were empty. However, as soon as the next pile was driven, the sand pile became saturated, and the water flowed out of the sand and flooded the surface of the fill. This phenomenon demonstrated the relatively high permeability of the matrix in the fill. After the first sand pile was installed, bailing from the well was begun. Records were kept of the quantity of water removed from each cell per day. They showed that the inflow into each well decreased at a decreasing rate. After about three weeks the inflow became constant, which indicated that the water content of the fill ceased to decrease. Hence, about ten days later bailing from this cell was discontinued. Information regarding the initial and the final rate of flow toward the wells is given at the bottom of Fig. 5.



LEGEND

A Entire Cell
B Lower Half of Cell Filled Directly from Shipway; Remainder from Borrow Pit
C Entire Fill Taken from Borrow Pits
D Random Mixture, 50% Marl + 50% Gravel
a Sand Piles Only in Outer Half of Fill
b Central Sand Pile Cluster + Single Sand Piles
c Sand Piles + Gravel Pockets as Shown in Fig. 4
• Water Content of Marl After Deposition
× Water Content of Marl After Consolidation

Fig. 5

A constant flow toward the wells can be maintained only by seepage from permanent sources of water into the cells, partly through the marl stratum which constitutes the base of the fill in the cells and partly through the locks of the sheet piles. Approximate computations based on the flow-net method indicated that the observed rate of flow and the measured hydraulic gradients can be accounted for only by assuming that the loss of head associated with the percolation of the water across the rows of sheet piles was negligible.[4]

In order to collect information on the effect of the sand piles on the water content of the marl fill in the cells, 22 Shelby-tube borings were made from the top of the dam to the base of the fill in the cells. From each hole a continuous set of 2-in. samples, 2.5 ft long, was recovered. In the cells of the north wall the borings were made in duplicate, one immediately after the cells were filled and one after bailing was discontinued. Fig. 6 shows the results of the water content and Atterberg limit determination for cell No. 108 in the north wall. Before the sand piles were installed the water content of many samples was high above the liquid limit. After consolidation the average water content was roughly equal to that of the parent material in its original state. Fig. 5 represents a digest of the results of the tests and observations.

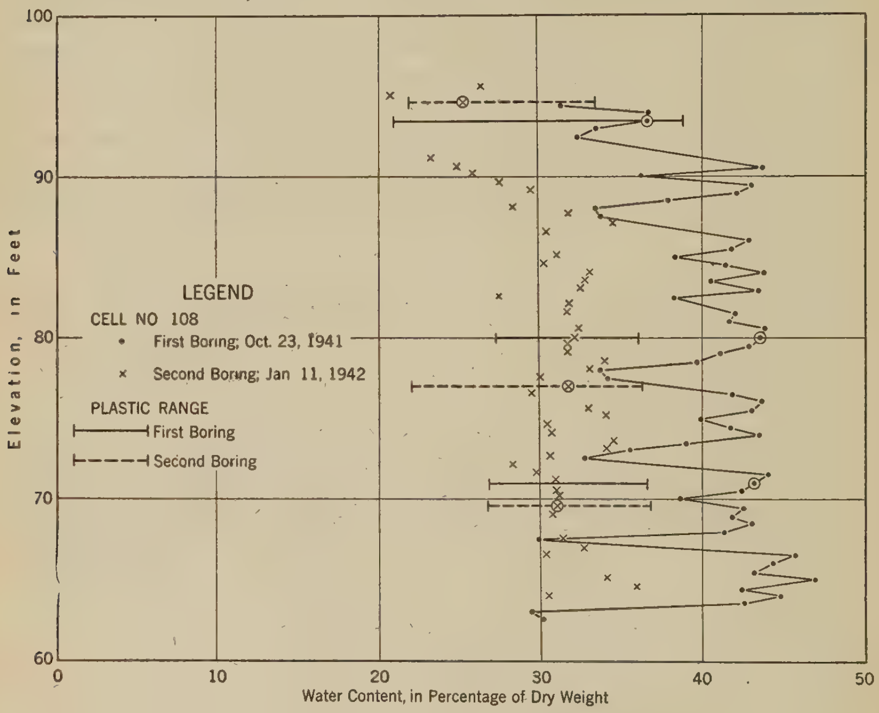


Fig. 6

After all the cells were filled and drained, test shafts were excavated in six cells of the south, middle, and north walls from El. 72 to El. 75. In all these shafts the marl appeared to be very firm and the matrix (which, at the outset, was a source of justified concern) had become as firm as the undisturbed marl.

HYDROSTATIC PRESSURE CONDITIONS BENEATH THE SHIPWAY FLOORS

Fig. 7 is a simplified vertical section through the middle wall and the adjoining shipways A and B. The shipway floors are tied to the foundation piles and the sum of the resistance of the individual piles against pulling is considerably greater than the weight of the soil between the piles. Therefore, the floors cannot be lifted without lifting, simultaneously, the major part of the body of soil between the piles. The lower ends of the piles are very thin and can readily be pulled out of the surrounding soil. Therefore, it is assumed that the potential plane of separation between the stationary mass of soil and the mass that goes up with the piles is 3 ft above the level of the points of the piles. This plane will be referred to as the "plane of separation." It is further assumed that the piezometric levels at the plane of separation are identical with those at the points at which the water pressure is being measured. These points are located at some distance above or below the plane of separation. Nevertheless the preceding assumption is justified because the observations on the shallow gages have shown that the hydraulic gradient in a vertical direction is very small.

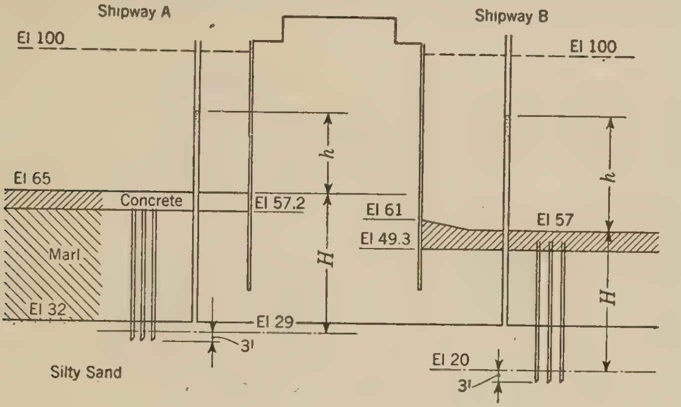


FIG. 7.—SIMPLIFIED SECTION THROUGH THE MIDDLE WALL

In the following p_t equals total submerged weight of soil, concrete, and water located between the plane of separation and the surface of the shipway floor (total weight reduced by full hydrostatic uplift), in pounds per square foot of the shipway floor; h equals hydrostatic head with reference to shipway floor (difference between the elevation of the piezometric level and the elevation of the shipway floor); H equals vertical distance between the plane of separation and the shipway floor; and γ_w equals unit weight of water. The force that tends to lift the submerged mass of soil and concrete above the plane of separation is $h\ \gamma_w$ and the force that resists this tendency is p_t per unit of area. Therefore, the factor of safety with respect to a rise of the floor is equal to the ratio

$$G_s = \frac{p_t}{h\ \gamma_w} \qquad \text{(2a)}$$

provided the shipway is empty. If the water rises in the shipway to an elevation h_1 above the floor, the resisting force remains unchanged while the uplift decreases from $h\ \gamma_w$ to $(h - h_1)\ \gamma_w$. Hence,

$$G_s = \frac{p_t}{(h - h_1)\ \gamma_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2b)$$

When computing the resisting unit force p_t it was assumed that the unit weight of the concrete is equal to 156 lb per cu ft and that of the saturated marl 110 lb per cu ft. Since it was found that the water escaping from the weep holes and sand wells is fresh, it was assumed that the unit weight of the water is equal to 62.4 lb per cu ft. The numerical values of the elevations of the boundaries between different materials are given in Fig. 7.

In shipway A the points of the piles are at El. 26, the plane of separation at El. 29, the base of the concrete at El. 57.2, and the surface of the concrete at El. 65; whence, $p_t = 28.2 \times 110 + 7.8 \times 156 - 36 \times 62.4 = 2{,}080$ lb per sq ft.

In shipway B the points of the piles are at El. 17, the plane of separation at El. 20, the base of the concrete at El. 49.3, and the surface of the concrete at El. 57; whence, $p_t = 29.3 \times 110 + 7.7 \times 156 - 37 \times 62.4 = 2{,}110$ lb per sq ft.

In Fig. 8, curve C_a represents the relation between the elevation of the piezometric level and the corresponding factor of safety G_s for the floor of the empty shipway A, and C_b represents the corresponding relation for the empty shipway B. By means of these curves one can determine, for any given piezometric elevation, and without any computation, the corresponding factor of safety G_s of the shipway floor, provided the shipway is empty. If the water stands in a shipway at an elevation h_1 above the floor, the corresponding G_s can be computed by means of Eq. 2b.

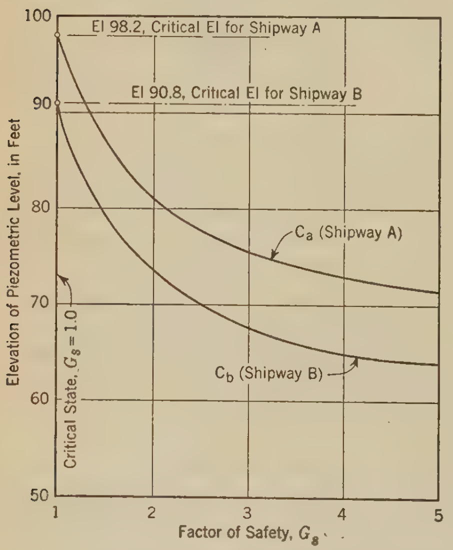
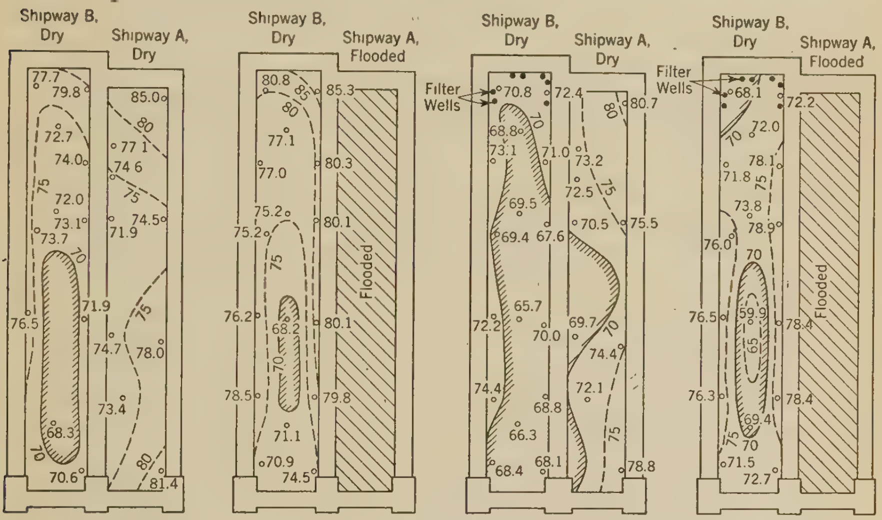


FIG. 8.—RELATION BETWEEN THE ELEVATION OF THE PIEZOMETER LEVEL AND THE FACTOR OF SAFETY

Fig. 8 shows that the danger of a heave of the floor of shipway A does not arise under any conditions of operation. However, for shipway B both the theoretical data given in Fig. 8 and the observations during the process of unwatering shipway B in January, 1942, indicated that the ultimate factor of safety with respect to the heave of the floor at the east end of shipway B would be considerably smaller than 1.5. To increase the factor of safety for this section of the floor, twenty-eight sand wells were installed during the process of unwatering in the following manner: 3-in. pipes were driven through 4-in.

openings in the eastern part of the concrete floor of shipway B and through the marl into the greenish sand to about El. 20. Then the interior of the pipe was emptied by means of an air jet and the pipe was pulled. While pulling the pipe its lower end was always kept full of coarse sand. Thus a vertical drain was formed with its lower end at about El. 20.

After shipway B was completely pumped out, the temporary system of pressure gages was replaced by a permanent one comprising thirty-one gages. The observation points were beneath the base of the marl, at about El. 25. Fig. 9(*a*) shows the curves of equal piezometric elevation, about six months after shipway B was pumped out for the first time. The location of each gage is marked by a small circle. The diagram indicates a fairly steep hydraulic gradient from the land end (east) toward the outboard end. Furthermore, the water that escaped from the weep holes was fresh. These observations show that the seepage into the shipways is ground water from the land side and not sea water from the estuary.


Shipway B, Dry
Shipway A, Dry
77.7
79.8
85.0
80
72.7
77.1
74.0
74.6
75
75
72.0
74.5
73.1
71.9
73.7
70
71.9
75
76.5
74.7
78.0
73.4
68.3
70
70.6
80
81.4
Shipway B, Dry
Shipway A, Flooded
80.8
85
85.3
80
77.1
80.3
77.0
75.2
80.1
75.2
75
Flooded
76.2
68.2
80.1
70
78.5
79.8
71.1
70.9
74.5
Shipway B, Dry
Shipway A, Dry
Filter Wells
70.8
72.4
80
80.7
70
68.8
73.1
71.0
73.2
75
72.5
69.5
70.5
75.5
69.4
67.6
65.7
72.2
70.0
69.7
70
74.4
74.4
72.1
68.8
66.3
75
68.4
68.1
78.8
Shipway B, Dry
Shipway A, Flooded
Filter Wells
68.1
72.2
70
72.0
78.1
71.8
75
73.8
78.9
76.0
70
Flooded
76.5
59.9
65
78.4
76.3
78.4
69.4
75
70
71.5
72.7

(*a*) SEPTEMBER, 1942 (*b*) APRIL, 1943 (*c*) JANUARY, 1944 (*d*) JUNE, 1944

FIG. 9.—CURVES OF EQUAL PIEZOMETRIC ELEVATION

The total leakage through the weep holes (630 gal per hr) was several times greater than the quantity computed on the assumption that the water flows toward the concrete floor only through the marl. This finding indicates that a considerable part of the leakage follows the rows of sheet piles in a vertical upward direction.

As often as a shipway is filled with water, the hydrostatic pressure at the base of the marl stratum goes up, but subsequent unwatering reduces it again to the initial value. The time lag between the change of the water level in the shipway and the corresponding change of the pore-water pressure at the base of the marl beneath the adjoining shipway is very small. It can be accounted for by a slight swelling of the marl due to the increase of the seepage pressure at unaltered total pressure during the filling of the shipway and subsequent slight reconsolidation while the shipway is again being pumped out. Fig. 10 shows the effect of flooding and unwatering shipway B on the readings of two of the gages in shipway A and on the water level in an observation well in the north wall. The abscissas represent the time; the ordinates of curve (1) indicate the water level in shipway B and those of curves (2) and (3) the piezometric levels computed from the gage readings. Curve (4) indicates the variations of the water level in the observation well. It plainly discloses the lag due to the resistance against the flow of the water into and out of the well. On account of this lag the well readings cannot be relied upon unless the hydrostatic pressure conditions change very slowly.

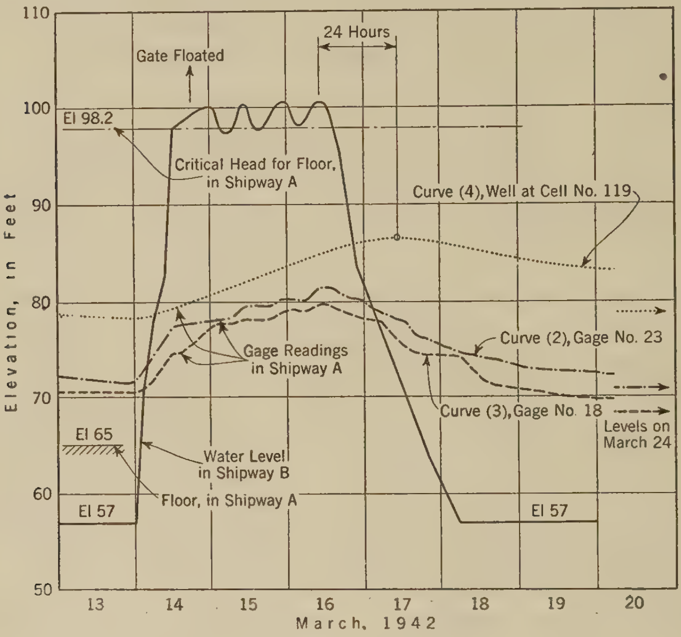


FIG. 10

Fig. 9(*b*) shows the curves of equal piezometric level at the base of the marl beneath shipway B while shipway A was flooded in April, 1943. In the vicinity of the east end of the shipway the piezometric elevation ranged between 80 and 85. According to Fig. 8 the corresponding factor of safety with respect to heave ranges between 1.4 and 1.1, which is rather close to unity. In order to increase the margin of safety, seven filter wells were installed in January, 1944, in the western end of shipway B (see Fig. 9(*c*)). Their lower ends are at about El. 0. To construct these wells, 3-in. extra-heavy casings were jacked into the ground through weep holes. The lower end of each casing was capped with a steel disk. The jack pressure required to force the capped casing to a depth of about 50 ft into the stiff soil was about 12,000 lb.

After a casing was jacked to the required depth, its interior was filled with pea gravel at a compressed air pressure of about 20 lb per sq in.; then the casing was pulled. The compressed air prevented the walls of the hole from collapsing while the pea gravel was dropping out of the casing into the hole. The steel plate that plugged the lower end of the casing during jacking remained in the ground. After each well had been installed, the discharge was measured with a water meter that was attached temporarily to the upper end of the well. Immediately after installing the well the discharge was greater than 20 gal per hr. It decreased at a decreasing rate and after about one week the discharge of every well became fairly constant and equal to about 15 gal per hr. The average discharge from the weep holes was less than 1 gal per hr per weep hole. The curves of equal piezometric elevation after the installation of all the wells, in January, 1944, are shown in Fig. 9(*c*). Although all the wells are close to the eastern end of shipway B, they reduce the piezometric elevation over the entire area covered by both shipways by about 5 ft. This was due to the fact that the wells intercepted the flow of seepage at the place where it entered the soil beneath the shipway floors. In the eastern part of the shipway, where the margin of safety with respect to a heave of the floor was too narrow, the specified safety requirements are now more than satisfied. Fig. 9(*d*) shows the curves of equal piezometric head in June, 1944, while shipway A was flooded.

By comparing Fig. 10(*d*) to 10(*b*) it can be seen that the sand wells increased the minimum factor of safety of the floor of shipway B from about 1.1 to more than 2.

DEFLECTION AND ELASTIC PROPERTIES OF THE WALLS

Fig. 11 represents the deflection of the crest of each of the three east-west walls immediately after the wall was acted upon for the first time by one-sided water pressure. The cause of the characteristic W-shape of the deflection curves has been discussed elsewhere.[4] Referring to Fig. 11(*a*), on November 20, 1941, the top of the fill was at about El. 100 and the gravel pocket was still filled with water. The excessive deflection of the eastern part of the south wall is due to the permanent deflection produced by the lateral pressure exerted by the clay backfill, prior to the removal of this fill and to the consolidation of the marl fill in the cells. The deflections that occurred after these events are represented by the ordinates of curve (*B*), Fig. 11(*a*). They are greatest near the western end where the closure cells are located. The excessive deflection of the cells near the west end of the middle wall is due to the fact that the fill in these cells rests on soft mud, whereas that in the other cells rests on stiff marl. The effect of the compressibility of the base of the fill on the deflection of cofferdams has been analyzed elsewhere.[4]

The most instructive data were obtained from observations on the middle wall which is periodically acted upon by water pressure either from the north or from the south. In Fig. 12(*a*) the plain curves indicate the relation between the unbalanced horizontal pressure and the deflection for cell No. 75, at which the deflection of the middle wall was greatest. Shipway A was pumped out in August, 1941, while shipway B was still flooded (curve ab). During the first twelve days after shipway A had been completely emptied, the deflection of cell No. 75 increased by more than 3 in., whereupon it was decided to reduce the free height of the wall in the proximity of cell No. 75 by means of a concrete sill, C in Fig. 12(*d*), with a height of 2 ft, bearing against the foot of the wall. Since the deflections continued to increase, although at a lesser rate, as shown in Fig. 12(*b*), the movement was stopped by struts which transferred the unbalanced pressure on to the floor of shipway A.

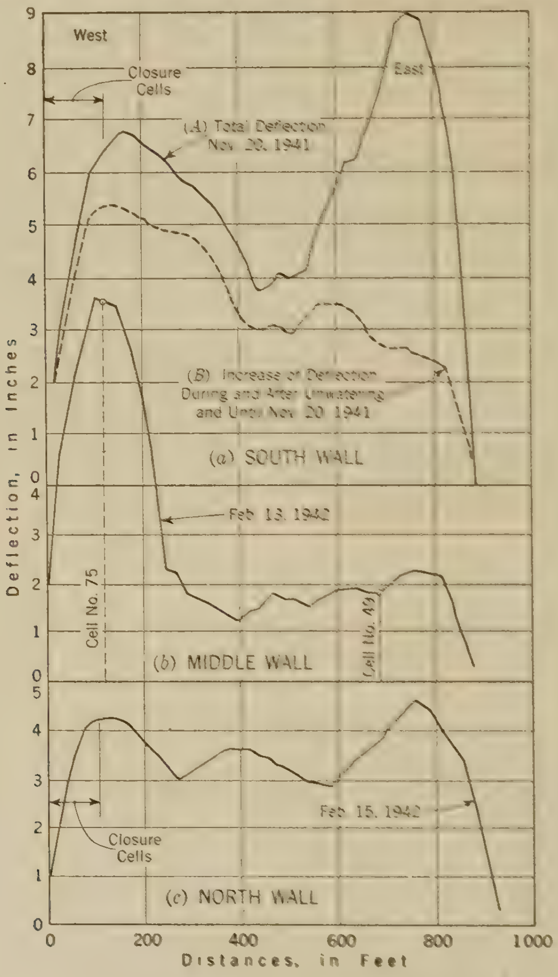


Fig. 11.—Deflections Along the Crests of Shipway Walls

The unwatering of shipway B, in January, 1942, caused the wall to move into the position marked d in Fig. 12(*a*). The line abb′c′cd represents the first cycle of loading produced by the application and subsequent removal of one-sided water pressure. This first cycle (August, 1941, to January, 1942) was followed by two others in March, one in August, and one in November, 1942. The crest movements produced by the first cycle in March and by that in

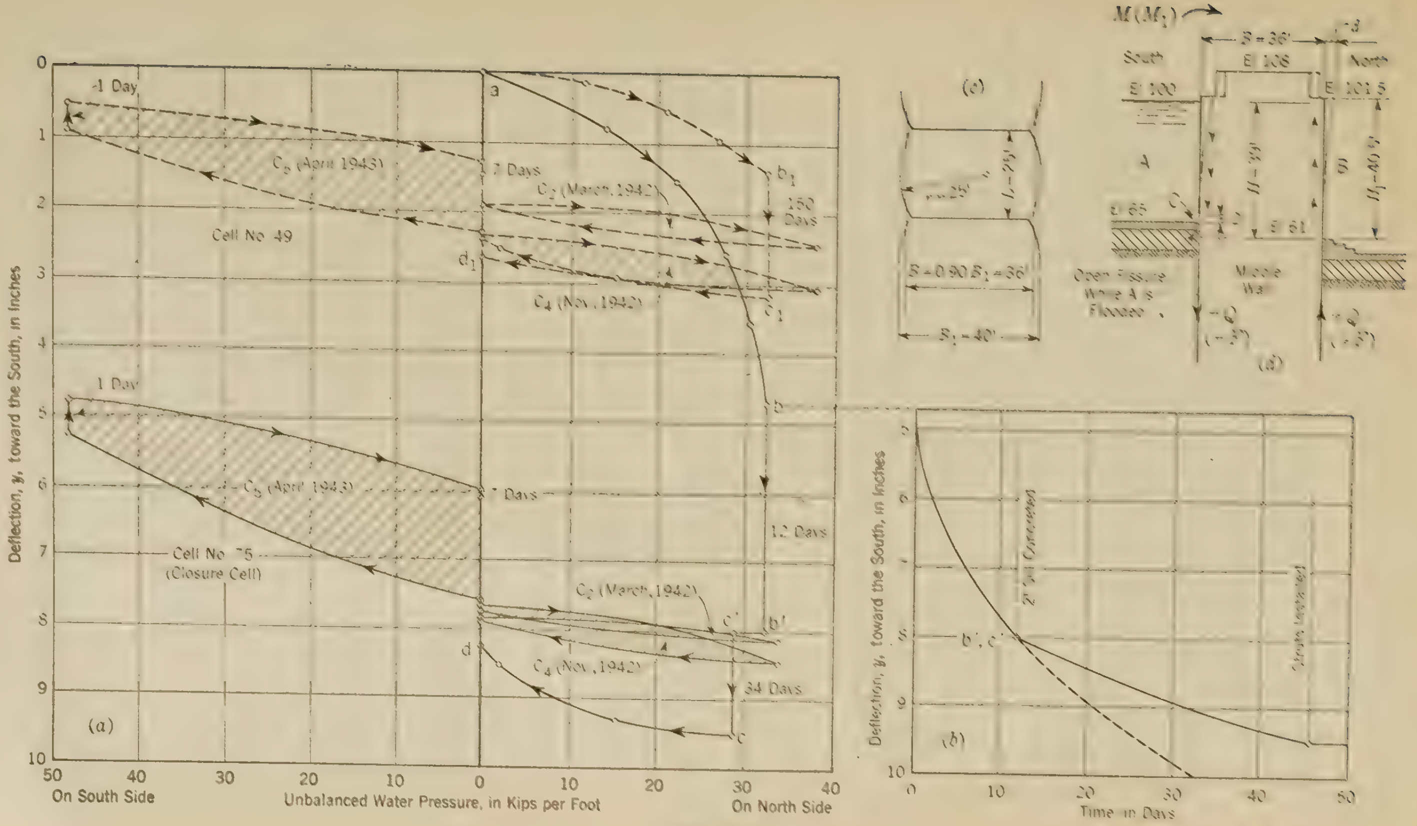

(a)
Deflection, y, toward the South, in Inches
Unbalanced Water Pressure, in Kips per Foot
50 On South Side
40
30
20
10
0
10
20
30
40 On North Side
0
1
2
3
4
5
6
7
8
9
10
-1 Day
a
b₁
7 Days
150 Days
C₅ (April 1943)
C₂ (March, 1942)
Cell No 49
d₁
c₁
C₄ (Nov., 1942)
1 Day
b
C₅ (April 1943)
7 Days
12 Days
Cell No 75
(Closure Cell)
C₂ (March, 1942)
c'
b'
d
C₄ (Nov., 1942)
34 Days
c
(b)
Deflection, y, toward the South, in Inches
Time in Days
0
10
20
30
40
50
b', c'
Struts Installed
(c)
L = 25'
H = 29'
S = 0.90 S₁ = 36'
S₁ = 40'
(d)
M(M₁)
S = 36'
South
North
E 108
E 100
E 101.5
A
B
H = 39'
H₁ = 40.5'
E 65
C
E 61
Open Fissure When A is Flooded
Middle Wall
-Q
+Q

Fig. 12

November are represented by the hysteresis loops C_2 (blank) and C_4 (shaded area), respectively. Finally, in April, 1943, shipway A was filled while B was empty (shaded area C_5).

Cell No. 75 is one of those cells of the middle wall in which the fill rests on mud. For comparison, the effect of cyclic loading on a cell with a stiff marl base (cell No. 49) is shown in Fig. 12(*a*) by dashed lines. The deflection produced by the first application of the one-sided water pressure on either cell No. 75 or cell No. 49 was much greater than that produced by any of the following applications. Similarly, the recovery of both cells due to the first removal of the horizontal pressure was much greater than that due to the following ones. These differences indicate a considerable increase of the stiffness of the middle wall between January and March, 1942. It is most likely due to a combination of several causes, including additional consolidation of the fill in the cells caused by pumping out both adjoining shipways, increase of lock friction due to rusting, and increase of stiffness of the marl fill by aging.

The data represented in Fig. 12 permit a rough evaluation of the average modulus of elasticity of the middle wall. If τ_s represents the average shearing stress at the base of the wall and θ denotes the angular deflection of a cell, the average modulus of elasticity in shear for the cell is

$$G = \frac{\tau_s}{\theta} \qquad (3)$$

According to the laws of mechanics the modulus of elasticity E for materials with a Poisson ratio of 0.25 to 0.5 ranges between 2.5 G and 3.0 G. The angular deflection of cell No. 49 due to the overturning moment $M = 630$ kip-ft per ft was 0.00195. On the basis of this value, the modulus of elasticity of the middle wall at cell No. 49 is approximately 190 tons per sq ft. The E-value for a clay with the stiffness of the marl ranges between 50 and 100 tons per sq ft. The E-value for a double-wall sheet-pile cofferdam is strictly equal to that of the fill. Hence, the average deflection of such a cofferdam would be from two to four times greater than that of a cellular cofferdam with equal dimensions. The difference is due almost exclusively to the lock friction in the cross-walls.

To supplement the results of the field observations the factor of safety G_s was computed for the middle wall, with respect to failure due to one-sided water pressure with one of the adjoining shipways flooded and the other empty. In a previous paper[4] it has been shown that a cellular cofferdam fails as soon as the shearing force on the neutral vertical plane of the dam becomes equal to the sum of the shearing resistance of the fill and of the friction in the locks which are located in this plane. The resisting moment M_s due to the lock friction can be computed on the assumption that the shearing resistance of the fill is zero. To compute the resisting moment M_f due to the shearing resistance of the clay it must be assumed that the lock friction is zero. The total resisting moment M_r per unit of length of the dam is then equal to

$$M_r = M_s + M_f \qquad (4)$$

It has been shown further that the overturning moment tends to pull the sheet piles on the water side out of the ground and to push those on the land side into the ground. At the instant of failure of the cofferdam by shear the force that acts on the sheet piles becomes equal to V_r per unit of width of the sheet piles.

The coefficient of lock friction f has not yet been reliably determined. Hence, when computing the numerical value of the factor of safety G_s, two extreme values, 0.2 and 0.3, were assigned to f. Finally, it was assumed that the fill in the cells rests on a solid base. For the eastern and the middle part of the wall this assumption is justified. However, in the western part of the wall the fill in the cells rests on a highly compressible stratum of soft clay. According to theory[4] this condition reduces the factor of safety considerably and it accounts for the excessive deflection of this part of the wall, shown in Figs. 11 and 12.

If M indicates the overturning moment produced by the one-sided water pressure and M_r the resisting moment given by Eq. 4, the factor of safety of the wall is determined by the equation

$$G_s = \frac{M_r}{M} = \frac{M_s + M_f}{M} \qquad (5)$$

Computations furnished the following results:

(a) For the case of water pressure from the north: $M_s = 418$ ($f = 0.2$) to 625 ($f = 0.3$) kip-ft per ft; $M_f = 486$ kip-ft per ft; $M = 454$ kip-ft per ft; and safety factor $G_s = 2.00$ ($f = 0.2$) to 2.45 ($f = 0.3$). On the water side of the neutral plane the sheet piles were pulled at the instant of failure with a force of $V_r = 18.4$ ($f = 0.2$) to 22.6 ($f = 0.3$) kips per ft. On the shipway side they were pressed down with the same force. The resistance of the piles against pulling or pushing was considerably greater than these values.

(b) For the case of water pressure from the south: $M_s = 516$ ($f = 0.2$) to 775 ($f = 0.3$) kip-ft per ft; $M_f = 532$ kip-ft per ft; $M = 562$ kip-ft per ft; $G_s = 1.86$ ($f = 0.2$) to 2.32 ($f = 0.3$); and $V_r = 21.2$ ($f = 0.2$) to 26.5 ($f = 0.3$) kips per ft.

At the instant of failure the shearing stresses at the boundaries of the fill would still be considerably smaller than the shearing strength of the clay. Hence, the failure would occur while the distribution of the pressure on the base of the fill is still fairly uniform. In both the loading cases (a) and (b) the overturning moment M produced by the one-sided water pressure is either slightly greater (if $f = 0.2$) or considerably smaller (if $f = 0.3$) than that part, M_s, of the total resisting moment of the wall which could be carried, if necessary, without the assistance of the shearing strength of the fill in the cells.

Observations Concerning the Sheet-Pile Walls of the Cells

The steel shell of the cells consists of straight-web, steel sheet piles with a web 1/2 in. thick. Fig. 13 is a section through a lock and Fig. 14 represents the relation between lock tension and unit elongation in the direction of the tensile force. The height of the specimen measured in the direction of the locks was 2 in. One of the ten specimens tested failed in the web, and the others failed in the finger or thumb at stresses ranging between 17,750 and 20,950 lb per in. According to Fig. 14 the failure strain is about 3.5%. Because of the small height of the specimen, the initial play in the lock was negligible. The initial play in the locks of a row of sheet piles is about 1/4 in. which increases the strain at failure in the direction of the hoop tension from 3.5% to about 5%. It should be noticed that the stress-strain curve in Fig. 14 is almost horizontal between a stress of about 5,000 and 10,000 lb per in.

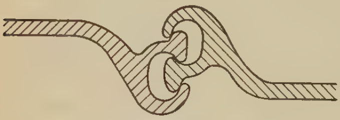


Fig. 13

The process of driving the sheet piles was described by Mr. Jansen.[6] A short time before shipway A was pumped out, it was discovered that a pair of sheet piles was driven out of groove, below El. 60, in each of four cells of the middle wall. Considering the fact that the cofferdams comprised 125 cells and that the marl was stiff, this record is very good. The damage was repaired easily by concreting the shipway floor to a distance of 20 ft from each end of each defective section in an east-west direction and to a distance of 20 ft to the north of it in advance of the remainder of the shipway floors. This operation was preceded by driving all the piles to be located within these areas.

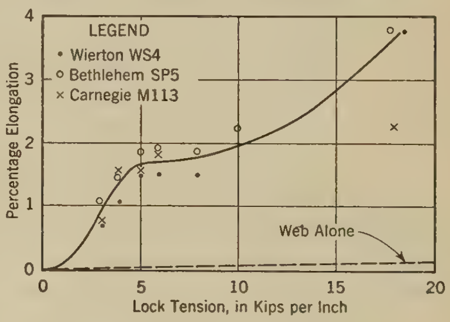


Fig. 14

To form some conception of the deformation of the steel walls of the cells, the following observations were made by the Dravo Corporation: Measurement of the length of the upper edge of the cross-walls of every cell after each operation which might change the tension in these walls; survey of the shape of two Y-piles and of the sheet piles at the middle third points of the arcs on the inside of cells No. 15 and No. 31 in the south wall, immediately after shipway A was pumped out and a few weeks later, after the piles for supporting the crane track south of the wall were driven; survey of the shape of two Y-piles, of the center sheet of the arc and of horizontal sections through the arc at El. 68 and El. 111 on cells No. 5 (south wall), Nos. 72 and 73 (middle wall), about three weeks after shipway A was pumped out and on cell No. 120 (north wall) about two weeks after unwatering shipway B; survey of the entire shape of cell No. 53 (middle wall), two weeks after shipway B was pumped out; and survey of the south face of cells Nos. 113, 116, and 124 of the north wall and of both faces of the cell No. 75 (middle wall), four months after shipway B was pumped out.

Table 2 contains the average elongation of the upper edge of the cross-walls, as well as the range of the extreme values. The table shows that the driving of wood and sand piles nearly doubled the elongation and increased the elongation of most of the cross-walls to more than 2%. It should be noticed that the driving of sand piles increased the elongation much less than the driving of

TABLE 2.—ELONGATION OF THE UPPER EDGE OF THE CROSS-WALLS

Cause of elongation	CELLS NOS. 1 TO 34		CELLS NOS. 41 TO 80		CELLS NOS. 86 TO 125	
	Average	Range	Average	Range	Average	Range
Filling	3.90	6.0 to 1.3	4.24	7.5 to 0.75	4.32	6.0 to 1.1
Driving wooden piles	0.67	4.1 to 1.5	2.41	7.5 to −1.4	3.38	6.0 to −0.75
Time effect	1.24	2.1 to 0.4	2.36	7.5 to 0	0.63	1.5 to −0.6
Driving sand piles	0.60[a]	....	0.60	1.6 to 0	0.63	1.5 to −0.6
Total:						
Inches	6.41	....	9.61	....	8.96	....
Percentages	1.9	....	2.42	....	2.26	....

[a] Estimated

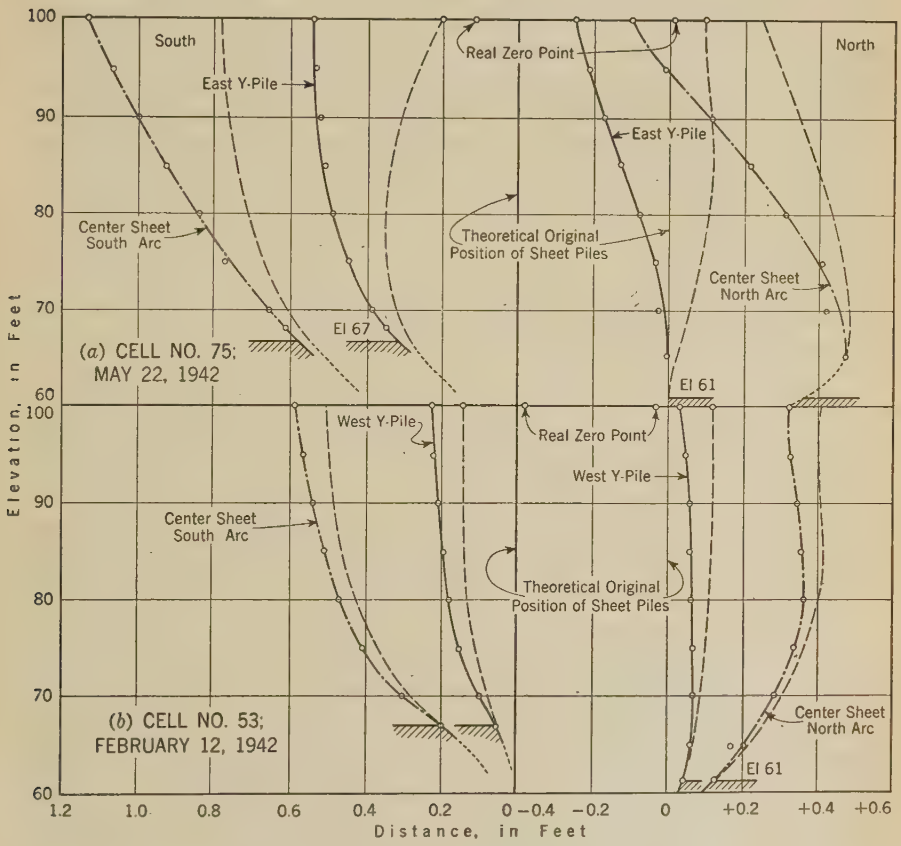


FIG. 15.—CHANGE IN THE SHAPE OF THE CELLS OF THE MIDDLE WALL CAUSED BY AN UNBALANCED HORIZONTAL PRESSURE

wood piles; yet the volume displaced by the sand piles is greater than that displaced by the wood piles. These facts suggest that the driving of wood piles increased the lock tension to a value greater than 5,000 lb per in. at which the stress-elongation curve in Fig. 14 flattens out. They also suggest that the lock tension prior to driving the wood piles was appreciably smaller than 5,000 lb per in.

The change of the shape of the cells caused by an unbalanced horizontal pressure is illustrated by Fig. 15. In each case the solid line indicates the shape and position of the outer edges of one of the cross-walls and the dash-dot line indicates the shape and position of the center sheet of the arcs, about four months after the unbalanced water pressure on the middle wall was removed by unwatering shipway B. Since the elastic recovery (curves cd and c_1d_1 in Fig. 12 (*a*)) was slight compared with the preceding deflection, the shape of the cells is almost identical to that of the cells prior to the removal of the unbalanced pressure. The position of the sheet piles was measured by offsets from the center line of the adjoining shipways. The surveyed points are indicated by small circles. It should be noted how close the real original position of the edges of the cross-walls (real zero point) is to that required by the construction drawings (origin of the abscissa). The thin dashed lines indicate the estimated position of the sheet piles prior to the application of the unbalanced pressure.

Over one half of the height of each cell between El. 80 and El. 100, the center sheet piles of the two arcs are almost parallel to each other. Hence, from the top of the cells down to about midheight, the lock tension was about the same everywhere; and below midheight, it decreased to zero at the base of the cells. This condition indicates that the hoop tension also decreased below midheight.

Fig. 16 is a horizontal section through the south arc of cell No. 124 at El. 65 surveyed after shipway B had been pumped out. Originally the arc had a radius r of about 24.4 ft. At the time of the survey the radius of curvature decreased from about 0.92 r at the crown to about 0.68 r on both sides. Since the tension at a given elevation is the same in every lock, the decrease of the radius of curvature from the crown toward the hinges indicates an increase of the lateral unit soil pressure in the same directions. This condition was typical for most of the cells, but in some of them the radius was smallest in the proximity of the crown. Hence, the distribution of the soil pressure over the arcs scatters very considerably from the statistical average. The same conclusion applies to the shape of the cross-walls. Since the average strain in the sheet pile depends to a considerable extent on the initial play in the locks, which is unknown, reliable information concerning the lock tension can be obtained only by means of strain-gage measurements on the web. The provisions for such measurements must be made before the sheet piles are driven. On account of imperfect fit in the locks, only the averages of readings on a great number of pairs of reference points can be relied upon. A practicable method for measuring the lateral pressure in the fill of the cells has been suggested elsewhere.[10]

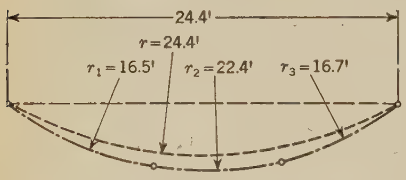


Fig. 16.—South Arc, Cell No. 124; El. 65; May 23, 1942

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[10] *Transactions*, ASCE, Vol. 110, 1945, p. 1187.

SUMMARY

1. Comparison between the cost of the shipways and the cost of other recently constructed dry docks showed that the design described in this paper was remarkably economical. Savings were achieved by providing the shipways with walls of the cellular type and by taking advantage of local soil conditions.

2. On account of the radical departure from conventional methods the design was inevitably based on various more or less uncertain assumptions. However, by systematic field observations during construction it was possible to correct the initial errors and to modify the design during construction in accordance with the findings. This procedure avoided the risk of a failure in spite of the novelty of the design, and the cost of the modifications did not exceed a small fraction of the total savings.

3. The observations during construction showed that the deflection of the walls was greater than the designers had anticipated. This condition was remedied by two independent measures. The stiffness of the fill in the cells was increased effectively by compaction and drainage, accomplished by means of sand piles. The unbalanced horizontal pressure on the outside walls was reduced by replacing the clay backfill by a drained gravel backfill.

4. The deflection observations confirmed the theoretical conclusion that the deflection of long cofferdams with fixed ends, resting on a firm base, should be greatest at a short distance from each end. They also showed, in accordance with theory, that the presence of a highly compressible layer beneath the fill in the cells increases the deflection very considerably, everything else being equal.

5. The relation between the unbalanced horizontal pressure on a wall and the corresponding deflection was found to be very similar to the relation between shearing stress and shearing strain for cohesive soils. The deflection increases more rapidly than the horizontal pressure, and at constant pressure the deflection increases at a decreasing rate. However, by trial computation it was found that the sheet-pile enclosure contributes very materially to the stiffness of the entire structure.

6. The factor of safety of the middle wall with respect to a failure by shear on a vertical plane was estimated by assuming that the coefficient of lock friction had a value between 0.2 and 0.3. On this assumption it was found that the factor of safety of that part of the wall which rests on solid marl ranged between 1.8 and 2.4. It was also found that the overturning moment had almost no effect on the distribution of the pressure on the base of the wall. This distribution was practically uniform. For the western part of the wall the factor of safety was considerably smaller than the foregoing values because this part of the wall rests on a very compressible base.

7. The results of a survey of the shape of the cells, after filling them with marl, showed that the hoop tension in the sheet-pile enclosures has almost the same value at any elevation within the upper two thirds of the height of the cells. Within the lower third it decreases in a downward direction to almost zero at the level of the base. Therefore, the unit elongation of the upper edge

of the cross-walls is practically equal to the maximum horizontal unit elongation of the sheet-pile enclosure of the cells.

8. While bearing piles and sand piles are being driven into the fill in the cells the lock tension is increased by an amount that cannot be predicted reliably. To make sure that the lock tension remains within safe limits the stretch in the upper edge of the cross-walls was measured repeatedly while driving the piles. The results of the measurements combined with those of previous tension tests with short sheet-pile sections indicated that the lock tension never exceeded 10 kips per in.

9. The design of the shipway floors was based on the assumption that the flow of the ground water toward the weep holes reduced the hydrostatic uplift on the base of the marl stratum which contained the piles; but the amount of reduction was not known. To observe whether the floors were safe with respect to heave, gages were installed which registered the water pressure at about the level of the points of the piles.

10. While the shipways were pumped out for the first time, gage readings were made at regular intervals. The readings showed that the safety of the floors with respect to heave will be adequate at any operating stage of the shipway, except at the eastern end of the floor of shipway B. By installing seven 3-in. filter wells beneath that part of the floor the piezometric head was reduced over the entire area covered by the shipways by about 5 ft. This local remedy served its purpose and it was much more economical than the customary method of preventing a heave by increasing the thickness of the floors.

ACKNOWLEDGMENTS

The submerged shipways described in this paper were a project of the Bureau of Ships, U. S. Navy, under the general supervision of O. L. Cox, Admiral, U. S. Navy, and appreciation is expressed to the Navy Department for permission to publish the data contained in the paper.

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Paper No. 2301

BALANCED DESIGN IN URBAN TRIANGULATION

By Charles D. Hopkins,[1] Esq.

With Discussion by Messrs. Walter S. Dix, Henry W. Hemple, L. G. Simmons, C. A. Whitten, Ronald F. Scott, and Charles D. Hopkins.

Synopsis

It seems hardly necessary to stress the desirability for selective design since this first step in erecting engineering structures is decisive. Of course, the design, the mathematical processes, and the actual construction must be given equal weight if the operation is to be successful, consistent with the greatest degree of accuracy within the limits of justifiable expenditure. Granted the establishment of adequate mathematics and the development of competent physical implements (made available in the field of geodetics by the efforts of many practitioners through the years), it seems timely to make an effort to discuss the too-little-mentioned subject of design.

This paper, placing primary emphasis on the subject of triangulation design, nevertheless contains several statements regarding the mathematical processes. These statements are included solely to suggest some of the provocative ideas that were encountered during the course of the study, in the hope that further and more searching discussion may be stimulated.

Actual field results of applying the theories presented herein were so favorable as to be almost beyond understanding. In its beginnings, the idea of "balanced design in urban triangulation" originated from curiosity concerning the striking similarity in characteristics (of which there are many) of bridge and roof trusses and triangulation networks. For example, in the English truss a particular panel is used repeatedly, whose counterpart in triangulation is the component figure; but, because of physical limitations in the field, it would rarely be possible to erect a network of urban triangulation using only a single type of figure, such as the completed quadrilateral. Therefore, for very practical reasons, other types of figures are used which should be as closely related as possible.

Note.—Published in November, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Geodetic Engr., Office, Chf. of Engrs., U. S. Army, Washington, D. C.

Chronologically, developments occurred in reverse order. The list of figure types discussed in Part 2 and designs for the field work mentioned in Part 3 were under way in 1937. In the interim, the field and final results (presented subsequently in Table 2) were being made available. As the results accumulated, the comparisons became apparent, and were deemed sufficiently unusual to warrant an attempt to find reasonable explanation for cause.

INTRODUCTION

An accurate triangulation scheme is the prime necessity as a foundation for the erection of a traverse structure to control property dimensions. This is particularly true in establishing geodetic control systems in urban areas since high property values prevalent therein are a most significant factor in determining the level of accuracy to be attained.

The desirable accuracy of horizontal urban control has been stated as not less than 1 part in 15,000 at the remote extensions of the traverse from triangulation.[2] This ratio of accuracy is based on the measurable precision within which a building can be constructed with relation to a given property line.

The control system, comparable to a local public utility to serve the needs of an area, should be capable of producing the required accuracy at any point within the jurisdictional limits where it may be demanded. Serious and unavoidable losses in accuracy must be anticipated in extending the control, from the triangulation through various phases of traverse, to define every property block in the area. It is desirable, therefore, that the basic triangulation be as strong as practical considerations will permit. Inherent weakness in the triangulation can cause almost insuperable difficulties, if the unpredictable distribution of discrepancies, in triangulation and traverse observations, is unfavorable and causes unacceptable localization of error. Localized error that cannot be corrected properly can rightly be called structural failure.

The necessity for well-designed figures at measured precision bases was recognized early, probably due to the fact that base lines were relatively short and the length element was undergoing rapid expansion to a longer average line of sight. The component figures of arcs of triangulation between measured bases, however, were not always so well designed, and in some cases simple triangles, singly or in groups, are found interspersed with much stronger geometrical figures. Similarly, local-area nets are sometimes characterized by mixed design. Absence of balanced design can produce only unpredictable results. For example, the triangulation nets in two adjacent cities, upon completion in the field, appeared equally acceptable; yet the final indexes of accuracy from the least-squares adjustments of each had little in common. One result was judged to be extremely good and the other very poor. Actually, both results were erratic accidents because selective predesign of the structures was not a controlling factor and consistency in design was lacking in both networks.

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[2] "Technical Procedure for City Surveys," *Manual of Engineering Practice No. 10*, ASCE, 1934, pp. 74 and 106.

By nature, the triangulation structure is difficult to analyze, because it is an indeterminate structure, and, therefore, has no rigid explanation. It is an engineering structure, subject in general to the same physical laws, but differing in that, visibly, its physical make-up can be appreciated only on a detailed layout map. The scattered monuments that perpetuate the station locations on the ground do not afford any understanding of the scheme. It is possible, however, to identify certain characteristics of the structure by a study of the pattern of the distribution of discrepancies resulting from observations, and to arrive at rational conclusions that should be helpful.

PART 1. EFFECTS OF CHANGES IN DESIGN

Obviously, any study of triangulation design based on actual arcs or nets would require enormous labor, and since many factors would modify final conclusions this idea was abandoned in planning the present study. By selecting types of actual component figures for individual examination, the outlook was made more promising, but even then certain difficulties remained, principally the lack of a reference base for common comparison. Although the absolute values of angles in actual triangulation can never be humanly known, discrepancies in observations can be distributed in the most consistent manner by a least-squares adjustment. Furthermore, changes in design would dictate the pattern of distribution. With these ideas in mind, the decision was reached to study the characteristics of design through comparison of the results from least-squares adjustments of ideal figures, whose absolutes are known.

For this purpose, four types of figures were selected—a central-point quadrilateral, a pentagon with and without central point, and a central-point hexagon. Using the perfect form for each figure wherein the absolute values of the angles are known, assumed discrepancies were applied to each angle. The assumed discrepancies were restricted in magnitude and range, and were uniformly applied—in tenths of seconds, 0.1 to 0.3, plus and minus successively, clockwise at each station and counterclockwise around the figure. By removing successive diagonal lines from the figure, the design could be changed by stages from saturation (that is, all diagonals being included) to the point where the figure becomes merely a combination of triangles. Thus, by making a series of adjustments for each stage of design, the various distributions of discrepancies could be tabulated and examined. In all, more than one hundred separate least-squares adjustments of the ideal figures were completed, and the results tabulated and graphed.

In the four types, the distribution of assumed discrepancies was made as similar as possible so that the final adjusted values would be closely comparable. Certain advantages are found in using the hexagon, however, since it covers wider range in design development and the results permit broader study and understanding. Therefore, the central-point hexagon is selected for discussion (see Fig. 1).

In making the series of least-squares adjustments for the hexagon (seven in all), the first involved the central-point figure without diagonals, and then the figure with the diagonals, included one by one in their numbered order.

Without diagonals the figure consists of six triangles in which the exterior lines lack double determination. With the inclusion of diagonal 1 (numbers denoting diagonal lines are circled), the figure consists of a quadrilateral (in which all lines have double determination; that is, all lines are sides of at least two triangles) and four simple triangles. Diagonal 2 produces two quadrilaterals and two simple triangles; but, with the inclusion of diagonal 3, every line in the entire figure has reached double determination. In other words—the design-development stage being indicated by the number of included diagonals—at 0 the design is that of simple triangles, at 1 and 2 the design is mixed, and at 3 the design has reached the initial state of complete double determination. At 4 and 5 the design is classified as double determination increasingly compounded, and at 6 the ultimate of design is reached where double determination is completely compounded. It is pointed out that the inclusion of diagonal groups either 1, 2, and 3, or 4, 5, and 6 would produce double determination.

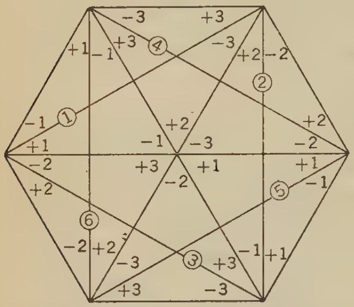

Fig. 1.—Hexagonal Figure, with the Pattern of Discrepancies given in Tenths of a Second

The assumed discrepancies applied to the angles of the hexagon (Fig. 1), and the pattern and regularity of their distribution, produce a series of triangle closures that are of consequence to the study.

In any figure or combination of figures, there is a total in closures that is a constant. Referring to Fig. 1, it will be noted that the discrepancies at the perimetric angles within each of the radial triangles counteract each other; therefore, the triangle closure in each radial triangle will be identical to the discrepancy at its central-point angle. The algebraic sum of the central-point-angle discrepancies is zero, and thus the algebraic total-triangle closure in the entire figure is also zero. This total-triangle closure for the entire figure will remain constant regardless of the number of diagonals that may be inserted. The insertion of diagonals serves to divide and subdivide the discrepancies in the angles, and forms and reforms the resultant triangle closures. Likewise, the complementary side equations state and restate the proportional sine-to-side discrepancies. The total closure to be reconciled in each of the seven least-squares adjustments, therefore, remains a constant throughout the seven stages of design, upon which can be based a study of the mechanical changes produced in the series of adjustments, as evidenced by variations in the distribution of discrepancies.

After completing the series of adjustments, the corrected directions were tabulated. It then was possible, since the ideal hexagon is a perfect geometrical figure, to ascertain how efficiently the assumed discrepancies were distributed and what degree of approach to the absolute was attained. Therefore, another series of terms was tabulated—the absolute directions minus the corrected directions—which are designated Δ. The Δ-tabulations, without regard to sign, were then plotted, as shown in Fig. 2. On these graphs, the vertical section shows the increase in magnitude; and the horizontal section, the occurrence of Δ-terms in equal magnitude, to the total number of terms

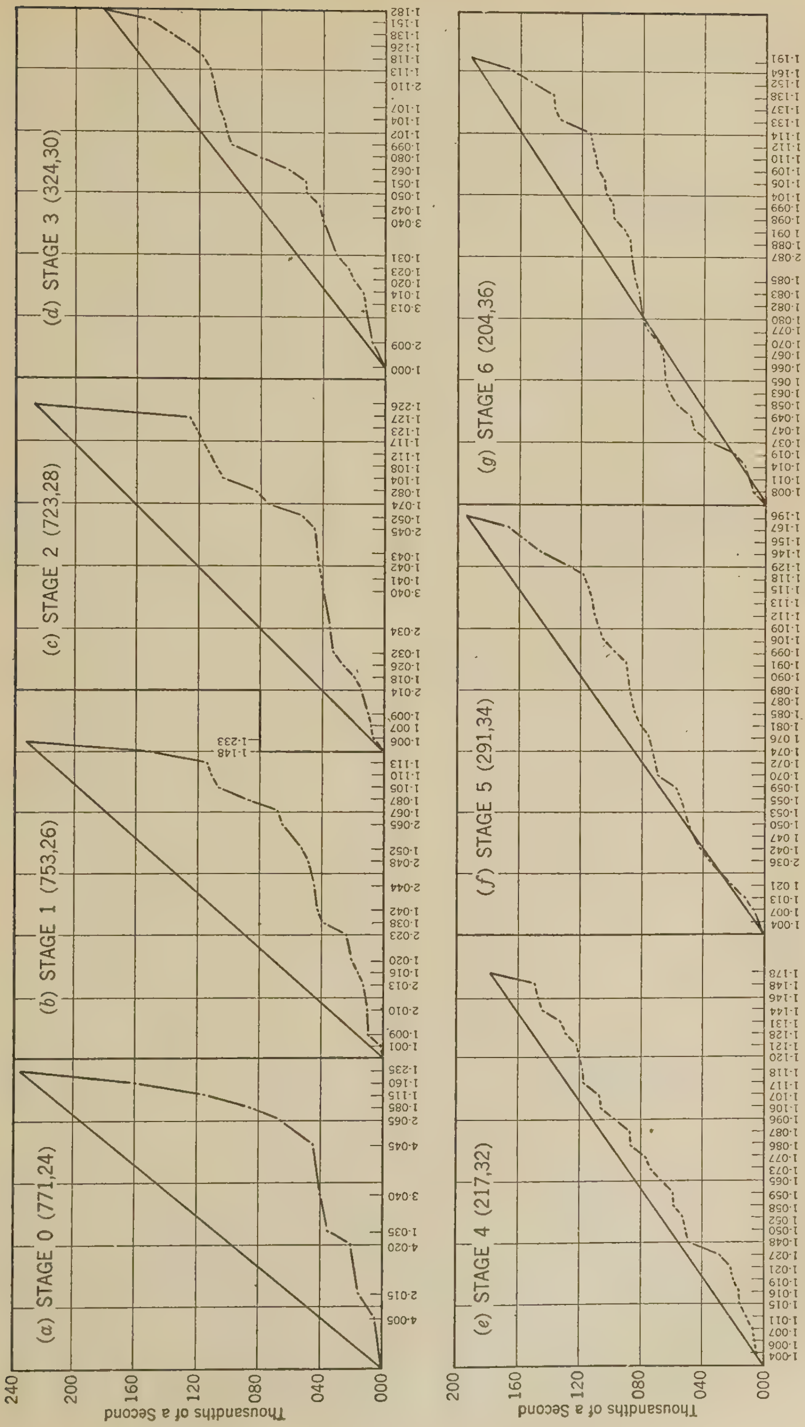

Fig. 2.—Graphs of Δ-Tabulations for Seven Stages of Design with Increasing Number of Diagonals

involved. For example, in Fig. 2(*a*), the abscissa 1–235 denotes one Δ-term, and 235 denotes its Δ-value in thousandths; the abscissa 4–005 indicates that four Δ-terms of the same value have occurred. The meaning of the sub-caption is explained as follows: In Fig. 2(*c*), for example, "stage 2" denotes a figure with two diagonals; "723" is the efficiency index; and "28" denotes the total number of Δ-terms. Subsequently, the writer will discuss these charts as comparative profiles depicting the state of geometry prevailing after the angle and side equations are satisfied—geometry upon which length equations are to be laid.

In order that a basis of comparison could be found to indicate a relative index of efficiency, an ordinate line was drawn connecting the extremes of the curve of the plotted Δ-terms. The ordinate areas were then scaled by planimeter and the resulting definitions of size indicated. Increased efficiency is thus approximated by the decrease in ordinate-area sizes. This unorthodox method of showing relative efficiency produced by changes in design was devised, and is believed to be a fair basis of comparison, because the inclusion of each successive diagonal introduces two more Δ-terms, and a realinement of all complementary Δ-terms is produced, with a tendency to approach progressively a straight-line distribution that will be more thoroughly compensated. This effect is very pronounced in the stages of Fig. 2, and would have ultimate influence on a length argument that passed through the figure.

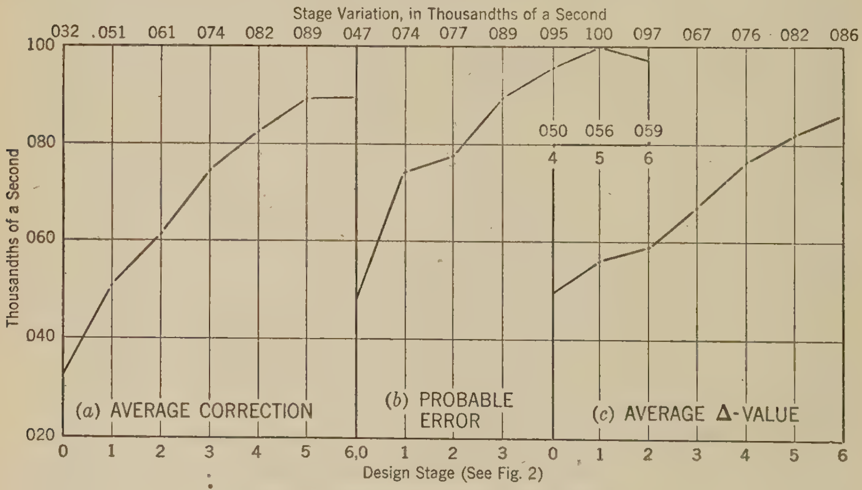

Fig. 3.—Summary Graphs of Criteria

Summary graphs of the average correction to a direction (Fig. 3(*a*)) and the probable error of an observed (assumed) direction (Fig. 3(*b*)), derived from the least-squares adjustments, and the average Δ-value from a direction (Fig. 3(*c*)) are inserted for use in conjunction with the Δ-ordinate-area graphs. These summary graphs are cross-referenced with the successive stages ((*a*), (*b*), etc.) in Fig. 2. For example, the ordinate-area index 771 (Fig. 2(*a*)), the average correction 0.032 (Fig. 3(*a*) stage 0), the probable error 0.047 (Fig. 3(*b*) stage 0), and the average Δ-value 0.050 (Fig. 3(*c*) stage 0) can be considered collectively to complete the criteria for stage 0.

Certain conclusions can be stated on the information outlined on these graphs (Figs. 2 and 3). Because the triangulation structure is indeterminate, the conclusions cannot be facts, in the strict sense of that word, but it seems reasonable to believe that they can indicate strongly the trend of cause and effect. Incidentally, the specimen hexagon in its stages of design development can be considered as a single figure or as a small network of triangulation. In either case, the principle remains the same.

(*a*) Simple-triangle design and mixed design are not acceptable in high-precision triangulation, misleading final data from adjustment to the contrary. Stages 0, 1, and 2 (Figs. 2(*a*), 2(*b*), and 2(*c*)) plainly point to an unconcluded distribution of discrepancies. Stages 0, 1, and 2 in Figs. 3(*a*), 3(*b*), and 3(*c*) as related to stages 0, 1, and 2 (Figs. 2(*a*), 2(*b*), and 2(*c*)) show values that are misleadingly small. It is obvious that lack of geometrical conditioning, produced by diagonals, might result in either favorable or unfavorable distribution of discrepancies that could adversely affect a base-to-base chain, or any part of a network—a matter which will be referred to in Part 3.

(*b*) The severe decrease in the ordinate-area index in stage 3 (Fig. 2(*d*)) can be expected since double determination is completed. At this stage of design, a minimum of lines produces a maximum of results. The ordinate-area index, average correction, probable error, and average Δ-value have reached the lowest common minimum in double determination.

(*c*) Stages 4, 5, and 6 (Figs. 2(*e*), 2(*f*), and 2(*g*)) show a relatively smaller decrease in the ordinate-area indexes, but the related stages 4, 5, and 6 in Figs. 3(*a*), 3(*b*), and 3(*c*) show continued increase—with but two exceptions in stage 6—in average correction, probable error, and average Δ-value. Unless very exceptional circumstances warrant the inclusion of any or all of diagonals 4, 5, and 6, the resulting increase in time and cost is not justifiable; also, these diagonals are in excess of the uniform-figure design to be treated in Part 2.

(*d*) It is possible to summarize conclusions (*a*), (*b*), and (*c*) by reference to the theory of least squares and the exercise of logic. As applied to triangulation, the theory is given as follows: The sum of the squares of the corrections to the directions shall be a minimum. These terms for the design stages are: At stage 0, 0.0340; at stage 1, 0.1083; at stage 2, 0.1442; at stage 3, 0.2273; at stage 4, 0.2996; at stage 5, 0.3708; and at stage 6, 0.3919. If graphed, these terms would have the characteristic curve of increase shown in Fig. 3 for other final terms. The least-squares theory is satisfied independently in each of the seven design-stage adjustments, but it is necessary, further, to decide which least-squares result (that is, which stage of design) shall be chosen as the specific one that will probably be the most useful. At this point, the logic of economy should be the only deciding influence; and, as expressed in conclusion (*b*), the logic of economy points rather directly to the first state of complete double determination, produced by diagonals 1, 2, and 3, providing the maximum of results with the minimum of lines.

In planning and executing the foregoing investigation, the components are greatly reduced in magnitude for convenience in handling. Similarly, greatly reduced models of trusses are used to study load stresses and subsequent reactions. Nevertheless, the process, however reduced, can guide thinking in connection with the much larger problem of planning and executing an actual urban network. The criteria for acceptance of field observations[3] constitute a restriction of the magnitude and range of discrepancies in observed angle values in somewhat the same manner that artificial restriction has been applied to the assumed discrepancies in the ideal hexagon. Of course, there is an infinite variety of patterns in the distribution of discrepancies since the structure is indeterminate, but the trends in results will be constant. Quite naturally, it should be remembered that the magnitude and range of actual discrepancies in a network are expected to be much larger and are correspondingly more decisive.

PART 2. UNIFORM UNIT FIGURES

To make the best use of advantageous trends, it is necessary to build up a reasonably complete series of closely related geometric figures, hereinafter to be called unit figures. They are designed to incorporate the useful characteristics suggested as regards the hexagon in Part 1. Such a series is presented in Table 1.

The ideal hexagon is the perfect form of the unit figure shown as type 4, Table 1. With shifts in the location of central points and changes in shape, the figures apparently become more difficult to identify; but this need not be misleading, since each type includes a definite number of physical members that is characteristic of one type only. In the series it will be seen that types 1 and 6 are used in varying combinations to make up the remaining composite types. All are equally useful in designing the urban net from reconnaissance operations.

The types of unit figures shown in Table 1 are in two groups—one with central point and the other with diagonals only. The completed central-point triangle and the completed quadrilateral are the basic unit figures. In the first of these one triangle overlaps three and in the second two triangles overlap two (overlap in each case for double determination). In the composite types, attention is directed to the central-point figures and the regular increase in the number of diagonals employed; and, in type 7, attention is directed to the pentagon with four diagonals. In that manner, types 2, 3, 4, and 5 combine a central-point triangle, in each, and one, two, three, and four quadrilaterals, respectively, in compound overlap (see crosshatched areas) needed for unit-figure geometry. Similarly, type 7 combines two quadrilaterals. The close relationship in visible characteristics is quite apparent in all these types. An indication of the close similarity in mathematical characteristics is shown by the reduced spread of the $\frac{D - C}{D}$ terms (see Table 1), in which:[4] D is the number of directions observed in the figure (always assuming that the starting line is held fixed); and C is the number of conditions to be satisfied in the least-squares adjustment.

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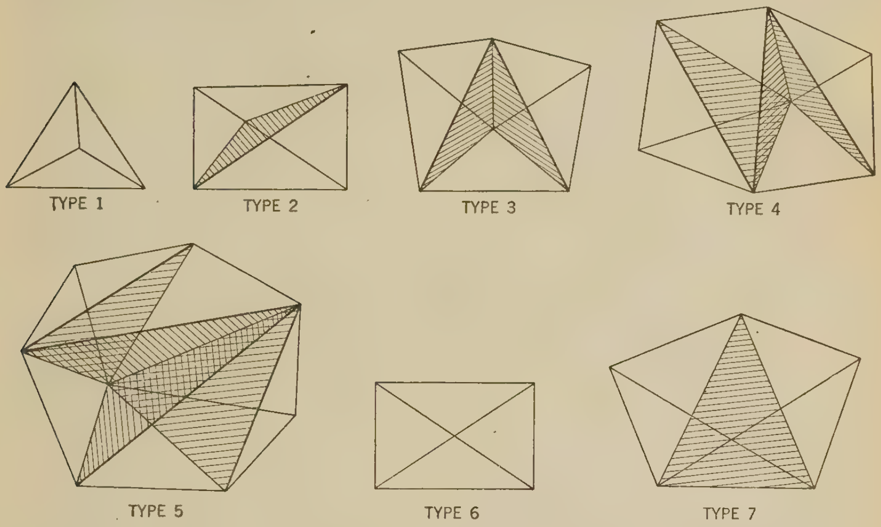
[3] "Technical Procedure for City Surveys," *Manual of Engineering Practice No. 10*, ASCE, 1934, p. 11.

[4] *Ibid.*, Eq. 1, p. 14.

It will be noted that certain diagonals are not used; therefore, various choices in selecting the required number of diagonals are possible. This is a very useful circumstance in resolving reconnaissance and designing difficulties, and in the elimination of poorly shaped triangles. It is thus possible to design the urban net by abutting component unit figures of relatively similar physical properties.

TABLE 1.—SERIES OF COMPONENT UNIT FIGURES AND THEIR CHARACTERISTICS



Type	Unit figure (C-p = "central point")	Diagonals	$\frac{D-C}{D}$	Adjustment Conditions	
				Angle	Side
1	C-p triangle	....	0.60	3	1
2	C-p quadrilateral	1	0.56	5	2
3	C-p pentagon	2	0.55	7	3
4	C-p hexagon	3	0.54	9	4
5	C-p heptagon	4	0.53	11	5
6	Quadrilateral	2	0.60	3	1
7	Pentagon	4	0.56	5	2

PART 3. ACTUAL RESULTS IN THE FIELD

During the years 1937 to 1941, the writer collaborated with local personnel engaged in the construction of several urban triangulation nets. At that time, even though the idea of balanced design for triangulation existed only as a line of logic, based upon past experience and research, it was possible in most instances to demonstrate successfully the usefulness of the design, which was used subsequently in designing several urban networks. As an example for discussion the basic triangulation net for the City of Minneapolis, Minn., is reproduced in Fig. 4.

This network has sixteen component unit figures, including nine quadrilaterals, two central-point triangles, four central-point quadrilaterals, and one central-point pentagon. Two precision base lines were measured—Main-Columbia and Holy Angel-Cedar. The line Aggie-Snelling, established in length and azimuth by the U. S. Coast and Geodetic Survey, was held fixed, being related to the measured bases by condition equations in the least-squares adjustment; azimuth for the rigidly adjusted net was derived from the federal definition on this line.

As is apparent in Fig. 4, the first visible effect of balanced design is the ease with which the net and its component unit figures can be studied. In this connection, a very significant feature is the absence of lines, possibly intervisible, that would cross over the heavy lines accentuating each unit figure. Such lines would be superfluous and could only serve to complicate dangerously a more orderly design. The second effect, not so apparent but present nevertheless, is the pattern of geometrical condition equations necessary in the least-squares adjustment to distribute discrepancies from observations.

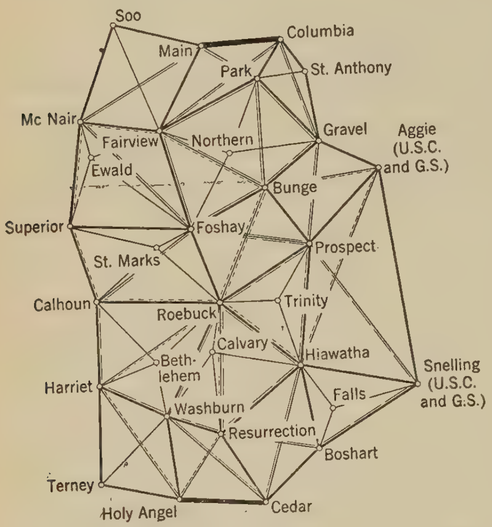


Fig. 4.—Triangulation Net, City of Minneapolis, Minn

Disregarding the length argument for the moment, there are sixty angle equations and thirty-one side equations. The angle equations, all of which occur as unit-figure conditions, require no special comment. There are two distinct varieties of side equations, however: (*a*) Figure-side equations, twenty two in number, that occur complementally to the angle equations in the individual unit figures; and (*b*) vertex-side equations, nine in number, occurring at the interior stations that are the vertices of abutting lines between unit figures.

In this net the vertex stations are: Fairview, Park, Foshay, Bunge, Roebuck, Prospect, Washburn, Resurrection, and Hiawatha. The limit lines of the vertex equations are shown in Fig. 4 by lines of three styles placed on the side of the limit lines nearest the vertex station. These vertex-side equations serve to knit the scheme geometrically, by relating each unit figure to every other, as the figure-side and angle equations relate the members of each unit figure.

Specifically, the function of the vertex-side equation is to establish coincidence of the abutting lines between unit figures at the pole—that is, at the vertex station. Likewise, the figure-side equations produce coincidence of certain unit-figure lines at selected figure poles, and, in conjunction with the angle equations, complete the integration of the individual unit figures. The process of integration proceeds from the individual unit figures, geometrically, through the vertex-side equations—relating figure to figure and vertex to vertex—to every part of the entire net. Each unit figure and its condition equations abut other unit figures and their conditions, and the vertex-side equations overlay each other and the unit-figure conditions in fish-scale fashion. There can be no doubt regarding complete integration and, furthermore, the highest possible degree of uniformity throughout the network.

Using transparent plastic sheeting, cutouts of the figure-side and vertex-side equations can be prepared; the selection of angle equations in each unit figure can be indicated on the layout map; and the cooperative effects, designed to produce mathematical rigidity, can be examined by overlay. In this way, also, the presence of an identical equation that would be fatal in the adjustment can be detected. Actually, this seems to be the graphic statics of triangulation whereby geometric integration becomes visible in form.

The length argument, superimposed as it is upon the geometrically conditioned network, can be considered separately. The necessary length equations, two in number, relate the measured bases to each other and to the fixed line through selected chains of triangles. There are discrepancies in passing from base to base and base to line, but, in view of the extreme accuracy of the measured lengths, these discrepancies after the satisfaction of the angle and side conditions can scarcely be considered as originating from the length measurements. Therefore, they represent the degree of failure in compensation in the angles used in the base-chain triangles, after these triangles have been effected by the angle and side equations, which are the geometrical conditions. Finally, the length equations (which are trigonometrical conditions) are superimposed and the total base-to-base discrepancy after adjustment is disclosed; and, if this final length discrepancy is highly acceptable, cannot it be considered as evidence that the geometrical adjustment, produced by the angle and side conditions, was but little disturbed by the length conditions? Furthermore, does it not afford evidence that all discrepancies were most consistently and favorably distributed, limited only by the mathematical process, throughout a network that is uniformly designed? In this connection, reference is made to the ultimate influence attributed to a compensated distribution of the geometrical discrepancies as suggested in Part 1.

Lastly, there are the final results from the least-squares adjustment that are accepted as the indicative proof. Criteria for the acceptance of urban triangulation schemes were presented in 1934 by the Committee of the Surveying and Mapping Division on City Surveys.[5] These specifications and the results of four actual operations, the networks of which were designed by abutting component unit figures of the types listed in Table 1, are summarized in Table 2.

In reviewing these somewhat surprising results, it should be remembered that, according to the theory of probability, any single result in this field can be called accidental, as can also any pair of results; but, when the third and the fourth are considered, the chance that the results are accidental becomes so remote that the circumstances call for another explanation—the inevitable conclusion in this case being cause and effect, with emphasis on balanced design as a basic cause.

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[5] "Technical Procedure for City Surveys," *Manual of Engineering Practice No. 10*, ASCE, 1934, p. 12.

To be sure, for all practical purposes, scheme C, Table 2, is just as useful as scheme B, despite the large difference in the adjusted base-to-base discrepancies. (The designations A, B, and C are used herein to identify three other city networks similar in design to the Minneapolis scheme.)

TABLE 2.—SUMMARY OF RESULTS, APPLIED TO URBAN TRIANGULATION

No.	Description	Manual 10[a]	Minneapolis[b]	Scheme A	Scheme B	Scheme C
	(a) RESULTS ATTAINED FROM FIELD OBSERVATIONS					
	Triangle Closures, in Seconds:					
1	Average	1.50	0.91	1.22	1.31	1.09
2	Maximum	5.00	3.26	3.21	2.94	2.50
	Sine-to-Side Discrepancies, in Seconds:					
3	Average	0.70	0.56	0.48	0.64	0.49
4	Maximum	2.00	1.83	1.65	2.13	1.60
5	Probable error of measured bases[c]	600,000	1,657,000	1,638,000	1,678,000	1,850,000
6		......	3,246,000	643,000	2,484,000	894,000
7		......	(and one fixed federal length)	1,862,000	3,853,000	978,000
8		......		2,051,000	....	....
9		......		2,234,000	....	....
	(b) FINAL RESULTS ATTAINED FROM THE LEAST-SQUARES ADJUSTMENT					
10	Base-to-base discrepancy[c]	100,000	1,478,000	2,782,000	3,378,000	378,500
11	Average correction to a direction, in seconds	0.75	0.61	0.64	0.54	0.49
12	Probable error of an observed direction, in seconds	0.75	0.75	0.54	0.64	0.59

[a] *Manual of Engineering Practice No. 10*, ASCE, 1934, p. 12. [b] Basic triangulation net for the City of Minneapolis, Minn. (see Fig. 4). [c] Ratios, 1 : 600,000, etc.

The significant fact is that, as a prerequisite to the other necessary phases of construction, it is possible by selective design to exceed the specifications for the length argument and to satisfy the other final criteria in four networks of similar design, without a single failure to date, at the same time producing the effect uniformly throughout each network. From networks of such consistent strength, extensions of traverse to provide adequately spaced control stations can be planned and executed very confidently, as has been amply demonstrated in practice.

It is disheartening to discover, during the development of a traverse layout from triangulation, the inexplicable presence of an area of weakness in an urban control system. It would seem to be only good judgment to build as strongly as is possible, rather than to be forced by circumstances to attempt the rationalization of disappointment that could have been avoided.

ACKNOWLEDGMENT

Fig. 4, the basic triangulation net for the City of Minneapolis, is reproduced with the kind permission of Frederick T. Paul, M. ASCE, City Engineer.

DISCUSSION

WALTER S. DIX,[6] ASSOC. M. ASCE.—The paper is a comprehensive analysis of a basic problem in urban area precision survey engineering, important to many surveyors and engineers, yet understood or considered by an alarming few. The author's research and effort on this paper are valuable contributions to the knowledge and advancement of professional surveying and engineering.

It is imperative that, for the degree of accuracy warranted by modern municipal precision surveys, not one single step in the direction of equilibrium for final values of an urban network should be neglected—that, whereas attention has been given in the past to full consideration of the laws of gravity, refraction, and trigonometry in city triangulation, not enough thought has been applied to equally important laws of balanced design, involving engineering philosophy with mathematical, geometrical, and algebraic analysis, to produce truly homogeneous accuracy.

In substance, the theory of homogeneity in this instance is that the reality of engineering has been added to the science of mathematics to produce the surprising data in the author's tabulation of final results.

The author is completely justified in comparing localization of error in an urban triangulation network, to the inherent weakness of a similarly localized error in structural design; and, as the author has indicated, experience shows that arbitrary acceptance of over-all closure adjustment in triangulation schemes heterogeneously composed of unsymmetric figure patterns of varying strength, as true index of homogeneous accuracy of the scheme, can quite conceivably conceal some local accrual of weakness.

The possibility of such concealment of local error in over-all closure values has previously been mentioned by G. D. Whitmore, M. ASCE, C. C. Miner, Assoc. M. ASCE, and W. O. Byrd who state:[7]

> "The writers have long felt that too much emphasis has been placed on the 'just happened' closure of the entire loop, and not enough on designing measuring instruments and measuring procedures that will insure uniform precision for each and every tangent of the entire traverse. Experience shows that traverses having circuit closures of high order, such as 1:10,000 or 1:20,000, may conceal errors, in individual courses, of the order of 1:1,000."

This statement follows closely the author's argument in favor of closer unit consideration in design, and solution of urban triangulation systems, to insure greater integral accuracies within the accepted over-all accuracy index.

Any precautionary measure taken toward eliminating the possible concealment of error is a step in the direction of optimum engineering design. The deliberate design of urban triangulation by compounding component unit figures, and the analytical selection of balanced equations for homogeneous solution with multiple determination alternatives, as propounded by the author

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[6] Cartographic Engr., Maps and Surveys Div., TVA, Washington, D. C.

[7] *Transactions*, ASCE, Vol. 108, 1943, p. 598.

(as compared with the over-all solution of an accidental heterogeneous triangulation pattern resulting only by intervisibility from conveniently available tall buildings, hill tops, tanks, etc.), is certainly a step in that direction. Furthermore, station towers should even be erected where required when found advantageous for optimum design strength in following out the overlapping unit-figure network in urban triangulation.

Portable towers are used by the United States Coast and Geodetic Survey to aid visibility and to help control figure strength in basic triangulation arcs of the United States; but, to the writer's knowledge, erected towers have seldom if ever been specially placed for urban triangulation, except scaffolding or platforms built up to augment convenient roof structures.

With reference to the author's closing statement, any latently discovered weakness in an urban triangulation system requiring rationalizing by arbitrarily adjusting the values of subsequent determinations is more than disheartening disappointment; it is an engineering tragedy. Urban networks, by characteristic unity, are capable of complete predesign. Other than false economy or meager budget (itself a false economy), there seems little argument contrary to urban triangulation network predesign.

Henry W. Hemple,[8] M. ASCE.—The subject of urban triangulation design has been presented by Mr. Hopkins in a vivid manner. Engineers engaged on this type of work will benefit by his general concept of design. The principles which the author recognizes have been in use for many years and are established as standard for precise surveys.

The comparison of triangulation design to that of structures is an apt illustration, but the comparison is analogous only to a certain point; beyond that the similarity does not exist. It should be understood, however, that specifications for design, observational procedure, computation, and adjustment are so rigid for urban and precise triangulation surveys that, if the basic measurements and observations were adequate and if the specifications were otherwise adhered to, there would be no weak determination of positions or lengths in the scheme.

In the design of structures there are various known factors, or those that may be correctly estimated for, producing stresses for which structural members whose strength characteristics are known can be provided. In triangulation work, unknown physical conditions may be of a nature to produce lateral refraction of light rays and introduce errors in the directional measurements. These effects cannot be called errors due to structural failures, and there is no element of design that can be introduced to guard against them. They are errors resulting from unknown causes which cannot be compensated for, and only by reobservation under good atmospheric conditions can they be eliminated.

If the computation and adjustment of a triangulation scheme have met the specifications, these discrepancies must be due to imperfect measurements or observations. The degree of failure that might be acceptable can be anticipated from the strength of any design. If the failure exceeds this limit, certainly the design is not at fault. In this concept, Mr. Hopkins' statement (see heading, "Introduction") that "Localized error that cannot be corrected properly can rightly be called structural failure" will have no basis.

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[8] Chf., Div. of Geodesy, U. S. Coast and Geodetic Survey, Washington, D. C.

Many factors enter into the formation of a scheme of triangulation. Topographic and other conditions will not always permit the location of the stations at points that will give the ideal design. Lines over which undue refraction may be expected must be avoided. Stations on opposite ends of the lines of the scheme must be tested for intervisibility. Base lines must be provided at the proper intervals so that adequate length control is available. Connections to adjacent federal control must be made so that there will be a continuous coordinated scheme throughout the area. Adequate strength of figures must be inherent in the triangulation figures formed. The scheme selected should be such that it can be carried through with economy and yet maintain the accuracy desired.

One is not justified in spending an undue amount of effort and time with the resulting increased cost, striving for ideal figures, when there are other possible solutions easier of execution which will satisfy the criteria for urban surveys. The competent engineer evaluates all these considerations and arrives at a scheme that can be executed economically and still satisfy the requirements of the specifications.

The U. S. Coast and Geodetic Survey has adopted the practice of providing at least one base line near the larger cities for length control of the federal net in that locality. Lines of the federal net and their azimuths may be accepted into the urban scheme with assurance that they are determined with adequate accuracy for city survey purposes. In cities where urban surveys exist at the time the federal net is carried to them, connections are made to the local surveys and their lengths are held if the work has been done with standard specifications for that type of control.

Mr. Hopkins has considered only the internal forces affecting a scheme of triangulation. When area triangulation is adjusted to an established scheme surrounding it, the maximum external forces are encountered. The engineer can be certain that, if the observational data will withstand the test of this adjustment, the fundamental control and the new survey have each been determined with high accuracy.

Mr. Hopkins has shown a keen appreciation for strong figures in triangulation and this fundamental principle must be recognized by all those engaged in urban surveys.

L. G. SIMMONS,[9] ESQ.—It is agreed that well-shaped figures with double determination in length are essential to high precision in geodetic triangulation. The writer does not agree on the importance that Mr. Hopkins attaches to the effects caused by the variations in his sample figure (Fig. 1). These effects are so small that they are likely to be completely obscured by more prominent effects arising from conditions entirely independent of figure design. It is quite possible that the writer has missed the point, but to him it seems that Mr. Hopkins' reasoning does not follow a logical course.

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[9] Prin. Geodetic Engr., U. S. Coast & Geodetic Survey, Washington, D. C.

For example, he begins with a symmetrical central-point hexagon and then assumes a set of observations closely approximating Fig. 1. As soon as he has assumed this set of observations, the perfect hexagon figure no longer has a place in the problem. It is nothing more than something he had in his mind before the analysis was started. The set of assumed observations is then adjusted by the method of least squares and the results tabulated as Δ-values or differences between the adjusted directions and the directions defined by the angles of the perfect hexagon. The significance of these Δ-values escapes the writer. There is no reason that the adjusted values of the angles approach those of a perfect central-point hexagon other than the fact that the assumed values approximated this figure.

Even if these Δ-values do have significance, it is not clear to the writer why the merit of the figure design should be judged by the closeness with which plotted values cling to a line drawn between the high and low values. On this basis, why does Mr. Hopkins state that stage 3, where complete double determination is first reached, is the most efficient figure design? Is it because the Δ-values, which have no meaning in the first place, occur in a uniformly ascending series of values? To the writer it is more important to keep the Δ-values low (if they do have a meaning), and on this basis it would seem that stage 0 is more efficient than stage 3. Moreover, the differences among all stages appear of small consequence, even if the assumed triangle closures were doubled.

As a practical proof of what figure design will do toward securing length agreement between measured bases, Mr. Hopkins cites several schemes of urban triangulation in which (in most cases) length closures were considerably better than 1 part in 1,000,000. The writer maintains that any length closures that are much better than 1 part in 200,000 are purely accidental and extra precision as implied by better figures is more apparent than real. The reason for this statement is twofold: First, lengths of the best standardized base tapes, with an accuracy of 1:200,000, are scarcely known. Certainly there are none more accurate than 1:500,000; and, second, the effects of systematic errors, which are present particularly in urban areas, preclude any certainty of a much better length closure. No amount of extra refinement in figure design will overcome the limitations already imposed by observing conditions (uncertainties in lateral refraction), and the limitations imposed by the ever-present systematic errors in triangulation work, too numerous to mention.

Of course, strong figures should be used and single triangles avoided where the highest precision is sought, but Mr. Hopkins is putting into figure design a refined value which cannot be extracted economically—if at all.

C. A. WHITTEN,[10] ESQ.—The selection of the type of mathematical analysis of a triangulation scheme is frequently dependent on the desires of the individual making the particular study. The possible types of investigations with all the variations which naturally develop present a problem for which there would be no end. When only one type of investigation or procedure is used, it must be established that the methods employed are rigid at least in principle.

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[10] Chf., Section of Triangulation, Div. of Geodesy, U. S. Coast & Geodetic Survey, Washington, D. C

A new method of analysis for a study of triangulation design is presented by Mr. Hopkins. In general, the results derived are not contradictory to established fact. Some of the author's conclusions are based no doubt on his experience, plus the "exercise of logic," rather than on the findings from a standard mathematical treatment. It is because of this that it seems necessary to caution those who would pursue this problem in further variations.

At the outset, the method of adjustment, whether by the angle or the direction method, should be established definitely. Both cannot be used simultaneously or interchangeably as Mr. Hopkins has done. Having established the method of adjustment to be used, it is permissible to make some assumptions. However, the writer questions the procedure of assuming a rigid figure and then assuming a set of observational errors. What will hold the true figure rigid? There could be a large number of mathematically consistent or rigid figures within the range of the assumed errors without creating an error greater than any of the original assumed errors. Thus, this method does not seem feasible for a general mathematical development. Even the distribution of the assumed errors was made in a systematic manner rather than according to probability. Why, then, use least squares?

Another unusual treatment is the development and tabulation of the Δ-terms. Since the assumed observational data are not truly observational, according to the laws of distribution, the graphs of the Δ-terms do not furnish conclusive evidence concerning the various stages. The ideal adjustment would be the one in which the Δ-values are small, and Mr. Hopkins has probably selected the wrong area of the graph for determining the "efficiency index." The mean Δ-term is a more logical value for an index.

The results obtained from the calculations of the seven stages of design should not be compared with each other without giving consideration to the mean of the assumed errors which were distributed by pattern. These mean errors are:

Stage of design	Mean error of angle (seconds)
0	0.067
1	0.088
2	0.097
3	0.113
4	0.122
5	0.126
6	0.127

Since there is a large change in the mean errors of the various stages, similar changes will be found in the probable errors and average corrections. However, the author has attributed the changes in probable errors and average corrections to the stages of design rather than to the magnitude of errors in the different figures. The mean errors have not been balanced sufficiently for a satisfactory comparison of the various stages.

Mr. Hopkins recognizes many of the difficulties that are encountered in determining the accuracy of any triangulation net or figure. When rigid specifications are used to limit the magnitude of errors of observations, the least squares method of adjustment will produce very satisfactory results. From a least squares adjustment, it is possible to determine the effect of an error of any one observation on the other lines of the scheme. Such a process as this, when applied to all lines of a scheme through various stages of development, would be very laborious and the value of the results obtained might not be worth the effort. However, such a method would produce results that could be interpreted with certainty since the procedure is rigid and conforms with accepted mathematical practice.

Ronald F. Scott,[11] Jun. ASCE.—It is hoped that this paper is the beginning of a new and wholesome approach to the problem of covering an area with triangulation of a more or less predictable accuracy. To this writer's knowledge all too little attention has been given to this very important phase of precise control surveys. The design of a triangulation scheme to provide uniformly precise control over a large area has been too much a matter of guesswork; or, perhaps, it has been classed as an art rather than a science. Much work has been done in adapting triangulation to the problem of a chain of quadrilaterals establishing control over long distances; but its use for an area has been insufficiently explored.

Those who have undertaken the design of a scheme for an area have too often been regarded as individuals apart from normal human beings with a special sense of things mathematical and geometrical. It will be very important to remove this phase of surveying from that highly specialized classification into the realm of things more common. Much work has been done to make the little-known facts of precise surveying available to a broader group of engineers. However, the profession is still a long way from providing the necessary and desirable tools to complete the numerous, carefully executed control surveys that will be required before the various systems of state-wide plane coordinates can come into general use. In the past decade some engineers have declined to undertake the supervision of precise-control projects involving triangulation simply because they have felt that the problems and techniques were far too involved for someone with ordinary engineering training. This concept of precise surveying can be done away with only by the development of design methods that will be suitable for widespread use.

The author's brief mention of a system of templets which will illustrate, graphically, the structural integrity in a triangulation scheme is the kind of thing which could well be expanded further. If engineers are required to take up this work they should be assisted to the greatest possible understanding of the significant factors, not only by mathematical reasoning but by utilization of all possible graphical and mechanical means. This relationship of geometric figures to one another in a triangulation scheme is the phase of the work that is the most difficult to understand. It is so intangible that it may too easily escape proper attention if, indeed, it receives any attention.

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[11] Director, Upper East Tennessee Office, Tennessee State Planning Comm., Johnson City, Tenn.

The hopelessly disorganized status of cadastral surveys, particularly in some parts of the eastern United States, will eventually bring about an insistent demand for sweeping reforms in surveying methods. It will be necessary for the engineering profession to have on hand, ready for use, the necessary criteria and design procedures in sufficient detail to enable the design of control surveys with the same assurance of success as can be had in the design of structures. Unfortunately, the advantages of sound surveying practice are apparent only to those who are most intimately connected with the very worst of the difficulties that arise from a short-sighted policy on control requirements. Even today a very few engineers, not to mention the laymen, realize the advantages and economy of a properly developed system of dimensional control for areas of extensive physical development.

Charles D. Hopkins,[12] Esq.—In concluding this paper, the writer wishes first to thank the persons who have contributed to the published discussion. A large measure of thanks is due, also, to those individuals whose advice continues to be a helpful factor in the development of a better understanding of the subject. It seems certain that every opinion that is brought to bear during the unfolding of the concept will materially aid progress.

The most significant reactions to this exploratory effort have come from the engineers in other fields. Without any doubt, the tenor of their remarks indicates a desire to understand this increasingly important subject and a belief that the subject material must be expressed in relative terms in the common language of the engineer. There is the "rub," since the person endeavoring to assist the growth of fellowship is confronted with the ever present difficulties: First, to conceive the nature of the problem and an approach to solution; and, second, to express findings so that the details and the relationships are clearly understandable.

It seems clear, in the light of knowledge acquired since this paper was written, that the first need for better understanding will be met when the subject material is properly classified in the conventional engineering manner—(*a*) the properties of the materials, (*b*) the structure in theory, and (*c*) the structure in place. The materials in triangulation, whose properties have been under close investigation for many years, are found in the field books recording the observations. The structure in place involves the use of the various projections to define the position of adjusted stations on the earth's surface and the data for projections are available in great detail through the long continued work of the mathematicians; but, to fully realize the relationship between the materials for construction and the structures in place, it is absolutely necessary to develop the theoretical concept for the structure. Otherwise, the engineer can never expect success in final results with any degree of certainty.

The writer appreciates to the full the complexities in the subject taken as a whole. That circumstance needs no elaboration. However, it is possible to reduce any subject to a degree of simplicity, given a point of attack and the required time. In this connection, an example of the reduction of a minor subject in this field, possibly encouraging, can be examined as follows:

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[12] Geodetic Engr., Office, Chf. of Engrs., U. S. Army, Washington, D. C.

In the quadrilateral with both diagonals, the geometric conditions are found stated in four angle equations and five side equations. These can be put into the adjustment as three angle equations and one side equation, or two angle equations and two side equations, or one angle equation and three side equations. The number of combinations of the four equations required in this simplest type of completed unit figure is somewhat surprising—a total of 126 combinations has been identified, of which 120 are true and useful and six are false and useless; but, of the large number of useful combinations, only one combination is required, usually three angle equations and one side equation being the selected choice.

This comparison suggests that an understanding of the minimum working essentials can be grasped with relative ease. In other words, the engineer who builds roads does not necessarily need to remember the knowledge required to make the machines used in the operation.

In the light of experience, it is the writer's personal conviction (no doubt shared by many others) that unsuspected facts, indispensable to success, are present in triangulation. It does not seem reasonably possible to consider triangulation other than as a structure; since it can be defined as a structure, an arrangement of parts is involved—parts being the lines between stations. Therefore, since an arrangement of parts is an operation in fact, an orderly reasoned arrangement of these parts is logically necessary. This most strongly suggests an engineering treatment; and, the subject being treated as engineering, it becomes essential that every required principle in engineering be incorporated in the structure.

In view of the natural difficulties and the conflicting opinions relating to the geodetic field, it seems fitting to close this paper with an appropriate statement which will sum up the problem and point the way to an understandable solution. In one of the truly profound sayings of modern times, Sir Norman Angell poses the question: "How, out of complexity, to distill simplicity; out of knowledge, essential understanding; out of confusion, clarity." It seems entirely reasonable to state that the last mentioned attributes—simplicity, essential understanding, and clarity—applied to their limits would go far toward making the geodetic subjects more understandable, and certainly more attractive, to the engineering profession.

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TORSION IN STEEL SPANDREL GIRDERS

By J. E. Lothers,[1] M. ASCE

With Discussion by Messrs. John E. Goldberg, I. Oesterblom, Robert V. Hauer, E. I. Fiesenheiser, Edward V. Gant, Oscar Hoffman, Phil M. Ferguson, and J. E. Lothers.

Synopsis

The computation of torsional stresses in rolled steel sections is not new. The application has been confined, however, to eccentric loads where the torque, or torsion moment, could be obtained by multiplying the load by the eccentricity. The case of hanger bolts suspended from a steel girder flange and supporting an intermediate stair landing might be cited as an example. A steel spandrel girder with a floor beam framed into its web presents a torsion problem of a different type. The deflecting floor beam twists the spandrel and a torque is introduced—the computation of which is analogous to that of the bending moment brought to a column by a rigidly connected beam. This torque has been neglected for the following reasons: (1) No simple method of computation has been advanced; and (2) it has not been considered of sufficient importance to justify an extended analysis.

The purpose of this paper is to propose simple formulas for the solution of the aforementioned problem and to justify them by the method of moment distribution. Simple empirical formulas are also proposed for computing approximate values of the torsional constant K for the design of rolled steel beams which involves only beam constants found in the ordinary steel handbook.

Introduction and Restatement of Principles

Notation.—The letter symbols in this paper are defined where they first appear, in the text or in illustrations, and are assembled alphabetically, for convenience of reference in the Appendix. Discussers are requested to adapt their notation to that of the paper.

Formulas Applicable to Cylindrical Beams.—In the ordinary textbook on mechanics of materials, torsion is applied to cylindrical shafts, that section being the most economical disposition of material to resist torsion. In other

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Note.—Published in March, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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words, if a beam is to be designed, the chief function of which is to resist torsion, a circular section is chosen. The following two equations apply:

$$s = \frac{T\,r}{J} \quad \text{(1a)}$$

and

$$T = \frac{J\,G\,\theta}{L} \quad \text{(1b)}$$

in which s is the torsional shearing stress at the surface of a cylindrical shaft or beam; T is the torsional moment or torque; r is the radius of the shaft or cylindrical beam; J is the polar moment of inertia of the section; G is the shearing modulus of elasticity (usually assumed to be $0.4\,E$); θ is the total angle of twist; and L is the length of the shaft or cylindrical beam over which θ has been accumulated.

Eqs. 1 may be used for noncircular sections, such as I-beams or channels, subjected to torsion provided that proper values are substituted for r and J. If the thickness of that part of the noncircular section for which the torsional shear is desired is substituted for r and if the torsional constant K for the section is used instead of J, a close approximation of the desired result will be obtained by Eq. 1*a*.

The Torsional Constant K.—The torsional constant K may be defined as the measure of the torsional rigidity and twisting deflections of the beam.[2] For circular sections, J and K are identical. For noncircular sections, K is always less than J, but there is no direct relationship between the two. The use of K in connection with noncircular sections is analogous to that of J for circular sections. Therefore, the following equation holds:

$$K = \frac{T\,L}{G\,\theta} \quad \text{(2)}$$

The value of K for any given beam may be found experimentally by observing simultaneous values of T and θ and substituting them in Eq. 2. For I-beams and H-beams it may also be found, for sloping flanges, by:[3]

$$K = \frac{b - t_w}{6}(m + n)(m^2 + n^2) + \frac{2}{3} t_w m^3 + \frac{1}{3}(d - 2\,m)\,t^3_w + 2\,\alpha\,D^4 - 4\,U_s\,n^4 \quad \text{(3a)}$$

and, for parallel-sided flanges—

$$K = \frac{2}{3} b\,n^3 + \frac{1}{3}(d - 2\,n)\,t^3_w + 2\,\alpha\,D^4 - 0.42016\,n^4 \quad \text{(3b)}$$

in which α is a factor that depends on two ratios, $\frac{t_w}{m}$ and $\frac{R}{m}$; and U is a factor depending on the $\frac{b}{n}$-ratio, with subscripts L and s to denote "large" and

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[2] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101 1936, p. 857.

[3] *Ibid.*, Eqs. 20 and 21, pp. 864–865.

"small" ends, respectively, of the flange. Symbol D is the diameter of the inscribed circle drawn tangent to the top of the flange and to the two adjacent fillets, and it may be computed or determined graphically by a large-scale layout of the beam.[3] The significance of the remaining symbols is shown in Fig. 1.

According to Saint Venant[2] and L. B. Tuckerman[4] the value of the torsional constant K for a structural steel section may be found approximately by dividing the section into three rectangular parts and taking one third of the long dimension times the cube of the short dimension for each part. By summation:

$$K = \frac{1}{3} dt^3_w + \frac{2}{3} (b - t_w) t^3_f \ldots\ldots\ldots\ldots (4)$$

in which t_f is the mean flange thickness. Finding that Eq. 4 gave results that are too small, Mr. Tuckerman multiplied the right-hand side by a factor based on tests. In using Eq. 3b, Inge Lyse and Bruce G. Johnston,[2] Members, ASCE, applied corrections to take care of end effects, the stiffening effect of fillets, and the added rigidity due to the connection of the flange and web. In a like manner, Eq. 3a provides for the K-values of sloping-flange sections.

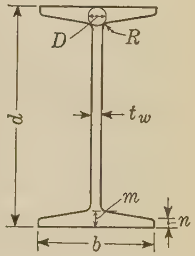

FIG. 1.—SYMBOLS

In an effort to arrive at simple formulas that would give practicable, approximate values for K and which would involve only those beam constants that are given in the ordinary steel handbook, the writer transformed Eq. 4 into the form,

$$K = c\, dt^3_w + 2\, c\, (b - t_w)\, t^3_f \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which c is an empirical constant to be evaluated by substituting known values for the other constants in Eq. 5. Rolled steel beams were divided into three groups—(1) The square column or H-sections with parallel-sided flanges, (2) the remaining wide-flange sections, and (3) the American standard I-beams. A value for c was computed for each group, resulting in a formula for each group.

To arrive at reasonably representative values for c, beam constants for twenty-seven, well-distributed sections for each group were substituted in Eq. 5. The values substituted for K were taken from the "Bethlehem Manual of Steel Construction"[5] and were originally computed by Eqs. 3. The average value of c computed for each of the three groups resulted in the following equations:

For square column sections or H-beams with parallel-sided flanges—

$$K = 0.36\, dt^3_w + 0.72\, (b - t_w)\, t^3_f \ldots\ldots\ldots\ldots\ldots (6a)$$

for the other, nonsquare, wide-flanged beams, including both parallel-sided and sloping flanges—

$$K = 0.39\, dt^3_w + 0.78\, (b - t_w)\, t^3_f \ldots\ldots\ldots\ldots\ldots (6b)$$

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[4] "Tests of I-Beams in Torsion," by L. B. Tuckerman, *Engineering News-Record*, Vol. 93, 1924, p. 882.

[5] "Bethlehem Manual of Steel Construction," Bethlehem Steel Co., Bethlehem, Pa., 1934, p. 279.

and, for American standard I-beams—

$$K = 0.43\, dt_w^3 + 0.86\,(b - t_w)\, t_f^3 \ldots\ldots\ldots\ldots\ldots\ldots (6c)$$

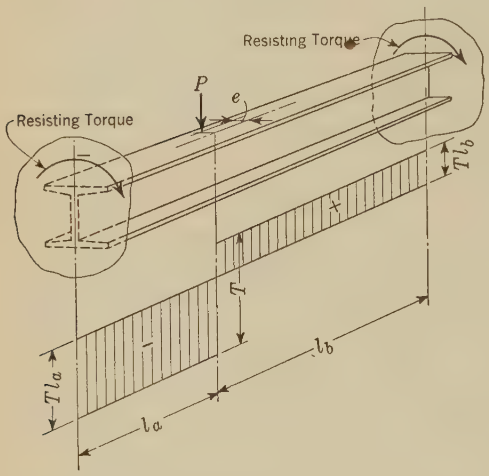
Resisting Torque
P
e
Resisting Torque
Tl_b
T
Tl_a
l_b
l_a

Fig. 2.—Distribution of Torque

Torsional Shear Equations for Rolled Steel Sections.—For H-beams and I-beams, the following equations were suggested by Messrs. Lyse and Johnston:[6]

$$s_w = \frac{T\,(t_w + 0.3\,R)}{K} \ldots (7a)$$

and

$$s_f = \frac{T\,(n + 0.3\,R)}{K} \ldots (7b)$$

in which s_w and s_f are the torsional shearing stresses in the web and the flange, respectively.

Distribution of Torsion in a Beam.—A beam with a torsional load applied at some point along its length is subjected to a torsion or torque, T, which is distributed between the two ends of the beam in inverse proportion to the lengths of the two legs (see Figs. 2 and 3). This follows from the fact that the distribution of torque is proportional to twisting stiffness which, in turn, is inversely proportional to length. The following equations apply and should be studied in conjunction

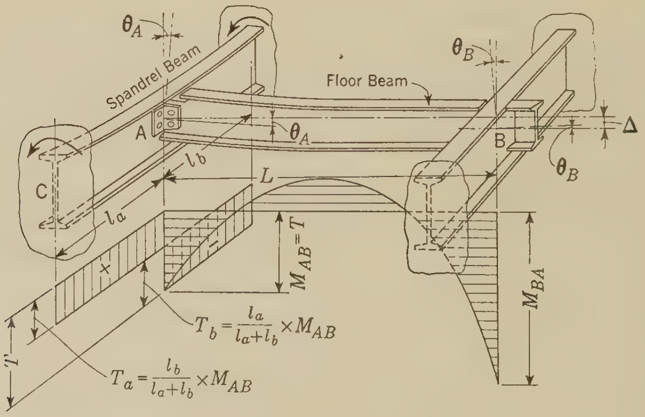
θ_A
Spandrel Beam
Floor Beam
θ_B
A
θ_A
B
Δ
θ_B
C
l_b
L
l_a
$M_{AB}=T$
M_{BA}
T
$T_b = \frac{l_a}{l_a+l_b} \times M_{AB}$
$T_a = \frac{l_b}{l_a+l_b} \times M_{AB}$

Fig. 3.—Distribution of Moment and Torsion

with Fig. 2:

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[6] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, Eqs. 27 and 28, p. 867.

$$T = P\,e \quad (8a)$$

$$T_a = \frac{T\,l_b}{l_a + l_b} \quad (8b)$$

and

$$T_b = \frac{T\,l_a}{l_a + l_b} \quad (8c)$$

TORSION IN STEEL SPANDREL BEAMS

Approach to the Problem.—In attacking the problem of torsion in spandrel beams several questions arise:

(1) How much moment can be developed at the end of a floor beam that frames into the web of a spandrel? In other words, what is the relative stiffness of the two beams, the one being in flexure and the other in torsion?

(2) How shall this moment be distributed among the two legs of the spandrel and the end of the floor beam?

(3) What assumptions shall be made as to the rigidity of end connections?

Relative Stiffness.—A beam is not nearly so stiff in torsion as it is in flexure. Therefore, since the end moment of a beam is proportional to its restraint, a relatively small moment is developed at the spandrel end of a floor beam. As shown in Fig. 3, the moment M_{AB} at the end of the floor beam must equal $T = T_a + T_b$. Also, assuming sufficient rigidity in the connection between the end of the floor beam and the spandrel web, the rotation θ_A of the end of the floor beam equals the angle of twist of the spandrel. From Eq. 3, $T_a = \frac{G\,K\,\theta_A}{l_a}$ and $T_b = \frac{G\,K\,\theta_A}{l_b}$. Therefore:

$$T = \frac{G\,K\,\theta_A\,(l_a + l_b)}{l_a\,l_b} \quad (9)$$

The slope-deflection equation for the moment at end A of the floor beam is:

$$M_{AB} = \frac{4\,E\,I\,\theta_A}{L} + \frac{2\,E\,I\,\theta_B}{L} - \frac{6\,E\,I\,\Delta}{L^2} - (\text{FEM})_{AB} \quad (10)$$

Comparing Eqs. 9 and 10, it is evident that the same θ_A which gives rise to a torque of $\frac{G\,K\,\theta_A\,(l_a + l_b)}{l_a\,l_b}$ in the spandrel contributes $\frac{4\,E\,I\,\theta_A}{L}$ to the end moment M_{AB} of the floor beam. Hence, the relative stiffness or flexural stiffness of the floor beam becomes:

$$S_F = \frac{4\,E\,I}{L} \quad (11a)$$

and the torsional stiffness of the spandrel girder is:

$$S_T = \frac{G\,K}{l_a} + \frac{G\,K}{l_b} = \frac{G\,K\,(l_a + l_b)}{l_a\,l_b} \quad (11b)$$

Moment M_{AB}.—The entire problem hinges on the moment M_{AB} at the spandrel end of the floor beam. This moment must be absorbed by the spandrel girder; it produces the twist or torsion in the girder. Therefore, if a formula for M_{AB} can be found, the major part of the problem is solved. The following remarks are pertinent to this moment:

(*a*) The sum of the torques in the two legs of the spandrel must equal the moment M_{AB} (see Fig. 3).

(*b*) In view of the relatively low value of the torsional stiffness of the spandrel as compared to the flexural stiffness of the floor beam, the end moment M_{AB} of the latter is small and, by the same token, the ordinary standard connection between the two may be considered rigid. Any error in this assumption is on the conservative or safe side.

(*c*) The moment M_{AB} depends greatly upon the degree of restraint at the inner end or end B of the floor beam. Three different conditions of restraint will be assumed and a formula for M_{AB} given for each case.

Case I.—The inner end or end B of the floor beam is assumed to be fixed:

$$M_{AB} = \frac{S_T}{S_T + S_F} \times (\mathrm{FEM})_{\mathrm{AB}} \dots\dots\dots\dots (12a)$$

Case II.—The inner end or end B of the floor beam is assumed to be free:

$$M_{AB} = \frac{2\,S_T}{S_T + S_F} \times (\mathrm{FEM})_{\mathrm{AB}} \dots\dots\dots\dots (12b)$$

Case III.—The inner end or end B of the floor beam is assumed to be 50% fixed:

$$M_{AB} = \frac{3\,S_T}{2\,(S_T + S_F)} \times (\mathrm{FEM})_{\mathrm{AB}} \dots\dots\dots\dots (12c)$$

In Eqs. 12, S_T represents the torsional stiffness of that part of the spandrel girder which resists the moment M_{AB}, and S_F signifies flexural stiffness and refers to the floor beam. The abbreviation $(\mathrm{FEM})_{\mathrm{AB}}$ denotes the fixed-end moment at end A of the floor beam. (See Eqs. 11 for values of S_F and S_T to be used in Eqs. 12.)

Eq. 12*a* holds for any type of loading on the floor beam so long as end B is fixed. Eqs. 12*b* and 12*c* for cases II and III, however, hold only for symmetrical loading on the floor beam AB. For unsymmetrical loading under cases II or III, moment distribution must be resorted to directly.

Distribution of Torsion.—The moment M_{AB} is divided in the form of torsion between the two legs l_a and l_b of the spandrel girder in the ratio of their stiffness factors. In other words, the torsion produced at end A in Fig. 3 by the member AB is divided between C and D in inverse proportion to their distances from end A. This is in accordance with Eqs. 8*b* and 8*c* and, in terms of M_{AB}, the following equations result:

$$T_a = \frac{l_b}{l_a + l_b} \times M_{AB} \dots\dots\dots\dots (13a)$$

and

$$T_b = \frac{l_a}{l_a + l_b} \times M_{AB} \dots\dots\dots\dots (13b)$$

in which the symbols have the significance shown in Fig. 3. The torsion is constant from end A to end C and from end A to end D, and of opposite sign on the two sides end A.

When two symmetrically placed floor beams frame into the spandrel girder both transmitting the same torsion, the distribution is as shown in Fig. 4. Each of the two equal outer legs of the spandrel carries all the torsion brought in by the floor beam and the middle leg has zero torsion.

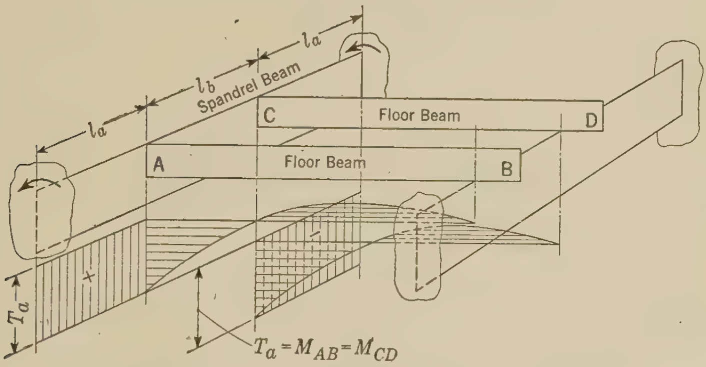

Fig. 4.—Torsion Distribution for Two Symmetrical Floor Beams

Sign Convention.—A sign convention is indicated in Figs. 2, 3, and 4 which follows that of ordinary vertical shear. This convention was used to distinguish between the directions of twist on the two sides of the applied torque and to illustrate the analogy that exists between torsional and vertical shear. This sign convention applies to the spandrel beam only when the beam is treated as an isolated or separate beam. It does not apply when the spandrel is treated as a part of a space structure for the purpose of distributing moments among the end of the floor beam and the two legs of the spandrel.

Where a single floor beam frames into a spandrel, the entire length of the latter resists the moment brought in by the floor beam. Therefore, for the purpose of distributing moments in the resulting space structure, the signs of the torque on the two sides of the floor beam will be like, and will be opposite to, that at the end of the floor beam.

In the three examples below, cases I, II, and III applying Eqs. 12 are illustrated in that order. The M_{AB} for each example is first solved by the appropriate, proposed formula and then checked by the method of moment distribution (see Fig. 5, and subsequently Figs. 6 and 8). L. E. Grinter, M. ASCE, originated this scheme of applying moment distribution to space structures.[7]

Illustrative Examples

Example 1.—A 12-in. WF 106-lb spandrel girder is 10 ft long and has a 10-in. I 30-lb floor beam framing into its web 3 ft from one end. The floor beam is one of two continuous 20-ft spans rigidly connected at its ends and carries a uniformly applied load of 1,000 lb per ft throughout the length of both spans. Find the maximum torsional shear in the spandrel web.

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[7] "Design of Reinforced Concrete in Torsion," by Paul Andersen, *Transactions*, ASCE, Vol. 103, 1938, p. 1503.

This example falls under case I and, hence, Eq. 12*a* applies. From Table 1 values of R and K for the 12-in. WF 106-lb spandrel beam are found to be 0.60 and 9.23, respectively; and $G = 0.4\,E = 11{,}600{,}000$ lb per sq in. Since only the relative values of S_T and S_F are needed for Eq. 12*a*, 29 will be used for E and 11.6 for G in Eqs. 11. The torsional stiffness factors for the two legs of the spandrel are, $S_{T3} = 1/3 \times 11.6 \times 9.23 = 35.7$ and $S_{T7} = 1/7 \times 11.6 \times 9.23 = 15.3$ (see Eq. 11*b* and Fig. 5).

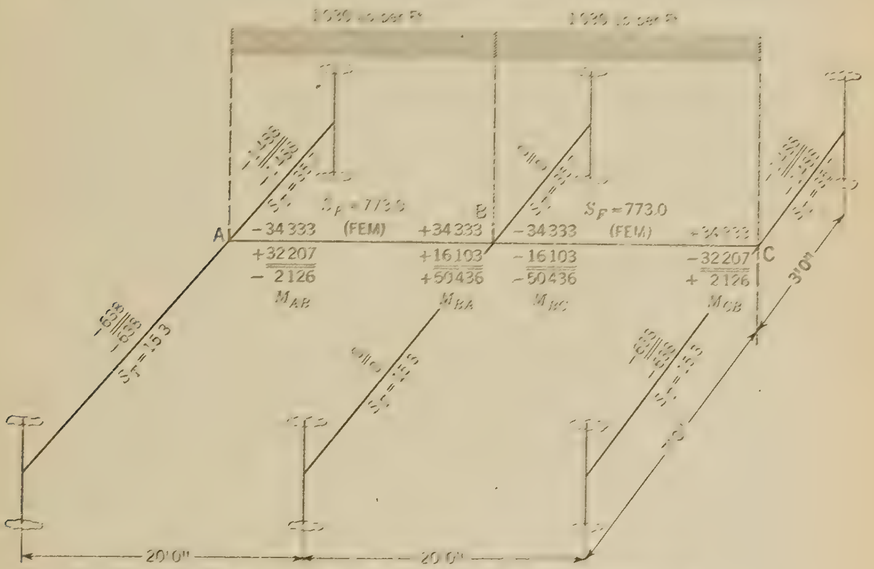
TABLE 1.—DIMENSIONS AND TORSION CONSTANTS FOR WIDE-FLANGE AND LIGHT BEAMS WITH PARALLEL-SIDED FLANGES

(Symbols Defined by Fig. 1)

Pounds per foot	INCHES^4	INCHES
	K	$m = n$
14 × 16 (R = 0.60)		
426	338.60	3.033
412	307.85	2.938
398	278.71	2.843
384	251.81	2.748
370	226.99	2.658
356	203.37	2.563
342	181.48	2.468
328	161.47	2.378
320	134.07	2.093
314	142.60	2.283
300	125.29	2.188
287	109.96	2.093
273	95.35	1.998
264	86.86	1.938
255	78.47	1.873
246	70.93	1.813
237	63.63	1.748
228	57.10	1.688
219	50.72	1.623
211	45.49	1.563
202	40.22	1.503
193	35.21	1.438
184	30.80	1.378
176	26.87	1.313
167	23.08	1.248
158	19.79	1.188
150	16.96	1.128
142	14.39	1.063
14 × 14½ (R = 0.60)		
136	13.69	1.063
127	11.22	0.998
119	9.31	0.938
111	7.56	0.873
103	6.09	0.813
95	4.75	0.748
87	3.72	0.688
14 × 12 (R = 0.60)		
84	4.48	0.778
78	3.57	0.718
14 × 10 (R = 0.60)		
74	3.92	0.783
68	3.06	0.718
61	2.22	0.643
14 × 8 (R = 0.60)		
58	2.54	0.718
53	1.97	0.658
48	1.47	0.593
43	1.06	0.528
12 × 12 (R = 0.60)		
190	49.96	1.736
176	39.89	1.606
161	31.20	1.486
147	23.84	1.356
133	17.95	1.236
120	13.13	1.106
106	9.23	0.986
99	7.54	0.921
92	6.09	0.856
85	4.87	0.796
79	3.90	0.736
72	2.98	0.671
65	2.21	0.606
12 × 10 (R = 0.60)		
64	2.81	0.701
58	2.14	0.641
53	1.61	0.576
12 × 8 (R = 0.60)		
50	1.82	0.641
45	1.34	0.576
40	0.97	0.516
10 × 10 (R = 0.50)		
136	26.69	1.498
124	20.37	1.368
112	15.31	1.248
10 × 10 (*Cont'd*)		
100	11.05	1.118
89	7.88	0.998
77	5.19	0.868
72	4.23	0.808
66	3.32	0.748
60	2.53	0.683
54	1.87	0.618
49	1.39	0.558
10 × 8 (R = 0.50)		
45	1.53	0.618
41	1.15	0.558
37	0.84	0.498
33	0.59	0.433
8 × 8 (R = 0.40)		
67	5.14	0.933
58	3.37	0.808
48	1.99	0.683
40	1.13	0.558
35	0.78	0.493
33	0.65	0.463
31	0.54	0.433
8 × 6½ (R = 0.40)		
27	0.49	0.448
24	0.35	0.398

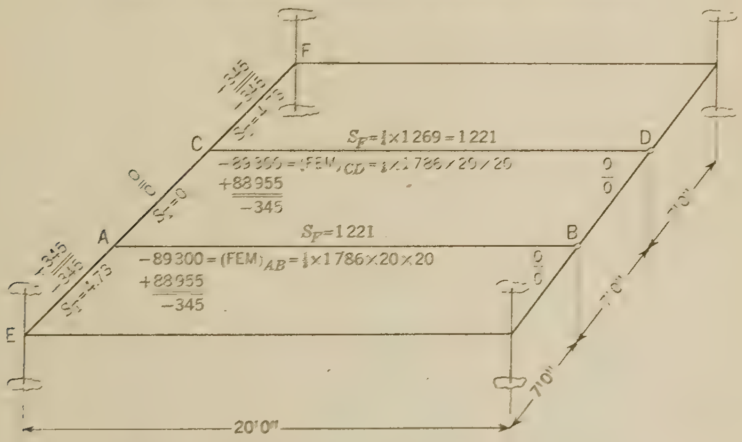
Also: $S_T = 35.7 + 15.3 = 51.0$; $S_F = \dfrac{4 \times 29 \times 133.5}{20} = 773.0$; (FEM) $= 1/12 \times 1{,}030 \times 20^2 = 34{,}333$ ft-lb; $M_{AB} = \dfrac{51}{51 + 773} \times 34{,}333 = 2{,}126$ ft-lb; $T_3 = 7/10 \times 2{,}126 \times 12 = 17{,}860$ in-lb; $T_7 = 3/10 \times 2{,}126 \times 12 = 7{,}650$ in-lb; and $s_w = \dfrac{17{,}860\ (0.62 + 0.3 \times 0.6)}{9.23} = 1{,}548$ lb per sq in. The values of M_{AB}, T_3, and T_7 are found by moment distribution in Fig. 5.

Example 2.—A 16-in. WF 64-lb spandrel girder, 21 ft long, has a 12-in. WF 36-lb floor beam framing into its web at each of the third points. The floor beams are 20 ft long, are freely supported at their inner ends, and each carries an applied load of 1,750 lb per ft. It is required to review the spandrel girder for maximum torsional shear in the web.



A
B
C
S_F=773.0
S_F=773.0
-34 333
(FEM)
+34 333
-34 333
(FEM)
+32 207
-2 126
M_{AB}
+16 103
+50 436
M_{BA}
-16 103
-50 436
M_{BC}
-32 207
+2 126
M_{CB}
S_T=153
20'0"
20'0"
3'0"

FIG. 5.—SOLUTION OF EXAMPLE 1 BY MOMENT DISTRIBUTION



F
C
D
A
B
E
S_F=⅛×1269=1221
-89 300 = (FEM)$_{CD}$ = ⅛×1 786×20×20
+88 955
-345
S_F=1221
-89 300 = (FEM)$_{AB}$ = ⅛×1786×20×20
+88 955
-345
0
0
-345
S_T=473
S_T=0
7'0"
7'0"
7'0"
20'0"

FIG. 6.—SOLUTION OF EXAMPLE 2 BY MOMENT DISTRIBUTION

As previously mentioned, only the torsional stiffness of one outer leg of the spandrel is opposed to the moment at the end of each floor beam in such a symmetrical arrangement of beams. This must be kept in mind when applying Eqs. 11*b* and 12*b*. For the 16-in. WF 64-lb spandrel (see Table 2), R, K, and t_w are 0.50, 2.85, and 0.443, respectively[5] (see Fig. 6).

TABLE 2.—DIMENSIONS AND TORSION CONSTANTS FOR WIDE-FLANGE AND LIGHT BEAMS WITH SLOPING FLANGES

(Symbols Defined by Fig. 1)

Pounds per foot	INCHES^4	INCHES	
	K	m	n
$36 \times 16\frac{1}{2}\ (R = 0.95)$			
300	68.80	1.876	1.484
280	56.43	1.766	1.374
260	44.78	1.636	1.244
250	39.68	1.576	1.184
240	35.13	1.516	1.124
230	30.94	1.456	1.064
$36 \times 12\ (R = 0.75)$			
194	23.69	1.402	1.118
182	19.61	1.322	1.038
170	16.03	1.242	0.958
160	13.23	1.162	0.878
150	10.77	1.082	0.798
$33 \times 15\frac{3}{4}\ (R = 0.90)$			
240	39.29	1.588	1.212
220	30.33	1.463	1.087
210	26.31	1.398	1.022
200	22.78	1.338	0.962
$33 \times 11\frac{1}{2}\ (R = 0.70)$			
152	13.18	1.192	0.918
141	10.36	1.097	0.823
132	8.35	1.017	0.743
125	6.88	0.942	0.668
$30 \times 15\ (R = 0.85)$			
210	30.69	1.494	1.136
200	26.53	1.429	1.071
190	22.85	1.364	1.006
180	19.58	1.304	0.946
172	17.01	1.244	0.886
$30 \times 10\frac{1}{2}\ (R = 0.65)$			
132	10.35	1.124	0.876
124	8.52	1.054	0.806
116	6.85	0.974	0.726
108	5.35	0.884	0.636
$27 \times 14\ (R = 0.80)$			
177	21.57	1.357	1.023
163	16.96	1.262	0.928
154	14.42	1.202	0.868
145	12.15	1.142	0.808
$27 \times 10\ (R = 0.60)$			
114	7.86	1.051	0.813
106	6.32	0.981	0.743
98	5.00	0.911	0.673
91	3.89	0.831	0.593
$24 \times 14\ (R = 0.70)$			
160	17.82	1.303	0.967
150	14.60	1.223	0.887
140	11.88	1.148	0.812
130	9.46	1.068	0.732
$24 \times 12\ (R = 0.65)$			
120	8.84	1.074	0.786
110	6.92	0.999	0.711
100	5.24	0.919	0.631
$24 \times 9\ (R = 0.50)$			
94	5.58	0.979	0.765
87	4.46	0.914	0.700
80	3.40	0.834	0.620
74	2.66	0.769	0.555
$21 \times 13\ (R = 0.65)$			
142	15.27	1.251	0.939
132	12.16	1.176	0.864
122	9.71	1.101	0.789
112	7.57	1.021	0.709
$21 \times 9\ (R = 0.55)$			
103	8.50	1.116	0.904
96	6.86	1.041	0.829
89	5.49	0.971	0.759
82	4.32	0.901	0.689
$21 \times 8\frac{1}{4}\ (R = 0.50)$			
73	3.23	0.838	0.642
68	2.62	0.783	0.587
63	2.04	0.718	0.522
59	1.67	0.673	0.477
$18 \times 11\frac{3}{4}\ (R = 0.60)$			
124	12.57	1.211	0.931
114	9.93	1.131	0.851
105	7.82	1.051	0.771
96	6.01	0.971	0.691
$18 \times 8\frac{3}{4}\ (R = 0.50)$			
85	5.82	1.015	0.807
77	4.42	0.935	0.727
70	3.32	0.855	0.647
64	2.56	0.790	0.582
$18 \times 7\frac{1}{2}\ (R = 0.40)$			
55	1.78	0.719	0.541
50	1.34	0.659	0.481
47	1.08	0.609	0.431
$16 \times 11\frac{1}{2}\ (R = 0.60)$			
114	11.01	1.173	0.897
105	8.71	1.093	0.817
96	6.75	1.013	0.737
88	5.19	0.933	0.657
$16 \times 8\frac{1}{2}\ (R = 0.50)$			
78	5.08	0.976	0.774
71	3.86	0.896	0.694
64	2.85	0.816	0.614
58	2.13	0.746	0.544
$16 \times 7\ (R = 0.40)$			
50	1.62	0.712	0.544
45	1.19	0.647	0.479
40	0.85	0.587	0.419
36	0.59	0.512	0.344
$14 \times 6\frac{3}{4}\ (R = 0.40)$			
42	1.16	0.654	0.492
38	0.86	0.594	0.432
34	0.61	0.534	0.372
30	0.41	0.464	0.302
$12 \times 6\frac{1}{2}\ (R = 0.35)$			
36	0.90	0.618	0.462
32	0.64	0.558	0.402
28	0.44	0.498	0.342
25	0.30	0.433	0.277
$12 \times 4\ (R = 0.30)$			
22	0.301	0.443	0.405
19	0.189	0.368	0.330
16.5	0.114	0.288	0.250
14	0.072	0.243	0.205
$10 \times 5\frac{3}{4}\ (R = 0.30)$			
29	0.62	0.569	0.431
26	0.46	0.519	0.381
23	0.31	0.459	0.321
21	0.23	0.409	0.271
$10 \times 4\ (R = 0.30)$			
19	0.239	0.413	0.375
17	0.160	0.348	0.310
15	0.106	0.288	0.250
11.5	0.050	0.223	0.185
$8 \times 5\frac{1}{4}\ (R = 0.30)$			
21	0.31	0.466	0.340
19	0.22	0.416	0.290
17	0.16	0.371	0.245
$8 \times 4\ (R = 0.30)$			
15	0.140	0.333	0.295
13	0.089	0.273	0.235
10	0.044	0.223	0.185
$7 \times 3\frac{1}{2}\ (R = 0.30)$			
12	0.112	0.364	0.282
$6 \times 6\ (R = 0.30)$[a]			
41	2.05	0.750	0.750
30	0.853	0.565	0.565
27	0.611	0.500	0.500
23	0.394	0.435	0.435
20	0.260	0.375	0.375
18[a]	0.177	0.343	0.285
15.5[a]	0.117	0.298	0.240
$6 \times 4\ (R = 0.25)$			
16	0.230	0.423	0.385
12	0.092	0.298	0.260
8.5	0.034	0.213	0.175
$6 \times 3\ (R = 0.30)$			
10	0.088	0.314	0.306
$5 \times 5\ (R = 0.313)$			
18.9	0.348	0.503	0.330
$4 \times 4\ (R = 0.25)$			
13	0.156	0.364	0.326
10	0.073	0.284	0.246
7.5	0.033	0.219	0.181

[a] $R = 0.25$ for the 6×6 at 18-lb and the 6×6 at 15.5-lb beams.

The remaining solution then is: $S_F = \dfrac{4\times29\times280.8}{20} = 1{,}629$; $S_T = \dfrac{11.6\times2.85}{7} = 4.73$; (FEM) $= 1/12\times1{,}786\times20^2 = 59{,}533$ ft-lb; $M_{AB} = M_{CD} = \dfrac{2\times4.73}{4.73+1{,}629} \times 59{,}533 = 345$ ft-lb; $T_{EA} = T_{CF} = 345 \times 12 = 4{,}140$ in-lb; and $s_w = \dfrac{4{,}140\ (0.443 + 0.3 \times 0.50)}{2.85} = 861$ lb per sq in.

Example 3.—Fig. 7 represents a typical floor plan of a multi-story, steel-framed, light factory building. It has a story height of 10 ft and 8-in. brick curtain walls. All steel connections are assumed to be rigid. The floor consists of a 4-in., reinforced-concrete slab supporting a live load of 125 lb per sq ft. All beams and girders are fireproofed in concrete.

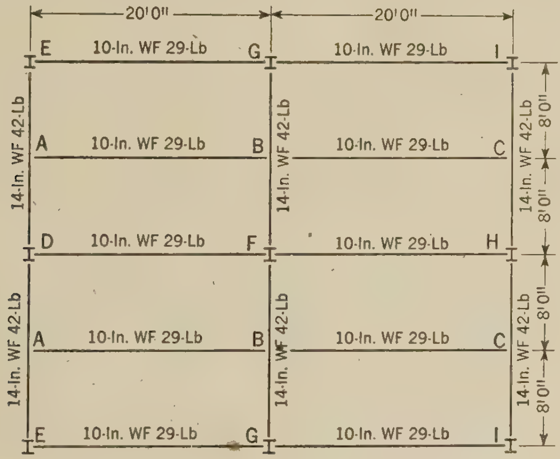

20'0"
20'0"
E
10-In. WF 29-Lb
G
10-In. WF 29-Lb
I
14-In. WF 42-Lb
A
10-In. WF 29-Lb
B
14-In. WF 42-Lb
10-In. WF 29-Lb
C
14-In. WF 42-Lb
8'0"
8'0"
D
10-In. WF 29-Lb
F
10-In. WF 29-Lb
H
14-In. WF 42-Lb
A
10-In. WF 29-Lb
B
14-In. WF 42-Lb
10-In. WF 29-Lb
C
14-In. WF 42-Lb
8'0"
8'0"
E
10-In. WF 29-Lb
G
10-In. WF 29-Lb
I

FIG. 7.—EXAMPLE 3

It is required to find the total maximum web shear in the spandrel girder DE (14-in. **WF** 42 lb) due to both direct and torsion loads. To insure maximum conditions the live load is to be omitted from the floor beam BC (see Fig. 8). It is required, also, to find the percentage of error involved in assuming this example to fall under case III.

The dead load carried by the floor beam, including fireproofing, is 520 lb per ft and the live load is 1,000 lb per ft. The K-value for the spandrel is 1.16 (see Table 2) and G is 11.6. The torsional stiffness factor for each end of the spandrel, the flexural stiffness factor for the floor beam, etc., are as follows: $S_T = \dfrac{11.6 \times 1.16}{8} = 1.68$; $S_F = \dfrac{4 \times 29 \times 157.3}{20} = 912.34$; $(\text{FEM})_{AB} = 1/12 \times 1{,}520 \times 20^2 = 50{,}667$ ft-lb; and $(\text{FEM})_{BC} = 1/12 \times 520 \times 20^2 = 17{,}333$ ft-lb.

From Fig. 8, M_{AB} is 246 ft-lb; the torque in the spandrel DE is 123 ft-lb on either side of the floor beam; and $s_w = \dfrac{123 \times 12\ (0.338 + 0.3 \times 0.4)}{1.16} = 583$ lb per sq in.

In computing the vertical shearing stress v in the web, the total shear V will be assumed evenly distributed over the web, the depth of the web being taken as the beam depth minus twice the mean flange thickness. That is,

$$v = \frac{V}{t_w (d - 2 t_f)} \quad \text{. . . . .}(14)$$

It can be demonstrated that Eq. 14 gives values for the average rolled steel beam that are much closer to the maximum v given by the laborious ratio $\frac{V Q}{I b}$ than do the usual approximate formulas.

Treating the floor beam AB as a free body and taking the end moments −246 ft-lb at A and +50,998 ft-lb at B (Fig. 8) into consideration, the maximum reaction that can be developed at A from the loaded floor beam AB is found to be 12,660 lb. The brick curtain wall weighs approximately 80 lb per sq ft of wall and the spandrel girder itself, including fireproofing, about 210 lb per ft; thus: $v = \frac{12{,}660 + 10 \times 16 \times 80 + 16 \times 210}{2 \times 0.338\,(14.24 - 2 \times 0.573)} = 3{,}257$ lb per sq in. The total shearing stress = 3,257 + 583 = 3,840 lb per sq in.

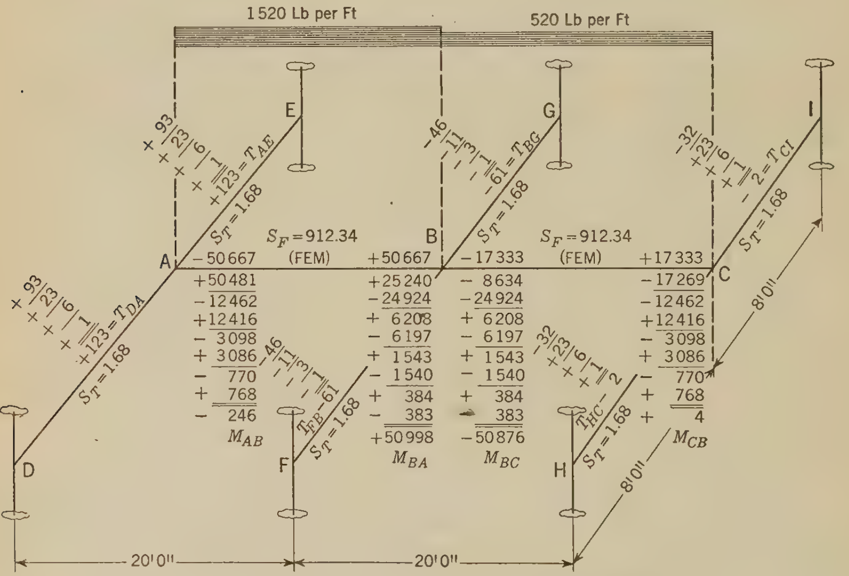

1520 Lb per Ft
520 Lb per Ft
E
G
I
A
B
C
D
F
H
S_F=912.34 (FEM)
S_F=912.34 (FEM)
ST=1.68
×93/23/6/1/×123=TAE
×93/23/6/1/×123=TDA
/46/11/3/1/61=TBG
/46/11/3/1/61=TFB
/32/23/6/1/2=TCI
/32/23/6/1/2=THC
−50667
+50481
−12462
+12416
−3098
+3086
−770
+768
−246
M_{AB}
+50667
+25240
−24924
+6208
−6197
+1543
−1540
+384
−383
+50998
M_{BA}
−17333
−8634
−24924
+6208
−6197
+1543
−1540
+384
−383
−50876
M_{BC}
+17333
−17269
−12462
+12416
−3098
+3086
−770
+768
+4
M_{CB}
8'0"
8'0"
20'0"
20'0"

FIG. 8.—SOLUTION OF EXAMPLE 3 BY MOMENT DISTRIBUTION

Short Method of Solving Example 3.—The assumption is made that loading one bay only with the live load will result in sufficient rotation of support B to give approximately 50% rigidity at that point and that, therefore, the example falls under case III. Then, $M_{AB} = \frac{3\,(1.68 + 1.68) \times 50{,}667}{2\,(1.68 + 1.68 + 912.34)} = 279$ ft-lb.

The percentage of error in M_{AB} is $\frac{279 - 246}{246} \times 100$, which is 13.4% on the conservative side. The shear is $s_w = \frac{279}{246} \times 583 = 661$ lb per sq in.; and the percentage of error in total shear is $\frac{661 - 583}{3{,}840} \times 100$, which is 2.03% on the conservative side. The percentage of error involved in neglecting the torsional shear is $\frac{583 \times 100}{3{,}840}$, which is 15.2% on the side of weakness.

Derivation of Equations for M_{AB}.—Eqs. 12*a* and 12*b* follow directly as a result of distributing the fixed-end moment of the floor beam among the end of the floor beam and the two adjacent legs of the spandrel girder. Their derivation would involve the basic principles of moment distribution. They are based on the following premises:

(1) The moment M_{AB} at the end of the floor beam is resisted entirely by the spandrel girder. In other words, the combined torsional stiffness of the two adjacent legs of the spandrel girder is opposed to the flexural stiffness of the floor beam.

(2) In order to balance the joint by the laws of moment distribution, the fixed-end moment at the end of the floor beam is distributed between it and the spandrel in the ratio of opposing stiffness factors.

(3) The sum of the torques in the two adjacent legs of the spandrel must equal the moment M_{AB} at the end of the floor beam.

(4) It can be demonstrated by moment distribution that, if the inner or end B of the floor beam, AB, is released, the resulting moment at the spandrel end is virtually twice what it would be with the inner end fixed; hence the factor 2 in Eq. 12*b*.

(5) Case III assumes 50% rigidity at the end B of the floor beam. Since this is the average of conditions assumed in cases I and II, the factor 3/2 is used in Eq. 12*c*.

CONCLUDING REMARKS

It is not good engineering practice to neglect any computable stress. Torsional shear derived from the foregoing cause may be as little as 15% of the total shear as shown in Example 3. Where the floor beam frames into the spandrel near the column, however, the shear from torsion is much more important (see Example 1). Again, it is conceivable that an intermediate stair landing might be suspended from a spandrel girder by means of hanger rods so connected as to produce torsion in the spandrel additive to that brought in by the floor beams. To neglect the torsional shear from the latter in such a case could result in an overstress.

Although this paper primarily concerns torsion in a spandrel girder, it provides, at the same time, a method for determining the moment M_{AB} at the end of the beam framing into the spandrel. It is obvious, therefore, that a method is indicated for taking the "guess" out of the end moment and the required top steel of an outside reinforced-concrete building slab (or beam) poured monolithically with its supporting spandrel girder.

Direct stress from secondary moment due to torsion was not considered. It may be found just as readily as the torsional shear by substituting the torque found by Eqs. 12 and 13, inclusive, in the appropriate equations, not included in this paper. Equations for direct stress from torsion are based on the assumption that the ends of the twisted beam are boxed in and welded to the end supports in such a manner as to prevent warping of the end sections, as well as relative movement of the two flanges at the ends. This type of connection is not used in ordinary building construction at the present time.

Other methods than moment distribution might have been used in this demonstration. The method of slope deflections, for example, would be quite adaptable. Another point about which discussion might arise is that of rigidity or extent of elasticity of beam to girder connections. The question as to whether the ordinary standard end connection is sufficiently rigid to absorb the amount of end moment involved is a legitimate one for discussion.

Some of the beam constants used in torsion equations are not given in the ordinary steel handbook. The important ones are the torsional constant K and the fillet radius R. Eqs. 6 give reasonably close values for K and may be used where tables of K-values are not available. Results that are sufficiently close for most purposes will be obtained if the following approximate values for R are used:

Depth d (in.)	Fillet radius R (in.)
20 or more	0.7
10 to 20	0.5
10 or less	0.3

APPENDIX. NOTATION

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

b = width of section;
c = an empirical constant evaluated by substituting known values for other constants in Eq. 5;
D = diameter of a circle inscribed in a beam section (see Fig. 1);
d = depth of a section;
E = Young's modulus of elasticity;
e = eccentric distance;
(FEM) = "fixed-end moment";
G = modulus of elasticity in shear;
I = rectangular moment of inertia;
J = polar moment of inertia;

K = a torsional constant; the measure of the torsional rigidity and twisting deflections of a beam;
L = length of shaft or beam;
l = lengths along a spandrel beam; l_a and l_b are distances from a load point to either end of the beam;
M = moment; (FEM) = "fixed-end moment";
m = the greater flange thickness t_f;
n = the lesser flange thickness t_f;
P = a concentrated load;
Q = area under the moment diagram;
R = radius of fillet;
r = radius of a shaft or cylindrical beam;
S = stiffness, subscripts f and t denoting "flexural" and "torsional," respectively;
s = torsional unit shearing stress at the surface of a shaft or cylindrical beam, subscripts w and f denoting "in the web" and "in the flange," respectively;
T = torsional moment or torque;
t = thickness, subscripts w and f denoting "web" and "flange," respectively;
U = a factor depending on the $\frac{b}{n}$-ratio, subscripts L and s denoting "large end of flange" and "small end of flange," respectively;
V = total vertical shear;
v = vertical unit shear stress;
α = a factor that depends on the ratios $\frac{t_w}{m}$ and $\frac{R}{m}$;
Δ = linear displacement; and
θ = total angle of twist, with appropriate subscripts to indicate the point of application.

DISCUSSION

JOHN E. GOLDBERG,[8] ASSOC. M. ASCE.—By coincidence, the writer has been working on a paper covering the same problem as that presented by Professor Lothers, and the incidental study of the problem has disclosed several circumstances worthy of mention.

Professor Lothers has stated that the flange bending stresses due to the torsional moment (stresses which, under "Concluding Remarks," he designates as "direct stress from secondary moment") may be neglected on the ground that warping of the spandrel cross sections is not prevented. On the basis of this statement, only the torsional shearing stresses are considered by the author. In the opinion of the writer, the assumption of lack of warping restraint is erroneous, and is without practical and physical foundation. It is difficult to find a spandrel in which considerable restraint against warping does not exist. Consequently, an analysis which overlooks the effects of warping is likely to be very much in error, particularly when the torsion is applied a short distance from the end of the spandrel.

An excellent illustration is found in Example 3 of the paper. Assuming (as did Professor Lothers) that each spandrel may be analyzed independently of its supports, then the midpoint of each spandrel defines not only the location of the applied torsion but also a plane of symmetry. Clearly, this implies perfect restraint against warping at this location. Consequently, the flexural rigidity of the flanges must be considered. If this is done by the method used by the writer, it is found that the true torsional strength of the beam is approximately 2.22 times the value used by Professor Lothers.

The short method developed by the author can be used to obtain the approximate value of the torsional moments and thus to determine the approximate magnitude of the ultimate discrepancy. For the spandrels, Example 3:

$$K' = 1.16\ (2.22) = 2.57$$

$$S_T = \frac{11.6 \times 2.57}{8} = 3.73$$

$$M_{AB} = \frac{3(3.73 + 3.73)\ (50{,}667)}{2(3.73 + 3.73 + 912.34)} = 617 \text{ ft-lb.}$$

By the same short-cut method, but neglecting bending stiffness of the flanges, Professor Lothers obtained 279 ft-lb. Since it must be admitted that warping is prevented at the plane of symmetry (that is, the midlength of the spandrel), it follows that this procedure leads to an error of 338 ft-lb in the value of the end moment of the floor beam. The entire torsional moment induced in the spandrel beam must be resisted, at or near the ends of the spandrel beam, by torsional shearing stresses the maximum values of which may be determined by Eqs. 7*a* and 7*b*, using the value of K taken from Table 1 or Table 2. As the analysis progresses from the end of the spandrel to the point of application of the torsional moment, a continuously smaller portion of the moment is carried by torsional shear stresses and an increased portion is carried by flange bending. At the midlength (the point of application of the torsional moment) practically the entire added moment of 169 ft-lb $\left(= \frac{338}{2}\right)$ is carried by flange bending and the distribution and magnitude of torsional shearing stresses are substantially as calculated by Professor Lothers.

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[8] Structures Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

The torsional moments increase as the length of the spandrel decreases. It may be noted that decreasing the spandrel of Example 3 from $L = 16$ ft to $L = 6$ ft (approximately) increases the torsional stiffness not to $16/6 = 2.67$ times its former value, but to 3.66 times that value.

The rotations at the ends of the spandrels are not necessarily zero and, therefore, have a great effect on the torsional moments distributed to the spandrel. A complete analysis should consider this effect which may be sufficiently great to reverse the torsional moment.

I. OESTERBLOM,[9] M. ASCE.—Many years ago it was known that the torsional effect on spandrel beams was severe; but designers were still thinking in terms of free static forces, and the procedure for computing these effects was very much a mystery. Slowly the idea that the strains in a structure were energy problems captured the consciousness of the engineers and they began to see ways out of many mystery problems—including the behavior of spandrels. It was a slow development, because most structures were highly indeterminate; and, even though one could write—if need be—any number of equations, it was only for simple cases that one was able to solve the system.

The spandrels are only a minor part of this difficulty; but—minor or major—the difficulty still exists, and the correct solution is well-nigh impossible if the designer is limited to classical theory and methods only.

Early in the writer's professional career he made attempts to find a solution of the torsion problem in concrete frames. He had to be satisfied with approximations. Nothing else at that time seemed feasible. Later, the subject was considered for steel spandrel beams by one of the large steel producers. Here also there was approximation in that one of the elements in the torque—namely, the eccentricity—was assumed. Although the scope was thus limited in both the foregoing attempts, and the methods were primitive, yet they were steps forward. There was a recognition not only that there was a problem, but that the problem was important. In fact it was discovered that torsional shears were heavy and often dangerous. It was important that they should be provided for.

Now Professor Lothers is urging—in fine style and with good practical sense—another step forward. He wants designers to evaluate the torsional effects of end restraints when secondary beams are framed into the spandrels. This is a very complicated problem if it is to be rationalized fully. Evidently, he knows this because he side-steps the difficulty by introducing an assumption.

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[9] Engr., Carbide and Carbon Chemicals Corp., South Charleston, W. Va.

"Three different conditions of restraint will be assumed * * * ," he states under the heading, "Torsion in Steel Spandrel Beams: Relative Stiffness (*c*)." Unfortunately, no one of the three (Eqs. 12) is likely to fit a specific problem very closely. In other words, it is again an approximation and not a rational solution. Although this is conceded it is not stated in a critical sense. The method is good; and with good judgment added in the selection of the proper case, the designer will be much closer to a safe and economic design than he would by the older method of judicious guessing—followed by a prayer that Providence would be kind in the matter of strain and stress.

With this grateful acceptance of the help that is offered in the paper, the structural engineer may yet be allowed to ask: Is that the best that can be done today? Does the profession have no means to fill the "wide open spaces" between the three cases offered?

In fact, such means are available; the problem of spandrels is merely a part of the larger problem of the space frame which can now be solved—if the designer can keep in good order the great mass of basic material from which he must start and the progressive steps in the process of relaxation that follow. So far, the torsional moments and reactions, and their behavior in an elastic framework, have been woefully neglected by authoritative writers of texts and technical papers. Can it be that these writers are still suffering from the confusion that attends the efforts of a novice to keep signs and computations in order during the process?

Surely that should not apply to experienced pundits of engineering science. This paper, therefore, is a challenge to the younger pundits to whet their imaginations, and take the next step forward courageously and in full stride. The ground ahead is neither soft nor hard; it is just fair to the one that has the wits and the will.

ROBERT V. HAUER,[10] ESQ.—The method of computing torsional stresses in spandrels, as described by the author, is sufficiently accurate, if applied to a concrete construction with the necessary modifications; but it is not satisfactory in the case of steel beams.

Even if the spandrel is assumed to be connected to the columns in such a way that the end cross sections are free to warp, there must be one section in the central part of the spandrel which remains plane and which is therefore the origin of a certain stress irregularity. In steel beams this causes a considerable increase of the torsional stiffness, changes the distribution of the shear stresses, and gives rise to normal stresses in the middle of the girder. The latter are of much more interest than the shear stresses, because they are additional to the bending stresses that generally govern the design of the spandrel.

Checking Example 3, by the use of the approximate method[11] of S. Timoshenko, shows that the torsional stiffness of the spandrel is 2.11 times as great as that computed by the author and that the additional normal stresses in the flanges amount to 2,780 lb per sq in.—a quantity which can hardly be considered negligible. The shear stress in the web due to torsion alone is found to be 43% higher than the value given by the author.

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[10] Structural Engr., Albert Kahn, Inc., Detroit, Mich.

[11] *Zeitschrift Mathematik Physik*, Vol. 58, 1910, p. 361.

In principle, the behavior of a rectangular spandrel as used in concrete is the same. However, the difference is that, in a rectangular beam, a stress irregularity in a certain section practically disappears within a very short distance from this section and therefore does not materially affect the member as a whole, whereas an I-beam, because of its particular shape, emphasizes such irregularities. In the quarter points of the spandrel investigated in Example 3—that is, at a distance from the center equal to about three and one-half times the depth of the beam—the additional normal stress in the flanges due to torsion is still 35% of its maximum value. It might be stated that an I-beam is somewhat reluctant to follow the well-known principle of de Saint-Venant.

Since the torsional stiffness of any part of an I-spandrel depends on the degree of restraint against warping of its end sections, this stiffness cannot generally be determined in advance, but is governed by the final distribution of the torsional moments in the entire spandrel. Therefore, both the method of moment distribution and the slope-deflection method fail in problems like those discussed by the author, except in the cases where only one floor beam or two symmetrical floor beams frame into the spandrel. In all other cases only the classical method of redundant moments offers a solution.

E. I. Fiesenheiser,[12] Assoc. M. ASCE.—The profession is indebted to Professor Lothers for presenting a method of analysis for torsional stresses in steel spandrel girders and particularly for Tables 1 and 2, which give torsion constants for use in design.

The analysis of steel frameworks for torsional stresses is a refinement not often used in steel building design, partly because shear stresses do not usually govern in the selection of floor beams. Ordinarily, attention is directed mainly to analysis for moment. Moreover, it is the writer's opinion that torsional analysis is not justified unless another factor of equal importance is also considered.

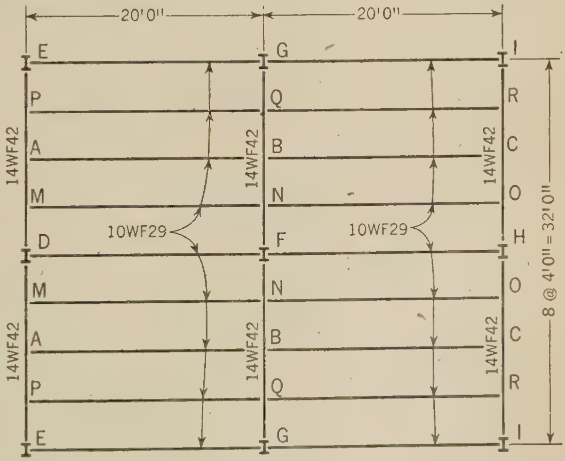

Fig. 9

The manner in which the ends of filler beams are connected to their supporting girders in actual practice is fully as important if greater refinement in design is to be used. Standard, riveted connection angles will probably be adequate to transmit the torque of a spandrel girder to its supporting column;

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[12] Associate Prof. of Civ. Eng., Illinois Inst. of Tech., Chicago, Ill.

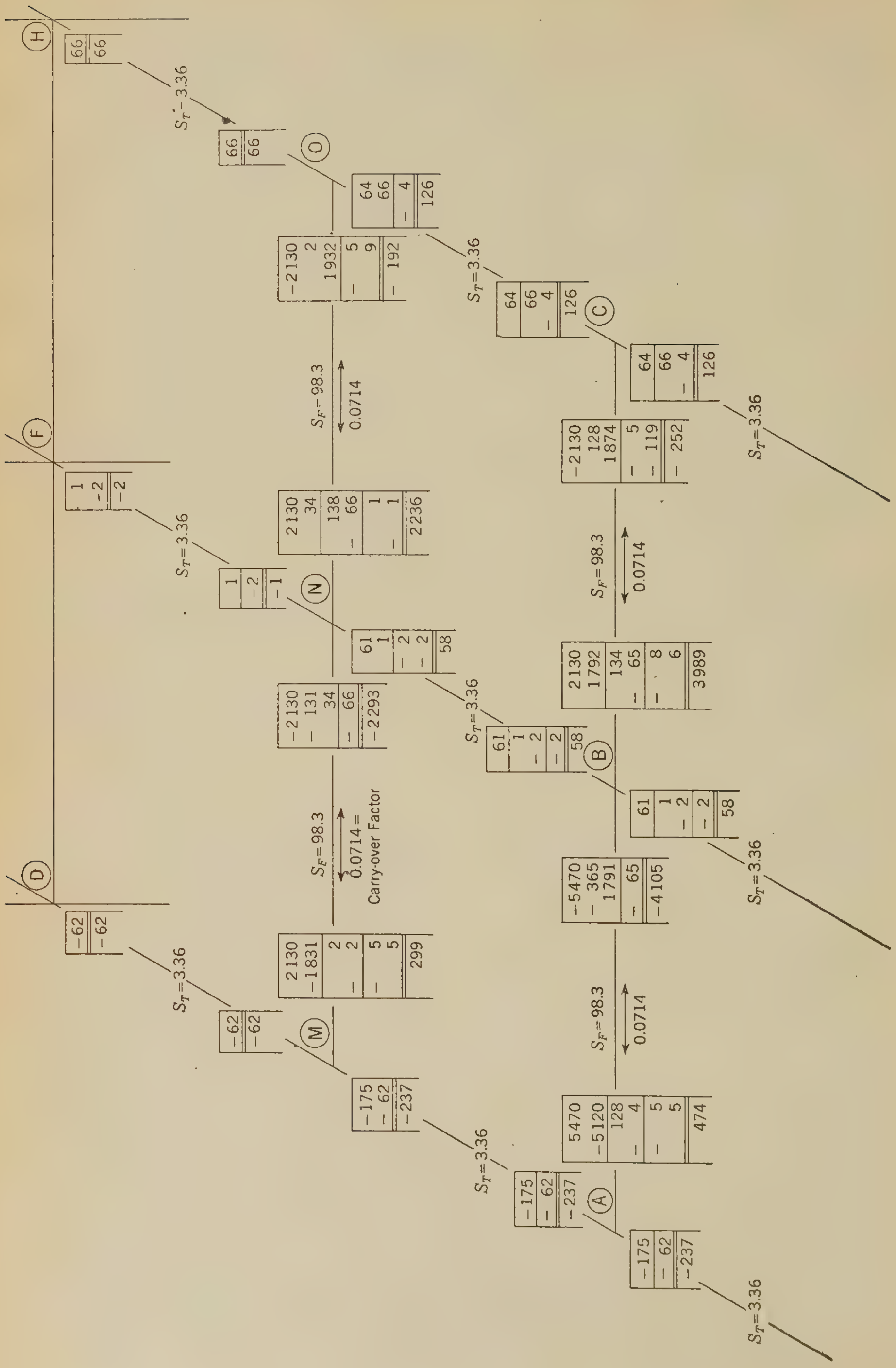
H
D
F
M
N
O
A
B
C
$S_T = 3.36$
$S_F = 98.3$
0.0714
0.0714 = Carry-over Factor

Fig. 10

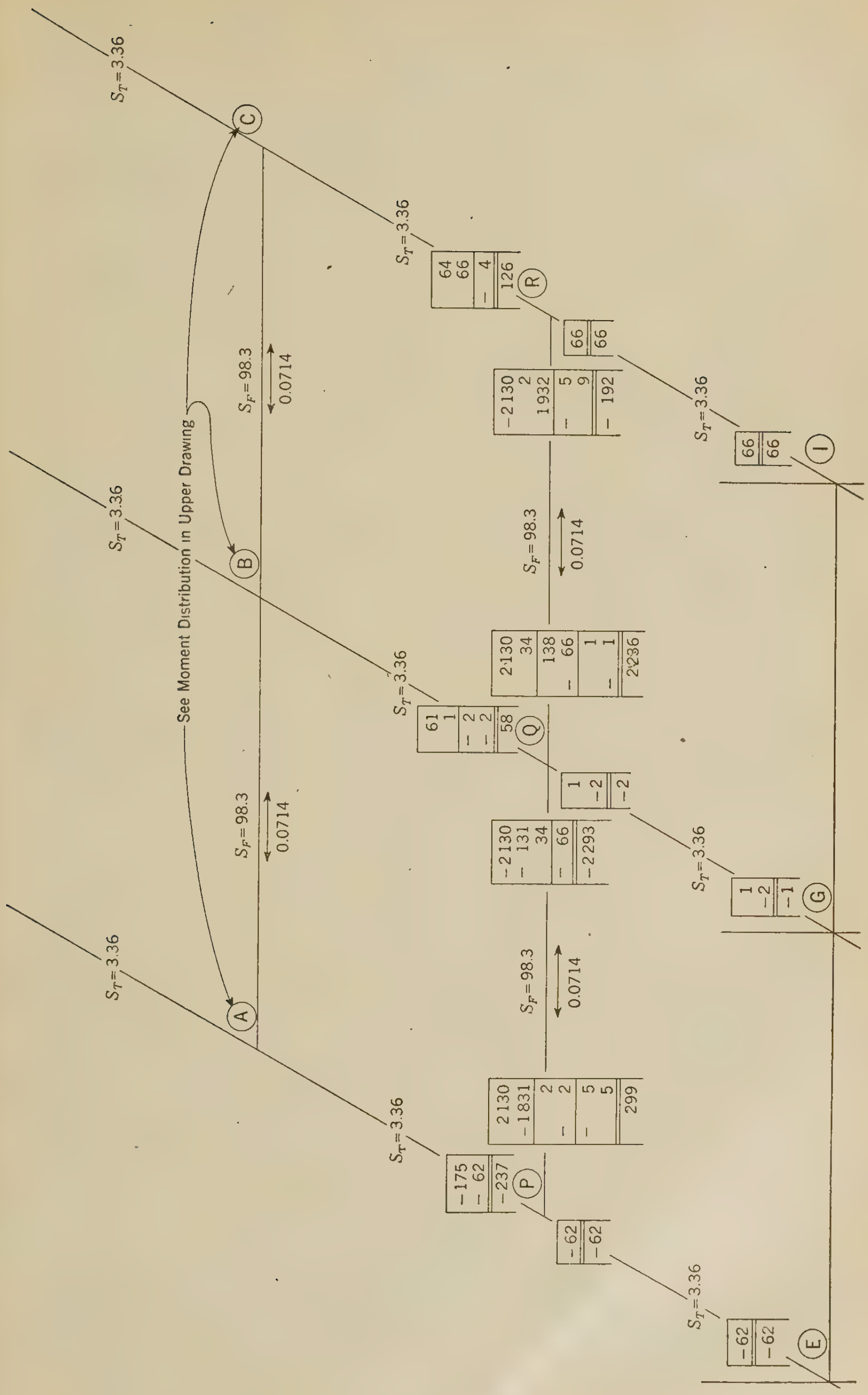
See Moment Distribution in Upper Drawing
A
B
C
P
Q
R
E
G
I
$S_T = 3.36$
$S_F = 98.3$
0.0714
−175
−62
−237
2130
−1831
−2
2
−5
5
299
−62
−62
−62
−62
−2130
−131
34
−66
−2293
61
1
−2
−2
58
1
−2
−2
1
−2
−1
2130
34
138
−66
−1
1
2236
−2130
2
1932
−5
9
−192
64
66
−4
126
66
66
66
66

Fig. 10.—(*Continued*)

but, in beam and girder framing, the end connection angles will not be sufficiently rigid to transmit a large end moment into a joint without some reduction. End connections should not be assumed rigid unless special attention is paid to their design to make them so. Ordinary connections develop only partial continuity. Hence, it is common practice to design steel beams as simply supported, a practice justified only as being safe, and not as being economical. The writer proposes to show how actual connection restraint may be included in an analysis, if desired.

Referring to Example 3, Figs. 7 and 8, the author considers the connection of the 10WF29 filler beams to the girders to be rigid. The end moment of 50,998 ft-lb will then have to be resisted by the connection at point B. If an ordinary connection, consisting of four rivets in bearing on the web, is used, it will fail to resist this moment. The connection angles will deform or yield, requiring beam AB to resist a larger moment near the center of the span than it was designed to carry. Consequently, the beam will be overstressed.

As an example, the writer proposes to use the 10WF29 beams at 4-ft centers, as shown in Fig. 9. The distribution of moments in three cycles is demonstrated in Fig. 10, in which the order of distribution is: Joints A, B, C, M, P, N, Q, O, and R. Although an economical design for the beams does not result, neither does an overstress of the standard connection. This particular connection was tested by J. Charles Rathbun,[13] M. ASCE, who explained how connection restraint may be taken into account in an analysis. After a decade, this excellent paper is still very much to the point, and the methods given will be applied to the example, using moment distribution.

Because an end connection is not as stiff as the connected beam, the fixed-end moment, the carry-over factor, and the flexural stiffness of the member should be computed by taking into account actual conditions. The decrease in member stiffness enters the analysis by the use of a revised length of span, transformed by the connection. This revised length is termed L_2, and is equal to $L + 3\,E\,I\,Z$. The term $Z = \frac{\theta_M}{M}$ is the ratio of the angle of rotation θ_M of the end connection to the moment M producing the rotation. Its value is determined by test.

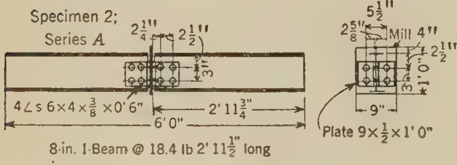

Fig. 11

A standard connection[14] for a 10-in. beam is to be used, as shown in Fig. 11. Assuming an average slope and with the assistance of available curves,[15] $\frac{1}{Z} = 9.5(10)^6$ in.-lb. The revised length,

$$L_2 = 240 + \frac{3(29)(10)^6 157.3}{9.5(10)^6} = 1{,}680 \text{ in.}$$

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[13] "Elastic Properties of Riveted Connections," by J. Charles Rathbun, *Transactions*, ASCE, Vol. 101, 1936, p. 524.

[14] *Ibid.*, p. 527, Fig. 1, Specimen 2.

[15] *Ibid.*, p. 538, Fig. 13, Specimen 2.

The fixed-end moment for symmetrical loading is given by the expression:

$$(\text{FEM}) = \frac{6\,Q}{L}\,\frac{2\,L_2\,\frac{L}{2} - L\,\frac{L}{2}}{4\,(L_2)^2 - L^2} \qquad (15)$$

in which Q, the area under the moment curve for a simple beam, is equal to $\frac{w\,L^3}{12}$ for a uniform load of w per unit of length. Substitution yields a fixed-end moment of $\frac{w\,L^2}{60}$ for the 10WF beams. The carry-over factor $\frac{L}{2\,L_2} = \frac{240}{2 \times 1{,}680} = 0.0714$. The flexural stiffness—

$$S_F = \frac{3\,E\,I}{L_2 - \frac{L^2}{4\,L_2}} \qquad (16)$$

—yields $0.431\,\frac{E\,I}{L} = \frac{0.431 \times 29 \times 157.3}{20} = 98.3$.

In applying moment and torque distribution for space frame structures to to this example, the writer uses the process described by L. E. Grinter,[16] M. ASCE. The sign convention for moment or torque assumes that a moment or torque tending to produce clockwise rotation of a joint is positive.

The stiffness factor $S_T = \frac{G\,K}{L} = \frac{11.6 \times 1.16}{4.00} = 3.36$ for the 14WF 42 girders subjected to torque. The carry-over factor for torque is -1.0.

The revised load carried by the filler beams is now 320 lb per ft dead load, and 500 lb per ft live load. In the example to follow, beam AB only is loaded with live load. For this beam the fixed-end moments are $\frac{(320 + 500)(20)^2}{60} = 5{,}470$ ft-lb. For the other beams the fixed-end moments are $\frac{320\,(20)^2}{60} = 2{,}130$ ft-lb.

The numerical work was done by slide rule since any refinements in accuracy greater than 1% or 2% are of no significance. The distribution begins at joint A in Fig. 10 and proceeds through three complete cycles. The original fixed-end moment at joint A is 5,470 ft-lb. When this joint is released, it is permitted to rotate in a clockwise direction. The accompanying balancing moments are $-5{,}120$ for member AB, and -175 for members AM and AP, which are in proportion to the S_F-factor and S_T-factor for the members. The clockwise rotation at joint A induces a counterclockwise moment at end B of $0.0714 \times 5{,}120 = 365$, which is written in with the minus sign at joint B to indicate the direction. The balancing moment or torque of -175, accompanying the clockwise rotation at joint A, induces a clockwise moment at joints M and P, of 175, which is written in as $+175$ at these joints. A single straight line is drawn under each balancing moment to indicate completion of distribution. After all moment and torque have been balanced, a double line is drawn before adding the columns at the various joints.

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[16] "Elastic Properties of Riveted Connections," by J. Charles Rathbun, *Transactions*, ASCE, Vol. 101, 1936, Vol. 103, 1938, p. 1512.

The maximum end moment to be carried by any connection is 4,121 ft-lb at end B of member AB, which is not too high for the ordinary connection to withstand.

The method of moment distribution applied to the analysis of space frames is more tedious, although scarcely more complicated, than is moment distribution applied to planar structures. Its use is justified in the investigation of torsional shearing stresses and in the study of connections.

Edward V. Gant,[17] Assoc. M. ASCE.—As the author has stated, one of the problems that arises in investigating the torsion of steel spandrel girders is the assumption regarding the rigidity of the connection between the floor beam and the spandrel girder. There is not much experimental information on this point available; but, according to recommendations made by the Steel Structures Research Committee of Great Britain,[18] no allowance should be made for a restraining moment at the end of a beam which frames into the web of a girder unless there is a similar beam framing into the girder on the opposite side at the same point. Thus, with ordinary steel construction there would seem to be little moment developed at the spandrel end of a floor beam, and consequently torsion in the spandrel girder from this source could be neglected. However, where welded connections are used, or where special provision is made for a restraining moment at the spandrel end of a floor beam, some moment may be developed; and an investigation of the resulting torsion in the spandrel girder would be desirable. In any case, a definite answer to the question of the rigidity of the connection between the spandrel girder and the floor beam would seem to await additional experimental evidence.

Assuming that the connection between the spandrel girder and the floor beam is rigid enough to transmit moment, Professor Lothers has proposed simple formulas (Eqs. 12) for the moment at the spandrel end of the floor beam. It should be emphasized that these formulas do not allow fully for the facts that the spandrel girder and the floor beam are units of a structural frame and that deformation of other members may influence their deformation. For instance, the rotation of the ends of the spandrel girder is a function of column deformation and could either relieve or increase the torsion in the spandrel girder, depending on the direction of rotation.

Another factor that may influence the validity of Eqs. 12 is the relative vertical deflection of the ends of the floor beam that frames into the spandrel girder. The effect of this deflection would be to add algebraically a moment to the fixed-end moment $(FEM)_{AB}$ in Eqs. 12. From Eq. 10 and a similar equation for the moment M_{BA} at the inner end of the floor beam, the corrections would be

$$\pm \frac{6\,E\,I\,\Delta}{L^2}; \qquad \pm \frac{2\,E\,I\,\Delta}{L^2}; \qquad \text{and} \qquad \pm \frac{3.3\,E\,I\,\Delta}{L^2} \quad \ldots\ldots\ldots (17)$$

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[17] Asst. Prof., Civ. Eng., The Univ. of Connecticut, Storrs, Conn.

[18] "Final Report of the Steel Structures Research Committee," Dept. of Scientific and Industrial Research, His Majesty's Stationery Office, London, 1936, p. 552.

for Eqs. 12*a*, 12*b*, and 12*c*, respectively. In many cases the magnitude of the correction will be small. However, the possibility of a large relative deflection modifying the moments in Eqs. 12 should not be overlooked.

Oscar Hoffman,[19] M. ASCE.—A better understanding of the actual behavior of structural frameworks, beyond that which is given by simple two-dimensional analysis, is contributed by this excellent paper. The methods presented are very straightforward and workable in cases similar or closely related to those illustrated by the author; but they would become somewhat impracticable and cumbersome for a larger number of floor beams, say, three or more, within a single spandrel girder span.

The writer wishes to call attention to the possibility of using finite difference equations in handling such cases. The fundamentals of the calculus of finite differences, and some interesting applications of it to structural problems can be found in a book by Theodor von Kármán and M. A. Biot[20] published in 1940.

The method of approach to the spandrel girder problem can best be illustrated by its application to a comparatively simple case—a single-span spandrel girder with fixed-end supports (which may be visualized as an interior span of a continuous girder supported by infinitely rigid columns) carrying a number of floor beams, placed at constant intervals, a. The girder span is $l = N a$, in which N is an integer. The floor beams are so arranged that there is one of them at the center line of each supporting column. All floor beams are assumed to be fixed at their inner ends, to have the same moment of inertia, I, and to carry the same uniform load, w, per unit length. Fig. 12(*a*) shows a view of the spandrel girder. Figs. 12(*b*), 12(*c*), and 12(*d*) show the torques, M_x, acting on the girder at each floor beam intersection; the torsional moment, T_x, along the girder; and the angle of twist, θ_x. Fig. 12 shows a complete analogy with the load diagram, shear diagram, and moment diagram of an imaginary simple beam having the span length l and being loaded with the imaginary concentrated loads, M_x. Fig. 13(*a*) represents the forces acting on a typical floor beam and Fig. 13(*b*) is the corresponding moment diagram.

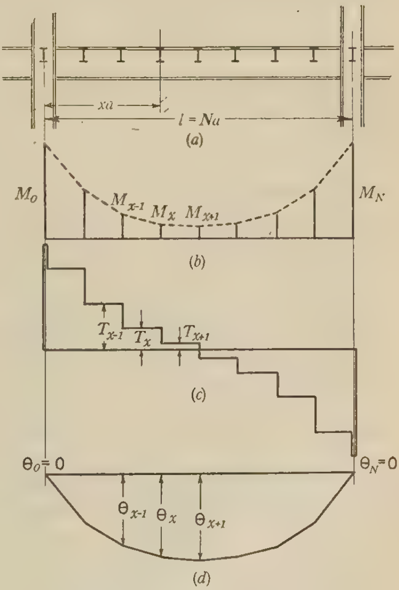

FIG. 12

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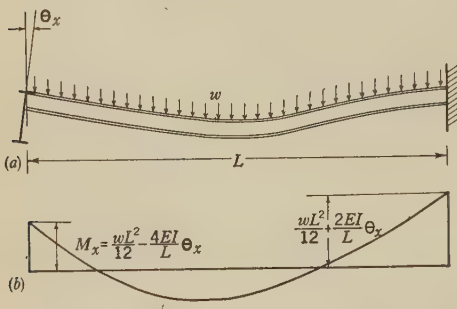
[19] Associate Prof., Civ. Eng., Case School of Applied Science, Cleveland, Ohio.

[20] "Mathematical Methods in Engineering," by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940, p. 437.

The relationship between the difference in the torsional moments between x and $x - 1$ and the change in the angle of twist between these points follows from the elastic properties of the spandrel girder, by using Professor Lothers' notations, G and K:

$$\theta_x - \theta_{x-1} = T_x \frac{a}{G\,K} \qquad \text{(18)}$$



θ_x
w
(a)
L
(b)
$M_x = \frac{wL^2}{12} - \frac{4EI}{L}\theta_x$
$\frac{wL^2}{12} + \frac{2EI}{L}\theta_x$

FIG. 13

At the same time, θ_x represents the outer end slope of the floor beam x, and M_x represents the outer end moment for the same floor beam. The relationship between those two quantities, readily obtained, for instance, from the slope-deflection equation, is:

$$\theta_x = \left(\frac{w\,L^2}{12} - M_x\right)\frac{L}{4\,E\,I} \qquad \text{(19)}$$

A third basic relationship is that the change in torsional moment from a point to the left to another point to the right of the intersection of a floor beam with the spandrel girder is equal to the torque, M_x, developed by the rigid connection of the floor beam with the girder. Then:

$$M_x = T_x - T_{x+1} \qquad \text{(20)}$$

Substituting in Eq. 18 the expression for θ_x given in Eq. 19 and a similarly built expression for θ_{x-1}, and solving for T_x, one obtains:

$$T_x = (\theta_x - \theta_{x-1})\frac{G\,K}{a} \qquad \text{(21)}$$

Substitution of this expression, and of a similar one for T_{x+1}, in Eq. 20 yields:

$$M_{x-1} - \left(\frac{4\,a\,E\,I}{L\,G\,K} + 2\right) M_x + M_{x+1} = 0 \qquad \text{(22)}$$

a linear difference equation of second order. The method of finding the solution of such an equation is discussed and abundantly illustrated by Messrs. von Kármán and Biot.[20] Its application to the present problem leads to the expression:

$$M_x = \frac{w\,L^2}{12}\,\frac{\beta^x + \beta^{N-x}}{\beta^N + 1} \qquad \text{(23)}$$

in which

$$\beta = 1 + \frac{2\,a\,E\,I}{L\,G\,K}\left(1 + \sqrt{1 + \frac{L\,G\,K}{a\,E\,I}}\right) \qquad \text{(24)}$$

The assertion that Eq. 23 satisfies Eq. 22 and the boundary conditions of the problem ($\theta_0 = 0$ and $\theta_N = 0$) can be verified by simple substitution.

The expression for the torsional moment between the floor beams $x - 1$ and x is to be mentioned because of a subsequent use of it:

$$T_x = \frac{w\,L^2}{12}\,\frac{\beta^{N-x+1} - \beta^x}{(\beta^N + 1)\,(\beta - 1)} \quad \text{(25)}$$

Eq. 23 has some general implications. It shows that the behavior of spandrel girders depends on β, which is given by Eq. 24 as a function of the stiffness ratio $\frac{a\,E\,I}{L\,G\,K}$.

A survey of the possible range of practical limits of values for the stiffness ratio in steel structures, with the usual low torsional stiffness of the spandrel girder, shows that, in general, β can be approximated satisfactorily by the expression:

$$\beta \approx \frac{4\,a\,E\,I}{L\,G\,K} \quad \text{(26)}$$

and, as the stiffness ratio is seldom less than 15, β will be larger than 60 in most cases.

In such cases, the end moment of the floor beam 1, M_1, and the end moment of the floor beam 0, M_0, will be related by the approximate formula:

$$M_1 \approx \frac{M_0}{\beta} \quad \text{(27}a\text{)}$$

and, similarly,

$$M_2 \approx \frac{M_1}{\beta} \approx \frac{M_0}{\beta^2} \quad \text{(27}b\text{)}$$

In other words, M_1 drops to less than 2% of M_0 and the end moments of the floor beams with order numbers from 2 to $N - 2$ are negligible, for all practical purposes, irrespective of the number of floor beams. Furthermore, the total torque transferred by the spandrel girder into the column can be estimated with an error of less than 2%, as being equal to M_0. In the example herein discussed, $M_0 = \frac{w\,L^2}{12}$ results from the assumption of infinitely stiff columns, but the foregoing considerations indicate the possibility of taking into account, with a fair approximation, a finite column stiffness in conjunction with a single floor beam —the one that frames directly into the column.

An essentially different situation arises with reinforced-concrete structures because of the higher torsional stiffness of the spandrel girders and of the consequently lower values of β. For such structures, β can vary from a little more than 1 to 5; but it seldom passes the latter limit. As a consequence, the moment, M_x, decreases slowly from its maximum value, at the columns; and its magnitude is no longer negligible in the design of the floor beams, even at the center of the spandrel girder span.

The total torque, transferred to the column, which forms an intermediate support for a continuous spandrel girder, is $M_0 + 2\,T_1$, and, in the specific example considered herein, the following expression can be found for it:

$$\frac{w\,L^2}{12}\,\frac{(\beta^N - 1)\,(\beta + 1)}{(\beta^N + 1)\,(\beta - 1)} = \frac{w\,L^2}{12}\,\nu \ldots\ldots\ldots\ldots (28)$$

In Eq. 28, the factor:

$$\nu = \frac{(\beta^N - 1)\,(\beta + 1)}{(\beta^N + 1)\,(\beta - 1)} \ldots\ldots\ldots\ldots (29)$$

is evidently the number of floor beams that develop the fixed-end moments to be transferred fully into the column. The product $\nu\,a$ can be considered as the width of the tributary strip of floor which is to be taken into account in cases of finite column stiffness, with its moment of inertia equal to $\nu\,I$.

The writer wishes to emphasize the simplifying assumptions upon which the foregoing specific example is founded, and which are primarily responsible for the reasonably simple results obtained; yet, the writer feels strongly that the method of approach illustrated herein may have some broader applications, although the mathematics involved in such applications might prove to be somewhat more formidable.

Phil M. Ferguson,[21] M. ASCE.—The writer has watched the development of the torsion formula for steel beams and the attempts to apply it to practical problems with a great deal of interest. A solution of this problem is needed in many situations and the trend of architectural development each year adds to the need; yet the freestanding steel beam subject to torsion is relatively a rare condition. In building construction, there is nearly always a restraint present that is not considered in the conventional treatment of the problem, for example, a concrete floor slab or a stiff wall encasement.

The author's solution of the problem is interesting and helpful. However, it should be emphasized that its field of usefulness is limited to cases in which the girder is free to rotate in accordance with the simple torsion formula (Eq. 2). This eliminates most cases in which a concrete slab frames into the girder, including all cases where the girder is encased in concrete, and probably most cases where the slab is simply poured on or around the top flange of the girder.

In ordinary design the assistance that the slab gives to the beam may be, and generally is, ignored. The slab is likewise considered as an entirely separate unit for design purposes. However, a concrete slab resting on a steel beam increases the stiffness and reduces the deflection of the beam very materially, even when not accompanied by concrete fireproofing of the beam. It would thus seem necessary, in dealing with spandrel girder torsional stresses that depend chiefly on beam deflection, to consider the probable composite action of steel beam and concrete slab. The slab modifies the beam action in several ways:

1. The stiffness of the composite beam is much greater than that of the steel beam alone;

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2. The increased stiffness greatly reduces the beam deflection and the end rotation that results, whether the spandrel girder is similarly reinforced or not; and

3. The neutral axis of the composite beam lies much higher—above the middepth of the steel beam.

The effect of these differences will be discussed after the action of the girder is considered.

The slab also modifies the girder action. Even when the girder is not encased, and the web and bottom flange are thus left almost entirely free from concrete restraint, the top flange is usually interlocked with the concrete. Lateral movements of the girder top flange that can result are largely determined by the movements of the concrete; these movements depend on the slab deflection and the extent to which the slab is shortened by its action as part of the compression flange of the beam.

A comparison between the author's assumed girder flange deflection and the probable deflection when a concrete slab is present will reveal the difference. The simple torsion formula assumes that the upper girder flange is displaced laterally by the end rotation of the beam it carries, and that this displacement decreases to zero at the end of the girder at essentially a linear rate. In other words, a plan view of the deflected flange is assumed to be essentially a straight line between the beam connection and the end of the girder. The concrete slab, on the other hand, shortens lengthwise, with the beam axis and perpendicular to the girder, in an amount that is dependent upon the extent to which it serves as a compression flange for the beam. Generally this flange action will correspond to nearly perfect adhesion between the concrete and the steel beam. It is not probable that this concrete shortening will decrease directly with the distance from the beam, as would be required to permit the steel girder to satisfy the simple torsion formula. Instead, it might be expected that this shortening would be nearly uniform for a considerable distance on each side of the beam. The upper girder flange must be displaced to conform to this concrete pattern, as if the beam had a very wide connection tied in to the top girder flange. The result seems to be that the top girder flange is displaced almost uniformly over a considerable length, leaving a much shorter length within which it must return to its normal position (if it does so return at the end of the girder). At the same time, the bottom flange retains greater freedom of action. The resulting torsional resistance to this type of rotation and displacement must be considerably different from that indicated by the simple torsion formula.

It would also appear that the rotation impressed by the connection of the composite beam is a twisting about an axis at the level of the composite beam axis, and this is several inches above the middepth of the beam and the girder.

The rotation of the girder over most of its length thus seems to be about an axis which varies from point to point, and which is generally not through the centroid of the cross section. The torsional stiffness will surely be increased by this shifting of the center of rotation, since it adds a lateral displacement of the girder to the rotation about an axis through the centroid. This aspect of the problem seems to have been largely neglected in the engineering literature that has come to the writer's attention. In the related field of beams curved in plan, elaborate formulas have been developed which assume a beam to be free to twist about a centroidal axis; but no mention is made of the concrete slab which usually enforces entirely different deformations. This seems more serious than approximations for the value of K.

The effect of the concrete slab can be evaluated more definitely in its relation to the bending stiffness of the beam, which is greatly increased even when the beam is not encased. Although the exact amount of increase may be subject to some uncertainties, a 4-in. concrete slab added to Example 2 would at least double the beam stiffness. This increase in beam stiffness would result in a reduction of approximately 50% in the total rotation of the girder envisioned in this paper; and girder web shears created by torsion would seem to be reduced proportionately. However, the writer is disturbed by the feeling that the complex type of rotation described herein would have a considerable effect on the resulting torsional stiffness and torsional stresses. For this reason, he does not state that torsional shears are necessarily less serious than those Professor Lothers would find; but they may be quite different.

If the spandrel girder is encased in concrete, its torsional action would be greatly modified, and an entirely different analysis would be necessary. Especially in this case, the slab moments on a section parallel and adjacent to the girder flange (caused by the deflection of the slab relative to the girder) would seem to be of considerable importance. When strictly considered, these slab moments complicate, very materially, the moment-distribution process indicated in Fig. 6.

If there is objection to the use of a composite beam as not the usual method of steel design, a different problem is presented, but one that is still without a simple solution. If the slab does not act as a flange of the beam, it would seem to follow that it would act as a relatively rigid spacer which would tend to hold the top flange of the girder in its initial position; and any girder rotation would be rotation about the top flange, not about the centroid of the girder.

In the form presented by Professor Lothers, the solution is thus subject to considerable question when concrete floors connect to the girder. Nevertheless, there are cases in which this analysis is directly applicable, such as open steel frameworks, timber flooring on steel beams, and the like. It is a step in the larger problem that must ultimately be solved, and the author is to be commended for presenting this step so clearly.

It seems to the writer that Eq. 12*b* is not mathematically correct, although the practical difference is not important. For Case II the effective stiffness of the beam (from joint A) is $\frac{3\,S_F}{4}$ and the end moment, before joint A is allowed to rotate, is $\frac{3}{2}\,(\text{FEM})_{AB}$. Then a moment distribution would give

$$M_{AB} = \frac{S_T}{S_T + \frac{3}{4}\,S_F} \times \frac{3}{2}\,(\text{FEM})_{AB} = \frac{2\,S_T\,(\text{FEM})_{AB}}{\frac{4}{3}\,S_T + S_F} \quad \ldots\ldots\ldots(30)$$

Eq. 30 differs from Eq. 12*b* only in the first term of the denominator, and this term is often so small that its complete omission would do slight damage.

J. E. Lothers,[22] M. ASCE.—The able discussion of this paper has been no mean contribution to structural engineering literature and its courteous flavor has been most flattering and gratifying. The paper itself was an attempt to present a more or less simple method for computing torsional shear in steel spandrel girders. The discussion was a masterful job of providing details and embellishments, calling attention to shortcomings and exceptions, and indicating the mitigating effects of fireproofing and other factors encountered in practice. In his closing discussion the writer can do little else, aside from expressing his appreciation to these accomplished writers, than to agree with most of the hypotheses.

Professor Fiesenheiser is correct, of course, in stating that a standard riveted connection would not suffice at joint B, Figs. 7 and 8. He gives a very complete demonstration of the application of Prof. J. Charles Rathbun's method of correcting for connection restraint to torsional analysis. In his introductory remarks Professor Fiesenheiser mentions that the ratio Z of the angle of rotation θ_M of the end connection to the moment M producing it is determined by test. The constant Z may be computed and the writer hopes to have equations for that purpose published. It can be demonstrated, for example, that the small connection mentioned by Professor Fiesenheiser may be subjected to a moment of 5,780 in.-lb without exceeding the allowable bending stress of 20,000 lb per sq in. in the outstanding legs. This condition would seem to refute Professor Gant's contention that a standard riveted connection would not develop the torsion in a spandrel girder.

Eq. 17 (Professor Gant) for correcting for the effects of deflection and Eqs. 18 to 29 (Professor Hoffman), inclusive, illustrating the application of the calculus of finite differences to the torsion problem are gratefully acknowledged.

Professor Ferguson calls attention to the modifying effects of the enveloping concrete slab and fireproofing. Since a steel beam without fireproofing is rarely found in important steel construction, his discussion is very pertinent. The writer "laid himself open" when he specified concrete fireproofing in Example 3. In so doing, however, he was endeavoring to arrive at practical and readily computed loads, and the stiffening effects of the resulting composite beams and girders were neglected for the purpose of simplicity in demonstrating the proposed method of solution.

Professor Ferguson also called attention to the fact that Eq. 12*b* is not mathematically correct and stated that the practical difference is not important. Eq. 30 can be further simplified to:

$$M_{AB} = \frac{6\,S_T}{4\,S_T + 3\,S_F} \times (\mathrm{FEM})_{\mathrm{AB}} \dots\dots\dots\dots\dots\dots (31)$$

The derivation of Eq. 31 was demonstrated in the oral presentation of the paper before the Oklahoma Section at Tulsa on April 28, 1945.[23] Undoubtedly, the derivation (indicated in Professor Ferguson's discussion) should have been included in the published version. It will be noted, however, that Eq. 12c was given for a condition of end fixity that was the average of the conditions covered by Eqs. 12a and 12b. To have averaged Eqs. 12 and 31 would have led to an awkward form for Eq. 12c that would not have been justified in view of the rather rough assumptions of end fixity for the three cases covered by Eqs. 12. As Professor Ferguson has stated, S_T is very small as compared with S_F; and, accordingly, the coefficient of S_T in the denominator of Eq. 31 may be changed from 4 to 3 without affecting slide rule computations. Eq. 12b results if this substitution is made.

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[23] *Civil Engineering*, June, 1945, p. 295.

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Paper No. 2303

LANDSLIDE INVESTIGATION AND CORRECTION

By Hyde Forbes,[1] M. ASCE

With Discussion by Messrs. T. W. Lambe, Jacob Feld, George S. Harman, Robert F. Legget, Rush T. Sill, Earl M. Buckingham, Alfred V. Bowhay, D. P. Barnes, and Hyde Forbes.

Synopsis

The occurrence, cause, and correction of landslides and earth movements in the San Francisco Bay region of California are described in this paper. Examples are cited of (1) a shear slide in which hydrostatic uplift on an unbalanced slope resulted in mass movement of the unconsolidated slope material; (2) a shear slide caused by the overstress of clayey material on a slope that had absorbed a water load; (3) a slide that developed along the contact between two rock formations; (4) a detrital slide caused by the seasonal saturation of soil; (5) a detrital slide resulting from a geochemical breakdown of rock cut slopes; (6) a slide of street fills founded upon soils subject to movement when saturated; (7) a detrital slide generated by earthquake shock; (8) slides started by ground-water pressure; and (9) soil creep.

The methods used for investigation, the procedure followed, the costs involved in the corrective work, and the results obtained are described in the instances where economic factors required the investigation and correction of slides. The geological and ground-water conditions that generate slides and earth movements are treated in the paper, which demonstrates that such conditions can be recognized and their effect predetermined. Plans for engineering construction in such locations can include the corrections.

Introduction

A localized, surficial movement of earth severe enough to destroy the structure of the soil, earth, or rock mass involved, and any structure in its path commonly compose a landslide. Landslides differ in intensity and characteristics because of variance in the natural conditions through which they are generated.

Ordinarily, the geologic structure and mineralogical characteristics of the

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Note.—Published in February, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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rock or soil formations affected by a landslide will contain a clew as to the cause of movement and the corrective measures to be adopted. The bedding planes within one formation and contact planes between two formations or types of material are frequently the planes of slippage. In geology the term "plane" is used to denote "surface of separation," and the sliding surface, although it is seldom a geometric plane, is commonly called the "slip plane." Under the influence of certain forces and conditions slip planes are developed, and the material above the slip plane moves from one position of stability or condition of equilibrium to a new position and condition of equilibrium.

When a rock mass is fractured, as through earthquake fault movement, or when it disintegrates through weathering, loose material may accumulate in sufficient unbalanced weight to produce "detrital" slides of dry material; or water may penetrate the loose material and accumulate to the extent that a sudden "shear" slide is generated.

All manner of ground-water occurrence, movement, and pressure phenomena is found involved in generating slides. The minerals present in some rock formations become readily hydrated in contact with water, producing a clayey material that deforms under load or begins to flow in a plastic state. The same is true of residual clay subsoil which retains moisture normally to within a small percentage of its lower plastic limit, and the limit is reached by contact with very little additional water or by subjecting the mass to pressure or additional loading.

Soil technology, including a study of the physical and mechanical properties of the mass, may be of value in the solution of a problem involving the stability of materials on slopes as they exist in nature and as modified by construction. However, most of the laws and experimental data in this field have been developed through the use of dry sand composed of from 95% to 99% stable silica. Sand is seldom found disassociated from silt and clay, which are largely composed of unstable hydroxides and hydrous silicates, and most generally found containing water. Furthermore, most products of rock disintegration are subject to physical change under the geochemical processes of oxidation, carbonization, and hydration. This change occurs more rapidly than is generally realized and results in the accumulation of unstable masses.

The stability of slopes, determined from experimental data and assumed values, is of obvious significance; but, in the case of sliding ground and the prevention of earth movement, in loading slopes or excavation of cuts proposed in engineering construction, the experimental determination or assumption of stability factors is not recommended as a substitute for either thorough preliminary investigation of all other determinate facts or the use of judgment based upon experience under similar conditions.

Landslides

Landslides may be classed as "shear" or "break in the ground," and "detrital" or "flow of loose earth," or "deformation of plastic material," according to the character of the material involved and the type of movement in a given case. Shear slides develop as the result of well-defined breaks through the soil or earth along which the mass has been sheared. The surface of shear is the slip plane and a single area may have one or several such surfaces along which the material is sliding. Shear slides are frequently associated with the unbalanced slopes that remain after material has been removed from natural slopes. The shearing force may be entirely the pull of gravity—that force for the particular slope being greater than the resistance offered by the cohesion and friction within the material involved. More often, however, ground water is the principal generating factor, especially when large bodies of material are involved.

For example, when a stable mass of material, protected from water seepage by a cover of topsoil and vegetation, is disturbed by excavation or grading, rainfall will penetrate and thus increase the ground-water content. Also, a well-drained mass of material may have its natural drainage outlet obstructed by construction activities with the result that ground water backs up behind the obstruction, thus increasing the ground-water content.

The weight of the added water alone may overstress the material in such cases; the fluid may reduce cohesion and friction within the mass; the percolating water may reach bedding planes or contact planes, acting as a lubricant to generate slides; ground-water levels may rise, causing increased hydrostatic pressure with resulting uplift forces and blowouts; and absorbed moisture may create geochemical changes that weaken and swell the solid particles so that they cannot support loads.

Experience has shown that the variety of conditions under which slides are generated are numerous. No two slides are the result of conditions exactly alike; nor can any one slide be classed as typical. Once generated, shearing occurs throughout the sliding mass destroying its structure. Movement of the loosened mass along the main plane results in an upper scarp and a lower heave or bulging which is characteristic. Borings show that, as a rule, slip planes are cylindrical in shape and a segment of a circle in longitudinal dimension, but within the moving mass, as stated, there may be several such planes.

Each slide requires the determination of the conditions unique to the area before plans are formulated for correction; but methods successful in one case may generally be adapted successfully to meet the conditions encountered in other cases. For that reason it may be of value to detail the conditions and corrective measures applied in connection with several landslides in the vicinity of San Francisco, Calif.

SHEAR SLIDES

Examples of shear slides, caused by unbalanced slopes, usually with a change in moisture conditions, are afforded by experience along Monterey Boulevard; Parker Avenue at Lone Mountain; at Market and Glendale streets in San Francisco; along the State Highway, north and south of San Francisco; and in some residential districts of Oakland, Berkeley, and Crockett in the San Francisco Bay area.

Parker Avenue Slide in San Francisco.—This slide occurred in the latter part of December, 1935. The maximum movement, shown in Fig. 1(*a*), was

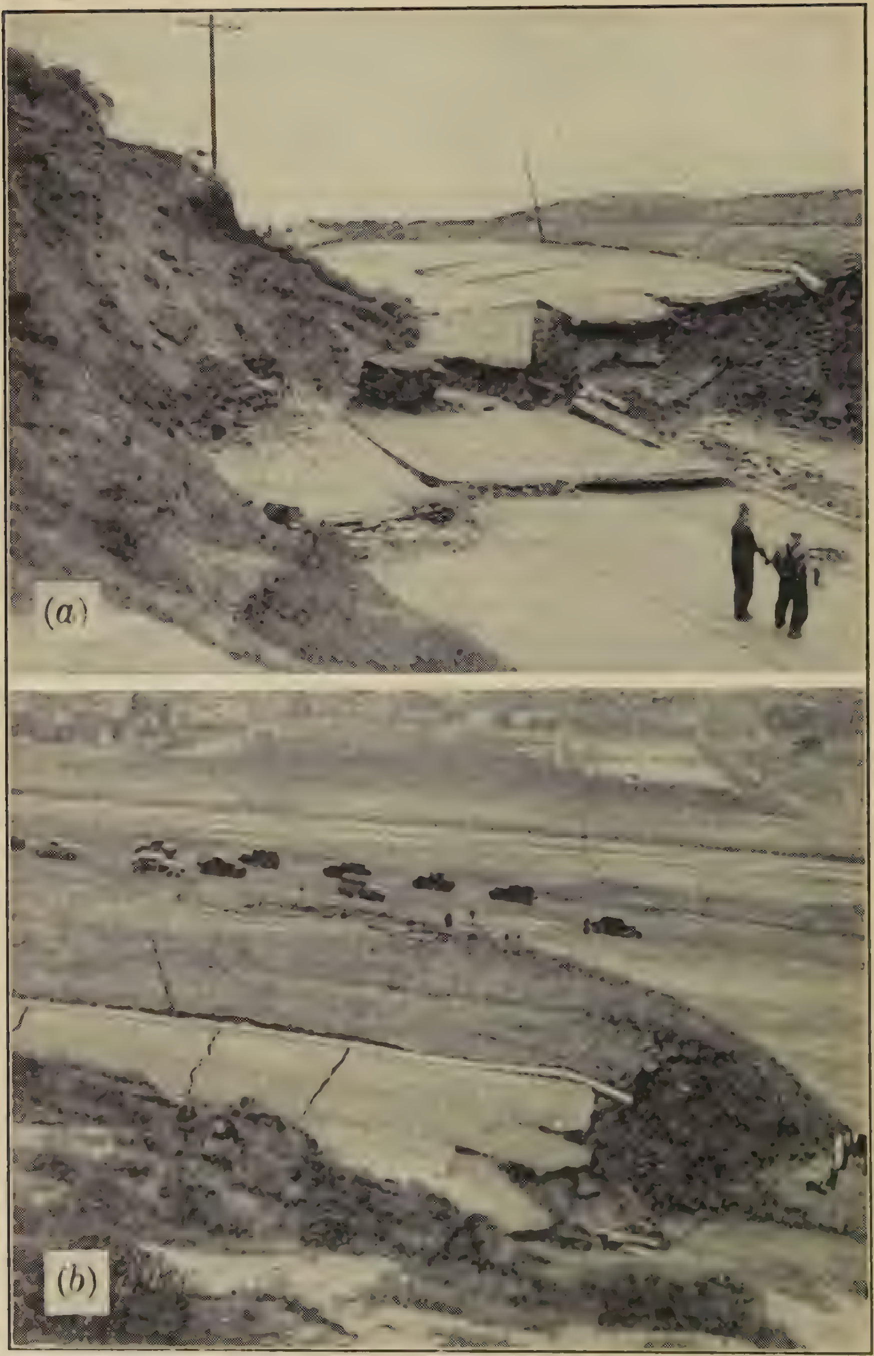


Fig. 1.—The Parker Avenue Slide, San Francisco, Calif., December 24, 1935

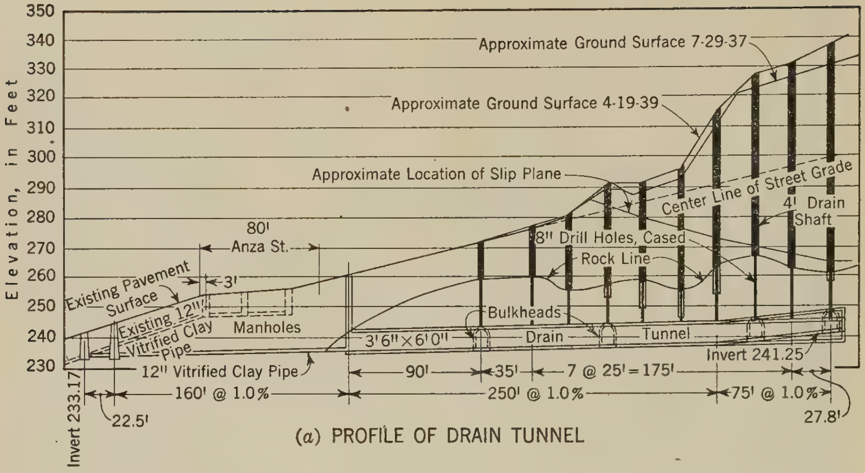
sudden. The noise was tremendous, the street was disrupted, and the million-dollar building of the San Francisco College for Women just above the scarp was threatened seriously. This movement produced a semicircular escarpment having a maximum height of 30 ft at the uphill boundary, with a corresponding heave of the downhill part. The downhill heave carried that section of Parker Avenue to a maximum of 20 ft above the finished grade, as shown in Fig. 1(*b*). The area affected extended about 450 ft east to west along the axis of the slide (at right angles to the street) and from 200 to 250 ft north to south.

In the preliminary investigation and exploration period of seven months, seventy-nine holes, totaling 2,855 ft, were bored. The drilling program was expanded or modified as the work progressed, to obtain an accurate picture of all subsurface conditions that may have influenced the movement. Water levels were measured in 2-in. pipes set in a gravel backfill of the boreholes at frequent intervals to determine the dynamic forces acting on the mass.

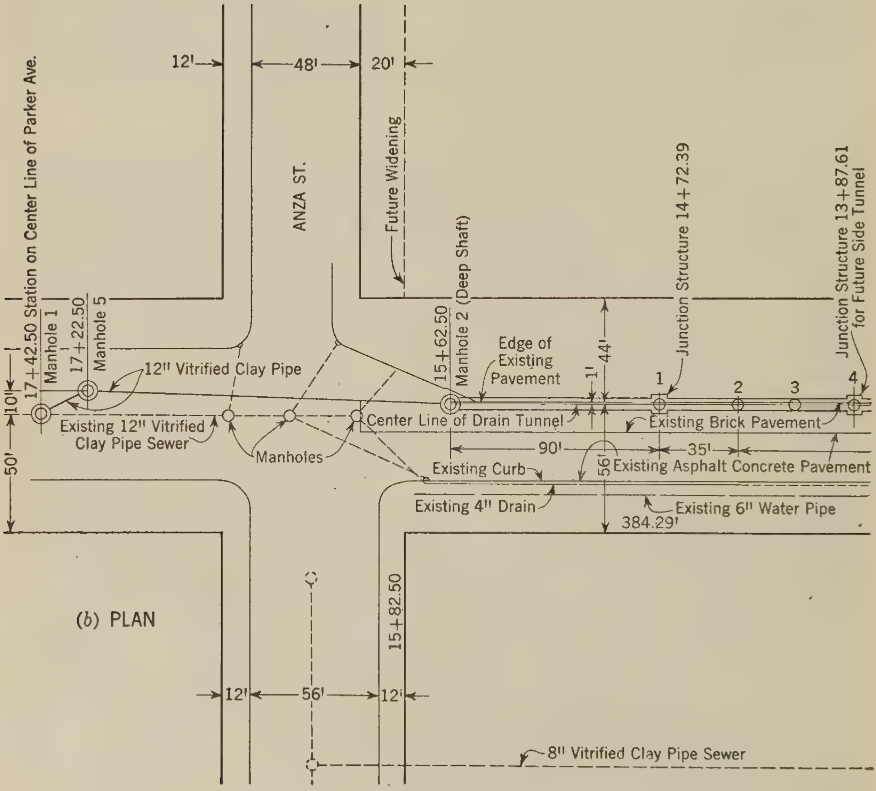
This investigation revealed that the original, somewhat stratified material was broken by internal shearing and movement—a given strata not being subject to the same amount of displacement from point to point, but the mass moving along a well-defined sliding plane. Movement continued throughout the period of investigation so that the main slip plane was located definitely. Ground-water levels indicated three different water horizons, each affected by its own drainage characteristics. Consequently, no single cross section could be drawn, and the position of the strata and water levels were studied by plotting contours of equal elevation from time to time. The approximate contour of the area before and after the slide is shown in Fig. 2(*b*).

The landslide was produced by a combination of causes, both natural and artificial. The natural causes were: The mineralogical and physical character of the serpentine bedrock of the region; the topographic development of that bedrock under the attack of ground water; the character, weight, and water content of the sand overburden; the position of the clay and other relatively impervious material, developed in the overlying mantle of unconsolidated material; and the effect of rainfall and other accretions to ground water upon the whole.

The artificial causes can be defined best by giving the sequence of engineering events affecting the area. During 1919 the right of way for Parker Avenue was excavated to grade through the base of the west slope of Lone Mountain. No movement had been recorded for fifteen years. Tile drains were placed along the right of way and the street pavement was completed that year. During 1931 the top of Lone Mountain was graded for the site of the San Francisco College for Women, and the excavated material was spread over the upper slope. In January, 1933, the construction of the college building was completed, and the grounds were landscaped, with provision for watering the lawn by a sprinkling system. Apparently, this construction work on the hilltop and upper slope did not overstress the supporting material or disturb its condition of stability, since no movement was noted in the succeeding three years. The accumulative effect of water applied by the sprinkler system may have been a contributing factor as it seeped through the surface and found its way underground.

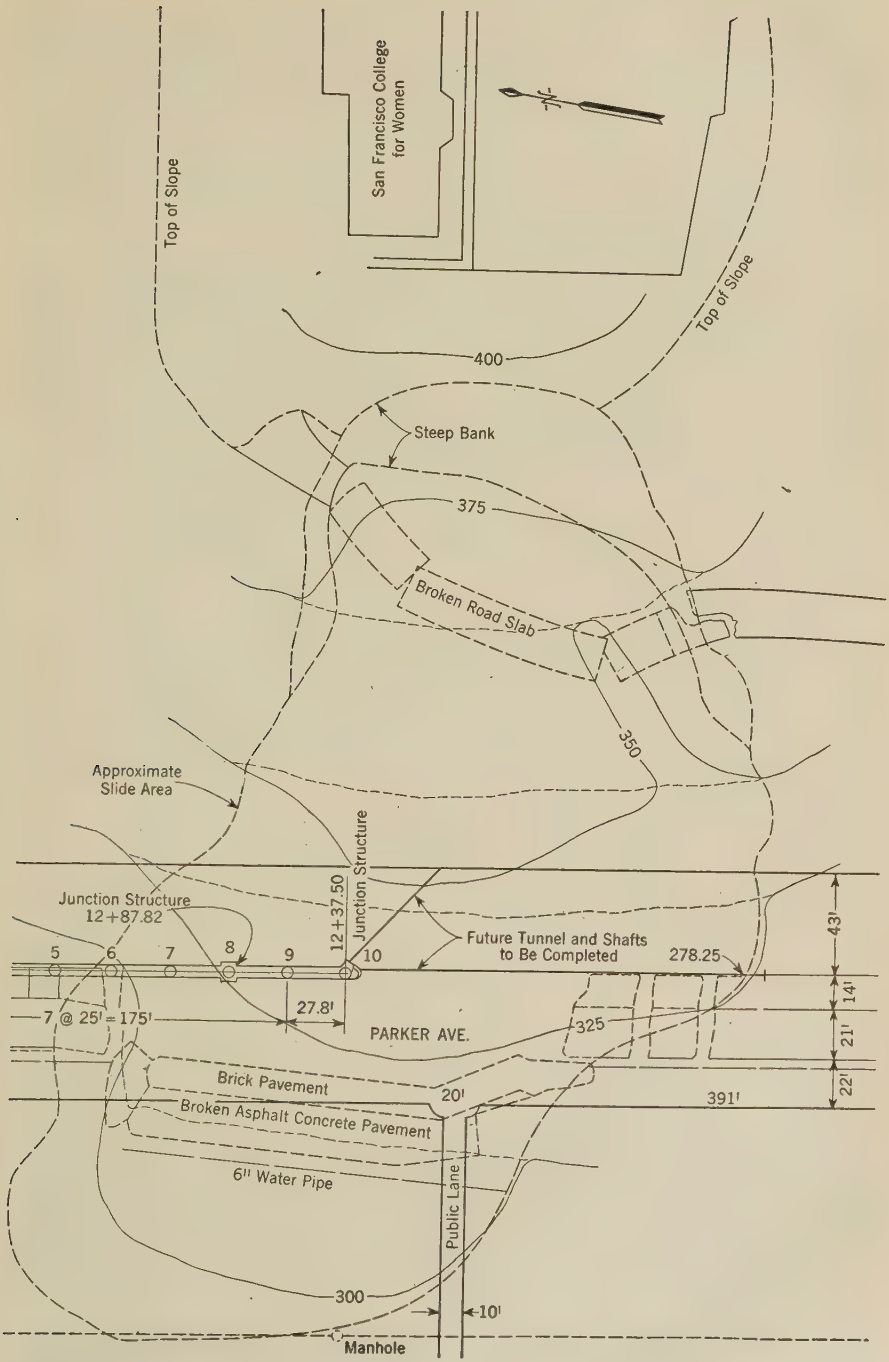

350
340
330
320
310
300
290
280
270
260
250
240
230
Elevation, in Feet
Approximate Ground Surface 7-29-37
Approximate Ground Surface 4-19-39
Approximate Location of Slip Plane
Center Line of Street Grade
4' Drain Shaft
80' Anza St.
8" Drill Holes, Cased
Rock Line
Existing Pavement Surface
3'
Existing 12" Vitrified Clay Pipe
Manholes
Bulkheads
3'6"×6'0"
Drain
Tunnel
Invert 241.25
12" Vitrified Clay Pipe
90'
35'
7 @ 25' = 175'
160' @ 1.0%
250' @ 1.0%
75' @ 1.0%
22.5'
27.8'
Invert 233.17

(*a*) PROFILE OF DRAIN TUNNEL


12'
48'
20'
ANZA ST.
Future Widening
Station on Center Line of Parker Ave.
17+42.50
Manhole 1
17+22.50
Manhole 5
15+62.50
Manhole 2 (Deep Shaft)
Junction Structure 14+72.39
Junction Structure 13+87.61 for Future Side Tunnel
12" Vitrified Clay Pipe
Edge of Existing Pavement
1'
44'
1
2
3
4
10'
50'
Existing 12" Vitrified Clay Pipe Sewer
Manholes
Center Line of Drain Tunnel
Existing Brick Pavement
90'
35'
56'
Existing Asphalt Concrete Pavement
Existing Curb
Existing 4" Drain
Existing 6" Water Pipe
384.29'
15+82.50
12'
56'
12'
8" Vitrified Clay Pipe Sewer

(*b*) PLAN

FIG. 2.—THE PARKER


San Francisco College for Women
N
Top of Slope
Top of Slope
400
Steep Bank
375
Broken Road Slab
350
Approximate Slide Area
Junction Structure
Junction Structure 12+87.82
12+37.50
Future Tunnel and Shafts to Be Completed
278.25
43'
14'
21'
22'
5
6
7
8
9
10
7 @ 25' = 175'
27.8'
PARKER AVE.
325
Brick Pavement
20'
391'
Broken Asphalt Concrete Pavement
6" Water Pipe
Public Lane
300
10'
Manhole

Avenue Slide

During October, 1935, a subdivision west of Parker Avenue was graded to street level, thus removing the remainder of the irregular surface of the west slope of Lone Mountain. The new slope then became overstressed, and the first rainstorms that occurred thereafter raised the ground-water levels, exerting an hydrostatic uplift—a moving factor—on the already overbalanced mass. Late in December shear cracks were noted in the surface; and, within two days, the mass gave way along a slip plane that developed just above consolidated rock, not more than 70 ft below the original surface.

Water was encountered at several horizons, the two deepest being a bed of sand between two clay beds and the disintegrated rock surface below a residual clay. At both levels considerable pressure head was recorded. The remedial work that could be done by the City of San Francisco was limited to city-owned property. Accordingly, a tunnel was driven into the serpentine bedrock in Parker Avenue, as shown in Fig. 2. At intervals shown in Fig. 2(*a*), ten vertical holes were bored, from the surface into the tunnel top. Through the overlying soil to rock the diameters were 36 in. and for the remaining depth, through rock, the diameters were 8 in. The latter (churn-drill holes) were cased with an 8-in., perforated pipe, which was then extended upward to the surface through the 36-in. boreholes. The space behind the 8-in. casing was backfilled with crushed rock.

A partial dewatering of the area and the relief of water pressures was accomplished during the construction of these vertical drains. The discharge from the tunnel reached a maximum of 1 mgd. Since the completion of the present tunnel section, the discharge has corresponded closely to the occurrence of rainfall, decreasing during the summer months to about 100 gal of water per hr. Although the total corrective work planned has not been completed, the movement has been arrested, and the street can be reopened when works providing additional safety factor are installed.

Earth Slide at Crolona Heights in Crockett, Calif.—Crolona Heights is a highly developed residential area situated on a steep hill composed of a bedded sedimentary formation in which shale predominates. The shale disintegrates rapidly when exposed to the atmosphere or to alternate wetting and drying. The product of disintegration is a clay that appears as soil on the hillsides and remains in place in a state of more or less unstable equilibrium for slope moisture content, and load. Any change in one or more of those factors upsets the equilibrium, and small movements are common over the hillsides each winter.

The large, sudden, and destructive slide that occurred in the early winter of 1936, was the result of geologic conditions not observed at any other location. A fault was found passing through the clay shale, approximately along its strike. The shale involved in the faulting had been sheared and jointed so that water had penetrated to a depth of from 10 ft to 20 ft below its surface. Disintegration had extended to a greater depth than in the shale beyond the fault zone.

Very little free water was found by boring, or in the excavations below the surface of the clay; but the clay had absorbed and retained a considerable water load. Samples taken from the boreholes on November 5, 1936, were tested for water content. The average results for the cross sections are noted at their central point in Fig. 3. The same samples were tested for other moisture characteristics with the following results: Plastic limit, from 20.27% to 26.27%; liquid limit from 35.79% to 38.96%; and moisture equivalent, from 22.39% to 32.11%—all in relation to the dry weight of the samples.

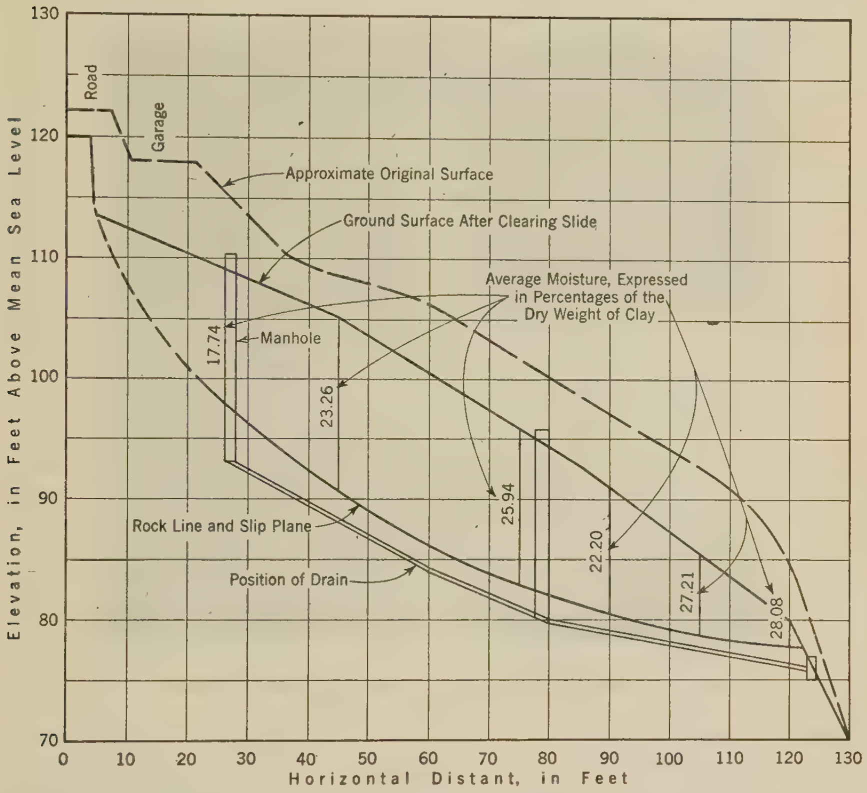


FIG. 3.—CROSS SECTION ALONG THE DEEPEST TROUGH OF THE CROLONA HEIGHTS LANDSLIDE AT CROCKETT, CALIF.

It was concluded that the water load had finally reached proportions that caused the material to shear. Cracks developed, allowing rainfall to penetrate to the shale, and a sudden slide occurred along a slip plane developed in the clay above sound rock. The borings made it possible to determine the depth to rock in place and the position and conformation of the slip plane. The slip plane was found to be at a maximum depth of about 15 ft below the surface of the slide and possibly 20 ft below the original ground surface (Fig. 3).

Corrective work consisted of excavating to rock below the slip plane (which was easily recognized by its striated and polished surface), installing a well casing preperforated over one half of its circumference in the sound rock below the slip plane (see Fig. 4), and backfilling with crushed rock to within 3 ft of the surface.

Vertical ventilating shafts, consisting of perforated corrugated pipe, were left in the backfill to aid in drying the clay. Reinforced concrete posts poured in holes bored 12 ft into the shale retained the fill for the main access roadway to the top of the hill and supported a garage. Drains were provided to keep surface water from accumulating in the area, and the loose material over the toe and street and sidewalk areas was removed; but the upper slope below the roadway was not brought back to its original grade and loading. During the intervening wet winters many small slides have developed at other points on Crolona Heights, but there has been no movement of the corrected slide.

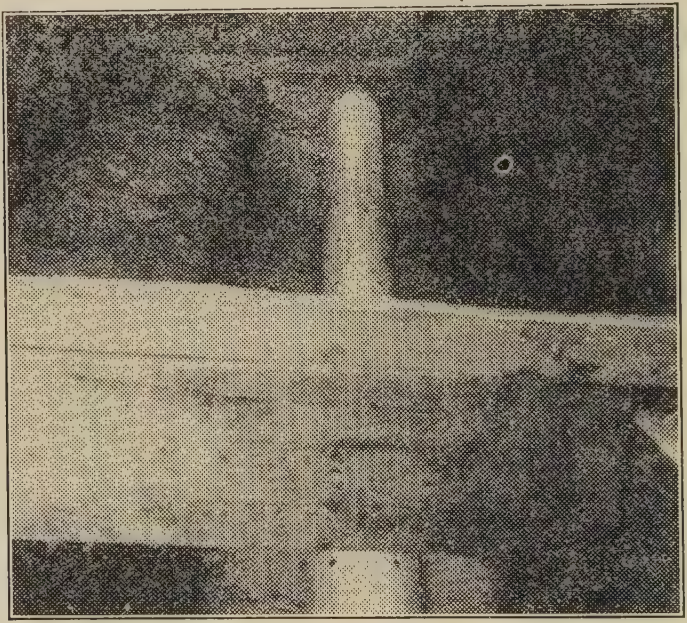


Fig. 4.—Drain Pipe Placed Below the Slip Plane to Correct the Crolona Heights Landslide

Plastic Material at Market and Glendale Streets, San Francisco.—The slide movement which occurred on Market and Glendale streets in San Francisco was also one caused by geologic conditions unique to the area. Market Street was graded through solid rock in a low cut on the slope of Twin Peaks. For several years the rock mass sheared and moved during the winter months, disrupting the pavement and blocking traffic on an important artery of the city. Each summer the street was regraded and the pavement replaced. The usual procedure for investigating subsurface conditions by borings was not adapted to the hard rock involved in the movement. In 1940 and 1941 surveys were made of the surface, the escarpments of the slide, and the movements of the ground. The results of these surveys are given in Fig. 5; and the remedial works, the limits and contours of the slide, and the reference line for measurements in Fig. 5 are shown in Fig. 6.

The axis of the slide was approximated from the results of Fig. 5, and exploration pits (an upper pit 6 ft by 8 ft and a lower pit 8 ft by 10 ft) were excavated accordingly. The top 15 ft or so of each pit was excavated through a metamorphosed sandstone, as hard as quartzite and as difficult to break. The bottom sound rock was a basic igneous rock, intrusive in the sandstone, which did not outcrop at the surface near the area. Between the two formations was a thick body of blue-black clay, the decomposition product of the basic rock.. Surface water had penetrated to the lower rock formation causing it to hydrate and swell with such force as to fracture the overlying quartzite.

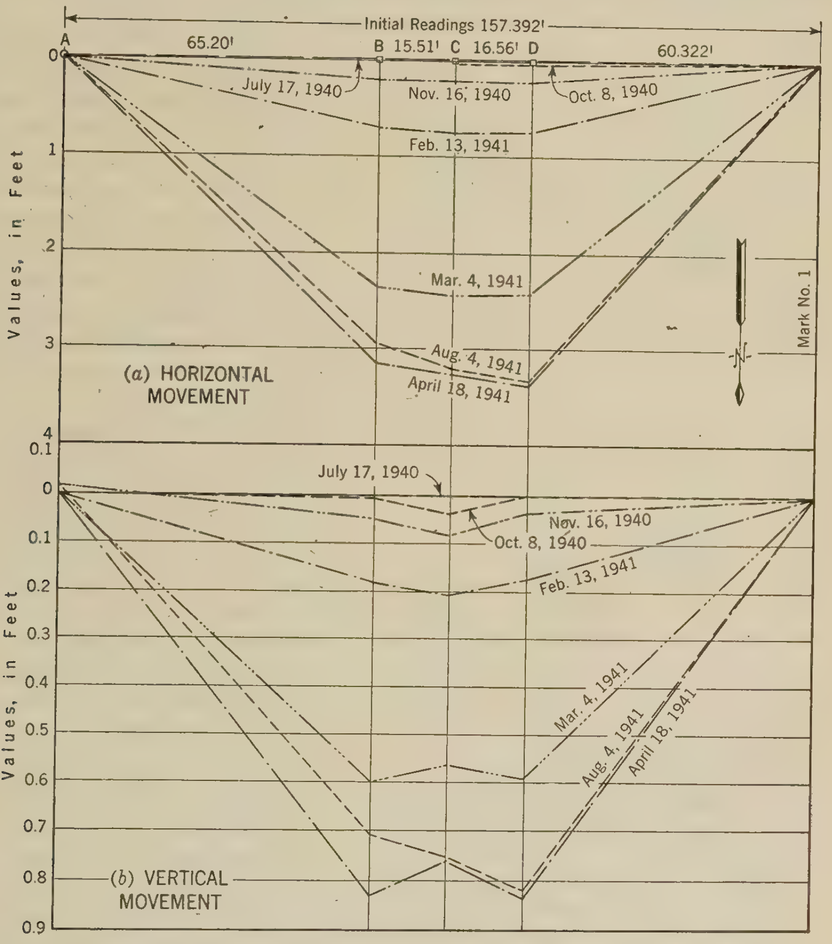


FIG. 5.—PROGRESSIVE MOVEMENT OF REFERENCE STAKES (SEE REFERENCE LINE IN FIG. 6), SLIDE AT MARKET AND GLENDALE STREETS, SAN FRANCISCO, CALIF.

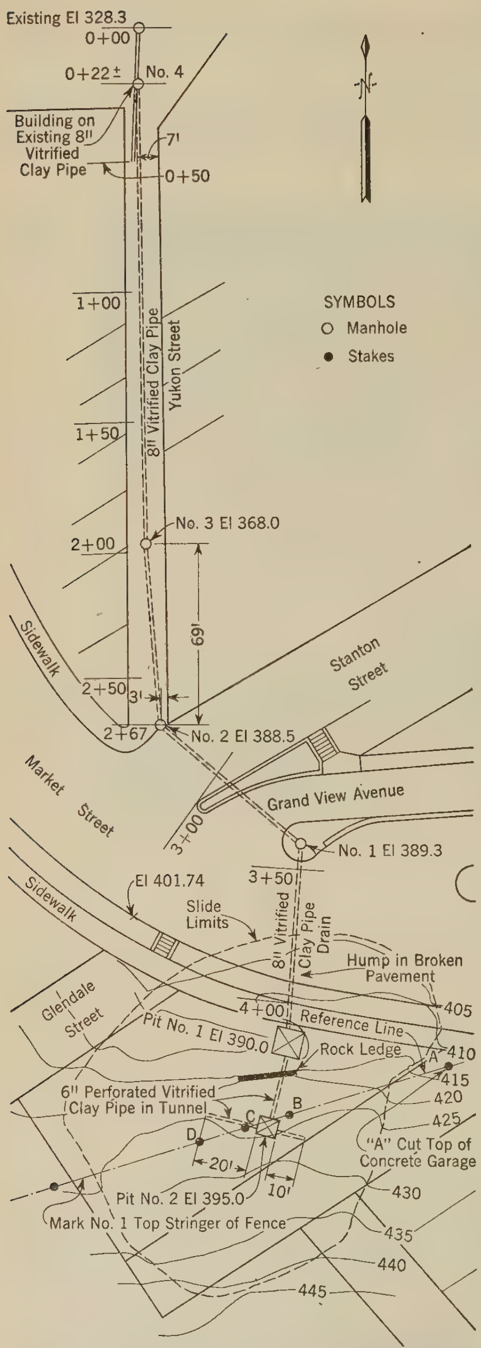

Existing El 328.3
0+00
0+22±
No. 4
Building on Existing 8" Vitrified Clay Pipe
7'
0+50
N
SYMBOLS
○ Manhole
● Stakes
1+00
1+50
8" Vitrified Clay Pipe
Yukon Street
No. 3 El 368.0
2+00
69'
Sidewalk
2+50
3'
2+67
No. 2 El 388.5
Stanton Street
Market Street
Grand View Avenue
3+00
No. 1 El 389.3
El 401.74
3+50
Slide Limits
8" Vitrified Clay Pipe Drain
Hump in Broken Pavement
Sidewalk
Glendale Street
4+00
Reference Line
405
Pit No. 1 El 390.0
Rock Ledge
A
410
415
420
6" Perforated Vitrified Clay Pipe in Tunnel
B
C
425
D
"A" Cut Top of Concrete Garage
20'
10'
430
Pit No. 2 El 395.0
Mark No. 1 Top Stringer of Fence
435
440
445

Fig. 6.—Limits of the Market and Glendale Slide, Showing Contours of the Slide, Reference Line for Observations, and Remedial Works

These fractures allowed a more ready access of water and contact with the basic rock and the process continued until the thick intervening clay had been produced through hydration.

The clay absorbed excess water with each winter's rainfall penetration, and became plastic and deformed. This action permitted the overlying rock load to slide toward, and push up, the subgrade and pavement of Market Street. The pits were completed late in 1941, as the seasonal rains were beginning, and water was pumped from them that winter. The time of contact between the water and clay was reduced materially and no movement was noted except during a 36-hr period, over a weekend, when the pumps failed. At that time the water rose in the pits and the pavement rose less than 1 in. The pits were connected by a drain laid in a tunnel and their bottoms, cleared to sound rock, were lined with concrete in 1942. A collector drain system was extended along the rock contact both ways from the upper pit, and the lower pit was connected with a sewer by a gravity drain in a tunnel under Market Street. The pits were filled with crushed rock and covered. The work completed has arrested all movement and water has been discharged to the sewer continuously.

Costs of Corrective Works.—Shear slides involve large quantities of material, and a considerable area, and the cost of corrective work applicable for a given case varies with the magnitude of the area in motion and with the extensions necessary to meet subsurface conditions and to provide drainage outlets. The hard serpentine rock encountered in tunneling below the Parker Avenue slide becomes slacked and swelled in the presence of air, so that a reinforced concrete lining was required. This raised the estimated total cost of the tunnel to about $100,000, the incompleted installation as of December, 1945, costing $65,000. At the Market Street slide, on the other hand, the cost of sinking, timbering, bottom lining, and backfilling the pits, on a unit bid basis, was $2,466. The tunnel connection and the intercepting drain connection to the sewer on Yukon Street (see Fig. 6) cost $5,126 on a lump sum contract.

A surface survey alone could not have revealed the geologic conditions affecting the Market Street slide, but several other shear slides have been analyzed as to the cause and to determine corrective methods by a geological survey. The experience of the City of San Francisco, in landslide correction work, has shown that a geological survey of surface exposures is a first requirement in directing the subsurface exploration and that such a survey can always shorten the time and thus control the cost of boring. Boring records and samples are of little value if they are not collected by an experienced geologist who can analyze them in light of the problem presented.

Preventive and Precautionary Measures.—Because the City of San Francisco had frequently been sued for property damage resulting from landslides and because, more frequently, the owner had no recourse in the event that his property suffered damage, the late A. D. Wilder, M. ASCE, then Director of Public Works, was prompted to require that plans be approved by his department in relation to surface and subsurface drainage, stability of fills, and street subgrade of proposed subdivisions whose streets would be taken over by the city for maintenance. Among projects submitted to the writer for investigation in October, 1941, was an area covering the slope between Silver Avenue and Alemany Boulevard, six blocks wide and about a quarter of a mile along Alemany Boulevard, with streets and utilities serving three hundred and eighty dwelling units. The slope, which was then being graded, gave some evidence of landslide material in several places and a few springs issued from the surface.

The cost of the preliminary investigation was divided between the city and the subdividers. The borings revealed the fact that alternate beds of dune sand and silty clay overlie a bedrock with an outcrop at an elevation 300 ft above the upper street level. Each sand bed contained water and the lower beds developed water under pressure at several points in line up the slope from spring outlets. The need for drainage works to stabilize the area and to prevent landslides and the saturation of street fills was anticipated. This work was begun before the street and building construction of the subdivision was completed.

The drainage system consisted of a continuous perforated tile 900 ft long laid in a trench from 4 ft to 8 ft below the surface in the sidewalk area of the highest street of the subdivision and rising 35 ft to El. 180 city base. The trench intercepted the water in a series of horizontally bedded sands and clays which yielded about 90 gal of water per hr in September, 1942. Three relief wells, 50 ft apart, were bored at the higher end of the trench—one to bedrock at El. 140 from which water flowed under pressure into the trench, one to a sand bed at El. 160 from which water also flowed, and one through clay to sand at El. 170 in which the water level just rose to the bottom of the trench. This work increased the discharge of the system to 180 gal per hr.

During the winter months, the discharge of the drain increased until in March, 1943, it was 515 gal per hr, decreasing again through the summer until in November, 1943, it was 225 gal per hr. That the effective radius of the drain is increasing is evidenced by the fact that the discharge in January, 1944, was 301,500 gal as against 172,700 gal in January, 1943. The comparable February measurements revealed a total of 357,200 gal for 1944 as against 204,800 gal for 1943 although the 1944 rainfall was less than that of 1943.

The drain is located close to the foot of the upper cut slope. The total cost of the drain construction was $3,350 on a lump sum contract, which was very economical insurance. No saturation of fills, heaving of pavements, or other forms of instability have been noted, but those experienced in these matters in the office of the city engineer know that the saturation of fills and development of water pressure would have resulted in slides if the quantities of water drained away had been allowed to accumulate underground in the area.

Detrital Slides

Detrital slides in natural earth masses have several causes. Masses of material will move after heavy rains have saturated or swelled them. Under certain conditions rainfall will penetrate to, and will lubricate, surfaces of contacting material such as soil or clay over rock (or soil over clay), which thus allows the top material to slide. This action frequently exposes the subsoil material to moisture penetration that generates slides. Weathering of, or geochemical changes in, earth or rock slopes so weakens the material that it moves under the force of gravity. Earthquake shock has been the cause of an initial movement in loose or sheared material of fault zones on the West Coast of the United States. Examples of each of these types of movement are to be found in the San Francisco Bay region.

O'Shaughnessy Boulevard Slides.—During construction, and after completion in 1941, two detrital slides of considerable extent occurred during the winter months along O'Shaughnessy Boulevard in San Francisco. The surface material became saturated, expanded, lost its cohesive properties, cracked, and slid to the boulevard right of way. One area is retained by a high and heavy wall, but the material flowed over and around the wall. Another area involves the subgrade of the boulevard and the wetted material moving along a solid surface has forced the pavement upward during the winter and spring months—a hazard to traffic which had to be corrected.

The exact contour of the solid rock or clay over which the latter slide recurred each winter could not be determined without an extensive and costly boring program. Surface surveys were made and, upon the basis of the experience gained in the correction of similar slides, specifications were prepared and a unit price contract was let in the fall of 1945 under which exploration and corrective work were done simultaneously. Three shafts were first excavated at the estimated location of the axis of the slide.

The material that was within from 10 ft to 12 ft of the surface consisted of loose soil and clay, carrying considerable rock float which had originated in the dumps from old quarry operations at the top of the slope and had moved as much as 500 ft down the slope. There was a definite sliding plane at the base of this material which curved to meet the pavement close to curb line. Movement along this plane had sheared a tile drain parallel to the roadway at a maximum depth of 7 ft below ground surface.

The bulge under the greater area of the pavement, however, was caused by the action of a clay subsoil when its moisture content increased above its plastic limit each year. Apparently, the top detrital material became saturated with rainfall penetration and seepage, allowing the water to reach the top of the clay subsoil, and thus lubricating its surface and generating surficial movement. Water was then in contact with the clay subsoil and absorbed by it; and this clay became viscous. Movement, causing the bulging up of the pavement, resulted from the integration of minute slippages between clay particles in plastic deformation.

The depth to which water had been drawn into the clay and the bottom of the clay involved in movement was clearly perceptible in the shaft walls. The solid, dry clay into which the shafts were driven exhibited its original residual structure and characteristics whereas that involved in movement was obviously "remolded." The lowest point in the latter material was determined from the first three shafts. A manhole was constructed at that point, and thirty-four shafts, averaging 20 ft in depth, and two additional manholes were excavated. The line of shafts was located along the axis and grade of the slide, as its location and depth were revealed by excavation from point to point. The function of the shafts and of their crushed rock backfill was to draw the ground-water levels down below the top material and to consolidate the clay through evaporation of its excess moisture.

TABLE 1.—UNIT COSTS ON CORRECTIVE WORK, 1945;
O'SHAUGHNESSY BOULEVARD SLIDES

Item	Description	Unit cost
1	Drain shafts; including excavation, hand finishing the bottom to fit the drain pipe, sheeting and bracing, disposing of surplus material, and backfilling with from ½-in. to 1½-in. crushed rock (per linear foot of depth)	$ 9.50
2	Drain tunnels, minimum section 15 in. by 15 in., connecting bottom shafts and manholes; including excavation, hand finishing the bottom for pipe installation, and tamping the crushed rock backfill (per linear foot of tunnel)	$ 6.00
3	Perforated drain; furnishing and installing 8-in. vitrified clay pipe; including the grout and mortar work necessary to produce a tight bottom and a rigid support (per linear foot of drain)	$ 3.00
4	Manholes, 4 ft in diameter; constructed complete with concrete foundation, steps, and other features in accordance with City Standard Specifications (per linear foot of depth)	$20.00

The shafts were connected by short tunnels through which a perforated drain tile was laid, in a manner similar to that illustrated by the Crockett slide drain (Fig. 4). The system was extended across the boulevard through solid clay and road fill to discharge on a rock slope. The unit costs were as given in Table 1. The total cost, units plus extras, is about $11,000.

The bedrock core of the San Francisco Peninsula is a geologically ancient complex (Franciscan formation) in which rock of sedimentary origin is metamorphosed to varying degrees through both intrusion of basic igneous magmas and crustal pressure. The basic igneous dikes, principally serpentine, characterize areas in which gulleys have developed in the bedrock through the geochemical breakdown of the rock. Two such areas in the cliffs facing the Golden Gate are producing slides that involve the El Camino del Mar subgrade and Phelan Beach recreation area, the serpentine hydrates forming a clay which, with the further absorption of moisture, becomes a viscous mass moving down the slope and carrying its overburden of dune sand and road fill.

The Franciscan formation has been subjected to many periods of faulting, which is the fracture relief of enormous compressive stress although, for the most part, the faults have been long "dead" and healed. In the area of the O'Shaughnessy Boulevard slide, however, a healed fault zone was found in which the fault gouge, although consolidated through time and pressure, was subject to comparatively rapid breakdown in the presence of ground water, resulting in a blue plastic clay. Free water was found in the jointed rock of the old fault zone and under pressure beneath the gouge clay, which made it necessary to stabilize a considerable part of the material by excavating shafts to depths as great as 27 ft and by flattening the gradient of the drainpipe across the zone.

There is nothing unique about the rock formations of the San Francisco Bay area that makes them particularly susceptible to the development of landslides other than the slopes resulting from the topographic development. All geologic formations contain zones of structural weakness and are subject to the attack of ground water and weathering agencies. Ground water occurs at such zones of weakness, in varying degrees, conditions, and quantities, often sufficient to produce local unstable masses in which slides are readily generated. In fact, the ground movement along the slope that supports O'Shaughnessy Boulevard is the normal process of topographic development common to all slopes and was initiated long before the boulevard was constructed.

Rock slopes disintegrate under the attack of weathering agencies to produce soil in place; or soil is moved to rock slopes by wind action (as in the dune sand areas of San Francisco) and the underlying rock then is disintegrated and decomposed by the geochemical action of ground water held in the overlying blanket. When the rock decomposes, the subsoil develops in place as a residual clay. The soil cover moves down the slope whenever its moisture content reduces cohesion and friction to less than the pull of gravity. This exposes fresh rock surfaces to weathering or underlying subsoil or residual clay to seepage, the clay then being subject to some form of plastic flow. The process is continuous until a new slope is formed upon which the material will remain in a stable position. However, detrital slides have been generated in San Francisco when fresh rock surfaces were exposed to weathering agencies and when lateral support for bedded materials was removed in excavating for streets or building sites.

Joost Street Slide in San Francisco.—Removal of lateral support was the cause of the Joost Street slide in 1928, when two layers of material of different character (sandy topsoil and sandy clay subsoil) moved at different rates down a slope whose lower toe had been disturbed by excavation on Monterey Boulevard. The material adjoining the excavated surface lost its lateral support and the topsoil started moving. The structure of the topsoil was broken and movement was accelerated during the winter when the topsoil absorbed an unusual quantity of rainfall. Water then could reach the clay subsoil to generate a plastic flow over solid material. The movement extended up the slope until the foundations of houses on Joost Street, parallel to and above Monterey Boulevard, were affected and the houses slid down the slope.

Broadway Tunnel Approach in Oakland.—Detrital slide development on cut slopes occurs frequently in the San Francisco Bay region even when roadways are cut through seemingly solid material. Dry soil, clay, shale, or rock composed of basic minerals all become weathered when they are subjected to temperature changes, alternate wetting and drying, and exposure to the atmosphere. The resultant disintegration allows the penetration of rain water, adding weight to the mass as well as weakening and lubricating the material. The cut for the approach to the Broadway Tunnel in Oakland, Calif., was one of those investigated.

The fresh cut, made in 1934, on a slope of 1 on 1, presented a rocky surface. It was brown in color because, in this crushed and sheared fault zone, there was an iron-oxide coating on many joint faces of the hard blue rock. Moist areas on the slope were characterized by the presence of partly decomposed rock. These clay pockets contained material that had developed polished and grooved joint faces (slickensided) produced by the force of the internal swell in hydration.

Samples of the crushed rock were obtained and tested for their action under hydration. The moist clay from the clay pockets was placed on sloping surfaces and further wetted. It was concluded, from these tests, that the rock fragments would break down to clay when subjected to rainfall penetration and that the clayey material would slide on slopes at angles greater than 25°. The material of the cut slope did reduce to clay and slide when wetted each winter, exposing new rock faces. This process was repeated until a predicted slope of 25° was reached. The progressive development, from 1934 through 1941, toward the predicted stable slope is shown by three typical sections in Fig. 7. The character of the slides generated and the breakdown of the property on the parallel and higher Buena Vista Avenue, with the destruction of trees, as it appeared in 1941 are shown in Fig. 8. This process occurs frequently along California highways. The rock at some cut faces decomposes with comparative rapidity and breaks down to clay when exposed to weathering and when penetrated by water. The clay retains moisture and increases in volume to a much greater extent than the rock from which it is derived. The inevitable result is that the clay cover slides progressively, each time uncovering fresh surfaces of rock. The process of deep disintegration and decomposition advances with each cycle until the slope becomes stable for the conditions prevailing at the time.

Street Fills Placed on Unstable Materials or on Soils Subject to Saturation.—Detrital slides develop slowly as compared with shear slides and are seldom as destructive to streets and property. However, damage results when material subject to such movement is utilized as a foundation for fills or for the subgrades of streets. An instance is afforded in San Francisco from experience on Monterey Boulevard. The fill-supported pavement settles on the downslope side in many places; and, at Detroit Street, a fill to street grade slid down the slope, destroying one house and damaging two other properties.

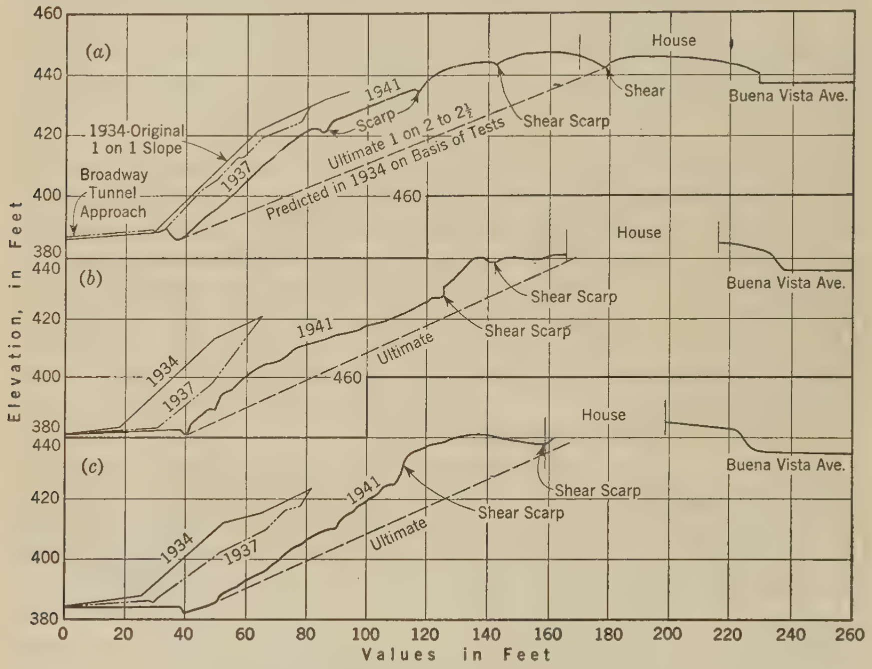


FIG. 7.—PROGRESS OF SLOPE STABILIZATION, APPROACH TO THE BROADWAY TUNNEL, OAKLAND, CALIF.

Burnham Street and 24th Street Slide.—A fill required for 24th Street and Burnham Street on the east slope of Twin Peaks settled from 2 ft to 4 ft each winter, disrupting the Burnham Street roadway to its center line. When the fill slid down the slope to force a wave of saturated soil ahead of it, the foundations of three houses on 24th Street were destroyed, and the material piled against the foundation of a fourth house at the end of the block.

Test boring in 1942 revealed a clay and rock fill 22 ft deep at the intersection, decreasing to 12 ft down and along the crossing streets. Water under pressure was encountered at various levels, the pressure being developed by the restraint exerted by the overlying clay fill on the original surface soil. The moisture content of soil samples recovered from parts of the fill, in July, 1942, was greater than the plastic limit, but no plastic deformation was found to have occurred; the street surface had settled when the downslope part of the fill had sheared; in other words, the displacement occurred because of the unstable underlying material. The base of the fill as well as the underlying soil—a total thickness of from 4 ft to 5 ft—was saturated and yielded free water. In many of the boreholes along Burnham Street, the water rose to within 4 ft of the street level when the original soil was penetrated.

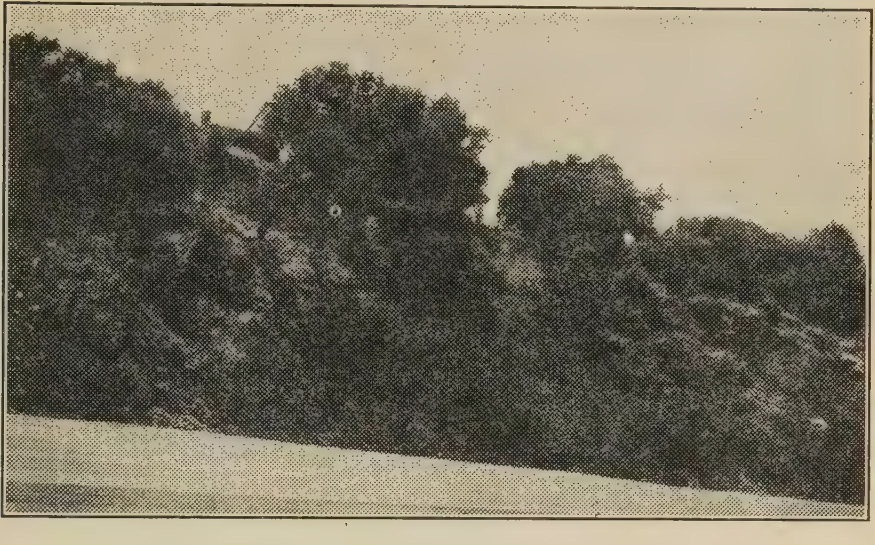


FIG. 8.—VIEW OF SLOPE, APPROACH TO THE BROADWAY TUNNEL IN 1941

The remedial works had to be installed within the city-owned street and sidewalk area. A total of thirty-five 4-ft by 5-ft shafts, varying in depth from 18 ft to 35 ft up the slope of 24th Street and both ways from the junction manhole on Burnham Street were excavated by a half-yard clamshell bucket on a 44-ft boom. The shafts were 6 ft apart so that a column of earth was left as a protection against movement. Tunnels about 3 ft by 3 ft in section connected the shafts, and 450 ft of perforated tile drainpipe were laid as a continuous drain along the bottom.

The drain was covered with a backfill of ¾-in. to 3-in. crushed rock in the tunnels and to within 6 ft of street level in the shafts. Three manholes were constructed to give access to the drain and to act as air vents at differing levels. The lump sum contract price for this work and for a short, trench-laid connection to a sewer, was $18,750 in 1943.

The water pressures were relieved by the drainage outlet, as evidenced by the lowering of the free surface water levels in the boreholes. The discharge of the system was as much as 300 gal per hr during construction; after the completion of the project and before the winter rains (November, 1943) the discharge was 100 gal per hr. The movement has been arrested and no recurrence of the settlement has been noted. With the passing of time, as the circulation of air causes water to evaporate, the fill will become consolidated and permanent pavement will be placed.

Construction Control and Preliminary Investigation in Private and Public Work.—Litigation with private parties, concerning private property, is a costly factor in city government. To correct this condition in San Francisco, legislation was initiated in 1941, by Director Wilder and John J. Casey, M. ASCE, then city engineer, to require that private parties obtain city permits to excavate, place fill, or dispose of waste material of specified minimum quantities. The need for such legislation was demonstrated in February, 1942, when two persons were killed in a house destroyed by flowing mud. A body of waste material dumped in a gully above the house had become saturated and flow of a considerable quantity of mud so formed was the result. Many slides could have been avoided, final slopes of excavations could have been determined in advance, the city could have been protected from damage suits and excessive maintenance costs, and the property owner could have been protected—if the facts determined later, by necessity, in the correction of such occurrences had been determined before construction was undertaken. Such considerations led to the adoption of a program of preliminary geological and subsurface investigation in connection with the city's plans for street, boulevard, and free way construction.

Earthquake Shock.—The Pacific Coast of the United States is a region of seismic activity. Small earthquake shocks occur frequently, and the shattered rock of fault zones has provided material for slides on natural slopes. The typical landslide topography along known fault lines in California is evidence of the fact that surficial ground movements accompany shock. These are the so-called "moving mountains" which receive newspaper publicity and are of general interest from time to time. One such slide was investigated because its effect brought about a damage suit. It occurred in 1936 along the right of way of the Western Pacific Railroad in the Altamont Pass. Its cause and effect may be of interest in connection with West Coast highway cuts.

The railroad cuts in that vicinity are excavated in hard, thin bedded shale at right angles to the bedding planes and with their dip 14° from the horizontal at the slide. The shale consists of cemented fines rather than of compacted fines as it contains intergranular binder, does not slack nor disintegrate readily in the presence of moisture, nor does it absorb moisture. Furthermore, the slide occurred on November 21, 1936, before any rain had fallen that season, and no water was encountered in boring. The usual conditions that cause slides were lacking in this instance and no slide had occurred there since the railroad cut was completed thirty years previously. It was reasonable to conclude that the slope adopted in excavation was safe, that no deformation or slide would develop as a result of the excavation, and that the cut slope would remain stable except for superficial sloughing.

The shale formation is faulted locally, due to varying compression in the geologic folding and distortion of the entire area. These faults are numerous and of minor importance as a rule, having little significance in the stability of the entire mass. For the most part, they cause no trouble in the cuts. However, at mile post 60 an extensive fault with considerable displacement of the beds cut across the shale beds in both the railroad cut and a fresh State Highway cut on the opposite side of the pass (see Fig. 9). The west fault of the zone made up the west slip plane of the slide. Vertical shears at the head of the slide were cross faults of the zone, and the shale within the fault zone was sheared and jointed. Even then, considerable stress must have been required, under those conditions, to overcome the stability inherent in the mass due to its natural state and position. The character of the dry slide material, after the main body had been removed, is shown in Fig. 10.

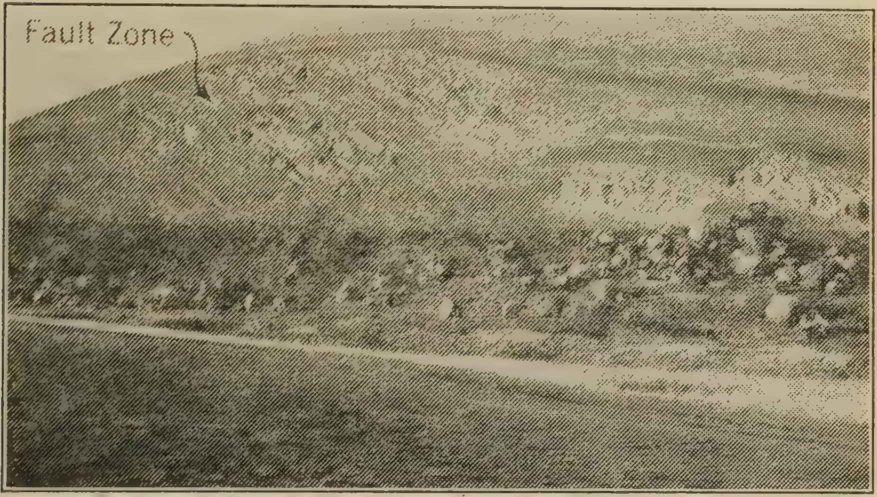


FIG. 9.—FAULT ZONE IN ALTAMONT PASS AS REVEALED BY A STATE HIGHWAY CUT

In order to observe subsurface conditions, a number of holes were bored after the loose material was removed by bulldozer. Seven of them were 30 in. in diameter, ranging from 10 ft to 20 ft deep. The line of break was located; and, because the dip of the displaced shale beds could be observed from the top to the bottom of these holes, it was possible to reconstruct the history of the movement. The flow of dry material occurred as an outward movement at the toe of the cut slope of the right-of-way excavation. This was followed by flow along the west fault boundary and a breakup of unsupported material toward the east boundary. The rotary movement continued, involving a mass of material overlying a bedding plane common to all boreholes and below which the material was still in place. The entire mass of shale fragments spilled on to the tracks from a wide hillside area as through a funnel. The flow would have continued if approximately 10,000 cu yd of loose material had not been removed in clearing and protecting the tracks.

A week before the main slide, a trackwalker reported a heaving and cracking of the soil that loosened fence posts and destroyed the right-of-way fence. This led to the conclusion that the stress involved was in the nature of an earthquake shock or series of shocks. The records of the nearest seismograph, at Berkeley, Calif., showed two shocks with an estimated epicenter not far from the location to have occurred about a week apart, the second one at the time estimated for the slide. The first shock is believed to have overcome the stability of the mass, loosening it and causing it to heave and then to settle. The shock that followed a week later caused the actual flow of material, rendered loose by the first, to be shifted from its position to the tracks.

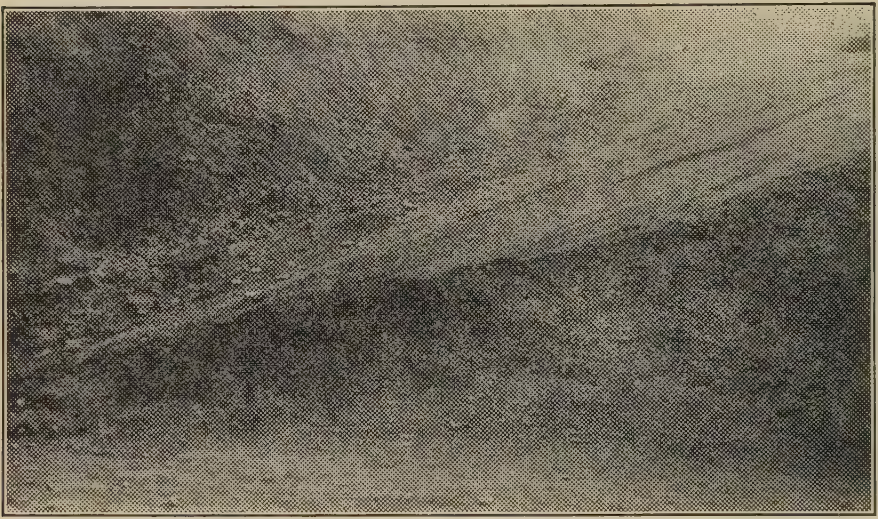


Fig. 10.—Fault Zone Material in a Slide of the Right of Way of the Western Pacific Railway

Slides Due to Water Pressure

Ground-water pressure is one factor in the generation of many slides, and it is the principal and only factor in some slides. Road cuts and grading operations on slopes have the effect of intercepting subsurface channels or zones through which underground water passes from higher to lower levels to be discharged as springs in a state of nature. Localized slides will develop in such areas, due to the pressure exerted by the water.

Bernal Avenue Cut Slide Corrected at Arlington Street.—A localized slide of this kind occurred in the Bernal Avenue cut in San Francisco. The surface of the slopes of the cut revealed an apparently uniform material. However, landslides occurred at two points on the west side. As a temporary expedient, timbers were driven into the slope and connected to concrete anchors by horizontal tie rods. Bulkhead timbers were bolted to the upright timbers. This arrangement failed and the slide continued. The disturbed material was saturated and moisture emerged from the scarp of undisturbed material at points above the slide.

A line of test holes was bored through the slide into the underlying material, and water flowed from a number of the lower holes. Local saturation and water pressure produced deformation that had weakened and unstablized the material to well below the timber bulkhead and the bottom of the cut. Continued sliding ground destroyed a retaining wall and some of the test borings. The latter were supplemented by others in which pipes were set for future water-level observation, and the borings were continued in line up the slope of the hillside for a distance of three blocks. The water level in the holes bored through clay into the disintegrated rock on the upslope sides of Arlington, Chenery and Laidley streets, parallel to Bernal Avenue, rose to from within a few inches to within a few feet of the sidewalk level. A profile along this line to Arlington Street is shown in Fig. 11. By observing the water levels in these pipes the hydraulic gradient of the water that caused the slide condition was found to intercept the slope of the cut, inducing flow from the lowest installed pipe at an elevation 25 ft above the pavement of Bernal Avenue. The water-table gradient before and after the installation of the Arlington Street drain, in relation to the position of the drain, is shown in Fig. 11.

Additional holes were bored at right angles to the first line, along the parallel, and next higher, Arlington Street; and water was found to be under pressure head in seams of fractured rock and along the disintegrated rock surface in line with the slide areas of Bernal Avenue cut. An intercepting drain, similar in construction to that described in connection with the Burnham Street and 24th Street slide, was laid in shafts and tunnels at a maximum depth of 26 ft (25 ft above the cut grade opposite the Bernal Avenue slide areas) along Arlington Street. The length of the drain, in shaft and tunnel, was 460 ft, with an additional 340 ft of trench-laid pipe connecting with a sewer in Bernal Avenue. The work, which included three manholes, cost $16,880 on a lump sum contract.

The Arlington Street drain provides an outlet for the ground water; it releases the water pressure that produced the slides on the Bernal Avenue cut slope. The discharge amounted to 300 gal per hr when the works were completed in April, 1943, decreasing through the summer months to 133 gal of water per hr in November, 1943. The water levels in the observation pipes were lowered almost immediately when the drain excavation and installation approached their vicinity. The saturated slide material was drained slowly by a shallow trench extended into it in 1944. Plans are being made for the replacement of the cut slope.

St. Mary's Playground Slide.—Another common form of landslide that results from water pressure concerns fills. Loose or compacted fills have been placed to provide streets or other level areas on hillsides without provision for draining all the natural spring flow or normal ground water. Such water finds its way through the soil along the contact of the base of the fill material or through disintegrated rock below that soil. The hydrostatic uplift exerted by the confined water against the base of the fill is the principal factor causing the fill to slide in some instances.

The St. Mary's Playground slide was the largest slide of this type that has occurred in San Francisco. The original fill was constructed as a project of the U. S. Work Projects Administration (WPA). Loose material was dumped over an area of about 80,000 sq ft, varying in thickness to 60 ft. Each winter parts of this fill slid down the slope until 15,000 sq ft of the highest parts of the fill were lost. The result of this movement, with large parts of the fill intact but displaced, is shown in Fig. 12.

Preliminary borings showed that the fill material was practically impervious and contained no free water; but, when the old surface soil or disintegrated rock was penetrated, the water rose in the boreholes, in some holes to within a few feet of the surface of the fill. The depth to bedrock, its elevation, and the elevation of the water levels, varied greatly in the boreholes. The contour of the original surface was not known. Its determination and the determination of the source and occurrence of the ground water and water pressures would have required an extensive boring program. A combination involving exploration and correction was adopted to meet these conditions, and the writer accepted a professional service contract for investigation and progressive design

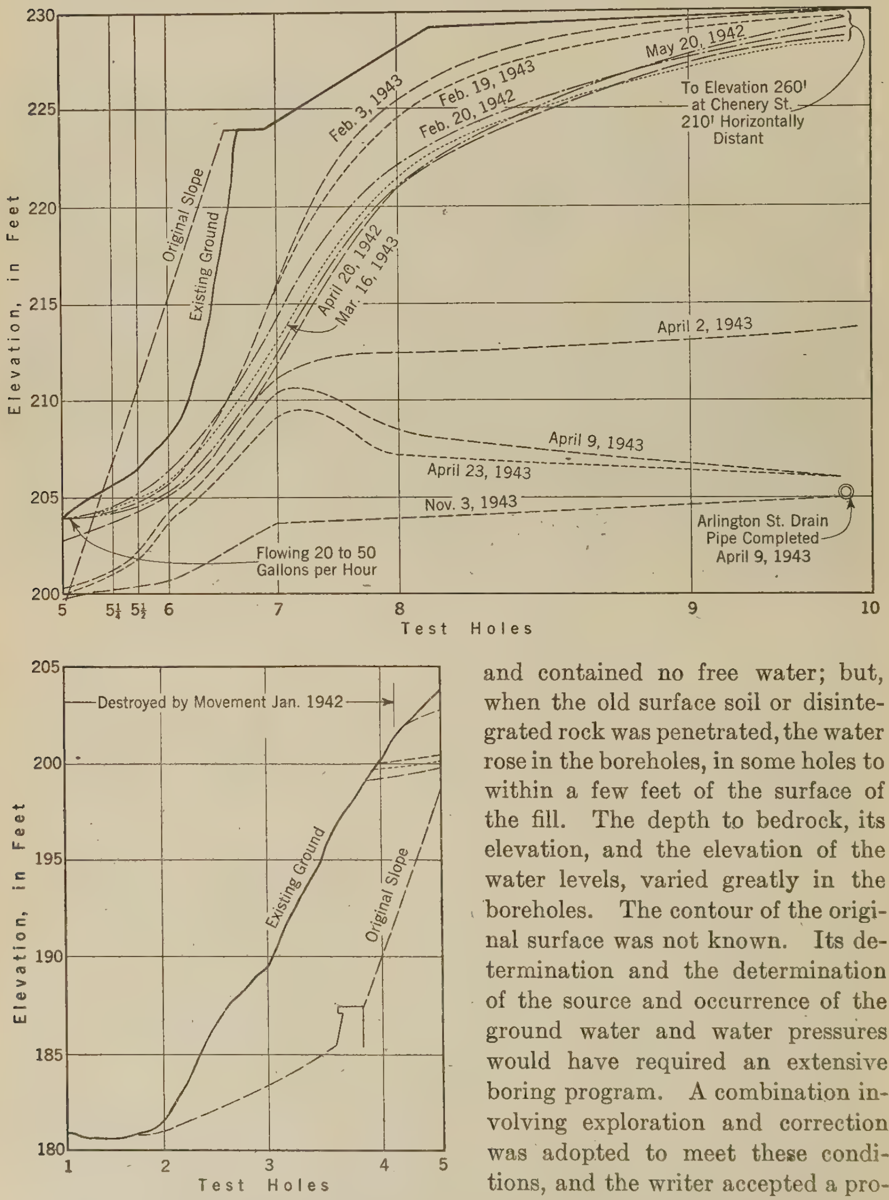


Fig. 11.—Hydraulic Gradients and Profile of Landslide, West Side of Bernal Avenue, 1942 and 1943

and direction of a work contract let on a unit basis. This procedure proved to be satisfactory and economical.

The work was started in the fall of 1941 before the winter rains. An exploration trench was first excavated from one side a distance of 25 ft into the slide area, shown in Fig. 12. This trench revealed a buried contact of serpentine rock with the siliceous sandstone making up the slope beyond the playground area. Decomposition of the serpentine rock had produced a "gully" in the bedrock surface which would not have been evidenced in the contour of the surface soil. The gully underlay the easterly toe of the fill and was an important factor in the instability of the whole. Ground water was concentrated in this region and was moving downward under head. The disintegrated serpentine rock below a blue clay was found to carry the greater quantity of water under pressure, although seepage occurred from the original soil cover of the slope underlying the fill.

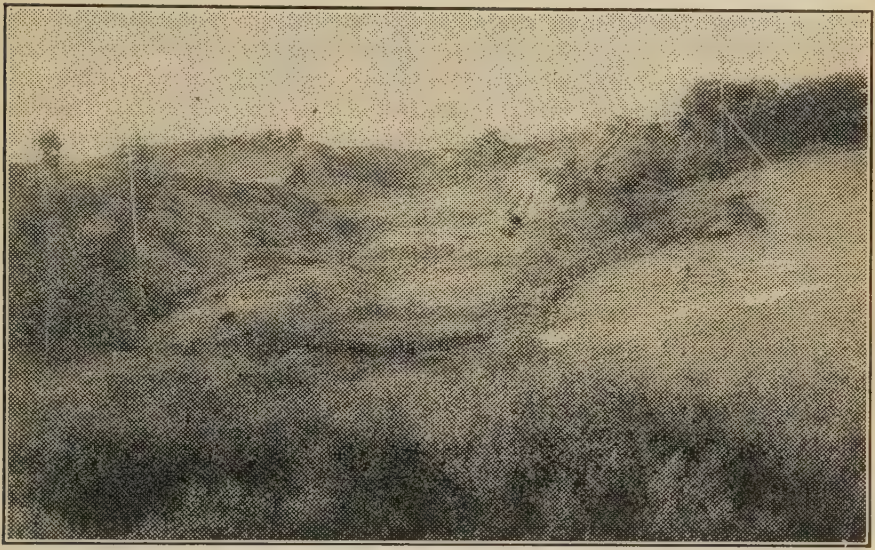


FIG. 12.—LANDSLIDES IN ST. MARY'S PLAYGROUND, SAN FRANCISCO, CALIF.

The depth to rock prohibited further trenching, and 30-in. holes were bored at 5-ft centers, 20 ft to 35 ft deep, through the slide material and through 2 ft of disintegrated serpentine rock to sound rock. Grades were established after drilling four holes. The fill material was dry and stood without casing, but water encountered at the rock line rose to varying heights in each hole. A 6-in. red-iron casing perforated one half of the circumference, was laid as a drain pipe in the trench, and was carried on grade from hole to hole through short tunnels so that water was drained off as the work progressed. The boreholes and trench were backfilled with crushed rock to 6 ft above the base of the fill and were finished with excavated material. The first twelve holes and the trench crossed the subsurface "gully" and produced a flow of 50 gal of water per hr in November, 1941.

The varying depth to sound rock below the original soil surface made it impossible to predetermine the line of the holes and still maintain the necessary grade. The location of each borehole depended upon conditions found at grade in the preceding holes projected. The irregular route[2] so determined, was such that the drain did collect all the water and relieve water pressures, which varied in amount from place to place. A second gully was encountered between holes 20 and 23 and the flow increased to 72 gal per hr. The most extensive gully development was found at about the axis of the slide where the westerly contact of the intrusive serpentine rock with the silicious sandstone was found. The holes bored in the sandstone were dry, so a turn was made to follow the gully up the scarp of the slide on to the undisturbed fill terrace. At this point the discharge had been increased by 86 gal per hr.

The St. Mary's Park area consisted of three terraces on fills, the middle terrace being 25 ft lower than the upper terrace. The lowest terrace was 40 ft still farther down the slope above Alemany Boulevard. The drain system was completed by locating the line of boreholes along the edge of the upper terrace and placing the drain in rock at depths decreasing from 30 ft to 15 ft below the surface in a horizontal distance of 200 ft. An intercepting drain, consisting of a 6-in. corrugated drain pipe, was laid from 4 ft to 6 ft below ground surface along the upper cut slope of the top terrace. The line of boreholes in the axis gully were discontinued at the division between fill to cut, and it was connected to the intercepting drain by a pipe laid in open trench. The discharge of water from the completed system has varied from 140 gal per hr at the end of summer to more than 500 gal per hr during the rainy season. No further movement of the fill has occurred.

The quantities and unit costs of the St. Mary's slide contract let in 1941 were as follows:

Item	Description	Unit cost
1	3,200 ft, 30-in. drain shaft; per linear foot of shaft	$ 2.95
2	400 ft, connecting tunnels; per linear foot of tunnel	$ 1.40
3	800 ft, 6-in. perforated casing; per linear foot of of casing	$ 1.50
4	450 tons, crushed rock (¾ in. to 1½ in.); per ton	$ 2.35
5	Two manholes, including 40 lin ft of 30-in. steel casing and covers; each manhole complete	$32.00
6	600 ft, 6-in. perforated drainpipe; per linear foot of pipe	$ 0.65
7	200 cu yd of excavation 4 ft to 8 ft deep, and backfill for item 6; per cubic yard	$ 2.40

The length of the connecting tunnels was limited by the difficulty of working, by the tools that had to be used in the cramped quarters at the bottom of 30-in. boreholes, and by the problem of disposing of the material removed from the tunnel with a man working in the space. However, the borehole system, even with borings spaced close together, is relatively economical and is best adapted to the correction of large active slides which would require a considerable number of preliminary borings to establish the grade and contour of the contact between moving and undisturbed material or costly shoring to protect workers and adjoining property if open excavation were used.

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[2] "Pipe Drains Stop an Earth Slide," *Engineering News-Record*, April 9, 1942, p. 67.

Success of Remedial Measures.—The success of remedial measures is demonstrated by the fact that the slides have been arrested. To December, 1945, the corrected slides described in this paper have passed through one or more winter rainy seasons without any further movement or signs of instability. The slides in which water was the moving force have been observed continuously, and the discharge of their drainage systems has been approximated from periodic measurement. The results of such determinations, and the corresponding monthly rainfall record at the San Francisco weather bureau station, are listed in Table 2. For the Parker Avenue slide, in addition to the values given in Col. 2, Table 2, the following data for 1939 are presented to complete the record and to simplify Table 2:

Month	Rainfall (in.)	Discharge (gal. per month)
July	Trace	69,200
August	Trace	55,500
September	1.06	48,800
October	1.17	45,700
November	0.20	32,700
December	1.05	41,700

SOIL CREEP

Soil creep is the type of movement characteristic of all soil-covered slopes in excess of 25° from the horizontal. Soil creep takes place on natural slopes such as topographic canyons, and is entirely the result of internal forces. During rainy seasons saturated or wet soil cover, over rock surfaces, gravitates downslope when the forces of cohesion and friction are reduced below that of gravity. Downhill soil creep ceases when soil dries sufficiently to overcome the pull of gravity—that is, as the forces of cohesion and friction increase. This common form of earth movement is evident over large sections of every sidehill and, next to stream-bed erosion, is the main factor in the geologic history of canyon or sidehill topographic development.

As a general rule, this movement is slow and is only destructive where the soil involved in movement supports a fill, roadway, or structure. Retaining walls alone may serve no useful purpose because the soil can creep to, and overflow, the wall. This was found to be the case with the Turk Street slides in San Francisco, and it is a natural action under natural processes. However, the City of San Francisco has been involved in damage suits as the result of such natural action. A sliding condition that developed on the slope above Laidley Street before 1925 resulted in litigation; and the city, at that time, was required to purchase the property and the houses damaged in the movement. The slopes were flattened by the removal of part of the accumulated soil;

shallow drains were installed; and an extensive area was covered with asphalt to shed rain water. This area is two blocks up the slope above, and half a block west of, the drain completed on Arlington Street in April, 1943. No water has discharged from the Laidley Street drain since May, 1943, which would indicate that soil creep can be controlled if the moisture content is lowered by diverting water into underlying natural drainage ways that are kept dewatered by works installed at lower levels.

TABLE 2.—Discharge Record of Drainage Systems in

Month	Rainfall[a] (in.) (1)	Parker Avenue (2)	Rainfall[a] (in.) (1)	Parker Avenue (2)	St. Mary's Playground (3)	Stonecrest subdivision (4)	Bernal-Arlington (5)
	(1940)				(1942)		
January	9.98	136,000	4.76	144,900	.....[b]	......	......
February	7.81	270,000	4.27	200,000	.....[b]	......	......
March	5.32	296,200	2.62	157,900	.....[b]	......	......
April	0.94	276,500	3.65	141,400	.....[b]	......	......
May	0.63	267,800	1.11	121,800	85,560	......	......
June	0.01	242,900	T	93,700	75,000	......	......
July	T	132,000	0.01	96,500	74,400	......	......
August	T	70,000	T	126,600	72,000	......	......
September	0.59	56,200	0.18	113,200	93,000	......	......
October	1.05	50,500	0.95	104,200	80,000	......	......
November	2.22	46,900	4.45	95,500	102,300	172,800	......
December	6.25	63,800	2.87	118,300	155,000	156,240	......
	(1941)				(1943)		
January	8.24	218,100	6.15	343,400	198,400	172,700	......
February	6.71	233,500	1.95	302,000	349,000	204,800	......
March	4.75	304,500	3.18	288,400	246,600	316,100	......
April	4.05	285,700	1.88	260,500	114,000	184,700	122,200
May	1.18	204,800	0.13	234,200	105,400	208,500	134,700
June	0.01	127,600	0.13	185,000	108,000	170,200	118,200
July	0.03	97,300	T	141,300	130,200	186,600	117,500
August	0.01	75,600	T	99,000	107,000	178,500	108,400
September	T	64,900]	0.02	120,100	96,000	169,000	100,900
October	0.93	57,400]	0.74	96,900	102,000	170,800	100,900
November	1.99	49,400]	0.80	68,100	114,000	155,400	92,300
December	7.29	101,800]	2.69	66,100	124,000	184,900	91,900

[a] U. S. Weather Bureau Station, San Francisco, Calif.; T denotes "trace." [b] In November, 1941, the discharge inclusive, 1942, nothing is recorded because

Correction Should Be Based Upon a Thorough Investigation

Experience is the only basis upon which landslides and earth movements can be predicted. In excavation or foundation work, they occur under certain recognizable conditions. Data concerning the distinctive characteristics of movement, and the topography developed through slides, are informative; but correction can be planned in detail only after the subsurface geologic and ground-water conditions are determined. Each type of slide and each set of conditions will present a particular problem in drainage, lateral support, or rebalancing of slopes with or without retention. Corrective works will have to be designed to meet the determined elements of a given problem, and no general treatment can be prescribed.

A geological survey, together with exploration boring, is the first requisite in the determination of the subsurface conditions and physical characteristics of the material involved. The saving in ultimate cost of the corrective work will repay such preliminary work many times. For example, drainage works were designed for the slide at Market Street near Glendale in San Francisco, described previously; and their cost was estimated at $25,000 before adequate investigation was made of the problem. After careful investigation, by standards described in this paper, the corrective work was finally done at a cost of one fifth of that of the original estimate.

Corrected Slides (in Gallons of Water per Month)

24th Street-Burnham Street (6)	Rainfall[a] (in.) (1)	Parker Avenue (2)	St. Mary's Playground (3)	Stonecrest subdivision (4)	Bernal-Arlington (5)	24th Street-Burnham Street (6)	Month
(1942)	(1944)						
......	4.31	113,800	161,200	301,500	103,900	151,200	January
......	5.34	220,000	171,100	357,200	120,400	291,900	February
......	0.83	189,100	136,400	309,000	105,000	156,700	March
......	2.07	100,800	129,000	182,700	81,900	82,000	April
......	0.94	81,900	108,500	186,700	82,000	65,200	May
......	0.12	46,800	96,000	168,300	72,800	45,000	June
......	T	48,200	91,776	152,700	72,200	51,200	July
......	0.02	58,300	93,696	142,600	67,300	67,300	August
......	0.00	56,900	129,600	124,000	62,600	77,500	September
......	1.73	64,000	120,144	111,700	84,900	100,900	October
......	6.24	71,500	135,120	158,100	74,900	133,100	November
......	3.97	121,900	130,120	236,500	72,900	192,200	December
(1943)	(1945)						
......	1.33	167,000	124,560	296,900	76,000	311,100	January
......	3.43	133,800	116,640	258,900	70,700	278,600	February
......	4.15	146,700	133,200	212,300	80,800	208,800	March
......	0.32	134,400	123,120	187,100	84,000	187,100	April
......	0.64	102,000	108,000	180,800	91,000	180,000	May
......	0.01	95,500	98,120	176,500	92,900	171,700	June
70,500	T	89,400	95,040	177,600	102,600	140,400	July
62,900	T	87,100	97,720	175,900	99,500	151,900	August
56,000	0.04	94,900	118,650	163,000	91,000	126,000	September
51,600	1.95	67,300	124,500	135,100	93,600	139,100	October
43,900	3.24	69,900	135,000	132,300	103,100	210,800	November
53,700	9.84	126,900	270,080	242,300	120,600	372,200	December

for St. Mary's Playground (Col. 3) was 46,800 gal. In December, 1941 it was 108,700 gal; and in January to April, surface water was diverted into the drain.

For determinations of moisture content and drainage characteristics, under some conditions of sliding ground, soil samples should be taken which are representative of the full soil column over the area. Ground-water sources and the seasonal water levels and pressures should be determined. If slip planes are observed in an upper scarp, throughout, or at the toe of a slide, they should be watched for in the borings. A series of such planes should be followed down carefully so that the bottom planes or the base of the present or prospective slide is recognized or so that the plane along which shearing has occurred, or is to be expected, is located definitely. Geologic conditions are controlling factors in the occurrence of ground water, and borings should be made sufficiently deep to determine those conditions and the condition and the occurrence of the contained ground water. Landslide corrective works have failed in a number of instances for lack of definite information on this score.

Continuing Investigation During and After Construction.—Considerable preliminary borings have been made in slide areas known to the writer, a plan of works has been designed on the basis of such borings, and later the subsurface was found to vary materially from that assumed on the basis of the borings. Excavation for the construction of subsurface works should always be considered as a continuation of the investigation. The writer requires three contracts or service orders in slide corrective work for the City of San Francisco: One covering the preliminary investigation, direction, and analysis of the exploration boring and a report with recommendations; one covering time spent in consultation with the City Engineering Department in design, preparation of specifications (which are always made sufficiently broad so that work needed to meet conditions as they are revealed can be done on a unit basis or as an "extra"), and cost analysis of alternate plans; and the third covering time spent in examining and directing the work of the contractor.

Subsurface work in engineering construction must be considered in the nature of research, and it is necessary for the investigator to "feel his way." The data resulting from subsurface observation, boring, or excavation are necessary in the preparation and adjustment of plans, for drainage, sloping, retention, or stabilization. Frequently, boreholes in sliding ground will be cut by movement on two or more planes in their depth. If the boreholes are closed by the movement of any part of the soil column along any plane, such planes and movement must be recognized and provisions must be made to meet the deepest movement as well as the moving horizons in the column.

In slide corrective work much depends upon whether the slide is one of shear or detrital movement. Shear slides must be stabilized by providing works below the slip plane. Detrital slides can often be stabilized by: (*a*) Drainage ways in the upper slope; (*b*) shallow drainage facilities below the ground surface; (*c*) excavation to reduce the slope; or (*d*) retaining walls along the toe of slope to reduce the slope.

However, when the earth's structure is once ruptured by movement and its surface is cracked so that water has ready access to the interior of the mass, causing it to swell and become unstable, or causing bedding or contact planes below installed shallow drains to become lubricated, movement will recur. In all cases, once a slide is generated under a force great enough to overcome stability due to structural compactness and position of the material, it will continue or recur under forces much less than the initial force. Earth slides are a menace to the safety of life and property in urban localities or along roadways until they are corrected, properly and permanently.

DISCUSSION

T. W. LAMBE,[3] JUN. ASCE.—Water has several effects on slopes: It erodes the slope; it changes the strength properties of the soil; it makes possible a liquefied condition when the soil undergoes certain volume changes upon shearing; and it exerts forces on the soil. The author describes the water forces in general terms which, to the engineer, do not present a clear picture of the true nature of the effects. He mentions (see heading, "Landslides") that "The weight of the added water alone may overstress the material in such cases * * *." This thought, which is expressed in several places in the paper, needs some clarification, since a slope submerged, with no seepage, is safer than one that is not submerged although the "weight of added water" is much more. The seepage forces—the forces tending to cause slope failure—result from the difference in water pressures (that is, a sloping water table). If a flow net can be drawn for any slope, the seepage forces can be evaluated.

The question as to whether a slope is safe against rupture is one of mechanics: Failure occurs when the forces actuating the possible failure exceed the forces resisting such a failure. There are several types of slope movement that do not lend themselves to a quantitative analysis because of the difficulty of evaluating, accurately, the actuating or resisting forces. An example of an indeterminate actuating force is the force that an earthquake exerts on a slope. Likewise it is sometimes impossible to determine the shearing resistance of the soil that exists at incipient failure. Geology is a science that is helpful in formulating a more complete understanding of factors affecting these two types of forces.

To present a picture of the forces acting on a slope, an element of soil from an infinite slope will be analyzed, first without seepage forces and, second, with seepage forces. (A description of the methods of analyzing slopes has been presented elsewhere by D. W. Taylor,[4] Assoc. M. ASCE.)

In Fig. 13, H is the height of a soil element; α is the angle of inclination of the slope; γ is the unit weight of soil; and ϕ is the angle of internal friction of soil. The actuating unit forces, which are shown by solid lines in Fig. 13(b) are then:

$$W = \frac{\text{weight of soil}}{\text{area}} = \frac{\gamma \times H \times 1 \times 1}{\sec \alpha \times 1} = \gamma\, H\, \cos \alpha \ldots\ldots\ldots (1a)$$

$$W_n = \text{normal component of } W = \gamma\, H \cos^2 \alpha \ldots\ldots\ldots\ldots (1b)$$

and

$$W_s = \text{shearing component of } W = \gamma\, H \cos \alpha \sin \alpha \ldots\ldots\ldots (1c)$$

and the resisting unit forces, which are shown by dashed lines in Fig. 13(b), are: C equals unit cohesion; and

$$F = \text{frictional resistance} = W_n \tan \phi = \gamma\, H \cos^2 \alpha \tan \phi \ldots\ldots (2)$$

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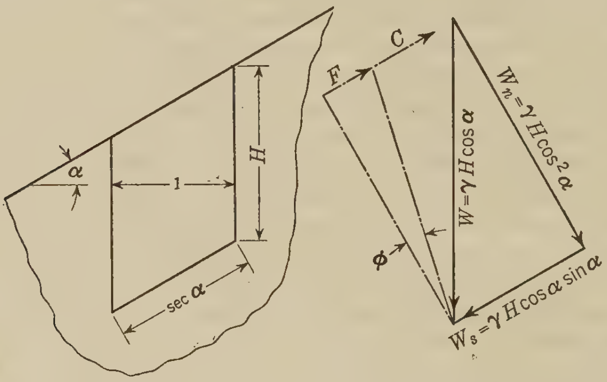
[3] Instructor of Soil Mechanics, Civ. Eng. Dept., Mass. Inst. Tech., Cambridge, Mass.

[4] "Stability of Earth Slopes," by D. W. Taylor, *Journal*, Boston Soc. of Civ. Engrs., July, 1937, p. 197.

For stability the resisting forces must exceed the actuating forces along the plane of failure; that is, $F + C > W_s$. The factor of safety (FS) with respect to strength is equal to

$$\frac{F + C}{W_s} = \frac{\gamma H \cos^2 \alpha \tan \phi + C}{\gamma H \cos \alpha \sin \alpha} = \text{FS} \ldots\ldots\ldots\ldots\ldots (3)$$

The only effect of complete submergence, with no seepage, in the expression for FS is to change γ (submergence may cause minor changes in ϕ). The value of γ for a slope above the water table is the unit weight of soil, plus any capillary moisture, as this total weight is carried by the soil structure. When the slope is submerged, the effect of buoyancy can be taken care of by using the submerged, or buoyed, unit weight of soil.



(a) INFINITE SLOPE (b) FORCE POLYGON

FIG. 13

The two limiting types of soil are the cohesionless type (which is approached by clean sands) and the highly cohesive type (which is approached by stiff clays). If the soil in question is cohesionless, C is equal to zero and Eq. 3 becomes

$$\frac{\cos \alpha \tan \phi}{\sin \alpha} = \frac{\tan \phi}{\tan \alpha} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The only criterion for stability of a cohesionless slope is that ϕ be greater than α; submergence or height of element has no effect.

If the soil is highly cohesive, submergence makes the slope have a higher FS against movement. This can be demonstrated from the fact that submergence decreases both F and W_s by replacing γ with the buoyed unit weight, whereas C remains unchanged in the expression $\text{FS} = \frac{F + C}{W_s}$. When F and W_s are decreased, C remaining constant, the factor of safety increases. Since all soils fall between the two limiting cases, the effect of submergence, without seepage, varies from not changing the FS to making the FS larger. This effect of submergence with no seepage is common knowledge that can be checked easily in the laboratory.

The water content of a soil can be increased by capillary action; this water content would make the slope less safe as the increased weight would have to be carried by the soil structure. In Eq. 3, F and W_s would be increased because of the increased γ although C remained unchanged. Drains within the slope would not remove the capillary moisture. This condition is not to be confused with analyzed conditions of complete submergence.

The detrimental forces of water are the seepage forces which the water imparts to the soil. Seepage forces act in the direction of the seepage and are equal to

$$J = i\,\gamma_w \text{.............................} (5)$$

in which J is the seepage force per volume; i is the gradient, or head lost per distance traveled; and γ_w is the unit weight of water. This seepage force when directed upward, having reached a magnitude greater than the weight of the soil, causes "quicksand." Seepage forces may be evaluated in one of two ways—(1) by consideration of the actual seepage force, or (2) by consideration of the boundary water pressures, which are the pressures that make up the seepage force. In most soil problems it is more convenient to use the boundary water pressures. The results obtained from the two methods are identical.

To show the fundamentals of the seepage force, an element from an infinite slope in which seepage is taking place throughout the mass parallel to the slope (that is, the top flow line coincides with the slope surface) will be analyzed by both methods. Any slope for which a flow net can be drawn can be analyzed, but the very simple infinite slope is selected as it shows the fundamentals clearly without unnecessarily complicating the problem. The forces acting on the element in Fig. 14(*a*) are determined by the two methods and superimposed in the force polygon shown in Fig. 14(*b*).

The forces on the element are first set up using the actual seepage force. Let γ_t be the total unit weight of soil and water; γ_s, the submerged, or buoyed, unit weight of soil; and γ_w, the unit weight of water. The actuating unit forces, shown by solid lines in Fig. 14, are then:

$$W_s = \frac{\text{submerged weight of soil}}{\text{area}} = \gamma_s\, H \cos\alpha \text{................} (6a)$$

$$W_{sn} = \text{normal component of } W_s = \gamma_s\, H \cos^2\alpha \text{................} (6b)$$

$$W_{ss} = \text{shearing component of } W_s = \gamma_s\, H \sin\alpha\cos\alpha \text{...........} (6c)$$

$$J = \text{unit seepage force} = \frac{i\,\gamma_w \text{ volume}}{\text{area}} = \gamma_w\, H \sin\alpha\cos\alpha \text{.....} (6d)$$

and the resisting unit forces, which are shown by dashed lines, are: C equals unit cohesion; and

$$F = \text{frictional resistance} = W_{sn}\tan\phi = H\,\gamma_s\cos^2\alpha\tan\phi \text{......} (6e)$$

The forces on the element are now set up using the total soil weight and boundary water pressures. The actuating unit forces shown by the dotted lines are then:

$$W_t = \frac{\text{total weight of soil and water}}{\text{area}} = \gamma_t\, H \cos \alpha \ldots\ldots\ldots (7a)$$

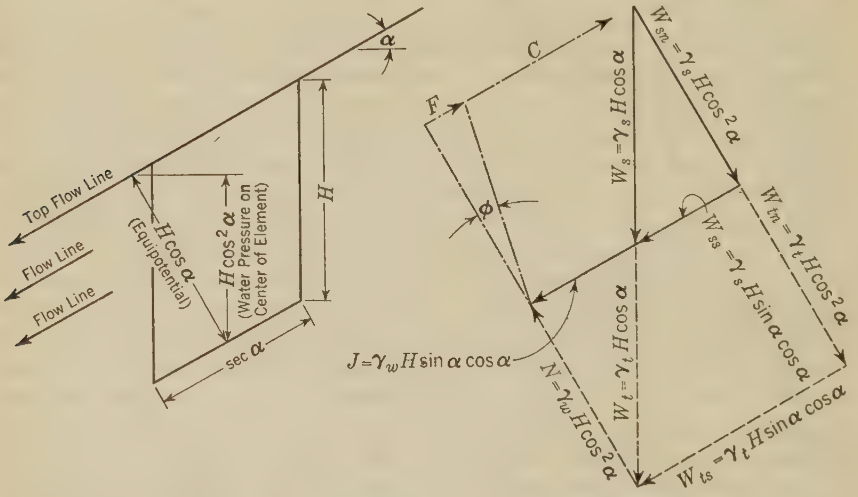
$$W_{tn} = \text{normal component of } W_t = \gamma_t\, H \cos^2 \alpha \ldots\ldots\ldots\ldots (7b)$$

$$W_{ts} = \text{shearing component of } W_t = \gamma_t\, H \sin \alpha \cos \alpha \ldots\ldots\ldots (7c)$$

$$N = \text{boundary water pressure} = \gamma_w\, H \cos^2 \alpha \ldots\ldots\ldots\ldots (7d)$$

and the resisting unit forces, which are shown by dashed lines, are: C equals unit cohesion, and

$$F = \text{frictional resistance} = (W_{tn} - N) \tan \phi = H\, (\gamma_t - \gamma_w) \cos^2 \alpha \tan \phi \ldots (7e)$$



α
Top Flow Line
Flow Line
Flow Line
H cos α (Equipotential)
H cos² α (Water Pressure on Center of Element)
H
sec α
C
F
φ
J = γ_w H sin α cos α
W_s = γ_s H cos α
W_sn = γ_s H cos² α
W_ss = γ_s H sin α cos α
W_tn = γ_t H cos² α
N = γ_w H cos² α
W_t = γ_t H cos α
W_ts = γ_t H sin α cos α

(a) INFINITE SLOPE (b) FORCE POLYGON

FIG. 14

Since $\gamma_s + \gamma_w = \gamma_t$, the seepage force plus the shearing component of the submerged, or buoyed, soil equals the shearing component of the total weight. For the same reason the friction components by the two methods are alike. The force polygon shows that the two methods give identical results. As before the criterion of failure is whether or not the sum of actuating forces exceeds the sum of the resisting forces.

The force polygon in Fig. 14(*b*) clearly indicates the effect of seepage. In this case seepage has added an actuating force of about the same magnitude as the gravity force since for most soils γ_s is close in value to γ_w.

The slope on the "West Side of Bernal Avenue," of course, is made much safer by the instalment of the "Arlington Street Drainpipe" (see heading, "Slides Due to Water Pressure: Bernal Avenue Cut Slide Corrected at Arlington Street"); but this added safety is provided by the removal of seepage forces, or differences of water pressure—not simply from the "release of water pressure." This is apparent from the fact that had the entire slope been submerged, although the water pressures have been increased, the slope would have been made more safe by this submergence than by the installation of the drain.

JACOB FELD,[5] M. ASCE.—The landslides reported in this paper are spectacular demonstrations of adjustments often occurring within soil bodies because of load changes or modifications of physical conditions. The internal adjustments result from (*a*) an increase in either static or dynamic loading externally, (*b*) a decrease in a lateral resistance or support, or (*c*) a modification of the soil constituents causing a decrease in physical resistance characteristics. Such internal adjustments very often do not exhibit any exterior result. When an exterior displacement becomes visible, it is called a landslide. However, the same phenomenon is encountered in engineering work and treatments similar to those described by the author become necessary even for cases which are not designated as landslides. For instance, when the fills in back of retaining walls or abutments settle because of either changes in external load or changes in internal characteristics, when structures and pavements along the side of railroad cuts and subway operations move either laterally or vertically, and when buildings constructed on hillsides or on filled-in ground covering sloping original surfaces show sign of distress—all these phenomena are landslides.

In general there are two types of correction: (*a*) The physical lateral resistance or support to a mass of soil can be increased by adding a retaining wall, sheet piling, or other structural additions; (*b*) further change of the physical make-up of the soil can be prevented by control of internal water, or by chemical injections, or by other methods that have been developed to alter the property of weak soils. For a number of cases in which building movements resulting from internal soil failures occurred, the methods of correction were published by the writer elsewhere.[6]

No general rules can be devised for sufficient and necessary methods of altering the physical characteristics of soils. Before any decision on such work can be reached, it is necessary to obtain complete information about the soils that exist in the locality; to know how these soils react with different moisture conditions under varying pressure and varying temperature; and to realize the effect of local chemicals that may be brought into the picture by the soil moisture. It is known that different chemical constituents in clays can be recognized and that the success of chemical control of clays depends upon the chemical constituents in the clays.

The effect of added absorbed moisture is described by the author (under the heading, "Landslides") as a weakening of the soil particles so that they cannot support loads. This is not always the case, as is exemplified in the description under the heading, "Shear Slides: Plastic Material at Market and Glendale Streets, San Francisco." The hydration of some soils retained within the materials which are not affected by water may increase the bearing capacity of the mass. This condition occurs when a formation swells from the absorption of water and integrates with adjacent layers. Some of these mixtures are similar to hardpans in their resistance to excavation.

Filling up cracks in clays overlying rock or other unalterable surfaces with rain water (under the heading, "Shear Slides: Earth Slide at Crolona Heights in Crockett, Calif.") is identical with the cause of failure of basement walls built in clay areas if surface waters are not properly collected. In both the Hartford, Conn., and Detroit, Mich., areas, the writer has seen basement walls pushed in by the pressure of water filling in a narrow but continuous crack when clay backfill dried out and left a gap along the exterior face of the wall. The use of underground stone drains in soils of this nature is only a temporary correction, since the clay soils find their way into the stone and seal up the ground. The only solution is to eliminate the water.

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[5] Cons. Engr., New York, N. Y.

[6] "Several Foundation Repair Jobs," *Proceedings*, Brooklyn Engrs. Club, April, 1930, p. 47.

In attempting to prophesy where slides, either of the spectacular type or of the equally troublesome internal adjustment type, may occur, it is necessary to have a complete picture of subsurface ground conditions. Trouble will almost always occur in areas where natural drainage channels have been buried. Near Newburgh, N. Y., where some fills almost 60 ft deep had been placed under proper supervision and by heavy equipment, an internal adjustment occurred quite suddenly, forming a crack similar to that resulting from earthquakes. Fortunately, the crack developed one day before a concrete pavement was placed on a prepared subgrade which had been tested for bearing capacity and had been approved. Borings taken of the soil adjacent to the point of failure indicated that an old drainage channel at the bottom of the fill was still in operation and that the material at a depth of 60 ft was too soft to obtain any kind of sample.

A reliable method of determining the nature, thicknesses, and direction of the slope of the subsurface layers of soil is by means of test pits. The cost of such procedure is much more than the cost of borings and the accuracy of the information obtained is proportionate to the cost of the two methods. In connection with the methods of determining subsurface conditions, it is difficult to agree with the author in his statement (under "Shear Slides: Costs of Corrective Works") that boring records and samples are of little value if they are not collected by an experienced geologist. In some soil locations, boring records are of little value regardless of who collects them. The presence of an experienced geologist does not increase the value of the soil sample which has been improperly taken. The writer has found a collective history of the vicinity and especially a comparison of maps showing topographical conditions over a period of years to be of great value. Such information has permitted the design of necessary corrective work for several subway operations in New York, N. Y., since the data indicated the direction and, to some extent, the amount of movement over a period of years. In this way artificial resistance can be provided to prevent incipient movement of adjacent structures. For instance, the record of surface elevations in the vicinity of 8th Avenue and the Harlem River in Manhattan (an area adjacent to the Polo Grounds) was a history of slowly decreasing movements in certain directions. Later excavations and test pits as well as borings provided sufficient information to prepare comprehensive subsurface maps indicating the slopes of the various layers. The movement predicted by this study agreed almost exactly with that found from the previous collection of data—on movement of monuments, amount of settlement of adjacent structures built at different times, and movements of streets. Movement of streets is not an unusual condition in cities built along waterfronts where almost always the marginal streets are on filled-in ground. In such locations corrective methods are often necessary and the methods used by the author in the correction of landslides in the San Francisco Bay vicinity have yielded very valuable data.

GEORGE S. HARMAN,[7] Assoc. M. ASCE.—The Recreation Commission of San Francisco, Calif., from time to time has acquired parcels of land which, because of topography, instability, or both, are not particularly adapted to commercial or residential development. However, playgrounds, which supply a space for children to play in an otherwise highly congested district, can be built on these lands.

In designing a playground, the recreation profession and recreation leaders desire areas as large and with as few levels and terraces as possible. Thus, since San Francisco is a very hilly city, some heavy cuts and fills are necessary when grading a parcel of land for playground purposes.

When it became incumbent on the writer to determine engineering policy for developing such lands into playgrounds, it soon became evident that special knowledge in geology and experience in underground water were required. This need was filled when Mr. Forbes was employed as a consultant; and, since the writer followed the investigation and construction phases closely, he is most pleased that the unique conditions and methods used to solve the problem at St. Mary's Playground have been presented to the Society as a whole.

One of the principal benefits to be derived from the section on "St. Mary's Playground Slide" is the very practical approach to the solution of a problem of engineering geology in a hilly area with very complex soil and rock composition. The fact that investigation of the problem and remedial engineering construction work were combined in one project saved time, work, and money. The construction itself was quite novel in that it allowed the author to feel his way and to control the underground water as soon as it was encountered. Many ideas were considered in trying to overcome the slide conditions, but all of them were discarded in favor of the method used because neither the depth of the underground water nor the location of the source were known.

Precise surveys and levels over a period of a year after the work on St. Mary's Playground have shown that the slide has been stabilized, whereas, previous to construction, each winter the slide was moving progressively many feet south.

The author mentions briefly the cause of the sliding at James D. Phelan Beach. Before this site can be used as a recreational area, the slide must be totally stabilized and a sea wall constructed to control the sandy beach and to prevent undercutting by high tides and wave action. As the author states, the underground water is causing the underlying serpentine rock to deteriorate and to disintegrate into a blue clay, thus producing a slip plane. This disintegration is not apparent on the surface, but borings and preliminary investigation have shown that this is the case. Sufficient remedial work has been done to prove that this slide can be stabilized by properly controlling the underground water. However, the partial installation also showed that time and money can be saved by having the advice of a consultant geologist during construction.

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[7] Engr., Recreation Comm., San Francisco, Calif.

Robert F. Legget,[8] M. ASCE.—The description, presented by Mr. Forbes, of a number of landslides typical of the San Francisco area is a most valuable addition to the published records of earth movements affecting civil engineering works. Since, as the author rightly observes (under the heading, "Correction Should Be Based Upon a Thorough Investigation")—"Each type of slide and each set of conditions will present a particular problem * * * and no general treatment can be prescribed"—it is impractical to offer any useful general comment upon the individual cases included in the paper. However, it is especially significant that serpentine is apparently predominant among the rock types encountered locally, being referred to (under the heading, "Shear Slides: Parker Avenue Slide in San Francisco") as "the serpentine bedrock of the region" and later (under the heading, "Detrital Slides: O'Shaughnessy Boulevard Slides") as "basic igneous dikes, principally serpentine." Accordingly, it is puzzling to read the author's statement (also under the heading, "Detrital Slides: O'Shaughnessy Boulevard Slides") that "There is nothing unique about the rock formations of the San Francisco Bay area that makes them particularly susceptible to the development of landslides * * *." In the strictly literal sense, serpentine cannot be described as unique, but it is encountered as bedrock in built-up areas so infrequently that this statement seems to require some qualification.

Serpentine has long been recognized as one of the most treacherous of rocks, at least by the civil engineer. The author himself comments (under the heading, "Detrital Slides: O'Shaughnessy Boulevard Slides") on "* * * the serpentine hydrates forming a clay which, with the further absorption of moisture, becomes a viscous mass moving down the slope * * *." (Presumably this is what is meant by the "landslide material" mentioned under the heading, "Shear Slides: Preventive and Precautionary Measures.") Very fortunately, there are few if any other varieties of solid rock which possess the undesirable features of serpentine. Therefore, it may be suggested that some of the earth movements so ably corrected and described by Mr. Forbes were due, at least in some measure, to a special cause—the presence of serpentine.

If this suggestion is true, it is even more difficult to draw any general conclusions from this study of selected cases. Despite such a limitation the author has suggested a classification of earth movements which has little to commend it as compared with previously published classifications.[9,10] Only unusual local circumstances could be held responsible for having led the author to make statements in the "Introduction" which are not quite in agreement with a large body of informed opinion previously expressed and recorded.

The great value of the factual descriptions presented by the author make the submission of any critical comment an invidious task. However, for example (see heading, "Introduction") it would be singularly unfortunate if it were recorded in the publications of the Society, without refutation, that "* * * most of the laws and experimental data in this field [soil mechanics] have been developed through the use of dry sand composed of from 95% to 99% stable silica." The contents of the *Transactions* of the Society alone suffice to question this assertion. Although, it may be true (see heading, "Introduction") that, in the San Francisco Bay area, "Sand is seldom found disassociated from silt and clay, * * *" there are at least a few other parts of North America where even this statement is not quite correct. Other statements in the "Introduction" could be questioned, and there would surely be few engineers who would urge the use of slope stability calculations "as a substitute for * * * thorough preliminary investigation of all other determinate facts." Rather such calculations can only be used in conjunction with a thorough study of site conditions. There are many valuable publications which substantiate this suggestion, of which only two need be mentioned—that by A. W. Skempton in 1945[11] and that by Alex Collin in 1846.[12]

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[8] Associate Prof., Civ. Eng., Univ. of Toronto, Toronto, Ont., Canada.

[9] "Landslides and Related Phenomena," by C. F. Stewart Sharpe, Columbia Univ. Press., New York, N. Y., 1938.

[10] "Landslides, Subsidences and Rock-Falls, as Problems for the Railroad Engineer," by C. E. Ladd *Proceedings*, A.R.E.A., Vol. 36, 1935, pp. 1091–1162.

With Mr. Forbes' emphasis upon the importance of ground water in many cases of earth movement, there will be very general agreement. Not the least valuable part of the paper, therefore, is the statement of actual costs for the several types of drainage installations used by the author. The actual records of drainage discharge are similarly interesting and show in no uncertain manner the significance of ground-water movement at the slides in question. When plotted graphically, the discharges show somewhat erratic correlations with rainfall records; it would be interesting to know the author's explanation of this condition. Water can affect slope stability in ways other than those outlined by the author. Another paper, published in 1945, gives useful relevant illustrations.[13]

RUSH T. SILL,[14] ESQ.—An extremely well-presented treatment of the causes of landslides and their correction is contained in this paper.

The writer had the opportunity to study and suggest corrective measures in a slide in which the movement took place along faults occurring in a large mass of homogeneous rock.

Water over a large surface area percolating through the soil entered the fault and softened the gouge material transforming it into softened lubricated mass on which the rock moved.

The spillway cut at the El Capitan Dam built by the City of San Diego, Calif., on the San Diego River, as a part of its water collecting and storage system, was made through the much faulted, fractured, and altered granitic rock which forms the north abutment of the dam. Along lines of rock weakness, fault, shear, and fractured planes, weathering had progressed, developing near the surface the boulder type of weathering so common to granite. The results of this type of weathering are boulders of almost fresh, unaltered granite surrounded by weathered and altered granite less durable than the boulders.

North of the dam area, in the spillway cut about opposite the west end of the weir is a prominent fault striking north 16° east with a dip of 69° to the south. Some distance below the top of the hill, this fault had been cut by a steep westerly dipping shear plane, and Fig. 15 shows a slide of several hundred tons of the crushed material in the fault zone. This slide occurred soon after the excavation in the spillway cut had progressed below the westerly dipping shear plane.

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[11] "A Slip in the West Bank of the Eau Brink Cut," by A. W. Skempton, *Journal*, Inst. C. E., Vol. 24, 1945, pp. 267–287.

[12] "Recherches Experimentales sur les Glissements Spontanes des Terrains Argileux," by Alex Collin, Paris, 1846.

[13] "The Stability of Natural Slopes," by W. H. Ward, *The Geographical Journal*, Vol. 105, 1945, pp. 170–177.

[14] Partner, Ruscardon Engrs., Los Angeles, Calif.

A mass of rock estimated to be 150,000 cu yd rested on a westerly steeply dipping fault plane which extended southeasterly along the spillway cut from this fault on the north to a small gulley formed on a shear zone to the south end of the spillway cut and east 400 ft to the first prominent shear planes strike north 58° west dipping steeply to the north. The wedge-shaped block of ground was left unsupported on the lower or big end of the wedge by the excavation of the spillway (see Fig. 15). During construction, in the winter wet season, this block of ground moved approximately 6 in. down the westerly dipping shear plane, opening a series of almost continuous cracks over the crest of the ridge between the small gully formed on the north fault and the gully formed along the south shear plane.

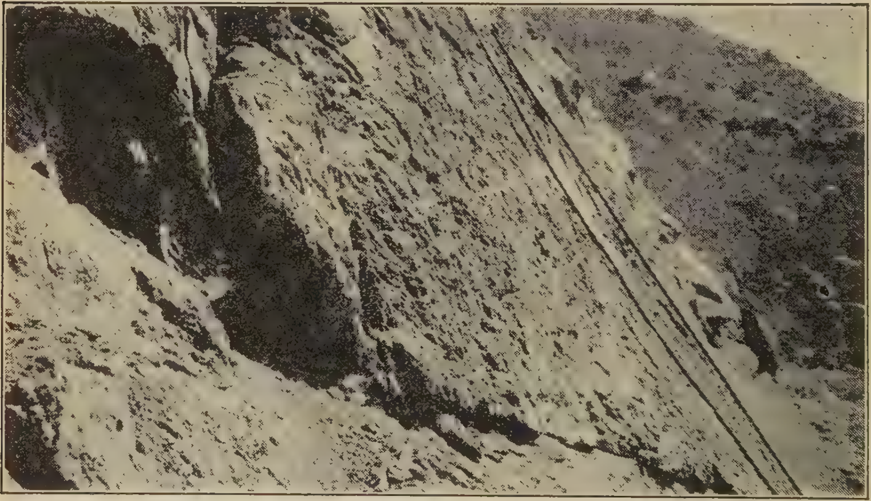

Fig. 15.—View of Slide, North Abutment, El Capitan Dam, San Diego, Calif.

These cracks in the surface soil are from 2 in. to 4 in. wide and some of them can be followed for a distance of from 40 ft to 50 ft. Continued slippage would have resulted in the movement of this 150,000 cu yd into the spillway cut. Its subsequent removal would have been a very expensive operation. Had the slide occurred and the spillway been plugged during a flood, overtopping of the dam might have resulted.

It was recommended that this block of ground be protected by well-constructed ditches to keep the water from entering this fault plane to lubricate it with resulting possible further movement.

The El Capitan Dam is in a dry section of Southern California, and further movement of this block of ground has been arrested.

The following investigations were instigated by the contractors to classify the material in the spillway cut and to confirm the surface indications as to the cause of this movement.

Crushing, specific gravity, and percolation tests were made on samples of the partly decomposed granite surrounding the boulders. This was done to establish the classification into which the material fell—solid rock (ledge rock in place that cannot be loosened except by wedging, barring, or blasting); detached masses of solid rock more than 1 cu yd in volume; or earth, or overburden. Samples were taken from seven sections spaced approximately at equal intervals along the north face of the spillway, none of which were free of fracture lines and thin gage seams which greatly reduced their strength. For permeability tests, samples of rock were cut to a thickness of 4 in. These were placed in a circular side form composed of galvanized sheet iron; then a mortar composed of one part of Portland cement and two parts of sand was puddled around the stone, which gave a finished specimen in the shape of a disk 4 in. thick and 8 in. in diameter. The metal side forms remained on the disks during the tests. The specimens were placed in a moist room as soon as they were molded to insure proper curing of the mortar.

The disks were placed for testing between cast-iron top and bottom pieces of holders to bring water under pressure in contact with the upper surface, and to retain for measuring all water passing through the disk.

These castings were made to make contact or bearing with the disk for a width of 1 in. around the circumference; a small chamber for water was left both above and below the disk; the water was free to pass through the center of the 6-in. disk; and the castings were drawn together by tightening the six bolts in the flanges.

The upper cap was provided with a water gage and air vent; the lower cap had an outlet valve and an outlet tube leading to a glass tube graduated to cubic centimeters.

In testing, water was poured into the lower chamber; outlet valves were filled and no air was present. The outlet valve was closed so that any water passing through the disk was caught in the graduated tube. Water was then admitted to the upper cap, excluding all air. The reservoir was so arranged that the head of water was kept constant throughout the test.

Readings of the water level in the container were taken at 15-min intervals, and the amount of leakage calculated. The results of these tests (Table 3) show a porous condition of this weathered granite to depths of between 200 ft and 250 ft. Near the surface percolation rates of 1,149 ft per yr were obtained from sample 4C.

TABLE 3.—AVERAGE LEAKAGE RATE THROUGH TYPICAL SAMPLES

Flow, in cubic centimeters	Specimen 1B	Specimen 4C	Specimen 5A
For a head of 80 in.	206	2,967	388
Per hour	824	11,868	1,552
Per day	19,776	284,832	37,248
Per sq in. per day	695	11,508	2,763

In making the tests on voids in rock fragments, the rock was first crushed into particles passing a No. 4 screen. The sample was poured loosely into a steel cylinder 6.17 in. in diameter and 7.93 in. high which was filled to about ½ in. below the top. A steel piston was placed on the rock fragments and brought to level bearing without pressure except its own weight; the specimen was placed in the testing machine and successive loads applied. After each load was applied and the piston had come to rest, measurements were taken of the height of the material in the steel cylinder. The increase of the load on the piston caused the rock material in the cylinder to be compressed; from the heights of the material in the cylinder, the volumes of the material at various loads were calculated; and the weights of the material per cubic foot were then calculated from the actual weight of the samples used. The apparent specific gravity of each sample (see Table 4) was determined by the "Standard Method of Test for Approximate Apparent Specific Gravity of Fine Aggregate" of the American Society for Testing Materials (A.S.T.M. C—68—30).

TABLE 4.—CHARACTERISTICS OF FINE MATERIAL

Description	Sample 1A	Sample 5C	Sample 6B
Specific gravity (approximate, apparent)	2.39	2.76	2.76
Percentage of Voids, Under Loads (Lb per Sq Ft) of:			
Piston only	44.46	44.16	43.54
2,500	44.76	43.12	42.85
5,000	43.74	42.50	42.22
7,500	42.93	41.91	41.74
10,000	42.18	41.30	41.32
12,500	41.49	40.89	40.99

The percentage of voids under each condition was determined according to the "Standard Method of Test for Determination of Voids in Fine Aggregate for Concrete" of the American Society for Testing Materials (A.S.T.M. C—30—22).

The soft decomposed granitic material from the spillway cut disintegrates rapidly when excavated and placed in the waste pile. In order to test the percentage of voids and apparent specific gravity of the material in the waste pile, parts of the rock samples 1A, 5C, and 6B were crushed to pass a ¼-in. opening. The compaction, under 12,500 lb per sq in., equal to a height of dump of 100 ft, varied from uniform grading of the crushed material, as shown in the screen tests. Grading of stone voids in the fragments before testing was made on all the samples from the north spillway cut, the results of sample No. 1 being as follows:

Sieve No.	Individual sieve	Cumulative
200	6.3	6.3
100	4.2	10.5
80	2.8	13.3
50	7.1	20.4
40	5.6	26.0
30	8.5	34.5
20	12.8	47.3
16	7.4	54.7
10	18.3	73.0
8	7.7	80.7
4	19.3	100.0
	100.0	

The weight per cubic foot was made on samples from the spillway cut as follows: Sample 5B, 136.66; sample 1B, 160.99; and sample 5C, 166.61.

The compression tests (see Table 5) were made on all the samples. Rocks with crushing strength of 388 lb per sq in., etc., represent material that can be excavated without blasting, but that in quarrying operations is drilled and blasted for economy and speed of operation.

TABLE 5.—SELECTED TESTS FOR TOTAL COMPRESSIVE STRENGTH OF STONE

Strength	Sample 4B	Sample 1B	Sample 5C
Pounds	38,220	38,220	10,380
Pounds per square inch	1,333	2,389	353

Where nests of boulders and hard-rock ribs are interspersed in this decomposed granite, it is necessary to drill and blast before excavating by a steam shovel, and should be paid for as rock.

EARL M. BUCKINGHAM,[15] Assoc. M. ASCE.—A very practical approach to a difficult problem is evidenced by the data in this paper, and it is not the intent of the writer to criticize the author for his excellent presentation. Rapid progress has been made in soil mechanics in recent years; much has been learned that is of value in slide investigation; but the fundamental problem is still one of determining the structure of the hill and the nature of the materials involved. Mathematical analysis has its place, particularly in dealing with slides that cannot be drained. However, in most cases the problem is simply one of locating and removing water. If this can be done, the increase in the strength of the soil will nearly always be more than sufficient to insure stability, and a stress analysis becomes merely an interesting academic exercise. No slide investigation is complete without a thorough determination of underground conditions, but it is frequently possible to simplify the boring program and materially reduce its cost.

Topographic Shape.—The landslide is a natural phenomenon, and one of the processes by which topographic forms are sculptured. An experienced topographer can frequently study the shape of the slide and the adjacent hillside and predict with considerable success the underground conditions that will be discovered by borings.

A long narrow shape, with movement approximately parallel to the original surface, indicates a shallow source of water, and a shallow bedrock which is also roughly parallel to the surface. (In this connection the writer broadens the term bedrock to include a firm and impervious clay.) Although it is easy to draw conclusions after the result is known, the St. Mary's Playground slide in San Francisco, Calif., appears to be typical of this class.

A broad sweeping curve at the head of the slide indicates a deep source of water, particularly if the outline of the slide is poorly defined and formed by a number of parallel cracks. This condition may be accompanied by a distinct rotary motion of the slide mass. The McKillop slide, in Oakland, Calif., shown in Fig. 16, falls in this class. The extreme breadth of the slide indicates either an unusually broad sheet of water or several more or less independent channels.

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[15] **Draftsman, City of Oakland, Oakland, Calif.**

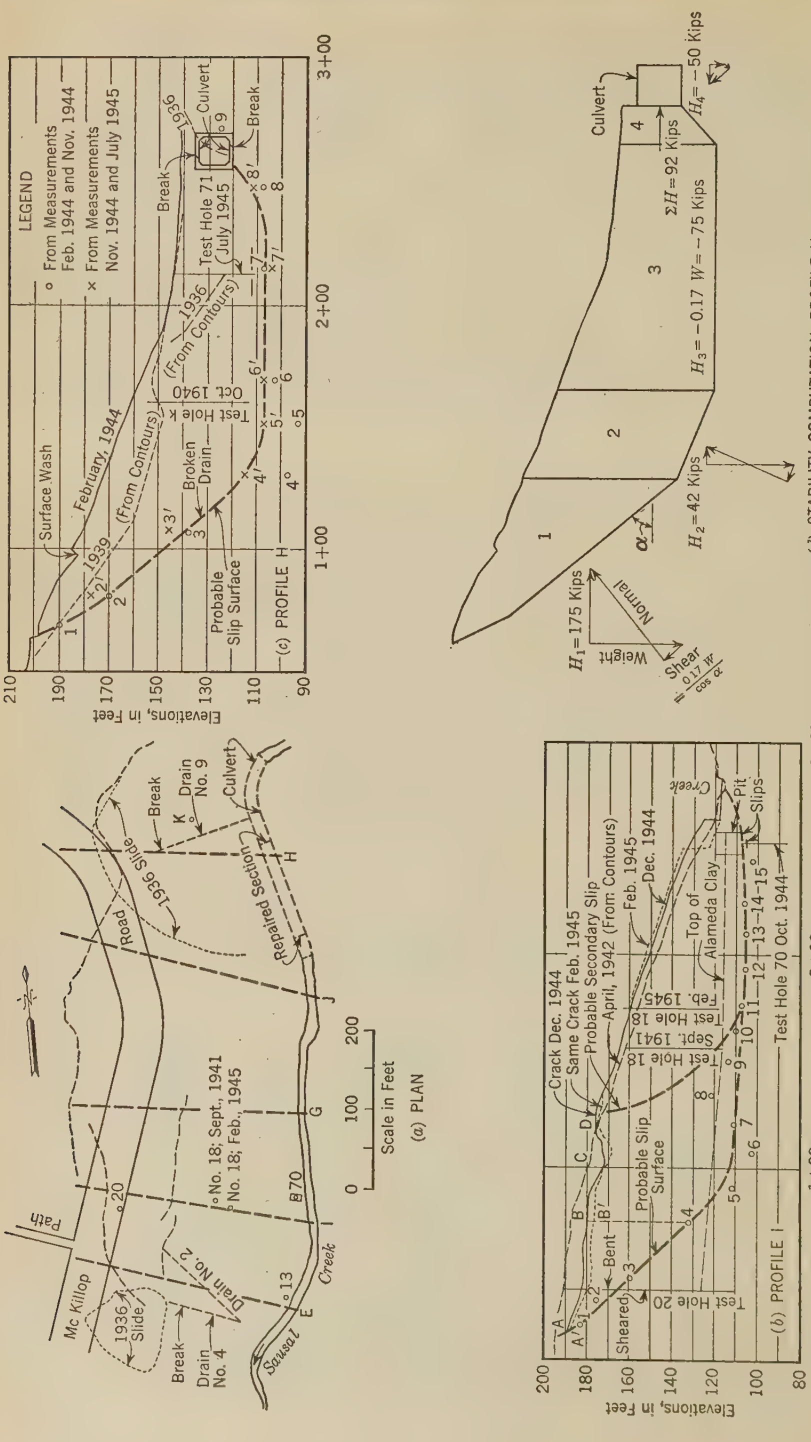
LEGEND
○ From Measurements Feb. 1944 and Nov. 1944
× From Measurements Nov. 1944 and July 1945
Surface Wash
February, 1944
1939
1936
(From Contours)
Break
Test Hole 71 (July 1945
Culvert
Test Hole K Oct. 1940
Broken Drain
Probable Slip Surface
(c) PROFILE H
Elevations, in Feet
210
190
170
150
130
110
90
1+00
2+00
3+00
Mc Killop
Path
Road
1936 Slide
Break
Drain No. 4
Drain No. 2
Drain No. 9
K
Culvert
Repaired Section
H
J
G
E
I
No. 18; Sept., 1941
No. 18; Feb., 1945
20
13
70
Sausal Creek
0
100
200
Scale in Feet
(a) PLAN
H₁=175 Kips
Weight
Normal
Shear = 0.17 W / cos α
α
H₂=42 Kips
1
2
3
4
ΣH=92 Kips
H₃=−0.17 W=−75 Kips
H₄=−50 Kips
Culvert
(d) STABILITY COMPUTATION, PROFILE H
Crack Dec. 1944
Same Crack Feb. 1945
Probable Secondary Slip
April, 1942 (From Contours)
Feb. 1945
Dec. 1944
Top of Alameda Clay
Test Hole 18 Sept. 1941
Test Hole 18 Feb. 1945
Test Hole 20
Test Hole 70 Oct. 1944
Probable Slip Surface
Sheared
Bent
Creek
Pit
Slips
A
A′
B
B′
C
D
(b) PROFILE I
200
180
160
140
120
100
80

FIG. 16.—MAP OF MCKILLOP SLIDE

At least four separate deep channels were located by borings. The small apex opposite the north arrow indicates an independent shallow channel. Shallow water was found in the slide mass, and opposite this point in the next street to the west, but the connection was not followed through private property to the head of the slide.

The almost semicircular outline of the head of the Parker Avenue slide (San Francisco) indicates a deep and fairly concentrated source, located near a luxuriant growth of shrubbery about midway between the "5" of the 375-ft contour and the head of the loop of the 350-ft contour in Fig. 2. The depth below the original surface is confirmed by the extent of the movement that has occurred; but it would be interesting if the author could comment on the location and relative importance of the channels that he found.

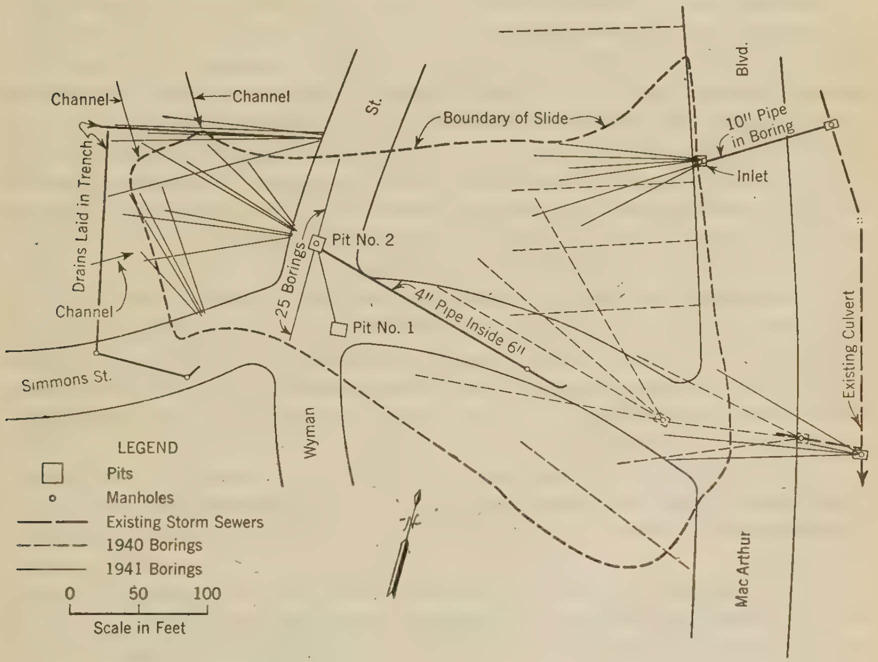
Channel
Channel
St.
Boundary of Slide
Blvd.
10" Pipe in Boring
Inlet
Drains Laid in Trench
Pit No. 2
25 Borings
Channel
Pit No. 1
4" Pipe Inside 6"
Existing Culvert
Simmons St.
Wyman
Mac Arthur
LEGEND
Pits
Manholes
Existing Storm Sewers
1940 Borings
1941 Borings
0
50
100
Scale in Feet

FIG. 17.—SIMMONS STREET SLIDE

A sharp apex forming the head of a slide indicates a narrow, shallow channel feeding into the apex. If the slide broadens considerably in the lower part, it is likely that either the slip surface is deep or there are other feed channels part way down the slide, or both.

Irregular embayments at the head of a slide are almost certain indications of more than one source of water; channels sometimes can be located within a few feet by this method alone. The Simmons Street slide, in Oakland, shown in Fig. 17, is a good example; the three channels shown were strongly suspected before they were located by borings. The apparent embayment on Simmons Street above Wyman Street is an upheaval or secondary toe, and does not indicate a channel.

The terms "shallow" and "deep" are relative, of course, and must be considered with respect to the other dimensions of the slide.

Displaced Volumes.—Under favorable conditions it is frequently possible to compute the shape of the slip surface with a fair degree of accuracy from surface surveys. The method is by no means infallible, but the writer has found it to be of considerable value, and when it fails the error will usually be obvious. The field work consists of running one or more profiles as nearly as possible in the direction of the slide motion, using marked points, and repeating the measurements after an appreciable movement has occurred. The McKillop slide will be used to illustrate the procedure.

Referring to Fig. 16(*b*) and considering a thickness of unity perpendicular to the paper, the area ABB′A′ equals the volume of material which has moved across the vertical line B-4, provided there has been no expansion or consolidation of the slide mass. If the rate of movement has been constant throughout the depth, this area divided by the horizontal component of the movement of point B will give the depth to point 4 on the slip surface. The process is repeated for each point on the survey, the discrepancies evaluated as well as possible, and the probable slip curve sketched in. There are obviously several sources of error in the method, but the operator can rapidly acquire enough skill to make fairly accurate allowances for most of them.

If the slide is fairly uniform at right angles to the plane of the profile and if the line of motion is comparatively straight, any reasonable skew between the profile and the direction of motion should not affect the result, as the cosine of the angle of skew appears in both the area and the horizontal movement. The method does not appear to be of value for a slide of irregular cross section moving on a meandering course.

If the toe of the slide is not being carried away by a stream or other agency, the over-all change in volume of the slide mass can be determined; a reasonable distribution can be made from the evidence presented by open cracks, shattered areas, etc.

An important local error may develop at an abrupt change in the slope of the slip surface if there is rotational motion present. Movement of the survey point will be less than the mean if the slip surface is concave upward, and the computed depth will be too great. This condition is illustrated by points 5, 6, and 15 of Fig. 16(*b*) and 4, 5, 6, and 8 of Fig. 16(*c*). Rotation may or may not be relieved by the development of a vertical crack such as formed at C and moved to D (Fig. 16(*b*)), apparently breaking the slide into two blocks moving on different slopes. A similar condition developed on a smaller scale over points 4 and 5 of profile H during the second measurement period, and may account for the apparent lack of rotational effect on points 4′, 5′, and 6′. In the somewhat unusual case of a slip surface which is convex upward, it is possible that the computed points would be too high, although expansion due to tension at the surface might introduce another type of error.

If there are two or more major slip surfaces separated by any considerable distance, the motion will not be uniform and neither slip can be located by computation. If the two slips join, the error will be localized, as at points 8, 9, and 10 of profile I. The occurrence of two or more parallel slippages within a narrow zone would not seriously affect the result; the computed position would fall between the actual surfaces and would be well within the zone of slippage. The existence of multiple slippages sufficient to cause appreciable error usually will be indicated by surface disturbances.

After the slip surface has been sketched, the result can be verified by a comparatively few borings. The major part of these can be sunk by ordinary drilling methods, with core samples required for only a few feet in the vicinity of the slip surface. As shown in Fig. 16, the check may be quite close. Point 2 of profile I is known to be too high because of a zone of multiple cracking at the head of the slide; if a similar allowance had been made at point 3, the sketched slip would have been very close to the known shear in test hole 20. The loss of depth of hole 18 is believed to be due to the infiltration of sand, and this well apparently does not reach the main slip surface. Point 15 is obviously too deep because of rotation at the toe; a smooth curve drawn from point 14 to the known toe in the creek passes directly through the upper of two slips found in hole 70. The checks on profile H are not so definite, but are reasonably certain. The broken drain was installed by boring, and may not be exactly on the theoretical grade shown. An abandoned culvert was encountered about 20 ft from the start; a note made at the time suggests the possibility of a downward deflection. Test hole k did not encounter the material in which the slip is known to occur, which merely proves that the slip is below the bottom of the hole. Hole 71 was driven with hand tools. It was nearing the limiting depth for the equipment when it encountered a shattered zone, very similar to one exposed in the pit in hole 70, extending through about 5 ft immediately above the slip surface. This was accepted as sufficient confirmation, and no attempt was made to reach the actual slip surface. If point 9 is disregarded because of a slight loss of volume due to incipient failure of the culvert, the slip drawn from point 8′ to the farthest measured surface movement intersects the culvert at a point which would account for an observed upward displacement of the left part of the failed section. It then continues up the original east bank of the creek, which would be a natural line of weakness.

Some confirmation was available for profile E from test hole 13, the behavior of drains 2 and 4, and from an exposure of the slip surface by creek cutting. Profiles G and J showed characteristics similar to the other profiles, and were accepted without confirmation.

The writer has had very little success in trying to fit circular arcs to actual slides, and therefore is not satisfied with the conventional methods of making stability computations. Most slip surfaces can be represented satisfactorily by a comparatively few straight lines and the section divided into prisms with plane bases. Then the only assumption necessary is the interfacial pressure between the prisms. The frequency of vertical shear displacements at important changes of slope of the slip surface indicates that the error in assuming this pressure to be horizontal would be comparatively small, particularly after vertical cracking has occurred. A force polygon can be drawn then for each prism, and the horizontal forces summed, as shown in Fig. 16(*d*). This procedure is not strictly correct in that the difference of elevation of the lines of interfacial pressure has been neglected, but the simplification is believed justified.

The greater part of the slip surface of the McKillop slide lies in a very firm silty clay locally known as the Alameda clay, which underlies the hill just above creek level. This clay has a shearing strength of well over 1,000 lb per sq ft in its undisturbed state, but becomes very plastic when remolded. Shear tests, by both direct shear and unconfined compression on completely remolded material extruded from the slip surface at the toe of the slide, indicated a shearing strength of 0.17 times the reconsolidation pressure. The material overlying the Alameda clay at first appearance seems to be entirely different in character, but an examination shows that its clay content gives it similar shear properties. The value of 0.17 was used, therefore, for the entire slip surface. (It should be noted that the writer carefully avoids any use of the expression—

$$s = c + p \tan \phi \qquad (8)$$

—which he regards as an unfortunate mathematical coincidence. In Eq. 8, s is the shearing strength; c is cohesion; p is the normal pressure; and ϕ is the angle of internal friction. Expressed in this nomenclature the test results would have been $c = 0$ and $\tan \phi = 0.17$.) The unit shearing strength on the slip surface is 0.17 times the vertical pressure per unit area; the total shear for the prism is 0.17 times the total weight of the prism divided by the cosine of the angle of inclination of the slip surface. If this analysis is correct, the conventional conception of cohesion plus friction introduces an appreciable error on the side of safety, which may help to explain why the thin factors of safety in current use have not more often resulted in disaster. The writer was led to his conception of shear by the close agreement between the results of direct shear and unconfined compression tests. It is difficult to reconcile the unconfined compression test with a zero cohesion and a coefficient of friction; whereas a simple shearing strength determined by consolidation pressure (or consolidation history of an undisturbed sample) would seem to fit both tests.

A valuable check on the computation is afforded by the behavior of the culvert. Profile H, Fig. 16, was established in the direction of previously recorded movement, and is somewhat skewed with respect to the culvert. On the other hand it is quite probable that the thrust on the culvert is increased by an arch action from the mass of the slide, indicated on the ground by a series of cracks which disappear about where they cross the line of the profile. The profile crosses the culvert near the upper limit of the failed area, and the computed slide thrust of 92,000 lb per ft compares with a value of 93,500 lb per ft for the failure strength of the culvert. This seems to be well within the accuracy of any current method of determining the shearing strength of soil.

Horizontal Borings.—In fields of construction that have not become standardized the engineer must of necessity adapt his methods to suit available equipment and the skill and experience of local contractors. Whereas the author has developed a technique of wells connected by short tunnels, the writer has had considerable success with a machine for making horizontal borings. The essential part of the machine is a reversible air motor with a hollow crankshaft which delivers water to a cutting bit through a string of hollow rods. When working in clayey material, it is capable of surprising accuracy, but is almost impossible to control in gravel. Progress is rapid, and in 1940 a 4-in. perforated pipe, in place, cost about $1.00 per ft for drains averaging 100 ft long. The cost increases somewhat for longer drains, and increases rather rapidly with the diameter of the pipe installed.

McKillop Slide.—The history of the McKillop slide illustrates several interesting processes of slide development. A competent observer reported topographic indications of considerable prehistoric movement of the northern part of the slide, between the culvert and the street. Cross sections taken prior to the construction of the street show a substantial gully at this point, which was filled by street grading. By 1935 the fill had been extended, presumably by loose dumping, far enough to permit the construction of three houses on the east side of the street. This overloading of the slope, possibly aggravated by some creek cutting, was sufficient to cause collapse during the wet winter of 1935–1936. Later in 1936, the culvert shown in Fig. 16(*a*) was constructed in firm ground along the east bank of the creek, and fill was placed to the height of the east bank.

At about this same time a small slide occurred in another filled gully on the east side of McKillop Road just south of the present limit of the main slide. This appears to be a superficial movement more or less independent of the main slide.

The loss of support due to the northern 1936 slide apparently caused a local concentration of stress sufficient to rupture the Alameda clay immediately south of the 1936 slide. Slow cracking of the houses on the west side of the street, progressing from north to south, indicated that this concentration of stress and consequent rupture traveled southward until it reached the small southern slide. These movements did not reach alarming proportions until the spring of 1940. During the summer of 1940 several drains were installed, and both of the 1936 slides were refilled in an attempt to buttress the slide mass. The culvert was located on developed property, and, therefore, it was not possible to place additional fill in that area. The corrective work was not successful; movement has continued to the present time, with the destruction or removal of all improvements in the area, including the partial failure of the culvert.

Drainage work included several trench drains in McKillop Road and a number of borings made from near creek level. A considerable volume of water was removed; attempts at drainage were abandoned when it was definitely determined that the major part of the slip surface was below creek level. The construction of a counterweight fill at the toe would require the condemnation of developed property on the east side of the creek, and does not appear economically justified. The culvert was considerably larger than required to carry the creek, and has been repaired by installing a steel plate liner backed with asphaltic concrete. There are indications that the slide is now (1946) heaving on the west side of the culvert and forming a new toe which may ride over the culvert instead of thrusting against it. In that case the present flexible culvert will probably not deform enough to cause failure.

Simmons Street Slide.—The Simmons Street slide (Fig. 17) developed in the winter of 1939–1940, and for two winters threatened to move across MacArthur Boulevard and block the main branch of U. S. route No. 50. There is considerable evidence that the slide was a renewal of an ancient movement of practically the same mass. A crack along the south edge of the slide and an upheaval near the southwest corner of Simmons Street and MacArthur Boulevard seemed to be duplicating a gully and knoll described by persons who had been familiar with the area before it was graded. If this former movement did occur it must have been very old, as all traces of a scarp at the head of the slide had disappeared.

During the summer of 1940 a number of drainage borings were made in the lower part of the slide, as shown by the dashed lines in Fig. 17. This work did not prove effective, and movement was resumed in January, 1941. This second movement was sufficient to outline the head of the slide definitely and to permit the location of the water channels shown, which were intercepted by a combination of borings and drains laid by ordinary trenching methods.

Two test pits were dug on Wyman Street and a line of test holes put down to establish the cross section of the slide. The deeper of the two pits was then drained by boring from a point on Simmons Street about 175 ft away. The boring was cased with a 4-in. pipe inside of a 6-in. pipe to provide a slip joint in case of further movement. The boring reached the center of the pit at a point about ½ ft above the planned elevation. It was planned to construct a shaft-and-tunnel drain across the slide, similar to the one described by the author at Burnham Street and 24th Street, but the one bid received was considered excessive. As a substitute twenty-five borings were made along the line of the proposed drain, working from a series of three stagings in the pit. In the critical places the borings were about 1 ft apart vertically, although staggered somewhat in plan. The cost of the work was approximately one fourth of the amount of the rejected bid, and the cutoff seems to have been complete. Some difficulty was encountered in sluicing the wash through the outlet boring. It would have been better to have deferred the installation of the inside 4-in. pipe until after the work in the pit had been completed had it been known that the cutoff would not be constructed as planned.

Additional test holes along the toe of the slide showed that the 1940 borings were not deep enough, and the outlet of the group in Simmons Street had been destroyed by the 1941 movement. A pit was opened over an existing culvert east of MacArthur Boulevard opposite Simmons Street and a new group of borings was made at a lower level. As the water in the north side of the slide was beyond boring range from the culvert it was necessary to bore from a pit over the culvert to another at the west curb line of MacArthur Boulevard and make the drainage borings from the latter. Since the drain connecting the two pits was required to carry surface water from the gutter, it was cased with a 10-in. pipe. Except as indicated, all other borings were cased with 4-in. perforated pipe.

Transit lines were established along all streets in the slide area in February, 1942, and remeasured at intervals until September, 1945, with no measurable movements detected other than those caused by local swelling of the clay soil.

This slide was stabilized before the method of displaced volumes was developed. It is somewhat irregular both in cross section and direction of movement, and exhibited a definite spreading action as it moved downhill. As both irregularity and lateral expansion introduce serious errors the method probably would not have been successful.

Barrows-Holman Slide.—This slide, which is shown in Fig. 18, is typical of several which have occurred in a rather limited area of Oakland. The tract was developed by the owners between 1922 and 1924. Apparently the fill on the north side of Holman Road was placed with very little compaction and no subdrainage. Minor repairs to sidewalks and drives on Barrows Road had been required for years, and considerable settlement had occurred along the north side of Holman Road; but major slide action was not recognized until the early summer of 1942. At that time the toe upheaval along Barrows Road was fairly well defined although no serious damage had resulted, and the head of the slide on Holman Road was partly outlined by a series of sidewalk and driveway cracks not wider than 1 in. The adjacent settlement delayed, somewhat, the determination of the exact limits of the slide but the shape was well enough defined to point to a small area on Holman Road where the cracks extended into the street. As the original cross sections showed a fill of about 7 ft on the north side of the street it was obvious that a spring or seep had been covered and that the percolation of the dammed water through the years was responsible for the slide. A series of test holes along the street located the channel almost exactly as anticipated, even to its width. Several 8-in. wells were then drilled in the channel and plans were made for drainage. The greatest known depth of the channel was 28 ft, which would have made a trench drain long and expensive. Two profiles were established in the upper part of the slide, and, although the recorded movements were only a few hundredths of a foot, and therefore

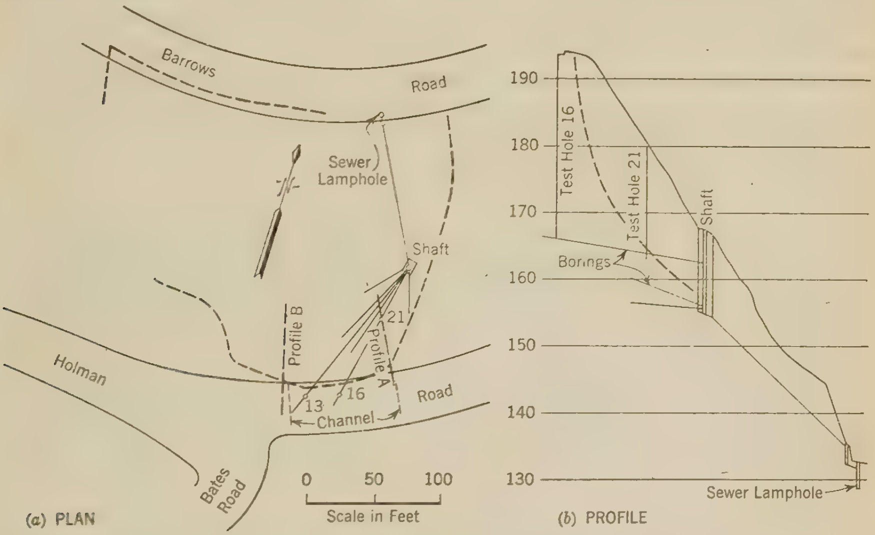

FIG. 18.—PLAN AND PROFILE, BARROWS-HOLMAN SLIDE, OAKLAND, CALIF.

scarcely larger than the inaccuracies of the survey, the result seemed reasonable. Test hole 21 (see Fig. 18) was then put down and the slip surface was found within a foot of the computed elevation.

As soon as financing arrangements could be completed, a shaft was excavated near one side of the slide to an elevation well below the slip, and drained by boring from Barrows Road. Borings were then made from the shaft to drain the channel and pick up such additional water as could be intercepted by a few random borings. The boring to hole 16, a distance of slightly more than 100 ft, actually intersected the well, and the drain pipe was clearly visible in the bottom of the 8-in. well. The boring to hole 13 (about 120 ft) was never seen from the surface, but welding fumes were quite strong at the well while the casing was being installed in the boring.

It was realized that the widening toward the west of the lower part of the slide indicated the possibility of a deeper water feed in this vicinity; but no such channel could be found readily. As it was necessary to keep costs as low as possible, it was decided to defer work in this vicinity until it became necessary; and the subsequent stability of the slide seems to have justified this decision. The slide was arrested after a movement of probably less than 6 in., and the damage caused was minor.

Conclusion.—In conclusion the writer wishes to emphasize the following points:

(1) The problem of landslides is one involving both geology and engineering, whether the investigation is conducted by a geologist with an appreciation of engineering problems or an engineer who has acquired a working knowledge of geology. In most cases the only corrective measure required is the removal of underground water.

(2) The science of soil mechanics has not yet reached the state of standardization where its methods can be accepted without careful analysis and review. The combined work of many men is yet required to bring it to a state of reliability comparable with structural design or hydraulics.

ALFRED V. BOWHAY,[16] Assoc. M. ASCE.—The subject of landslide correction is one that merits a complete review such as Mr. Forbes presents. It is of particular interest to those engaged in municipal engineering as well as in highway engineering.

The municipal engineer is vitally concerned because varied types of municipal improvements involve the removal of support through excavation and the deposit of materials in constructing embankments, both of which (through the action of moisture) are potential slide situations. With a better understanding of the action of the forces of nature, effective measures can be taken to prevent, or to minimize, the destruction of valuable public and private works. It is not within the scope of the civil engineer's practice in municipal engineering to apply any corrective measures other than ordinary drainage. The engineer is competent to study borings and strata so that drains may be placed at the proper locations and depths, thus insuring drainage of excess water. He is also competent to apply surface measures such as consolidation by rolling and the application of asphalt or cement coatings to divert surface drainage to the regular storm drainage system. In this manner the sliding material may be relieved of seasonal rainfall, and will have an opportunity to drain and dry out, so that it may again reach a state of stability.

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Beyond this practice the civil engineer cannot reasonably go—especially in municipalities where only a few slides occur from year to year. In San Francisco, Calif., there have been some eight or ten major slides in a period of thirty years. This record is too limited to establish a standard norm of behavior. To obtain adequate knowledge of the causes and remedies, it is necessary to procure the services of a competent engineering geologist, experienced in the particular procedures peculiar to landslide correction. Such a geologist should have had experience on from fifty to one hundred landslides.

There is a technical aspect of the procedures of the geologist that Mr. Forbes mentions—namely, the tests, in the Crolona Heights slide, of plastic limit, liquid limit, and moisture equivalent. These tests indicate the type of soil or clay, the amount of surplus moisture, and, more particularly, the moisture retention qualities. Because of these factors, some ground loses its moisture by drainage so gradually that three or four years are required for it to dry out to a condition of stability. This slow progress toward stability, and the continued sliding after drains are installed, discourages engineers from trying to stop slides. However, a geologist, with wide experience, through knowledge of comparative conditions, can advise the correct type, number, and location of drains to suit the conditions that these tests indicate. Thus, dewatering or drying out can be reduced to the minimum time economically justified.

The correction of slides involves procedures of great interest to engineers because of the complexity of the problems involved. Often the land is worthless before the slide. There is little value in a cut slope, for example; and, after a slide occurs, the value of the land involved is reduced to a negative sum in most instances. Highway or street frontage must be cleared, and it is essential to repair the area to prevent a recurrence. The question naturally arises as to whether such work will warrant an expenditure greater than that necessary to clear up the debris. In this respect, "the wish is father to the thought," and the general thought is "perhaps it will not slide again." In nearly every instance, cities proceed on this basis. They clean up the debris, put in a minimum of subdrains, and repave or replace the improvement if possible.

The nominal cost of a geologist's services is well worth the public confidence it inspires. Public administrators cannot justify the expenditures frequently called for, except with the best advice obtainable. Even private individuals, homeowners, or those in charge of institutions threatened by slides cannot readily question the technical advice of an engineering geologist, as they quite often question proposals by engineers, made without the benefit of geological study and advice.

The term "maintaining a slide" appears to be on the facetious side, but to maintain a slide has been found to be a necessary and often valuable procedure. In the Laidley Street slide, one of the first major correction jobs of the City of San Francisco, after the drains were installed, the surface was smoothed and rolled with a heavy tractor (used because of a grade of about 20%) and then given a coating of about 0.5 gal of heavy road oil per square yard. This surfacing acted as an umbrella, and it shed the water to the foot of the slide where it was conveyed to the storm sewer. Naturally in a few years the surface deteriorated, and insufficiently compacted pockets developed; but, by regular annual inspections and study, the conditions were noted and remedied. It was even observed that there were points where additional drains should be installed.

A part of the maintenance work is to keep a discharge record of the drainage system. This record is important in connection with observations of rainfall. Reference stakes that show the progressive horizontal and vertical movement must also be maintained and observed. The study of these data gives valuable aid in improving design by such measures as added drains and improved surfacing, as well as in showing the degree of improvement produced by the corrective work.

It seems advisable to direct attention to the possibility of prescribing general treatment, which Mr. Forbes indicates cannot be done (see heading, "Correction Should Be Based Upon a Thorough Investigation"). Since 1926 the City of San Francisco has used the general treatment of drainage by the construction of ditches with the familiar perforated metal pipe. Later on this drainage was accomplished by chimneys. These chimneys or vertical drains are usually extended downward to a nearly horizontal drain, where the usual metal pipe and rock backfill is installed. This type of drainage was installed in the Parker Avenue slide in San Francisco as reported by Mr. Forbes. The design of the Parker Avenue slide drainage system was prepared by, and under the immediate direction of, the writer. This system was the first of its type, subsequent to which the 3.5-ft by 6-ft drain tunnel was replaced by a very small tunnel (minimum 15 in. by 15 in.) connecting the bottom of the shaft. The latter work was done on the O'Shaughnessy Boulevard slide (see Table 1 and supporting text) also in San Francisco. A perforated metal drain with crushed rock backfill is usually installed in this small temporary tunnel. Lately the shafts have been increased in size to 4 ft by 5 ft, so that they could be timbered and still admit a small clamshell bucket. The cost of this type of shaft is less than that of the 36-in. circular shaft, the excavation for which is all handwork and requires a metal 36-in. shell for lagging. The 36-in. shell sometimes cannot be recovered during the placing of the center drain and rock backfill, generally constructed to within a few feet of the surface.

A great advantage of this general procedure is that a suitable contract can be drawn so that competitive bids may be taken. Quantities can be determined satisfactorily in advance, necessitating a minimum of change in the actual work. This change is always well within the 10% allowance for extras, the limit required by city ordinance. Of course, this contract preparation must be preceded by borings (by contract) and sample studies and tests. From such data, profiles and sections of the strata can be made for study so that, as Mr. Forbes states, corrective works can be designed to meet these determined elements of the problem.

The nine classifications presented in the "Synopsis" are interesting, although they appear to overlap slightly. No doubt there is ample geological reason for differentiation. However, of particular interest is "(1) a shear slide in which hydrostatic uplift on an unbalanced slope resulted in mass movement of the unconsolidated slope material." This class of slide is further subdivided under the heading, "Landslides"): "Shear slides are frequently associated with the unbalanced slopes that remain after material has been removed from natural slopes." Although Mr. Forbes does not apply this type to the Parker Avenue slide, there were many persons who did. The idea that the particular slope in question was unbalanced, or that there was an unbalanced condition, seems to vitiate the theory of safe slopes, which is still popular in the practice of highway engineering. Under the theory of safe slopes, certain materials will stand at a slope of 1 on 1, others on a slope of 1 on 1.5, and the most unstable excavated materials on a slope of 1 on 2. On Parker Avenue these slopes stood at a slope of 1 on 0.5 for nearly fifteen years. There was some seepage which was provided for, showing a wet ground condition. However, it could be concluded that the slope was safe, particularly on the Lone Mountain side of Parker Avenue, which had a concrete and brick pavement, 35 ft wide. Across Parker Avenue, in a sidehill situation, was the low slope of the cut which was removed or "daylighted" during the subdivision work. This construction was claimed to have unbalanced the material across the street and up the hill. If this were true, then the material could not have been classified as safe for a slope of 1 on 0.5 fifteen years previously. Actually, unbalancing was not a primary factor in this situation, but it was a minor factor in resisting hydrostatic uplift.

In the Parker Avenue slide, water was the primary factor, as the millions of gallons drained from the slide annually will attest (Table 2). The material was stable but highly absorptive, and had a high retention value. There was also the familiar slipping plane of plastic material, which had no resistance to sliding when wet. The great mass of saturated material had an unusually heavy weight, the resultant of which was parallel to the slope of the sliding plane. The weight of the counteracting dry material, which was similar to that of a dam, held this resultant force of the wet material stable. As the saturation extended down the slope, and the slipping plane became further lubricated, the resistance of the dry material to sliding became so low that, like a poorly designed dam or retaining wall, it was thrust aside. The plan of the slide, as well as the photographs, shows this lateral movement. The apparent uplift resulted because the slipping plane turned upward and the material was also thrust upward along this plane. The plan also shows the trend to the northwest of the gulley which formed the bed of the slipping plane.

Unbalancing, therefore, entered into this slide problem only in the sense that the weight of counteracting material, acting as a dam, may have been reduced. The heavy weight of the wet material, and the moisture-lubricated slipping plane of plastic character were the prime causes of movement, which could have occurred whether the excavated material was or was not removed. Slide problems will be more clearly understood if this distinction is kept in mind. Although engineers may speak of slopes in balance and discuss them in such a manner, actually in engineering problems of this kind they are referring primarily to the forces that are in balance.

One of the more important factors in the solution of slide problems is the force that resists sliding. As usual, this force depends on the weight of the resisting material and the coefficient of friction, which is naturally very low when the sliding surface lies along or within a seam of wet plastic material.

Landslides not only damage highways, streets, and public works, they also damage homes. Some fortunate homeowners can remove their houses to a more stable location. The remaining foundation on private property rapidly deteriorates and becomes a public nuisance and eyesore, a kind of testimonial of the inability of the community to cope with the situation. The less fortunate homeowners, whose houses are so badly wracked and damaged by the slide action, generally appeal to the local governing agency for help or assistance. This help the local agencies, supervisors, or councilmen feel obligated to give. The repairs or replacements are too often makeshift in character because temporary measures are popular in slide correction. Thus, most of the slide areas become not only vacant land but also eyesores. This is a poor testimonial to the engineer and his ability to sell his measures toward a more complete remedy.

More education and enlightenment are needed on this subject. Engineers and engineering geologists must collaborate to convince their communities that these eyesores can be corrected, and that the cost of the remedies is not out of proportion to the benefits. Then one may look forward to less disfiguring areas in the rapidly growing modern cities and along the modern highways of the United States.

D. P. Barnes,[17] M. ASCE.—A wide variety of landslides, together with an explanation of the success achieved by various remedial measures, is discussed in this interesting and practical paper.

The tendency of water to precipitate an earth movement can be explained in different ways. As in so many analytical matters, the choice of a suitable illustration with which to begin the analysis may mean the difference between a simple concept easily applicable to practical cases, or a complex concept with little useful application.

An illustration that appears to have some merit in providing a simple concept of the forces exerted by seepage water is that presented in Fig. 19. Consider the equilibrium of a soil prism of convenient length, *du*, and width, *dv* (thickness unity), with sides parallel and perpendicular, respectively, to lines of equal pressure. The soil in the prism is restrained on all sides by the contacts between soil particles and is acted upon by the hydrostatic pressures and

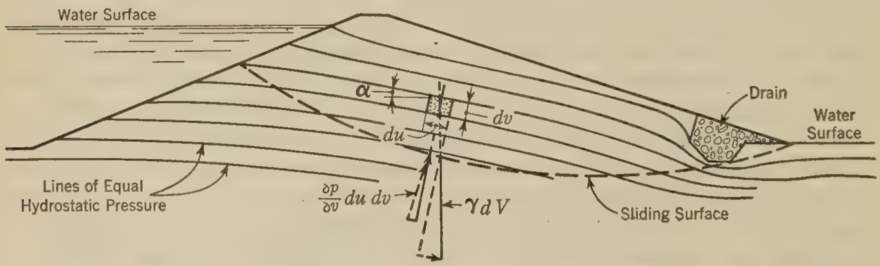
Water Surface
α
dv
du
Drain
Water Surface
Lines of Equal Hydrostatic Pressure
$\frac{\partial p}{\partial v} du\, dv$
$\gamma d V$
Sliding Surface

Fig. 19.—Seepage Forces in an Earth Dam

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[17] Engr. Asst. to Asst. Chf. Engr.—Civ., Branch of Design and Constr., U. S. Bureau of Reclamation, Denver, Colo.

its own weight. The water in the prism is acted upon by hydrostatic pressures, by the resistance of the soil particles to change in hydrostatic pressure, and by its own weight.

Isolating the prism, with its soil and water content intact (see Fig. 20), and applying the general equations of static equilibrium, the forces transmitted from the water to the soil are readily determined. For static equilibrium (that is, assuming that no masses are being accelerated), the sum of the components in any direction of all the forces acting on the body must be zero.

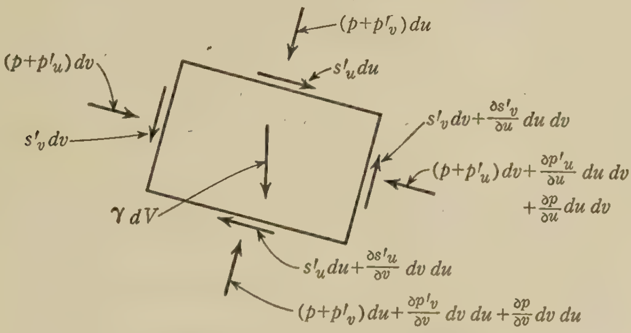
$(p+p'_v)\,du$
$(p+p'_u)\,dv$
$s'_u\,du$
$s'_v\,dv+\frac{\partial s'_v}{\partial u}\,du\,dv$
$s'_v\,dv$
$(p+p'_u)\,dv+\frac{\partial p'_u}{\partial u}\,du\,dv+\frac{\partial p}{\partial u}\,du\,dv$
$\gamma\,dV$
$s'_u\,du+\frac{\partial s'_u}{\partial v}\,dv\,du$
$(p+p'_v)\,du+\frac{\partial p'_v}{\partial v}\,dv\,du+\frac{\partial p}{\partial v}\,dv\,du$

FIG. 20.—ELEMENTARY SOIL PRISM

Let p be the hydrostatic pressure; p' be contact pressure between soil particles; s' be the contact shearing stress between soil particles; V equal the volume of the soil prism; and γ equal the wet unit weight of soil. Adding the components in the u-direction, therefore, yields:

$$\frac{\partial p'_u}{\partial u}\,du\,dv+\frac{\partial s'_u}{\partial v}\,dv\,du+\frac{\partial p}{\partial u}\,du\,dv+\gamma\,du\,dv\sin\alpha=0\ldots\ldots(9a)$$

or, since $\frac{\partial p}{\partial u}=0$,

$$\frac{\partial p'_u}{\partial u}\,dV+\frac{\partial s'_u}{\partial v}\,dV=-\gamma\sin\alpha\,dV\ldots\ldots\ldots\ldots\ldots\ldots(9b)$$

Similarly, considering the components in the v-direction:

$$\frac{\partial p'_v}{\partial v}\,dv\,du+\frac{\partial s'_v}{\partial u}\,du\,dv+\frac{\partial p}{\partial v}\,dv\,du+\gamma\,dv\,du\cos\alpha=0\ldots\ldots(10a)$$

or

$$\frac{\partial p'_v}{\partial v}\,dV+\frac{\partial s'_v}{\partial u}\,dV=-\frac{\partial p}{\partial v}\,dV-\gamma\cos\alpha\,dV\ldots\ldots\ldots\ldots(10b)$$

The right-hand members of Eqs. 9b and 10b represent the forces acting on the prism due to gravity and to changes in the hydrostatic pressure. Adding the equations vectorially yields:

$$\left(\frac{\partial p'_u}{\partial u}\,dV+\frac{\partial s'_u}{\partial v}\,dV\right)\oplus\left(\frac{\partial p'_v}{\partial v}\,dV+\frac{\partial s'_v}{\partial u}\,dV\right)$$
$$=-\gamma\sin\alpha\,dV\oplus\left(-\frac{\partial p}{\partial v}\,dV-\gamma\cos\alpha\,dV\right)\ldots\ldots\ldots\ldots(11)$$

The right-hand and left-hand members, however, need be concurrent only if $\frac{\partial s'_u}{\partial v} = 0$, and $\frac{\partial s'_v}{\partial u} = 0$, which is unlikely. (The sign $\oplus$ in Eq. 11 indicates a vector sum.)

In making a graphical analysis, it is convenient nevertheless to neglect $\frac{\partial s'_u}{\partial v}$ and $\frac{\partial s'_v}{\partial u}$. The error of this procedure is of the same character as that introduced by assuming, in the absence of percolating water, that the weight of the soil is transmitted vertically downward indefinitely in the same line of action. As Karl Terzaghi,[18] M. ASCE, has shown, its effect is only to cause a slight redistribution of pressures without affecting their total. For practical purposes, therefore, the increment in soil pressure or contact pressures through the soil prism, $du\, dv$, is equal to, and has the opposite direction to, the vector sum comprising the right-hand member of Eq. 11, and may be considered to act through the center of gravity of the prism. The simplicity of this approach arises from the fact that in any rectangular element it is necessary to deal with only the wet weight, $\gamma\, dV$, acting downward and the hydrostatic pressure difference, $\frac{\partial p}{\partial v}\, du\, dv$, acting normal to the lines of equal pressure.

To obtain the total pressures between soil particles along a typical sliding surface in an embankment, therefore, it is necessary only: (*a*) To divide the area intercepted by the sliding arc into convenient finite rectangular elements, (*b*) to determine the forces exerted by the hydrostatic pressure difference $\left(\text{corresponding to } \frac{\partial p}{\partial v}\, dV\right)$ from the equal pressure lines and the forces of gravity from the wet weights, and (*c*) to add their several effects vectorially. In Fig. 19, the forces $\frac{\partial p}{\partial v}\, du\, dv = \frac{\partial p}{\partial v}\, dV$ and $\gamma\, dV$ are shown projected to the sliding surface and there resolved into their tangential and normal components.

It would seem that the mere presence of water, except in so far as it may affect the cohesion or coefficient of friction within the soil mass, is not a direct indication of an increased susceptibility to sliding. For example, on the upstream slope of an earth dam through which water is percolating toward the downstream drain, the action of the water is to shift the resultant of all the forces acting on a prism of soil slightly downstream from vertical, and, thus, to reduce the components tending to produce sliding in the upstream face. The downstream face, however, toward which the water is flowing, has an increased tendency to slide as a result of this same shift in the resultant of the forces acting on each soil prism.

The case of submergence without flow can be deduced by an inspection of Fig. 19 if it is imagined that the downstream water surface is raised to the level of the upstream surface, and that the lines of equal hydrostatic pressure are correspondingly rotated to their normal horizontal position as found in any body of static water. The resultant forces on the soil prism become concurrent and vertical, and the forces tending to oppose sliding, while reduced, bear the same relation to the frictional resistance to sliding as before.

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[18] "The Mechanics of Shear Failures on Clay Slopes and the Creep of Retaining Walls," by Karl Terzaghi, *Public Roads*, December, 1929.

Most cases with which the engineer or geologist is likely to deal, with the sole exception of artificial embankments constructed for the purpose of impounding water, will, nevertheless, probably resemble the examples cited by Mr. Forbes in that the presence of water will contribute significantly toward the danger of movement. It may well be, also, that, although an approximate analysis of the case of static submergence indicates no increase in susceptibility to sliding, for many practical construction problems such as those encountered in the installation of piers or underwater footings, the secondary factors such as reduction in cohesion or in coefficient of friction and swelling may outweigh abstract conclusions. Any predictions with respect to susceptibility of a completely inundated embankment to sliding should, therefore, be attempted with caution.

Hyde Forbes,[19] M. ASCE.—The discussions have presented material and considerations which will point the way for the application of methods of landslide investigation and correction developed in one region to conditions obtaining in other regions—which is the primary purpose of the preparation of the paper. Professor Legget states that the writer placed emphasis on the development of instability of San Francisco (Calif.) areas due to the presence of serpentine, yet states that the rock formations are not unique. Large areas in San Francisco are underlain with serpentine and are stable, providing sound foundations. The writer had in mind the fact that all rock disintegrates and decomposes under certain conditions, the ultimate product of that decay being largely hydrated material—clay.

Rock containing a high percentage of magnesian minerals has an affinity for water which gives it a tendency to hydrate. This characteristic makes the peridotite of the San Francisco area a ready subject for metamorphism into serpentine, and its surface is subject to comparatively rapid geochemical change when in contact with water. Other common basic igneous rocks such pyroxenite, diabase (Market and Glendale slide), and some gabbros have exhibited metamorphism and extensive decay; sedimentary formations (Crolona Heights, Bernal Avenue cut, and approach to Broadway Tunnel) develop a blanket of clay that becomes unstable with the absorption of moisture. The adaptation of the methods described to areas of unstable clay accumulation is believed possible regardless of the origin of that clay. Test boring and sampling has shown clay to resist penetration and become consolidated to a greater extent with depth below its surface. Clay becomes stabilized by the drainage of overlying or underlying materials, or by the substantial reduction of the time the clay surfaces are exposed to water, together with provisions for drying the clay.

It is of value to know that the horizontal borehole method of installing drainpipe, described by Mr. Buckingham, has been effective in arresting slides in Oakland, Calif. The method is particularly adapted to tap water under "channel "conditions and the ability to locate such channels appears to be the important factor. The writer used that method successfully to tap free water in fairly coarse granular materials in a gully under a fill. The base of the fill had become saturated and the fill was sloughing. In a similar situation, in fine dune sand, the perforations of the drainpipe soon became clogged and the diffused ground water continued to accumulate. The limitations of the percolation or drainage area presented by a small diameter pipe where ground water is diffused through the material, and the possibility of clogging when installed through plastic clay, made the method not applicable to the conditions found to exist in the clay involved in movement at most slides examined. The clay could only be stabilized through drying out, as described as a function of the shafts installed (O'Shaughnessy Boulevard slide).

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[19] Cons. Engr. and Geologist, San Francisco, Calif.

Mr. Bowhay discusses the "maintenance" required in the earlier work done by the City of San Francisco in connection with landslide correction and stabilization. The object of the later work described by the writer was to avoid such maintenance and to provide permanent stabilization, which seems to have been accomplished. The point is well made by Mr. Bowhay that expedient or temporary measures are never satisfactory, and they are subject to criticism and complaint by taxpayers when eyesores or hazards continue to exist. The design of the slide correction works on Parker Avenue followed the recommendations of the consultants who had been investigating and observing conditions for a period of about one year but were not retained during the design and construction operations. Subsequently, the procedure of advisory and supervisory employment outlined by the writer (see heading, "Continuing Investigation During and After Construction") was adopted by the engineering department. In regard to the future improvement of Parker Avenue, it is believed the necessary extensions can be made at far less cost per unit of length than that of the original work. Mr. Feld emphasizes the advantages of test pits over borings in determining subsurface conditions. It is true that there are advantages; and the continued observations of those conditions in shafts excavated as part of the slide correction works described has revealed necessary modification of original plans based on preliminary investigation. This has also resulted in savings in cost below original estimates.

The lack of correlation of drain discharge from various slide areas with the San Francisco rainfall record noted by Professor Legget was, in part, attributed (see heading, "Preventive and Precautionary Measures") to the increase in effective radius of the works with time, as natural subsurface channels are developed to carry water toward them. In certain areas a decrease in the discharge has been noted as the materials are dewatered. Becoming more compact or consolidated, the surfaces tend to shed the water that falls upon them rather than to absorb it. In further answer to Professor Legget's discussion, the writer's reference to the development of certain laws and values through the use of dry silica sand applied to the treatise by William Cain[20] presented in 1916, using the theories of Rankin and Coulomb in relation to the factors of cohesion and internal friction.

Values had been determined experimentally, for these factors, in connection with clean sand. The writer has been impressed by the number of abstract considerations presented since which were adaptations of that treatment involving those two factors. Values for cohesion and internal friction had been assigned arbitrarily within the range of those determined by Mr. Cain, or through experiments made on samples, the approach being in part theoretical, in part experimental, or, frequently, a combination of the two. The criticism has been made that geologists lack the training in mathematics and mechanics necessary to comprehend such treatment fully, but empirical methods of stability analysis as applied to materials in natural occurrence have been questioned by geologists because nature does not act in an empiric manner. The factors of cohesion and friction are not static, being affected by slight changes in moisture content and, principally, by the continued geochemical change occurring within a body of material.

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[20] "Earth Pressure, Retaining Walls and Bins," by William Cain, John Wiley & Sons, Inc., New York, N. Y., 1916.

Mr. Barnes recognizes this as he states, "* * * the secondary factors such as reduction in cohesion or in coefficient of friction and swelling may outweigh abstract conclusions." The colloidal silt and clay particles present or developed in accumulations of sand or other natural material tend to lessen internal friction and, being favorable to absorption of moisture, may render it more cohesive or unstable dependent upon the amount and the mineral character of such material and its absorptive capacity. Water percolating through sand separates the grains, thus reducing friction; and its presence tends to facilitate a change in form from crystalline or fragmental to the amorphous or hydrated form. Mr. Feld enumerates, as one of the internal adjustments causing landslide displacement, "(*c*) a modification of the soil constituents causing a decrease in physical resistance characteristics." Such modification is due to geochemical change and the point the writer wished to make was that the change is continuous.

The admonition for caution in the use of mathematical solution for stability problems, commented upon by Professor Legget, resulted from the writer's experience. Called upon to investigate landslide and other conditions of instability and present findings in court in connection with claims for damages, the writer has been confronted with mathematical computations, asserted to be supported by texts or other publications and to prove the effect of certain acts. In one case a slide occurred because of geological conditions affected by a long period of unusual rainfall. The court took judicial notice that the same conditions generated numerous slides in the same region during the same period. Quoting from the decision of the judge:

> "The expert testimony of Mr. Forbes, together with his factual observations of the locality in and around the slide, afford an adequate and scientific explanation of the cause of the slide, with which the court is in accord and to which it gives credit."

In another case involving a claim for damages in a condemnation suit the decision of another judge stated:

> "I am not at all in accord with defendant's witnesses. I do not believe the big slide * * * was due to excavations of the plaintiff. I believe it was due to natural causes, including the steep slope of the ground, the character of the soils, the rains, and the drainage. * * * the description [by the plaintiff's witness] of the geological conditions of the areas under examination, and reasons for the slips and slides there found, seemed to me to be essentially sound."

Mr. Buckingham points out that landslides are the natural "processes by which topographic forms are sculptured," and the writer has found the cause to lie more often in geologic reasoning based on investigation, which was the basis of the decisions cited, than in abstract consideration of the effect of near-by work. One of the San Francisco slides was treated theoretically, values of unknowns being determined experimentally on samples of the material taken from the slide, and a mathematical solution of the factors and evaluation of the acts generating the slide was published in a bulletin of a public agency. The writer was later employed to investigate the slide and recommend methods of correction. That investigation revealed the slide to have been generated, principally, through ground-water accumulation and uplift, which were not considered in the published treatment. The writer appreciates the value of abstract treatment of stabilization problems, as well as the limitations; he studies the problems and frequently uses them as a tool in analysis and, much less frequently, in making graphic presentations to illustrate the principles involved and the reasoning followed in the design of works for which he recommends the expenditure of money. Also, in order to obtain some idea of the safety factor the works provide, the writer adopts the abstract treatment, with the revision of values assigned to unknown factors as he "believes" they may become changed through the installation of works. He does not use it as a substitute for adequate geological investigation, however. Mr. Buckingham's comment in this regard finds the writer in full agreement. The writer's comment has been previously presented.[21] The uncertainty in applying the results of laboratory determinations of values to field conditions lies in the fact that it is never true that a body of natural material will exhibit uniformity of physical properties consistent with tested samples; nor will those properties remain constant for any considerable time.

Mr. Lambe presents an interesting treatment in relation to the effect of submergence which, in some respects, is confirmed by Mr. Barnes' analysis. In that connection, it may be of interest to note that a subaqueous landslide is a common occurrence. The foundation for the south pier of the Golden Gate Bridge was originally planned as a caisson to be sunk and landed at a predetermined depth below rock surface. Excavation was carried on within a prescribed area into which the caisson was to fit and the caisson was built. When the matter was referred to the writer by the contractor's engineer, it was estimated that about twice as much material had been excavated from the subaqueous site as could be accounted for by the volume of material within the boundaries of the excavated area. Divers were sent down and found that landslides had occurred from all sides. The caisson was abandoned, towed out to sea, and sunk, and a new foundation was designed to fit the conditions resulting from the sliding walls of the excavation. The problem of possible underwater slide shearing structures was presented in connection with the intake towers at the end of the new (1945) adits for the San Andreas Reservoir (Crystal Springs Lakes) of the San Francisco Water Department. Many alluvial fans and detrital accumulations on the floor of the reservoir were found to have suffered landslides with saturation on submergence, and these old slides were revealed when the reservoir water level was lowered. The slide surfaces were examined and the slopes found to be as flat as 1 on 2½ to 1 on 3. Similar material surrounding the towers was graded to those slopes, therefore, before the water level was raised again.

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[21] *Transactions*, ASCE, Vol. 110, 1945, pp. 340–341.

Slide analysis on the basis of displaced volumes described by Mr. Buckingham has its value as well as its sources of error. The moving mass most generally breaks up to a number of units horizontally and seldom is the movement uniform in rate from top to bottom. A large slide along Camino del Mar, the scenic drive following the Golden Gate in San Francisco, was investigated in 1946. Certain lower areas of the slide exhibited movement at rates in excess of those taking place in adjoining areas, due to a higher moisture content of the clay, and so that striated scarps were left as banks to each "glacier-like stream" of clay. During the winter 2-in. pipes were set in boreholes, and carried through the displaced fill, underlying sand, and clay into bedrock in some instances, and in others ending in the clay above bedrock. By late summer the pipes reaching into rock had been sheared at rock line and those ending in clay were dragged so their projecting ends pointed up the slope; the clay in which they were landed moved at a faster rate than the overlying sand and fill. The maximum thickness of moving ground was 125 ft, composed of 35 ft of fill, 35 ft of moist sand, 10 ft of saturated sand, and 45 ft of clay. The latter was the plastic product of the decomposition of the bedrock which had accumulated to that thickness in the bottom of a bedrock gully through movement from the sides and down the slope of the gully. Such differences in rates of movement are important in the location, depth, and type of stabilization works selected. The effect of movement on the many observation pipes set in the Parker Avenue boreholes showed that the moving mass was separated vertically in horizons of varying character of material and varying groundwater conditions, each horizon with a distinct character and rate of movement; all horizons had to be drained and stabilized down to bedrock. This has been usual and the stabilization works have been carried into the bedrock and drainage pipes have been set in that rock in most of the installations. That is an added reason why the horizontal borehole treatment does not apply to the slides discussed, although successfully applied to the Oakland slides described by Mr. Buckingham.

Mr. Feld adds greatly to the value of the paper in citing the application of drainage methods to conditions of instability developing within a body of material, although exterior displacement, or landslide, does not result. Mr. Feld agrees with the writer as to the necessity of obtaining a complete picture of subsurface conditions and it is evident that he has a good understanding of geology and geological phenomena; but he finds it difficult to agree with the writer that boring records and samples are of little value if not collected by an experienced geologist. It is probable that the writer did not make himself clear on the point.

Since about 1917 contractors and engineers have submitted for interpretation, driller's logs, in some instances accompanied by samples and frequently attached to specifications. Such material was of no value, except when taken from localities familiar to the writer through previous work or in connection with structural requirements which presented simple problems, as it was impossible to produce an interpretation having the reliability necessary when a contractor is to risk his money in making a bid, or upon which an engineer must depend in design; and the services sought had to be refused. In years past, the City of San Francisco has let many contracts for boring which produced only driller's logs and bottled samples and for which there was left no record of interpretation. The writer has found, and wishes to convey in his statement, that a geologist experienced in problems presented, which require further subsurface information, after making a careful study of the surface conditions and history of the locality in question and its vicinity, can direct the location of a minimum number of borings or test pits to produce the maximum working data. During the progress of drilling and sampling, he can observe mechanical actions which are informative; he can determine ground water and soil conditions at sufficiently close intervals so that he can project ahead and tell drillers and inspectors what to watch for and note during his absence; and he can make current geological interpretation while matters are fresh in mind. The saving in cost of exploration in such instances has more than covered the cost of services rendered and the results obtained have been used with confidence which proved warranted. Mr. Harman mentions the value of geological investigation and direction of work in accordance with the results of such investigation; but an amplification of his experience with landslide correction and stabilization work on city-owned land conducted as "relief projects" would have explained his views.

Mr. Buckingham presents the reasoning he follows in the analysis of landslides through surface topographic features. As a rule much can be derived from such observation, but the exceptions are important, and occur frequently enough to make subsurface observation a requisite. No evidence was found that sliding had occurred on the slope at St. Mary's Playground before the the fill was placed. The fill compressed the soils through which water previously passed freely down the slope; pressure was developed and the fill slid out of place. At Parker Avenue the uniform sand slope of Lone Mountain gave no evidence of the underlying bedrock gully. Water was found in disintegrated bedrock underlying residual clay in that gully, and in sand beds between clays which had been developed when those sands were surfaces. The flow net, as modified by the escape of water from the disrupted mass during a period of more than a year following the slide, is shown by contour lines in Fig. 21. The variation of water level in the holes to bedrock (circled) from those not registering the deeper water at that time is noted in numerals.

Mr. Lambe designates the writer's consideration of the ground-water hydrology involved in the generation of landslides as being described "in general terms which, to the engineer, do not present a clear picture of the true nature of the effects." This view is undoubtedly warranted, as a consideration of the subject, even in connection with but one of the slides, proved to add too greatly to the length of the paper. The continued observation of the water levels in pipes set in preliminary boreholes has provided very necessary data for the correction and determination of the effectiveness of installed works at several of the slides. The graphic analysis of such observations at the

Parker Avenue slide was quite extensive, largely through the use of contour lines of equal water elevations as movement and drainage continued. Fig. 21 is such an analysis for a specific date. It would have been of interest in Mr. Harman's discussion of the St. Mary's Playground slide had he presented his analysis of the water levels measured in pipes set in many borings around

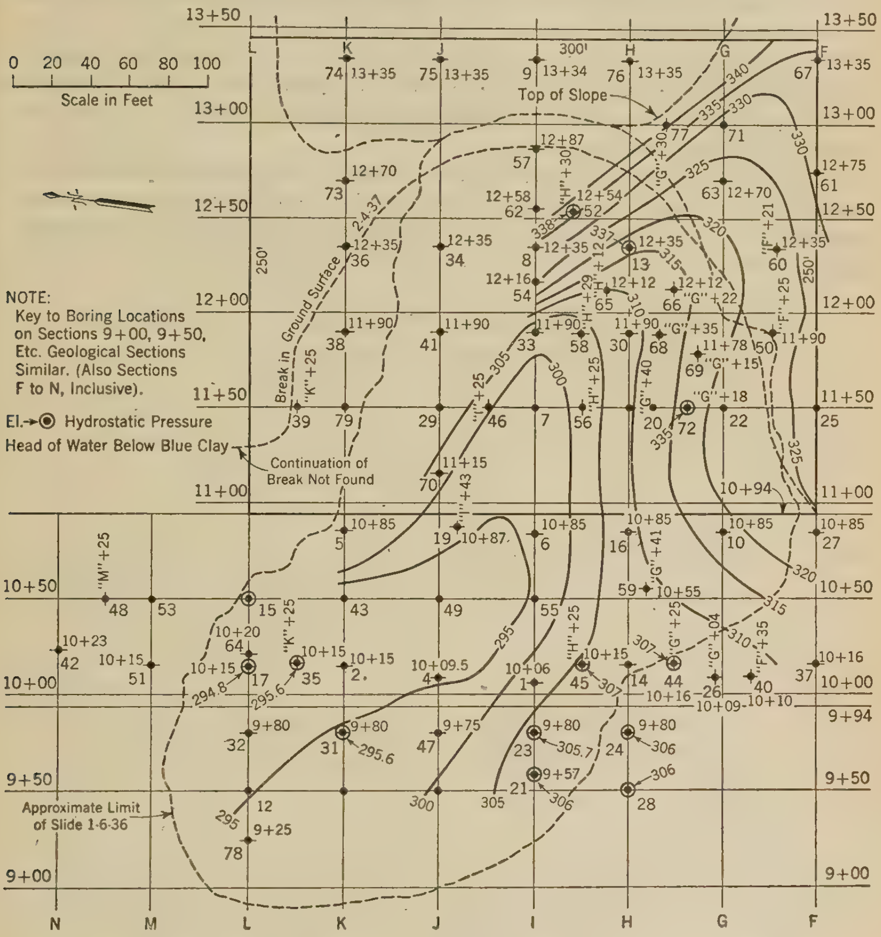
0 20 40 60 80 100
Scale in Feet
NOTE:
Key to Boring Locations on Sections 9+00, 9+50, Etc. Geological Sections Similar. (Also Sections F to N, Inclusive).
El.→◉ Hydrostatic Pressure
Head of Water Below Blue Clay
Top of Slope
Break in Ground Surface
Continuation of Break Not Found
Approximate Limit of Slide 1-6-36
250'
300'
13+50
13+00
12+50
12+00
11+50
11+00
10+50
10+00
9+94
9+50
9+00
N
M
L
K
J
I
H
G
F
"M"+25
"K"+25
"H"+30
"H"+25
"G"+30
"G"+40
"G"+41
"G"+25
"F"+21
"F"+25
"F"+35
340
335
330
325
320
315
310
307
306
305
300
295
295.6
294.8
305.7
243

FIG. 21.—FLOW NET AND GENERALIZED CONTOUR OF PERCOLATING GROUND WATER; PARKER AVENUE SLIDE, APRIL 1, 1937

the slide and covering several years. A water table could be defined for the area; but many inconsistencies were found in individual measurements which were not explained until the boring for the correction work was under way.

Mr. Sill describes in detail the geological conditions resulting from the weathering of granitic rock, and the slippage of masses of such rock lying between a steeply dipping fault plane and the excavated face of a spillway. It is of interest to note that movement was arrested by preventing the entry of surface water. It has been the writer's experience that finely divided fault gouge acts as a lubricant as well as a plane of parting, and that dry masses will part from the main body and slide along the slanting footwall of a fault zone if excavation leaves them unsupported. A similar experience was had with micaceous schist. The State Highway Department blasted a cut in that rock over a Western Pacific Railroad tunnel near Keddie, Calif. The blasting caused the mass to part along planes of schistosity dipping about 70° from the horizontal toward the cut and tunnel which, although dry (summer of 1931), allowed segments to slide along the slick planes into the highway cut. Other segments created a thrust against the tunnel lining at a depth of about 60 ft below the bottom of the cut, buckling it.

The writer is gratified by the interest shown in the subject matter of the paper, and appreciates the value of the comment and material produced by discussion in its application to the general problem of landslide investigation and correction.

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UPLIFT PRESSURE IN AND BENEATH DAMS
A SYMPOSIUM

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EXPERIMENTS ON EFFECTIVE UPLIFT AREA IN GRAVITY DAMS

BY SERGE LELIAVSKY BEY[1]

SYNOPSIS

In a gravity dam, water filters through the natural pores of the material of the structure and develops an internal pressure of uplift. This paper describes tests made to determine the effective area of pores, to which this internal uplift pressure is applied. To that end, water was driven into the pores of masonry and concrete specimens, which were, at the same time, subjected to directly applied, mechanically developed loads, and tested to destruction on a machine of original design. These results permit the designer to replace conventional assumptions of design by physically measured uplift coefficients.

INTRODUCTION

In the design of masonry or concrete dams the uplift due to the penetration of water into the structure is a factor that has never been completely investigated. There are two essentially different schools of thought in connection with this problem. According to the earlier group of theories—which comprised Maurice Lévy's law (communication to the French Academy of Sciences submitted by Maurice Lévy, on August 5, 1895), the Lieckfeldt method,[2] the Link theory,[3,4] and others[5]—the uplift pressure was assumed to penetrate into the material through cracks (see Fig. 1), and the horizontal or hydrostatic pressure of water was assumed to be exerted externally on the face of the dam. These uplift theories were very popular in Europe in the earlier years of the twentieth century and were incorporated in some of the larger projects of the period; but as experience and knowledge accumulated they were gradually abandoned.

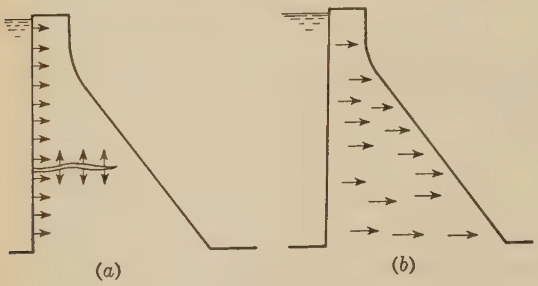
(a) (b)

FIG. 1

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[1] Chf. of Designing Office, Projects Dept., Ministry of Public Works, Cairo, Egypt.

[2] "Die Standfestigkeit von Staumauern mit offenen Lagerfugen," by G. Lieckfeldt, *Zentralblatt der Bauverwaltung*, Berlin, Vol. 18, February 26, 1898, p. 105.

[3] "Die Bestimmung der Querschnitte von Staumauern und Wehren aus dreieckigen Grundformen," by E. Link, Berlin, 1910.

[4] "Über Sohlenwasserdruck bei Staumauern mit entwässerter Grundungsfläche," by E. Link, *Zeitschrift für Bauweisen*, 1919, p. 525.

[5] "Profils des Barrages en Maçonnerie," by Pelletrau, *Annales des Ponts et Chaussées*, 1897.

The modern principle, accepted by Karl Terzaghi,[6] M. ASCE, P. Fillunger,[7,8] O. Hoffman,[9] and many other research workers in all parts of the world, is that water filters through the natural pores of the material of the dam. For that purpose the structure need not be considered cracked or injured but may be perfectly sound and strong, at the same time being more or less pervious. (Messrs. Terzaghi, Fillunger, and Hoffman are chiefly responsible for the latest developments in the theory of internal uplift in masonry dams. The serious student of this subject is referred to their several published contributions.) That the two effects (effects of pressure in the cracks and in the pores) might be superimposed has been suggested from time to time but the idea does not seem to have gained popularity because in one case there is a purely abstract conception of a hypothetical crack whereas the other concerns a physical phenomenon, capable of being investigated by means of the usual methods of experimental science.

As a corollary to the new conception it appears that the horizontal water pressure is not applied to the face of the material, as was formerly believed to be the case, but is totally or partly transferred into the body of the dam (shown by the arrows in Fig. 1(*b*)).

This change in the ideas concerning the point of application of the hydrostatic pressure necessarily must have affected the methods of stress analysis. It may be shown, however, that the normal stresses on horizontal planes and the shearing stresses are not altered, the normal stresses on vertical planes being the only ones that need to be revised. Although an exhaustive reply to this question might call for model tests, preliminary calculations tend to show that there is no cause for apprehension, at least in so far as the usual type of sections is concerned.

It follows from the foregoing that "uplift" can be defined as a "pressure in the pores" or an "interstitial pressure." The first and obvious problem in this connection is to find the intensity to which such a pressure should be raised, in order to cause the material to be broken.

Curious as this might seem, the published information on this subject is very scarce and rather inconsistent. In other branches of engineering it is standard practice to test specimens to failure in the laboratory, and thus to ascertain the effect of load upon the resistance for use in practical design; but very little evidence of similar character has been supplied concerning the force of uplift and its effect upon concrete or masonry.

The first set of experiments to be cited in this connection is the well-known investigation, as early as 1900, by Prof. August Föppl.[10] The apparatus he used is shown in Fig. 2.

Water under pressure supplied through the intake was driven into the pores of the material, and so caused the concrete to burst. The following are the pressures, in atmospheres, which were required in order to produce failure of exactly identical specimens: 11.0, 3.8, 12.0, 6.4, 15.0, 29.0, 7.0, 8.0, and 2.0. Obviously, with such irregular results no conclusions could be reached.

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[6] "Die Wirksame Flächenporosität des Betons," by Karl Terzaghi, *Oesterreichischer-Ingenieur-und-Architekten-Verein, Zeitschrift*, Vol. 86, 1934, Heft 1/2, pp. 1–9.

[7] "Der Auftrieb in Talsperren," by P. Fillunger, *Oesterreichische Wochenschrift für den öffentlichen Baudienst*, Vol. 19, 1913.

[8] "Neuere Grundlagen für die statische Berechnung von Talsperren," by Paul Fillunger, *Oesterreichischer-Ingenieur-und-Architekten-Verein, Zeitschrift*, 1914, p. 441.

[9] *Die Wasserwirtschaft*, 1929–1930.

[10] *Mitteilungen aus dem Mechanisch-Technischen Laboratorium der K. Polytechnischenschule in München*, No. 27, 1900, Heft 24, p. 16.

The technical consensus tends to attribute the inconsistency of these experiments to secondary influences and accidental circumstances, the effect of which had not been properly dealt with in designing the apparatus.

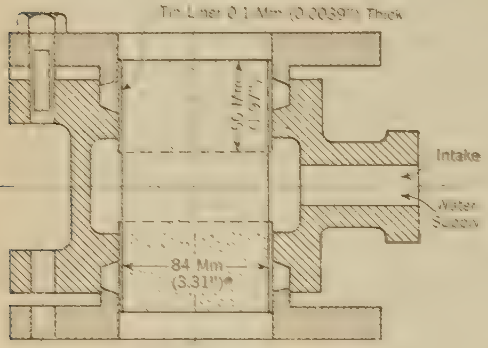

FIG. 2

In addition to Professor Föppl's tests, several others might be cited in which the test pieces were broken (or were intended to be broken) by the direct action of the uplift force: A test at the Bull Run Dam in Oregon; an experimental investigation which formed the subject of an unpublished thesis at the California Institute of Technology at Pasadena; a set of tests made by S. H. Woodard, M. ASCE, and reported by the late D. C. Henny,[11,12] M. ASCE.

Although the details of the mechanical devices used in these experiments were not published it appears that they were based upon the same general principle as the Föppl apparatus; but, in addition to this, the tensile breaking stress of the materials experimented upon was determined, in every case, by means of control specimens, on testing machines of the usual type.

The difference in the results derived in these interesting experiments seems to have been very great: The ratio of the interstitial area affected by the uplift pressure to the total sectional area was found to be 8% in one of these tests but appeared to increase to as much as 183% in another experiment.

One might naturally be reluctant to use these results in either design or theory, because the numerical value of the ratio in question must lie logically in between the coefficient of volumetric porosity, as a minimum, and 100%, as a maximum. It is rather difficult to interpret what actually occurred, particularly as the published information about these tests is indeed very scanty; but, according to Mr. Henny (as interpreted by the writer), the fact that the results in this case were far from consistent was most probably due to two causes: (*a*) The control briquettes did not furnish the true value of the tensile breaking stress, and (*b*) the pressure of the water injected into the pores was not uniformly distributed over the entire area of rupture (being undoubtedly less at the center than at the periphery).

These technical causes might have been effective in the two other cases as well, but the main source of the trouble seems to be deeper than that, as explained subsequently herein.

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[11] "Problems in Concrete Dam Design," by D. C. Henny, *Engineering News-Record*, Vol. 106, No. 11, March 12, 1931, p. 433.

[12] "Stability of Straight Concrete Gravity Dams," by D. C. Henny, *Transactions*, ASCE, Vol. 99, 1934, p. 1041.

Four other experimental investigations on uplift should be cited—namely, those of Max Rudeloff and Panzerbieter,[13] Fillunger,[14] Terzaghi[15] (1934) and N. Kelen.[16] An example of the testing devices used in these experiments is shown in Fig. 3. These four series of tests differed greatly in value, but they had one point in common—although the uplift force was present in every case, and the results of the tests were more or less dependent upon it, the failure itself was due to another agency, a mechanically developed load. This led to some ambiguity in interpreting the results obtained, thus rendering their significance difficult to assess, and leading consequently to much controversy as to the true interpretation of the tests.

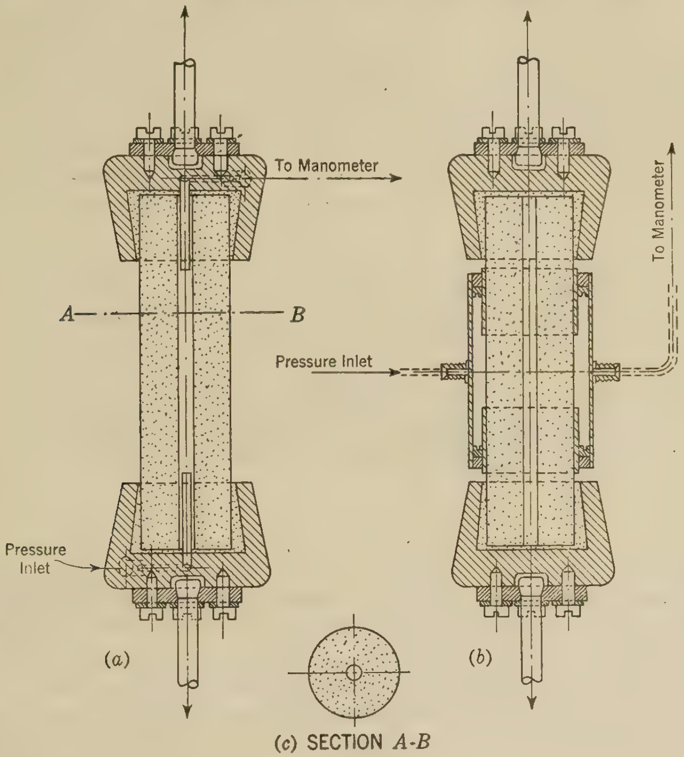

FIG. 3.—TESTING DEVICE USED IN THE EXPERIMENTS BY RUDELOFF AND PANZERBIETER

The existence of uplift (as an interstitial hydraulic force which was capable of causing failure) was still considered in certain circles as not being fully proved; and, even if it existed, opinions were often expressed that the magnitude of that force was a matter of chance and could not be dealt with as a measurable physical quantity.

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[13] "Versuche über den Porendruck des Wassers im Mauerwerk," by Max Rudeloff and Panzerbieter, *Mitteilungen aus deutschen Königlichen Materialprüfungsamt zu Gross Lichterfelde West*, Berlin, 1912.

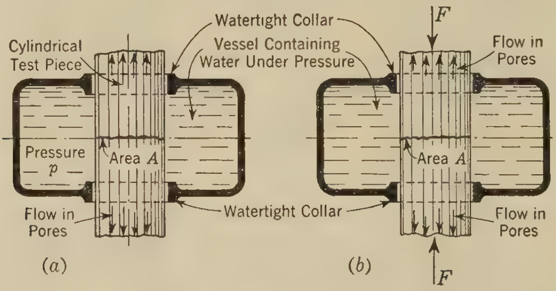
[14] "Versuche über Zugfestigkeit bei allseitigem Wasserdruck," by Paul Fillunger, *Osterreichische Wochenschrift für den öffentlichen Baudienst*, Vol. 21, Pt. 2, July, 1917, Heft 29, p. 443.

[15] "Simple Tests Determine Hydrostatic Uplift," by Karl Terzaghi, *Engineering News-Record*, Vol. 116, June 18, 1936, p. 872.

[16] "Experimental Determination of the Tangential Foundation Resistance of Gravity Dams," by N. Kelen, *Engineering*, Vol. 144, August 20, 1937, p. 215, and August 27, 1937, p. 242.

Therefore, it was believed that, in order to determine a convincing conclusion about the uplift pressure, the first and obvious step in solving the problem was to cause the material to be broken by driving water into the pores of the specimen, as was done by Föppl, but also to design the testing apparatus in such a way as to avoid the irregularity of the results he obtained.

Consequently, a more advanced type of apparatus was called for, but this alone would not have supplied the required solution, because the principle itself, upon which Föppl's device was based, did not contain the elements required for solving the problem. In this connection, attention is called to Fig. 4. The unit pressure of water being p and the sectional area of the cylindrical specimen, Fig. 4(a), the total pressure in the pores will be $p\ A$ multiplied by a certain coefficient of reduction n_a which represents the ratio of the interstitial area subjected to the hydraulic pressure, to the total area of the section, and is the main unknown value to be determined by these tests. According to the modern school of thought n_a is believed to be a physical constant depending on the arrangement of pores. It is variously described in different works on the subject either as the "uplift factor," or the "reduction coefficient," or as the "effective porosity." At the moment of failure, when the specimen bursts, the force $n_a\ p\ A$ must be equal to the resistance z of the specimen; that is,



(a) (b)

FIG. 4

$$n_a\ p\ A = z \qquad (1)$$

Unfortunately, Eq. 1 contains two unknown factors, n_a and z, and it is obvious that it cannot be used for finding either of them. Apart from technical details, therefore, the general layout of Föppl's testing arrangement did not satisfy the conditions of the problem.

In other instances, mentioned previously herein, z was determined by means of control specimens, so that apparently, had the causes of excessive dispersion been removed, these experiments might have supplied at least a partial solution to the problem. It will be shown, however, that, for reasons concerning the range of conditions experimented upon as compared with the probable deviation, consistent results would have been exceedingly difficult to obtain, if this method had been followed.

Proposed Solution of the Problem

The principle of the author's solution of this problem is shown in Fig. 4(*b*). The material is placed under the same conditions as it is in a dam; that is, the specimen is first subjected to the effect of a force F, so that, when the uplift begins to produce its effect, it must not only overcome the resistance of the material, but must also counteract the effect of a certain compressive force, which in a full-sized dam is due to its structural weight. Suppose that, having tested several identical specimens subjected to various forces F_1, F_2, F_3, etc. it is known that the pressures at which they broke were p_1, p_2, p_3, etc. The following series of formulas can then be written:

$$\left.\begin{aligned} n_a\,p_1\,A &= z + F_1 \\ n_a\,p_2\,A &= z + F_2 \\ n_a\,p_3\,A &= z + F_3 \\ \cdot\ \ \cdot\ \ \cdot\ &\ \ \cdot\ \ \cdot\ \ \cdot \\ n_a\,p_n\,A &= z + F_n \end{aligned}\right\}\tag{2}$$

from which

$$\left.\begin{aligned} n_a &= \frac{F_2 - F_1}{p_2\,A - p_1\,A} \\ n_a &= \frac{F_4 - F_3}{p_4\,A - p_3\,A} \\ n_a &= \frac{F_n - F_{n-1}}{p_n\,A - p_{n-1}\,A} \end{aligned}\right\}\tag{3}$$

The result is not only one, but several equations, from which n_a can be determined. The more general form of Eqs. 3 is:

$$n_a = \frac{\Delta F}{A\,\Delta p}\tag{4a}$$

or

$$n_a = \frac{dF}{A\,dp}\tag{4b}$$

The physical interpretation of Eqs. 4 is as follows: The additional pressure Δp is absorbed in overbalancing the additional force ΔF; therefore, Δp multiplied by the effective area $n_a\,A$ must be equal to ΔF.

This solution is valid only when a relationship is found to exist between $p\,A$ and F. In the absence of definite information on the subject, the problem of proving the existence of this relationship was at least as important as that of estimating it quantitatively; but, once the relationship had been established empirically, the new solution was independent of the precise nature of the physical causes which determined the value of the coefficient n_a. Had the problem been considered from the standpoint of the results of the earlier tests, the existence of this relationship between $p\,A$ and F might have appeared rather doubtful.

The value of n_a may be conceived either as being correlated with the interstructural arrangement of the pores in the normal condition of the material tested; or, alternatively, this material may be assumed to be already affected before the actual failure takes place, being at that moment in the state of "agony" observed in certain earlier tests. A third interpretation could also be imagined; in fact, one may visualize the effective area as being affected by the crack itself, or, in a strict sense, by the process of its gradual development, so that the failure of the less resistant or more accessible parts facilitates the penetration of water into the material, until, at the moment when the final fracture occurs, a certain (possibly quite a large) part of the area of rupture is already open to the direct action of the pressure.

So long as a measurable relation is found to exist between $p\,A$ and F, it would not really matter which of these three interpretations was taken to be true; or, should another physical explanation be suggested, the results of the tests would still remain valid and applicable to the statical analysis of the dam section. Nevertheless, when analyzed, the tests themselves supplied a definite indication that the first of the three suggested explanations is the correct one, but this point will be considered subsequently in this paper.

A certain correction must be introduced into the value of n_a obtained as suggested herein, in order to account for the fact that, apart from producing uplift, the water contained in the pores increases the weight of the material. For a more or less triangular profile the diagram of this additional weight, for any horizontal section considered, is nearly triangular; which means that, at any point, this effect may be assumed to be proportional to the uplift pressure. Therefore, it may be taken into account by a corresponding reduction in the value of n_a. The final value of the uplift factor would then be found from:

$$n = n_a - n_v \qquad \text{(5)}$$

in which n_v is the volumetric porosity of the material.

This method of calculation seems to be justified for the reason that n_v is very small. It is evident, however, that an alternative path could also be followed—namely, to take n_v into account by a corresponding adjustment of the structural weight of the dam, and then to multiply the interstitial, pressure intensities by the unreduced value of n_a. The first procedure seems to be more usual because it sets off the difference in the relative values of the two coefficients and therefore gives a better idea of the total effect which the filtering of water may produce upon the mechanical stability of the work.

CALCULATION OF PRESSURE INTENSITIES OF FILTERING WATER

This problem is not specifically discussed in this paper. It is rather more simple and more obvious than that referring to the value of n_a. It has been treated extensively in modern technical literature and may now be considered as belonging to the scope of a textbook. For example, Professor Terzaghi has written on the subject in various papers,[17] as have E. W. Lane,[18] M. ASCE, W. Weaver,[19] Hoffman[20] and many others.

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[17] "Beanspruchung von Gewichtstaumauern durch das strömende Sickerwasser," by Karl Terzaghi, *Die Bautechnik*, Vol. 12, July 6, 1934, Heft 29, p. 379.

[18] "Security from Under-Seepage Masonry Dams on Earth Foundations," by E. W. Lane, *Transactions*, ASCE, Vol. 100, 1935, p. 1235.

[19] "Uplift Pressure on Dams," by W. Weaver, *Journal of Mathematics and Physics*, M.I.T., Cambridge, Mass., June, 1932, p. 114.

[20] "Permeazioni D'Aqua e loro Effeti nei Muri di Ritenuta," by O. Hoffman, Milan, 1928.

In the study of streamline flow as in many other cases of physical analysis the investigator is fully dependent on boundary conditions, which forces him to adopt graphical and approximate arithmetical solutions, or to use model tests and electrical-analogy methods in connection with the design of weirs and dams built on granular soils because the structural details of the foundations of such works may often create exceedingly complicated boundary conditions in so far as the seepage of water is concerned. As compared with these involved cases, the triangular profile of a gravity dam resting on rock is usually very simple to analyze mathematically. In fact, Hoffman has shown[21] that in this case the field of pressures tends to be delimited by a plane, which means that the pressure diagram is nearly triangular. Many times before Hoffman defined it, this principle was used more or less intuitively, in designs, theoretical analyses, and specifications.

To convert from the conditions applying to an ideal triangular profile to those of a practical dam section, the effect of the roadway at the top and the interruption of continuity at the base must be taken into account. These are chiefly local effects, however, which are of rather secondary importance as compared with the main tendency toward a linear pressure distribution. There is no great difficulty in finding the numerical values of the corresponding corrections.

Occasionally, departures from the accepted theory have been reported in connection with pressure-pipe observations in actual dams. There are reasons to believe that the cause of the trouble is partly in the accidental differences in the value of Darcy's coefficient; but, as in every case reported the number of observation points was far too small for the application of scientific statistical investigation methods, the personal element seems to have played a large part in the interpretation of these results. One must distinguish, however, between the accidental differences in the texture of the material in the dam itself, and the systematic differences in Darcy's coefficient over the line of contact with the foundations. Departures from the theoretical triangle are accidental in the first case, but might be systematic in the second case. They should then be taken into account by means of the same methods of calculation as were adopted for granular foundations. Galleries, drains, and differences in the mixtures and methods used in casting the toe and heel areas of the dam are some other causes that may account for systematic variations.[22,23,24]

After much preliminary work, the engineers working on granular foundations in India have succeeded in showing that theory, model tests, and pipe observations were all in agreement.[25] There is no apparent reason why the same should not apply to rock foundations, except that, in the latter case, the investigator is faced[26] with the much more difficult problem of computing the value of n_a, which is the specific subject of this paper.

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[21] "Permeazioni D'Aqua e loro Effeti nei Muri di Ritenuta," by O. Hoffman, Milan, 1928, p. 41.

[22] *Ibid.*, pp. 49–73.

[23] "Pressures due to Percolating Water and Their Influence upon Stresses in Hydraulic Structures," by J. H. A. Brahtz, *Transactions*, 2d Cong. on Large Dams, U. S. Government Printing Office, Washington, D. C., Vol. V, 1938, p. 43.

[24] "Beanspruchung von Gewichtstaumauern durch das strömende Sickerwasser," by Karl Terzaghi, *Die Bautechnik*, Vol. 12, July 6, 1934, Heft 29, p. 3.

[25] "Design of Weirs on Permeable Foundations," by A. N. Khosla, Simla, 1936, p. 6.

[26] "Measuring Dam Behavior," by Douglas McHenry and Roy W. Carlson, *Engineering News-Record*, Vol. 122, No. 13, March 30, 1939, p. 58.

General Principle Underlying the Design of the Experimental Machine

In detailing the testing apparatus for the proposed study it was essential to insure that, when the test piece broke, the pressure p had already penetrated into the thickness of the material and was the same over the entire area of the surface of rupture; or, alternately, if the specimen failed before the penetration was complete, means had to be provided to ascertain, subsequently, the true intensity that the pressure in the pores had actually attained at the moment of failure. To solve this problem, the part of the test piece in which the failure was intended to take place was made hollow, similar to a ring or a pipe with very thick walls, as shown in Fig. 5. (Rudeloff[13] had previously adopted a somewhat similar arrangement for his experiments, but this was not known to the writer until 1938 when the finished drawings of his own testing machine had been completed.)

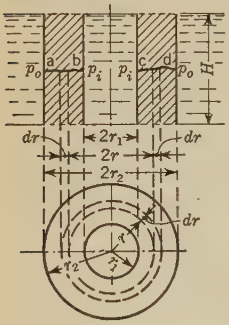

Fig. 5

The sequence of the operations performed in the course of an experiment on the writer's apparatus was as follows: At the beginning of the experiment the test piece was subjected to a certain force F and the water pressure in the container was raised to the intensity p_o, which, hereafter, is denoted as the "outer pressure." At the same time, the empty space inside the test piece was filled with water, connected to a manometer, and closed. The pressure in that space ("inner pressure") was then allowed to rise naturally, until it became constant, or nearly so. In order to prevent this rise from becoming too rapid, or from being abrupted, an air vessel was introduced into the system of pipes connected with the inner space of the specimen.

By that time, if the test piece still remained unbroken, the outer pressure p was increased, and the apparatus was left standing until the inner pressure became constant again. The same procedure was repeated, the pressures being stepped up, until failure finally occurred. Thus, the rise of the inner pressure p_i (which was recorded throughout the duration of the entire test) depended on the quantity of water percolating through the walls of the specimen, and on the velocity of infiltration. It indicated the intensity of the pressure in the pores in immediate contact with the inner periphery of the specimen. A reverse order might have also been adopted—namely, keep the pressure p_o constant and reduce the force F gradually until the specimen broke. This sequence would have been easier but was believed to be inadmissible, because in actual practice the material in the dam begins by being compressed by the weight of the superimposed layers; and then only is it subjected to the filtering effect of the water. It was considered essential, therefore, that the sequence of the various causes of stress should be reproduced in the test in the order it occurs in nature.

Had failure occurred when p_i and p_o were equal, the conclusion would be that, at that moment, the pressure in the pores on the outer and inner surfaces of the ring-shaped specimen were the same. In such cases the resultant pressure was calculated by multiplying the sectional area of the specimen by

$p_i = p_e$ (as was originally assumed in explaining the principle of the solution); yet, more often than otherwise, the specimen broke when p_i was less than p_e. (In fact p_i never equals the full value of p_e. A slight difference always remains; but in many cases its effect on the results is almost imperceptible.) Then the resultant depended on the law controlling the distribution of the pressure intensities over the section:

$$p = \frac{Q}{2\pi H K} \log_e r + C \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which p is the unit pressure at the radius r, Q is the discharge; K is Darcy's filtering coefficient; and H is as indicated in Fig. 5.

Eq. 6 is the standard formula for radial filters, which is given in various textbooks on the subject,[7] and may be transformed into:

$$p = (p_e - p_i) \frac{\log_e \frac{r}{r_1}}{\log_e \frac{r_2}{r_1}} + p_i \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The average value of pressure over the cross section is then as follows:

$$p_a = \frac{1}{A}\int_0^A p\, dA = \frac{1}{A}\int_{r_1}^{r_2} 2\pi r p\, dr \ldots\ldots\ldots\ldots (8)$$

Substituting for p its value obtained from Eq. 7, and integrating:

$$p_a = p_i + (p_e - p_i)\left\{\frac{r^2_1}{r^2_2 - r^2_1} - \frac{1}{2 \log_e \frac{r_2}{r_1}}\right\} \ldots\ldots\ldots (9)$$

or

$$p_a = p_i + (p_e - p_i)\Delta \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which

$$\Delta = \frac{r^2_1}{r^2_2 - r^2_1} - \frac{1}{2 \log_e \frac{r_2}{r_1}} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

In conducting the tests the average pressure p_a calculated from Eq. 10 was substituted for p in all the cases in which, at the moment of failure, the inner and outer pressures were not the same.

The physical meaning of Eq. 10 is perfectly clear—the average pressure intensity is equal to the sum of two terms p_i and $(p_e - p_i)\Delta$. The first represents the limit reached if the outer pressure falls and becomes equal to p_i; or, conversely, if the inner pressure rises and attains the value p_e. The second term shows the effect of the difference $p_e - p_i$ and depends on the value of Δ which, in turn, is a function of the ratio $\frac{r_2}{r_1}$ of the outer and inner radii. This function is shown graphically in Fig. 6. Eq. 10 applies also to the case in which $p_i > p_o$ except that the flow is reversed, and, therefore, instead of being convex the curve of pressures must be concave.

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[7] "Hydraulik," by Philipp Forchheimer, B. G. Teubner, Leipzig and Berlin, 1914, p. 432.

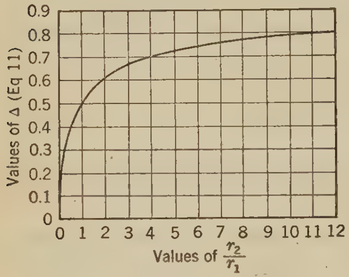


FIG. 6

Referring again to the arrangement actually adopted in the tests (that is, $p_o > p_i$), attention is called to the fact that, thus far, no proof has been offered that n_a is constant. Since this was one of the questions that the experiments were intended to answer, the investigation was conducted in such a manner that the results would have remained significant regardless of whether n_a was found constant or variable. With this object in view the results of each series of tests were represented graphically on a chart, the values of F being plotted as ordinates against $p_a A$ as abscissas.

A series of points was thus obtained, which one could expect to define either as a straight line, or a curve. When the points define a straight line, n_a is shown to be constant (and equal to the tangent of the angle between this straight line and the horizontal). When they define a curve, the variable slope of the curve represents the value of n_a corresponding to various values of $p_a A$ and F. Should the observed points be irregularly dispersed over the entire diagram, defining neither curve nor straight line, this would indicate either that the apparatus was improperly designed or that the uplift was not controlled by any regular law.

In such computations, the greater the range of variation of the variables, the more convincing is the result. In perfecting the details of the testing apparatus, therefore, it was believed convenient to use a device which allowed for reversing the direction of the force F. Thus it was possible to obtain positive as well as negative values of that force, which did not affect the substance of the conclusions, but permitted an increase in the range with the same testing equipment.

THE TEST SPECIMEN

One of the reasons for adopting a ring-shaped type of test specimen has been given. The ring shape also solves the problem of air drainage, as the air contained in the pores in the critical part of the specimen can escape freely into the inner space.

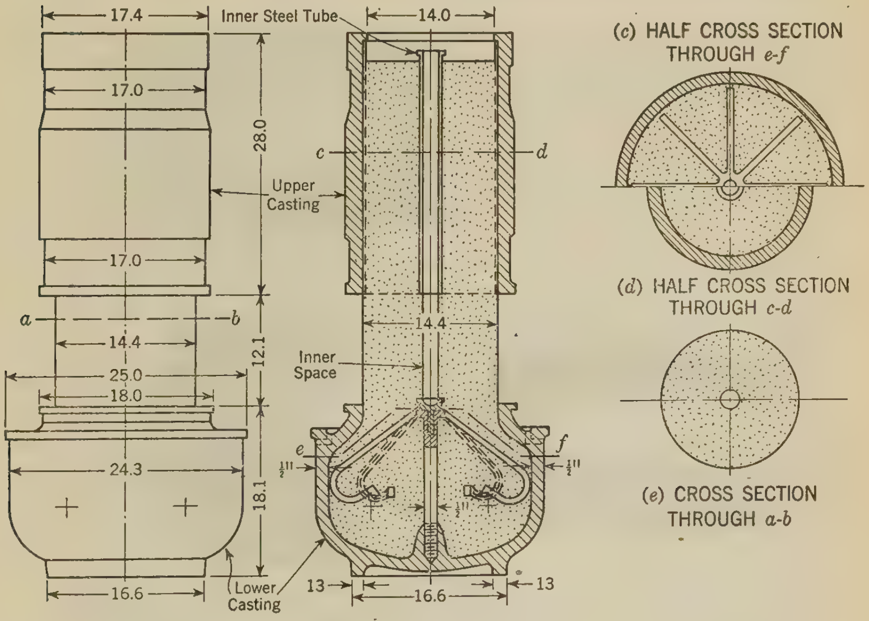
The diameters of the specimens adopted in earlier experiments on uplift were:

	Diameter (cm)
Föppl	8.4
Rudeloff and Panzerbieter	9.0
Terzaghi	8.0
Woodard	5.0

Although sometimes referred to as "concrete," the materials used in these experiments were actually "mortar," being made of cement and sand without the addition of the coarser aggregate. Since the new experiments were intended to explore a wider range of conditions, it was considered essential to introduce broken stone or gravel into the mixtures tested.

To derive consistent results from samples of such a nonhomogeneous substance, the dimensions of the specimens must be much greater than the irregularities that cause the nonhomogeneity of the material. For that reason, the section of the test piece in the new experiments was made slightly greater than 150 sq cm, the outer diameter being 15 cm when cast, and somewhat less than that when tested, owing to the removal of the "skin"—an operation which was always performed before placing the specimen in the apparatus.

As no marked difference was discovered, later, between the dispersion of the individual tests in the groups prepared, respectively, with and without coarse aggregate, there is sufficient reason for believing that the adopted dimensions of the specimen were adequate.



(a) ELEVATION (b) VERTICAL SECTION

(c) HALF CROSS SECTION THROUGH *e-f*

(d) HALF CROSS SECTION THROUGH *c-d*

(e) CROSS SECTION THROUGH *a-b*

Fig. 7

The general arrangement of the apparatus is shown in Fig. 7, and Fig. 8 is a view of the appearance of the specimen before and after failure. The cast-iron collars by means of which the two castings are held together, when the test piece is cast, are shown in Fig. 9. It will be observed that the apparatus consists of: The inner core, made of the material tested; the upper lining; the lower casting; the pipe in the upper part of the inner space; and the reinforcing bars in the lower part of the core.

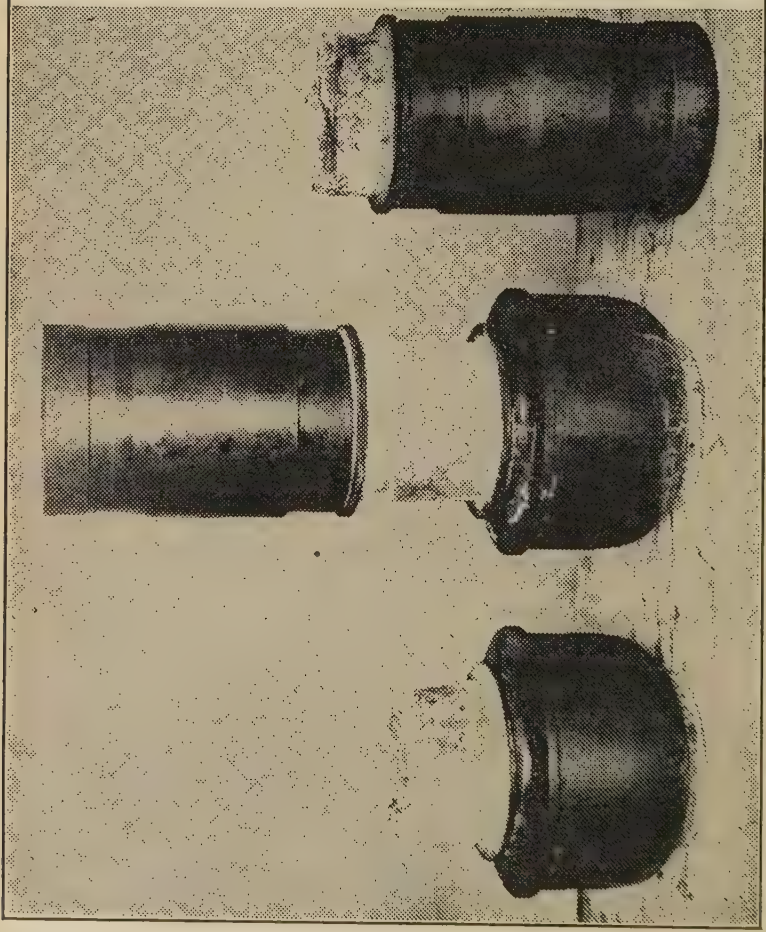


FIG. 8

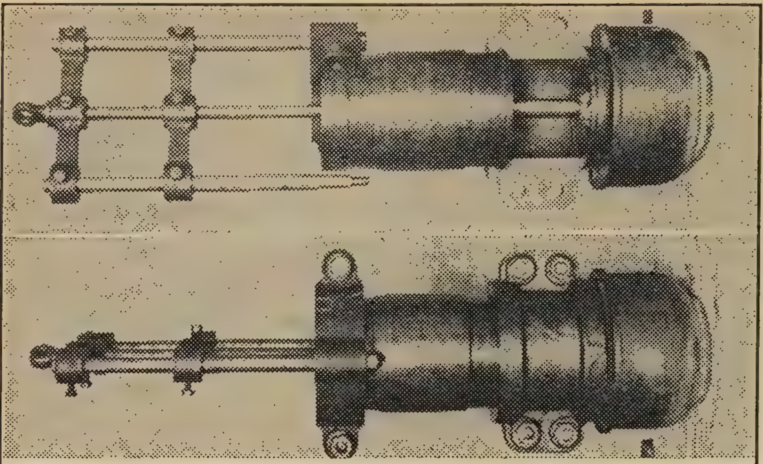


FIG. 9

The first point that calls for comment is the unsymmetrical layout of the specimen. Instead of having both its ends projecting beyond the specimen container, as was done in Föppl's experiments and Rudeloff's experiments, the test piece is entirely enclosed on its lower side, so that only one watertight collar is required instead of two. This arrangement is preferable for technical reasons, both in regard to simplicity of design and facility in conducting tests.

The function of the upper casting is twofold: (1) To provide a watertight and smooth surface for the leather collar to slide over; and (2) to transmit to the material tested the force F—compression or tension as the case may be.

It is essential that this force should not be applied directly to the concrete by means of any object placed on its top surface, because such an object would obstruct the exit from the pores through which the water is filtering during the experiment. Should this occur, the conditions would not be the same as in an actual dam. Therefore, the design adopted was as follows: The force F is transmitted by a cup and a brass collar to the upper rim of the top casting, and retransmitted to the specimen through the shearing effect from the inner deformed surface of the lining to the concrete adhering to it. The lower casting (which resembles in shape a common kitchen kettle) has no other function than to maintain the specimen in position.

Both castings were made heavy and strong, and all surfaces that control the position of the specimen relative to the container were machined carefully, thereby avoiding the secondary effects that caused the failure of Föppl's experiments. The remaining parts of the test specimen are of less importance. The pipe in the upper part of the inner space (diameter, 2 cm) is required for fixing the connections to the flexible pipe leading to the air vessel and to the manometer that measures p_i. The reinforcing bars in the lower part of the tested material are introduced in order to prevent the failure from occurring along a curved surface, subject to critical inclined stresses.

A pair of pincers, made of $\frac{3}{4}$-in. round iron, was used to lift and transport the specimens. They were arranged in such a way as to engage with the upper rim of the lower casting, so that no tension stresses, even of the smallest intensity, were set up in the tested material at any moment before the test proper was actually started.

The Testing Machine

The testing machine consists of the four parts: (*a*) Test vessel, or container, in which the specimen is placed for testing; (*b*) lever system, required to develop the compressive or tensile force F; (*c*) accumulator with inverted plunger, frame for carrying the loads, counterweight, manometer for measuring p_o, etc.; and (*d*) air vessel, with manometer for measuring p_i, and a system of flexible pipes designed to connect with the inner space of the specimen.

The lever system (item (*b*)) comprises the main lever with bearings, tie pieces and rocker, the secondary lever with adjusting screws and counterweight, and the cup-and-collar device arranged to engage with the upper lining of the specimen when the test is in progress.

As seen from Figs. 10 and 11, all these parts are erected on a main frame built of riveted channels and gussets. For the tests, this frame was fixed on

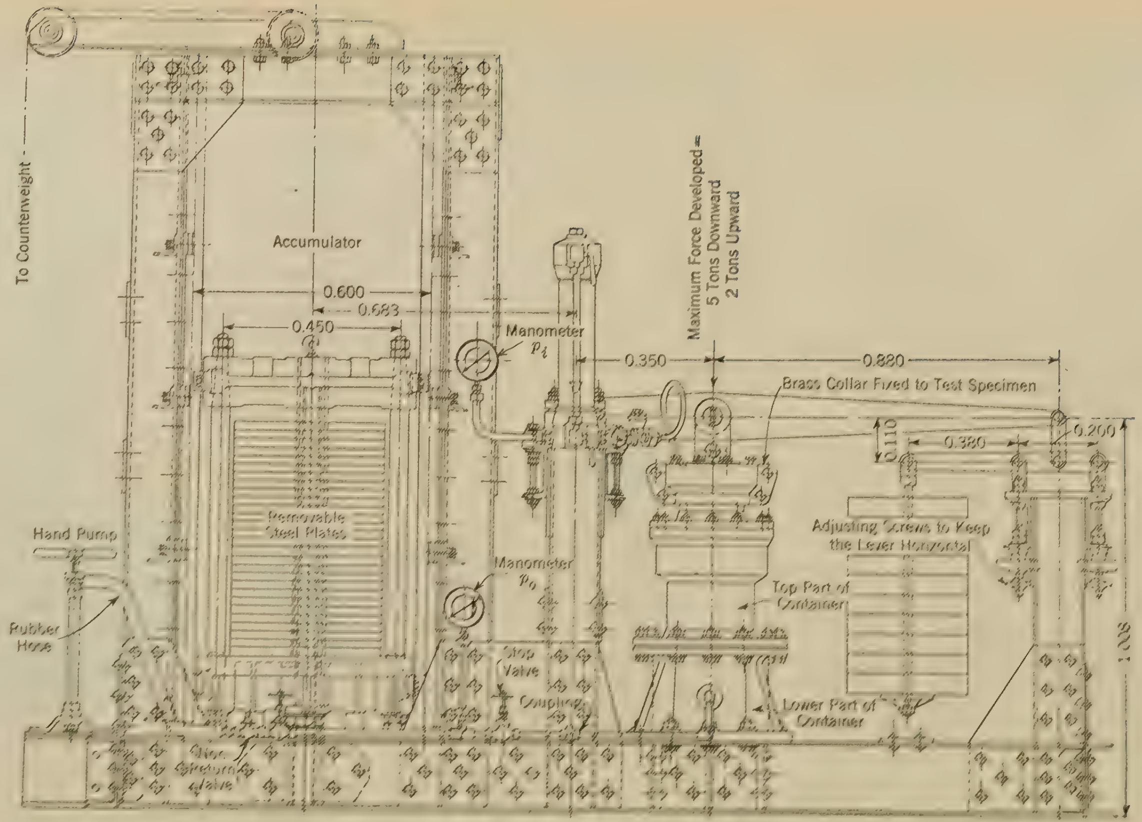

To Counterweight
Accumulator
0.600
0.683
0.450
Removable Steel Plates
Hand Pump
Rubber Hose
Manometer p_i
Manometer p_o
Stop Valve
Coupling
Maximum Force Developed = 5 Tons Downward 2 Tons Upward
0.350
0.880
Brass Collar Fixed to Test Specimen
0.110
0.380
0.200
Adjusting Screws to Keep the Lever Horizontal
Top Part of Container
Lower Part of Container
1.008

FIG. 10.—GENERAL ARRANGEMENT OF TESTING APPARATUS

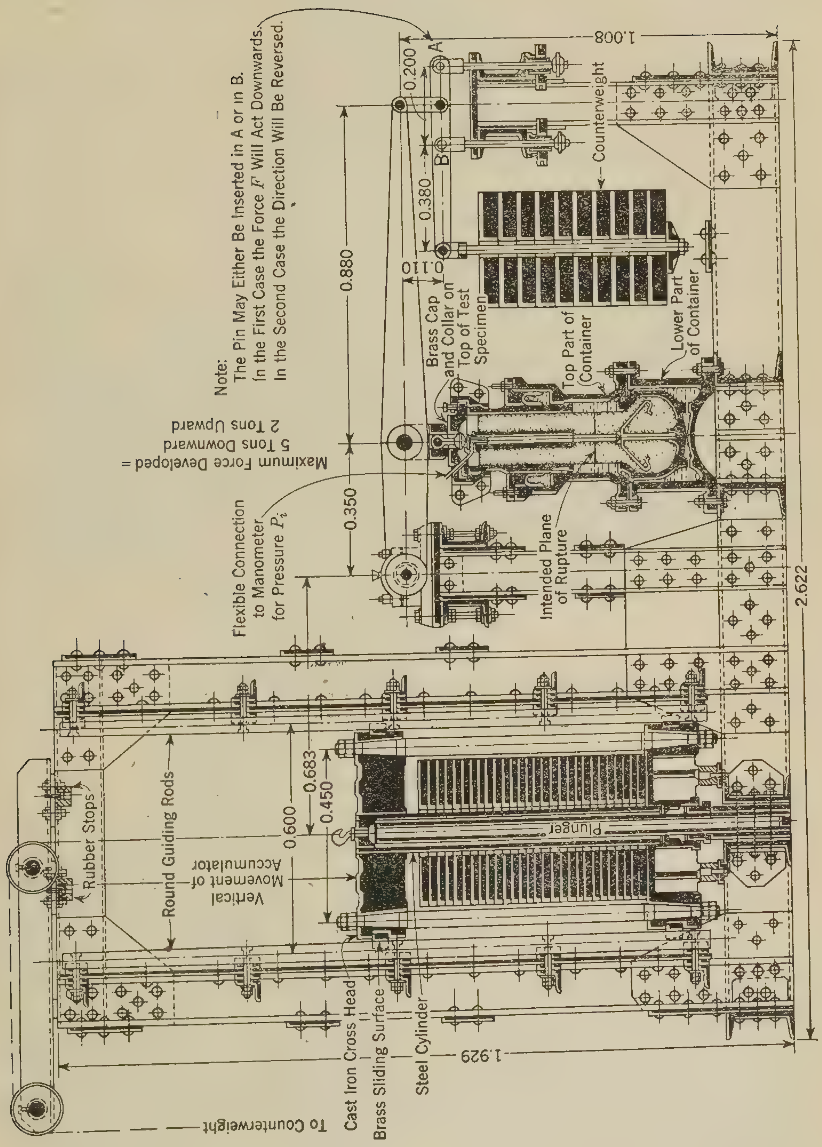

Note:
The Pin May Either Be Inserted in A or in B.
In the First Case the Force F Will Act Downwards.
In the Second Case the Direction Will Be Reversed.
Maximum Force Developed =
5 Tons Downward
2 Tons Upward
Flexible Connection to Manometer for Pressure P_i
A
B
0.200
0.380
0.110
0.880
0.350
1.008
2.622
1.929
0.600
0.683
0.450
Counterweight
Brass Cap and Collar on Top of Test Specimen
Top Part of Container
Lower Part of Container
Intended Plane of Rupture
Rubber Stops
Round Guiding Rods
Vertical Movement of Accumulator
Plunger
Cast Iron Cross Head
Brass Sliding Surface
Steel Cylinder
To Counterweight

Fig. 11.—Vertical Section Through Axis of Testing Apparatus

an independent block of concrete, about ten tons in weight, which was constructed separately from all other foundations of the building, so as to avoid the effect of vibrations. Fig. 12 contains two views showing the machine fully erected in the laboratory.

The supply pipe through which the container is supplied with water is connected at the lower part of the apparatus. This pipe is not shown in Fig. 11 but appears in Fig. 10. It connects the accumulator with the lower part of the container. From this lower part the water passes directly into the top part, and thence, through the pores in the walls of the specimen, it percolates into the inner space of the latter. A part of the water escapes from the upper surface of the concrete or masonry test piece. During the experiments, it was found necessary to introduce a small sieve in this supply pipe connection, to prevent small particles of the broken concrete from being carried backward by the effect of the pulsations set up when the specimen failed.

The levers are designed in such a way that they can be used to produce, at will, either compression or tension. For that purpose the smaller lever is provided with two bearing pins; depending on which one of them is used, the force transmitted to the test piece is either directed downward or upward.

Had the position of the axles been fixed rigidly, the slope of the levers, due to the deflection of the various parts of the apparatus, would have been so great as to affect, seriously, the precision of the results of the experiment. This was avoided by using the adjusting screws shown on the right side of Figs. 10 and 11—in such a way that the levers were permanently maintained horizontal.

The object of the rocker which transmits the stress from the main lever to the specimen is to prevent horizontal stresses from occurring in the test specimen either during loading (caused by the effect of elastic deformations) or later (caused by temperature variations).

In calculating the force F, in addition to the force transmitted by this rocker, the operator also takes into account the weight of the upper part of the test piece (above the surface of rupture), and the hydraulic pressure acting on the metal linings attached to the specimen; that is, the effect of the pressure p_1 on the under surface of the top casting and that of the pressure p_2 on the pipe bent connected to the inner space. Both these hydraulic forces can be determined easily since the pressure intensities and the surfaces over which they are effective are all known precisely.

An accumulator was included in the design in order to develop the required intensity of the water pressure and to keep it permanently controlled during the test. It was fed periodically with an ordinary pump of the type commonly used in testing boilers.

The object of the air vessel was to avoid an irregular, or too rapid, rise of the pressure. Its volume was calculated accordingly. It was considered of interest to measure the quantity of water that entered into the air vessel while the test was in progress. Accordingly the vessel was made rather high, as compared with its diameter, and was provided with a glass gage similar to the gages commonly used in boilers.

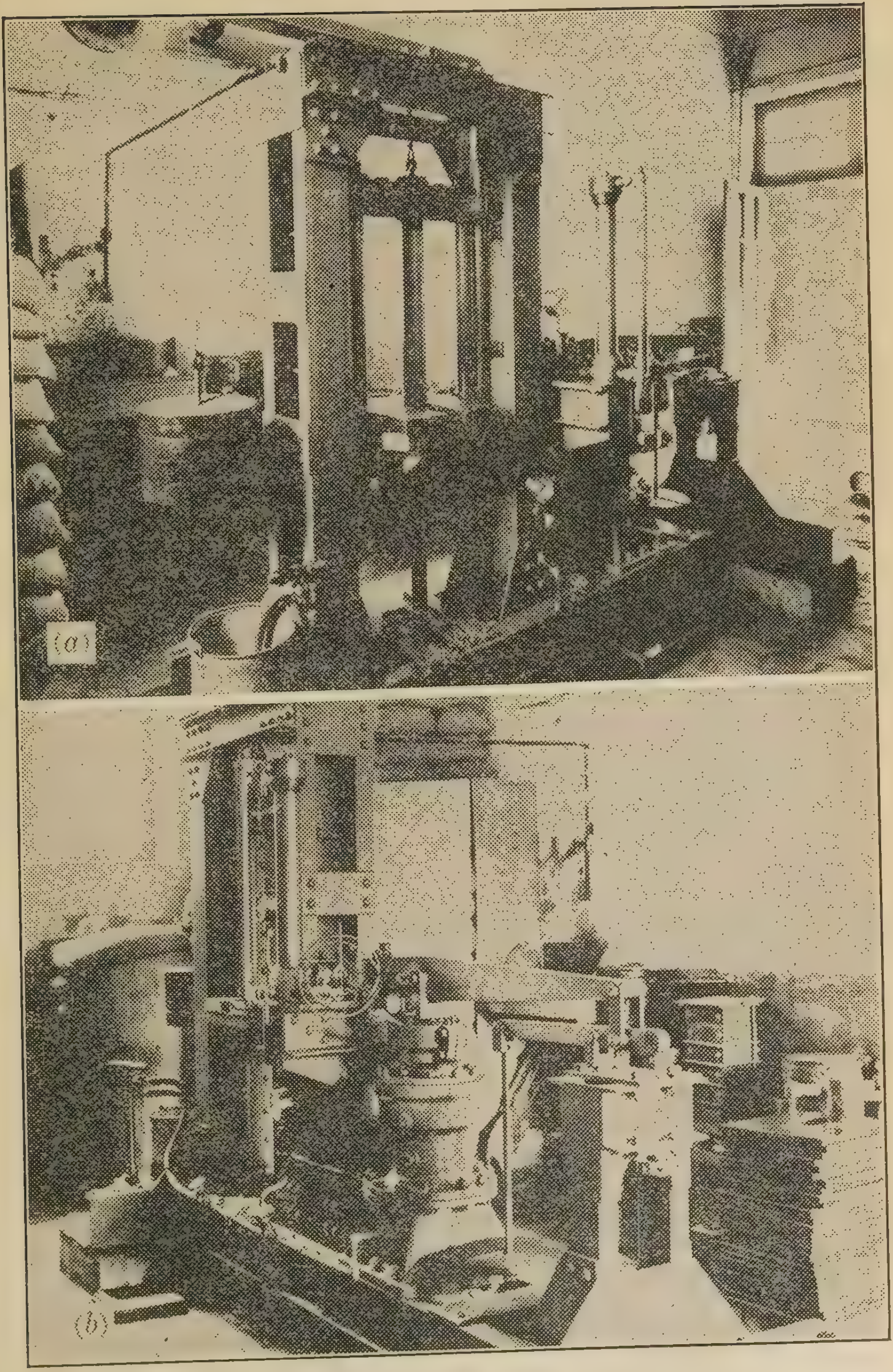


FIG. 12.—VIEWS OF THE LABORATORY WITH TESTING MACHINE FULLY ERECTED

PROCEDURE AND SIGNIFICANCE OF TESTS

Detailed shop drawings for the apparatus and for the entire testing equipment were completed and transmitted to the Cairo Government Workshops, at Bulaq, Egypt, on November 2, 1937. In the autumn of 1939 the apparatus was completed and installed in the same workshops.

The novelty of the design was such that some adjustments were anticipated. A number of preliminary experiments were conducted, therefore, to verify the efficiency of the machine, on the one hand, and to study the best procedure for making the test specimens, on the other.

In making the specimens the writer was ably assisted by the engineers of the Bridges Service, Egyptian State Railways. Various procedures were investigated, the object being to obtain uniformity of texture and absence of leaks.

Two methods were finally adopted. The first was based on standard practice in making concrete. In the second method, the particles of the large aggregate were pressed into the layers of mortar after the latter had been deposited in the mold, precisely as in building a masonry dam. The entire operation was completed at such a speed, and with such efficiency, as to form one single, uninterrupted, and continuous process.

The specimens were left standing on a shelf until the following day, when the upper collar and the steel core were dismantled. At the same time the "skin" was removed from the exposed parts of the tested material. For the outer surface this was done by means of a steel brush, whereas in the inner space a ratchet fixed on a long flexible rod was found to be the most convenient tool for the purpose.

Subsequently, the test specimens were allowed to cure for six weeks or more. While curing, the specimens were kept thoroughly wet, for a period of two weeks. The program of each series of experiments was arranged in such a way as to avoid any correlation between the age of the specimens and the force F.

The time required for testing one specimen was at least three days and often more, because the pressure p_o was raised only very slowly, so as to make certain that both p_o and p_i were truly representative of the pressures existing in the pores in immediate contact with the outer and inner peripheries of the tested material. In this connection attention is called to Fig. 13 which shows three characteristic instances selected from among the diagrams representing the rate of the increase in pressure in the individual experiments. Similar diagrams were prepared for all the tests. They are typical in representing conditions regarding the interdependence between p_o and p_i.

For example, Fig. 13(*a*) shows a case in which the material appeared to be fairly permeable, so that p_i followed very rapidly after an increase in p_o. In Fig. 13(*b*), however, the effect of p_o upon p_i was less rapid, whereas Fig. 13(*c*) is characteristic of the conditions that are the extreme opposite to those represented in Fig. 13(*a*). In this last experiment, the pressure p_i began rising toward the end of the test only, when p_o had already attained a high intensity.

In spite of such differences in the values of p_o and p_i, the results of the individual experiments were quite consistent with each other. It follows, therefore, that Eq. 10, which defines the relative effect of p_o and p_i upon p_a, is not only correct theoretically, but is also applicable to the case under review. As will be shown subsequently, the results attained by Rudeloff supply another proof in support of this conclusion.

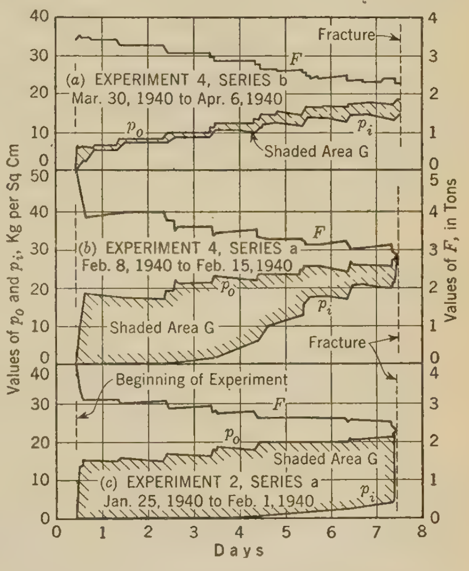


Fig. 13.—Rate of Increase in Pressure

Attention is also called, in this connection, to the diagrams shown in Fig. 14. The two experiments which they represent were conducted with the special object of verifying Eq. 10, and of ascertaining the effect of variations in the program of loading on the results of the test. These experiments were made generally similar, therefore, but the programs of loading—that is, the rates of increase of the pressure p_o adopted in the two cases—were essentially different. In spite of that, these experiments gave practically the same result. This may be of some importance, as it shows that the program adopted in filling the reservoir does not affect the uplift in the dam.

Another point, which was also rather important in the tests, was to ascertain the mechanical friction losses in the bearings of the lever systems and in other parts of the apparatus, in order to find the effect of friction on the force *F*. On several occasions during the experiment the machine was loaded with a special test specimen, which was generally similar to the standard specimen but was filed across (before being placed in the test vessel) in about the same spot in which failure usually occurred in other specimens. No inner space was provided in this specimen. Consequently, the pressure p became instantly effective over the full section filed, as soon as the accumulator had been loaded with the corresponding number of plates.

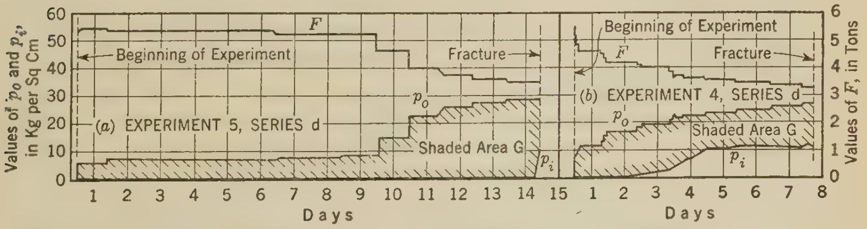


Fig. 14

It follows, therefore, that in this special experiment water had direct access to every point of the area that had been filed through. Thus, it was possible to compute the unreduced effect of the water pressure precisely. By comparing the resultant pressure so determined to the lever load required to balance it, the friction losses could be found easily.

The results of one of such tests are shown in Fig. 15. The experiments represented by circles are intended to reproduce conditions similar to a standard test in which a certain number of cast-iron weights are placed on the lever and the pressure p_o is gradually increased until the micrometer shows that the top of the specimen begins moving upward. In this case the friction forces are directed in the same way as in a standard test; that is, they all tend to increase the force F. It is only logical, therefore, that the corresponding points are below the "no-friction line," the recorded pressures $p\,A$ being always greater than the calculated force F. This experiment was repeated many times and for various loads, first in the rising and then in the falling order, the entire set of tests being completed in 48 hr.

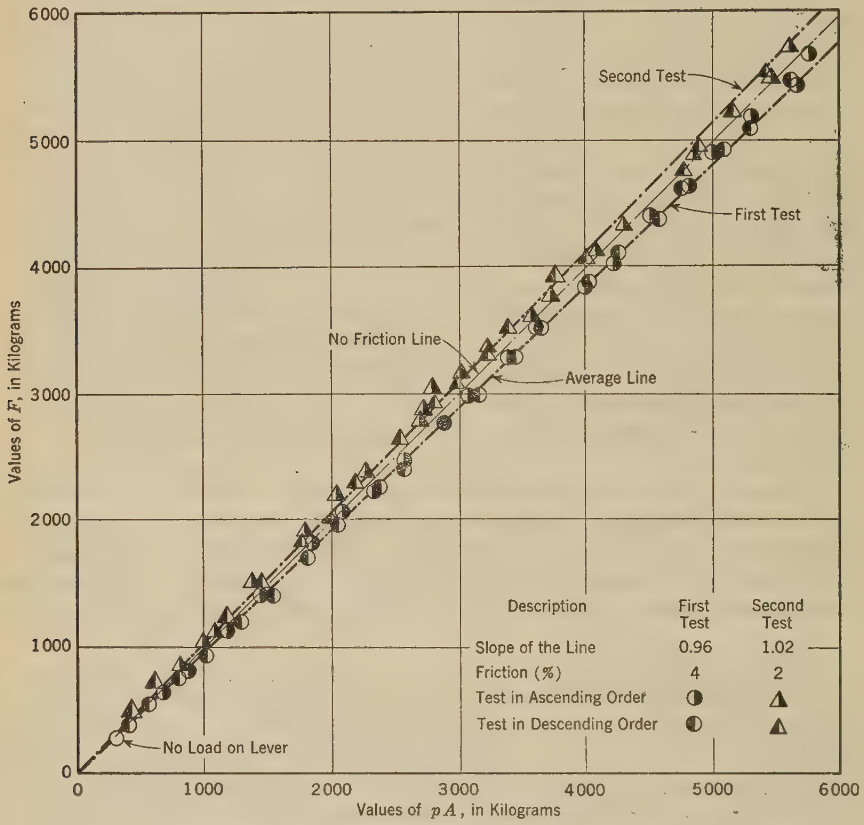


Fig. 15.—Test to Determine the Effect of Friction

The tangent of the angle between the mean line drawn through these points and the vertical is calculated by the formula,

$$\tan \theta = \frac{\Sigma(x\,y)}{\Sigma y^2} \qquad (12)$$

which is derived from the "least-squares" principle.[28] The symbols y and x appearing in this equation represent the individual values of F, and the corresponding recorded pressures $p\,A$, which are used respectively as ordinates and abscissas in plotting curves in Fig. 15. The slope of the calculated line is 4% less than 1 on 1. Accordingly, a correction of 4% was introduced in all standard tests when calculating the value of F.

Although this might have sufficed for the main object of the tests, it was nevertheless considered of interest, as a matter of general information, to conduct another friction test, similar to the first but with the order of the operations reversed; that is, instead of loading the lever system and then gradually raising the water pressure (applying a load to the accumulator) as was done during the first test, the pressure was kept constant while the lever load was gradually increased, until the top of the test piece began moving downward.

It is clear that, in this case, the friction forces must act in a direction opposite to F. This was actually confirmed by the results, since all the points representing this test (triangles in the diagram) were found to be above the zero-friction line; the slope of the mean line, calculated for these points in the same manner as for the first friction test, was 2% steeper than 1 on 1.

The magnitude of the friction forces is different in this case, as compared with the first test, because the leather collar in the upper part of the container, which is shaped in section as an inverted **U**, is capable of developing different resistances depending on whether the legs of the **U** tend to become opened or closed. This means that the friction must depend on whether the test specimen moves upward or downward, being greater in the first case than in the second.

It was a satisfaction to observe that, in general, the friction forces were reasonably low, as compared with the main forces applied to the specimen; thus the friction correction introduced into the calculation of the force F was more in the nature of a refinement, than a substantial factor that may have influenced the conclusions. It is also of some importance to realize that the consistency of the results of the friction tests supplies another proof of the exactness of the calculation of the force F, because the deviations of the points appearing in Fig. 15, from their average line, give a measure of the precision of the results of this calculation. For example, the standard deviation for forty-two points is found to be 41 kg, which is only about 1% of the average value of F. This is an independent confirmation of both the principle and the details of the method applied in calculating this force.

Apart from, and parallel with, the main experiments, a second series of tests were conducted in order to determine the porosity of the tested material. This porosity is required for finding the correction term n_v in Eq. 5.

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[28] "Calcul de l'inclinaison des minarets," by S. Leliavsky, *Report on the Reconstruction of the Mosque Mohammed Aly Pasha*, Ministry of Wakfs, Cairo, 1931–1933, p. 95.

All these tests were made in the laboratory of the Bridges Service of the Egyptian State Railways. The specimens were taken from the test pieces used in the uplift experiments, after their failure. In all the cases, the porosity tests were repeated daily with the same specimen, for several months. According to the standard program this period was intended to be subdivided into four sections, as follows: (*a*) Specimen kept in water; (*b*) specimen kept in the room; (*c*) specimen exposed to the effect of wind and sun; and (*d*) specimen placed in the heater. Each operation was continued until the weight of the sample became constant. The results of two cycles, on seven samples, in series d, e, f, and g, are shown in Table 1.

TABLE 1.—RESULTS OF POROSITY TESTS

Sample and series	Critical pressure (kg per sq cm)	Time from failure to first test (hr)	POROSITY (%) AFTER SUCCESSIVE STAGES OF THE TEST						Total duration of test (days)
			First Cycle			Second Cycle			
			Room temperature	Exposure to sun	Placed in heater	Immersion in water	Room temperature	Placed in heater	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A, d	27	4	8.2	10.7	15.6	1.3	14.1	....	112
B, d	32	2	6.6	8.7	....	1.5	8.2	14.3	105
C, d	8	2	7.9	8.5	....	0.6	8.5	14.5	100
D, d	16	40	9.6	11.3	18.7	....	....	....	99
E, e	12	11	3.1	3.7	7.0	. .	....	....	35
F, f	30	2	8.3	6.1	10.3	....	....	...	114
G, g	15	2	3.5	5.7	12.0	....	....	....	67

In certain cases, it was believed of interest to reimmerse the dried samples. After a period of ten to twenty days in the tank, they absorbed almost the same quantity of water as when subjected to the exceedingly high pressure in the container; there was only 1% to 2% difference in the moisture content corresponding to either case. This result is instructive as it shows that the capacity of absorption is independent of the pressure applied.

The conclusion to be drawn from Table 1 tends to diminish the importance of the porosity on the stress analysis of dam sections—at least in so far as Fillunger's ideas on the subject are concerned. Fillunger believed[7,8] that the uplift factor was equal to the difference between the porosity of pure cement paste (when set) and the average porosity of the composite material (including cement, sand, and either gravel or stone). These two constants were supposed to represent respectively the coefficients n_a and n_v in Eq. 5. This conclusion was derived theoretically, assuming that the surface of fracture was an ideal plane which passed through the structural joint without intersecting any of the particles of either the fine or the coarse aggregates. In the light of more recent interpretation (which appears to be chiefly due to Hoffman[29]), the surface of rupture, being a surface of least resistance, is itself dependent on the structure of pores. In other words, the percentage area of the voids intersected by this irregular surface is greater than the volumetric porosity of the material. This upsets the Fillunger assumptions, because they are based upon the hypothesis of Delesse[30] according to which the superficial and volumetric porosities are the same (as is actually the case for a plane section). (In 1930, Fillunger restated his theory in a paper read before the Second World Power Conference.[31] It is substantially the same theory as in his earlier publications, but instead of the common symbols of Cartesian algebra he introduces the vectorial notation, which makes the discussion rather more general, although this notation is less familiar to the average engineer.)

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[29] "Permeazioni D'Aqua e loro Effeti nei Muri di Ritenuta," by O. Hoffman, Milan, 1928, p. 28.

The moisture content under normal conditions appears to be rather high in the writer's tests (see Table 1), leaving a very small margin for additional absorption. This means that, when samples of the material destined for use in a dam are being tested in order to find their specific gravity, they already contain such a large percentage of water that their weight is almost the same as when fully saturated. In this connection attention is called to the porosity coefficients appearing in Col. 4, Table 1. It will be observed that these are the values that must be included in the second term of the uplift formula (Eq. 5), as representing the factor n_v. They are found to vary from 3.7% to 11.3%, with an average of about 7%.

These values are very low, corresponding to from only 2% to 4% of the total weight of the material. It appears, therefore, that a 7% effective volumetric porosity can be accepted as a sufficiently close and safe approximation for all cases. Of course, in a large structure the porosity would tend to be slightly greater than in a small block made of the same material; but then the efficiency of evaporation would be higher in the second case, so that a certain compensation must occur. Since the effect of this coefficient on the results of the stress analysis is indeed very small, there is every reason for accepting $n_v = 0.07$ as a sufficiently accurate basis for the calculation of stability. On the other hand, where the construction of a new dam is contemplated, the coefficient n_v can easily be determined experimentally before the profile is designed.

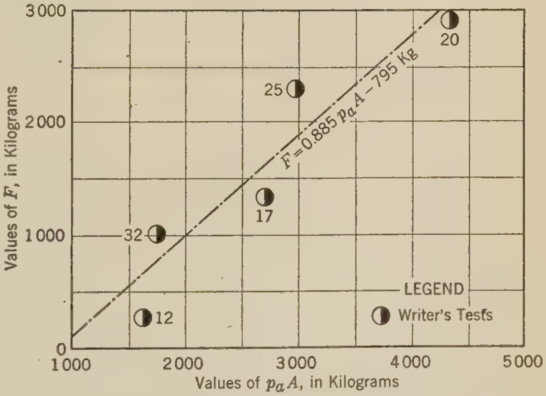

FIG. 16.—SERIES a; CAST DECEMBER 2, 1939, AND TESTED JANUARY 17 TO FEBRUARY 28, 1940 (SEE TABLE 2)

RESULTS

In describing the results of the tests in this section, the writer has followed the chronological order in which the conclusions were revealed. This was found advisable, because the object of and the particular problems to be solved by some of the later experiments originated from the analysis of the earlier tests.

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[30] "Procédé Mécanique pour Déterminer la Composition des Roches," by M. Delesse, *Annales des Mines*, 4 Série, Tome XIII, 1848, pp. 379–388.

[31] *Transactions*, 2d World Power Conference, Vol. 9, Paper No. 421, Berlin, 1930, p. 323.

The results of the first series of tests (referred to as series a) are shown in Fig. 16. The ingredients used in making these test pieces were: Broken Assuan granite and Assuan sand—both taken from the same quarries which were previously used in building and heightening Assuan Dam on the Nile River in Egypt. They were mixed with Portland cement from the Turah Factory, the proportion of cement to sand to stone (by volume of dry material) being 1.0:1.6:1.1. Descriptive data and comparison of results of twelve series of tests are presented in Table 2.

TABLE 2.—Results of Tests on Twelve Series of Specimens

Series (see Figs. 16 to 24)	Number of tests	Method of preparation[a]	Cement (kg per cu m)	Water-cement ratio (by weight)	Type of coarse aggregate	Average age at test (days)	Uplift Coefficient		At Failure		Probable deviation of a point (tons)	Range of F-values (tons)	Ratio, $\frac{\text{Col. 11}}{\text{Col. 12}}$
							n_a	Probable error	Resistance (tons)	Average stress (kg per sq cm)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
a	5	C	530	0.40	Granite	70	0.885	0.098	0.795	5.24	0.215	2.597	0.083
b	5	S	290	0.93	Granite	68	0.916	0.030	0.138	0.88	0.069	2.673	0.026
c	5	C	500	0.50	Granite	101	0.954	0.093	0.458	2.91	0.204	2.630	0.078
d	6	S	280	0.72	Granite	137	0.991	0.040	0.474	2.98	0.115	3.611	0.032
e	9	C	300	0.58	Granite	68	0.871	0.038	0.845	5.37	0.173	4.174	0.041
f	7	C	470	0.40	Gravel	82	0.957	0.062	0.929	5.78	0.278	4.232	0.066
g	8	R	435	0.44	....	115	0.853	0.033	0.326	2.14	0.132	3.477	0.038
h	7	C	390	0.40	Fine gravel	218	0.967	0.080	0.978	6.26	0.344	4.030	0.085
i	9	C[b]	325	0.50	Granite	151	1.005	0.034	0.791	5.01	0.136	4.056	0.034
j	8	C	325	0.40	Granite	299	0.900	0.054	0.947	6.02	0.299	4.049	0.074
k	10	S[c]	330	0.68	Granite	179	0.888	0.016	0.404	2.58	0.078	3.500	0.022
l	16	C	300	0.43	Granite	126	0.858	0.017	1.058	6.64	0.111	4.284	0.026
Weighted mean	9.0	..	359	0.52	....	139	0.913	0.043	0.727	4.61	0.171	3.756	0.046

[a] "C" denotes "concrete"; "S" denotes "masonry"; and "R" denotes "mortar." [b] Concrete cast in two days. [c] Dressed masonry work.

The position of the points shown in Fig. 16, both in respect to each other and relative to the axes, leads to two fairly obvious conclusions: (*a*) A definite relation exists between the values of $p_a A$ used as abscissas and the corresponding values of F represented as ordinates, and (*b*) this relation is almost certainly linear. Although one single series of tests would not have been sufficient as a definite proof for these conclusions, the tests of that one series were, nevertheless, found to be corroborated by the results of all other tests, and as such were not devoid of interest—first, because these conclusions confirmed the general principle of the experiments and proved the efficiency of the testing apparatus; and, second, because they showed that the coefficient n_a was constant. It was also apparent from Fig. 16 that the slope of the line representing the coefficient n_a was very steep.

In finding the slope in question (which was the main problem to be solved by these experiments) it was believed advisable, in order to exclude the personal equation, to adopt the same method for all the series tested; and with this object in view the following simple formula embodying the "least-squares" principle was used:

$$n_a = \frac{N\,\Sigma(x\,y) - \Sigma x\,\Sigma y}{N\,\Sigma x^2 - (\Sigma x)^2} \quad \text{.....................} (13)$$

in which x and y are the coordinates of the points and N is their total number. Also,

$$z = \frac{\Sigma x^2\,\Sigma y - \Sigma(x\,y)\,\Sigma x}{N\,\Sigma x^2 - (\Sigma x)^2} \quad \text{.....................} (14)$$

The values of the constants defining the average line were calculated by means of Eqs. 12 and 13, applied to each of the series tested. For series a (Fig. 16) the coefficient n_a was found to be equal to 0.885 (see Table 2). This is considerably more than is usually assumed in designing dam profiles (excluding exceptional cases, of course), but it tends to approach the limit suggested by Professor Terzaghi. On the other hand, such a high numerical value completely disproves the Fillunger theory, because the volumetric porosity of the materials tested was only about 0.15.

One other point that Fig. 16 seems to demonstrate is the effect (on the results of the tests) of the ratio of the areas occupied, respectively, by the binding material and the particles of the aggregate appearing on the surface of rupture. In fact, when the average line is taken as a basis of comparison, the points with higher ratios move to the left. Had this been a general rule, it might have been interpreted in two ways—the higher percentage of the stone surface may have meant either a larger uplift factor n_a or a smaller resistance z. In both cases, the primary cause would have lain in the different microstructural characteristics of solid material as compared with a surface of contact between two different substances.

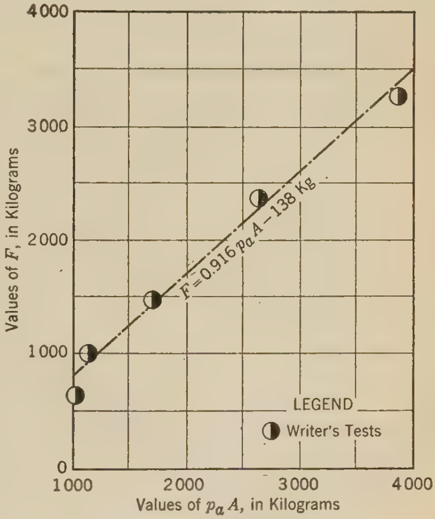

FIG. 17.—SERIES b; CAST JANUARY 17, 1940, AND TESTED MARCH 9 TO APRIL 9, 1940 (SEE TABLE 2)

In order to explore this subject further, two parallel sets of experiments (see series f and g, Table 2) were conducted, with and without coarse aggregate, respectively.

The proportion of cement in the first series of specimens was rather high. It was considered advisable, therefore, to use a relatively small proportion of cement in the second series (1.0:2.7:3.3), the ingredients being the same as in series a. The results are shown in Fig. 17.

The characteristics of the line in Fig. 17 confirm the results of series a; that is, the relation which the data represent is obviously linear and the slope of the line is again very steep. In spite of the difference in the proportion of cement, the factor n_a calculated for series b, the same as for series a, is found to be equal to 0.916, which is only 3% greater than series a (see Table 2).

Apart from that, Fig. 17 is also interesting in that it reveals a case in which the individual points are very close to the calculated line. Should the "probable" deviation (namely, 0.6754 times the standard deviation) be taken as a measure of the dispersion of the points, regardless of its physical significance (which, with such a small number of entries, is naturally very limited), the following values may be of interest:

Series	Probable deviation (tons)
a	0.215
b	0.070

(The term "deviation" is used in preference to "error," for in this case the cause of the differences between the observed and theoretical values lies in the irregularities in the texture of the material tested, and not in the inaccuracies of the measurements.)

For other series, the calculated standard deviation tended to approach the conditions shown in Fig. 16, and therefore Fig. 17 must be considered exceptional. No physical reason is given to explain the exception, the cause being most probably a matter of chance.

In series c, Fig. 18, the conditions of the test were the same as series a and b except that the aggregate proportion was 1.0:1.1:2.2. The results are similar to those demonstrated by Figs. 16 and 17 except that these data might also be interpreted as representing a curve, AB. This was the only case in any series in which such a definite curve could have been drawn through the points obtained; but, even here, the curve was absurd because the slope of end B would have been much steeper than 1 on 1 (which was obviously impossible since n_a could not have been greater than 100%).

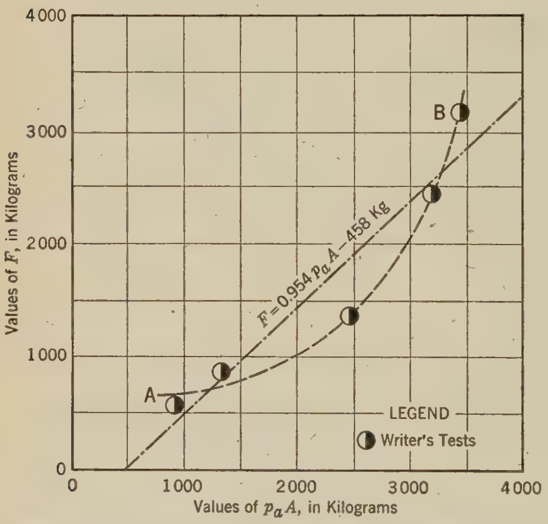

FIG. 18.—SERIES c; CAST MARCH 4, 1940, AND TESTED MAY 22 TO JULY 1, 1940 (SEE TABLE 2)

It follows that in series c, as in all other series, the variation was linear and $n\ a = 0.954$ was computed in the same manner as in the earlier series (see Table 2).

In order to avoid conditions such as those that produced Fig. 18, and also to study the effect of extending the data into negative values of F, the number of specimens in all subsequent series was increased. For instance, series d, Fig. 19, comprised as many as twelve experiments. The larger number of plotted points in Fig. 19 offers a more strict proof of the main conclusions derived from the earlier tests—namely, that there is a systematic relationship between the product $p_a A$ and F, that the relationship is linear, and that the slope of the calculated line is rather steep.

It must be observed, however, that, although this general result was supported by the twelve experiments represented in Fig. 19, the calculation of the average line was based on six records only, because the "weight" of the other six points (shown by dotted circles in Fig. 19) was believed to be less; in other words, the data for these experiments were considered to be less certain than those for the other experiments. In regard to the two points in Fig. 19, at $p_a A = 0$ (mechanical tensile tests with no hydraulic pressure, conducted with identical specimens, and using the same machine), it was found that the time factor affected the conditions of failure of the material to a far greater extent than many engineers are inclined to accept. The importance of this factor was discovered almost accidentally during the test run for series d. During one period the machine was left with the specimen still in position, with a constant load. After a lapse of about 15 min the specimen failed suddenly. This conclusion concerning the time factor refers chiefly to short periods. If the duration of the test exceeds two or three days, the effect of the rate of loading appears to vanish, or is almost imperceptible.

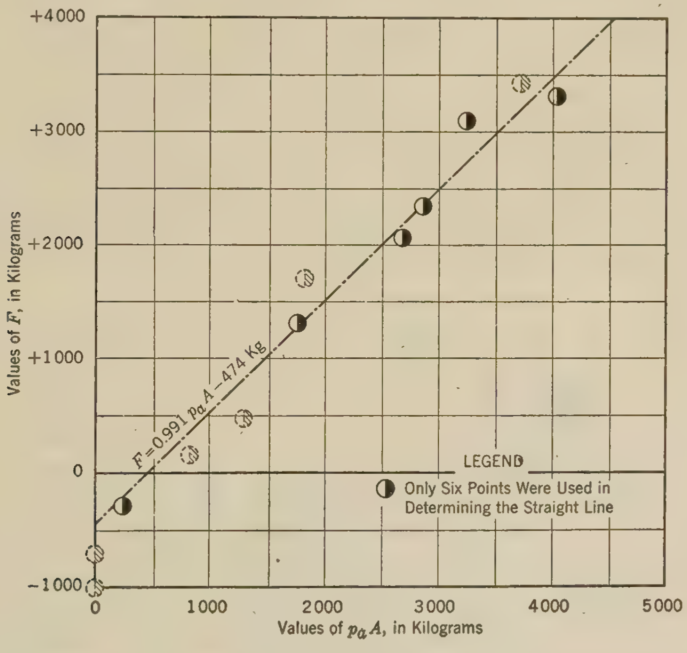

FIG. 19.—SERIES d; CAST APRIL 14 AND 15, 1940, AND TESTED JULY 9 TO SEPTEMBER 23, 1940 (SEE TABLE 2)

The next test under similar conditions was conducted more slowly and consequently the breaking tensile limit was found to be lower. Thereafter it was realized that, in order to obtain consistent results, the duration of a purely mechanical tension test must be the same as that of a standard uplift experiment—about three days. This rule was adopted for all subsequent series and the results were found to be quite reliable.

Four other tests in the same series were also considered as being less precise than the remainder, because the specimens broke at night. (Subsequently, orders were issued to remove two loads from the accumulator, before closing the room for the night, and to replace them again in the morning. Failures at night were thus avoided.) The four corresponding points were calculated from the last values of p_i recorded in the log book during the day, and were shown by dotted circles in the diagram; but, of course, they were not included in the calculation of the line. Nevertheless, they fell close to the latter.

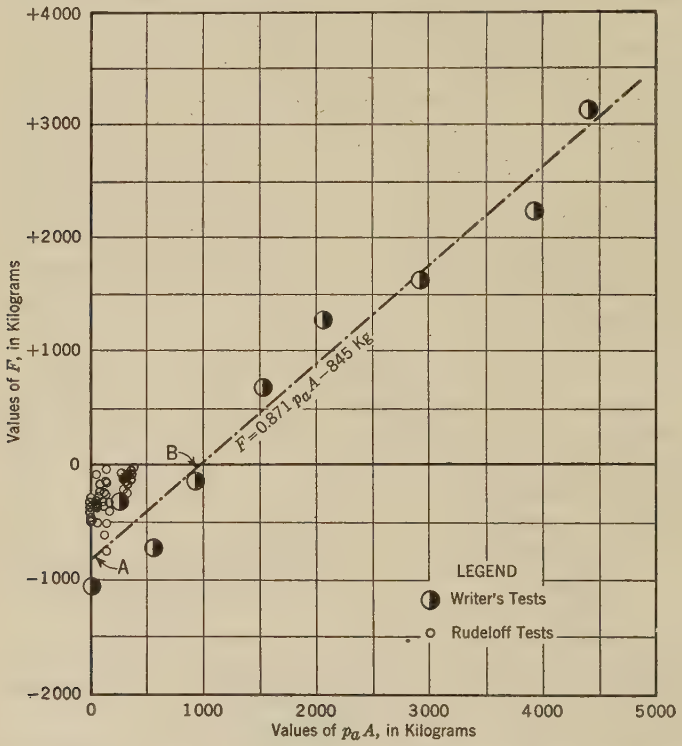

FIG. 20.—SERIES e; CAST AUGUST 7 AND 10, 1940, AND TESTED SEPTEMBER 28 AND NOVEMBER 4, 1940 (SEE TABLE 2)

At this stage of the investigation, in comparing the results determined, the most striking point appeared to lie in the fact that the coefficients n_a calculated for various groups were found to be almost the same. It was decided therefore to alter the ingredients in the new series in such a way as to ascertain whether the coefficient in question was affected by the nature (if not by the quantities) of the materials used in making the specimens. With this object in view a coarser sand (obtained by washing the sand that had been used before) was used in making the fifth series.

The results of these tests are illustrated in Fig. 20. Although the coefficient n_a is found to be still high (0.871), it is nevertheless slightly lower than the average calculated for the first four groups, which amounts to 0.937 (see Table 2). Thus, by removing the finest particles of the sand it appeared that the uplift was reduced by about 7%, as compared with the average for other series.

To illustrate the importance of the range of conditions under which uplift experiments are to be made, it was decided to plot, on any one of the diagrams representing the writer's experiments, the tests of Rudeloff and Panzerbieter.[13] While Fig. 20 was selected for this purpose, any of the other series might have been used instead with equivalent results.

The coordinates of the small circles, which are used to represent the Rudeloff tests, are computed as follows: The values of the mechanical forces F (which, in these experiments, are all negative) are read off directly from the tables appearing in the published report. The values of $p_a A$ are calculated in accordance with the particular arrangement adopted in each series of these experiments. For the arrangement shown in Fig. 3(*a*), the formula used is

$$p_a = p_i (1 - \Delta) \tag{15a}$$

and, for the arrangement in Fig. 3(*b*), the formula is

$$p_a = p_o \Delta \tag{15b}$$

Substituting in Eqs. 15 the breaking values of p_o and p_i published by Rudeloff, the required values of p_a are found in a form that represents his experiments in the writer's graphs. It is obvious from Fig. 20 that the forty-four points so obtained could not have been used for developing empirical formulas or for finding physical constants, because the range of variation of the coordinates was too narrow. The small circles are arranged in a meaningless group, which cannot be defined as a curve or a straight line. In other words, the effect of accidental circumstances that caused the dispersion of points was too large as compared with the range of variation of the main factors; and the entire set of Rudeloff's records was equivalent, in the weight of the information it conveyed, to only one point in Fig. 20.

In the investigation reported herein the conclusions are based on the difference between the individual tests, and not on their average value; and, therefore, in spite of all precautions that had been taken in designing the writer's apparatus and planning his experiments, greatest care and utmost diligence had to be exercised subsequently in making the specimens, and in handling them, in order to reduce the dispersion of the points. Only the most competent, experienced men, fully realizing the importance of regularity in their work, should be employed in such experiments, to insure consistent results. In fact, it was observed that, whenever changes in laboratory assistants occurred, the dispersion of the points immediately became greater, and two or three series had to be discarded for this reason. The maximum ratio of probable deviation to range of force that can be allowed reasonably in such tests is 1:10. Beyond this ratio the results should be considered as untrustworthy.

Another point, calling for comment in regard to Rudeloff's tests, as they are represented in Fig. 20, is that the ratio of the outer to the inner diameters of the test pieces used in this investigation (the test pieces in question were made hollow as in the writer's tests) was 6.43. When this value is used in Eqs. 15 it will be found that, for Fig. 3(*a*), $p_a = 0.25\, p_i$; and, for Fig. 3(*b*), $p_a = 0.75\, p_o$.

Thus, with equal pressures, the effect in the second case (Fig. 3(*b*)) was three times as much as in the first arrangement (Fig. 3(*a*)), in spite of which the points (which included both arrangements) fell close to each other in Fig. 20. In other words, in order to equalize the conditions that caused failure, the inner pressure had to be raised to an intensity three times as high as that of the outer pressure. In this respect the agreement between test and formula is found to be fully satisfactory.

This goes a long way to show that the formula assesses correctly the relative effect of the two pressures p_o and p_i; and the principle upon which it is based, therefore, is not only correct theoretically, but is also consistent with actual facts.

Another illustration of the importance of a wide range of conditions is afforded by the experiments of Woodard,[12] and by the experiment conducted at Bull Run Dam. The physical interpretation of these experiments was intended to be based on the difference between the conditions that cause the failure of the specimen in the testing machine of the common type (that is, without uplift pressure) and effect of uplift pressure acting alone (without any additional mechanically developed stress). In Fig. 20, these two cases correspond to points A and B, in which the average line intersects, respectively, the vertical and horizontal axes. A glance at Fig. 20 will show that the section of the line defined by these two points is relatively very short. As they are too close to each other, minor errors in determining the position of these two points experimentally must largely affect the slope of the line drawn through them, the calculated value of n_a being uncertain and possibly misleading. Had the same precautions been taken in preparing the test pieces and in conducting the test proper as in the writer's experiments, the ratio of the probable deviation to the range of forces would still have been high. For instance, under conditions similar to those of Fig. 20, this ratio would have been 1:5 (as compared with 1:25 in the writer's test). There is little wonder, therefore, that the results of the earlier uplift tests did not supply the solution to the problem. More precision might have been attained by repeating the test several hundred times, and then taking the averages of the results so obtained; but even so, with two points only, it would have been impossible to obtain a clear picture of the true relation between the variables, and many factors affecting the experiment (such as the predominant effect of time in the mechanical-tension test) might have remained undetected, thus aggravating the inaccuracy of the final calculation.

Fig. 21 includes two series of tests, referred to as groups f and g, respectively. In representing them on the same chart the object was to facilitate comparison because, in this case, the problem was concerned with the difference (if any) in the results obtained with two mixtures. To ascertain the effect of the large particles, these two series were made identical in regard to the composition, except as to one point—the presence in one mixture, and the complete absence in the other, of the coarser aggregate (gravel).

The ingredients were proportioned in such a way as to provide for nearly the same quantity of cement per cubic meter of finished material in both cases.

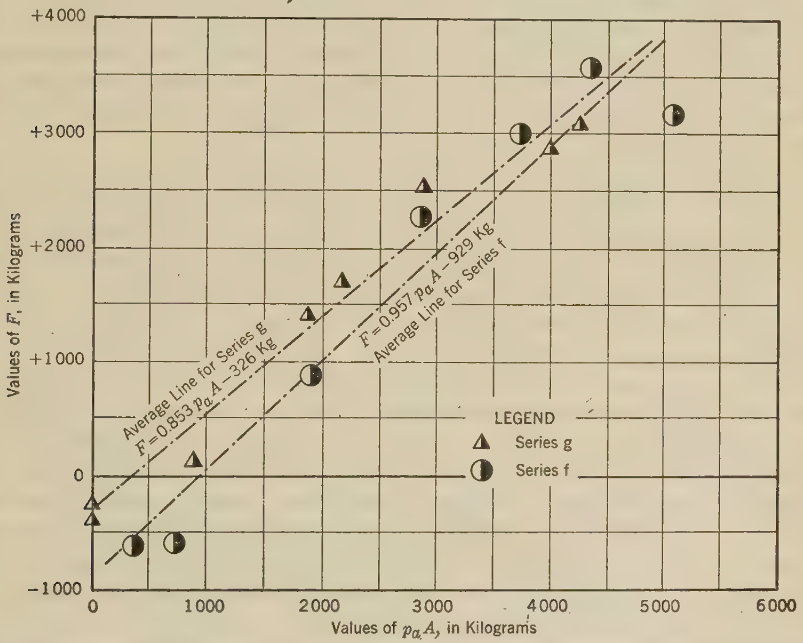

Fig. 21.—Series f (Cast October 13, 1940, and Tested December 17, 1940, to January 20, 1941) and Series g (Cast October 16, 1940, and Tested January 26 to February 18, 1941) (see Table 2)

In interpreting Fig. 21 attention is called to what was stated in connection with the first group (Fig. 16), on the possible action of the coarser aggregate, which was supposed to be affecting either the uplift factor (in which case the slope of the two lines in Fig. 21 should have been different) or the resistance of the material against tension. In the latter case the lines would have been parallel, but at some distance from one another.

The results of the tests, as represented in Fig. 21, appear to show that these two effects exist simultaneously, but that they are combined in such a way that they often escape attention because, in a sense, they balance each other. The addition of the aggregate appears to increase the factor of uplift (as one might naturally have expected) but, at the same time, the resistance of the material against tension is also increased, which causes the two groups of the experimental points to be superimposed upon each other.

On the other hand, the differences in the uplift coefficients are found to be so small as to be largely influenced by accidental causes, so that the true conclusion to be derived from Fig. 21 is that the effect of the aggregate is rather of secondary importance.

Another seven specimens were later prepared from fine gravel instead of the coarse gravel that had been used in mixing group f. They were referred to as series h. As the results of these tests were very close to the average conditions, they do not call for any particular remark.

The next series tested (series i) was intended to reveal the effect (if any) of the interruptions in the casting process. Following a special program, nine test pieces were prepared, a lapse of time of twenty-four hours being allowed between the casting of the lower parts of these specimens, and the completion of their top portions. The surface of interruption was located about halfway between the lower and the upper linings fixed to the specimens (middle part of the critical section).

Of all the tests this series gave the highest coefficient. In fact, the value of n_a calculated for this group of specimens, by means of the same formula that had been used before, was found to exceed the theoretical limit—unity. The excess (0.005) was small, however, as compared with the probable error of the coefficient (0.034), and it is interpreted as being the result of the fluctuations of sampling and unavoidable errors. This excess, therefore, is not consistent with the general theory on which the experiments are based.

On the other hand, the fact that the effective area for this particular series was found to be greater than for all other specimens may tend to show that interruptions in the casting of concrete create critical conditions for the uplift pressure, as they do in regard to many other structural characteristics of the material. This conclusion, although based upon relatively small numerical differences, is supported nevertheless by the fact that the surfaces of rupture in these specimens tended to lie closely to the surfaces of interruption. In other groups the surfaces of rupture were irregularly distributed over the height of the critical portion of the broken test pieces.

The eight specimens belonging to series j were intended to supply more information about the effect of the size of the sand particles. With this object in view a sand coarser than that in series e was selected in making them. The reduction in the value of the uplift coefficient that had been observed in series e, and had apparently been due to the removal of the smaller grains in that series, did not seem to have been increased or maintained when the medium particles were also removed and large particles alone were present. The value of such conclusions is very much reduced, however, by the fact that the numerical differences, upon which they are based, are of the same order of magnitude as the probable error.

All the groups thus far described were prepared according to one of the two standard methods ("concrete" or "masonry") discussed under the heading, "Procedure and Significance of Tests" (see also Table 2, Col. 2). On the other hand, the system adopted in making the test pieces of group k differed substantially from all that had been done before. An objective attempt was made, in this case, to duplicate the methods commonly used in building masonry dams.

The critical sections of these test pieces were built in alternate courses, consisting of three to four pieces of granite laid on approximately horizontal beds. Consequently, the size of the individual stones was larger than in all earlier series, and each stone was dressed roughly so as to reduce the thickness of the joints and to minimize the quantity of mortar per unit volume of the finished material. To make the stones fit more accurately, the critical sections of the test pieces were built before placing the cast-iron collars in position.

Although the value of the coefficient of uplift deduced from these experiments was again very near to the average ($n_a = 0.888$), the points representing the individual tests were closer to the average line than in the earlier series (see Fig. 22).

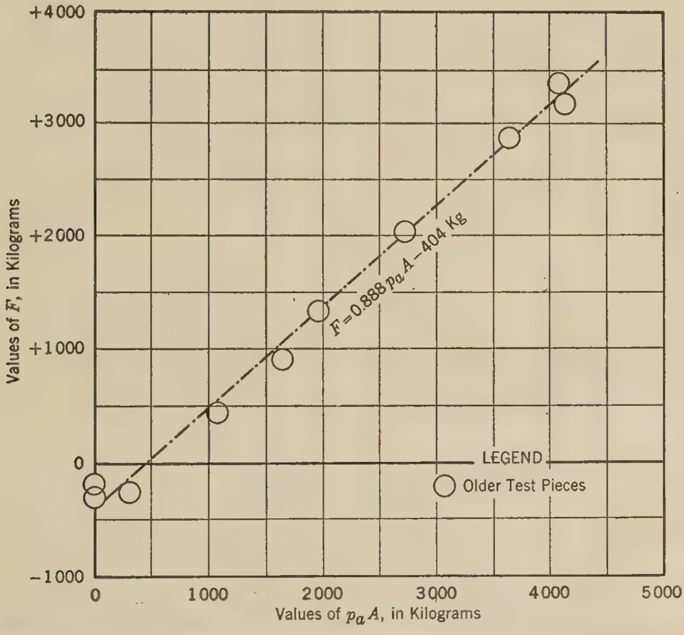

FIG. 22.—SERIES k; CAST FEBRUARY 2 AND 3, 1942, AND TESTED JULY 15 TO AUGUST 25, 1942 (SEE TABLE 2)

Consequently, with this particular method of construction the results of the experiments were more regular and, being less affected by accidental circumstances, conformed more strictly to the theoretical conditions of an ideal case. The reason for the better results obtained with this series might be twofold:

(*a*) The horizontal beds may have presented more uniform conditions for the filtering of water and for the development of the uplift pressure; and

(*b*) The dressing of the stones (such as it was) and the greater regularity in the joints may have resulted in more uniform tensile resistances.

In concluding the program of experiments described in this paper, a set of tests was run with a greater number of specimens. The range of ages of these specimens was increased in such a way as to accentuate the effect of the time factor, should such effect be actually present. Accordingly, a series of sixteen test pieces was cast from August 8 to August 10, 1942; four of them were tested from September 28 to October 28, 1942; and the remaining twelve were tested from December 15, 1942, to February 12, 1943.

As demonstrated by Fig. 23, no difference whatever can be detected between the earlier and later experiments, and the entire series is fully satisfactory in so far as regularity of results is concerned. The coefficient n_a is equal to 0.858 and the probable error of this factor is 0.024.

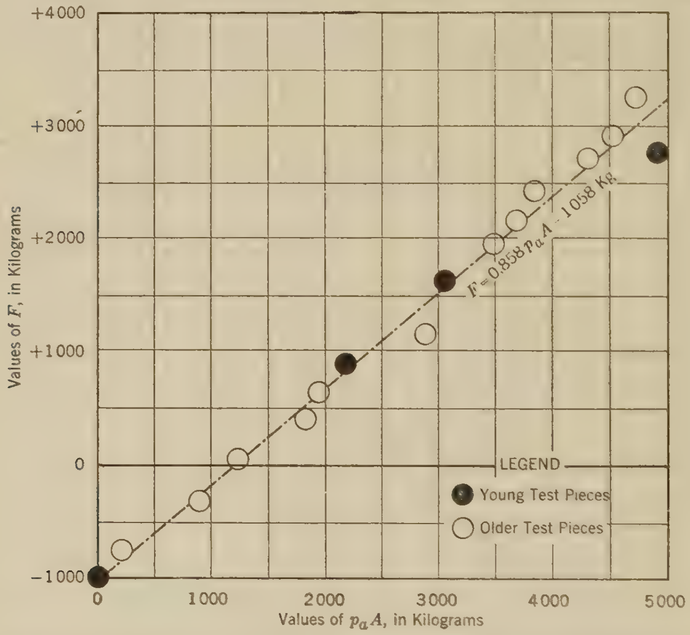

Fig. 23.—Series 1; Cast August 8 and 10, 1942, and Tested September 28, 1942, to February 12, 1943

In spite of a relatively smaller cement percentage, the tensile breaking limit for this series was found to be greater than for all other groups. This was most probably due to the more intensive ramming and to a lower water-cement ratio. Also, the workmen had acquired a greater ability in preparing the test pieces, so that a generally better class of material was obtained.

In addition to the data in Table 2 that have already been discussed, this table also contains information such as the water-cement ratios, which are calculated in the usual way (by weight).

In regard to the values in Col. 11, Table 2 (probable deviations of a point), and Col. 8, Table 2 (probable errors of the coefficient n_a), it should be realized that these terms are used conventionally. Little physical significance can be attributed to statistical parameters based upon so small a number of points as that which is contained in each series taken individually (with the possible exception of series 1 which comprises sixteen tests). These values must be interpreted as representing results of certain well-known arithmetical operations. As such they are of some interest as a means for comparing the dispersion of the points in the different series, provided the word "probable" is not taken too literally.

Another point to be mentioned in connection with Table 2 concerns the tensile resistances of the material. The average breaking intensity of the tensile unit stress for all the series tested was found to be about 4.6 kg per sq cm, whereas for the experiments in which only mechanical tension was used it was about 3 kg per sq cm. Should individual series be considered, the two values will be found to be very close. For instance, in series 1 the tensile resistance calculated from the average line was 6.6 kg per sq cm, whereas under mechanical tension alone it was found to be 6.5 kg per sq cm.

The fact that the resistance of the material determined under mechanical tension was actually similar to, or less than, that which was indicated by the uplift tests explained much concerning the physical cause of the failure of the test pieces. At the instant when fracture occurred in the uplift tests, the material was still found capable of withstanding considerable tension (at least as much as when tested under mechanical tension only). This is evidence that the specimen had not yet cracked. At that instant the uplift force had already reached its full value, and consequently it was concluded that the channels through which water penetrated into the material were natural "pores"—not failure cracks.

The point may be explained further by reference to the special "friction tests" presented in Fig. 15. In comparing this diagram with the several curves of uplift tests, it will be observed that in Fig. 15 the average line passes through the origin of coördinates, whereas in all other cases it intersects the axes at some distance from the origin. This is due to the fact that the abscissas of the points of the standard uplift tests are always greater than those of the corresponding points of the special "friction tests." In other words, it was always found necessary to raise the pressure to a higher intensity in the uplift tests than with the same force in the "friction tests." The obvious reason for this condition was that in "friction tests" the specimen was filed crosswise beforehand, so that during the experiment its two parts were held together by the application of the force F only. In uplift tests, on the other hand, the two parts formed one solid block, and, therefore, in addition to the force F, there was also the resistance of the material to be overcome by the uplift pressure. Had a crack existed in the "uplift test" the results must have been the same as in the "friction test."

Thus, although the principle of these experiments was compatible with both the old and the new schools of thought, the numerical results tended to show that the modern group of theories supplied the true explanation of the phenomenon (in so far, at least, as the tests themselves were concerned).

Elsewhere, in this paper, the writer has shown that the differences in the values of the uplift factors obtained for various series were surprisingly small. The possibility was also explored that the entire body of tests might not be analyzed as a single composite group of records. This approach would have appeared unorthodox if it had been advanced in the earlier part of the investigation; but it suggested itself later, as a natural conclusion derived from the numerical results of the tests. (The differences between the individual values of n_a calculated for each series are of the same order of magnitude as the corresponding "probable" errors. This means that these differences may be attributed to accidental circumstances.)

It should be realized, however, that the value of z in the different series (the ultimate resistance of the specimens against tension) is not, and cannot be, the same (*a*) for all the various proportions of cement used, (*b*) for the differences in the periods of curing, and (*c*) for differences in the water-cement ratios. Consequently, had the tests relating to different series been superposed upon each other as they occurred, the differences in z would have exaggerated the dispersion of the points and obscured the issue. Therefore, they were removed prior to combining the tests in a single group, and values of $F + z$ were plotted as ordinates instead of F.

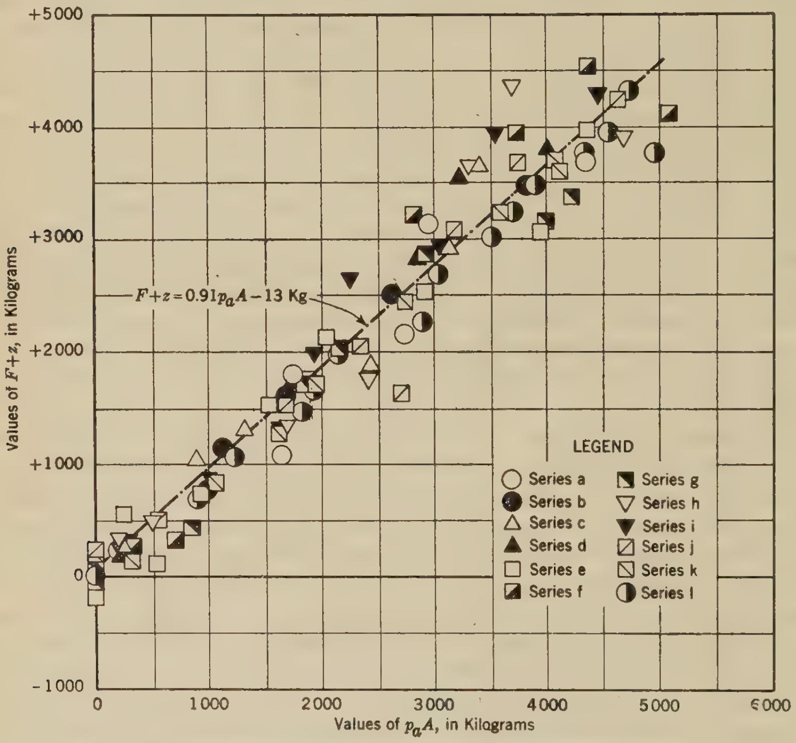

FIG. 24.—COMBINATION OF NINETY-FIVE EXPERIMENTS

The diagram so obtained (Fig. 24) may also be conceived as being derived from Figs. 16 to 23, by using different origins of the vertical scale for different groups in such a way that all the calculated lines intersect at the origin.

The parameters of the new average line calculated for all the ninety-five points taken together, by means of the same formulas as before (Eqs. 13 and

14), are: $z = 0.013$ ton and $n_a = 0.91$. The constant z is so small that, to all intents and purposes, the average line may be assumed to pass through the new origin of coordinates (see Fig. 24). This tends to prove that the principle upon which the adjustment of the ordinates is based is generally reasonable.

The standard deviation for the new combined diagram is 0.305 ton, which is almost the same as the average of the values computed for the individual series (see the line of weighted means in Table 2).

On the other hand, the advantage of the composite diagram, as compared with Figs. 16 to 23, is that, with almost the same standard deviation, it contains a much larger number of observations. From the viewpoint of the statistician a record comprising ninety-five entries may be regarded as very meager, but it represents, nevertheless, a substantial improvement on the individual series which contain only five observations. Thus, it is possible to apply some statistical methods of investigation to the composite group, which can never be applied to the smaller individual groups, considered separately.

The correlation method was adopted for the statistical analysis of the composite data, with the following results:

Description	Correlation	Regression
Coefficient.	$r = 0.974$	$b = 0.91$
Probable error.	$\Delta r = 0.0035$	$\Delta b = 0.014$
Approximate ratios.	$\frac{\Delta r}{r} = 1:300$	$\frac{\Delta b}{b} = 1:70$

These coefficients appear to be very satisfactory. With the number of experiments available, therefore, the curve in Fig. 24 can be accepted as final, and the value $n_a = 0.91$ can be considered as a fairly reliable average, the corresponding probable error being only about 1.5%. The same values of b are found by the least-squares method. In a preliminary investigation, with seven groups only, the same computation gave $n_a = 0.92$. These seven groups have also been combined without excluding z (that is, using the values of F and $p_a A$ as in the individual diagrams) and almost the same results were obtained.

Fig. 24 has also served as a general basis for detecting the presence of systematic errors, if any. Two main sources of such errors may be anticipated: (*a*) The effect of time; and (*b*) the effect of the difference $p_o - p_i$.

(*a*) *Time.*—Great care was exercised in arranging the test program so as to avoid any apparent or concealed association between the age of the specimens and the main mechanical characteristics of every individual test. Additional evidence was gathered confirming the fact that the time factor had no influence on the test results. The first step in the check analysis was to compute the deviations from the average line, for every test, by the formula:

$$\Delta y = y - n_a x + z \text{ . . . . }(16)$$

in which x and y are the coordinates of the point; and n_a and z are the constants of the average line.

The correlation and regression coefficients between these ninety-five deviations and the age of each of the ninety-five specimens tested were then computed. The result ($r = 0.059$ and $b = 0.00025$ ton per day) is obviously negative. The coefficients are so small as to warrant the conclusion that the tests were not influenced by the time factor.

(*b*) *Difference* $p_o - p_i$.—The effect of the difference $p_o - p_i$ is to create compressive stresses in the material at right angles to the geometrical axis of the specimen and to the main tensile stress that causes its failure. Since the critical breaking value of the tensile stress in question may be influenced by the compressive stress at right angles to it, an effort was made to ascertain whether the results of the experiments were actually affected by the magnitude of the difference $p_o - p_i$.

Messrs. Fillunger and Terzaghi have demonstrated beyond any reasonable doubt that the effect of an interstitial hydraulic pressure upon the resistance of a porous solid is negligible, so long as this pressure is equally distributed over the walls of the pores. Since the existing evidence confirming this statement is conclusive, it follows, in so far as the present tests are concerned, that the ultimate resistance of the test piece is not affected, in all the cases in which $p_o = p_i$. However, the cases in which p_i is smaller than p_o remain to be investigated. When this happens the ring-shaped part of the specimen is subjected to stresses similar to those in a pipe of a boiler. Since these stresses are compressive, z could be smaller for higher values of $p_o - p_i$.

The solution of this problem had been attempted previously by Rudeloff who undertook a special set of tests in which an India rubber envelope was placed around the specimen, thereby preventing water from penetrating into the pores. Since in his experiments the pressure was applied on only one face of the wall of the test piece (see Fig. 3) the effect of the ring stresses, in the sense referred to herein, could have been more pronounced than in the tests reported by the writer. In any case, the ring stresses were present and were the same as in the main tests, but there was no water in the pores and, consequently, no interstitial pressure. Hence, the entire effect, had it been found to exist, would have been attributable to the ring stresses alone.

Such special experiments were conducted by Rudeloff for every type of material and testing device used in these experiments; but the present concern is with the apparatus shown in Fig. 3(*b*) because it is mechanically similar to the device used by the writer. Similar to the writer's apparatus, the main stress causing failure in the Rudeloff apparatus is tension, but the secondary ring stress is compression.

The average results obtained by Rudeloff are represented graphically in Fig. 25. The average line is calculated by the least-squares method; its slope (regression coefficient) is found to be 0.034 (or 0.002 ton per atmosphere) which is too small to be detected. Clearly, the effect of compressive ring stresses on the ultimate resistance against tension is negligible.

In the writer's experiments, had the effect of the difference $p_o - p_i$ on z been present, it would have resulted in points with larger values of $p_o - p_i$ tending to move upward, the average line being taken as the basis for comparison. Then, the deviations Δy computed by Eq. 16 must have been related to, and affected by, the differences $p_o - p_i$. This possibility was explored analytically, the results being: Correlation coefficient $r = 0.068$; and regression coefficient $b = 0.0016$. (In order to represent the regression coefficient as an absolute number, the deviations Δy were divided by the sectional areas of the specimens.)

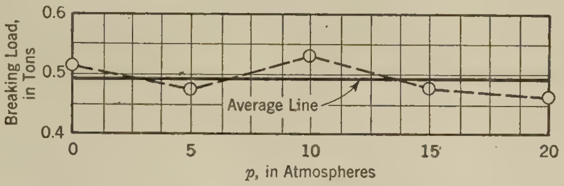

FIG. 25.—RESULTS OF RUDELOFF'S EXPERIMENTS (NO INTERSTITIAL PRESSURE)

Rudeloff's results are thus confirmed. The correlation and regression coefficients are found to be so low that the existence of an interdependence between $p_o - p_i$ and the resistance of the material z is definitely disproved. Although the physical interpretation of this conclusion does not directly concern the subject of the present inquiry, it may be suggested as a most plausible explanation that in Mohr's graphical criterion the lines are curved, as assumed by certain authors, and not straight. This would substantially reduce the effect of the transverse compressive stresses on the resistance against the main tension stresses, particularly in the negative zone, close to the vertical axis of Mohr's diagram.

To complete the description of the writer's tests the following observations should be recorded:

(*a*) The deviations of the ninety-five points from the average line agreed in a general manner with the bell-shaped Gaussian frequency curve.

(*b*) Darcy's coefficient calculated for the various test pieces varied within rather wide limits and did not indicate any correlation with the uplift factor or with any other physical constant. An approximate value of this coefficient can be obtained from the equation:

$$K = \frac{\log_e r}{2\pi H} \frac{\int_0^T Q\,dt}{\int_0^T (p_o - p_i)\,dt} \qquad (17)$$

in which $\int_0^T Q\,dt$ is the total quantity of water percolating through the specimen during the test and $\int_0^T (p_o - p_i)\,dt$ is found from a diagram in which p_o and p_i are plotted against time (see shaded area G in Figs. 13 and 14).

(*c*) In most cases the failure occurred abruptly, without any warning, but certain specimens suddenly became very leaky an appreciable time before failure occurred. The state of the material during this period (which varied from 2 min to 30 min) could be described as "agony." Although capable of withstanding a large amount of tension, the walls of the specimen allowed water

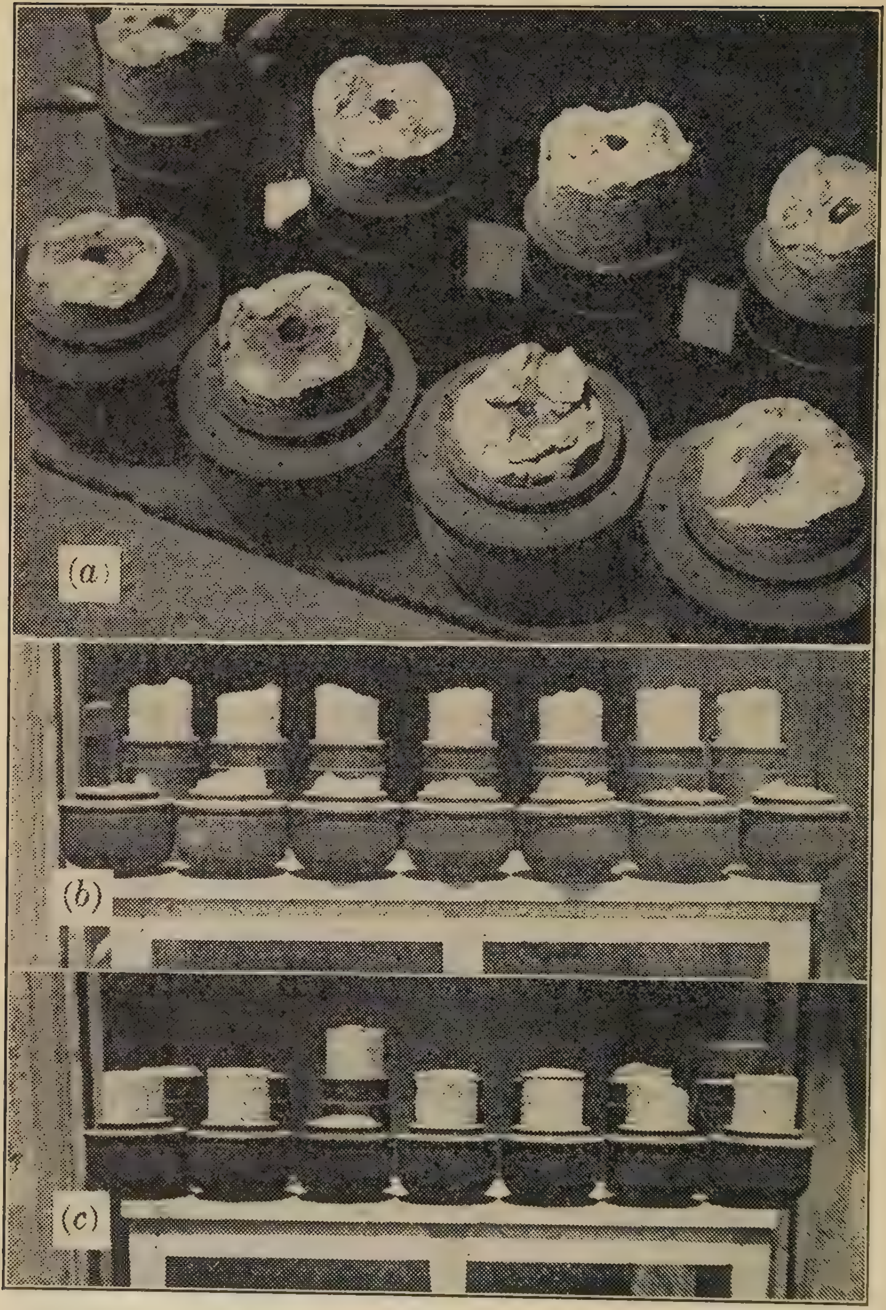

Fig. 26.—Views of Test Specimens After Failure

to pass freely through them, and the deformations became sometimes excessive. A somewhat similar phenomenon had been observed earlier by Kelen.[16]

Some views of the test pieces after failure are reproduced in Fig. 26.

CONCLUSIONS

Although the investigation began with the analysis of individual diagrams, some of which contained five points only, it was later found possible to construct a combined chart having ninety-five points to support the empirical relations which it represented. The conclusions thus derived may be summarized as follows:

(1) Uplift was shown to exist, sufficient to cause failure, by tension, of perfectly solid and sound material. This was true despite the fact that, at the moment this failure occurred, the block was subjected to a mechanically developed compression of variable intensity which tended to oppose the effect of uplift. Sometimes this pressure intensity was as much as 18 kg per sq cm, which is equivalent to a height of superimposed masonry of about eighty meters.

(2) It may be even more important to know that the uplift force was proved to be a measurable quantity, subject to certain laws and capable of analysis by laboratory experimentation—perhaps even more directly measurable than some of the forces that are usually believed to be known, such as the impact effect. This conclusion deserves emphasis since there are many who draw pessimistic comparisons between the alleged "uncertainty" of the uplift effect in dams, as opposed to the precision of the stress analysis in steel framework.

(3) The average value of the "effective superficial porosity" n_a, which was the main unknown to be determined by these experiments, was found to be 0.91, with a probable error equal to 0.014.

(4) Within narrow limits (about three times the standard error) this coefficient appears to be dependent upon certain physical characteristics of the ingredients used in making the material. Such influences must be considered as secondary, however, because the value of the coefficient changes only very slightly, and the causes of such very small changes might be accidental. For practical purposes the coefficient n_a can be assumed to be almost constant.

(5) In general, n_a is much greater than the porosity of the material tested (regardless of the assumption under which the latter may be determined), which completely disproves the ideas of Fillunger, and strongly supports the theory of Terzaghi. According to the latter theory, large values of the coefficient n_a are explained by the fact that the microstructure of a porous solid (see Fig. 27) is an agglomeration of a large number of small blocks, with points of contact so minute as to belong to a still higher order of minuteness. In three series of observations Terzaghi determined values of n_a, respectively, as 0.97, 0.97, and 0.998. The surface of rupture (line AB, Fig. 27) is an irregular surface of "least resistance" which passes almost entirely through the micro-

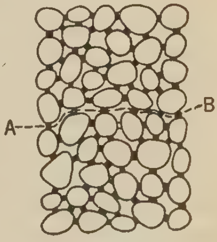

FIG. 27

voids. This seems to supply a possible explanation of the very large difference between n_a and the porosity, observed in the writer's tests.

(6) Although the logic underlying the writer's experiments is compatible with any reasonably evolved theory concerning the characteristics of the uplift pressure, the results derived contain evidence that favors the modern school of thought on this subject.

(7) The factor n_a is found to bear no relation to Darcy's coefficient of filtering, which means that, other things being equal, leakage and sweating cannot serve as evidence that uplift forces are present. It follows that the division of dams into "leaky" and "dry" categories is illusory in so far as uplift is concerned.

(8) On the other hand, if a dam which is known to have been "dry" should suddenly show signs of becoming "leaky," this might be a very strong warning that an "agony period" is impending. Leakage must be prevented or reduced as much as possible in any case, because it may be the cause of the gradual deterioration of the material.

(9) The coefficient n_a is independent of the stress to which the material is subjected (linear relationship between $p_a A$ and F). It is also independent of several other factors which, apparently, might have affected it, such as the cement and water percentages, the temperature, the rate of rise of the pressure, and the consecutive order of loadings and pressures applied.

(10) The increase in the weight of material due to the penetration of water into the pores is less than might have been expected because, in the natural state of the material, the pores are already partly filled with water. The water that may be added, over and above the normal moisture content, represents an average of about 7% of the volume of the materials tested. Therefore, this is the value which, according to the modern views on the subject, must be subtracted from n_a in order to obtain the final, net value of the uplift factor.

(11) It follows that the uplift factor, by Eq. 5, is $0.91 - 0.07 = 0.84$. The writer recommends 85% as a fairly accurate average round figure.

Suggestions for Further Research

The experiments described in this paper should serve as a starting point for further studies. The writer's interpretation of the tests is based upon the conditions that cause the rupture of the specimen, which indeed are the critical conditions to be taken into account in evaluating the safety of a project. The deformation of the test piece could be recorded throughout and, in particular, during the final period before rupture. Such records might reveal more truth about the mechanics of failure and help in explaining the circumstances that cause the "agony" conditions.

To study deformations would require certain alterations in the mechanical details of the apparatus. Had time and money been available, it was the writer's intention to proceed with these additional tests, but under the conditions extant during World War II this was impossible, and some appreciable time will elapse before such results can be obtained and analyzed.

ACKNOWLEDGMENTS

The officials of the Egyptian Government have encouraged the experiments described in this paper and have helped in the investigation. In particular the writer is indebted to H. E. Hurst, Director General of the Physical Department, Ministry of Public Works, for valuable assistance and advice, and to the past and present staff in the writer's office, for help with the experiments and computations.

He wishes also to record his admiration for the excellent work done by the Cairo Government Workshops in making the testing machine, according to his drawings, and the expert assistance of the engineers of the Bridges Service, Egyptian State Railways, who helped in preparing the test specimens.

DAMS ON POROUS MEDIA

BY MOHAMED AHMED SELIM,[32] ASSOC. M. ASCE

SYNOPSIS

When a dam or weir is constructed on a porous foundation, some of the water will percolate underneath it toward the downstream side. The purpose of this paper is to present experimental data on the uplift forces, acting on a dam built on porous media, and on the accompanying loss of head. The electric-analogy method of analysis was chosen because of its simplicity, economy, and reliability. Different conditions of substrata and various combinations of simple horizontal floors and cutoff walls are considered.

INTRODUCTION

Two empirical methods have been used in designing dams on porous foundations—the first that proposed by W. G. Bligh[33] and the second that proposed by E. W. Lane,[18] M. ASCE.

The Bligh Theory.—Mr. Bligh[33] treated the course of seepage beneath a dam on the assumption that the water follows a path along the line of contact of the dam foundation (including the cutoff walls) with the foundation material. This contact between dam and foundation is sometimes called the line of creep, and the method may be called the line-of-creep method. The head is assumed to drop along a straight line from the headwater to the tailwater (if there is any) in proportion to the distance along the line-of-creep. According to the Darcy law for a fluid of unit dynamic viscosity,

$$Q = K \frac{h\,A}{L} \qquad (18)$$

in which Q is discharge in cubic feet per second; h is head in feet; A is gross cross-sectional area in square feet; K is a dimensional coefficient; and L is length of path in feet. From continuity, $Q = A\,V$; hence,

$$L = K \frac{h}{V} \qquad (19)$$

For a given size distribution of material, there appears to be a definite maximum velocity, V_m, at which water can emerge below the dam without carrying away the foundation material, thus causing failure of the structure. Combining this value of V_m with K, which also depends on the material, to form a new constant, $c = \frac{K}{V_m}$, Eq. 19 becomes

$$L_m = c\,h \qquad (20)$$

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[32] Lecturer "A," Irrig. Dept., Faculty of Eng., Fouad First Univ., Giza, Egypt; formerly Instructor in Mech. Eng., Univ. of California, Berkeley, Calif.

[33] "The Practical Design of Irrigation Works," by W. G. Bligh. D. Van Nostrand Co., New York, N. Y., 1910.

in which L_m is minimum safe length of travel path (length of creep), and c is a constant depending on the foundation material. Mr. Bligh assumed that horizontal length of contact was just as effective as a vertical length. In other words, for a dam with horizontal floor of length b and an upstream cutoff wall of depth d_1:

$$L_m = b + 2\,d_1 = c\,h \quad \text{(21a)}$$

For a dam with horizontal floor of length b, and upstream and downstream cutoff walls of depths d_1 and d_2:

$$L_m = b + 2\,d_1 + 2\,d_2 = c\,h \quad \text{(21b)}$$

The Lane Theory.—The Bligh theory was accepted widely because of its simplicity until, in 1934, Professor Lane introduced his empirical "weighted-creep" theory. In this theory it is assumed that the line of flow will follow the line of contact between a dam and its foundation; but the vertical contact is considered more effective than the horizontal contact, and the coefficient c is different from that given by Mr. Bligh. After a study of the designs of a large number of dams in the United States and of the failure of some others, Professor Lane recommended that a unit horizontal length of contact be considered to be one third as effective as a unit length of vertical contact.

For a dam with horizontal floor of length b and upstream cutoff wall of depth d_1:

$$L_m = \frac{b}{3} + 2\,d_1 = c_1\,h \quad \text{(22a)}$$

For a dam with a horizontal floor of length b and upstream and downstream cutoff walls of depths d_1 and d_2:

$$L_m = \frac{b}{3} + 2\,d_1 + 2\,d_2 = c_1\,h \quad \text{(22b)}$$

From Eqs. 22,

$$\frac{\text{Loss of head per unit length of vertical}}{\text{Loss of head per unit length of horizontal}} = 6 \quad \text{(23)}$$

Throughout this paper, this ratio has been called the "efficiency of the vertical."

R. B. Khosla has developed quite an ingenious method of solving problems in this field. For comparison, some of his results are mentioned in this paper; but, because of limited space, the reader is referred to the original source[34] for more complete information.

THEORY OF THE FLOW OF WATER THROUGH HOMOGENEOUS POROUS MEDIA

Flow may be either laminar or turbulent depending for the most part upon the resistance offered by the material through which the liquid is flowing. The resistance of a porous medium composed largely of grains smaller than gravel is sufficient to prevent turbulent flow if natural channels are prevented from enlarging. Hence, for ordinary practical purposes, only laminar flow need be considered.

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[34] "Design of Weirs on Permeable Foundations," by R. B. Khosla, N. K. Bose, and E. Taylor, *Publication No. 12*, Central Board of Irrig., Simla, India, September, 1936.

For two-dimensional flow, the general differential formula for the steady flow of water through homogeneous materials has the form of a LaPlace equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \ldots\ldots\ldots\ldots (24)$$

Eq. 24 is obtained by substituting the Darcy law of flow—

$$u = k\frac{dp}{dx} \quad \text{and} \quad v = k\frac{dp}{dy} \ldots\ldots\ldots\ldots (25)$$

—into the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \ldots\ldots\ldots\ldots (26)$$

in which p is the pressure, k is a permeability coefficient which includes the effect of viscosity, and u and v are the two components of the discharge per unit gross area in the directions x and y, respectively. Eq. 26 is dependent on the following conditions: (1) The voids are completely filled with water; (2) the porous medium is uniform in all directions; (3) the quantity of flow entering any small element of volume is equal to the quantity flowing out of this element of volume during any given time; and (4) both water and material are incompressible.

Eq. 24 represents two families of curves, intersecting at right angles, which are usually referred to as the flow lines and equipotential lines (or lines of equal head loss). Four methods are in common use for the solution of Eq. 24; namely—(*a*) the mathematical method, (*b*) the graphical method, (*c*) the method of model experiments, and (*d*) the electric-analogy method. The writer's paper is confined to the electric-analogy method. For treatment of the other methods, beyond the scope of the present paper, the reader is referred to the work of W. Weaver;[19] A. Casagrande,[35] M. ASCE; E. W. Lane and F. B. Campbell, Members, ASCE, and W. H. Price;[36] R. Rhoades and M. N. Sinacori,[37] Assoc. M. ASCE; M. G. Ionides;[38] and B. F. Reltov.[39]

ELECTRIC-ANALOGY METHOD

The method introduced by N. N. Pavlovsky[40] in 1920 is more advantageous than other methods because: (1) It is simple, economical, and rapid; (2) it gives results that correspond to the exact theoretical solution, even in complicated examples where the theoretical solution is very difficult or even unreliable; and (3) it can be used in certain instances to investigate cases of nonhomogenous media. Development of this method was possible because of the analogy between the Ohm law for flow of electricity and the Darcy law for the flow of water in granular material. Thus, if the components of the current are i_x and i_y, in the x-direction and y-direction, respectively, the equation of continuity is

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[35] "Seepage Through Dams," by A. Casagrande, *Journal*, New England Waterworks Assn., June, 1937, pp. 131–172; see also *Bulletin No. 209*, Harvard Univ., Cambridge, Mass.

[36] "The Flow Net and the Electric Analogy," by E. W. Lane, F. B. Campbell, and W. H. Price, *Civil Engineering*, October, 1934, pp. 510–514.

[37] "Pattern of Ground-Water Flow and Solution," by Roger Rhoades and M. N. Sinacori, *The Journal of Geology*, November-December, 1941, pp. 785–794.

[38] "A Method of Determining the Flow-Net in Soil Seepage," by M. G. Ionides, *Engineering*, August 30, 1935, pp. 211–212.

[39] "Electrical Analogy Applied to Three-Dimensional Study of Percolation Under Dams Built on Pervious Heterogeneous Dams," by B. F. Reltov, *Transactions*, 2d Cong. on Large Dams, Washington, D. C., Vol. V, 1936, pp. 73–85.

[40] "Motion of Water Under Dams," by N. N. Pavlovsky, *Transactions*, 1st Cong. on Large Dams, Stockholm, Sweden, 1933, Vol. 4, pp. 179–192.

$$\frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} = 0 \qquad (27)$$

but, according to the Ohm law,

$$i_x = C\frac{\partial E}{\partial x} \quad \text{and} \quad i_y = C\frac{\partial E}{\partial y} \qquad (28)$$

in which C is a coefficient and $\partial E/\partial x$ and $\partial E/\partial y$ are the components of voltage gradient in the x-direction and the y-direction, respectively. Hence, substitution of Eq. 28 into Eq. 27 gives the following LaPlace equation:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = 0 \qquad (29)$$

The similarity of Eqs. 24 and 29 is the basic principle by which the electric-analogy method is applied. Thus, if an electrolyte is placed in a shallow tank in which a set of boundary conditions are introduced which are analogous to a certain hydraulic system, the flow lines of electricity and the equipotential lines can be plotted. The details of this procedure have been described previously by L. F. Harza,[41] engineers of the Bureau of Reclamation,[42] and many others. A complete description of the general methods used need not be repeated. Certain improvements in the technique of the experimental equipment have been developed, however, and are described in the section to follow.

Experimental Equipment and Technique

In the study reported herein, all experiments were conducted in a wooden tank (40½ in. by 31 in. by 2½ in. deep) coated inside with a waterproof nonconductive paint. In most electric-analogy experiments made in the past, a Wheatstone bridge and earphones have been used to detect the equipotential lines. When these experiments were conducted for a considerable time, however, the experimenter often experienced difficulty in recognizing the null points. Instead of using the Wheatstone bridge and earphones in the present experiments a vacuum-tube voltmeter was used. This instrument was equipped with six accurately calibrated scales covering various ranges to a maximum of 16 v. It was also equipped with a 10,000-ohm resistance and could be used to measure the voltage at any point in the tank without using an appreciable amount of current. During an experiment the various equifrom enlarries were detected by means of a probe attached to a pantograph. For example, if the 90% line was to be plotted, the voltmeter was maintained at 10 v, and the probe was moved along that potential curve to various closely spaced points corresponding to a voltmeter reading of 9 v.

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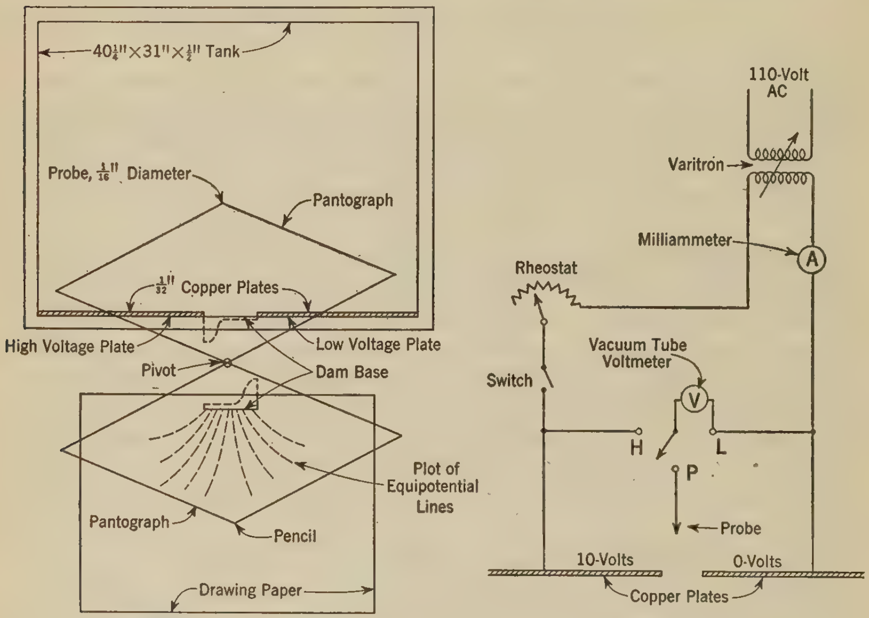
[41] "Uplift and Seepage Under Dams on Sand," by L. F. Harza, *Transactions*, ASCE, Vol. 100, 1935, pp. 1352–1385.

[42] "Model Studies of Penstocks and Outlet Works," *Bulletin No. 2*, U. S. Bureau of Reclamation, Denver, Colo., 1938, pp. 107–118.

Fig. 28(*a*) shows the arrangement of the pantograph, equipped with probe and pencil, and the experimental tank. As an illustration, the setup for mapping the equipotential lines under a dam with only a horizontal floor is shown. In this example, the sides of the tank upstream and downstream from the base of the dam were covered with copper plates. Therefore, current flows through the electrolyte from one plate to the other in a manner analogous to that of the flow of water from the upstream bed to the downstream bed.


40¼"×31"×½" Tank
Probe, 1/16" Diameter
Pantograph
1/32" Copper Plates
High Voltage Plate
Low Voltage Plate
Pivot
Dam Base
Plot of Equipotential Lines
Pantograph
Pencil
Drawing Paper
110-Volt AC
Varitron
Milliammeter
A
Rheostat
Switch
Vacuum Tube Voltmeter
V
H
L
P
Probe
10-Volts
0-Volts
Copper Plates

(*a*) APPARATUS

(*b*) WIRING DIAGRAM

FIG. 28.—EXPERIMENTAL SETUP FOR PLOTTING EQUIPOTENTIAL LINES FOR A DAM WITH A HORIZONTAL FLOOR AND NO CUTOFF WALLS

In mapping the flow lines for the dam shown in Fig. 28(*a*), the position of the copper plates and that of the nonconductive surfaces was reversed—that is, the base of the dam was constructed of copper plate and the entire perimeter of the tank was covered with copper, with the exception of that part of the sides which represented the upstream bed and downstream bed of the stream. The equipotential lines which are mapped with this latter arrangement then represent the flow lines of the system.

Fig. 28(*b*) shows the general wiring diagram of the apparatus. Alternating current (120 v, 60 cycles) was used in the experiments to avoid polarizing effects of the electrodes. Using a variable-potential transformer, a potential of only

10 v could be maintained on the circuit. The supply of current was regulated by a rheostat and measured by a 200 milliammeter. The control switch shown in Fig. 28(*b*) was set on "L" (low) when "zeroing" the voltmeter, set on "H" (high) when the over-all voltage between high and low was selected, and set on "P" (probe) while mapping the equipotential lines. The over-all voltage for determining the potential distribution was measured by placing the probe very near the high voltage plate. This latter step was necessary to eliminate oxidizing effects on the plate.

The base length of all model dams was maintained at 6 in. The model floor and cutoff walls were constructed of $\frac{1}{32}$-in. copper plates when the flow lines were mapped and of $\frac{1}{32}$-in. bakelite plates when the potential lines were mapped. The electrolyte, $\frac{3}{4}$ in. deep, was a solution of copper sulfate in distilled water acidulated with sulfuric acid (50 g of copper sulfate and 25 cc of sulfuric acid per 5 gal of water). In plotting the flow lines, the large current required by the relatively large copper area necessitated a dilution of the solution to half the foregoing concentration.

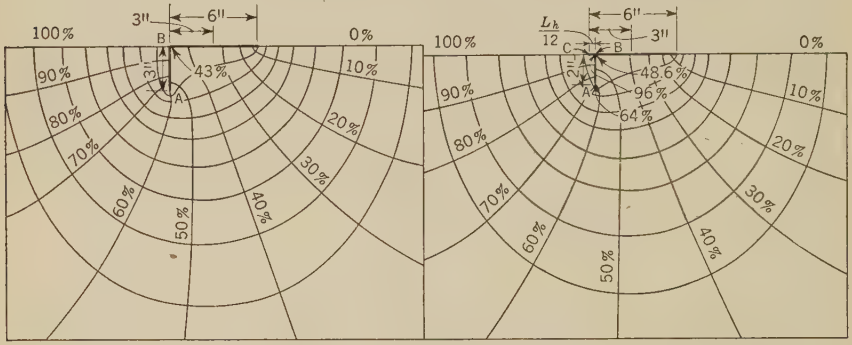
PROBABLE SOURCES OF ERROR

The sides of the tank represent streamlines for the flow system. Consequently, the ends of the tanks should be semicircular, since it can be proved mathematically that the streamlines approach semicircles at a relatively large distance from the dam. However, the influence of the sides of the tank was very small for the uplift and head losses considered. Also, the experimental results with this tank, whose breadth was 5.2 times the dam width, approached very closely the results calculated mathematically for a dam on a homogeneous material of infinite depth. The possible errors may be summarized as follows: Boundary effects, copper plates not flush with the surface at the tank sides, difference in thickness between bakelite and copper plates, evaporation of the electrolyte, and possible defects in the electric instruments.

EXPERIMENTAL RESULTS OF STUDIES ON LOSS OF HEAD

Dams on Permeable Strata of Infinite Depth.—Four cases were investigated in this category: (I) Horizontal floor and upstream cutoff wall, (II) horizontal floor and upstream cutoff wall at one twelfth of the base width from the upstream toe, (III) horizontal floor and sheet-pile cutoff walls at both upstream and downstream toes, and (IV) horizontal floor only.

Case I. Dam with a Horizontal Floor and Upstream Cutoff Wall.—Fig. 29(*a*) shows a typical example of the plot of flow lines and equipotential lines for the set of experiments in which the depth of cutoff wall d_1 was varied to give values of the ratio, b/d_1, between 1 and 6. These data are summarized in Table 3. The reading at point A, Col. 2 (see Fig. 29(*a*)), represents the percentage of head remaining to be lost after subtracting the percentage lost along the upstream face of the sheet piling. Similarly, the reading at point B (Cols. 3 and 4) represents the percentage of head remaining to be lost by the horizontal floor. Subtraction of the reading at point B from 100 gives the percentage of head that is lost because of the cutoff wall (that is, lost in the length 2 d_1). Losses of head per unit length of path, expressed as percentages of h, are given in Cols. 5 and 6, subscripts v denoting the depth of cutoff wall, or vertical path and subscript b denoting the horizontal path. In other words, h_v (Col. 5) $= \frac{100 - \text{Col. 3}}{d_1}$; and h_b (Col. 6) $= \frac{\text{Col. 3}}{b}$. In Cols. 7 and 8, Table 3, the efficiency of the vertical is computed as the ratio $\frac{\text{Col. 5}}{\text{Col. 6}}$.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A
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60%
50%
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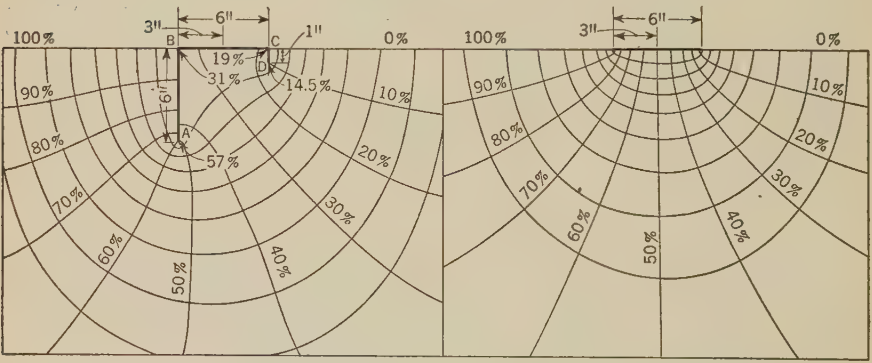
$\frac{L_h}{12}$
6"
C
B
3"
100%
0%
2"
48.6%
96%
A
64%
90%
10%
80%
20%
30%
70%
60%
40%
50%

(*a*) CASE I, UPSTREAM CUT-OFF WALL; $\frac{b}{d_1} = 2$

(*b*) CASE II, UPSTREAM CUT-OFF WALL AT ONE TWELFTH OF THE BASE WIDTH FROM UPSTREAM TOE; $\frac{b}{d_1} = 3$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(*c*) CASE III, CUT-OFF WALLS AT BOTH UPSTREAM AND DOWNSTREAM TOES; $\frac{b}{d_1} = 1$ AND $\frac{b}{d_2} = 6$

(*d*) CASE IV, HORIZONTAL FLOOR ONLY; $b = 6$

Fig. 29.—Flow Lines and Equipotential Lines, Dam with Horizontal Floor on Permeable Material of Infinite Depth

Case II. Dam with Horizontal Floor and Upstream Cutoff Wall at One Twelfth of the Base Width from the Upstream Toe.—A typical example of a plot of the flow line and equipotential lines is shown in Fig. 29(*b*), and the data are summarized in Table 4. Similarly to Table 3, the reading in Col. 2, Table 4, is that part of the head which remains to be lost by the cutoff wall and the remainder of the horizontal floor. The loss of head along the upstream part of the horizontal floor ($b/12$ in this case) is 100—Col. 2.

From Table 4(b) it is obvious that the efficiency of the vertical cutoff wall is decreased by moving the wall slightly toward the downstream toe. From an economical point of view, the cutoff wall should be as close as possible to the upstream toe of the dam.

TABLE 3.—READINGS FOR CASE I (SEE FIG. 29(a)), PERMEABLE STRATUM OF INFINITE DEPTH
(Expressed as Percentages of h)

$\frac{b}{d_1}$	(a) Loss of Head			(b) Comparative Loss Between Vertical and Horizontal Lengths			
	Point A	Point B		Loss Per Unit Length of Path		Efficiency of the Vertical	
		Selim	Khosla	h_v	h_b	Selim	Lane
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
6	76	68.5	64	31.5	11.42	2.75	6.0
3	68.5	49.5	51	25.25	8.25	3.06	6.0
2	60	43	42	19.0	7.16	2.65	6.0
1.5	60	35	....	16.25	5.84	2.79	6.0
1.2	60	33.7	....	13.30	5.61	2.38	6.0
1.0	60.2	31.6	....	11.40	5.27	2.17	6.0

Case III. Dam with Horizontal Floor and Sheet-Pile Cutoff Walls at Both Upstream and Downstream Toes.—A typical example of a plot at the flow lines and equipotential lines is shown in Fig. 29(c). A relatively large number of combinations of b/d_1 and b/d_2 were investigated and the data summarized in Table 5.

TABLE 4.—READINGS FOR CASE II (SEE FIG. 29(b)), PERMEABLE STRATUM OF INFINITE DEPTH

$\frac{b}{d_1}$	(a) Loss of Head[a]			(b) Comparative Loss Between Vertical and Horizontal Lengths		
	Point C	Point A	Point[b] B	Loss per Unit Length of Path[a]		Efficiency[c]
				h_v	h_b	h_v/h_b
(1)	(2)	(3)	(4)	(5)	(6)	(7)
6	92	72	62.8	29.2	11.8	2.47
3	96	64	48.6	23.7	8.77	2.70
2	96.5	59	38.5	19.33	7.0	2.76

[a] Expressed as percentages of h. [b] Compare with Col. 3, Table 3. [c] Efficiency of the vertical, h_v/h_b; compare these values with Col. 7, Table 3.

The general effect of the downstream cutoff wall on the readings at points A and B for a particular b/d_1-ratio can be found quickly by comparing the top line where $b/d_2 = \infty$ with the other b/d_2-ratios. In an effort to summarize these data for the designer, Col. 11, Table 5, was computed to give the efficiency of the cutoff wall under various values of b/d_2 and b/d_1. These values show the fallacy of using excessive lengths of downstream cutoff wall, because in such cases the horizontal length of the floor has relatively little effect as part of the so-called path of percolation.

TABLE 5.—READINGS FOR CASE III (SEE FIG. 29(*c*)) PERMEABLE STRATUM OF INFINITE DEPTH

$\frac{b}{d_2}$	(*a*) LOSS OF HEAD[a]				(*b*) COMPARATIVE LOSS BETWEEN VERTICAL AND HORIZONTAL LENGTHS					
	Point A	Point B	Point C	Point D	Loss per Unit Length of Path[a]			Efficiencies h_v/h_b		
					h_v upstream	h_v downstream	h_b	Upstream[b]	Downstream[b]	Total[c]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(*a*) $b/d_1 = 1.0$										
∞	60.2	31.6	0	0	11.40	0	5.27	2.17	0	....
6	57.0	31.0	19.0	14.5	11.50	19.00	2.0	5.75	9.50	7.0
3	57.8	35.0	25.0	19.6	10.82	12.50	1.67	6.50	7.50	7.0
2	59.5	39.5	35.0	25.0	10.08	11.67	0.75	14.40	15.50	15.0
1.5	59.5	43.0	39.5	29.5	9.50	9.875	0.584	16.40	16.90	15.0
1.2	62.0	47.0	44.0	32.0	8.84	8.80	0.50	17.68	17.60	15.0
1.0	64.0	51.0	49.0	37.0	8.16	8.19	0.33	24.7	24.7	25.0
(*b*) $b/d_1 = 1.2$										
∞	60.0	33.7	0	0	13.30	0	5.61	2.38	0	....
6	57.0	34.5	17.0	15.0	13.10	17.00	2.92	4.50	5.83	5.0
3	58.4	39.0	31.0	23.0	12.20	15.50	1.34	9.10	11.57	10.0
2	65.0	44.0	39.0	28.0	11.20	13.00	0.834	13.42	15.60	14.0
(*c*) $b/d_1 = 2.0$										
∞	60.0	43.0	0	0	19.00	0	7.16	2.65	0	....
6	70.0	47.0	26.0	16.0	17.70	26.00	3.50	5.05	7.43	6.0
3	67.0	52.0	39.0	27.0	16.00	19.50	2.17	7.38	9.00	8.0
2	69.0	55.0	45.0	31.0	15.00	15.00	1.67	9.00	9.00	9.0
(*d*) $b/d_1 = 3.0$										
∞	63.5	49.5	0	0	25.25	0	8.25	3.06	0	....
6	70.0	54.5	30.0	20.0	22.75	30.00	4.08	5.57	7.34	6.0

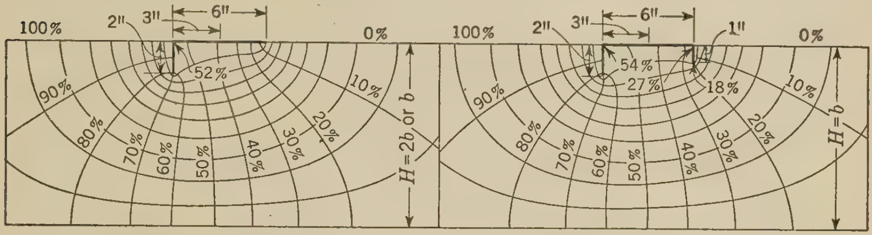
[a] Expressed as percentages of h. [b] Efficiency h_v/h_b of the upstream and downstream cutoff walls. [c] Efficiency h_v/h_b of the total path; that is, both cutoff walls and the horizontal floor. E. W. Lane, M. ASCE, reported this value as 6 in all cases.

Case IV. Dam with Horizontal Floor Only.—The plot of the flow lines and equipotential lines for this case is shown in Fig. 29(*d*). Data from this plot are discussed subsequently in the section on uplift diagrams.

Dams on Permeable Stratum of Finite Depth H.—Only three cases were investigated in this category, Case II being omitted. Experiments were conducted on two depths (H) of permeable stratum—namely, with $H = 2\,b$ and $H = b$.

Case I. Dam with Horizontal Floor and Upstream Cutoff Wall.—A typical example of a plot of the flow lines and equipotential lines is shown in

Fig. 30(*a*). Comparing Cols. 3 and 4, Table 6, with Cols. 2 and 3, Table 3, it is evident that a finite depth of permeable layer has only a small effect on the head-loss readings. The effect of the finite depth of porous material in reducing the efficiency of the vertical lengths as compared with that for an



(*a*) CASE I, UPSTREAM CUT-OFF WALL

(*b*) CASE III, CUT-OFF WALLS AT THE UPSTREAM AND DOWNSTREAM TOES; $\frac{b}{d_2}=6$

Fig. 30.—Flow Lines and Equipotential Lines, Dam with Horizontal Floor on Permeable Stratum of Finite Depth $H = 2b$ ($b/d_1 = 3$)

TABLE 6.—Readings for Case I (See Fig. 30(*a*)) Permeable Stratum of Finite Depth *H*

Depth, *H*, of permeable stratum (1)	$\frac{b}{d_1}$ (2)	(*a*) Loss of Head[a] Point[b] A (3)	(*a*) Loss of Head[a] Point[c] B (4)	(*b*) Comparative Loss Between Vertical and Horizontal Lengths: Loss per Unit Length of Path[a] h_v (5)	Loss per Unit Length of Path[a] h_b (6)	Efficiency[d] h_v/h_b (7)
2 *b*	6	80.0	69.5	30.5	11.59	2.63
	3	70.0	52.0	24.0	8.67	2.77
	1	56.0	32.0	11.25	5.41	2.08
b	2	62.0	60.0	19.68	6.84	2.88

[a] Expressed as percentages of *h*. [b] Compare Col. 2, Table 3. [c] Compare Col. 3, Table 3. [d] Efficiency of the vertical, h_v/h_b; compare Col. 7, Table 3.

TABLE 7.—Readings for Case III (See Fig. 30(*b*)), Permeable Stratum of Finite Depth *H*

$\frac{b}{d_1}$ (1)	$\frac{b}{d_2}$ (2)	(*a*) Loss of Head (Percentages of *h*)[a] Point A (3)	Point B (4)	Point C (5)	Point D (6)	(*b*) Comparative Loss Between Vertical and Horizontal Lengths: Loss Per Unit Length of Path[b] h_v upstream (7)	h_v downstream (8)	h_b (9)	Efficiency h_v/h_b Upstream[c] (10)	Downstream[c] (11)
2	2	70.0	55.5	44.5	30.0	14.85	14.85	1.67	8.9	8.9
3	6	70.0	54.0	27.0	20.0	23.0	27.0	4.5	5.12	6.0

[a] Compare values in Cols. 3, 4, 5, and 6 with appropriate lines of Table 5—Cols. 2, 3, 4, and 5, respectively. [b] Expressed as percentages of *h*. [c] Compare values in Cols. 10 and 11 with appropriate lines of Table 5—Cols. 9 and 10, respectively.

infinite depth of material is demonstrated by comparing Col. 7, Table 6, with Col. 7, Table 3.

Case III. Dam with Horizontal Floor and Sheet-Pile Cutoff Walls at Both Upstream and Downstream Toes.—For this arrangement of floor and cutoff walls, only the case in which $H = 2\,b$ was investigated. A typical plot is shown in Fig. 30(*b*), and the data are summarized in Table 7.

Case IV. Dam with Horizontal Floor Only.—The data from these plots are discussed subsequently in the section on uplift diagrams.

PERMEABLE STRATA UNDER THE DAM COMPOSED OF TWO MATERIALS WITH DIFFERENT PERMEABILITIES AND THE DEPTH OF THE UPPER LAYER IS FINITE

A dam with horizontal floor only was investigated for this condition. Because of the difficulty of adapting the upper plates to the various positions of the glass false bottom, only the equipotential lines were drawn (see Fig. 31). At the boundary between isotropic soils of different permeability, the equipotential lines are deflected in such a manner that the angles α and β and the permeabilities as defined by Eq. 18 or 25 are related as follows (see Fig. 31):

$$\frac{\tan \beta}{\tan \alpha} = \frac{k_1}{k_2} = \frac{K_1}{K_2} \quad \ldots\ldots (30)$$

Eq. 30 states merely that the deflection of the equipotential lines occurs in such a manner that the tangent of the intersecting angle with the boundary is inversely proportional to the coefficient of permeability.

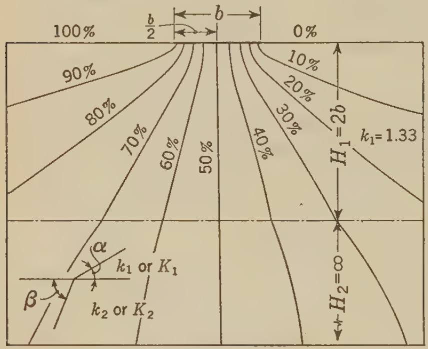


FIG. 31.—DAM WITH HORIZONTAL FLOOR ON TWO TYPES OF PERMEABLE STRATA ($H_1 = 2\,b$ AND $H_2 = \infty$)

UPLIFT DIAGRAMS

The distribution of uplift force on the base of a dam may be conveniently represented in the form of a diagram. In the present investigation, uplift diagrams were prepared from the various graphs of equipotential lines by plotting, as ordinates, the values of head at the intersection of the potential lines with the base of the dam. The maximum ordinate of 100% h will be that at the upstream toe and the minimum ordinate at 0% h will be at the downstream toe. The following definitions are useful in discussing uplift diagrams: A_F is the area of diagram expressed in terms of the base width b and the head h (representing the total uplift force); $\bar{X}$ is the distance at the center of gravity of the uplift diagram from the downstream toe of the dam; and M is the moment of uplift force about the downstream toe of the dam. "Bligh's uplift diagram" is a triangle with the maximum ordinate at the upstream toe and the zero ordinate at the downstream toe. This diagram is for a dam with a horizontal floor only and zero depth at the downstream toe. "Theoretical uplift (horizontal floor only)" is the uplift calculated by the formula:[19]

$$P = \frac{h}{\pi} \cos^{-1}\left(\frac{2\,x}{b}\right) \ldots\ldots\ldots\ldots\ldots (31)$$

in which the origin is at the midpoint of the floor and P is the value of the force in percentage of the total head, h, acting on the dam.

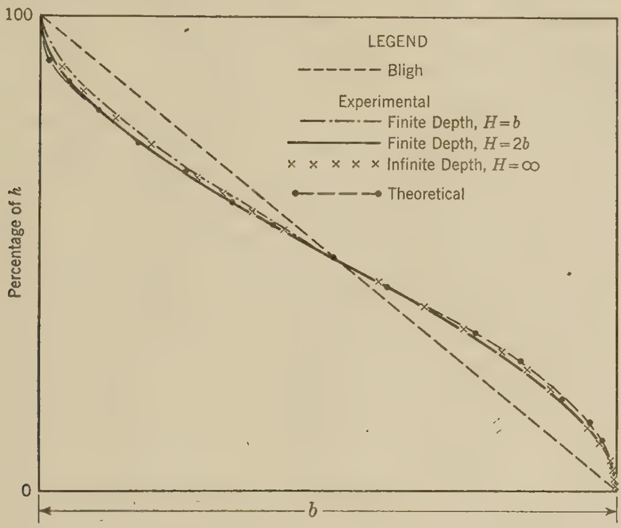


FIG. 32.—UPLIFT DIAGRAM, DAM WITH HORIZONTAL FLOOR ONLY

Dam with Horizontal Floor Only.—Fig. 32 shows the uplift diagram according to Mr. Bligh, the Weaver theoretical formula, and the author's experiments for a dam with a horizontal floor only. A comparison of uplift forces and the resulting moments for the various conditions of thickness of permeable strata are given in Table 8. The coincidence between the theoretical (that is, mathematical) and experimental results is evident. The effect of finite depth of permeable layer is very small and the results still approach the theoretical values. In connection with Table 8, as well as in the data to follow, the values of moment are given only for comparative purposes, because the type of dam to which this information would be applied nearly always has such a wide base that overturning is no problem.

TABLE 8.—COMPARISON OF RESULTS, DAM WITH HORIZONTAL FLOOR ONLY

H	A_F	$\bar{X}$	M
(*a*) THEORETICAL			
∞	$0.5\,b\,h$	$0.625\,b$	$0.3125\,b^2\,h$
(*b*) EXPERIMENTAL			
∞	$0.504\,b\,h$	$0.626\,b$	$0.313\,b^2\,h$
b	$0.496\,b\,h$	$0.629\,b$	$0.312\,b^2\,h$
$2\,b$	$0.492\,b\,h$	$0.630\,b$	$0.310\,b^2\,h$

Dam with Horizontal Floor and Upstream Cutoff Walls (Permeable Strata of Infinite Thickness).—Uplift diagrams for the cutoff wall at the upstream toe are shown in Fig. 33(*b*) and in Fig. 33(*c*) when the sheet piling is one twelfth of the base width from the upstream toe. Data from these diagrams are summarized in Table 9, which also shows the effect of the upstream cutoff wall in reducing the uplift force and its moment about the downstream toe. Table 10(*a*) offers a comparison between the two cases (Tables 9(*a*) and 9(*b*)), taking into consideration the experimental value of uplift force for a dam on horizontal floor ($H = \infty$) only as a basis. It is obvious that the cutoff wall is more effective at the upstream toe than if it is moved back from the toe toward the downstream toe.

Dam with Horizontal Floor and Upstream Cutoff Wall (Permeable Strata at Finite Depth).—The uplift diagrams and values of moment are shown in Fig. 33(*a*) for the cases of finite thickness, $H = b$ and $H = 2\,b$. The effect of the upstream cutoff wall on the uplift force for various thicknesses of the substratum can be noted in Table 10 by comparing Col. 2 ($H = \infty$) with Cols. 6 and 7 and Col. 4 ($H = \infty$) with Cols. 8 and 9. The basis of comparison is the uplift force obtained experimentally for a dam with horizontal floor only, on a very deep permeable layer.

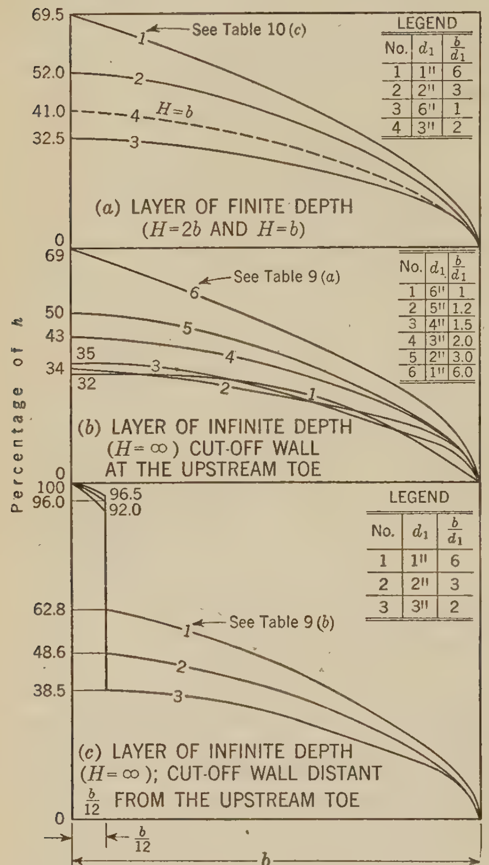


FIG. 33.—UPLIFT DIAGRAMS, DAM ON A PERVIOUS FOUNDATION, WITH HORIZONTAL FLOOR AND UPSTREAM CUTOFF WALL (b = 6 IN.)

Dam with Horizontal Floor with Upstream and Downstream Cutoff Walls (Permeable Strata of Infinite Depth).—The uplift diagrams for various lengths of the downstream cutoff wall for a length of upstream cutoff wall (d_1) equal to the base width (b) are shown in Fig. 34(*a*). A summary of data for several arrangements of upstream and downstream cutoff walls is shown in Table 11. Clearly, the effectiveness of the upstream cutoff wall, in reducing both the uplift force and its moment about the downstream toe, is offset by the use of a downstream cutoff wall. This effect is heightened by increasing the depth of the downstream cutoff wall. Consequently, a cutoff wall at the upstream toe only is better for design as far as the uplift force on the dam and the moment about the downstream toe are concerned. The downstream cutoff wall may be valuable, however, in reducing the exit gradient to such a value that material will not be disturbed by the outflow from under the dam.

TABLE 9.—UPLIFT FORCE AND ITS MOMENT ABOUT THE DOWNSTREAM TOE, DAM ON A PERVIOUS FOUNDATION OF INFINITE DEPTH, WITH HORIZONTAL FLOOR AND AN UPSTREAM CUTOFF WALL

$\frac{b}{d_1}$	Curve	(a) CUTOFF WALL AT UPSTREAM TOE (b = 6 IN., SEE FIG. 33(b))				(b) CUTOFF WALL DISTANT $b/12$ FROM UPSTREAM TOE (SEE FIG. 33(c))		
		d_1	A_F	$\bar{X}$	M	A_F	$\bar{X}$	M
∞	....	....	$0.5\ b\,h$	$0.626\ b$	$0.313\ b^2 h$	$0.5\ b\,h$	$0.626\ b$	$0.313\ b^2 h$
6	6	1	$0.44\ b\,h$	$0.604\ b$	$0.266\ b^2 h$	$0.453\ b\,h$	$0.627\ b$	$0.283\ b^2 h$
3	5	2	$0.336\ b\,h$	$0.564\ b$	$0.189\ b^2 h$	$0.393\ b\,h$	$0.623\ b$	$0.245\ b^2 h$
2	4	3	$0.304\ b\,h$	$0.559\ b$	$0.17\ b^2 h$	$0.324\ b\,h$	$0.663\ b$	$0.214\ b^2 h$
1.5	3	4	$0.244\ b\,h$	$0.480\ b$	$0.117\ b^2 h$	....	....	....
1.2	2	5	$0.238\ b\,h$	$0.552\ b$	$0.132\ b^2 h$	....	....	....
1.0	1	6	$0.236\ b\,h$	$0.567\ b$	$0.134\ b^2 h$	....	....	....

Dam with Horizontal Floor with Upstream and Downstream Cutoff Walls (Permeable Strata of Finite Depth).—Experiments on this arrangement of cutoff walls were made with a thickness of permeable stratum equal to twice the base width and with only two combinations of floor and depth of cutoff walls.

TABLE 10.—COMPARISON BETWEEN THE EFFECTS OF AN UPSTREAM CUTOFF WALL IN DIFFERENT DEPTHS OF PERVIOUS STRATA

$\frac{b}{d_1}$	(a) CUTOFF WALL IN RELATION TO UPSTREAM TOE[a] ($H = \infty$)				(b) VARIATION IN FINITE DEPTH OF PERVIOUS LAYER[a] (FIG. 33(a))				(c) UPLIFT DIAGRAM (FIG. 33(a))			
	At the toe	Not at the toe	At the toe	Not at the toe	$H = b$	$H = 2\,b$	$H = b$	$H = 2\,b$	d_1 (in.)	A_F	$\bar{X}$	M
	Uplift force		Moment		Uplift force		Moment					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
6	12.0	9.4	15.0	9.6	....	14	....	14.4	1	$0.43\ b\,h$	$0.622\ b$	$0.268\ b^2 h$
3	32.8	21.4	39.6	21.7	....	31.2	....	31.4	2	$0.349\ b\,h$	$0.616\ b$	$0.215\ b^2 h$
1	....	....	....	....	....	....	....	....	6	$0.225\ b\,h$	$0.611\ b$	$0.137\ b^2 h$
2	39.2	35.2	45.7	31.7	44	55	42.8	56.1	3	$0.28\ b\,h$	$0.64\ b$	$0.179\ b^2 h$

[a] Percentage reduction in uplift forces and moments about the downstream toe.

The uplift diagrams from these experiments are shown in Fig. 34(b). A comparison of these results with those for similar arrangements of floor and cutoff walls for an infinite thickness of permeable stratum is given in Table 12. The same conclusions appear to apply for the case of finite depth of permeable layer (Table 12) as for that of infinite depth (Table 11).

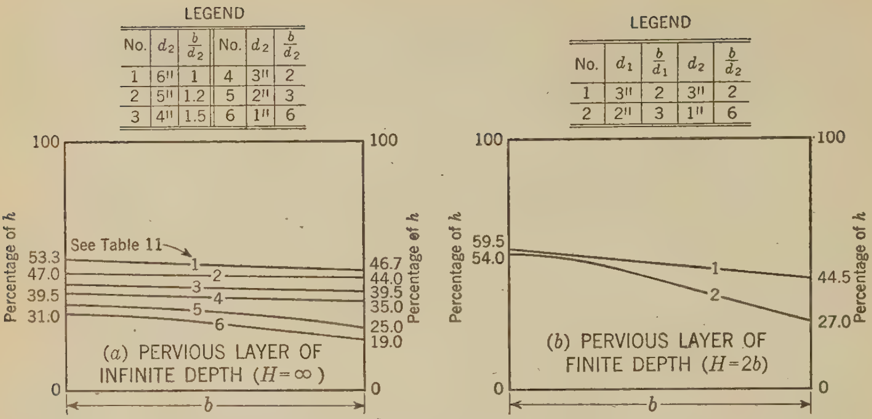


Fig. 34.—Uplift Diagrams, Dam on a Pervious Foundation with Horizontal Floor and Both Downstream and Upstream Cutoff Walls (b = 6 In.)

TABLE 11.—Summary of Data, Dam with Horizontal Floor on an Infinitely Deep Stratum of Impervious Material ($H = \infty$) and with Cutoff Walls at the Upstream and Downstream Toes ($d_1 = b$)

$\frac{b}{d_2}$ (1)	Uplift Force (See Fig. 34(a)) Area A_F (2)	Uplift Force (See Fig. 34(a)) Reduction (%) (3)	$\bar{X}$ (4)	Moment about Downstream Toe M (5)	Moment about Downstream Toe Reduction (%) (6)
		(a) $b/d_1 = \infty$			
∞	$0.5\ b\,h$	0	$0.626\ b$	$0.313\ b^2h$	0
		(b) $b/d_1 = 1$			
∞	$0.236\ b\,h$	52.8	$0.567\ b$	$0.134\ b^2h$	57.1
6	$0.256\ b\,h$	48.8	$0.555\ b$	$0.140\ b^2h$	55.2
3	$0.305\ b\,h$	39.0	$0.5444\ b$	$0.1659\ b^2h$	47.0
2	$0.373\ b\,h$	25.4	$0.51\ b$	$0.1903\ b^2h$	39.3
1.5	$0.406\ b\,h$	18.6	$0.515\ b$	$0.2094\ b^2h$	33.2
1.2	$0.455\ b\,h$	9.0	$0.505\ b$	$0.2299\ b^2h$	26.5
1	$0.50\ b\,h$	0	$0.51\ b$	$0.2554\ b^2h$	18.5
		(c) $b/d_1 = 1.2$			
∞	$0.238\ b\,h$	52.4	$0.552\ b$	$0.132\ b^2h$	57.8
6	$0.276\ b\,h$	44.8	$0.56\ b$	$0.154\ b^2h$	50.8
3	$0.350\ b\,h$	30.0	$0.516\ b$	$0.181\ b^2h$	42.2
2	$0.412\ b\,h$	17.6	$0.526\ b$	$0.217\ b^2h$	30.7
		(d) $b/d_1 = 2$			
∞	$0.304\ b\,h$	39.2	$0.559\ b$	$0.17\ b^2h$	45.7
6	$0.382\ b\,h$	23.6	$0.533\ b$	$0.204\ b^2h$	34.8
3	$0.461\ b\,h$	7.8	$0.518\ b$	$0.239\ b^2h$	23.6
2	$0.50\ b\,h$	0	$0.516\ b$	$0.258\ b^2h$	17.6
		(e) $b/d_1 = 3$			
∞	$0.336\ b\,h$	32.8	$0.664\ b$	$0.189\ b^2h$	39.6
6	$0.425\ b\,h$	15.0	$0.622\ b$	$0.251\ b^2h$	15.7

TABLE 12.—SUMMARY OF DATA, DAM WITH HORIZONTAL FLOOR ON A FINITE LAYER ($H = 2\,b$) OF IMPERVIOUS MATERIAL AND WITH CUTOFF WALLS AT THE UPSTREAM AND DOWNSTREAM TOES

$\frac{b}{d_2}$	UPLIFT FORCE (SEE FIG. 34(b))		$\bar{X}$	MOMENT ABOUT DOWNSTREAM TOE	
	Area A_F	Reduction (%)		M	Reduction (%)
(1)	(2)	(3)	(4)	(5)	(6)
		(d) $b/d_1 = 2$ (CORRESPONDS WITH TABLE 11(d))			
∞ 2	.... $0.50\ b\,h$	55 0	.... $0.512\,b$	.... $0.256\,b^2\,h$	56.1 18.3
		(e) $b/d_1 = 3$ (CORRESPONDS WITH TABLE 11(e))			
∞ 6	.... $0.424\,b\,h$	31.2 15.2	.... $0.58\ b$	.... $0.246\,b^2\,h$	31.4 21.4

Effect of Layers of Different Permeability on the Uplift Diagram of a Dam with Horizontal Floor Only.—These experiments were made to show the effect on the uplift force and the resulting moment about the downstream toe when adjacent layers with different permeabilities are present. The depth of the upper stratum was changed from $H = b$ to $H = 2\,b$; and the ratio of permeabilities, k_1/k_2, had values of 1.33 and 1.5. The uplift diagram for one such experimental condition is shown in Fig. 35 along with the comparable curve by Mr. Bligh and the theoretical curve by Mr. Weaver. The difference between the theoretical and experimental curves appears to be negligible. Data from the various diagrams for dams with horizontal floors only are summarized in Table 13. The change in uplift force and moments did not exceed ± 2% for $k_1/k_2 = 1.33$ or ± 3% for $k_1/k_2 = 1.50$. However, because of the limited number of experiments, a general conclusion cannot be reached.

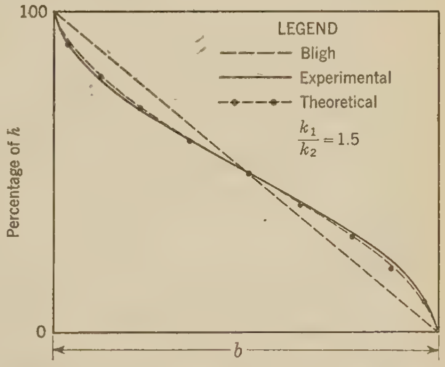


FIG. 35.—UPLIFT DIAGRAM, DAM ON A FINITE PERVIOUS LAYER ($H_1 = b$ and $k_1 = 1.50$) OVERLYING AN INFINITELY DEEP PERVIOUS LAYER ($H_2 = \infty$ AND $k_2 = 1.0$)

APPLICATION OF EXPERIMENTAL RESULTS TO PRACTICAL EXAMPLE

Example 1.—A weir 10 ft high, with a horizontal floor ($b = 60$ ft) and with a cutoff wall ($d_1 = 20$ ft) at the upstream toe rests on an impervious foundation as shown in Fig. 36. The ratio b/d_1, therefore, has a value of 3; and, according to the data in Table 3(a), the loss of head due to the sheet piling is 50.5% at the head, h, and the loss of head due to the horizontal floor is 49.5% of h.

From Table 3(b), the ratio of the loss of head per unit of vertical length to the loss of head per unit of horizontal length is 3.06. If γ is the weight per cubic ft of water, the uplift force on this dam (see Table 9) is $0.336\, b\, h\, \gamma = 201.6$ lb and the moment of this force about the downstream toe is $0.189\, b^2\, h\, \gamma = 6{,}804\, \gamma$ ft-lb.

TABLE 13.—COMPARISON OF RESULTS, DAM WITH HORIZONTAL FLOOR AND WITHOUT CUTOFF WALLS, ON STRATA OF VARIOUS DEPTHS

$\frac{k_1}{k_2(=1)}$ (1)	Condition of substratum (2)	UPLIFT FORCE*		MOMENT ABOUT DOWNSTREAM TOE	
		A_F (3)	Change (%) (4)	M (5)	Change (%) (6)
		(a) $H = \infty$			
1.0	Uniform	$0.5\ b\, h$	0	$0.313\, b^2\, h$	0
		(b) $H = b$			
1.33	Stratified	$0.492\, b\, h$	−1.6	$0.310\, b^2\, h$	−0.96
1.50	Stratified	$0.491\, b\, h$	−1.8	$0.319\, b^2\, h$	+1.92
∞	Impermeable	$0.496\, b\, h$	−0.8	$0.312\, b^2\, h$	−0.32
		(c) $H = 2\, b$			
1.33	Stratified	$0.496\, b\, h$	−0.8	$0.316\, b^2\, h$	+0.96
1.50	Stratified	$0.486\, b\, h$	−2.8	$0.322\, b^2\, h$	+2.78
∞	Impermeable	$0.492\, b\, h$	−1.6	$0.310\, b^2\, h$	−0.96

These results differ markedly from those determined by the weighted-creep method.[18] For example, the length of the path of percolation is $L_m = \frac{b}{3} + 2\, d_1 = \frac{60}{3} + 40 = 60$ ft; and, by the weighted-creep method, the loss at head due to the sheet piling is $\frac{20}{60}\, h\, 100 = 66.66\%$ of h; but the loss of head due to the horizontal floor is 33.4% of h (Fig. 36(b)); and the uplift force on the dam is $0.167\, b\, h\, \gamma = 100.2\, \gamma$ lb. Finally, the moment of this force about the downstream toe is $0.111\, b^2\, h\, \gamma = 3{,}996\, \gamma$ ft-lb.

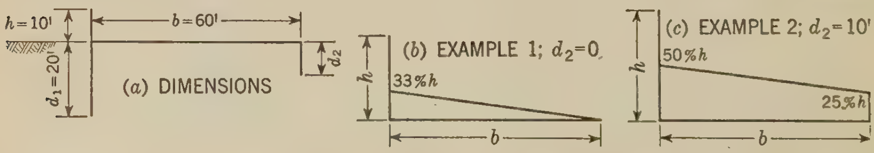


FIG. 36.—PRACTICAL APPLICATION OF EXPERIMENTAL RESULTS

Example 2.—For a weir with both upstream and downstream cutoff walls, assume the same weir as in Example 1, except that a 10-ft downstream cutoff wall is added ($d_2 = 10$ ft). According to Table 5(a), for the case of $b/d_1 = 3$ and $b/d_2 = 6$:

Path	Loss (percentage of h)
d_1	45.5
d_2	30.0
b	24.5

These data give a ratio of head loss per unit vertical length to head loss per unit horizontal length of 5.57 for the upstream cutoff wall and 7.34 for the downstream cutoff wall (see Table 5(*b*)). The uplift force from Table 11 is $0.425\ b\ h\ \gamma = 255\ \gamma$ lb; and the moment of this force is $0.251\ b^2\ h\ \gamma = 8{,}835\ \gamma$ ft-lb.

In comparison, the weighted-creep analysis yields a length of the path of percolation equal to $L_m = 2\ d_1 + \frac{b}{3} + 2\ d_2 = 40 + 20 + 20 = 80$ ft and the losses of head are:

Path	Loss (percentage of h)
d_1	50
d_2	25
b	25

The ratio of loss of head per unit of vertical path to the loss per unit of horizontal floor is assumed to have a value of 6. The pressure distribution is assumed to be that shown in Fig. 36(*c*); and the uplift force then is $0.375\ b\ h\ \gamma = 225\ \gamma$ lb and the moment of this force about the downstream toe is $0.208\ b^2\ h\gamma = 7{,}488\ \gamma$ ft-lb.

SUMMARY AND CONCLUSIONS

A new experimental technique is presented in this paper. A vacuum-tube voltmeter is used instead of earphones and a Wheatstone bridge to detect the lines of equal voltage, and a pantograph is used for the direct plotting of the points.

The advantage of an upstream cutoff wall in reducing the uplift force is demonstrated. A downstream cutoff wall tends to increase the uplift force and hence to offset the value of the upstream cutoff wall. The downstream cutoff wall may be necessary, however, to prevent the formation of channels beneath the dam. It is recommended that only a relatively short downstream cutoff wall be used in connection with a relatively deep upstream wall.

When the thickness of the stratum of porous material under a dam is greater than the base width, the uplift force and moment are essentially the same as when the porous media is of infinite depth.

ACKNOWLEDGMENT

The data on which this paper is based are included in an academic thesis by the writer, entitled "Underflow and Uplift Pressure for Dams and Weirs on Porous Media by Electric Analogy." It was presented to the University of California at Berkeley in May, 1941, in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Irrigation Engineering. Copies of the thesis are filed at the University of California and at Engineering Societies Library in New York, N. Y.

DISCUSSION

W. H. Holmes,[43] Assoc. M. ASCE.—Designers of dams should find the paper by Mr. Leliavsky extremely interesting and important because it indicates that the effective area on which uplift acts may be larger than is usually assumed. Any solution of a problem that gives results different from the conventional should be critically examined.

From simple mechanics, neglecting friction in the apparatus, the sum of the vertical forces is equal to zero, or

$$\Sigma V = 0 \qquad (32)$$

In this case, at any section of the test cylinder, forces acting to resist uplift are weight, W, of that part of the cylinder above the section; externally applied force, F, on top of the cylinder; and the tensile strength, z, of the test cylinder. The forces tending to rupture the cylinder are the average liquid pressure, p_a, acting on the area, $n\,A$ (in which A is the total area of the test cylinder and n is the effective area on which the pressure is applied).

For the condition just prior to the instant of rupture, Eq. 32 becomes:

$$W + F + z - p_a\,n\,A = 0 \qquad (33a)$$

Neglecting the weight of the cylinder, Eq. 33*a* reduces to:

$$F = n\,p_a\,A - z \qquad (33b)$$

Stress, f, in terms of load on a unit area and unit tensile strength, c, of the material may be obtained by dividing Eq. 33*b* by the cross-sectional area, A, as:

$$f = n\,p_a - c \qquad (33c)$$

For demonstration, series 1 will be examined since, as Mr. Leliavsky states (in introducing Fig. 23), "* * * the workmen had acquired a greater ability in preparing the test pieces." The results of series 1 (Fig. 23) were given, in kilograms, as:

$$F = 0.858\,p_a\,A - 1{,}058 \qquad (34)$$

Since the area A was about 150 sq cm, the value of f for a unit area equals $0.858\,p_a - \dfrac{1{,}058 \text{ kg}}{150 \text{ sq cm}} = 0.858\,p_a - 7.06$ kg per sq cm. In pounds per square inch, therefore,

$$f = 0.858\,p_a - 100.4 \qquad (35)$$

This equation is similar to Eq. 33*c*. Eq. 35 is redrawn in Fig. 37. For the case in which the oil pressure is zero, the force, f, necessary to rupture a cylinder is equal to the tensile strength, or 100.4 lb per sq in. This tensile strength of the concrete, for some unknown reason, is lower than that expected for good concrete.

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[43] Superv. Engr., State Dept. of Public Works, Sacramento, Calif.

Experience has demonstrated that construction materials do not have perfect physical properties as assumed in simple mechanics. Forces acting in one direction have deformations at right angles to the direction of force. The analyst becomes involved in nomenclature when he refers to strains without an accompanying stress, but the foregoing deformations and stresses are real to the extent that failure occurs on a plane when loadings at right angles to the plane become excessive. The various theories as to the cause of failure will not be discussed, but this type of failure has been demonstrated repeatedly.

Frank E. Richart and Anton Brandtzaeg, Members, ASCE, and Rex L. Brown[44] have demonstrated that an external force on the circumference of cylinders, even with all pores sealed, ruptures them in manner similar to the ruptures produced by Mr. Leliavsky's experiments. For a concrete mix of 1:3:5 on 4-in. by 8-in. cylinders, they found that,[45] for oil pressures of 3,830, 5,460, and 6,790 lb per sq in., the unit axial loads at rupture were 660, 1,215, and 1,700 lb per sq in., respectively.

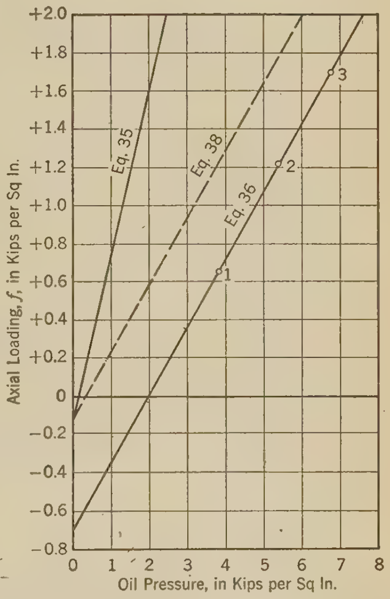

FIG. 37.—(1 KIP = 1 "KILO-POUND" = 1,000 LB)

The equation for these three points, in lb per sq in., is:

$$f = 0.351\, p_o - 689 \dots\dots\dots\dots (36)$$

From Eq. 36 (see Fig. 37), the ratio μ, commonly called Poisson's ratio of the coordinate stresses, can be determined. The external oil pressure may be resolved into two pairs of equal compressive forces, p_o, acting upon two planes at right angles through the axis of the cylinder. Since each pair of forces produces a tensile force, f, acting along the axis of the cylinder which amounts to $\mu\, p_o$, the total force tending to rupture the cylinder is

$$f = 2\,\mu\, p_o \dots\dots\dots\dots (37)$$

Therefore, from Eq. 36 if p_o is assumed equal to p_a (which is never exactly true), $2\,\mu = 0.351$, or $\mu = 0.175$.

If the concrete for the 1928 and 1942 tests had the same tensile strength, the curves of Eqs. 35 and 36 would have intersected the x-axis at a common point. If a line is drawn through the intersection of Eq. 35 and the x-axis and parallel to Eq. 36 (Fig. 37), the result for such a line is the equivalent of making the tensile strength of the two materials the same. The equation (see Fig. 37) is:

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[44] "A Study of the Failure of Concrete Under Combined Compressive Stresses," by Frank E. Richart, Anton Brandtzaeg, and Rex L. Brown, *Bulletin No. 185*, Univ. of Illinois, Urbana, November, 1928.

[45] *Ibid.*, p. 72.

$$f = 0.351\, p_o - 100.4 \quad \text{.....................} (38)$$

Curves representing Eqs. 35 and 38 (Fig. 37) differ only in that uplift was encouraged in one and no pore pressure or uplift was permitted in the other. The area on which uplift acts must be the difference between 0.858 A and 0.351 A, or $n = 0.507$ for this case.

A more nearly correct value of the uplift factor can be found by subtracting $2\,\mu$ from Mr. Leliavsky's proposed value of n; or:

$$n = 0.85 - 2\,\mu \quad \text{.....................} (39)$$

Other defects in the physical properties of construction material are illustrated by the fact that (1) the modulus of elasticity, E, and (2) Poisson's ratio, μ, are not constants over a large range of loading. If E and μ were constant, tests made at the point of rupture would be sufficiently accurate for use in the working range of construction material. The value of μ for the working range of concrete is usually considered lower than indicated for this particular test. Under "Suggestions for Further Research," Mr. Leliavsky recognized the value of making additional studies based on deformation instead of "at point of rupture."

Mr. Leliavsky should be commended for using the statistical application of the "least-squares" method to avoid the personal equation in "averaging" given data. If engineers would use this method, instead of "averaging by eye" a line through a series of plotted points, they would be able to obtain more consistent results from experimental work.

Since Mr. Leliavsky's reference to the "least-squares" principle is not readily available, a brief derivation follows: If the data available have the form $x_1\, y_1,\ x_2\, y_2,\ \ldots,\ x_N\, y_N$ which plot approximately as a straight line, the equation may be written as:

$$y = a\,x + b - r \quad \text{.....................} (40)$$

in which a is the slope of the line; b is a constant; and r is the error of regression or a coefficient of correlation. Then, $r = a\,x + b - y$; $r^2 = a^2\,x^2 + b^2 + y^2 + 2\,a\,x\,b - 2\,a\,x\,y - 2\,b\,y$; and

$$\Sigma r^2 = a^2\,\Sigma x^2 + N\,b^2 + \Sigma y^2 + 2\,a\,b\,\Sigma x - 2\,a\,\Sigma(x\,y) - 2\,b\,\Sigma y \quad \text{....} (41)$$

Let Σr^2 approach zero and take derivatives of Σr^2 with respect to a and b:

$$\frac{d\,\Sigma r^2}{da} = 2\,a\,\Sigma x^2 + 2\,b\,\Sigma x - 2\,\Sigma(x\,y) = 0 \quad \text{............} (42a)$$

and

$$\frac{d\,\Sigma r^2}{db} = 2\,N\,b + 2\,a\,\Sigma x - 2\,\Sigma y = 0 \quad \text{..............} (42b)$$

or

$$a\,\Sigma x^2 + b\,\Sigma x - \Sigma(x\,y) = 0 \quad \text{..................} (43a)$$

and

$$b\,\Sigma x + N\,b - \Sigma y = 0 \ldots\ldots\ldots\ldots\ldots\ldots (43b)$$

Solving Eqs. 43 for a and b:

$$a = \frac{N\,\Sigma(x\,y) - \Sigma x\,\Sigma y}{N\,\Sigma x^2 - (\Sigma x)^2} \ldots\ldots\ldots\ldots\ldots\ldots (44a)$$

and

$$b = \frac{\Sigma x^2\,\Sigma y - \Sigma x\,\Sigma(x\,y)}{N\,\Sigma x^2 - (\Sigma x)^2} \ldots\ldots\ldots\ldots\ldots\ldots (44b)$$

The slope of the curve representing Eq. 36, through the plotted points 1, 2, and 3, Fig. 37, was determined by the computations listed in Table 14. In Eqs. 44, let $N = 3$, $(\Sigma x)^2 = 16{,}080 \times 16{,}080 = 258{,}566{,}400$; and, substituting other values from Table 14,

$$a = \frac{3 \times 20{,}704{,}700 - 16{,}080 \times 3{,}575}{3 \times 90{,}584{,}600 - 258{,}566{,}400} = 0.351$$

and

$$b = \frac{3{,}575 \times 90{,}584{,}600 - 16{,}080 \times 20{,}704{,}700}{3 \times 90{,}584{,}600 - 258{,}566{,}400} = -\,689.4$$

The resulting equation for a straight line (the form $y = a\,x + b$) then is (see Eq. 36): $f_1 = 0.351\,p_o - 689$.

The writer concurs with Mr. Leliavsky that this paper should serve as a starting point for further studies, and he wishes to emphasize that it is an excellent start toward the solution of an important problem.

TABLE 14.—COMPUTATIONS FOR THE EQUATION OF A LINE TO REPRESENT OBSERVED DATA (EQ. 36, FIG. 37)

Points, N	x	y	x^2	$x\,y$
a	3,830	660	14,668,900	2,527,800
b	5,460	1,215	29,811,600	6,633,900
c	6,790	1,700	46,104,100	11,543,000
Σ	16,080	3,575	90,584,600	20,704,700

W. A. PERKINS,[46] M. ASCE.—Valuable information on the subject of uplift in dams has been added to engineering literature by Mr. Leliavsky. Much thought and care are shown in the design of the testing machine and in the testing procedure. Before a definite method can be established for utilizing these data in the design of gravity dams, however, certain additional information is desirable.

In the laboratory procedure the average pore pressure within the specimen is increased by the control of the fluid pressure, p_i, in the hollow core of the specimen, and by the convergence of the paths of flow of the fluid through the cylindrical test piece. Within a dam, however, the paths of flow are parallel, and there is no counterpressure corresponding to p_i. Consequently, the intensity of pore pressure producing uplift within the body of the dam may decrease, with increase of distance from the upstream face, much more rapidly than is indicated by the data produced by the experiments. Valuable information concerning this law of pressure variation could be obtained by the installation of suitable tubes and gages in the galleries of existing dams.

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[46] Hydr. Engr., Sacramento, Calif.

The results of the tests give tensile stresses much lower than are usually obtained with a briquette machine. In Col. 10, Table 2, the average of ninety-five tests was 4.61 kg per sq cm or 65.5 lb per sq in. This is only 20% or 25% of the average values of the tensile strength of mortar specimens of average mixes when broken in the usual manner of determining tensile strength. Furthermore, no allowance is made in the computations for the coordinate tensile stress that is produced by the compressive force on the exterior and interior faces of the cylinder. An application of Mr. Leliavsky's suggested uplift value is described briefly, as follows: Assume a gravity dam with vertical upstream face and uniform batter on the downstream face. Let H_d be the height of dam; γ be the unit weight of water; G be the specific gravity of concrete; m be the downstream batter equal to $\left(\frac{\text{horizontal}}{\text{vertical}}\right)$; and s equal unit stress.

It is easily proved that

$$s = \frac{\gamma H_d}{2}\left(G \pm \frac{2 - G\,m^2}{m^2}\right) \dots\dots\dots\dots\dots\dots\dots (45)$$

When $G = 2.4$ (an average value) and $m = 0.85$, s at the upstream face $= 75\,H_d - 11.5\,H_d = 63.5\,H_d$.

Taking Mr. Leliavsky's recommended value of 0.85 as the average area subject to uplift it is proper to state that the unit liquid uplift value is $0.85 \times 62.5 = 53$ lb per sq ft, or the total uplift for a height of H_d ft is 53 H_d lb per sq ft, leaving a favorable reserve of 10.5 H_d lb per sq ft.

Furthermore, the differential between this type of uplift and the vertical downward pressure increases with distance from the upstream face. As previously indicated, it is logical to assume that the uplift decreases with this distance, whereas the vertical downward pressure increases to 86.5 H_d at the downstream toe.

From the evidence at present available it appears that a gravity dam with a conservative design of thickness equal to 0.85 H_d, which is the practice at the present time, is safe. Likewise, it is the belief of the writer that further information about the law of variation of the pore pressure, due to upstream water pressure, within the body of the dam be obtained before abandoning present conservative methods of estimating uplift.

S. P. WING,[47] M. ASCE.—The comprehensive and costly experiments, reported by Mr. Leliavsky, on the percentage area of concrete that is effective to hydraulic uplift will remain for many years basic data to which engineers will refer in discussing this factor in dam design. Although future interpretations of his data may change the coefficients he has given by a few per cent, the scatter of his individual determinations from his final value is but $16\frac{1}{2}\%$ as measured by the coefficient of variation. This indicates a reproducibility of results which compares favorably with other types of concrete testing.

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[47] Civ. Engr., U. S. Bureau of Reclamation, Denver, Colo.

The most startling feature of the tests is the extremely low tensile strength of the concrete (Table 2, Col. 10). The derived mean tensile strength of concrete with a water-cement ratio of 0.52, as determined by breaking the cylinders hydraulically, was but 70 lb per sq in., and but 45 lb per sq in. when tested mechanically. (The writer takes the liberty of converting Mr. Leliavsky's metric units to the more familiar English system.)

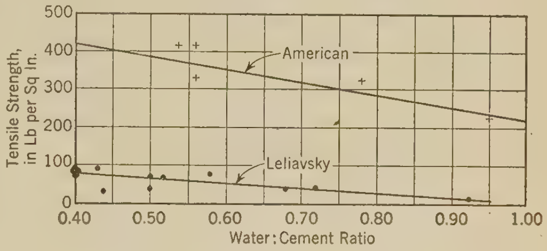

FIG. 38.—TENSILE STRENGTH OF CONCRETE CYLINDERS

American tensile tests of concrete (Fig. 38) with 6-in. by 18-in. cylinders, moist cured 180 days, gave unit strengths about seven times the values reported by Mr. Leliavsky. Although some European cements have shown tensile strengths of but 50% of the usual American product,[48] it is possible that the low strengths indicated by Mr. Leliavsky's data may be related either to the form of his test specimen or to some other factor not taken into account in his analysis.

The essential procedures of the uplift test, eliminating details, consisted in placing an external compressive load on a 6-in. concrete cylinder equipped with a metal water jacket at its midsection, arranged with a slip joint. By next applying a comparatively high water pressure through the water jacket until the concrete was saturated and then by slowly reducing the external compression load, the concrete was brought to failure in tension by the bursting pressure of the water within the concrete pores. Several cylinders of the same mix, tested at different hydrostatic loads and breaking at different values of the external loads, provided the data needed to relate the two variables. Plots of Y, the external load, as a function of X, the water pressure, multiplied by the gross area of the section, yielded data to which straight-line graphs were fitted by the least-squares method, their equations being of the form:

$$Y = B X + C \quad \text{........................} \quad (46)$$

It is to be noted that this procedure defines a purely empirical relationship whose validity for use under other conditions depends upon the goodness of fit and upon analysis. It relates the compressive loads to the water pressures at the time of failure in such a manner that errors in estimates of the magnitudes of the compressive loads acting at the time of failure, if made from known applied water pressures, are less on the average than would be obtained were any other straight line fitted to the data. The numerical values of the constants of the equations depend purely on the least-squares solution and not on their interpretation in terms of physical phenomena. For example, if, instead of expressing Y as a function of X, the pressure X is expressed as a function of Y so as to give the best estimates of the magnitudes of the water pressures needed to break a cylinder under varying external loadings, then a "least-squares" fit of the data locates different lines from those given by Mr. Leliavsky; and their equations solved for Y have different X-coefficients (different uplift coefficients). Alternatively, if, instead of minimizing either the Y-deviations or the X-deviations, the deviations normal to the trend line are minimized, thus giving "the line of best fit,"[49] a third value of the X-coefficient results. If all these coefficients (following the form used by Mr. Leliavsky) were called "uplift" coefficients, their different values might be seen from the ($p_a A$)-coefficients in the following equations: For estimating external loads—

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[48] "Comparative Tests of 52 Brands of Portland Cement from 15 Countries," by Eugene V. Barrett, *Proceedings*, A.S.T.M., Vol. 44, 1944, p. 797.

$$F + z = 0.90\, p_a A + 50 \quad \text{(47}a\text{)}$$

For estimating bursting water pressures—

$$F + z = 0.97\, p_a A - 130 \quad \text{(47}b\text{)}$$

and, for the line of best fit—

$$F + z = 0.93\, p_a A - 30 \quad \text{(47}c\text{)}$$

in which F is in kilograms. Unavoidable errors in scaling data from the graph prevent Eq. 47*a* from agreeing precisely with the author's equation for a straight line given in Fig. 24.

A fourth equation, obtained from Eq. 47*c* by transforming into English units and substituting the mean value of the indicated tensile strength z, has been added for later discussion:

$$\bar{p}_a = 1.08\,(f + 70) \quad \text{(47}d\text{)}$$

Eq. 47*d* is thus a function of the unit bursting pressure $\bar{p}_a$, in pounds per square inch, substituting a mean value of z. The line of best fit, Eq. 47*c*, indicates an uplift coefficient of 0.93, 0.03 units greater than that given by Eq. 47*a*, the method used by Mr. Leliavsky.

If a superficial interpretation is placed on the coefficients of Eq. 47*d*, this formula indicates that unit water pressures of only 8% in excess of the average unit pressures due to external load plus 70 lb per sq in. were needed to break the concrete. In short the pressures, if acting on the full cross section, were so nearly those required simply to lift the external load that one wonders whether the data do not apply to the condition within the pore space just prior to ultimate failure rather than to the condition at working loads.

Fig. 39, derived from two-dimensional photoelastic studies of tension briquettes,[50] shows a rough estimate of what may have been the distribution of stresses across the section before rupture was reached, and at a state of loading when there was no net resultant tension acting on the cross section, the mean intensity of water pressure just balancing the mean external load. The diagram indicates that, even if the intensity of water pressure across the section were uniform and its resultant equal to the resultant of the external load for a specimen shaped like that used by Mr. Leliavsky, the interior of the cylinder would be subjected to a tensile unit stress of approximately 25% of the applied water pressure. If it be assumed that the true ultimate unit tensile strength of pressure-saturated concrete (0.7 of dry concrete) broken under slowly applied tensile loads (0.8 of quick break) is 0.7 × 0.8 × 350 = 200 lb per sq in., then, because of nonuniform stress distribution, balanced interior and exterior loads approximately 800 lb per sq in. would be sufficient to break the concrete without the potential tensile resistance of the entire cross section ever having been effective. Thus a straight line fitted to the data of the higher loads would intersect the axis at the origin, indicating zero tensile strength for the concrete. The data of Fig. 24, if averaged in small groups, suggest that a sag curve (concave downward), conforming with the foregoing comment, might fit the data better.

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[49] "Economic Control of Quality of Manufactured Product," by W. A. Shewhart, D. Van Nostrand, Co., Inc., New York, N. Y., 1931, p. 105.

[50] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Univ. Press, Cambridge, England, 1931, p. 574.

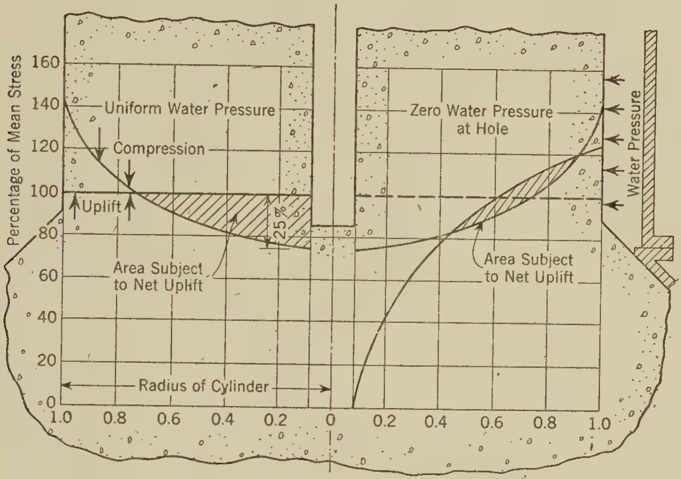

Fig. 39.—Probable Unit Stresses Within Cylinder with Superimposed Load and Water Pressure Balanced

The writer, finally, would like to touch upon certain practical considerations in connection with "uplift" tests and assumptions based upon permeability experiments made on concrete cylinders up to 12 in. by 24 in. and at pressures up to 400 lb per sq in.

No one who has seen concrete saturated under high pressure and weeping water like a squeezed sponge when placed under load in a testing machine, will question the reality and importance of uplift phenomena. However, it is not generally realized that a surprisingly long time is required to attain complete saturation and to stabilize internal hydraulic pressures, and until these are stable, tests may lead to wrong inferences. Six days were required in permeability tests on 6-in. by 12-in. cylinders to balance the inflow and outflow of the specimens. In some of the tests by Mr. Leliavsky, with low pressures used during the early stages of the test it is doubtful whether static conditions were reached at the time of break. The time required to saturate a large dam through homogeneous concrete, assuming no cracking of the structure, has been estimated at many thousands of years. During the economic life of the structure, it may be doubted if water pressure ever penetrates as much as one third of the base width. Stresses computed on the assumption the concrete is fully saturated would then appear to bear little relation to the facts.

On the other hand, there is adequate evidence to show that so-called impermeable foundations of granite and the like are actually hundreds or even thousands of times more permeable than the concrete or rolled earth materials above them. Permeability coefficients computed from the discharges of rock tunnels under construction, or from the leakage of from 1 cu ft per sec to 5 cu ft per sec past grouted dam sites, or from the yields of small wells drilled in rock to form the water supply of mountain cottages, all yield permeability coefficients hundreds of times higher than those derived from either laboratory tests of small cylinders of homogeneous rock or than those obtained from concrete and compacted earth tests made in a laboratory. Open rock quarries usually show hair cracks in three dimensions at spacings of from 2 ft to 50 ft apart. Grout cannot be forced any distance into a crack smaller than about 0.005 in. because of the limiting size of the biggest cement particle. Furthermore, the flow from crack at spacings of 2 ft to 50 ft reduced to permeability coefficients, on the basis of gross surface exposed to percolation, yields coefficients that are hundreds of times greater than the laboratory tests. In view of these two facts, the greater permeability of the foundation compared to that of the structure built upon it should be more generally recognized and taken into account in studies dealing with the effect of uplift. There can be little question that, within the foundation of a dam, uplift acting on nearly 100% of the area is a pertinent design assumption; although, of course, the intensities and the distribution of the unit pressures acting may be modified by grouting and drainage, at the option of the designer.

Finally, the writer feels that the extremely low tensile strength of concrete indicated by the Leliavsky bursting tests warrants additional analysis and laboratory investigation unless adequate explanations can be offered in the closure. Many large spillways are now being contemplated in which concrete surfaces are expected to withstand high-velocity flow. At their bends, deflection of the water builds up high hydraulic heads which are reduced immediately downstream to negative values or, at most, to heads corresponding to the depth of the flowing sheet of water. The intervening concrete is thus continuously subjected to a high-head permeability test. If the tensile strength of the concrete is no more than shown by Mr. Leliavsky's tests, the internal bursting forces plus the shear from the high-velocity water may well result in failure of the surface concrete.

It is hoped the following data, needed to clear up certain points of the tests, described by Mr. Leliavsky, can be given in the closure:

(*a*) Fineness of cement and tensile strengths from briquette tests with the corresponding mix proportions and water-cement ratios.

(*b*) Size of coarse aggregate in concrete; and method of cure to the time of testing after the initial period of wet curing.

(*c*) Compressive strength of concrete as tested in standard cylinders or cubes with corresponding mix proportions and water-cement ratios.

(*d*) Were the reported mechanical tension tests made before or after the concrete specimens had been water saturated?

(*e*) What was the percentage reduction in tensile strength for those tests where the breaking loads were applied slowly in terms of rapid load strengths?

(*f*) How was the point of loading at which the concrete failed determined? Fig. 13(*b*) suggests that failure had started prior to the test loading marked "fracture."

(*g*) How was the quantity of water percolating through the cylinders measured?

(*h*) What were the computed values of the permeability coefficients at different water-cement ratios? (Coefficients, in terms of cubic feet per second per square foot per unit gradient, obtained by the Bureau of Reclamation[51] for 6-in. by 12-in. cylinders with $1\frac{1}{2}$-in. aggregate have ranged from 10×10^{-12} for a water-cement ratio of 0.50, to $7{,}000 \times 10^{-12}$ for a water-cement ratio of 0.90).

(*i*) Give the number of cylinder breaks which occurred near the junction of the lower container, the number in the middle of the test section, and the number near the top sleeve.

(*j*) Give tabulation of experimental values of F and p_a.

Gordon V. Richards,[52] Jun. ASCE.—The impressive thing about the work reported by Mr. Selim is the extreme simplicity of the method as compared to the complexity of the problem. It requires no great amount of equipment and could be used for problems far more complicated than those presented.

One is led to wonder if it would be possible to find the uplift pressures under a dam where drains were installed. It seems likely that this could be done. As with the models in the paper, the stream-bed pressure upstream from the dam would be represented by a plate maintained at, say, 10 v. The downstream plate could be maintained at zero potential. Between these two (that is, under the foundation of the dam) a third conductor, with a size and shape depending on the type of drainage installed in the dam, might be maintained at some intermediate potential depending on the efficiency of the drainage under the prototype.

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[51] Closure of discussion on "An Investigation of the Permeability of Mass Concrete with Particular Reference to Boulder Dam," by Arthur Ruettgers, E. N. Vidal, and S. P. Wing, *Proceedings*, A.C.I., Vol. 32, 1936, p. 385.

[52] Instr., Mech. Eng. Dept., Univ. of California, Berkeley, Calif.

Possibly, also, the method could be applied without undue difficulty to three-dimensional problems of laminar flow. The only requirements to be met are that the model and prototype be geometrically similar and that the boundary pressures existing in the prototype be duplicated in the model as electric potential. In the case of a three-dimensional model it would be necessary to measure the potential at a point, instead of at a vertical line as is allowed in the two-dimensional problem. Perhaps this could be accomplished by using, for the probe, a well-insulated wire with an exposed copper sphere (a sixteenth or an eighth of an inch in diameter) on the end. With the proper measuring instruments, a three-dimensional model might be explored with the copper sphere. One difficulty with this idea will be the construction of the model in such a way that the liquid is maintained in the right shape. For a dam in a steep ravine, for example, the best method might be to invert the model so that the bottom of the tank containing the liquid represents the ground surface inverted.

Serge Leliavsky Bey.[53]—Those who took part in the interesting discussion of this paper have focused attention on certain important points which otherwise might have remained obscure. In doing so, they have contributed to presenting the subject of this investigation more clearly, thus helping to avoid possible misinterpretation.

The original manuscript from which the paper was condensed was about three times longer than the text finally accepted for publication, with the result that certain points of the solution were left, if not "in the dark," at least "in the shade." This applies particularly to the historical part, which, in the original version, preceded the descriptive account of the tests. Since, in the printed text, this part was deleted, the interpretation of the writer's method lost its historic perspective, and some aspects of the conclusions reached became, therefore, less obvious.

The theoretical "three-dimensional" elastic effect, treated in Mr. Holmes' valuable discussion, was one of these aspects.

According to Mr. Holmes, the failure of the specimens (which is basically due to axial forces) is also affected by the external pressure exerted by the liquid on the lateral surfaces of the porous material. The uplift factor would then be an amount 2μ less than the value computed by the writer (μ being the reciprocal of Poisson's ratio). Assuming that the Brandtzaeg experiments were indeed applicable to the case, the writer's coefficient, $n = 0.85$, would thus be reduced to $n = 0.507$.

Mr. Holmes' point is rather obvious and might easily have been taken into account in the tests, if there had not been specific reasons militating against this procedure. In fact, the objections against introducing the three-dimensional effect in the interpretation of these experiments, as suggested by Mr. Holmes, might have been inferred from the third paragraph following Eq. 16:

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[53] Director, Designing Service, Reservoirs and Nile Barrages, Dept., Ministry of Public Works, Cairo, Egypt.

> "Messrs. [P.] Fillunger and [Karl] Terzaghi [M. ASCE] have demonstrated beyond any reasonable doubt that the effect of an interstitial hydraulic pressure upon the resistance of a porous solid is negligible, so long as this pressure is equally distributed over the walls of the pores. Since the existing evidence confirming this statement is conclusive, it follows, in so far as the present tests are concerned, that the ultimate resistance of the test piece is not affected, in all the cases in which $p_o = p_i$. However, the cases in which p_i is smaller than p_o remain to be investigated. When this happens the ring-shaped part of the specimen is subjected to stresses similar to those in a pipe of a boiler. Since these stresses are compressive, z could be smaller for higher values of $p_o - p_i$."

Perhaps this text was not sufficiently clear. Perhaps it would have been better to include the more explicit explanation of Professor Terzaghi's tests in the published version, as it was in the original paper. The fact is that, contrary to the views expressed by Mr. Holmes, unprotected porous specimens, subject to hydraulic pressure, do not behave in the manner described in the standard textbooks on elasticity.

The difference between the results obtained by F. E. Richart, A. Brandtzaeg, Members, ASCE, and R. L. Brown at the University of Illinois, Urbana (cited by Mr. Holmes[44]) with "protected" specimens (provided with thin metal mantels), and the behavior of Professor Terzaghi's own "unprotected" test pieces, was used by Professor Terzaghi as the starting point for his uplift theory. It is not the writer's intention to discuss this theory as such, except to state that there is probably no living man whose factual results could carry greater weight than those of Professor Terzaghi. These experiments, published in 1934,[6] have demonstrated that, when the water is allowed to penetrate into the pores, the behavior of the concrete specimen under axial loads is not affected by the lateral, hydraulically applied, external pressure, referred to by Mr. Holmes.

The Terzaghi test pieces (see Fig. 40) were placed in a vessel, with water under various pressures ranging from zero to 400 kg per sq cm (that is, ten times greater than in the writer's tests), and subjected, at the same time, to an axial load varying from 2.6 tons to 25 tons. The results, at breaking point, are given in Table 15. The values in Cols. 1 are from Professor Terzaghi's experiments with unprotected specimens (that is, similar, in this respect, to the writer's specimens). There is no marked correlation between these data and the pressures, p, appearing in the first column (under which the test pieces were broken). On the other hand, the stresses in Cols. 2 are intended to portray the behavior of concrete specimens provided with thin metal jackets, such as those of the Illinois tests.[44] These values are scaled from the Terzaghi dia-

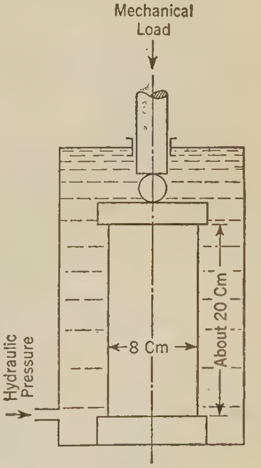

FIG. 40

grams, based upon the Illinois tests, in the well-known graphical computation devised by Mohr. The rapid rise of the breaking limit in Cols. 2 is characteristic of the three-dimensional effect referred to by Mr. Holmes. The contrast between Cols. 1 and 2, Table 15, supplies a quantitative negative reply to the question raised by Mr. Holmes.

TABLE 15.—COMPARISON OF COMPRESSIVE BREAKING STRESSES IN KILOGRAMS PER SQUARE CENTIMETER

(Test Pieces Unprotected in Cols. 1 and Protected in Cols. 2)

Pressure[a] p	SERIES I		SERIES II		SERIES III	
	(1)	(2)	(1)	(2)	(1)	(2)
0.......	580	580	532	532	66.3	66.3
100.......	630	860	..	..	67.8	353
200.......	646	1,148	534	1,102	67.0	636
300.......	404	1,431	645	1,390	66.5	922
400.......	604	1,722	454	1,678	67.1	1,210

[a] Water pressure in the test vessel, in kilograms per square centimeter.

The physical explanation of these results will be almost obvious if the analyst will ignore conventional assumptions made in textbooks on elasticity, and will follow the modern views on the subject. In other words, instead of the imaginary, continuous structureless elastic solid contemplated by the mathematicians since the time that Navier, Lamé, Cauchy, and Poisson laid down the bases of classical elasticity, the analyst must visualize the material as an elastic structure, containing both solids and voids (see Fig. 41). This has been the approach of Messrs. Fillunger, Terzaghi, Hoffman, and other modern thinkers. Fig. 1 illustrates the significant aspect of the difference between the two conceptions.

It is evident from Fig. 41 that the stress conditions in porous elastic solids must depend substantially on whether the pores are empty or full.

Fig. 42(a) is reproduced from Professor Terzaghi's paper[6] and represents schematically the conditions of the Illinois tests. In this case the external hydraulic pressure does not differ in its action from a mechanically applied load. The pressure is taken entirely and exclusively by the solid material and, therefore, causes the particles to be pressed against each other. The accumulative effect of their relative displacements results in the elastic deformation of the whole, leading, in its ultimate stage, to the three-dimensional effect exemplified by the failures of test pieces in the Illinois tests (see Cols. 2, Table 15).

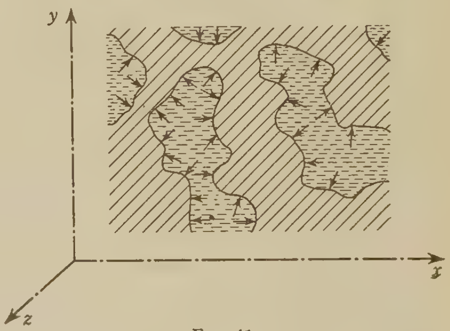

FIG. 41

The second case, illustrated by Fig. 42(b), demonstrates the principles materialized in Professor Terzaghi's experiments. The external pressure is here resisted not only by the particles but, also, by the water contained in the pores. The water thus participates in resisting the externally applied load. The effect on the solid material must therefore be much smaller; but what is even more important is the fact that, although each solid particle is subjected to a certain uniform compression, there is now no prevailing tendency for one particle to be pressed against the other (except in the proportion of $(1 - n)$ to 1, which is very small indeed). Thus, the chief factor producing strain and fracture is negligible in this case; therefore, the pressure can be raised almost indefinitely, without affecting the mechanical resistance of the material (as shown by the Terzaghi tests[6] listed in Cols. 1, Table 15).

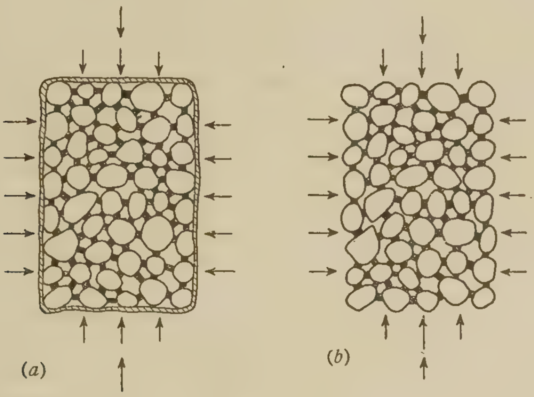

FIG. 42

The point, thus demonstrated by Professor Terzaghi—namely, the absence of the three-dimensional effect in unprotected porous materials subject to uniform hydraulic pressure—was observed by Fillunger as early as 1913;[7] except that in the latter case, instead of an axial compression, the hydraulic effect upon unprotected concrete specimens, was combined with axial tension. The conclusions were precisely the same, however, as might be seen from the following data, which represent average breaking stresses, for twelve specimens each, of:[14]

Hydrostatic pressure, in atmospheres	Tensile resistance, in kilograms per square centimeter
0	36.1
100	32.4
200	31.6

The significant conclusion is that the resistance of concrete against axial overload is practically the same at the bottom of the sea as in the free air; the fact, therefore, that in the writer's machine the specimen is surrounded by water under a certain pressure, has no elastic effect on the axial resistance of the material tested. There remains only the main axial effect of the interstitial water pressure in the pores of the material, which varies gradually from p_a at the plane of rupture to zero at the top of the specimen, and therefore produces an accumulative vertical action on the walls of the pores, causing the specimen to rupture. Under these circumstances, the writer cannot concur in the idea of introducing a secondary "three-dimensional" effect in the interpretation of his tests, in the manner suggested by Mr. Holmes.

There is another aspect of the same problem, however, which Mr. Holmes does not mention. The writer has shown that, when the pressure, p_o, in the container (see Figs. 7 and 11), and the pressure, p_i, in the inner space of the specimen, are both the same, it is not necessary to take the three-dimensional effect into account. On the other hand, when these pressures are not equal, this effect might possibly become a significant consideration (to the extent of the difference $p_o - p_i$), because the interstitial pressures would then vary, from pore to pore, in the horizontal direction, and the accumulative action of these differences would result in a general horizontal compression of the entire cylinder. This compression, in turn, might possibly affect the axial vertical tensile resistance of the specimen, according to the "three-dimensional" principle referred to by Mr. Holmes.

This explains, more clearly, the object of the specific statistical examination of the results of the tests, described in the last few paragraphs of the paper preceding "Conclusions"—namely, the calculation of the correlation and regression coefficients for Δy (vertical distance from a point to the average line in Fig. 24) and the difference $p_o - p_i$.

Had such a correlation been found to exist it might easily have been taken into account, by a correction based on the regression coefficient; but, as stated in the text, the results of the calculation showed that the correlation in question did not exist. (The calculated values of these coefficients were, respectively, 0.068 and 0.0016.) Even from this rather remote standpoint, therefore, the three-dimensional effect was found to be absent.

This conclusion is also confirmed by the results of Rudeloff's experiments,[13] represented in Fig. 25. In this particular case the specimen (which in other respects was generally similar to Fig. 3) was provided with an impermeable mantel, on the same principle as in the Illinois tests; but, in spite of that, the three-dimensional effect did not exist. The axial tensile breaking load was found to be independent of the hydraulic pressure in the container. The following average values of the tensile resistances of "protected" cylindrical specimens were obtained by Rudeloff:[13]

Hydrostatic pressure, in atmospheres	Tensile resistance, in kilograms per square centimeter
5	7.50
10	8.35
15	7.55
20	7.32

Although these are actual facts which, in themselves, constitute an exhaustive reply to the question under review (in so far as the main uplift problem is concerned), it will nevertheless be relevant to the discussion to explain the apparent contradiction, between the evidence of the Illinois tests on the one hand, and Rudeloff's and the writer's experiments on the other. In fact, the first point that comes to one's attention is that the main axial effect is compression in one case, and tension in the other two cases. The failure is therefore a "shear failure" or a "pseudo shear failure" in the Illinois tests, and a "tension failure" in Rudeloff's and the writer's experiments. Apart from all other considerations, a glance at Fig. 26 will suffice to show that the plane of fracture in the writer's tests is almost exactly perpendicular to the axial breaking force, F, which substantiates the principle of a tension failure, in contradistinction to a rupture by compression in the Illinois tests.

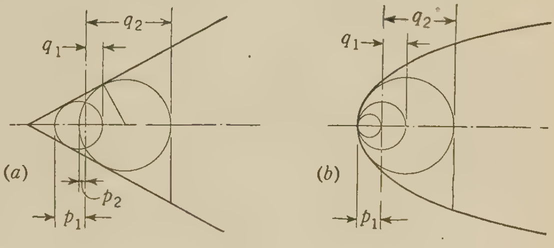

Fig. 43

The difference in the mechanics of the two cases is shown in Fig. 43, of which Fig. 43(*a*) shows the earlier views, with Mohr's critical lines being represented by straight lines. For every compressive breaking stress, q_i, acting at right angles to the main load, there is a different axial tensile limit, p_i. L. Prandtl appears to have been the first to suggest[54] that the critical lines in such diagrams should not be straight, as in Fig. 43(*a*), but curved, as in Fig. 43(*b*). It will then be obvious that the compressive limit, q_i, may vary in rather wide limits without affecting the corresponding tensile breaking stress, p_i. When both stresses are compression, however, the interdependence between the critical limits remains valid, as in the earlier diagrams.

This explains, theoretically, the actually observed difference between the Illinois tests[44] and the writer's experiments (with reference to the difference $p_o - p_i$). Thus, to represent both these experiments in the same chart as is done in Fig. 37 is definitely objectionable. From whichever point the question is considered, there is, consequently, no reason for altering the original conclusions, in order to take into account the "three-dimensional" effect. The writer is indebted to Mr. Holmes, nevertheless, for having raised a most interesting point, and thereby provided an opportunity for an explicit reply.

He is also grateful to Mr. Holmes for having explained in detail the derivation of the formulas he used in finding the empirical straight lines representing the results of his tests for every individual group of specimens. To conserve space, this development, which was based on the least-squares principle, was not given in the writer's paper, for it was believed to belong to elementary mathematics. It is rather unfortunate that in developing these formulas Mr. Holmes changed the signs in basic Eq. 2, which resulted in the second constant of the final equation becoming negative. There is no advantage in this sign convention because the force z is essentially positive, and a negative sign may be misleading.

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[54] Report to the Congress of Physicists and Physicians, by Ludwig Prandtl, Dresden, 1908.

In his exceedingly interesting contribution, Mr. Perkins raised a number of important questions. In regard to the convergence of the paths of filtration in the specimen of the paper, as opposed to the would-be parallel lines in a dam, it should be noted that the main flow lines, correlated with the process of failure in the writer's specimen, are vertical. They occur in the material enclosed in the upper lining of the test piece, and are created by, and are a function of, the full difference $p_a - 0$. On the other hand, the convergent streamlines referred to by Mr. Perkins, are only a secondary consideration, depending on the difference $p_o - p_i$, which, as already stated, is negligible as a cause of failure. Apart from that consideration, the lines of seepage are only important in relation to the pressures and it should be understood that the object of the writer's experiments was to determine the effective area on which the interstitial pressures act, and not the pressures themselves, for these pressures have been investigated frequently, and "may now be considered as belonging to the scope of a textbook" (as stated in the second paragraph following Eq. 5). In addition to the references cited in the paper,[17,18,19,20] a comprehensive investigation on the subject has been presented by I. Houk, M. ASCE, and B. A. Bakhmeteff, Hon. M. ASCE.

Regarding the low breaking stress under mechanical tension, the specific series which were marked "S" in Col. 2, Table 2, were meant to constitute conditions occurring in masonry dams (in which indeed the resistance against tension was very small) and were prepared accordingly (see heading, "Procedure and Significance of Tests"). Even in the other series, the general tendency was to approximate current field practice, which does not always coincide with the usual laboratory methods.

The more important point, however, was not in the preparation of the test pieces, but in the testing procedure. As reported in connection with Fig. 19, the effect of the time factor had a large influence upon the observed breaking stress. The dotted points at $p_a = 0$ in Fig. 19, in contradistinction to the point of intersection of the average line with the vertical axis, gave a measure of this effect. Assuming that the compound material was from 60% to 80% as strong as the corresponding laboratory mortar briquette, and that the effect of the time factor was about 2.5 (see Fig. 19), the series marked "C" (concrete) (Col. 2, Table 2) represented a nominal tensile resistance of from 21 to 28 kg per sq cm, which harmonized well with the standard values adopted in Egypt. On the other hand, the general consistency of the observed tensile resistances was confirmed by their obvious quantitative correlation with the various factors on which they must logically depend, such as the method of mixing and the intensity of ramming, the water-cement ratio, etc.

The writer wishes, further, to concur with Mr. Perkins' conclusion, that "* * * a gravity dam with a conservative design of thickness equal to 0.85 H_d * * * is safe." This, indeed, is an excellent summary of the writer's numerical results.

Turning now to Mr. Wing's discussion, the writer wishes to place on record the pleasure he had in reading and studying this pertinent contribution. The first question raised by Mr. Wing—the apparently low resistance of concrete—has already been answered; but his second point concerning the application of the "least-squares" method, deserves to be explained in greater detail. Eqs. 13 and 14, which are used for finding the constants n and z appearing in the equations of the average lines, are not symmetrical with respect to x and y. The result may, therefore, be affected by the assumed notation; it thus appears to depend on an arbitrary element.

In certain specific cases of engineering practice, this might indeed be of importance, particularly when the recorded observations of the two variables investigated, are not equally precise. As an instance, consider the case of a discharge-level curve, for a section of a large river. In applying the "least-squares" method to this case, the levels should be taken as the independent variable, x, and the curve will then be computed in such a manner that the sum of the squares of the deviations of the individual discharges, y, is a minimum.

Furthermore, when the subtangent angle of the average line ("regression coefficient") differs greatly from 45°, the smaller and larger amplitudes must correspond, respectively, to the y-axis and the x-axis. In the case under consideration, however, the precision of both variables is practically the same, and, also, the subtangent angle of the average line is almost 45°. Consequently, the choice of the variables in this case is immaterial.

The differences in the values of the coefficients in Eqs. 47*a*, 47*b*, and 47*c*, are not substantial numerically, and would not have affected the argument even if they had been physically significant; but, such as they are, they depend on an obvious inconsistency—namely, whichever of the two variables is denoted, respectively, by x or y, the entire computation, from start to finish, must be based on the same notation, whereas Mr. Wing uses the values of z, determined by the writer (for the individual series) on one assumption, but calculates the average curve for ninety-five points, on another assumption. This must naturally affect the result (particularly the second constant).

In regard to Fig. 39, attention is called to the difference between load and stress. For point loads or surface loads, this is indeed elementary and obvious, but in examining the effect of volume loads (such as gravity and uplift) it frequently occurs that one is mistaken for the other, as is the case with the convex curve on the right side of the quoted diagram. This curve is acceptable as representing the uplift pressure, but not as the stress, due to this uplift pressure.

These stresses could agree with the curve assumed by Mr. Wing only if the block of concrete above the section analyzed, were built of an infinite number of concentric rings, having no fixed contact with each other, and capable, therefore, of transmitting no vertical shear. Apart from the inherent physical impossibility of such an extreme assumption, it is also objectionable from the purely theoretical, formal standpoint, because, in that case, the compressive load which is mechanically applied to the upper lining could not be transmitted to the concrete core.

The argument represented by the compression curve appearing in the same figure, is also unacceptable, because overstressing at the edges of the section is the result of re-entrant angles, and the degree of "re-entrancy" is much greater in a standard concrete briquette, considered by E. G. Coker and L. N. G. Filon,[55] than in the writer's specimen. Had this effect indeed been significant, it must have resulted in all the writer's test pieces breaking close to the lower lining, which is by no means the case.

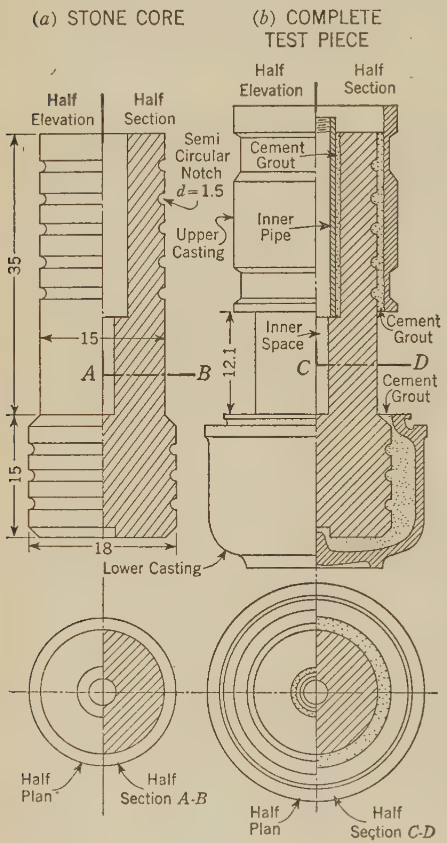
(a) STONE CORE
(b) COMPLETE TEST PIECE
Half Elevation
Half Section
Half Elevation
Half Section
Semi Circular Notch
d=1.5
Upper Casting
Cement Grout
Inner Pipe
35
15
A
B
12.1
Inner Space
C
D
Cement Grout
Cement Grout
15
18
Lower Casting
Half Plan
Half Section A-B
Half Plan
Half Section C-D

Fig. 44

Thirdly, and finally, the stress distribution in Fig. 39, even if it had been true, would not have altered the writer's conclusions, because of the flow of concrete which causes the redistribution of stress intensities when local overstressing occurs. This is the chief point of the modern theory of re-entrant angles in dams, expounded by John H. A. Brahtz.[56]

The next point in Mr. Wing's discussion concerns the time factor, in so far as it affects the conditions of seepage in actual dams. The contention is that, according to laboratory tests with specimens of concrete, the speed of the percolating water is so slow that it would require years for a filtering particle to reach the downstream face of the dam. Had this indeed been the case, the effect of the uplift pressure would have been confined to only a part of the total area of the section—close to the upstream face of the dam.

This contention cannot be accepted for the simple reason that water is actually seen to appear on the downstream face of many dams, in the form of seepage and sweating, in spite of the fact that quite a large percentage must be evaporated. This does not refer to narrow dams only, but even to such conservative designs as the Assuan and Gileppe dams.

It is suggested that Mr. Wing may possibly be overestimating the value of laboratory results, as a practical forecast of the conditions of seepage in nature. In this connection attention is called to a statistical examination of filtering tests with concrete specimens reported by M. Mary[57] in which a wide dispersion of results was observed. What actually occurs in nature seems to be that, adjacent to an almost impermeable section of the dam, there may be a very pervious mass of material, and the designer, therefore, must consider the worst conditions.

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[55] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Univ. Press, Cambridge, England, 1931, pp. 578–583.

[56] "The Stress Function and Photo-Elasticity Applied to Dams," by John H. A. Brahtz, *Transactions*, ASCE, Vol. 101, 1936, p. 1240.

In regard to the next point raised by Mr. Wing—the importance of the filtering in the rocky foundations beneath the dam, as compared to seepage in the dam itself—the writer wishes to explain that his method is not confined to compound materials only, such as those used in building dams, but applies also to natural rocks forming the foundations. The specimen is then prepared in the manner shown in Fig. 44, the core being made of the rock to be tested, and grouted in the respective castings. The results obtained in this manner in the writer's machine for Cairo limestone are shown in Fig. 45. These experiments were conducted after the completion of the investigation described in the paper, and, therefore, were not included in the ninety-five points analyzed therein; but they are consistent with the earlier tests.

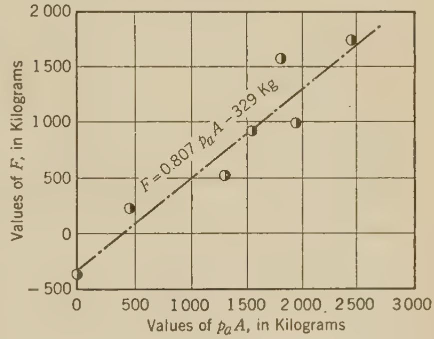

FIG. 45

It was rather interesting to observe that a core prepared from the ignaceous granite of Assuan, and tested in the same manner, proved to be the only material not affected by internal uplift. The writer agrees, nevertheless, with Mr. Wing that, because of the numerous fissures and decomposed seams existing in natural foundations, the uplift must be included in the design even in this case.

Acknowledgment.—For the experiments on which this paper is based, in June, 1946, the writer was awarded the degree of Philosophiae Doctor by Fouad First University, Giza, Cairo, Egypt.

MOHAMED AHMED SELIM,[58] ASSOC. M. ASCE.—By his discussion, Mr. Richards has contributed to the practical application of the electric-analogy method in solving problems of two-dimensional and three-dimensional nature. The writer might add that such an application was made at the Laboratory of the Mechanical Engineering Department, University of California at Berkeley, during and after the period of his research; namely, the fall of 1940. Two problems were set up, one concerned with the velocity of air currents and their effect on weather forecasts, and the second with the oil stored in petroleum reservoirs. It is unfortunate that the results of such work have not been presented to the engineering world because of wartime restrictions.

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[57] "Note on the Accuracy of Permeability Tests," by M. Mary, *Bulletin Periodique No. 7*, Commission Internationale des Grands Barrages, December, 1938.

[58] Lecturer "A," Irrig. Dept., Faculty of Eng., Fouad First Univ., Giza, Egypt; formerly Instructor in Mech. Eng., Univ. of California, Berkeley, Calif.

Meanwhile, much work has been done since 1933 by the Ministry of Public Works in Egypt in connection with the lengthening of floor for Assiut Barrage on the Nile. Recent books have shown an increasing confidence in the electric-analogy method of solving the difficult problems of flow through porous media.

The writer is very grateful for Mr. Richards' interest and comments.

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TRANSACTIONS

Paper No. 2305

STRENGTH OF THIN STEEL COMPRESSION FLANGES

BY GEORGE WINTER,[1] M. ASCE

WITH DISCUSSION BY MESSRS. FRED T. LLEWELLYN, JACOB KAROL, ROBERT L. LEWIS AND DWIGHT F. GUNDER, L. C. MAUGH AND L. M. LEGATSKI, BRUCE G. JOHNSTON, EDWARD L. BROWN AND DON S. WOLFORD, AND GEORGE WINTER.

SYNOPSIS

The production of light structural steel shapes from sheet steel, by cold forming and spot welding, necessitates the development of special design methods adapted to the peculiarities of such members. One of the questions of most practical importance is that of the strength and behavior of thin, wide flanges in compression. This paper reports the results of, and conclusions drawn from, an extensive experimental investigation of this problem. The strength, general behavior, and deformation of two types of structural elements are investigated—(A) members with compression flanges both of whose longitudinal edges are stiffened adequately (such as top flanges of inverted U-beams); and (B) members with compression flanges only one of whose longitudinal edges is stiffened (such as either half of a flange of an I-beam). Results of these tests are evaluated in terms of formulas and charts by which the strength, deformation, and general behavior of such members can be predicted under load conditions.

GENERAL

The thickness of ordinary hot-rolled structural steel sections cannot be decreased beyond a certain minimum, because of limitations inherent in the rolling process. This fact, in the past, has all but prevented the economical use of steel for small-scale structures such as residences, small commercial and manufacturing buildings, and for many secondary members such as floors with moderate loads in larger structures. In view of the low loads occurring in such members, available rolled sections are unduly heavy for the purpose and therefore are unable to compete with other materials more adaptable to such conditions.

NOTE.—Published in February, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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However, great advances have been made in forming structural shapes from sheet steel which can be obtained in all desirable thicknesses.[2] Outstanding examples of this type are the many kinds of steel roof and floor decks that are manufactured in various shapes and with thicknesses ranging from 14 gage (0.0766 in.) to 24 gage (0.0245 in.). The development of automatic spot welding has further increased the possibilities of using sheet steel for structural purposes. It is now possible, technically and economically, to form extremely thin I-shapes by joining two cold-formed, sheet-steel, channels by automatic spot welding in the web. Other shapes, such as thin-walled U-beams and box beams, likewise can be formed from cold-bent sheet steel. This development opens new applications for the use of steel in buildings and other structures. The possibilities of mass production inherent in the processes of cold forming and of spot welding make such members particularly suitable for prefabrication.

A number of design practices developed over the years in connection with the usual rolled sections will need revision in their application to thin-walled members. Although the problems of this kind are familiar in airplane manufacturing, they are new to the designer of the conventional type of steel structures. One of the most important of these problems is that of the behavior of thin steel members in compression. Thickness is a factor of very minor importance in members subject to tension; but thin sheets in compression will buckle at rather low stresses. This fact must be taken into account in the design of the compression flanges of thin-walled beams or in the design of the entire cross section of thin-walled columns. An investigation of the behavior and strength of such thin-walled compression members is reported in this paper. The findings are based in part on those of other investigators in this field, particularly in airplane design, and in part on tests reported in detail hereinafter.

Two types of compression flanges were investigated: (A) Flanges that are stiffened along both longitudinal edges such as the compression flanges of thin-walled I-beams with stiffening lips at the outer edges and the flanges of inverted U-beams and box beams which are prevented by the two webs from buckling out of their initial plane along both joints (that is, along both longitudinal edges); and (B) flanges that are stiffened only along one longitudinal edge such as either half of the compression flange of a thin-walled I-beam, stiffened only by the web, but free to distort at the outer, unsupported edge. The ranges of dimensions for the specimens (identified herein as types A and B) are given in Table 1. Since the behavior of these two types of flanges was found to be fundamentally different, the two types will be discussed separately.

TYPE A. COMPRESSION FLANGES STIFFENED ALONG BOTH LONGITUDINAL EDGES

Notation.—The letter symbols introduced in this paper are defined where they first appear and are assembled alphabetically, for convenience of reference, in the Appendix. Discussers are requested to adapt their comments to the notation as published herein.

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[2] "Light-Gage Steel for Peacetime Building," by Milton Male, *Engineering News-Record*, October 18, 1945, p. 525.

Equivalent Width of Thin Flanges.—The thin rectangular plate subjected to compressive forces in the plane of the plate has been analyzed extensively by G. H. Bryan and others. The theoretical critical stress at which buckling occurs was thus found to be:[3]

$$s_c = \kappa \frac{\pi^2 E}{12\,(1 - \mu^2)} \left(\frac{t}{b}\right)^2 \quad \dots\dots\dots\dots(1)$$

in which E is the modulus of elasticity; μ is Poisson's ratio; t is the thickness of plate; b is the width of plate perpendicular to the direction of compression; and κ is the numerical factor depending upon the ratio of length to width of the plate and the conditions of edge support.

TABLE 1.—RANGE OF FLANGE DIMENSIONS (IN INCHES) FOR THE TWO TYPES OF STRUCTURAL ELEMENTS INVESTIGATED

Type	WIDTH, b		THICKNESS, t		DEPTH, h		RATIO, b/t	
	From:	To:	From:	To:	From:	To:	From:	To:
A	3.40	10.10	0.0237	0.1478	1.49	8.00	14.3	170.0
B	1.43	12.0	0.0368	0.1077	2.0	8.00	9.3	108.0

These investigations are based on the assumption of ideally plane plates loaded precisely in the central plane. They use the "small deflection theory of plate bending"; that is, they assume that the significant deflections at buckling are of the order of the thickness of the plate, or less. In principle the formulas are derived on the same grounds as is the Euler formula for column buckling.

Experiments, with plates in compression, conducted by several investigators,[4,5] reveal, however, a fundamental difference between the practical significance of the Euler critical stress for columns and the critical stress given by Eq. 1 for plates. Whereas long columns actually fail at, or slightly below, the Euler stress, plates supported longitudinally along both edges have been found to carry compressive stresses considerably above the critical. L. Schuman and G. Back[4] summarize their experiments qualitatively as follows:

> "It is seen that for only the very narrow and thick plates do the Bryan loads approach or exceed the maximum loads found in the tests. For the wide, thin plates, the Bryan load is as low as 1/30 of the maximum load * * *.[6]
>
> "In most of the plates the buckling [that is, the formation of waves] was gradual * * * showing no sudden change. In some of the thick and narrow specimens, however, there was no appreciable buckling until the load approached the maximum. Owing to lack of ideal conditions, such as initial curvature, all plates buckled [but did not fail] before the Bryan load was reached."[7]

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[3] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 324.

[4] "Strength of Rectangular Flat Plates Under Edge Compression," by L. Schuman and G. Back, *Technical Report No. 356*, National Advisory Committee for Aeronautics, 1931.

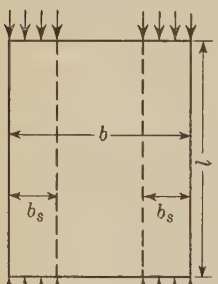
[5] "The Ultimate Strength of Thin Flat Sheet in Compression," by E. E. Sechler, *Publication No. 27*, Guggenheim Aeronautics Laboratory, California Inst. of Technology, Pasadena, 1933.

[6] "Strength of Rectangular Flat Plates Under Edge Compression," by L. Schuman and G. Back, *Technical Report No. 356*, National Advisory Committee for Aeronautics, 1931, p. 528.

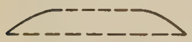
[7] *Ibid.*, p. 531.

Thus, while in the column field the Euler load is actually the maximum load a column can reach, for thin plates the Bryan load[8] is found to be of little practical consequence, except for very narrow ones.

Physically, the difference between the significance of the critical loads for free columns and for edge-supported plates is easily visualized as follows: Once a column starts deflecting at or near the Euler load, the deflection continues to increase because of the increasing bending moment caused by that same deflection. The column, then, fails very quickly, literally "getting away from its load" by unrestrained motion. On the other hand, in an edge-supported plate deflections are possible only in the central parts, the edges and parts adjacent to them being kept straight by the supports. Thus, the plate cannot "get away from its compression load" by unrestrained deflection such as that in columns. The central, more highly distorted, regions of the plate decrease in their resistance, thus throwing more of the total compressive force toward the stiffened edges. The plate reaches the limit of its carrying capacity only when these stiffened outer parts are stressed to the yield point. This action occurs at a load higher than the critical—that is, higher than that load at which, theoretically, small deflections start to occur in the plate. For these reasons, in columns, a small deflection at the Euler load produces almost immediate failure, whereas in plates small deflections merely result in a redistribution of the compressive stresses across the width of the plate. The "small deflection" (Bryan) theory of plate buckling, therefore, gives results of only very limited practical significance.



(a) PLAN



(b) CROSS SECTION

FIG. 1.—THIN PLATE IN COMPRESSION (ACCORDING TO THEODOR VON KÁRMÁN, M. ASCE)

An exact mathematical treatment of plates in compression on the basis of the "large deflection theory" is extremely difficult. Attempts in this direction have failed to give practically significant results, except for the one case of the circular plate uniformly compressed around the perimeter.[9] However, in 1932 Theodor von Kármán,[10,11] M. ASCE, suggested a semi-empirical approach to this question which, subsequently elaborated by others, proved to be extremely useful. Professor von Kármán took account of the fact that, in a plate compressed beyond the Bryan stress, the central strip (Fig. 1), most heavily distorted by wave formation, cannot be counted on to carry an appreciable part of the compressive load. On the other hand, two strips, adjacent to the edges, are held in line by the edge stiffeners and will withstand most of the compression. Thus, with only the two strips of width b_s each being effective, the actual plate of width b is equivalent to the fully effective narrower plate of equivalent width $b_e = 2\,b_s$ (Fig. 1). By rather intuitive analytical reasoning this width was assumed to be

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[8] "On the Stability of a Plane Plate Under Thrust in Its Own Plane," by G. H. Bryan, *Proceedings*, London Mathematical Soc., Vol. 22, 1890, pp. 54–67.

[9] "Buckling of the Circular Plate Beyond the Critical Thrust," by K. O. Friedrichs and J. J. Stoker, *Journal of Applied Mechanics*, March, 1942, p. A-7.

[10] "The Strength of Thin Plates in Compression," by Theodor von Kármán, E. E. Sechler, and L. H. Donnell, *Transactions*, A.S.M.E., Vol. 54, 1932, p. 53.

[11] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 396.

$$b_e = 1.9\,t\sqrt{\frac{E}{s_y}} \quad \text{(2a)}$$

in which s_y is the yield point of the material. To test the validity of this hypothesis, tests were made by E. E. Sechler[5,12] on thin plates of various metals. He found Eq. 2*a* to hold except that instead of the fixed constant 1.9 a variable coefficient C resulted in better agreement with the tests. Thus:

$$b_e = C\,t\sqrt{\frac{E}{s_y}} \quad \text{(2b)}$$

This coefficient was found to depend on the parameter $\sqrt{E/s_y}\,(t/b)$. Fig. 2(*a*) shows the results of Mr. Sechler's tests, representing the experimental values of C plotted against this parameter. (The considerable scattering noticeable in Fig. 2(*a*) is a feature common to all compression tests on thin plates and is probably caused by the influence of inevitable deviations from true shape of such specimens.) Only for the very small values of the parameter (that is, for extremely wide and thin plates) does C approach 1.9; for the

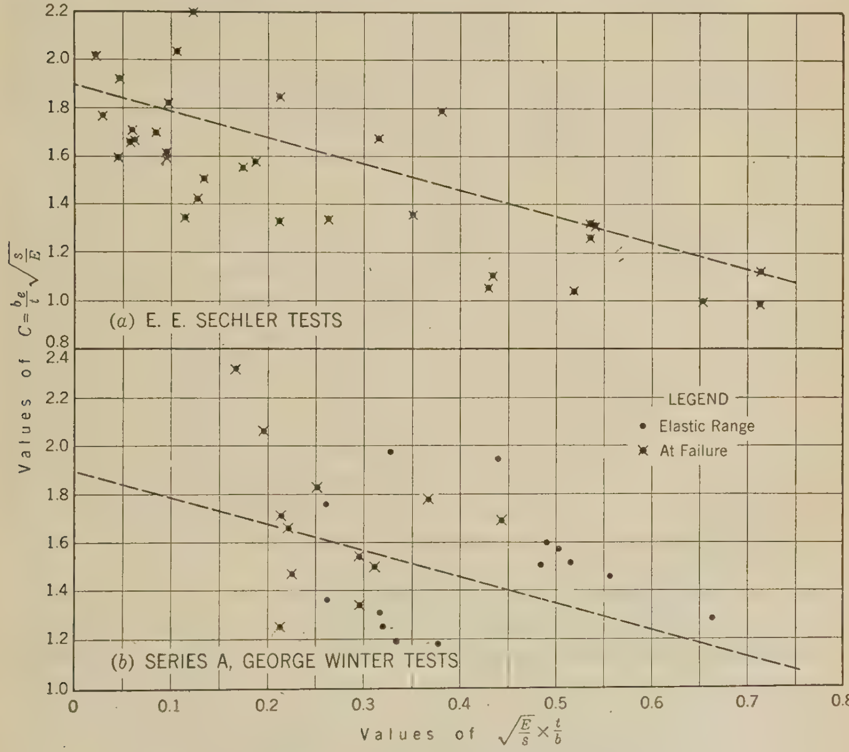

FIG. 2.—EXPERIMENTAL DETERMINATION OF EQUIVALENT WIDTH

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[12] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 402.

remainder of the range a smaller C is applicable. In airplane design it is common practice either to use an average value of $C = 1.7$ or to determine an appropriate value from the results of Mr. Sechler's tests.[13]

The tests by Messrs. Schuman and Back and those by Mr. Sechler were made on individual thin compression plates. To determine whether the equivalent width approach could be recommended for wider use in the field of structural engineering, it appeared desirable to institute further tests to investigate: (1) Whether thin compression plates representing component parts of structural members (such as compression flanges of thin-walled beams) performed in the same way as disjointed, individual plates; (2) whether a simple sufficiently accurate expression could be found for determining the equivalent width for design purposes; and (3) whether such an expression applied to the state of ultimate failure only or whether the same, or similar, expressions could be used at stresses below the yield point. (The original information by Messrs. von Kármán and Sechler leading to Eqs. 2 pertained to failure stresses only.) The test results are summarized in Fig. 2(*b*).

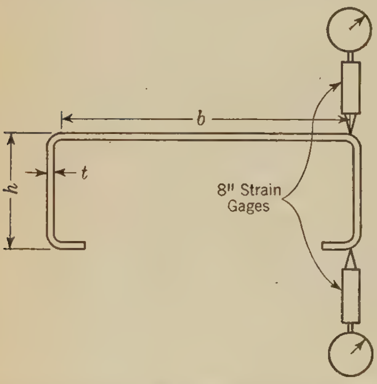

FIG. 3.—SHAPE OF INVERTED U-BEAMS OF TEST SERIES A

In test series A twenty-five inverted U-beams representing thirteen different types of sections of the general shape shown in Fig. 3 were tested as simple beams with quarter-point loading. In these specimens b varied from 3.40 in. to 5.33 in., h from 1.49 in. to 2.46 in., and t from 0.0237 in. to 0.0589 in. The span was 80 in. for all tests, equal loads being applied 20 in. from either support. Two 8-in. strain gages were used for strain measurements, mounted on the outer surfaces of the specimens, as close as possible to the web (see Fig. 3). In addition to strains, midpoint deflections were recorded. Mechanical properties of the various sheet steels were determined from tension tests on coupons cut from the steels from which the beams were formed.

The effective width of the compression flanges of each specimen was determined at two different loads: (*a*) In the wholly elastic range—that is, before the bottom (tension) flange began to yield; and (*b*) at, or as close as possible to, the failure load. For loads in the elastic range, the position of the neutral axis was determined from the strain readings. Once the location of the axis is known, the equivalent area, and subsequently the equivalent width, of the top flange is determined from the usual condition that the moment of the area of the cross section about the axis is zero; that is,

$$\Sigma(A_n y_n) = 0 \qquad (3a)$$

in which A_n represents the subareas into which the section is divided for convenience of computation; and y_n, the distances of the centroids of these areas from the neutral axis. In Eq. 3*a* all quantities are known except the effective area of the compression flange and, consequently, it is possible to determine the equivalent width of that flange. For determining the equivalent width at the failure load the position of the neutral axis was again determined from strain measurements. In this case strains were measured as close to failure as possible, at loads which were 5% or less below the ultimate. However, since the stress distribution at failure is that in Fig. 4(*b*) rather than that in Fig. 4(*a*),[14,15] Eq. 3*a* cannot be used in this case. Since the stress at this stage of loading is of uniform yield-point intensity over the entire section (compression above the axis, tension below), the condition that the sum of all stresses over the entire section is zero yields the simple expression,

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[13] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1943, Vol. 1, pp. 291 and 329.

$$\Sigma A_n = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

in which A_n is positive for the areas below the axis and negative for those above. Eq. 3*b*, then, can be solved for the effective area of the compression flange. (This method tacitly assumes equal yield points in tension and compression, an assumption which cannot be checked experimentally. However, the consistency of results obtained on this basis seems to justify such an assumption.)

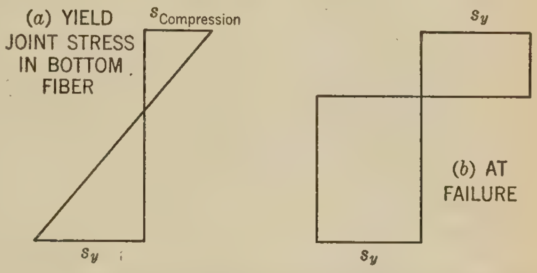

Fig. 4.—Stress Distribution in an Unsymmetrical Section

Once the equivalent widths were obtained experimentally, the coefficients C were computed from Eq. 2*a*. These coefficients are plotted in Fig. 2(*b*) in the same manner as are Mr. Sechler's data in Fig. 2(*a*). Comparing these two sets of data, the following conclusions can be drawn:

1. The experimental points in Fig. 2 were obtained by three fundamentally different methods. Mr. Sechler's results (Fig. 2(*a*)) refer to ultimate loads of individual plates. On the other hand, the points in Fig. 2(*b*) refer to compression flanges representing integral parts of structural shapes; and, whereas half of the latter refer to ultimate loads (at the yield point), the other half are obtained at stresses far below the yield point, ranging from 7,700 lb per sq in. to 21,500 lb per sq in., depending on the shape of the specimen. Despite this diversity, the agreement between the results of the two investigations is remarkably close.

2. Whereas Mr. Sechler's results (Fig. 2(*a*)) refer to failure stresses only, the fact that the points in Fig. 2(*b*) obtained at low stresses are located in the same general way as those obtained at the ultimate load indicates that Eq. 2*b*

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[14] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N.Y., 2d Ed., 1941, Pt. 2, Fig. 234, p. 371.

[15] Discussion by George Winter of "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, ASCE, Vol. 105, 1940, Fig. 24, p. 674.

can be used not only at the yield point but likewise in the elastic range, that is:

$$b_e = C\,t\sqrt{\frac{E}{s}} \quad \text{(4)}$$

in which s is any stress at or below the yield point. (It is likely that at very low stresses the equivalent width so obtained is somewhat on the conservative side.)

Once the relationship between the coefficient C and the parameter indicated in Fig. 2 is rather firmly established experimentally, it appears desirable to express the equivalent width b_e in a simple explicit formula rather than to determine it by using Fig. 2 in conjunction with Eq. 4. For this purpose the averaging straight line was drawn in Fig. 2. This line was so drawn as to represent the average values of both Mr. Sechler's tests (Fig. 2(*a*)) and the present tests (Fig. 2(*b*)). The experimental values scatter rather regularly around that line. The equation of this averaging straight line is

$$C = 1.9 - 1.09\sqrt{\frac{E}{s}}\left(\frac{t}{b}\right) \quad \text{(5)}$$

By substitution of this expression for C in Eq. 4, the following equation is obtained for the equivalent width b_e:

$$b_e = 1.9\,t\sqrt{\frac{E}{s}}\left[1 - 0.574\left(\frac{t}{b}\right)\sqrt{\frac{E}{s}}\right] \quad \text{(6)}$$

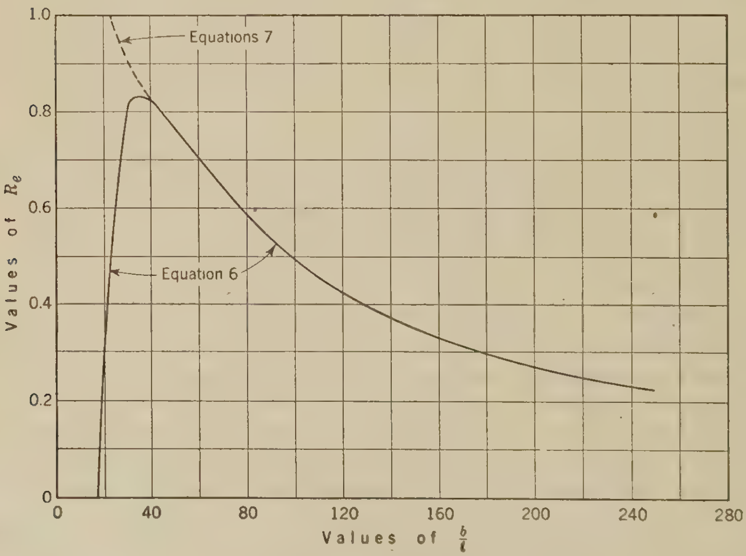

FIG. 5.—RATIO $R_e = b_e/b$ FOR THE PARTICULAR CASE OF s = 30,000 LB PER SQ IN. AND E = 30,000,000 LB PER SQ IN.

Thus, b_e, expressed as a multiple of the plate thickness t, depends on two parameters, the thickness-width ratio t/b of the flange and the strain δ at the supported edge (since $E/s = 1/\delta$).

The relationship expressed by Eq. 6 is shown graphically in Fig. 5 for the particular case $E = 30{,}000{,}000$ lb per sq in. and $s = 30{,}000$ lb per sq in. In Fig. 5 the ratio of the equivalent to the actual width, $R_e = b_e/b$, is plotted against the width-thickness ratio of the flange, b/t. In the medium and high range of b/t the equivalent width ratio decreases with increasing b/t, as would be expected. For the low range of b/t, however, b_e computed from Eq. 6 decreases with decreasing b/t, which is obviously impossible physically. It must be concluded, therefore, that Eq. 6, which was established from tests in the medium and high range of b/t, is limited in its application to that same range. (In series A the range of b/t tested extended from 64 to 170.)

To investigate the behavior of compression flanges with rather small width-thickness ratios b/t, a special series B of tests was instituted. The specimens were built-up I-beams formed by two channels, of the general shape shown in Fig. 6, with width-thickness ratios ranging from 14.3 to 56.0. The depths of these specimens varied from 3.94 in. to 7.97 in.; the widths, from 3.93 in. to 8.01 in.; and the thicknesses, from 0.0599 in. to 0.1478 in. For each half of the flange, stiffening along one edge is provided by the web, whereas the other edge is stiffened by the turned-down lip, in contrast to the U-beams of series A, where both longitudinal edges were stiffened by webs. Beams were loaded in a manner similar to that of series A except that the two equal loads were applied at distances of about 0.4 L from the respective supports rather than at the quarter points of the span L. The magnitudes of the equivalent widths were determined experimentally from strain-gage readings in the same manner as discussed previously. The results of this investigation are given in Table 2(a) where average results of two tests of each type of beam are recorded. In Table 2, b is the free, horizontal width of half of compression flange measured between toes of transition radii; t is the thickness of metal; and s is the stress, in pounds per square inch, at which strain measurements were made to determine the test values of b_e. The "test" values of b_e (Col. 4, Table 2(a)) are those determined from strain measurements; and the "chart" values are those determined from Fig. 7 for the b/t-values and the s-values of the table.

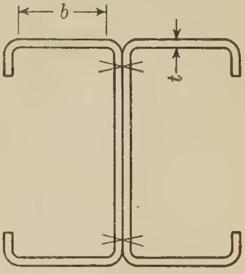

FIG. 6.—SERIES B

The accuracy of the determination of b_e is somewhat impaired in this series by the following fact: In these beams the compression flanges represent a rather small percentage of the total cross-sectional area. For this reason a moderate change in the equivalent area of the compression flange will change the position of the neutral axis by a much smaller percentage than the area itself changes, and will therefore cause a very small change in the strains recorded by the gages. Consequently, the accuracy of determining b_e from such tests is much smaller than the accuracy of the strain-gage readings proper.

The most important conclusion to be drawn from the b_e-values in Col. 4, Table 2(a), is that flanges of this type are fully effective ($b_e = b$) for values of b/t smaller than from about 20 to 30. From these values upward, the equivalent widths b_e are consistently smaller than the actual width b. For small

TABLE 2.—TESTS ON I-BEAM SECTIONS IN THE LOW RANGE OF b/t

$\frac{b}{t}$	Type[a]	(a) EQUIVALENT WIDTHS (SERIES B)				(b) ULTIMATE MOMENTS (SERIES B AND C)		
		s (lb per sq in.)	b_e		Col. 5 / Col. 4	s_y	M_1/M_t	M_2/M_t
			Test	Chart				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
14.3	I-1	26,300	14.1 t	14.3 t	1.01	35,700	0.98	0.98
16.3	I-2	27,200	16.5 t	16.3 t	0.99	33,100	0.94	0.94
16.4	I-3	26,600	16.6 t	16.4 t	0.99	33,100	0.93	0.93
19.2	I-4	27,600	19.2 t	19.2 t	1.00	35,100	0.90	0.90
22.9	I-5	27,200	21.8 t	22.9 t	1.05	33,100	0.95	0.95
23.6	I-6	26,300	23.4 t	23.6 t	1.01	36,200	0.95	0.95
24.0	I-7	29,200	22.6 t	24.0 t	1.06	35,100	0.95	0.95
28.9	I-8	27,400	27.4 t	27.0 t	0.98	30,200	0.98	0.95
32.0	I-9	27,200	31.7 t	29.0 t	0.91	36,200	1.06	0.99
33.5	I-14[a]	....	....	....	....	37,900	1.07	1.00
36.0	I-15	....	....	....	....	36,400	0.84	0.76
38.3	I-10	25,600	37.2 t	32.5 t	0.87	30,200	1.02	0.92
42.6	I-11	27,400	39.6 t	35.0 t	0.88	37,300	1.14	1.02
45.0	I-12	23,500	40.8 t	37.0 t	0.90	30,300	1.04	0.92
49.5	I-16[a]	....	....	....	....	37,900	1.20	1.04
49.9	I-17	....	....	....	....	36,400	0.98	0.84
51.6	I-18[a]	....	....	....	....	32,200	1.08	0.95
56.0	I-13	28,000	48.8 t	41.5 t	0.85	37,300	1.17	0.97
77.7	I-19[a]	....	....	....	....	37,900	1.55	1.12
86.6	I-20[a]	....	....	....	....	32,200	1.47	1.10

[a] Average results of two tests of each type of beam, except types I-14, I-16, I-18, I-19, and I-20 which are the average of three tests each.

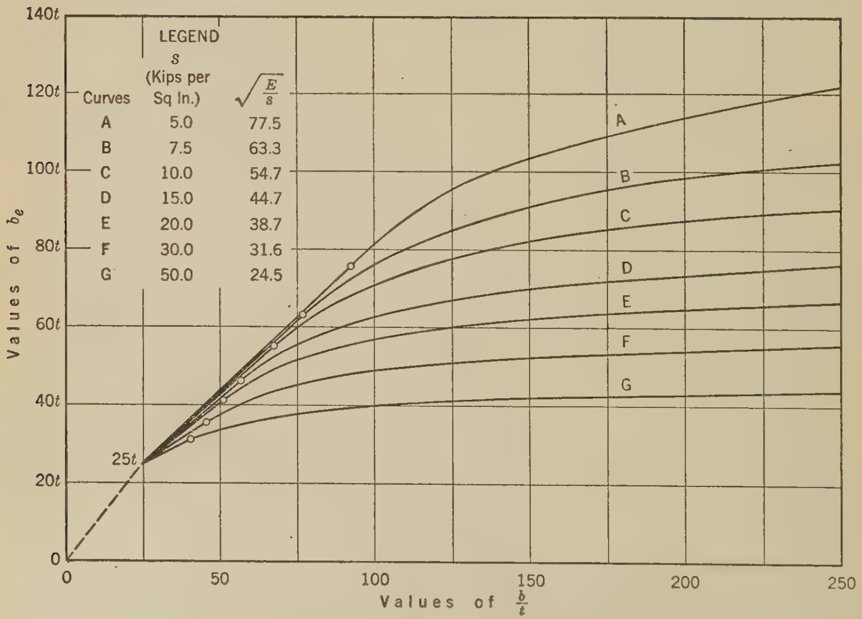

FIG. 7.—CHART FOR DETERMINING THE EQUIVALENT WIDTH b_e

values of b/t it appears reasonable, therefore, to draw a transition curve from $b/t = 25$, for $R = 1$, tangent to the curve representing Eq. 6 as is done in broken lines in Fig. 5. It is not unlikely that a more extensive investigation of b_e in this range of b/t would show a somewhat different shape of this transition curve. However, since in this region compression flanges are nearly fully effective (b_e close to b), the details of the shape of this transition line are of little practical consequence.

In practical design the use of Eq. 6 in the medium and high range of b/t in conjunction with the transition line in the low range just discussed is somewhat cumbersome. For this reason the chart in Fig. 7 was drawn for design purposes. The curves in Fig. 7 allow the equivalent width b_e to be read directly for any ratio b/t and for any practically important value of the stress s. The curved part of each line in this chart represents the values obtained from Eq. 6 and the straight parts in the low range were drawn as lines tangent to the curves and ending at $b_e = 25$ for $b/t = 25$. (The point of tangency, indicated by small circles on Fig. 7, is found from

$$\left(\frac{b}{t}\right)_1 = \frac{1.0906\,\frac{E}{s} + \sqrt{\left(1.0906\,\frac{E}{s}\right)^2 - 27.265\,\frac{E}{s}\left(1.9\sqrt{\frac{E}{s}} - 25\right)}}{1.9\sqrt{\frac{E}{s}} - 25} \quad \text{..}(7a)$$

and the equation of the straight lines in the range of low b/t is

$$b_e = \left[\frac{1.0906}{(b/t)^2_1}\,\frac{E}{s}\left(\frac{b}{t} - 25\right) + 25\right] t \ldots\ldots\ldots\ldots\ldots (7b)$$

in which $(b/t)_1$ is defined in Eq. 7a.) The chart is drawn for steel members, with $E = 30{,}000{,}000$ lb per sq in., but it can also be used for compression flanges made of other metals. In this case, instead of using the stress s indicated by the respective curves, the value $\sqrt{E/s}$ should be determined for the particular case and the corresponding curve found in the chart.

For the medium and high range of b/t the validity of Eq. 6, and of the curves derived from it, appears to be well established by the tests summarized in Fig. 2. Nevertheless, it is desirable to obtain more information on the low range—that is, on the validity of the straight-line parts of the respective curves. For this reason for all tests given in Table 2(a), the equivalent width ratios b_e, as obtained from Fig. 7, were determined and entered in Table 2(a). A comparison of these values (Col. 5, Table 2(a)) with those obtained from the tests (Col. 4, Table 2(a)) shows, with a very few exceptions, good agreement throughout this range (see Col. 6, Table 2(a)). Any significant deviations are on the conservative side; that is, the equivalent width, determined from Fig. 7, is equal to or somewhat smaller than that obtained from tests. For this reason it is believed that the chart, over its entire range, can safely be recommended for design purposes.

For practical use the findings incorporated in Fig. 7 should enable the designer to determine: (a) The ultimate load for a member of given configuration and material; and (*b*) for such a member, the deformation for a given loading, particularly the deflections of transversely loaded beams. In Tables 2(*b*) and 3 test results are reviewed in the light of these practical requirements.

Ultimate Loads.—To ascertain whether the equivalent-width concept permits a satisfactory prediction of ultimate loads, Table 2(*b*) gives the comparison of ultimate loads obtained from tests with those computed on the basis of the full, unreduced cross section and with those obtained by considering the equivalent width of the compression flanges. Tension tests were made on specimens cut from the steel from which the beams were formed. Equivalent widths were then determined from Fig. 7 for the yield points so obtained. Section moduli S were computed both for the full, unreduced cross section and for the reduced cross section—the latter by using the equivalent instead of the actual width of the compression flanges. The ultimate moments were then obtained from

$$M_u = s_y S \qquad (8)$$

Table 2(*b*) contains not only the results of series B but also those of a special, additional series C of I-beam tests (types I–14 to I–20). These beams were shaped in a manner similar to those of series B except that the tension flange was made about 20% wider than the compression flange and the beams were loaded at the third points. The depth of all beams of series C was 8 in.; the widths of the top flanges varied from 4.50 in. to 10.10 in.; and the thicknesses, from 0.0447 in. to 0.0755 in.

In Cols. 8 and 9, Table 2(*b*), M_1 is the ultimate moment computed from the section modulus of the full, unreduced cross section; M_2 is the ultimate moment computed from the section modulus obtained by substituting the equivalent width of the compression flange for the actual width; M_t is the average test-failure moment of the "identical" beams of the given type.

It is to be noted from Col. 9, Table 2(*b*), that the behavior of beams I–15 and I–17 is rather different from that of all others in that these beams failed at moments considerably above both M_1 and M_2. These two beams were formed of the same steel and this steel was the only one tested which showed the following peculiarity in the tension tests: Instead of revealing a definite yield point by the usual criterion of arrest of load or drop of the beam, this steel showed only a more rapid increase of strain with increasing stress at about 36,400 lb per sq in. In other words, the stress-strain curve of this steel had no horizontal part at the yield point. The absence of this horizontal part seemingly accounts for the fact that the outer fibers were stressed considerably higher than the rather indefinite yield point, before the beams actually failed.

A study of Table 2(*b*) reveals that, in the low range of b/t, the difference between M_1 and M_2 for this type of I-beams is rather insignificant. This is caused by the fact that the equivalent width in this range is very close to the actual (b_e is close to b in Fig. 7). Since, in addition, the compression flanges are only a small part of the total cross section, a small change in width changes the section modulus only insignificantly. In this range (say, to $b/t = 40$) both M_1 and M_2 are rather close to M_t and somewhat on the conservative side. (For other types of cross sections, if the compression flange represents a more substantial part of the entire area, the reduction in width may become practically important even in the low range of b/t.)

For b/t larger than about 40, M_1 becomes significantly and increasingly larger than M_t, whereas M_2 agrees with M_t rather satisfactorily throughout this range. If the values for the beams with b/t larger than 30 are averaged (with I–15 and I–17 omitted for reasons previously cited) the following average values are obtained: $M_1/M_t = 1.18$ and $M_2/M_t = 1.00$. In other words, if beams in this range were designed on the basis of the full, unreduced cross section, they would be underdesigned by an average amount of 18%, whereas, if they are designed on the basis of the equivalent width, a satisfactory and safe design would be obtained.

A similar comparison of ultimate moments for the inverted U-beams of series A is somewhat complicated by the nonsymmetry of the cross sections. Since the neutral axes of such sections are rather close to the compression flange, the stress in this flange will be rather low at loads at which the tension flange reaches yield-point stress. For this reason the beam will continue to carry increasing loads until the top flange, and with it practically all the cross section, is stressed to the yield point. The stress distribution is that of Fig. 4(b) and only then does the beam fail. (This somewhat novel concept of failure of steel beams has gained wide acceptance in recent years and was verified extensively by E. Volterra[16] and others. For the I-beams of series B and C discussed previously, this concept is of no importance because (a) these sections are closer to symmetry so that both flanges reached yield-point stress at about the same load; and, more important, (b) once the compression flange and a considerable part of the turned-down lips begin to yield, the latter become unstable and cause the ultimate breakdown of the beam. In the beams of series A, however, restraint of both edges of the compression flange is provided by complete webs which, therefore, do not become unstable until practically all the cross section reaches yield-point stress.) For these reasons the ultimate moments for series A are computed on the basis of the stress distribution of Fig. 4(b) by methods outlined previously.[14,15]

The symbols in Table 3(a) are the same as those in Table 2(b) except that M_1 is the ultimate moment computed for the stress distribution of Fig. 4(b) for the full, unreduced cross section; and M_2 is the ultimate moment computed in the same manner but by substituting the equivalent width of the compression flange for the actual width. A study of Table 3(a) reveals that M_1 in all cases is considerably larger than M_t, whereas M_2 in all cases is in rather satisfactory agreement with the test value. The average values are: $M_1/M_t = 1.36$; $M_2/M_t = 1.03$.

Summarizing the results of these seventy-one tests on thirty-three different types of I-beams (Table 2) and inverted U-beams (Table 3), it is notable that a conventional design procedure based on the full, unreduced cross section would result in definitely, and in many cases highly, unsafe values whereas an analysis based on the equivalent width concept (Fig. 7) results in very satisfactory agreement between computed and observed values.

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[16] "Results of Experiments on Metallic Beams Bent Beyond the Elastic Limit," by E. Volterra, *Journal*, Institution of Civ. Engrs. (London), Vol. 20, 1942–1943, p. 1.

Deflections.—Any rational method should enable the designer to predict not only the strength of his structure, but also the magnitude of its deformation under load. In this case, then, the equivalent width approach should furnish a method of predicting deflections of beams with thin, wide compression flanges. For such a prediction, it is only logical to compute the moment of inertia on the basis of the equivalent width of the compression flange, rather than on the basis of the full width, the equivalent width being determined from Fig. 7 for the stress corresponding to the load at which the deflection is desired. This approach, however, is somewhat approximate for the following reason: The stress in the compression flange of a freely supported beam is not constant along the span; it varies from zero at the supports to a maximum at or near the center. Fig. 7 indicates that the equivalent width decreases with increasing stress. Consequently, a beam with constant, thin-walled cross section actually has a varying effective moment of inertia, depending on the local magnitude of the compression stress, with a maximum at the supports and a minimum at the place of maximum moment. A full analysis of the test data taking account of this variation not only would be cumbersome but would also exceed the amount of effort a designer may reasonably be expected to devote to determining deflections. For this reason, deflections for the reduced cross section in Table 3(*b*) were determined for the minimum equivalent width—that is, the width corresponding to maximum moment. In the tests of series A, because of quarter-point loading, since the entire center half of the beam is stressed uniformly by the maximum moment, the error so introduced is rather small and the results obtained should be expected to be somewhat on the conservative side; that is, the computed deflections should be somewhat larger than the measured ones.

TABLE 3.—TESTS ON THE U-TYPES OF BEAMS; COMPARISON OF ACTUAL AND COMPUTED VALUES, SERIES A

Type	No.	$\frac{b}{t}$	(*a*) FAILURE MOMENTS			(*b*) DEFLECTIONS			
						In Elastic Range		At Yield Point	
			s_y (lb per sq in.)	$\frac{M_1}{M_t}$	$\frac{M_2}{M_t}$	d_1/d_t	d_2/d_t	d_1/d_t	d_2/d_t
U-1	1	64.0	39,100	1.16	0.99	0.96	0.99	0.93	1.04
U-2	1	78.2	36,400	1.23	1.01	0.95	1.04	0.88	1.05
U-3	1	95.5	32,800	1.39	1.06	0.95	1.04	0.84	1.02
U-4	3	103.5	30,330	1.20	0.97	0.88	1.00	0.88	1.02
U-5	1	104.7	42,970	1.47	1.09	0.92	1.05	0.78	1.00
U-6	3	107.5	40,030	1.29	0.98	0.93	0.98	0.84	1.05
U-7	3	111.0	30,330	1.22	0.96	0.90	0.96	0.91	1.02
U-8	3	111.0	30,330	1.18	0.91	0.95	1.02	0.91	1.05
U-9	3	142.0	31,880	1.53	1.09	0.84	1.11	0.81	1.05
U-10	2	143.0	35,770	1.49	1.07	0.76	0.90	0.72	0.96
U-11	2	147.0	27,820	1.44	1.07	0.82	1.03	0.85	1.05
U-12	1	148.0	28,470	1.47	1.07	0.65	0.92	0.67	0.95
U-13	2	170.0	34,300	1.65	1.15	0.69	0.91	0.67	0.95

In Table 3(*b*) actual and computed deflections are compared at two different loads: (*a*) In the purely elastic range (at loads of from about 75% to 90% of those which cause the bottom fibers to reach yield-point stress); and (*b*) at the loads that cause the bottom fiber to yield. It will be remembered that at this latter load practically all the cross section including the top flange is stressed considerably below yield-point intensity.

Table 3(*b*) gives average results of the same number of specimens of each type as does Table 3(*a*). The deflections d are defined as follows: d_1 is the deflection computed on the basis of the moment of inertia of the full, unreduced cross section; d_2 is the deflection computed on the basis of the moment of inertia determined by substituting the equivalent width of the compression flange for the actual width; and d_t is the deflection measured in the test.

Table 3(*b*) shows that deflections computed from the full, unreduced cross section throughout are smaller than measured deflections, particularly for the larger values of b/t. On the other hand, deflections computed on the basis of the equivalent width agree very satisfactorily with the measured deflections throughout the entire investigated range of b/t.

A similar comparison for the tests of series B and C is less revealing in its details. The b/t-ratios for these series are rather low (see Table 2(*b*)), causing the equivalent widths to be only slightly smaller than the actual widths. Also, the compression flanges of the I-beams represent a much smaller part of the cross section than do those of the U-beams of series A. For both these reasons the moments of inertia of the beams are little affected by the decrease of the equivalent width—indeed, less than the section moduli of these same beams. Therefore, only the average values of the deflections in the elastic range are given for these two series. For the beams of series B and C with b/t larger than 30 (that is, those for which the difference between equivalent and actual width is at all significant), the following averages were found: $d_1/d_t = 0.925$ and $d_2/d_t = 0.996$. For these beams, too, the deflections computed from the equivalent widths agree much better with the measured ones than do those computed from the unreduced cross section. (In analyzing the deflections of series B and C, account was taken of the part of the total deflections caused by shear stresses. This part amounted in some beams to as much as 6.7% of the total deflection. In the tests of series A the part of the deflection caused by shear is negligible because of the large span:depth ratios of these beams.)

Summarizing, the determination of deflections from the unreduced cross section would lead to a design which (just as with regard to strength) is in error on the dangerous side, whereas deflections determined from the equivalent width of the compression flanges are confirmed very satisfactorily by experiment.

Distortion Under Load and at Failure.—The compression flanges of all beams in the range of medium and large b/t developed very slight waviness at loads considerably less than the ultimate. At loads of the order of half of the ultimate the amplitude of these approximately quadratic waves, however, was so small that the distortion produced could not be considered of any structural consequence. In fact, so minute was the distortion that it escaped photographic recording, although wave formation could be detected by touch and was visible by careful inspection.

Failure occurred when one of the waves developed into a definite wrinkle; usually such development occurred within a very small load increment. Two views of such beams in the medium range of b/t (I–15 and I–17, Table 2) after failure are given in Fig. 8. The local, wavelike distortion of the compression flange is clearly visible. It is also noticeable that adjacent parts of the flanges are practically undistorted: The minute waves, which, before failure, were present over the entire flange of the beam between loads did not cause any permanent distortion and disappeared after unloading, except for the one wave that resulted in failure. Turned-down lips remained straight up to failure loads, except for the widest of the I-beams (I–19 and I–20) in which the lips showed some slight wavelike departure from the initially straight shape all

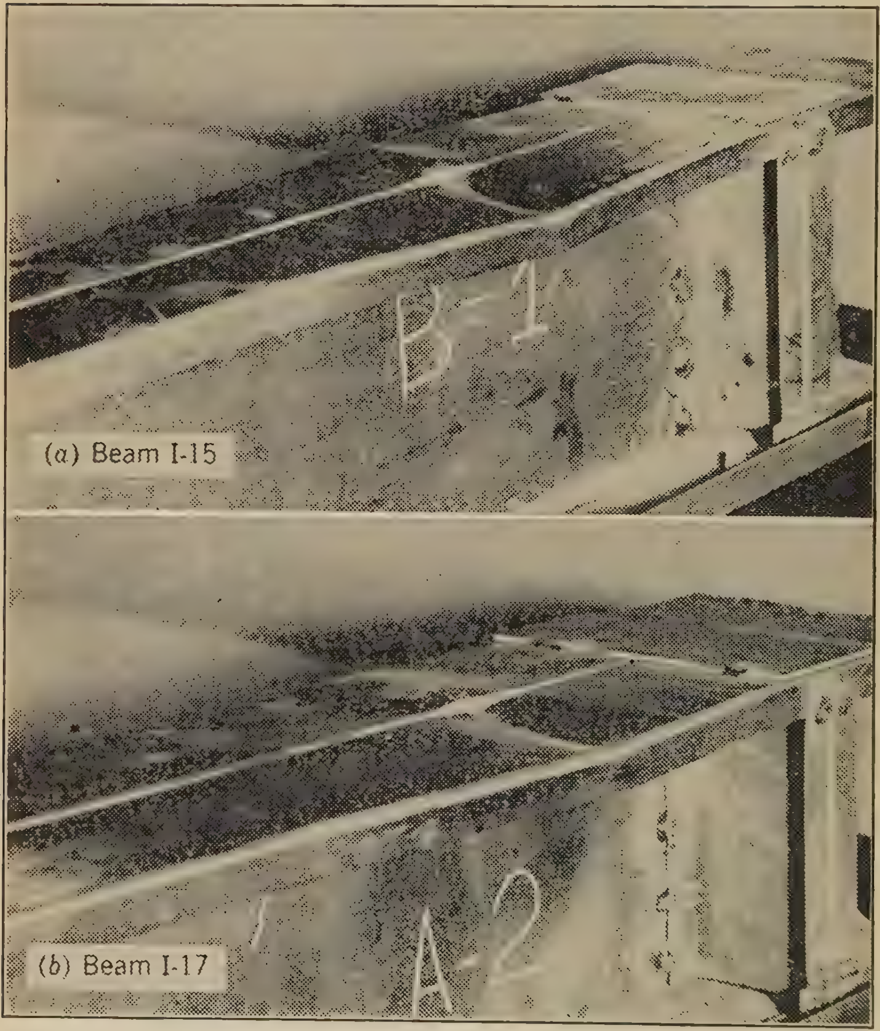

Fig. 8.—Examples of Failure Caused by Local Buckling

along the center parts of the beams at loads below the ultimate. Possibly the lips of these two types of beams were not rigid enough to fully stiffen the flanges and somewhat stiffer lips may have raised the ultimate loads slightly (see Table 2(*b*)).

Type B. Compression Flanges Supported Along One Longitudinal Edge

Elements of this type occur in I-beams or channels not furnished with stiffening lips at the outer edges, so that flanges are stiffened by the webs along the joints of these two elements, but are free to distort out of their planes at the other, unstiffened edge.

To investigate the behavior of such flanges, two series of tests were made: (1) I-beams (designated by I–B); and (2) struts (designated by I–S).

1. In series D, seventeen types of beams of the general shape shown in Fig. 9(*a*) were tested—principally three "identical" specimens of each type. Most of the beams were 8 in. deep; a few of them were 4 in. deep, within shaping tolerances. Widths of top flanges ranged from 1.43 in. to 9.90 in., widths of bottom flanges from 1.93 in. to 10.18 in., and thicknesses from 0.0368 in. to 0.1035 in. The beams were tested on various lengths of span, depending on their width, to avoid beams so slender that they might buckle laterally before reaching full local strength. The beams were loaded by two equal loads, symmetrically located with respect to the center of the beam, at a distance from each other varying from 21 in. to 36 in. Ultimate loads, deflections, and strains were measured in these tests in the same manner as in the preceding series.

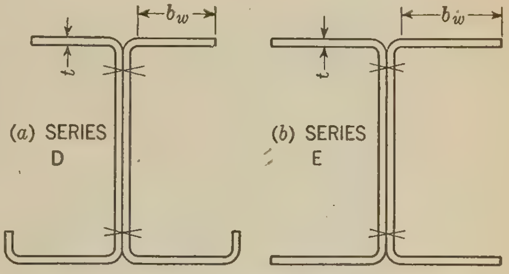

Fig. 9.—Shapes of I-Sections with Unstiffened Outer Edges of Compression Flanges

2. In series E, twenty-two types of struts (designated by I–S) of the general shape shown in Fig. 9(*b*) were tested in compression—mostly three "identical" specimens of each type. The narrower struts were 2 in. deep and the wider ones 4 in. deep, within shaping tolerances. Widths of flanges ranged from 1.56 in. to 12.00 in.; thicknesses, from 0.0368 in. to 0.1077 in.; and lengths of struts, from 15 in. for those with the narrowest flange widths to 63 in. for those with largest flange widths. The struts were tested with end attachments providing knife-edge support centered as precisely as possible in the plane of the web; and the aforementioned lengths were so adjusted as to keep the L/r-ratio in the direction of the least radius of gyration within the range of from 35 to 50. This was done to prevent failure of the struts due to column buckling rather than due to local failure. As a further safeguard against failure by bending or column buckling, deflections were measured at midlength both in the direction of the web and perpendicular to it. Thus, eccentric loading and conse-

quent bending, if they occurred, were detected at low stresses; and, in such cases, the loading was realined before the strut was tested to failure. This procedure, in conjunction with the small L/r-ratios insured the full development of local compressive strength rather than over-all buckling strength in these tests. In addition to ultimate loads and deflections, strains were measured by applying two 8-in. gages opposite each other in the center line of the web (see Figs. 10 and 11).

Since the results of both series of tests with regard to the behavior of the compression flanges were identical, the strut tests will not be separated from the beam tests in the following discussion, except for the nomenclature.

General Behavior and Types of Failure.—The tests revealed three rather different types of behavior under load and of final failure, depending on the ratio b_w/t of the compression flanges, in which b_w is the free projection of compression flange, measured from toe of radius to outer edge.

Flanges in the lowest range of b_w/t, up to about 12, as expected, failed by simple gradual yielding with little or no distortion perpendicular to the plane of the flange.

Flanges with b_w/t in the range of from 12 to 33 behaved and failed in a manner illustrated by Figs. 10(*a*), 10(*b*), and 10(*c*), in which a strut specimen I–S–10 with $b_w/t = 27.1$ is shown in three consecutive stages of loading. Fig. 10(*a*) shows the stud under a low load of 1 kip; flange faces are plane and their edges straight except for slight inherent inaccuracies obtained in the forming process. Fig. 10(*b*) shows the stud under a high load of 14 kips, with no perceptible distortion of the flanges. At a somewhat higher load (16 kips), definite distortions are seen in the upper part of the front flange of Fig. 10(*c*). These "kinks" are purely localized, the remainder of the flanges being essentially undistorted. The strut failed finally at a load of 18,600 lb after developing some more kinks of the same character. This strut and other struts (particularly in the range of b_w/t from about 20 to 33) carried considerably larger loads at failure than those in which the first kinks developed. Nevertheless, the load at which such kinks first appear must be regarded as the limit of structural usefulness for the particular element, and the subsequent discussion, therefore, is based on this local buckling rather than on the ultimate strength.

Finally, compression flanges in the range of b_w/t from 42 to 109 (the largest ratio of all specimens tested) behaved in the manner illustrated in Fig. 11. (No data are available for the range of b_w/t from 33 to 42.) Fig. 11 presents views of a strut specimen I–S–15 with $b_w/t = 42.6$ under increasing load; and Fig. 11(*a*), with a load of only 1 kip applied, shows the flanges being plane except for slight manufacturing distortion. However, at one half of the ultimate load, Fig. 11(*b*), taken at a load of 21 kips, shows slight regular waves beginning to develop in the flanges along the entire length. At further increased loads (31 kips and 39 kips), Figs. 11(*c*) and 11(*d*) show the amplitude of these waves gradually increasing, the regularity of the pattern being preserved. Finally, the strut failed at a load of 42 kips; and Fig. 11(*e*), taken after failure, shows that one of the previously developed waves had suddenly enlarged into a definite "kink" and produced failure. Whereas this particular strut did not

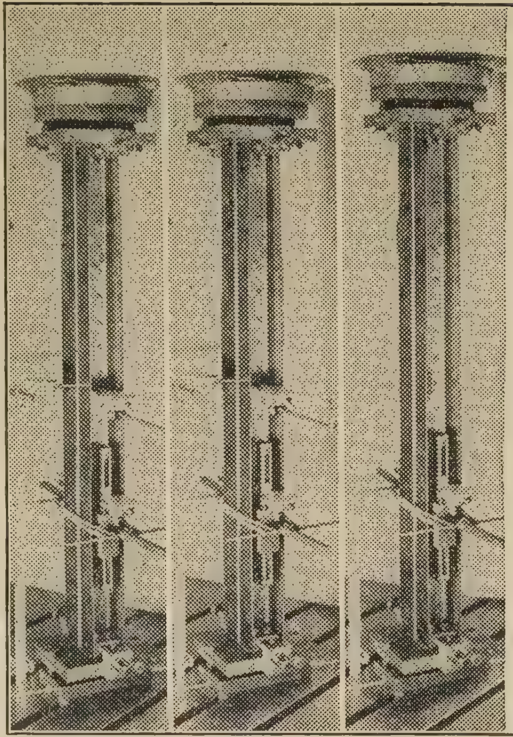

FIG. 10.—DISTORTION OF STRUT I-S-10 ($b_w/t = 27.1$) UNDER INCREASING COMPRESSION LOAD P

(a) $P = 1$ KIP
(b) $P = 14$ KIPS
(c) $P = 16$ KIPS

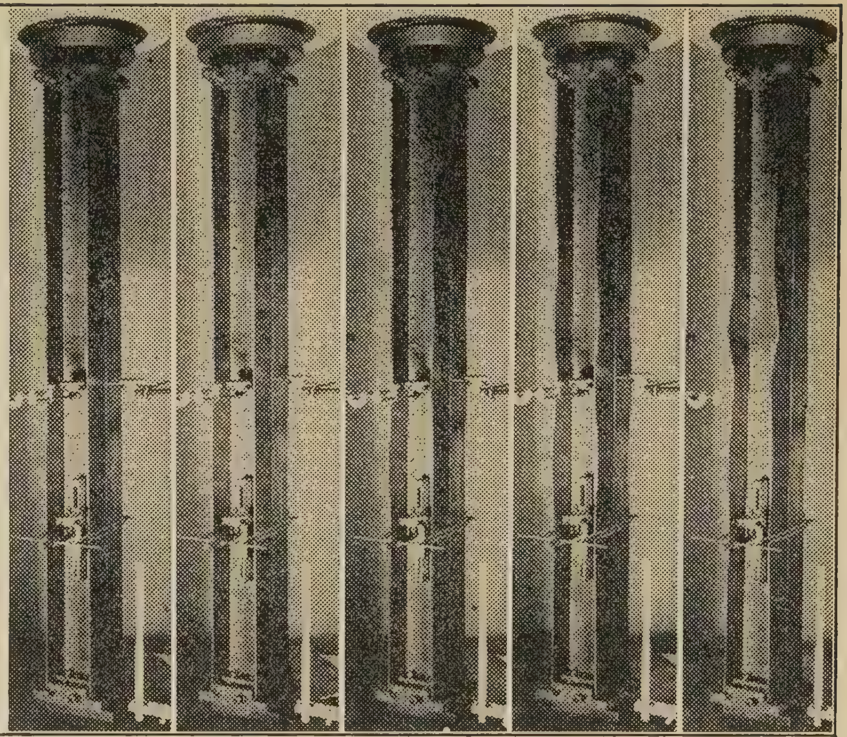

FIG. 11.—DISTORTION OF STRUT I-S-15 ($b_w/t = 42.6$) UNDER INCREASING COMPRESSION LOAD P

(a) $P = 1$ KIP
(b) $P = 21$ KIPS
(c) $P = 31$ KIPS
(d) $P = 39$ KIPS
(e) $P = 42$ KIPS

develop any perceptible distortions at loads less than about 50% of the ultimate, struts in the higher range of b_w/t developed distinct regular wave patterns at much smaller fractions of the ultimate load, down to less than 20% of the ultimate for the largest values of b_w/t.

The compression flanges of the beams of series D behaved in precisely the same manner in the respective ranges of b_w/t as did those of the struts of series E.

Three practical qualitative conclusions are to be drawn from these findings:

1. Flanges with b_w/t smaller than 12 do not buckle; they fail by yielding and can therefore be designed for full yield-point strength (for quantitative verification see the next section).

2. Flanges with b_w/t from 12 to about 30 (to stay slightly on the conservative side since no data are available for the range of from 33 to 42) will remain undistorted and serviceable until sudden buckling (kinking) occurs in the flanges. Despite the fact that such flanges may carry higher loads after kinking, the design criterion in this range of b_w/t should be the local buckling stress.

3. Flanges in the still higher range of b_w/t, although capable of carrying considerable stress, become distorted so seriously at loads far below the ultimate that they cannot be regarded as structurally useful except if used at extremely low design stresses.

The series of photographs in Fig. 11 is interesting in still another respect: It illustrates pictorially the concept of the equivalent width of thin flanges. Regular, wavelike distortion does not preclude considerable carrying capacity of the flange. On the other hand, a flange so distorted is not able to carry as much stress as the same flange if it were kept straight, say, by a system of stiffeners. After distortion occurs, moreover, the parts of the flange close to the web, kept essentially plane by the latter, will carry the main part of the total compression force in the flange, whereas the most highly distorted outer regions carry a rather small part of this force. Precisely this situation is idealized in the von Kármán concept of the equivalent width (Fig. 1). Compression flanges with both edges stiffened, like those discussed in the preceding section, behave in exactly the same manner, the only difference being that there the amplitude of the waves is limited to a much smaller magnitude by the combined action of both stiffeners as compared with that of flanges with one edge free. The idealization of this situation by the von Kármán concept is, simply, as follows: In the case of a flange with, say, both edges stiffened, the actual distribution of compression stresses over the width is of the character shown in Fig. 12. For convenience of design computation, the area under this stress curve is replaced by the equal area of the sum of the two dotted rectangles, whose altitude is the maximum actual stress at the stiffeners. Consequently,

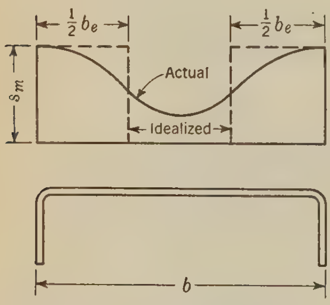

Fig. 12.—Stress Distribution Over the Width of a Stiffened Compression Flange

the maximum computed stress of a beam designed or analyzed according to this concept is equal to the actual maximum, and therefore governing, stress of the real, nonuniform stress distribution—all that is required for a rational design procedure.

Determination of Buckling Stresses.—In all cases in which formation of kinks was observed, the term "limiting stress" in this paper pertains to that stress at which the kink or kinks were first noticed. In the low range of b_w/t, failure occurred either by simple yielding, or by buckling (bending) of the entire specimen, or, in the case of specimens I–B–4, by an intermediate action between kinking and yielding. In these cases the limiting stress was determined as that at which irregularity of behavior appeared as a marked departure from the straight line in the load-deflection curve and the load-strain curve. In all cases stresses were computed by standard procedure—that is, by dividing the actual load (for struts) or moments (for beams) by the area or section modulus, respectively. These cross-sectional properties were computed from the full, unreduced dimensions of the sections. The justification for this procedure is given subsequently. Table 4, gives the results of the tests for the specimens with b_w/t less than 33.1—that is, for those which showed sudden formation of kinks (flange buckling) without preceding noticeable wave distortion. Table 4, therefore, covers the structurally useful range of b_w/t.

TABLE 4.—LIMITING STRESSES, s_l, AND ULTIMATE STRENGTHS, s_u, OF COMPRESSION FLANGES SUPPORTED ALONG ONE EDGE ONLY

Type	No.	b_w	STRESSES, IN POUNDS PER SQUARE INCH				$\frac{lC}{s_{lt}}$
			s_y	s_u	s_{lt}	s_{lC}	
I–S–2	3	9.3	35,400	34,600	33,400	35,400	1.06
I–S–3	3	10.1	49,400	35,800	35,800	49,400	1.38
I–B–3	3	10.1	37,300	30,200	29,600	37,300	1.26
I–B–4	3	17.5	36,800	40,300	30,400	30,200	0.99
I–S–6	3	18.5	35,400	31,800	25,600	28,000	1.09
I–S–7	3	19.0	34,500	26,100	22,800	27,000	1.18
I–S–8	3	19.1	49,400	38,800	35,400	36,500	1.03
I–B–5	2	20.3	37,300	29,400	23,600	27,000	1.14
I–B–6	3	20.8	34,000	29,200	26,100	24,700	0.95
I–B–7	3	21.6	32,600	28,800	23,400	23,100	0.99
I–S–9	3	21.6	34,000	25,500	23,900	23,700	0.99
I–B–8	3	25.2	38,700	30,000	21,200	21,200	1.00
I–S–10	3	27.1	34,500	22,900	16,700	18,000	1.08
I–S–11	3	27.8	34,000	23,900	15,600	17,200	1.10
I–S–12	3	27.8	34,500	29,200	23,700	17,300	0.73
I–S–13	3	28.3	49,400	29,200	17,600	18,200	1.03
I–B–9	1	28.9	29,200	26,200	19,700	15,900	0.81
I–B–10	3	29.9	32,600	24,600	15,200	15,200	1.00
I–B–11	3	30.6	34,900	25,700	17,400	14,500	0.83
I–B–12	2	31.2	37,300	28,300	14,200	13,800	0.97
I–B–14	3	33.1	34,000	23,000	15,200	12,300	0.81

In Table 4, b_w is the free projection of compression flange measured from toe of radius to the outer edge; s_u is the ultimate strength computed for struts from $s_u = P_u/A$ and for beams from $s_u = M_u/S$, A and S, respectively, being the area and the section modulus of the full, unreduced cross section; s_{lt} is the limiting stress, computed in the same manner as s_u but from the test loads at which local buckling occurred; and s_{lC} is the limiting stress computed from Fig. 13 which is discussed in detail subsequently.

Type I–S–3 was the smallest in cross section of all those given in Table 4—its total flange width being only 1.56 in.; its depth, 2.01 in.; and its length, 15 in. These small dimensions made an accurate centering of the load extremely difficult with the available apparatus and, in addition, made it impossible to control the accuracy of loading by measuring deflections in the direction of the least radius of gyration because the strain gages (see Fig. 10) left no space for the deflection-measuring device. The low value of s_u as compared with that of s_y seems to indicate that this strut was subject to considerable bending caused by eccentric loading, which provoked premature failure as compared with pure concentric loading.

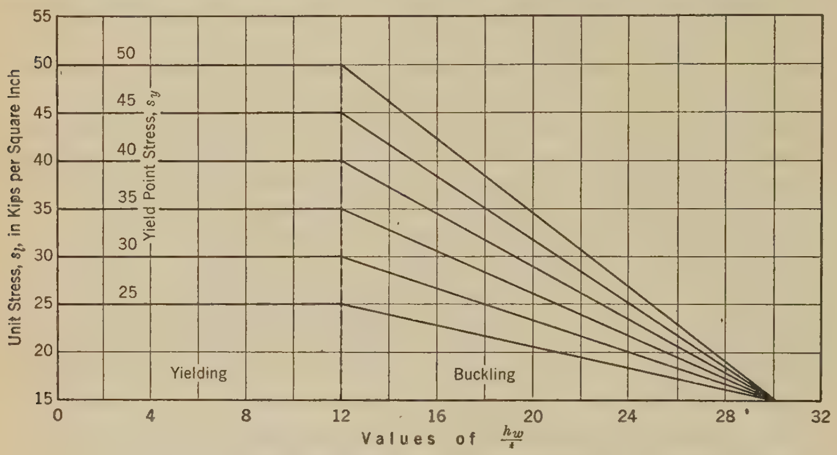

FIG. 13.—CHART FOR DETERMINING THE LOCAL BUCKLING STRENGTH FOR A GIVEN YIELD POINT s_y, FREE WIDTH OF PROJECTION b_w, AND FLANGE THICKNESS t, AS GIVEN BY EQ. 10

Type I–B–3, the narrowest of those included in Table 4, with a total flange width of 1.48 in. and a depth of 7.95 in., failed very markedly by lateral bending (due to the large difference of the two principal moments of inertia) rather than by local buckling or yielding. This is the reason for the low values of s_u and s_{lt} in this case (see Table 4).

In Fig. 14, the values of s_{lt} are plotted against b_w/t. The curve representing the formula,

$$s_c = 0.5 \frac{\pi^2 E}{12 (1 - \mu^2)} \left(\frac{t}{b_w} \right)^2 \dots\dots\dots\dots\dots\dots\dots (9)$$

is shown by the solid line. Eq. 9 gives the theoretical, critical buckling stress for a long, narrow plate simply supported along one longitudinal edge and unsupported along the other.[3] Fig. 14 suggests that, in the higher range of b_w/t—from about 25 upward, local buckling (kinking) occurs at stresses equal to, or somewhat larger than, those given by Eq. 9. At lower values of b_w/t, however, buckling occurs at stresses considerably below s_c. This situation is analogous to that in the case of column buckling where failure loads are found to be considerably below the Euler loads for values of L/r less than about 100. To develop a procedure for predicting limiting stresses in this range of b_w/t, the average yield point was computed from Table 4—specimens with $s_y = 49{,}400$ lb per sq in. being excluded from the averaging because of the high value of this yield point as compared with all others. The average value of s_y so obtained is 34,800 lb per sq in. Since flanges with b_w/t less than about 12 failed by yielding rather than by buckling, it seemed logical to presume that the dotted straight line in Fig. 14 (starting with the average s_y at $b_w/t = 12$ and ending with s_c at $b_w/t = 30$) would approximately represent the values of s_{lt}. The distribution of experimental points around that line indeed suggests that such may be the case. Fig. 14, however, is not sufficient evidence to prove the accuracy of this approach, because the yield points of the test specimens varied over a wide range from 29,200 lb per sq in. to 49,400 lb per sq in., whereas the dotted line in Fig. 14 refers to the average yield point only.

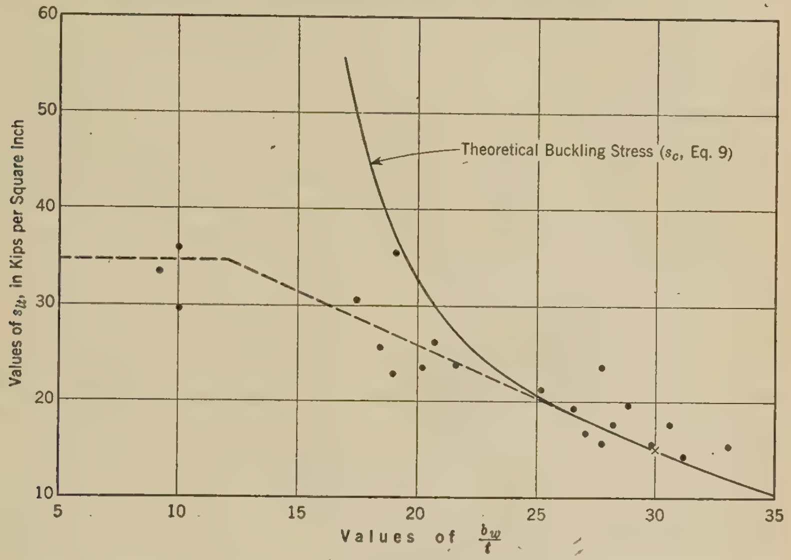

FIG. 14.—OBSERVED VALUES OF LOCAL BUCKLING STRENGTH s_{lt} PLOTTED AGAINST THE RATIO b_w/t; SERIES D AND E

If the assumption is correct that a straight line, drawn from s_e at $b_w/t = 30$ to s_y at $b_w/t = 12$, determines the limiting stresses satisfactorily, then the limiting stress for each specimen should be found, according to its particular yield point, from Fig. 13. (The equation of the straight lines in Fig. 13, between $b_w/t = 12$ and $b_w/t = 30$ is

$$s_{lC} = s_y - \frac{s_y - 15{,}050}{18}\left(\frac{b_w}{t} - 12\right) \qquad (10)$$

from which the limiting stress can be computed in that range. For the few specimens with b_w/t larger than 30, the limiting stress s_{lC} entered in Table 4 is the critical stress according to Eq. 9 (see also Fig. 14).) The values so computed from Fig. 13 are entered under the heading s_{lC} in Table 4 and a comparison of these with the test values is given in the last column of Table 4.

This last column (Table 4) reveals rather satisfactory agreement, except for types I–S–3 and I–B–3. The reasons for the peculiar behavior of these two types was discussed previously. The influence of the yield point on the limiting stress, very pronounced in the low range of b_w/t, but gradually decreasing in the higher range (see Fig. 13), is well demonstrated by types I–S–8 and I–S–13 with their unusually high values of s_y. This indicates that a prediction of failure stresses solely on the basis of the theoretical s_c, as often attempted, is certain to lead to erroneous results.

It should be noted that the agreement between test results of "identical" specimens of the same type is not as satisfactory for flanges of this kind as for flanges stiffened along both edges. Of the twenty types of sections for which more than one specimen of each type was tested (mostly three specimens of a type, see Table 4), the maximum deviation from the mean for a given type did not exceed ±15% for fifteen types. In the remaining five types the maximum deviation on the safe side was +30%; and, on the unsafe side, −17%. The data refer to s_u. This scattering of test results, no doubt, is caused by inevitable inaccuracies of forming and possibly nonuniformity of material. (To give only one practical comparison, the scattering is of about the same order of magnitude as that found from strength tests on concrete specimens mixed to identical specifications and cut out of a finished structure.[17])

Equivalent Width.—From the discussion of Fig. 11, it is apparent that the concept of equivalent width must be expected to apply not only to flanges stiffened along both edges but also to those with one edge free. Therefore, should not the evaluation of test series D and E be based on the equivalent areas of the compression flanges rather than on the unreduced cross sections as was done in Table 4?

In an academic thesis prepared under the writer's supervision,[18] E. A. Miller, Jun. ASCE, has computed the equivalent widths from the strain observations of series D and E in a manner similar to that previously discussed in connection with series A. This investigation was not limited to the practically useful range of b_w/t but included all specimens of these series with ratios of b_w/t up to 109.

Mr. Miller showed that expressions similar to Eq. 6 could also be developed in this case to express the equivalent width b_e. He found that the expression,

$$b_e = 1.25\, t \sqrt{\frac{E}{s}} \left(1 - 0.333 \frac{t}{b_w} \sqrt{\frac{E}{s}}\right) \quad \text{(11a)}$$

which is valid to about $t/b_w \sqrt{E/s} = 1.55$, represented rather accurately the average values found experimentally. Since results showed considerable scattering, a more conservative expression was developed, which, with a very few exceptions, gives a good approximation for the lowest values of b_e obtained experimentally. This expression is

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[17] "Properties of Job-Cured Concrete at Early Ages," Report of Committee 107, *Journal*, A.C.I., September–October, 1936, Tables 2 and 5, pp. 46 and 51.

[18] "A Study of the Strength of Short, Thin Walled Steel Studs," by E. A. Miller, a thesis presented to the faculty of the Graduate School of Eng., Cornell Univ., Ithaca, N. Y., in October, 1943, in partial fulfilment of the requirements for the degree of Master of Civil Engineering.

$$b_e = 0.8\,t\sqrt{\frac{E}{s}}\left(1 - 0.202\,\frac{t}{b_w}\sqrt{\frac{E}{s}}\right)\ldots\ldots\ldots\ldots(11b)$$

which is valid to about $t/b_w \sqrt{E/s} = 1.75$.

On the basis of these findings it is possible to decide whether, in determining buckling stresses by the procedure of the preceding section, the equivalent or the full, unreduced flange width should be used. This question, obviously, is of greater practical significance for large than for small values of b_w/t. It will therefore be investigated for the suggested upper limit of the useful range of b_w/t—namely, for $b_w/t = 30$. For this value the local buckling stress, according to Figs. 13 and 14, is found from Eq. 2*b* to be 15,100 lb per sq in. For this value of s, Eqs. 11*a* and 11*b* for $b_w/t = 30$ give, respectively, $b_e = 28.2\,t$ and $b_e = 25.2\,t$. In other words, in the practically important range of b_w/t the maximum reduction of flange width is only 6% on the average (Eq. 11*a*) with a maximum of 16% from the more conservative expression Eq. 11*b*.

Hence, in the range of b_w/t to about 30, computations based on the full, unreduced flange width are as accurate as can reasonably be expected—considering that (1) the degree of scattering of the values of the experimental buckling stresses makes a high accuracy of computation illusory; and (2) a reduction in width of 6% or even 16% results in a much smaller reduction of the significant values of A and S for ordinary sections in which the compression flanges of type B represent only a moderate part of the entire cross section. The introduction of an equivalent width, in this case, would unduly complicate computations without any significant improvement of the validity of results.

Thus, Eqs. 11 appear to be practically valuable only in establishing the foregoing conclusion. They may become significant in connection with further analytical research in this field, in testing future theoretical expressions for the equivalent widths of flanges of this type.

CONCLUSION

The experimental evidence presented in this paper indicates that the strength and general behavior of thin steel compression flanges is satisfactorily accounted for in the following manner:

1. *Flanges Stiffened Along Both Longitudinal Edges.*—Flanges with values of b/t to about 25 were found to fail by simple yielding, the full area of the flange being effective—that is, the stress is of uniform magnitude across the width. For flanges with larger values of b/t, deflections and ultimate loads computed by considering the compression flanges fully effective were found to err on the dangerous side.

Replacement of the actual flange width by an equivalent width determined from Fig. 7 resulted in very satisfactory agreement between observed and computed ultimate loads and deflections. Locations of neutral axes, as determined experimentally from strain measurements, showed good agreement with locations computed on this basis.

Within the investigated range of dimensions, distortions of such flanges at loads below the ultimate were very limited and of little, if any, practical significance.

2. Flanges Stiffened Along One Longitudinal Edge.—Flanges with ratios of b_w/t to about 12 failed by simple yielding, the full area of the flange being effective. In the range of b_w/t from 12 to about 30, local sudden and pronounced flange buckling was found to occur at stresses determined from Fig. 13. Local flange buckling did not precipitate immediate failure; indeed, ultimate loads of some specimens were as high as twice the load that produced local buckling, stresses being computed in all cases from the full, unreduced flange width. For practical purposes, however, it was held that loads resulting in local buckling must be regarded as reaching the limit of structural usefulness.

In the range of b_w/t larger than about 30, wavelike gradual flange distortion was observed to occur at stresses rapidly decreasing with increasing values of b_w/t. The distortions, even at rather small fractions of the ultimate loads, were so pronounced as to make such members useless for most structural applications, except at extremely low design stresses.

From strain measurements it was found that the area of such flanges was only partly effective. This phenomenon resembles the behavior of the flanges under conclusion 1. An expression for the equivalent width, as determined from strain measurements, is given in the text.

It is not contended that the findings of this investigation, as expressed chiefly in Figs. 7 and 13 necessarily represent the final answer to the question of the strength of thin compression flanges. Being rather analytically inclined himself, the writer is perfectly aware of the limitations inherent in such purely experimental results, unsupported as they are by any more rigorous theoretical analysis.

It so happens, however, that the rigorous treatment of buckling of thin plates along classical lines of elastic stability[19] proved to be of little practical value. The reasons for this situation have been stated in the paper. Attempts by various investigators to go beyond the simple, classical approach have so far failed to give practically significant results and have led, in most cases, to rather contradictory conclusions.

Under these circumstances the only feasible approach seemed to be that of a rather extensive experimental investigation and an attempt to express the results of such test work by formulas or graphs that would agree reasonably well with test results. The situation is comparable with that of earlier column investigations in the range in which the Euler formula is invalid. The simple formulas then developed, mostly of straight-line character, served their purpose rather well until such time as more accurate and more involved approaches, such as the secant formula, could be developed.

It is hoped that a similar situation may obtain with regard to the topic of this paper and that the limited results of this investigation may serve their practical purpose until such time as more general and exact treatments will become available.

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[19] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936.

ACKNOWLEDGMENTS

The tests and conclusions reported herein are part of an extensive investigation of thin-walled steel structures sponsored jointly by the American Iron and Steel Institute and Cornell University, in Ithaca, N. Y.,[2] under the general supervision of Prof. W. L. Malcolm, M. ASCE, director, School of Civil Engineering, the writer being in active charge of the work. The writer wishes to express his sincere appreciation of the unfailing patient cooperation and valuable suggestions of Director Malcolm and of the members of the technical subcommittee of the Committee on Building Codes of the Steel Institute—in particular that of Milton Male, M. ASCE, chairman; B. L. Wood; and F. E. Fahy, Assoc. M. ASCE.

In conducting the test work and the computations the writer was ably assisted by the following McMullen Graduate Scholars (in chronological order): C. A. Dunn, Assoc. M. ASCE; Capt. G. G. Green, R. L. Lewis, and E. A. Miller, Juniors, ASCE; and R. H. J. Pian. Their individual contributions, overlapping as they are as to time and topic, cannot readily be separated. The writer wishes to thank all these collaborators for their conscientious and devoted cooperation and their many helpful suggestions.

APPENDIX. NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = area of cross section; A_n = area of one of n subareas;
b = width of a plate of type A perpendicular to the direction of compression:
  b_e = equivalent width;
  b_s = width of a strip, part of the total width;
  b_w = width of the free projection of a compression flange of type B, measured from the toe of the radius to the outer edge;
C = a variable experimental coefficient; as a subscript, C denotes "computed";
d = deflection, with subscripts t denoting "by test," etc.;
E = Young's modulus of elasticity;
h = depth of beam section;
L = span lengths;
M = moment, with subscripts t denoting "by test" and u denoting "due to ultimate loads";

P = axial compression load (P_u = load at ultimate strength);
r = least radius of gyration;
S = section modulus;
s = unit stress:
  s_C = computed stress;
  s_c = critical stress;
  s_l = limiting stress;
  s_m = maximum stress;
  s_u = ultimate strength;
  s_y = yield-point stress;
t = thickness of plate;
y = coordinate distance; y_n = centroidal distance to an area A_n;
δ = strain, or unit elongation;
κ = a numerical factor depending upon the ratio of length to width of the plate and the conditions of edge support; and
μ = Poisson's ratio.

DISCUSSION

Fred T. Llewellyn,[20] M. ASCE.—This scholarly, yet not pedantic, study is peculiarly interesting to the writer because, prior to 1939, he was a member of the American Iron and Steel Institute committee sponsoring the Cornell University (at Ithaca, N. Y.) tests on which the paper is largely based. In 1933, the writer suggested the concept of effective (or equivalent) widths as applied to thin metal sections, such as the one illustrated by Fig. 12. He first collated the published proportions of a number of such sections, whose efficiency was fairly well established by industrial use; and next he endeavored to harmonize the results with the nonorderly and all-too-few tests then available.[21]

A comparison of the values then tentatively suggested by the writer, with the much more adequately supported values now given by Professor Winter, is offered in Table 5. The conditions of fixity are not quite identical in both cases, but an effort has been made to classify them along comparable lines. The author is asked to state whether the surprising value, $\frac{E\,t}{s\,b}$, is correctly assigned in Table 5.

TABLE 5.—Comparison of Equivalent Widths b_e/t

Investigator	Conditions of Fixity[a]		
	(1)	(2)	(3)
George Winter	12	25	$E\,t/(s\,b)$
F. T. Llewellyn	10	30	100

[a] Conditions of fixity:
(1) One edge fixed and one edge free;
(2) One edge fixed and one edge lipped; and
(3) Both edges fixed. The ratio $E\,t/(s\,b)$ is based on Eq. 2b and Fig. 2.

Although this rough comparison tends to confirm the safety of many miscellaneous sections in commercial use, the design of all thin-walled sections, of course, should be placed on a common and broad foundation. The present paper is welcomed as a valuable step in this direction—the development of a standard design specification for the products in question.

In view of the author's expressed inclination toward analytical methods, his restraint in, and happy application of, their use are to be applauded. Professor Winter's academic citations are few, but pertinent. In former years, when the writer was tempted to try and go "highbrow," he used to recall a warning from the field of art criticism. On one page of a revised edition of John Ruskin's works, the text was footnoted about as follows:

> "In the first edition of this book there was here a quotation from Aristotle, in the Greek. I inserted it to show I had read Aristotle. Having accomplished that purpose, it is now omitted."

The present paper affords a very suitable balance of the academic with the practical, to which Mr. Ruskin's self-irony could not possibly apply. The author does not need to apologize for approaching the subject from an experimental rather than a classical standpoint. Is it not true that all physico-mathematical theory must be converted into conventional forms that are valid only within the boundaries of everyday practice? Thus, the writer would expect Figs. 7 and 13 to receive most attention from actual designers of thin-walled sections.

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[20] Baton Rouge, La.

[21] "Light-Gage Flat-Rolled Steel in Housing," by F. T. Llewellyn, A.I.S.C., 1937, p. 29.

It is hoped that these casual, but quite appreciative, comments by an "old timer" may be of as much interest to the author, and fellow engineers, as the paper was to the writer.

Jacob Karol,[22] Assoc. M. ASCE.—Despite the fact that millions of formed sheet channels with and without stiffening lips have been used as bending members in airplane control surfaces, there has been no definite design procedure whereby a stress analyst could determine their ultimate strength. The paper by Professor Winter, therefore, is a timely and important contribution to the field of elastic stability.

The writer's procedure in determining the strength of symmetrical formed sheet channels in bending has been to compute the ultimate strength of the component elements of the section on the compression side, to obtain the resisting moments about the centroidal axis, and then to double their sum to obtain the total resisting moment. It is interesting to note that this procedure is similar to that presented in the paper.

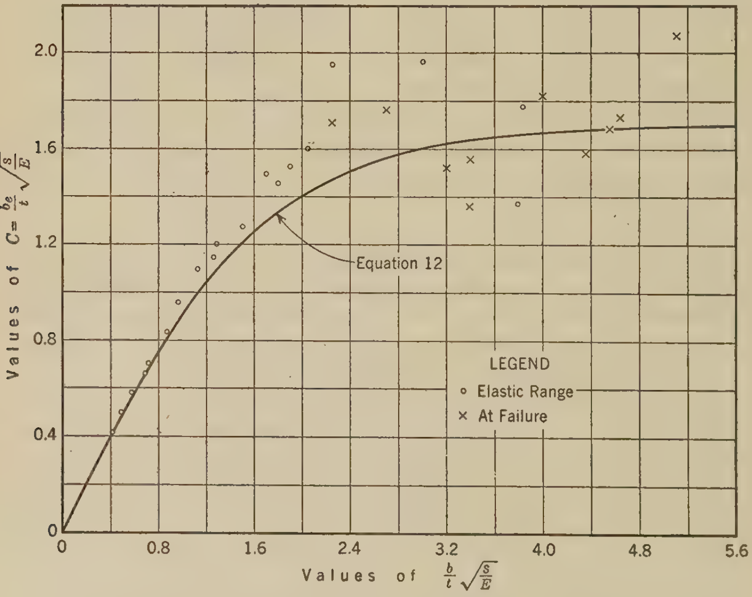

Fig. 15.—Experimental Determination of Equivalent Width

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[22] Design Engr., Howard, Needles, Tammen & Bergendoff, Kansas City, Mo.

It will be shown that, by plotting the test data for the type A members (compression flanges stiffened along both longitudinal edges) in a slightly different manner, the relationship between the variables is clarified, and it becomes readily apparent that a simple expression can be written for the experimental coefficient C which is applicable over the entire range of $\frac{b}{t}$.

The data from Fig. 2 have been plotted with C as ordinates, but with the inverse parameter $\frac{b}{t}\sqrt{\frac{s}{E}}$ as abscissas in Fig. 15. To indicate the relationship in the low ranges of the inverse parameter, the writer has also plotted the results of the special series B tests, given in Table 2, in Fig. 15. Since $C = \frac{b_e}{t}\sqrt{\frac{s}{E}}$, by definition (see Eq. 4, the ratio of C to the inverse paramenter is really $\frac{b_e}{b}$ for each test point. The data indicate that, for the low ranges of the inverse parameter, the value of C should be nearly equal to the inverse parameter, whereas, for the high ranges of the inverse parameter, the value of C is practically constant. The following expression fulfils these conditions and gives results practically identical with Eq. 6:

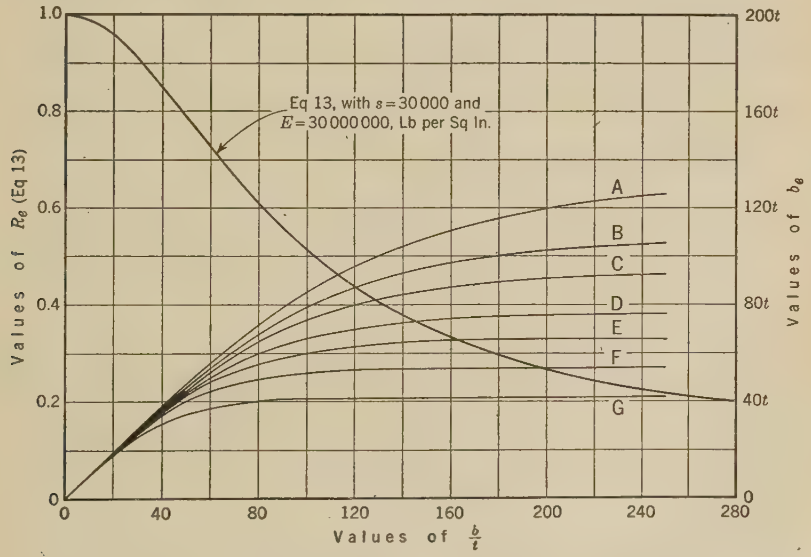

FIG. 16.—CHART FOR DETERMINING EQUIVALENT WIDTH b_e

$$C = 1.7 \tanh\left(\frac{1}{1.7}\frac{b}{t}\sqrt{\frac{s}{E}}\right) \qquad (12)$$

Eq. 12 is shown as a solid line in Fig. 15. The ratio of the ordinates to the abscissas in Fig. 15 is

$$R_e = \frac{b_e}{b} = \frac{\tanh\left(\frac{1}{1.7}\frac{b}{t}\sqrt{\frac{s}{E}}\right)}{\frac{1}{1.7}\frac{b}{t}\sqrt{\frac{s}{E}}} \qquad (13)$$

Eq. 13 is plotted for the particular case of s = 30,000 lb per sq in. and E = 30,000,000 lb per sq in. in Fig. 16, using $\frac{b}{t}$ as abscissas and R_e as ordinates. This curve (marked "Eq. 13") corresponds to Fig. 5 and shows that the value of R_e approaches 1 for small values of $\frac{b}{t}$, as it should.

Eq. 13 has been used to plot a family of curves similar to those in Fig. 7 (the legend in Fig. 7 applies to Fig. 16), and the resulting chart for equivalent width is also shown in Fig. 16. Although the author recommends the use of the chart for other metals than mild steel, it should be noted that, for metals with an indefinite yield point such as aluminum and magnesium, the value of E is a function of the stress s and is only equal to the initial modulus of elasticity for stresses below the proportional limit of the material. Hence, for stresses beyond the proportional limit, an effective modulus of elasticity corresponding to the particular stress must be used in determining the value of the parameter $\sqrt{\frac{E}{s}}$. The behavior of beams I-15 and I-17 indicates that, for material with an indefinite yield point, the ultimate stress in the curved corners of the flange is considerably beyond the yield point, and this must be considered in calculating the parameter $\sqrt{\frac{E}{s}}$ for such materials.

For the type B members (compression flanges supported along one longitudnal edge), an expression similar to Eq. 12 can undoubtedly be developed which would be applicable to the entire range of $\frac{b}{t}$. Since sufficient data are not presented in Table 4 from which to do so, it is suggested that the author derive the expression in his closing discussion.

ROBERT L. LEWIS,[23] JUN. ASCE., AND DWIGHT F. GUNDER,[24] ASSOC. M. ASCE.—A basis for the design of structural members fabricated from thin sheets of steel has been presented by Professor Winter. Any design procedure should be kept as simple as is consistent with safe and economical practice. With this thought in mind the writers have examined the proposed development, hoping to find a somewhat simpler approach for the designer. It should be emphasized that it is infinitely more simple to view a problem of this nature in retrospect, and that any contributions which this paper may make will owe their origin to the basic work of Professor Winter.

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[23] Prof. and Head, Civ. Eng., Colorado Agri. and Mech. College, Fort Collins, Colo.

[24] Prof. of Graduate Eng., Colorado Agri. and Mech. College, Fort Collins, Colo.

The author presents experimental evidence that confirms E. E. Sechler's data showing that the constant term in the von Kármán formula, Eq. 2*a*, should be replaced by a variable coefficient C which is a function of $(t/b)\sqrt{E/S}$. This leads to the expression $b_e = C\,t\sqrt{E/S}$ (Eq. 2*b*). Dividing both sides by b, Eq. 2*b* reduces to the dimensionless form,

$$\frac{b_e}{b} = \frac{C\,t}{b}\sqrt{\frac{E}{S}} \qquad \text{(14)}$$

As stated by the author, C appears to be a function of $(t/b)\sqrt{E/S}$. Hence Eq. 14 can be expressed as

$$\frac{b_e}{b} = \frac{t}{b}\sqrt{\frac{E}{S}}\,f_1\,\frac{t}{b}\sqrt{\frac{E}{S}} \qquad \text{(15)}$$

which merely states that

$$\frac{b_e}{b} = f\left(\frac{t}{b}\sqrt{\frac{E}{S}}\right) \qquad \text{(16)}$$

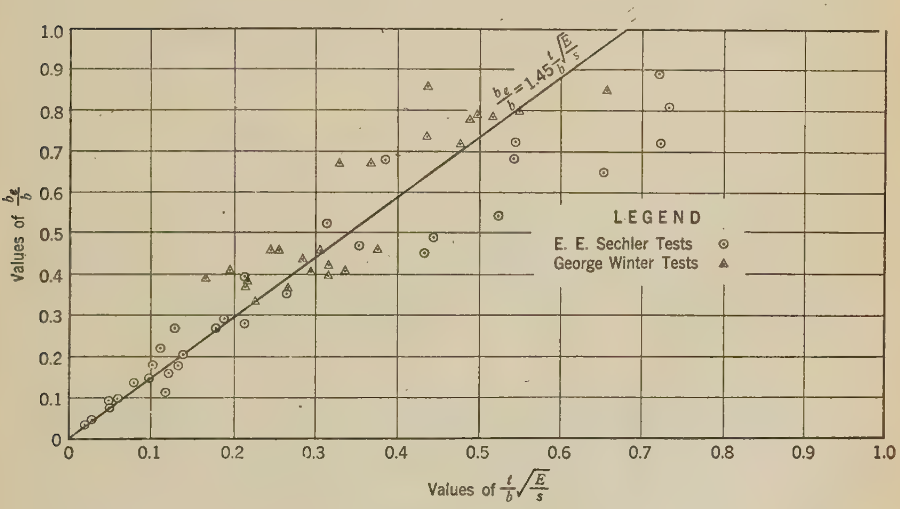

FIG. 17

Referring to Fig. 2 in which experimental values of the coefficient C are plotted against the parameter $(t/b)\sqrt{E/S}$, it will be noted that

$$\frac{C\,t}{b}\sqrt{\frac{E}{S}} = \frac{b_e}{t}\sqrt{\frac{S}{E}} \qquad \text{(17a)}$$

and

$$\frac{t}{b}\sqrt{\frac{E}{S}} = \frac{b_e}{b} \text{.........................(17}b\text{)}$$

The experimental values shown in Fig. 2 were read as carefully as possible from the graph and all values of the coefficient were multiplied by corresponding values of the parameter, thus obtaining the experimental values of b_e/b.

These values were plotted against the parameter $(t/b)\sqrt{E/S}$ as shown in Fig. 17. As a first approximation to the function $f[(t/b)\sqrt{E/S}]$ the best straight line was fitted to these data by the method of least squares. Its equation is:

$$\frac{b_e}{b} = 1.45\frac{t}{b}\sqrt{\frac{E}{S}}; \qquad 0 \leqq \left(\frac{t}{b}\right)\sqrt{\frac{E}{S}} \leqq 0.7 \text{............(18)}$$

In order to compare Eq. 6 with Eq. 18 the test values of Fig. 2 were plotted in Fig. 5 and on a graph of Eq. 18. Standard deviations from each curve were computed. The standard deviation from Professor Winter's curve was found to be 0.084 and that of the straight line was 0.113. The difference in standard deviations is caused by the wide scatter of three points of the Sechler data for which the value of b_e/b was approximately equal to 1. Although it is felt that with the limited data available the straight-line approximation is probably as satisfactory as any other, the F-test for goodness of fit does show that Professor Winter's curve is significantly better than Eq. 18. As a refinement to Eq. 18, a second-degree expression for b_e/b was fitted directly to the given data, as follows:

$$\frac{b_e}{b} = 1.8\frac{t}{b}\sqrt{\frac{E}{S}} - 0.9\left(\frac{t}{b}\sqrt{\frac{E}{S}}\right)^2; \text{ standard deviation} = 0.082 \text{...(19)}$$

Eq. 19 corresponds to Eq. 6 of the paper.

Although this curve is a slightly better fit than Eq. 6, it is not significantly so. Noting the apparently wide discrepancy in coefficients in Eq. 19 and Eq. 6 which causes no appreciable change in fit, one sees that the scatter of the points is so large for a b_e/b-ratio of nearly unity that more exact methods are scarcely justified.

In view of the foregoing discussion and the fact that a more careful study of the behavior of b_e/b is needed in the range $0.7 \leqq b_e/b \leqq 1$ it is suggested that, until more data are available, Eq. 18 be used for design purposes. This eliminates the necessity for the design curves given in Fig. 7 and permits direct use of Eq. 18 itself in design problems.

In conclusion the writers would like to call attention to a few mechanical improvements which could be made in the paper:

1. Fig. 7 would perhaps be more useful if the ordinates were values of b_e/b rather than of b_e/t.
2. The number of significant figures is misleading in several places, as, for example, in Eqs. 7 where an approximate value of 25 is used in combination with coefficients such as 1.0906.

3. Averages of the ratios d_1/d_t and d_2/d_t were the criteria used to judge the validity of using the equivalent width as a basis for determining deflections. Actually, it is the standard deviations of the values of d_1/d_t and d_2/d_t about these means which are the more significant measures. In the light of these deviations, the use of b_e for the calculations of deflections is far more superior than is indicated in the paper.

4. Finally, not only in this paper but in all publications it would be extremely helpful if standard deviations were included whenever mean values or fitted curves are used, since these give the reader a much more accurate picture of the reliability of the results.

L. C. Maugh[25] and L. M. Legatski,[26] Associate Members, ASCE.—For many years structural engineers have recognized the danger of subjecting thin elements to high compressive stresses; but, in general, they have avoided, rather than solved, the problems that are associated with the instability of such elements. However, industry is demanding that engineers remove the restrictions imposed by specifications and take advantage of any economy that may be obtained by the use of thin sheet metal. This interesting and timely paper is unusually important, therefore, inasmuch as it is related to the development of new specifications for the design of such structural members. Consequently, any discussion of the paper cannot logically ignore the problems that are presented, if design methods follow the procedures recommended by the author.

The basis of the solution recommended by Professor Winter is the equivalent width method, originally developed for the design of aircraft structures from tests on thin sheet panels with and without stiffeners. Most of these tests were made on specimens with large b/t-ratios and with E-values much less than those for steel. Consequently, the boundary effect was so large as compared to the primary resistance of the plate element that the latter effect could be ignored. The edge or boundary effect has commonly been evaluated (as the author states) by considering the strength of two-edge strips that are assumed to be unaffected by the central section between them. Even for such elements with large b/t-ratios, it would be more logical to consider each edge strip, in Fig. 1, as a separate plate rather than to combine the strips to form a single plate.

The principal question involved in the paper seems to be whether the equivalent width transformation is the best solution to the problem. Although the author rejects the Bryan formula[8] for calculating the resistance of the actual element, he does use it for determining the strength of the equivalent plate. Certainly, Eq. 2b is derived from the same basic differential equation as the general Bryan formula, that is,

$$\frac{\partial^4 \eta}{\partial x^4} + 2 \frac{\partial^4 \eta}{\partial x^2 \, \partial y^2} + \frac{\partial^4 \eta}{\partial y^4} = - \frac{s}{D} \frac{\partial^2 \eta}{\partial y^2} \dots\dots\dots\dots\dots\dots (20)$$

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[25] Associate Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[26] Research Engr., Dept. of Eng. Research., Univ. of Michigan, Ann Arbor, Mich.

in which η is the lateral displacement and $D = \dfrac{E\,t^3}{12(1 - \mu^2)}$. Furthermore, there is no fundamental reason why the differential equation cannot be satisfied by superimposing the wave displacements involved in the edge resistance on the sine series ordinarily used for deriving the primary resistance of the plate.

The question arises, therefore, as to whether the ultimate strength of a thin element does not also depend on the primary resistance, as well as on the edge effect. Strain measurements have been made by various investigators which indicate a considerable increase in axial strain at the edges after the element has buckled. Strain measurements that were made by the writers on steel C-sections, 24 in. long, and that were tested in direct compression are shown in Fig. 18. These measurements were taken with standard electrical-resistance gages fastened in pairs to both faces of the plate elements and connected in

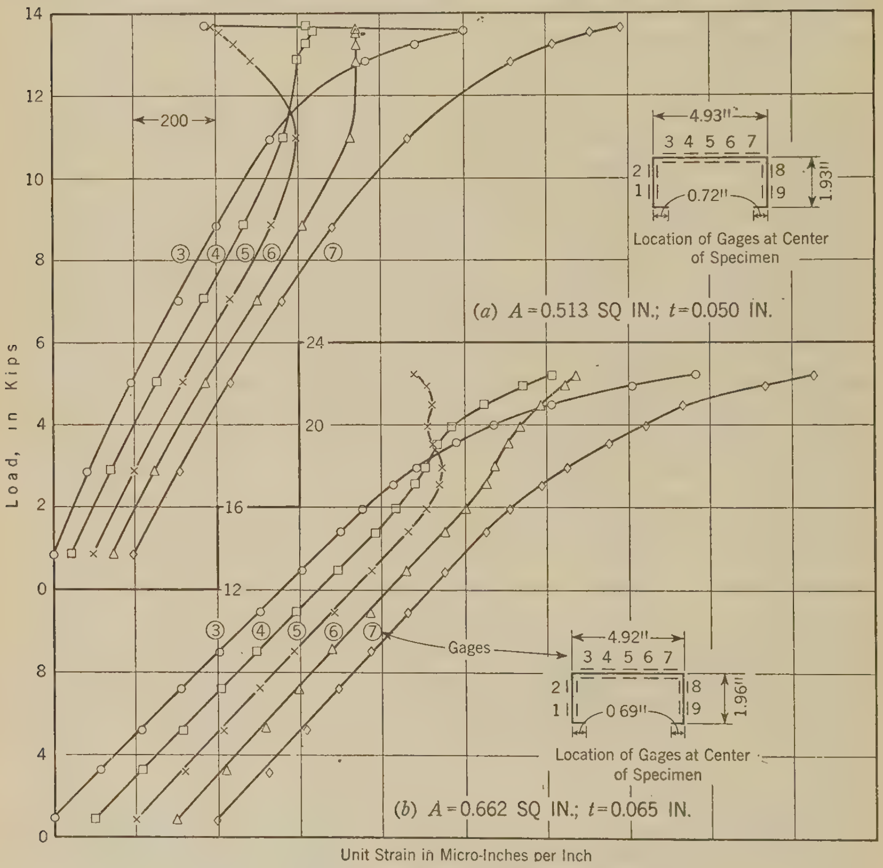

Fig. 18.—Strain Measurements on Steel C-Sections (L = 24 In. and L/r = 31.2)

series to give the average axial strain. In Fig. 18(*a*) where the web has a b/t-ratio of about 98, gage 5 at the center shows some reduction as the load approaches its ultimate value. Gages 4 and 6, adjacent to the center, remained practically constant, whereas gages 3 and 7 near the corners increased considerably. In Fig. 18(*b*) where the b/t-ratio of the web is about 75, gage 5 remains practically constant, gages 4 and 6 show some increase, and gages 3 and 7 show considerable increase as the ultimate strength of the specimen is approached. Because of the biaxial state of stress on elements near the center, the longitudinal stress is not directly proportional to the strain measurements shown. Nevertheless, the action in these specimens not only indicates an addition to the strain close to the corners but also shows that the primary resistance of the element is a definite and continuous phenomenon that may fluctuate but does not disappear as the ultimate strength of the specimen is reached. The author has not shown any distribution of strain across the compression flange of his specimens; nor has he stated whether such information was obtained.

A better approach would be to superimpose the edge effect on the primary resistance, whenever the element becomes unstable at a unit stress less than the proportional limit of the material. Above the proportional limit, a formula that considers the variation in E should be used and the edge effect neglected. For such assumptions, the ultimate strength of an element would be

$$s_u = s_c + s_e \quad\text{.........................}(21)$$

in which s_c is the critical stress of the element, and s_e is the average edge effect. It is assumed that s_c is less than s_p, the proportional limit. To determine s_e, the edge effect, it will be assumed that the resistance of the material near the boundary is approximated by that of a thin plate, supported at one edge and free at the other. The strength of this plate will be calculated from the usual formula; and, furthermore, the average unit stress on the edge plate will be taken equal to the proportional limit s_p. A mathematical investigation has indicated that this condition approximates closely the maximum resultant force that can be sustained by the assumed edge plate. Therefore, the width, b_s, of this edge effect will be such that:

$$s_p = \frac{K_1 E}{(b_s/t)^2} \quad\text{.........................}(22)$$

or

$$b_s = t\sqrt{\frac{K_1 E}{s_p}} \quad\text{.........................}(23)$$

The total additional resistance F' of both edges is, therefore,

$$F' = 2\,(s_p - s_c)\,t^2\,\sqrt{K_1}\sqrt{\frac{E}{s_p}} \quad\text{..................}(24)$$

and the average increase in stress over the element, due to the edge effect, is

$$s_e = \frac{F'}{b\,t} = \frac{2\,(s_p - s_c)\,\sqrt{K_1}}{\left(\frac{b}{t}\right)} \sqrt{\frac{E}{s_p}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25a)$$

The critical stress s_c is given by the Bryan formula:

$$s_c = \frac{K\,E}{\left(\frac{b}{t}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

The values of K and K_1 vary with the degree of restraint at the edges. For simply supported edges, $K = 3.62$ and $K_1 = 0.452$. These values give the following equation for the ultimate strength of an element that is simply supported at the edges:

$$s_c = \frac{3.62\,E}{\left(\frac{b}{t}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26a)$$

and

$$s_u = s_c + \frac{1.35\,(s_p - s_c)}{\left(\frac{b}{t}\right)} \sqrt{\frac{E}{s_p}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26b)$$

It is suggested that the proportional limit, s_p, be taken as five eighths of the yield-point stress, s_y, to allow for variation of material in the specimens.

The values obtained from Eq. 26*b* have been compared with the experimental results that were obtained by L. Schuman and G. Back in their comprehensive series of tests. The theoretical values as shown by the solid lines[27] in Fig. 19 agree satisfactorily with the test results for large values of b/t, but are somewhat high for the smaller values. This condition is not surprising as the initial curvature in the specimens would have the greatest effect on the primary resistance of the stiffer elements. For steel plates with b/t-ratios less than 80, the edge effect could well be ignored. The dotted lines in Fig. 19 represent the critical stress s_c. The equations of the solid lines in Fig. 19 are, for $s_c > s_p$—

$$s_u = \frac{3.62\,E\,\dfrac{s_y}{s_y - s_p}}{\dfrac{3.62\,E}{s_y - s_p} + \left(\frac{b}{t}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

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[27] "Strength of Rectangular Flat Plates Under Edge Compression," by L. Schuman and G. Back, *Technical Report No. 356*, National Advisory Committee for Aeronautics, 1931, Figs. 9(*b*) and 10.

and, for $s_e < s_p$—

$$s_u = \frac{3.62\,E}{\left(\frac{b}{t}\right)^2} + \frac{1.35\,(s_p - s_e)\,\sqrt{E/s_p}}{\frac{b}{t}} \quad \text{.............. (28)}$$

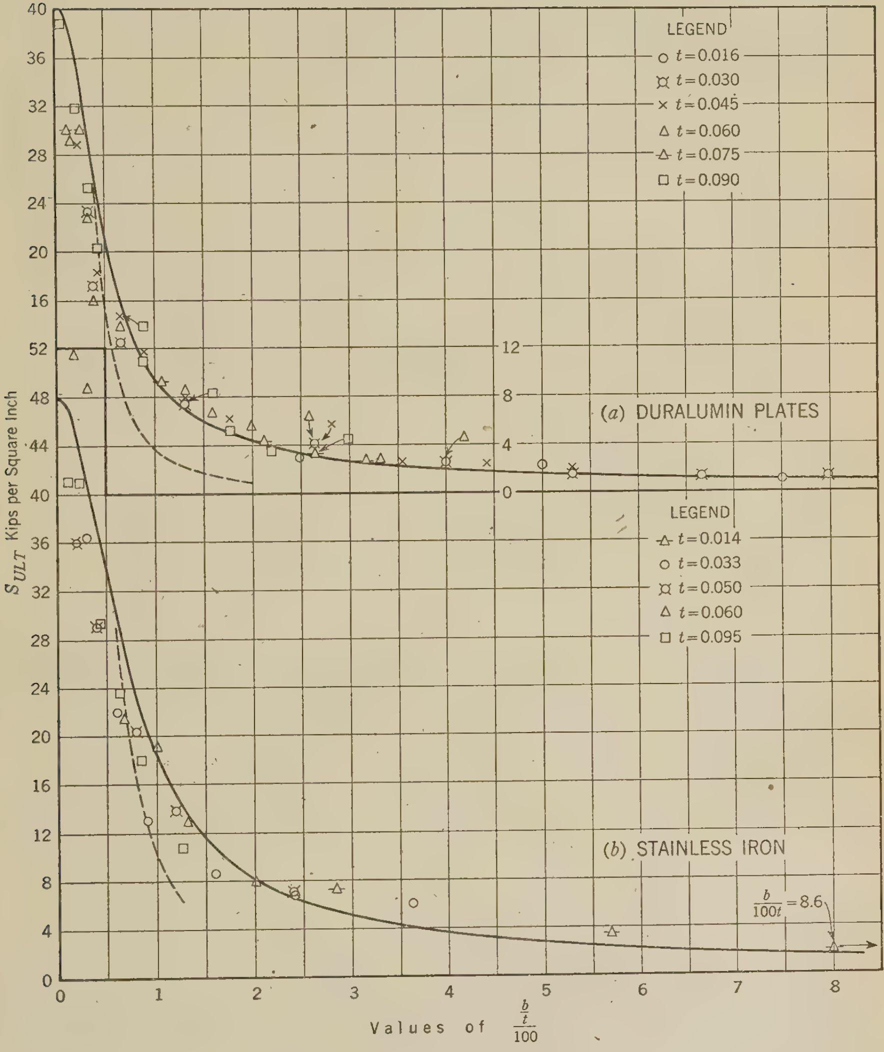

FIG. 19

The author has based his design procedure on the use of Eq. 2b in which the coefficient C is represented by a function of $\sqrt{\frac{E}{s}} \times \frac{t}{b}$. To the writers, this variation is tantamount to assuming that the coefficient must correct the equation for other factors as well as for imperfections in the specimens and apparatus. The scatter of points in Fig. 2 could indicate many things besides deviation from shape such as variation in edge restraint of the element, inaccuracies in the determination of the neutral axis and of stress distribution over the beam, and neglect of the primary resistance of the element.

Without accepting the effective width concept as the most logical approach to the stability problem in light-gage members, the writers would question several points in the author's method of determining the effective width experimentally. The instrumentation appears to be inadequate for the purpose of locating the neutral axis. With the gages on only one side of the member, as shown in Fig. 3, the effect of lateral bending and twisting action, which could introduce considerable error, is ignored. The fact that these effects were not nullified by duplicate gages on the opposite edge casts some doubt on the accuracy with which the neutral axis was located from strain measurements. The author states that at the failure load the neutral axis is again located from strain measurements. The only possibility of securing this result from the strains measured is by assuming a triangular strain distribution under conditions that approach the assumed rectangular stress distribution. For unsymmetrical sections such as those of test series A, the writers cannot agree that the neutral axis can be located accurately in this manner.

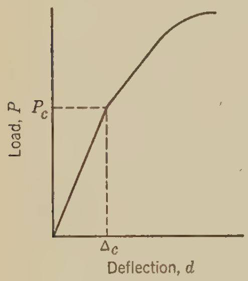

FIG. 20

In calculating deflections of light-gage beams, the use of an effective section appears to offer the best solution when the compression elements are stressed beyond the critical buckling stress. Fig. 20 shows a typical load-deflection curve for such a beam. The ordinate P_c indicates the load at which the flange elements reach the critical stress. Up to this load the properties of the full section should be used in calculating deflections. For higher loads an additional increment of deflection should be added to d_c, and this increment should be calculated using a reduced moment of inertia. For steel sections, sufficiently accurate results can be obtained by calculating the reduced moment of inertia using a flange width of 20 t adjacent to each element that stiffens the flange.

Perhaps the author was led to the erroneous conclusion that the Bryan formula cannot be used to determine the ultimate buckling strength of thin plates—by attempting too close an analogy between the Bryan formula and the Euler formula for columns. Both formulas give a critical buckling load. The Euler load is the ultimate load for the column since there is no other element of strength to prevent failure. The Bryan load is the load at which the plate element forms into buckling waves, but it is not the ultimate load because a boundary or edge effect remains as an additional element of strength. If the Bryan load alone is compared to the ultimate load, the agreement is good for b/t-values up to about 50 for duralumin and 80 for steel, nickel, and monel metal. For larger b/t-values, the Bryan load is low (as indicated by the statement of Messrs. Schuman and Back, referred to by the author under the heading, "Type A. Compression Flanges Stiffened Along Both Longitudinal Edges:

Equivalent Width of Thin Flanges"). This condition occurs because the boundary effect is small as compared to the effect of the Bryan load within the foregoing values of b/t, whereas, for larger b/t-values, the reverse is true.

Bruce G. Johnston,[28] **M. ASCE.**—The tests[29] made at Lehigh University at Bethlehem, Pa., by Lloyd Cheney, Jun. ASCE, and the writer, supplement the results presented in Fig. 14 of the paper by Professor Winter. In the Lehigh University tests, the flanges of both carbon and silicon steel 10WF49 sections, each from a single rolling, were planed to different thicknesses, thereby providing a range of width to thickness ratios with a minimum variation in yield point.

Details of the tests will not be discussed, since they have already been reported,[29] but the results are summarized in Fig. 21.

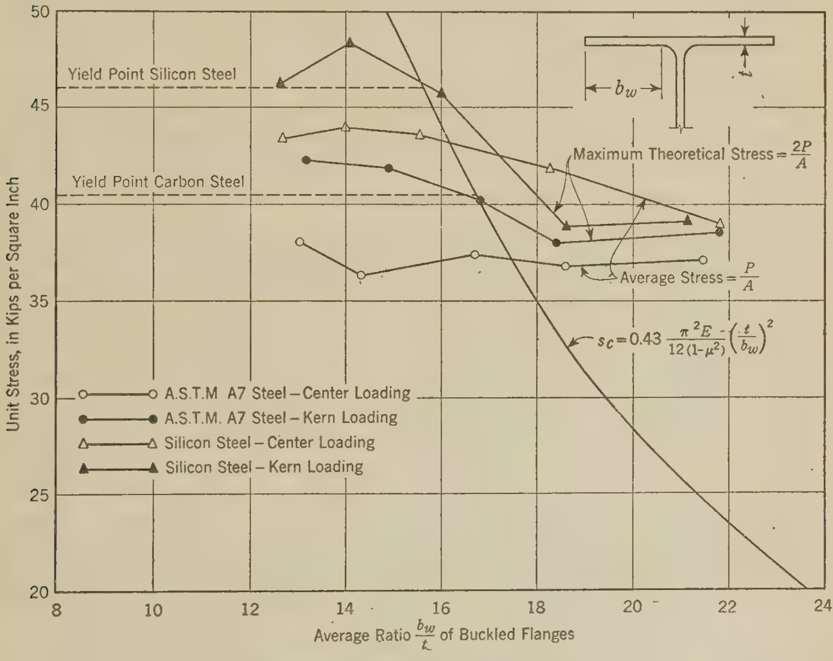

Fig. 21.—Stress in Buckled Flange at Maximum Load

For a very long plate, with one longitudinal edge simply supported and the other free, S. Timoshenko[30] gives a coefficient of 0.46 in the elastic buckling formula, whereas the author quotes a factor of 0.50 in Eq. 9. Professor Timoshenko's coefficient is based on Poisson's ratio of 0.25, corresponding to neither steel nor aluminum, for which coefficients of 0.43 and 0.41 are obtained[31,32] for Poisson's ratios of 0.30 and 0.33, respectively. These differences are somewhat academic, since the buckling strength will be affected by the torsional stiffness of the thick section at the juncture of the web and flange, and by the bending stiffness of the web, which may be negative or positive. However, if no account is to be taken of these effects, the question arises as to whether a coefficient of 0.43 is not preferable to one of 0.50.

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[28] Associate Director, Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[29] "Steel Columns of Rolled Wide Flange Section," by Bruce Johnston and Lloyd Cheney, *Publication No. 190*, A.I.S.C., November, 1942.

[30] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 340.

In the tests at Lehigh University, in the case of large values of $\frac{b_w}{t}$ the web was thicker than the planed flanges, providing a partly fixed edge, and yielding effective coefficients even greater than 0.5; in fact, the buckling in all the tests was plastic rather than elastic. The results presented in Fig. 21 are self-explanatory and show that flanges of rolled shapes may be expected to develop at least 90% of the yield point of the material, within the range of $\frac{b_w}{t}$ in which plastic buckling is predicted by the elastic buckling formula.

Edward L. Brown,[33] Esq., and Don S. Wolford,[34] Esq.—More accurate methods for computing strength and other properties of structural sections formed of thin flat-rolled metals have been needed for some time. Previous methods were mainly deficient because they did not adequately evaluate the strength of compression flanges. Designers have become accustomed to assuming certain maximum multiples of the thickness to be fully effective in stiffened flanges, which has been a fairly successful practice in reasonably compact sections. However, it was evident that such simple relations did not adequately define behavior for all cases. Professor Winter presents relations in his paper by which such flanges may be evaluated, taking both b/t-ratio and stress level into account. Sections discussed in this paper cover flanges with b/t-ratios up to 170. The purpose of this discussion is to present test data made on sections containing compression flanges with b/t-ratios ranging from 242 to 429.

These sections were formed of 18-gage and 20-gage mild steel. All were 3 in. deep with compression flanges of 12-in. and 16-in. widths. They were similar to the U-beam in Fig. 3, except that the lower flanges were lipped and one flange was turned outward to provide a joint for adjacent sections. These tests were sponsored jointly by the American Iron and Steel Institute and Cornell University at Ithaca, N. Y., and were made under Professor Winter's supervision. They were witnessed by the writers of this discussion who also made the calculations and analysis of the results.

Four identical pairs of each type of section were subjected to quarter-point beam loading using an 80-in. span. Deflections and strains in top and bottom

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[31] "Theory of Elastic Stability Applied to Structural Design," by Leon S. Moisseiff and Frederick Lienhard, *Transactions*, ASCE, Vol. 106, 1941, p. 1059.

[32] "Chart for Critical Compressive Stress of Flat Rectangular Plates," by H. N. Hill, *Technical Note No. 773*, National Advisory Committee for Aeronautics, August, 1940.

[33] Engr., American Rolling Mill Co., Middletown, Ohio.

[34] Senior Research Engr., American Rolling Mill Co., Middletown, Ohio.

fibers were observed as load was applied and released at successively higher values. The yield-point load was assumed to be that which first caused a large increase in residual deflection upon release of load.

Section properties were computed for each type of section using (1) gross section and (2) effective section. The effective widths were determined from curves computed by Eq. 6, carried out to b/t-ratios in the range needed. The measured tensile yield strengths of the steels used were taken into account. Computed and observed deflections and loads are compared in Table 6.

TABLE 6.—COMPARISON OF ACTUAL AND COMPUTED VALUES

Type	DIMENSIONS, IN INCHES			DEFLECTION,[b] IN INCHES			LOAD, IN POUNDS[c]		
	Width	Thick-ness	Ratio[a] b/t	Observed in test	Computed		Observed in test	Computed	
					Effective width	Gross section		Effective width	Gross section
320.....	12	0.037	321	0.25	0.30	0.16	1,267	1,212	1,912
328.....	12	0.049	242	0.25	0.28	0.17	1,725	1,610	2,070
360.....	16	0.037	429	0.24	0.30	0.15	1,350	1,179	1,880
368.....	16	0.049	323	0.28	0.28	0.16	1,725	1,683	2,168
Average ratio, $\frac{\text{computed value}}{\text{test value}}$				...	1.14	0.63	...	0.935	1.340

[a] The ratio of width (between webs) to thickness. [b] Deflections are at working stress level, assumed as the yield point divided by 1.85. [c] Loads are for a single section and are at the yield point.

Computed loads based on effective-width properties averaged 6.5% lower than observed loads, indicating good correlation tantamount to safe and efficient design. Gross section properties led to expected loads that were higher than actually obtained, and were therefore misleading.

Deflections shown in Table 6 are at working stress levels, determined by dividing the yield point by 1.85. It is the deflection at this stress level that is pertinent in design. The moment of inertia used to determine deflection is based on the effective width of the compression flange at the working stress level. A larger part of the flange is effective at this level than is effective at the yield-point stress level.

The calculated deflections averaged 14% greater than those observed in the test, which is reasonable. Deflections calculated by gross section properties were only 63% of those observed in test, which shows that such an approach is not reliable.

Top and bottom strains were approximately equal at the yield loads, placing the neutral axes near mid-depth where computations using properties based on effective widths indicated they should be, rather than near the compression flange as indicated by gross properties.

In summary, these additional tests and correlations show definitely that the relations given by Professor Winter for determining the effective widths of compression flanges, stiffened along both edges, enable the designer to compute design loads and deflections quite accurately at b/t-values at least as high as 429.

George Winter,[35] M. ASCE.—Since the publication of this paper, the concept of the effective width has been adopted in two structural design specifications: The Specifications for the Design of Light-Gage Steel Structural Members, American Iron and Steel Institute, April, 1946, and the February, 1946, edition of the Specifications for the Design, Fabrication, and Erection of Structural Steel in Buildings, American Institute of Steel Construction. In the latter code, Sections 18*c* and 18*d* make use of an effective width, although in a greatly simplified manner. The writer does not claim any credit for these latter sections. He merely wishes to note the increasing acceptance of this concept in structural design.

The gracious comments of Mr. Llewellyn are deeply appreciated, particularly coming, as they do, from an "old-timer," with such thorough and long-standing interest in this particular field. To Mr. Llewellyn, credit is due for having first proposed the use of an effective width in structural design of light-gage members.[21] Although the values originally suggested by Mr. Llewellyn underwent inevitable correction by subsequent investigation, they were amazingly close to over-all averages, considering the dearth of information on the subject more than a decade ago. Although Mr. Llewellyn differentiated between edges supported by webs, and those stiffened by lips, the writer found that flanges stiffened either way developed the same strength, provided the rigidity of the lip proper was sufficient to furnish full support. This statement holds at least for the range of b/t of the beams of Table 2—that is, for the lipped flanges tested in this investigation.

The only contribution that undertakes to challenge the writer's approach to the problem (merely with regard to flanges stiffened along both edges) is that by Messrs. Maugh and Legatski. To evaluate the contentions contained therein, it appears necessary to clarify some obvious misunderstandings, the source of which the writer is unable to trace.

In summarizing their discussion, Messrs. Maugh and Legatski state that "Perhaps the author was led to the erroneous conclusion that the Bryan formula cannot be used to determine the ultimate buckling strength of thin plates * * *." Three lines farther, however, they write:

> "The Bryan load is the load at which the plate element forms into buckling waves, but it is not the ultimate load because a boundary or edge effect remains as an additional element of strength."

This is exactly what the writer maintained in his discussion of fundamentals (paragraph preceding Eqs. 2), where he stated that,

> "The central, more highly distorted, regions of the plate decrease in their resistance, thus throwing more of the total compressive force toward the stiffened edges * * *. This action [failure] occurs at a load higher than the critical—that is, higher than that load at which, theoretically, small deflections start to occur * * *."

These two statements are exactly identical in content, and the writer is at a loss to understand wherein lies his "erroneous conclusion."

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[35] Associate Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

In the concluding paragraphs, the statement, "Without accepting the effective width concept as the most logical approach * * *" seems to indicate that Messrs. Maugh and Legatski know of, or have developed, a method which dispenses with that concept. Actually, the approach they propose is likewise based on an effective width, implicitly for stress determinations (see Eq. 23), and explicitly for deflection computations. The writer, again, is at a loss to understand the quoted statement.

From an analytical point of view, contrary to the contributors' contention, Eq. 20, from which the Bryan formula is derived, is no longer applicable, once the critical stress is exceeded. Under such conditions, it is necessary to consider the so-called large deflection theory; that is, Theodor von Kármán's differential equation,[36,37] in which F is the stress function:

$$\frac{\partial^4\eta}{\partial x^4} + 2\,\frac{\partial^4\eta}{\partial x^2\,\partial y^2} + \frac{\partial^4\eta}{\partial y^4} = \frac{t}{D}\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2\eta}{\partial y^2} - 2\,\frac{\partial^2 F}{\partial x\,\partial y}\,\frac{\partial^2\eta}{\partial x\,\partial y}\right)\ldots(29)$$

which depends not only on the external compression force, but also on the deformations. It is the extreme complexity of Eq. 29 which is the reason that a solution, so far, has been obtained for circular plates, only. In other words, once the critical stress is exceeded, the plate assumes a new state of equilibrium, which is governed by factors greatly different from those which determine the Bryan load. Eq. 20 does not hold for this state of equilibrium, because, in this state, the stress s is not uniaxial and constant throughout the flange.

An analytical investigation[38] of the stresses above the critical, by means of approximate strain energy methods, not only shows the stress distribution to be exactly of the type of Fig. 12; it also indicates that, contrary to the assumption made by Messrs. Maugh and Legatski, the stress in the center is not constant and equal to the Bryan stress. For very wide and thin plates (as Professor Timoshenko has shown[39]), the center strip of an edge-compressed plate is subject to tension, rather than to the Bryan compression stress. The same result, in a rigorous manner, was obtained by Messrs. Friedrichs and Stoker for a circular plate.[9]

To summarize: The writer did not maintain, as is stated by Messrs. Maugh and Legatski, that the center strip is free of stress. This is evident from Fig. 12, whose validity is borne out by the contributors' own strain measurements (for which the writer is grateful indeed as collateral evidence). However, he does maintain that the stress at the center strip is not constant and equal to the Bryan stress, which is again confirmed by the contributors' own stress measurements (Fig. 18(*a*)). These show a decrease of the center stress by about 40% of its maximum value with increasing loads, even at a b/t-ratio as moderate as 98.

In view of the theoretical complexity of the problem, the writer pursued a frankly semi-empirical method. Messrs. Maugh and Legatski propose instead

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[36] "Encyklopaedie der Mathematischen Wissenschaften," Vol. IV/4, 1910, p. 349.

[37] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 323.

[38] *Ibid.*, pp. 390–395.

[39] *Ibid.*, p. 395, Fig. 207.

a semi-analytical approach, based on a number of arbitrary assumptions, one of which is discussed herein (that is, center-strip stress equals Bryan stress). Another such assumption is that of taking the proportional limit as five eighths of the yield point, which was apparently necessary to make the equations fit the Schuman and Back results. Even for steels, the so-called "proportional limit," is a completely fictitious quantity, whose value depends primarily on the investigator and his instrumentation.[40] In sharply yielding steels, in addition, the error introduced by equating the proportional limit to the yield point is almost always negligible as compared with inevitable errors, due to other unknown but important factors (edge restraint, initial distortion, etc.).

In their method, Messrs. Maugh and Legatski like the writer, introduce an effective width, b_s, relating it, however, only to the part of the stress in excess of the supposedly constant critical stress. Their final answer is given in terms of a mean stress, s_u, related to the entire unreduced flange area.

For designing members in uniform compression, it may be irrelevant whether the actual stress distribution, Fig. 12, is replaced by a uniform mean stress, or by the writer's equivalent width, related to the maximum stress.

For designing members in flexure or eccentric compression, however, it is necessary to determine the actual maximum edge strain; because it is that strain which governs the location of the neutral axis, without which stresses cannot be computed. The writer's method allows this determination, the contributors' does not. The latter, therefore, would determine flange stresses by the ordinary flexure formula, using the centroidal axis. How far the actual neutral axis, as determined by strain measurements, can deviate from the

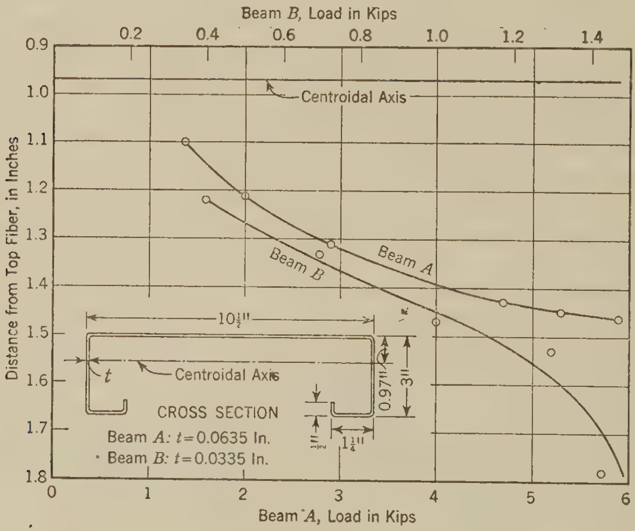

FIG. 22

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[40] "Stress, Strain and Structural Damage," by H. F. Moore, *Bulletin No. 10*, Univ. of Illinois, Urbana, Vol. 37, 1939, pp. 17–18.

centroid is shown in Fig. 22. The results there given were obtained on two beams of practically identical over-all dimension, but with different sheet thicknesses. The error that would result in using the centroidal axis instead of the actual neutral axis for design computations is evident from the figure.

This situation is implicitly conceded by Messrs. Maugh and Legatski in their statement that, for deflection computations, an equivalent width must be used. For reasons not indicated this width, for deflection computations, is different from the width used implicitly by the contributors for stress computations, as given in Eq. 23. Effective widths, locations of neutral axes, and deflections are interrelated purely geometrically, inasmuch as they are all functions of the magnitudes of the extreme fiber strains. If an effective width approach is necessary for determining deflections (that is, neutral axes), it is physically contradictory to maintain, on the other hand, that stresses can be computed from unreduced widths. This is tantamount to stating that a beam has two neutral axes—one, referred to the unreduced section, governing stresses, and another, referred to the reduced section, governing deflections. In the writer's proposed method, the same approach applies to both stress and deflection determinations.

Parenthetically, the contributors' method (computing deflections by (*a*) using an unreduced moment of inertia to find part of the deflection, (*b*) computing a critical stress, and (*c*) determining a second, reduced moment of inertia for finding the additional deflection) is certainly more cumbersome than that proposed in the paper. It results, qualitatively, in the same behavior as that found by the writer; that is, the effective moment of inertia (in the contributors' case, the weighted mean between the full and the reduced one) decreases with increasing b/t and increasing stress. It is rather doubtful, however, whether the arbitrary effective width of 40 t for the reduced moment of inertia, given without justification or empirical verification, leads to reasonably accurate results. For the wider flanges (as used in many commercial decks with high b/t) where the critical stress is very low, the use of 40 t will be found to lead to erroneous results on the conservative (uneconomical) side in determining deflections at design loads—that is, at stresses of from 10 to 20 kips per sq in. (For such conditions effective widths in this investigation were found to be of the order of 60 t to 80 t as compared with the contributors' 40 t.)

With regard to the contributors' criticism of test methods, the writer will agree to the extent only that (1) strain measurements on light-gage sections are plagued by various disturbing influences not present in more solid members; and (2) that it is always possible to think of other methods of instrumentation that could have been used. With regard to the latter, however, the time element is an important consideration. In this investigation, it was necessary to cover a very wide range of dimensions, which can be done on a large number of specimens, only. The investigator is limited, therefore, to such measurements as will furnish the most pertinent data. This, it is believed, was achieved by the methods selected. The collateral, confirmatory evidence provided by Messrs. Maugh and Legatski, by different test methods, is therefore doubly welcome.

The contributors' statement that twist and lateral deflection may have distorted the measurements, can be discounted, except for possible microscopic effects. Beams of the shape of Fig. 3 have no tendency toward lateral deflection, particularly if loaded through rollers with axes perpendicular to the axis of the specimen. In fact, to deflect laterally, they would have to overcome the contact friction at the load points. Not only can it be shown by computation from known friction coefficients that this is impossible, but any such sliding motion would immediately manifest itself by a scraping sound, as the writer observed frequently in tests of an entirely different nature. Twist, on the other hand, was prevented by a loading arrangement, which forced the beam into parallel vertical displacement. It is evident that effects of the magnitude of those of Fig. 22, consistently obtained on a great number of specimens, cannot be ascribed to details of instrumentation. Finally, the problem of the linearity of strain distribution in such specimens, even if it were open to question, appears to be irrelevant. The location of the neutral axis, as used in the proposed methods, is merely a means to determine, in design computations, the relative magnitude of the top and bottom strains. Since these strains were measured directly, even a curvilinear transition would not affect the results of such design computations. In addition, however, no reason for a curvilinear strain distribution is given by the contributors; nor is any apparent to the writer, particularly since strain measurements were made in the center part of the quarter-point loaded beams, that is, in a region of pure bending.

Two of the contributions (Messrs. Lewis and Gunder's, and Mr. Karol's) do not question the findings of this investigation, but concern themselves mainly with the mathematical form of Eqs. 6 and 7. Formulas such as these are developed with due consideration to simplicity of application. In this particular case, it was imperative, for simplification of design, to delimit a range of b/t, for which the full width can be used (up to $b/t = 25$). Although, theoretically, the establishment of such a definite limit is questionable, it is completely justified practically. This is why seemingly elaborate mathematical expressions had to be chosen, which result in $b_e = b$ for this limit, independently of the stress. The mathematical complexity of the formulas is relatively irrelevant, since designers will work from graphs such as Fig. 7, rather than from equations. In addition, Mr. Karol's Eq. 13, which involves a hyperbolic function, for that reason, will not be found very convenient by most designers.

Mr. Karol's cautioning remark regarding the use of the writer's chart (Fig. 7) for materials other than steel is to the point, and should be considered in such applications.

The writer gladly accepts Messrs. Lewis and Gunder's justified criticism of the statistical method used in this paper. It is quite true that more sensitive devices, such as standard deviations, should be used in determining the accuracy of fit of empirical formulas.

The writer appreciates the painstaking work of Messrs. Lewis and Gunder in replotting Fig. 2. He does not believe that Eq. 18 should be used practically, despite its simplicity. On the one hand, as the contributors state, goodness of fit of Eq. 18 within the tested range is not as satisfactory as that of Eqs. 6

and 7. Also, tests conducted since the publication of the paper showed Eq. 6 to be applicable satisfactorily for values of b/t up to 400 and more (see contribution by Messrs. Brown and Wolford). In this range of high b/t, Eq. 18 would err on the conservative side up to about 25%. On the other hand, Eq. 19 is identical in structure with Eq. 6, except for minor differences in the constants, and, as stated by the contributors, is not significantly better. The writer is grateful for the fact, established by Messrs. Lewis and Gunder, that, despite his somewhat cursory methods of curve fitting, he apparently managed to arrive at expressions which are as accurate as those developed by more sensitive statistical means.

Professor Johnston furnishes interesting additional material on flanges stiffened along one edge, by reference to his and Professor Cheney's earlier tests on wide flange sections.

The failure criterion used by Messrs. Johnston and Cheney differs from the writer's in that they noted merely the ultimate loads, whereas the writer considered also the stress at which local wrinkling was first noted. Therefore, Professor Johnston's results should be compared with the values of the ultimate stress, s_u in Table 4. In the range of b/t, common to both investigations (that is, up to about 21), it is seen from Table 4 that the writer obtained ultimate stresses of 0.8 to 0.9 of the yield point, with very few exceptions. This is in general agreement with Professor Johnston's findings of a 90% yield-point strength. The somewhat lower values obtained by the writer on thin gage specimens are probably due (a) to greater edge restraint in the H-sections, as noted by Professor Johnston, and (b) to the better forming accuracy of the milled H-beam flanges, as compared with cold formed sheet steel elements. The writer's colleague, Professor Cheney, volunteered the information that in many of the tests cited by Professor Johnston, development of slight waves and kinks was noticed at loads below the ultimate, which is in agreement with the findings of this investigation.

The writer wishes to thank the American Rolling Mill Company for releasing for publication the test data given in Messrs. Brown and Wolford's contribution. These data furnish important, supplementary evidence, since the b/t-ratios of these specimens, 242 to 429, are far beyond the range of those discussed in the paper. The test results given in Table 6, as well as conclusions by Messrs. Brown and Wolford, confirming the validity of the proposed method in this high range of b/t, speak for themselves. It is seen that the determinations made on the basis of Eq. 6, if compared with test results, are slightly on the conservative side, both for deflection and for strength. The deviations are small and within the range of discrepancy observed in most structural testing, but they point to one factor that should be emphasized in closing.

The behavior of thin compression flanges is naturally influenced by the amount of rotational edge fixity provided by adjoining elements, such as the webs in Fig. 3. A reasonably simple design method cannot be expected to take explicit account of this involved factor, particularly since, in the great number of tests, its numerical effect was found to be small. The amount of restraint provided in the very wide flanges by the shallow webs of Messrs. Brown and Wolford's specimens is rather large, as compared with a number of other beams in this investigation, with a consequent strengthening effect reflected in the test results.

The proposed methods, therefore, merely claim to represent average conditions of restraint. However, it is important to note that, despite a great variety of conditions of support and restraint obtained in these tests, deviations computed from observed results were within narrow limits, acceptable in practical work.

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TRANSACTIONS

Paper No. 2306

FACTORS CONTROLLING THE LOCATION OF VARIOUS TYPES OF INDUSTRY

By Charles P. Wood,[1] Esq.

With Discussion by Messrs. Harold M. Lewis, R. F. Goudey, and Charles P. Wood.

Synopsis

Any discussion of factors controlling the location of industry must take into account new and potent influences, resulting from World War II, which have disrupted a normal way of life and have changed the scope of future industrial activities. Revolutionary improvements in methods and equipment will be the constructive results of research and manufacturing experience growing out of the war effort. New reservoirs of labor and new potential manufacturing areas have been created by the development of war industries remote from established industrial centers. A reorganized distribution system, already geared up to export tremendous quantities of material from all parts of the United States through both Pacific and Atlantic ports, will facilitate decentralized and diversified industrial operations.

New opportunities, improved processes, and other causes of progress are familiar experiences in the American business and manufacturing field. The period from 1914 to 1940 seemed at one time to include a cycle of war and depression and recovery. The greatest war in history promptly followed, and resources and markets, once thought to be assured for the future, are no longer to be taken for granted. Factors that affect the location of industry are subject to changeable influences.

The Prospect of Changed Conditions

The rearrangement of the industrial pattern in the United States has been predicted from time to time without sufficient allowance for practical considerations. In defining the elements that have most to do with the location of industries, care must be taken to avoid categorical statements. The younger generation of engineers should study the history and economics of industries,

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as well as the technical theories, to guard against the mistakes of their predecessors.

Too many industrial plans are still dependent on the same way of life that was in effect during the past generation. The most important changes affect markets, transportation, labor, and management. Changes in processes and equipment are radical and far reaching, but they will be much easier to make than the changes that depend on providing a new kind of industrial environment for workers and a new system of distribution. Perhaps the greatest difference between the old and the new is the conception of time.

The soldiers, sailors, Wacs, and Waves of World War II will form the dominant element of management and of the working force as well as of the domestic market. Some of them made the trip from India, the Philippines, Japan, and China within two days—the same time their parents were brought up to think would be required to go from New York, N. Y., to Houston, Tex., or from Chicago, Ill., to San Francisco, Calif. This conception of time is affected by other innovations developed during World War II. Short cuts made possible by electronics and radio, seeing in the dark with radar, automatic devices for doing what formerly depended on the human senses and labor, and airborne freight—all these have combined to change much that once made up the requirements which affect the location of industries.

Fundamental Requirements

In spite of all these innovations and improvements, many old difficulties remain unsolved and the factories of the future will require power and labor, transportation facilities, and a supply of materials. Discrimination should be made between the type of enterprise that is tied to rigid requirements and the one that is free to follow the lines of least resistance.

The basic industries, which process raw materials in large bulk, are still confined to locations where transportation of raw materials to the plant and transportation of finished products to the market can be accomplished with greatest economy. Transportation in the broad sense includes power transmission lines, pipe lines for oil and gas, and facilities for assembling labor, as well as freight carriers. The conversion from raw material to the final product generally proceeds in separate steps which may occur at several different locations, depending on the nature of the product and the market to be served.

The Steel Industry

The steel industry offers the best example of integration, because the manufacturing process can be traced from definite sources of raw material to a variety of products without departing from the main channels of commerce and industry. Ore and coking coal for the manufacture of pig iron can be assembled most economically at certain blast furnaces. The larger plants, of course, carry the processes through to the steel ingot and on to the rails, plates, sheets, structural shapes, rods, and wire. Steel plants away from the blast furnaces take advantage of local supplies of scrap and fuel in combination with transportation facilities. Plants must be located to insure economical distribution of the product, as well as delivery of pig iron from the blast furnace to the open hearth or electric furnace whenever the local supply of scrap is not sufficient.

After steel has been made into rods, wire, plates, sheets, and structural shapes, it is sent to the fabricating plant, whose location depends entirely on the local market or on local resources that make manufacturing most economical. For example, Detroit, Mich., consumes steel for automobiles; New Britain, Conn., steel for hardware, and Newport News, Va., steel for ships. The location of these factories and shipyards is governed by factors different from those that govern the location of the steel plant. In some places conditions are favorable for blast furnaces and steel mills (Fig. 1), as well as for miscellaneous manufacturers of steel products, including ships—for example, Sparrows Point, Md., where blast furnaces on the coast are supplied with imported ore and domestic fuel. The Great Lakes area, using domestic ore and fuel, offers similar advantages, except that the size of ships built on the lakes is limited by the width and depth of the Welland Canal. Blast furnaces and steel plants operating on the Pacific Coast are handicapped by high transportation cost of fuel, and to a lesser extent that of ore. The scale of operations in peacetime, however, will depend on local markets and the location of bases for freight rates, as well as on transportation costs and competition.

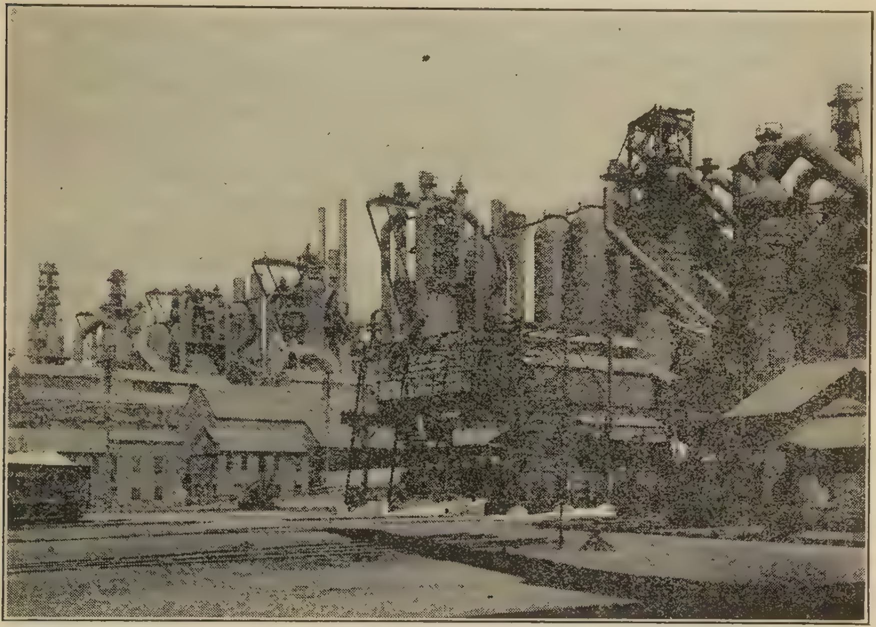

Fig. 1.—"In Some Places Conditions Are Favorable for Blast Furnaces"
(General View of Blast Furnaces, Carnegie-Illinois Steel Company)

Processing Industries

Portland cement, lime, sugar, flour, and coffee roasting offer examples of processes in which transportation again is a controlling influence. Milling-in-transit rates have served to prevent concentration of such processing industries around the comparatively few inland sources of raw materials or at seaports. The effect of these rates is to make it possible to manufacture the product somewhere between the source of material and the market—the exact location depending on the loss in bulk during the various processes and the best centers of distribution.

THE AIRPLANE INDUSTRY

The airplane industry grew up on the Pacific Coast very much as the automobile industry did in Detroit—that is, through the ability and enterprise of local individuals. The war accelerated the manufacture of airplanes at inland points for strategic reasons, but this industry was operating successfully in Wichita, Kans., St. Louis, Mo., and Buffalo, N. Y., as well as on the Atlantic and Pacific coasts, before 1940. Engines, instruments, and some other parts are made in separate plants, less widely distributed, because they require more skilled mechanics. Material required for airplanes is comparatively lightweight and the cost of transportation is not so important with airplanes as, for instance, with the heavy machinery. Furthermore, planes are delivered on their own power independently of existing surface transportation. Both of these factors tend to make the location of airplane manufacturing plants more or less independent of the sources of material and of surface transportation. Nevertheless, progressive management is probably the main reason why airplane manufacturers have broken away from traditional limitations.

The airplane industry developed its own force of skilled labor more effectively than did other industries which followed one another into districts where a supply of skilled labor was known to exist. Most of the workers in airplane factories are young men and women who have become sufficiently skilled without going through a long apprenticeship. Even airplane engine factories, where a nucleus of highly skilled mechanics is required, were operated in wartime at places where few comparable precision operations had been performed before. The industry deserves the distinction of leading the way to decentralization of manufacturing by a combination of boldness and ability which is characteristic of the younger element in management.

MISCELLANEOUS SMALL INDUSTRIES

Miscellaneous machinery manufacturing may follow the example of the airplane industry to a certain extent, and may show a preference for locations that serve the market although retaining other essential advantages, such as a supply of skilled mechanics. Again, improved transportation is a factor and skilled labor will find it easy to follow opportunities for employment and better living conditions. This discussion should include also industries which are composed of small units, comprising a large number of individual plants with a large combined volume of products and a large number of employees. In a plan for the industrial development of a district, these smaller, diversified industries become more important than the industries characterized by large and impressive individual plants. Manufacturers of garments and textile specialties—shoes, radio, electrical appliances, styled products, and novelties—

are dependent primarily on labor, markets, and distribution. The cost of transporting raw materials and the cost of power (which are vital to the success of some industries) are not among the most important factors in these industries. Studies to determine the location of some of the comparatively small plants are more difficult than corresponding studies in which a few large items clearly determine the most important considerations. In these smaller plants, involving smaller expenditures for each of a larger number of items, every slight advantage has to be considered, because success or failure may depend on any one of a number of slight advantages over competitors.

The Textile Industry

The textile industry came to New England from England because management and mechanical skill were available and the climate was supposed to be suitable. However, the effect of climate has been eliminated by humidifying and air conditioning systems, which are standard equipment for textile mills. In the Southeast the textile industry began to take advantage of lower operating costs, and the Southeast became competitive with New England as New England had become competitive with Europe. The cost of labor is important in the textile industry which is one of the industries that can train skilled operators within a comparatively short time. Therefore, locations where labor is plentiful, with a consequently low cost, are sought. The Southeast offered a supply of good textile labor at comparatively low wages, with other conditions favorable to the industry; and so cotton mills migrated from New England to the South, causing southern mills to constitute the greater part of the cotton textile industry. When these mills were built, wages were extremely low in the South and the regulation of working hours was comparatively lax or liberal. The costs of power, construction, food, and shelter also were low. Since the adoption of uniform national laws governing wages and working hours, southern mills no longer enjoy a competitive position based primarily on low wages and long hours. The better mills do not depend on these differentials, because good management and productive labor can overcome them. Uniform wage-and-hour laws have put the textile industry on a different basis with respect to location. In fact, markets and distribution facilities should be more important factors in the location of textile mills.

The textile industry is comparable to the steel industry with respect to the relationship between manufacturing yarns and fabrics in the basic plants and to the distribution of these materials throughout a wide variety of manufacturing operations, whose locations are controlled by influences different from those which affect the basic plants. For example, cotton mills convert raw cotton into yarn. Some mills make a variety of yarn to be shipped to plants that weave specialities in comparatively small quantities. The larger mills, making large quantities of standardized products, combine weaving with spinning in a completely balanced plant. After the fabrics have been woven, they are finished in bleaching, dyeing, and printing plants which are generally separate from the mill. The products of these finishing plants are distributed to the garment makers and miscellaneous fabricators and consumers. The manufacture of rayon yarn is a chemical operation with locational requirements different from the requirements of weaving plants. Rayon yarn is made in comparatively large plants and rayon weaving is widely distributed. Both rayon and wool manufacturing fall into the same general divisions as cotton—namely, yarn manufacturing, weaving, finishing, and the conversion of finished fabrics into marketable products.

Textile yarn manufacturing requires power and labor at competitive rates, together with large manufacturing sites in localities where conditions contribute to reasonable living expenses. Weaving plants have the same requirements for power and labor at competitive rates, especially in the larger units, but their requirements are more flexible. Textile finishing plants require large supplies of soft water and good transportation facilities from the mills and to the market—thus they are found in, or accessible to, distribution centers. The final products are made in plants of various sizes generally situated where they serve the market to best advantage. This accounts for the concentration of needle trades in the large cities, especially New York, which is the largest market for textile products.

Shoes, Radio, and Appliances

The shoe industry, which began in New England and later expanded westward, enjoys more freedom than the textile industry in the choice of plant location, because it consists in the main of smaller units. It is another major manufacturing industry in which plant location will be influenced, by markets and distribution facilities, in the future more than it has been in the past.

Manufactures of radios, household appliances, and numerous other attachments for the convenience and amusement of the individual provide a variety of employment for labor released by war industries and by the Army and Navy, irrespective of manufacturing locations. This condition should have a stimulating effect on the creation of new enterprises and on employment in areas not formerly considered active in the industrial field.

Statistical References

The "Census of Manufactures"[2] includes statistics from which industries can be classified according to their requirements for labor, fuel, and power. More than four hundred classes of industry are listed including the sum spent for wages, fuel, and power; the number of male and female employees; and the value added by manufacture in each class. Another convenient reference is "Industrial Location and National Resources."[3]

Statistics from the census can be used to compute the number of wage earners, the wages paid, and the cost of fuel and power per unit of value added by manufacture. A study of these results will show which industries employ the largest number of workers, pay the highest wages, or spend the most for fuel and power on the basis of value added by manufacture. It may be assumed that these industries will seek locations where labor, fuel, and power are available on the most economical basis. Intelligent inspection of the industries so segregated should eliminate cases for which statistical results are not confirmed by practical requirements. Further study of these computations will show certain coincidences. Two or three of these elements—for example, wages paid, number of employees, and cost of fuel and power—may combine to indicate preferences for locations having corresponding advantages.

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[2] "Census of Manufactures," U. S. Dept. of Commerce, Washington, D. C.

[3] "Industrial Location and National Resources," National Resources Planning Board, U. S. Govt. Printing Office, Washington, D. C., 1943.

When the number of wage earners alone is relatively high and the wages paid do not exceed the average, it may be assumed that the industry is not especially dependent on highly paid or skilled labor. Where the reverse is true and the wages paid are comparatively high although the number of employees is comparatively low, the industry probably requires skilled or high-priced labor.

The cost of fuel and the cost of electric power should be combined for general classifications because the census does not go into sufficient detail to discriminate between industries that generate all or part of their power and others in which the processes or local conditions make purchased power more economical. Generally, power is cheap where fuel is cheap, and so the combined expenditure for power and fuel is an index to the advantages of locations where these two items are available at a low cost.

The census also shows the number of male and female employees in various classes of industry. Selecting those industries in which more than 50% of the employees are female and then comparing the list with others showing comparatively large numbers of employees, or comparatively large sums paid for wages, will lead to interesting conclusions on the characteristics of employment in existing industries. These conclusions, considered in connection with the local population and labor supply, should serve as a basis for determining which industries are most desirable in a community.

Application of Fundamental Requirements

Familiar examples can be cited to confirm conclusions from these census studies. Products of mass production and products in which the value of material is relatively high do not appear in the list showing large numbers of employees and high costs of fuel and power per unit of value added by manufacture. Automobiles, airplanes, precision machinery, and jewelry are industries comparatively free from other restrictions on location but dependent on a supply of skilled labor. Woodworking, canning, food products, soap, and cheap novelties are industries requiring a comparatively high number of employees. Boatbuilding, railway car building, styled clothing, foundries, leather, and musical instruments are industries with comparatively high wages. Some of the industries in which both the number of employees and the wages paid are comparatively high are buttons, textiles, clothing, drugs, and medicines.

A comparatively high cost of fuel and power is indicated for the manufacture of electrochemical and electrometallurgical products, ceramic products, steel products, refined sugar, chemicals, vegetable oils, textile finishing, paper, and ice. The last item is included to illustrate an exception. The location of an ice plant is determined more by accessibility to the market, on account of expensive handling and loss during shipment, than by the availability of cheap power.

Ceramic and leather products are the most important industries showing a combination of heavy pay rolls and high cost of fuel and power per unit of value added by manufacture. The combination of a comparatively large number of wage earners with a heavy pay roll and high cost of fuel and power is characteristic of the textile industry. The foregoing examples can be multiplied by increasing the scope of the study, but these few will suffice to show the wide variety of industrial activities that share the same fundamental requirements with respect to power, fuel, and labor.

Transportation is a primary requirement for nearly every industry, whether involving raw materials, finished products, communications, or means for taking employees back and forth to work. Transportation costs include, besides the usual freight and passenger charges, the costs of time consumed and of damage to perishable or fragile products, such as ice. The time element, during the nonproductive transportation period, affects the amount of working capital tied up by work in process and by materials and products in transit. Transportation facilities, however, are not limited to railroads, automobiles, airplanes, or other locomotive appliances. They include parking space, landing fields, shelter for passengers, and shipping and receiving facilities. Limited accommodations for parking and poor transportation equipment between passenger terminals and factories are marks of obsolescence that apply to many of the older plants.

Influence of Climate

The effect of climate on the location of industry is often the converse of what might be expected. The most highly developed industrial districts are in the more severe climates, when it might be expected that temperate climates would have attracted population and provided economical conditions under which industry would prosper more than in places subject to severe weather. The conclusion is that life in severe climates makes people more adaptable to industrial work than they would be if they lived in mild or warm climates, which are particularly favorable to agriculture. There are many exceptions to this rule and the influence of fuel and mineral deposits is recognized. Still, the volume and variety of manufacturing in Scandinavia, Northern Europe, New England, and the Great Lakes district, not approached in other more temperate or warmer climates, proves that climatic conditions are not controlling factors in the location of manufacturing plants.

Opportunities for Planning

There is an important relation between city planning and factors controlling the location of various types of industry. By city planning, desirable local conditions can be created where they might not exist otherwise.

The activities leading to the adoption of a city plan, as well as the execution of the plan itself, should have a wholesome effect on industrial prospects. Major considerations, such as power, freight rates, availability of materials and accessibility of markets, wage scales, supply of labor, and even housing and living conditions, may balance about equally when comparing several desirable places. The advantages of one over the other will then depend upon plant sites, local laws and taxes, local utilities, amusements, and local transportation —to mention only a few of many possible items that are susceptible of intelligent treatment in a comprehensive plan.

Industrial districts can be laid out with railroad sidings and highways arranged so that factory sites of various sizes can be accommodated and so that the individual plants are relieved from building long spur tracks and paved roads, water, gas, and sewerage lines, and electric power lines to connect with the nearest existing system.

It is a mistake to select industrial locations where the cost of land might restrict the size of the site. Room is needed for parking space, future expansion, protection against hazardous storage, and for other reasons which may develop long after the site has been chosen. For example, helicopter landings, as yet unusual, promise to become typical features of future industrial sites. The principal reasons for the use of sites that are too small are high unit cost of land and prohibitive cost of providing transportation and utilities at other sites. City planning can eliminate these major difficulties by the proper location and development of industrial districts, as well as by proper zoning of land conveniently situated for industrial and commercial use.

The disposal of waste from industrial processes has been neglected in the past but such neglect is no longer permissible. There are few sites, however isolated, where it is legitimate to allow an industry to dump offensive waste. It is much better to face the problem wherever new plants are built than to evade it in the hope that somebody else will do something about it later. The pollution of streams and bathing beaches and the contamination of water supplies have created serious problems wherever provision has not been made for the treatment or disposal of objectionable waste. At the same time, regulations which are too stringent cause the industries concerned to favor locations where the cost of compliance will be more reasonable. Therefore, it is no service to a community to formulate restrictions or to enforce penalties which discourage desirable development. Neither is it any service to an industry to allow it to become a nuisance and then to prosecute it for an offense which could have been avoided.

Fair taxation should be inherent in any plan for the most desirable as well as the most profitable use of land. Industries may seek locations outside city limits primarily to avoid city taxes but, at the same time, they get more land without paying too much for the plant site. It should pay the city to let industry have plenty of room. The city's income is not necessarily reduced, in such cases, by an amount corresponding with what the industries could have been taxed if within the city limits. The city gets its income indirectly, because wages paid by the industry are spent in the city and support the expansion of retail business and services, the construction of homes, and the consumption of goods sold within the city. Sound economics would prescribe locations outside the city limits for industries needing large sites or room for expansion, so the city eventually would get the benefit of increased expenditures for wages and materials.

There is a large and important class of industries which seek locations within the city limits. Such industries show a preference for the congested districts accessible to concentrated labor reservoirs, markets, and services, and so they provide tenants for loft buildings, warehouses, and buildings specially constructed to accommodate small manufacturing tenants. Some of these industries continue to be comparatively small, although others expand later and seek larger sites and more commodious facilities in outlying areas.

The so-called incubator building, or manufacturing loft building, is a favorite subject for discussion in this connection. Experience has proved that such buildings are important assets to communities when they are built correctly and in the right location. Failures may be caused by spending too much on these buildings as well as by poor building design and construction and by improper location. It should be assumed that the tenant wants only the essentials for manufacturing convenience and comfort of employees. These loft buildings should be designed with liberal floor loads, with flexible systems of power, water, steam, and gas connections, and with provision for ample entrances and exits and for compliance with insurance regulations. The wiring, piping, and ventilating systems should have a capacity sufficient for air conditioning equipment, which can be installed later as needed. Railroad sidings and shipping platforms for cars and trucks, protected from the weather and served by conveniently placed elevators, are other important requirements. City planning, making it possible to find desirable sites for these buildings and access to them without blocking traffic in adjacent streets, will create inducements for important new industries. More than that: It will improve the appearance and the convenience of the city and increase the value of central property.

SUMMARY

The location of industrial plants is subject to new influences resulting from World War II.

Certain fundamental requirements, including the supply of materials, labor, power, fuel, and transportation, retain their relative importance.

A trend toward decentralization is apparent in the effort to relieve congestion, to serve new and expanding markets, and to utilize new reservoirs of labor and improved transportation facilities.

Local and regional planning should provide for improved and economical sites for both large and small industries.

DISCUSSION

Harold M. Lewis,[4] M. ASCE.—The part of the civil engineer in the location of industry has been, primarily: Supplying some of the accessories to a successful site, such as transportation facilities; laying out site utilities, such as water supply, drainage, streets, and terminals; and designing foundations and other structural elements of the plant itself. The fact that this paper was presented at a joint meeting of the City Planning Division of the American Society of Civil Engineers and the American Institute of Planners indicates that civil engineers are taking an active interest in some of the broader phases of the problem. Instead of simply having an industrial site handed to them, they should play an increasing rôle in helping their clients determine where that site should be.

Mr. Wood has indicated that the location of industry has followed certain laws based on accessibility to raw materials, convenience for manufacturing (power and labor), and accessibility to markets. These essentially physical factors are being supplemented by what might be called social and psychological factors. Sites which might not have been considered under the old standards may become desirable because the workers (labor) find the locality a pleasant place to live in and one which affords higher standards of housing, health, recreation, and education. The owners and executives (business) may find that better municipal government and sound city planning provide savings which more than counterbalance some disadvantages in accessibility of raw materials, power, and markets. In other words, a community that is well planned and administered as a result of good city and regional planning may find certain industries coming to it primarily because of those characteristics.

There will always be certain industries that are naturally better adapted to central urban areas, others that tend to gravitate to suburban areas adjoining large cities, and still others that require enough space and employ a sufficient variety of workers to enable them to go out into new areas and establish communities of their own. The economic and industrial survey undertaken by the Committee on a Regional Plan of New York and Its Environs,[5] classifying the industries in the region along these lines, was a pioneer study of its kind.

In the first, or urban, group were those small-scale industries with a seasonal labor force and therefore a high turnover of employees, or in which the time factor in contacting the consumer or purchaser was important. The group included certain types of men's and women's clothing, high-grade jewelry, photo-engraving, job printing, cosmetics, and perishable foods. In the second, or surburban, group were many of the heavy metals and chemical industries, refineries, printing of periodicals, bookbinding, textile finishing, lumber mills, and women's underwear. In the third group the writer would place large-scale munitions manufacture, mining, and the assembly and testing of airplanes. Among the cities that have been built up, or greatly expanded, as a result of the establishment of new industries on new sites are: The mining cities of the west such as Leadville, Colo.; the steel cities such as Gary, Ind.; the automobile cities such as River Rouge, Mich.; the rubber cities such as Akron, Ohio; and the iron and steel cities such as Birmingham, Ala.

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[4] Cons. Engr., New York, N. Y.

[5] "Major Economic Factors in Metropolitan Growth and Arrangement," Regional Survey of New York and Its Environs, Vol. I, 1927, pp. 19–30 and 104–107.

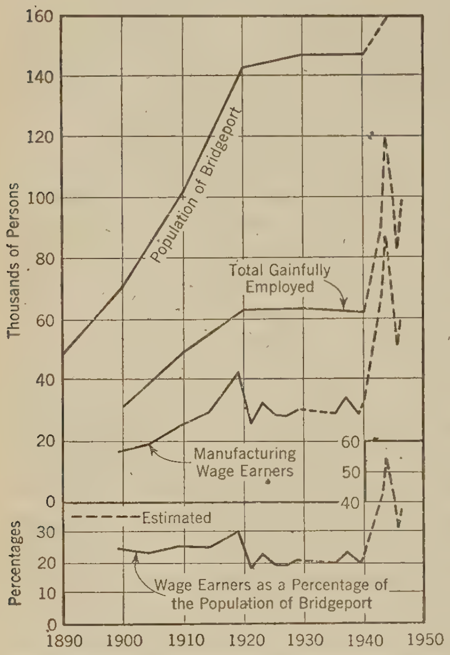

Fig. 2—Comparative Trends in Population and Wage Earners in Bridgeport, Conn., 1899–1946

Mr. Wood referred to the present postwar readjustment period. The writer recently had occasion to study industrial trends in Bridgeport, Conn.—a fairly typical manufacturing center in the 100,000 to 500,000 population range. Like many other cities, it is going through such a period; but, again like many other cities, it had a somewhat similar experience following World War I. The curves in Fig. 2 indicate the similarity and the differences in these two experiences. The manufacturing boom of World War I showed a peak in 1919 in the number of manufacturing wage earners. Then came a sudden drop, with relatively minor changes up to 1939. During the succeeding four years, all four curves showed second wartime jumps, the outstanding feature being the increase in the total number gainfully employed from 62,266 in 1940 to a peak of 119,500 in the fall of 1943, paralleled by similar increases in the number of manufacturing wage earners. The inevitable downward trend came in 1944, but the City of Bridgeport had anticipated this trend. Through its city administration, the Chamber of Commerce, and a representative planning council organized by the Chamber of Commerce in 1943, the problems of both the city and region of which Bridgeport forms a center had been attacked by eleven committees. It was found that, although the city area affords few sites for new large-scale industries, there are ample sites for both industry and port development within the region. These were mapped; their relative advantages and disadvantages were analyzed; and harbor improvements to make them more accessible were proposed.

It is expected that, whereas postwar trends will result in more production per employee, new manufacturing buildings will provide greater floor area per worker, more light and air within the buildings, and more open ground about them. The open space should be used to provide both automobile parking and recreation facilities for employees.

In 1946 a recovery from the low point of 1945 had already occurred and the Bridgeport Chamber of Commerce stated:

> "With the experience of World War I to look back on the industrial leaders and business men anticipated a mass exodus of the huge tide of in-migrant workers that flooded this area to reap the harvest of high war-time wages. This exodus did not take place, only a trickle left. * * * With the easing of parts, supplies and raw materials, Bridgeport is looking forward to its greatest peace-time production records."

In this case, advance planning seems likely to stabilize employment at a level considerably above that of the prewar period.

R. F. Goudey,[6] M. ASCE.—That industries should be located on the basis of sound engineering evaluations of source of raw materials, availability of labor, location of markets, transportation facilities, and availability of power has been emphasized by Mr. Wood. Rarely are such studies made and it is more common to have industries located by representatives of a local chamber of commerce and by public officials not familiar with the critical problems involved.

In too many instances, industries locate in new territory through decisions based largely on misinformation and even ignorance. Usually when an eastern industry is about to move westward, someone in its organization who is ready for a vacation takes a trip west to make a casual survey; or a written inquiry is forwarded to a local chamber of commerce or a business division of a power utility. In one large metropolitan area, the representative of a prospective industry, if routed through chamber of commerce channels, might have to meet fourteen public officials of the municipality before obtaining correct information on such items as taxes, cost of water, facilities for sewage and waste disposal, power cost, housing facilities, and availability of labor.

On the other hand, if the inquirer entered the area through contact with a large private power utility, he might be induced to locate in the county area instead of the city, in which case he would have to interview twelve or more county officials to obtain information on the points involved in the selection of a new site. In any event, he is kept ignorant of the advantages which other near-by areas might have over the one he first investigated. In many cases the preliminary stages have been passed and a final decision has been made to locate the plant at a given place when, for the first time, engineers for the industry learn that the quality of water is not satisfactory for their purposes, or that they are unable to dispose of liquid waste without polluting the underground water supply, or that the cost of water as indicated by the metered rate schedule did not disclose that there was an additional district tax of 50¢ per $1,000 valuation, or that adequate protection from floods could not be obtained except at exorbitant cost. Such information was not obtained in the preliminary stages because there was no proper coordination of engineering information from the different departments involved in the problem.

Cheap water is essential for cooling water, heat exchange purposes, processing, and fire protection for rayon plants, rubber products plants, woolen mills, paper pulp plants, and tanneries. In cities where domestic water must be imported from long distances, it may be desirable to reclaim sewage for a cheap water supply of ample volume to serve plants having large water requirements. Interesting instances where sewage has been conserved for commercial purposes are as follows:

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[6] San. Engr., Dept. of Water and Power, Los Angeles, Calif.

1. El Tovar, Grand Canyon, Arizona, has reclaimed sewage for boiler, toilet flushing, and engine use since 1925;
2. The Bethlehem Steel Company at Baltimore, Md., uses treated city sewage for cooling water;
3. The Barnsdall Oil Company at Corpus Christi, Tex., uses reclaimed sewage for cooling purposes;
4. The Shell Oil Company at Signal Hill, Calif., at one time used all the treated Signal Hill sewage for industrial purposes;
5. The Kaiser Steel Mill at Fontana, Calif., re-uses all its wastes so that there is little actual waste;
6. In Pasadena, Calif., treated sewage is discharged into the Rio Hondo and is the only supply for certain irrigation ditches in the Riviera District; and
7. In Fresno, Calif., sewage is discharged into aquifers from which irrigation water is developed.

These cases show clearly that, where the original cost of water is high or where it must be transported long distances or be developed at a high cost, reclamation is not only practical but economical.

Many industries attempt to relocate because of trouble experienced with the matter of waste disposal at their old location. Industries have a habit of trying to make little of their waste disposal problems and so make it difficult for consulting engineers to advise them as to a proper location. Although many industries have spent considerable sums in attending to their liquid waste problems, the amount spent by communities has been much greater. There has been considerable duplication of expense. Therefore, these matters should be given consideration during the period when the original location of industries is being considered. In California, the following experience with industrial waste has been particularly troublesome:

(*a*) In twenty cities there has been trouble with peach waste where lye wastes, spread on the land, resulted in mineral pollution of ground waters;
(*b*) Fourteen cities have had trouble with tomato waste in which lye waste has been discharged on to the land;
(*c*) Fourteen cities have had trouble with salt and lye waste from olive packing plants and these wastes have been placed in underground water supplies;
(*d*) Twelve cities have had trouble with creamery waste, particularly where whey was involved;
(*e*) Six cities have had trouble with packing pimientos and strong alkali waste with garbage has been placed on to the land;
(*f*) Five cities have had trouble with slaughterhouse wastes and have had to install grease removal facilities;
(*g*) Five cities have had difficulty with refineries and separate disposal was necessitated;

(*h*) Five cities have had trouble with winery waste which is now disposed of by land;

(*i*) Four cities have had problems with sugar plant waste, two having put in separate systems and two having ceased operation; and

(*j*) Three cities have had trouble with fish waste and in all instances special works have had to be installed.

The following are typical of the many special problems involved in practically every type of industry:

Anaheim, Calif.—The United States Industrial Alcohol Company discharged molasses waste in the Orange County Outfall Sewer, causing such a terrific production of hydrogen sulfide that the company was forced to disconnect from the sewer and install local treatment works. This situation resulted, however, in pollution of underground water supplies with mineral constituents.

Betteravia, Calif.—Betteravia has its own waste disposal system independent of any municipal system.

Corona, Calif.—The By-Products Exchange Plant at Corona discharged wastes into the Corona Sewer System, resulting in the pollution of several private wells. The company installed a separate system in cooperation with the City of Corona for adequate treatment and proper disposal. There is still some pollution of underground supplies, but no wells so far have been affected.

Guadalupe, Calif.—The creameries in Guadalupe installed a sewer system at their own expense to dispose of the industrial waste without affecting any underground water supplies.

Lemoore, Calif.—The city constructed a biofiltration plant consisting of aeration and with recirculation and a rock filter based on a year's experiment with creamery wastes; but, when the plant was put in operation, it proved to be a complete failure because of the acid lactose splitting organisms which thrived in the aeration tank and its recirculated sludge. Separate disposal had to be provided.

Los Angeles, Calif.—Los Angeles has had trouble with the disposal of alkali, salt brine, chromium, phenol, and solvents which have reached certain strata of local underground water supplies.

Ontario, Calif.—The Citrus By-Products Plant at Ontario installed a separate sewer to seepage beds on land, resulting in the pollution of near-by wells. Suit was brought against the company which was settled outside court; but the city extended City of Ontario water to these water consumers. In the meantime, however, there is continued pollution of the underground water supply.

Santa Ana, Calif.—Two sugar plants of the Hollywood Sugar Company and of the Santa Ana Sugar Company used to have a sewer system carrying the combined wastes to the ocean. These plants discontinued operation for several reasons—one of which was that their sewer facilities were no longer available.

Tulare, Calif.—Tulare built an activated sludge plant with the idea of treating domestic sewage. The chamber of commerce invited a creamery which produced whey waste to Tulare. The volume of waste amounted to 245,000 gal per day and gave an equivalent load of 80,000 people. This demand was about ten times what the treatment plant could meet and the plant became a complete failure. Separate disposal had to be provided.

Summary.—It is evident that the question of waste disposal should be given far more attention than has been the case in the past. It would greatly assist managers who are considering the problem of relocating industries to deal with a coordinating committee representing all sources of information that management should have in arriving at a final decision as to where industries should finally be located. In some metropolitan areas this committee might require representatives from public and private utilities as well as the heads of city and county departments supplying services to industries. It is strange that large industries appear to avoid the services of consulting engineers in deciding on new locations. The problem is sufficiently involved to warrant such consultation.

Charles P. Wood,[7] Esq.—Discussion, by amplifying the original paper, has emphasized some features that merit more attention. However, the discussion may be clarified by further definition of the underlying reasons for the location of industries: First, there are entirely new projects in which management is free to choose the best location; second, there are branches of existing industries which serve to increase capacity and to improve distribution facilities without affecting the operations of plants previously built; and, third, there are plants established for the purpose of decentralizing an industry.

Plants in the third class relieve the pressure on the parent plant and also provide the advantages mentioned in the case of branch factories. Distinction should be made between the migration and expansion of industries to take advantage of new opportunities and the decentralization of industries to avoid impairing or exhausting the resources of certain localities.

The New York economic report,[5] mentioned by Mr. Lewis, classifies industries dependent on a metropolitan location and discriminates between them and the industries needing more room than they can find in a great city.

It should be noted that New York (N. Y.) and other large cities are, in effect, industrial incubators for the less thickly populated sections of the United States. Many of the industries that started as small enterprises in the lofty buildings of great cities, which are the only places where they can find suitable rental space and skilled labor, later will expand into their own factories built either in the suburbs or at remote locations.

Reference to employment and industrial activity in Bridgeport, Conn., throughout the period which includes both World War I and World War II, is a good illustration of how planning in advance served to stabilize employment.

Bridgeport is a highly developed manufacturing community with diversified industries and a large, well-housed colony of skilled labor. It is also a convenient distribution point for the great northeastern markets and especially for the New York metropolitan area. For these reasons, it is not comparable with southern or western cities of the same size or larger, which have comparatively few established industries and comparatively small colonies of skilled labor. Despite its many advantages, Bridgeport has found it profitable to have its resources studied with a view to revealing opportunities that otherwise might have been neglected. Absence of large sites was not allowed to obscure prospects for the development of smaller sites for industries particularly adaptable to conditions at Bridgeport.

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[7] Industrial Engr., Lockwood Greene Engrs., Inc., New York, N. Y.

Mr. Goudey's timely comment on the treatment of waste and the reclamation of water directs attention to the increasing importance of the water supply. It is not surprising that this comment should come from the West, where the conservation of water resources is of more consequence than it is in the East. However, the abundance of water east of the Mississippi River is no reason for neglecting the treatment of polluted waste or the conservation of pure water. Polluted streams have become characteristic of industrialized regions, because of lax enforcement or the absence of regulations in the early days of industrial development. Pollution of ground water resulting from spreading industrial waste on the land, also mentioned by Mr. Goudey, shows the necessity for treating dry waste as well as effluents. There are other familiar cases where polluted effluent is collected in pits, whence it gradually seeps into the ground, and where the resultant pollution may not be apparent until there is a need for pure well water in the vicinity.

Reclamation of sewage and other polluted waste water for boiler feed, cooling, and similar purposes will become more important as progress is made with the industrial development of arid sections and as the water supply is depleted elsewhere.

The suggestion that the location or relocation of industries could be facilitated by consulting local coordinating committees fails to take into account that the resulting publicity may nullify the committees' efforts. Competition among industries and localities interested in almost every industrial location prospect makes it necessary to keep the project confidential until a decision has been reached. In such cases, consulting engineers function because they can assemble information impartially without disclosing the identity of their client.

Similarly, the increasing importance of markets as a factor controlling location of industries requires the use of marketing data in appraising the characteristics of an industrial location. In this respect, however, engineers can get assistance from sales and distribution management.

A summary of new plant locations chosen since 1940[8] by six large companies manufacturing different products includes the following observations:

The total number of plants built or locations selected is eighty three, divided thus: Aluminum Company of America, 2; E. I. du Pont de Nemours and Company, 11; General Electric Company, 35; General Motors, 19; Philco, 7; and U. S. Rubber Company, 9. Practically all the new locations are in states already heavily industrialized. Only twenty three are in cities larger than 100,000; the preference for smaller cities and towns being in the ratio of 3 to 1. The division by states, according to the number of plants added since 1940, is as follows:

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[8] "Industry Fans Out," *Business Week*, November 23, 1946, p. 31.

New York	13	North Carolina	2
Ohio	12	Tennessee	2
Pennsylvania	8	Virginia	2
Indiana	7	Delaware	1
Massachusetts	7	Georgia	1
Illinois	5	Iowa	1
California	4	Kansas	1
Michigan	4	New Hampshire	1
Kentucky	3	Rhode Island	1
New Jersey	3	Washington	1
Texas	3	West Virginia	1

There is evidence of a tendency toward smaller plants. For example, the General Electric Company's thirty-five new plants are described as engaging from 30 to 1,500 employees each.

The controlling factors mentioned in the selection of these locations are labor, markets, transportation, raw materials, power, and fuel. Reasons given for changes from former locations include a continuing policy of decentralization, decentralization for better control of operations, social and economic benefits to employees, and lower costs.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2307

THE PLANNING OF AERIAL PHOTOGRAPHIC PROJECTS

By F. J. Sette,[1] M. ASCE

With Discussion by Messrs. T. W. Norcross, James M. Cultice, Marshall S. Wright, Robert H. Randall, and F. J. Sette.

Synopsis

Although there is a progressively developing literature in the field of photogrammetry, until the studies herein described were initiated, there was practically no information in regard to planning and estimating the cost of aerial photographic projects. This paper discusses the establishment of a photographic day, zones of similar weather characteristics, and a method of scheduling projects. The reliability of the representative photographic day is discussed. A method of estimating the cost of aerial photography is also presented.

Introduction

The extensive use of aerial vertical photography by the U. S. Department of Agriculture for crop control, soil conservation, flood control, forestry studies, and other important purposes led to an examination of that program with a view to reducing its cost if at all possible. More than 2,900,000 sq miles of the United States have been photographed—approximately 700,000 sq miles of which has been photographed twice because of cultural and other changes on the ground. For certain purposes it may be necessary to rephotograph an area more than once. A decision to rephotograph would be based upon the extent of cultural changes since the date of the original photography, the requirement of greater pictorial detail, or the need for more precise, or larger scale photographs. Aerial photography offers the engineer a useful and economical tool in the planning and execution of engineering projects; in fact, it is a necessary tool which will be increasingly useful in the future.

Unit costs developed in this paper are based upon 1938 prices; upon a photographic scale of 1 to 20,000 (1,667 ft per in.); and the use of a camera with a focal length of $8\frac{1}{4}$ inches. For this scale and camera, the flying altitude is 13,750 ft above the ground.

Note.—Published in March, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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SCHEDULING OPERATIONS

Photographic Day.—The number of days in a month that an aerial photographer could reasonably expect to utilize for photographic purposes had to be determined. The Soil Conservation Service (SCS) of the Department of Agriculture had obtained from the United States Weather Bureau a 10-yr record of the number of days for some fifty stations throughout the United States when the sky was cloudless or less than 10% overcast—or, assuming the perfectly clear sky to be unity or 1, when clouds covered 0.1 of its area or less as an average of three observations taken at 7 a.m., noon, and 7 p.m. Flying reports from the field seemed to indicate that the number of days having a sky covered with 0.1 or less of clouds could be used as a very close index of the number of photographic days that might be expected. Accordingly, the number of stations was extended to include all stations for which records were available. The years of record varied from 26 to 37, the largest number of stations having a 37-yr record.

TABLE 1.—METHOD OF GROUPING[a] STATIONS OF SIMILAR WEATHER CHARACTERISTICS

Weather station	Days	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
(*a*) REGION 2 (SEE MAP IN TABLE 2)													
New York, N. Y......	4.5	91	102**	109**	96	102**	71	71	87	120**	158†	100**	91
Atlantic City, N. J....	6.3	97	98	97	89	103**	71	84	89	113**	157†	116**	94
Baltimore, Md........	5.4	93	104**	100**	94	94	70	81	78	118**	168†	111**	100**
Washington, D. C.....	5.3	100**	100**	100**	93	93	72	72	83	117**	164†	111**	100**
Norfolk, Va..........	6.2	95	106**	100**	105**	106**	74	65	68	92	157†	139**	102**
Cape Henry, Va......	6.4	91	86	91	105**	106**	77	81	89	106**	152†	123**	87
Hatteras, N. C.......	7.0	86	87	104**	107**	110**	86	86	79	91	137**	126**	93
Philadelphia, Pa......	5.4	104**	107**	100**	96	104**	67	72	74	117**	156†	107**	89
Harrisburg, Pa........	4.6	89	89	109**	98	100**	63	81	91	143**	163†	91	76
Elkins, W. Va........	2.6	81	65	115**	119**	142**	64	58	64	108**	154†	127**	77
Richmond, Va........	6.5	97	100**	95	106**	100**	66	65	69	108**	165†	118**	100**
Lynchburg, Va........	4.0	88	90	102**	107**	95	60	90	83	112**	163†	105**	95
(*b*) REGION 3 (SEE MAP IN TABLE 2)													
Raleigh, N. C.........	6.5	105**	102**	102**	109**	91	52*	46*	55*	98	182†	155†	102**
Chattanooga, Tenn....	5.3	94	104**	110**	85	91	58	45*	55*	96	196†	160†	106**
Charlotte, N. C.......	6.5	103**	103**	111**	97	94	43*	37*	46*	98	186†	160†	120**
Columbia, S. C.......	6.0	95	108**	105**	107**	92	57	38*	52*	93	176†	167†	118**
Atlanta, Ga..........	6.0	105**	110**	120**	97	88	60	32*	40*	88	175†	163†	117**
Birmingham, Ala......	6.4	103**	108**	117**	100**	102**	64	44*	47*	81	177†	153†	97
Montgomery, Ala.....	6.7	100**	98	112**	104**	91	66	42*	51*	102**	184†	157†	102**
Mobile, Ala..........	5.7	103**	98	102**	105**	104**	65	30*	42*	102**	186†	156†	103**
Meridian, Miss.......	6.4	95	91	108**	95	87	86	48*	62	103**	186†	145**	97
Vicksburg, Miss.......	6.4	100**	91	105**	100**	89	84	50*	56	102**	181†	148**	95
Wilmington, N. C.....	7.0	109**	109**	116**	109**	101**	63	43*	51*	71	153†	160†	124**
Charleston, S. C......	6.3	111**	118**	124**	119**	102**	56	35*	43*	70	151†	167†	116**
Savannah, Ga........	6.6	106**	112**	129**	126**	106**	64	36*	41*	68	139**	160†	114**
Augusta, Ga..........	6.7	109**	109**	107**	109**	103**	61	37*	54*	91	167†	154†	110**
Macon, Ga...........	7.2	103**	108**	117**	100**	102**	64	44*	47*	81	177†	153†	111**
Pensacola, Fla........	6.3	100**	102**	110**	110**	108**	67	29*	40*	92	173†	159†	111**
New Orleans, La......	5.2	112**	102**	112**	117**	100**	63	19*	23*	92	190†	170†	105**

[a] First column shows the average number of days per month that the sky is 0.1 or less overcast. The numerals in the other columns show the percentage of the average which may be expected for the stated month. Groups suggesting similar weather patterns may be identified as follows—*, 55% or less; **, 100 to 149%; and †, 150% to 199%—of an average month.

When this decision was made, it was also appreciated that an aerial photographer could utilize other days when he often found himself unable to photograph on so-called "clear days" because of adverse local conditions such as haze, fires, heavy winds, or mechanical and other difficulties. Therefore, the designation of a photographic day as that day in which clouds covered 0.1 or less of the total sky served largely as an index.

Regions of Similar Weather Characteristics.—For each weather station, the average number of days per month for the year ($\frac{1}{12}$ average number of days in a year) was divided into the actual average for each month which, multiplied by 100, gave the percentage that each month was of the average for the year. In other words, the values in Table 1 are percentages of the average days in the first column of the table. For example, Table 1 shows that, in an average year, New York, N. Y., has 4.5 days per month when the sky is 0.1 or less overcast—clear days. The average March weather is clearer than that, being 109% of 4.5 days or an average of 4.9 days. Stations with similar characteristics are indicated by appropriate superscripts. The number of stations (144) covering the entire United States made it difficult to ascertain the boundaries between regions. To do so, probability curves were plotted for each station. The curves for some of the stations were of the normal frequency distribution type whereas those for other stations were not. It was assumed, therefore, that, for the former stations, the weather pattern was consistent whereas the latter stations were subject to two or more weather patterns. Such irregular areas, therefore, were assumed to be boundary stations. Figs. 1(*a*), 1(*b*), and 1(*c*) illustrate the consistent type of station; and Figs. 1(*d*), 1(*e*), and 1(*f*), the boundary type of station.[2] There seems to be no definite boundary line, but rather a transition zone between the regions. Since there was no need to be precise about region boundaries, these

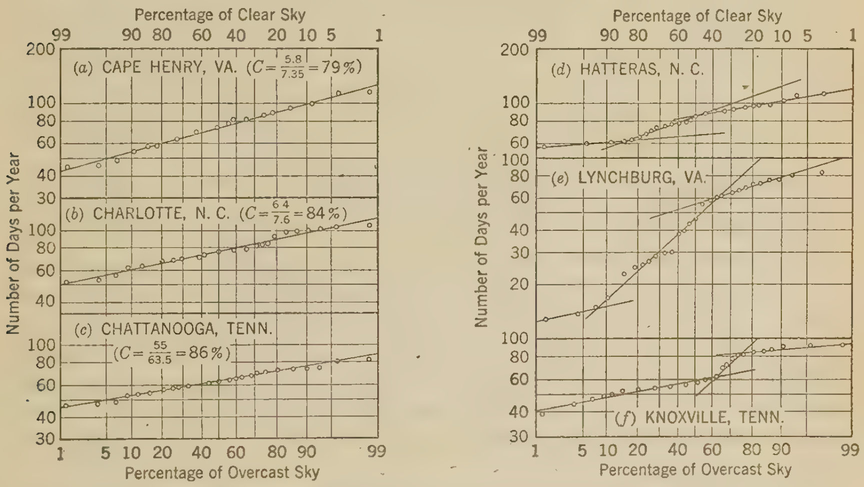

FIG. 1.—DETERMINATION OF BOUNDARIES OF AREAS OF SIMILAR WEATHER CHARACTERISTICS BY PROBABILITY CURVES
(C = COEFFICIENT OF VARIATION)

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[2] "Method of Estimating Time Required on Aerial Photographic Projects," by F. J. Sette, *Photogrammetric Engineering*, Vol. IV, No. 1, p. 42.

were drawn through those stations which showed the greatest variation in normal regional distribution.

As a result of these studies, Table 2 and Fig. 2 were prepared. Table 2 offers a quick method of scheduling projects. For example, regions 3 and 4 show very poor conditions for the months of June through September when two or three days per month is the average expectancy of good photographic weather, whereas in November, six to eleven days may be expected. Consequently, considerations of cost should dictate the scheduling of photographic projects in these areas from October through April. Equipment released from these areas could be used in regions 5 and 6 through October.

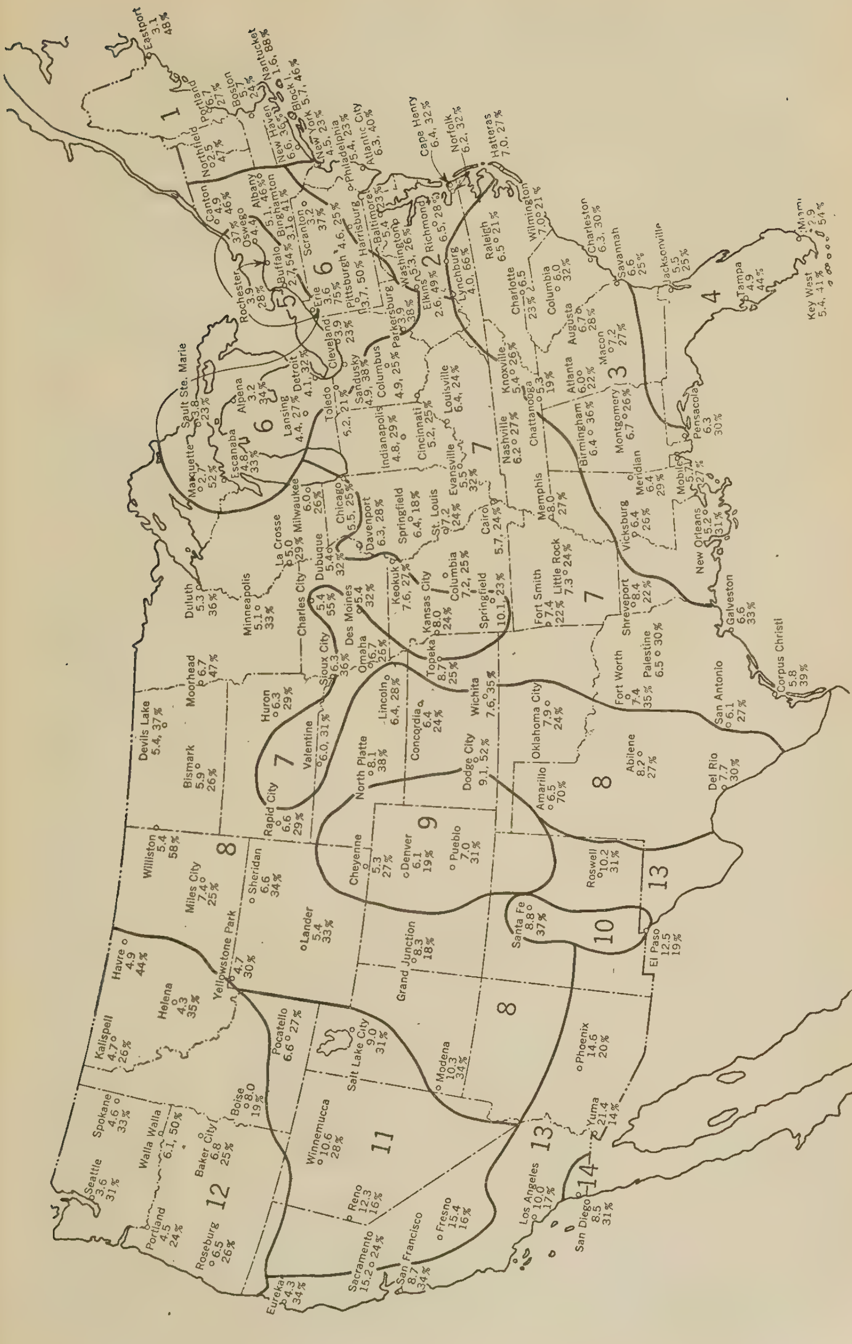
TABLE 2.—DATA REQUIRED FOR PREPARING A SCHEDULE OF OPERATIONS IN AERIAL PHOTOGRAPHY[a]

Region	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	100**	112**	125**	109**	95	66	69	94	115**	141**	85	89
2	94	95	102**	101**	105**	70	76	80	112**	158†	115**	92
3	104**	104**	112**	106**	97	62	36*	47*	90	175†	159†	108**
4	121**	149**	162†	139**	97	52*	28*	28*	44*	97	152†	131**
5	27*	38*	86	121**	155†	137**	150†	138**	154†	127**	43*	24*
6	79	73	100**	110**	124**	113**	122**	117**	131**	122**	60	49*
7	91	84	85	86	83	80	93	100**	125**	162†	118**	93
8	94	86	82	82	81	87	118**	115**	124**	135**	100**	96
9	116**	91	84	64	55*	79	74	68	142**	160†	135**	132**
10	115**	88	94	92	98	111**	42*	47*	104**	148**	134**	127**
11	50*	52*	65	75	86	127**	159†	160†	151†	132**	87	56
12	38*	53*	58	77	78	102**	226††	212††	155†	110**	51	40*
13	90	77	93	99	103**	113**	85	82	116**	129**	110**	103**
14	123**	100**	108**	94	63	52*	56	76	99	128**	159†	142**

[a] Groups that suggest similar weather patterns may be identified as follows: *, 55% or less; **, 100% to 149%; †, 150% to 199%; and ††, 200% or more of average month.

The numeral below the average number of photographic days per month shown for each station in the map, Fig. 2, is an index of variability of the weather (C), and is that percentage above or below the average which may be expected in a 10-yr period. At Yuma, Ariz., for example, the weather conditions are fairly constant. One year in every ten years, a deviation of approximately 14% below or above the average may be expected. On the basis of below average conditions, 18.4 (= 0.86 × 21.4) days per month is a normal expectation.

Validity of Index.—Although some preliminary observations had indicated that the number of days 0.1 overcast or less may be equivalent to the number of photographic days that could be expected, it was decided to test this assumption on the large-scale operations of the 1938 aerial photographic program of the Agricultural Adjustment Administration (AAA). Weekly weather reports of daily cloud conditions over the project areas were obtained by arrangement with the Weather Bureau. Four such areas are defined in Table 3(*a*). The study that follows is based largely on weekly reports of operation received from the contractors and from the Weather Bureau observers nearest the project locations. The reports from the crews were really estimates and were considered as such. Also, Weather Bureau stations were often located at a distance from the area being photographed; it was possible for perfect weather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Eastport 3.1 48%
Portland 6.1 27%
Boston 5.7 24%
Nantucket 1.6, 88%
New Haven 6.6, 36%
Block I. 5.7, 46%
New York 4.5, 23%
Philadelphia 5.4, 23%
Atlantic City 6.3, 40%
Northfield 2.5 47%
Canton 4.9 46%
Albany 5.1, 46%
Oswego 4.4 37%
Binghamton 3.1, 41%
Scranton 3.2 37%
Harrisburg 4.6, 25%
Baltimore 5.4 23%
Cape Henry 6.4, 32%
Norfolk 6.2, 32%
Hatteras 7.0, 27%
Richmond 6.5, 28%
Washington 5.3, 26%
Rochester 3.9 28%
Buffalo 2.7, 54%
Erie 3.6 75%
Pittsburgh 3.7, 50%
Parkersburg 3.9 38%
Elkins 2.6, 49%
Lynchburg 4.0, 66%
Raleigh 6.5 21%
Wilmington 7.0 21%
Charlotte 6.5 23%
Columbia 6.0 32%
Charleston 6.3, 30%
Savannah 6.6 25%
Jacksonville 5.5 25%
Miami 2.9 54%
Key West 5.4, 41%
Tampa 4.9 44%
Augusta 6.7 28%
Macon 7.2 27%
Atlanta 6.0 22%
Knoxville 5.4 26%
Chattanooga 5.3 19%
Montgomery 6.7 26%
Birmingham 6.4 36%
Pensacola 6.3 30%
Mobile 5.7 27%
New Orleans 5.2 31%
Meridian 6.4 29%
Vicksburg 6.4 26%
Shreveport 8.4 22%
Galveston 6.6 33%
Corpus Christi 5.8 39%
San Antonio 6.1 27%
Del Rio 7.7 30%
Palestine 6.5 30%
Fort Worth 7.4 35%
Abilene 8.2 27%
Oklahoma City 7.9 24%
Amarillo 6.5 70%
Roswell 10.2 31%
El Paso 12.5 19%
Santa Fe 8.8 37%
Fort Smith 7.4 22%
Little Rock 7.3 24%
Memphis 8.0 27%
Nashville 6.2 27%
Louisville 6.4, 24%
Evansville 5.5 32%
Cairo 5.7, 24%
Cincinnati 5.2, 25%
Indianapolis 4.8, 29%
Columbus 4.9, 25%
Sandusky 4.9, 38%
Cleveland 3.9 23%
Toledo 6.2, 21%
Detroit 4.1, 32%
Lansing 4.4, 27%
Alpena 3.2 34%
Sault Ste. Marie 3.8 23%
Marquette 2.7 52%
Escanaba 4.8 33%
Milwaukee 6.0 26%
Chicago 5.5, 25%
Davenport 6.3, 28%
Springfield 6.4, 18%
St. Louis 7.2 24%
Columbia 7.2, 25%
Springfield 10.1, 23%
Keokuk 7.6, 27%
Kansas City 8.0 24%
Topeka 8.7 25%
Wichita 7.6, 35%
Dubuque 5.4 32%
La Crosse 5.0 29%
Charles City 5.4 55%
Des Moines 5.4 32%
Omaha 6.7 26%
Sioux City 6.3 36%
Lincoln 6.4, 28%
Concordia 6.4 24%
Dodge City 9.1, 52%
North Platte 8.1 38%
Valentine 6.0, 31%
Huron 6.3 29%
Rapid City 6.6 29%
Duluth 5.3 36%
Minneapolis 5.1 33%
Moorhead 6.7 47%
Devils Lake 5.4, 37%
Bismark 5.9 26%
Williston 5.4 58%
Miles City 7.4 25%
Sheridan 6.6 34%
Cheyenne 5.3 27%
Denver 6.1 19%
Pueblo 7.0 31%
Lander 5.4 33%
Grand Junction 8.3 18%
Havre 4.9 44%
Helena 4.3 35%
Yellowstone Park 4.7 30%
Kalispell 4.7 26%
Pocatello 6.6 27%
Salt Lake City 9.0 31%
Modena 10.3 34%
Boise 8.0 19%
Spokane 4.6 33%
Walla Walla 6.1, 50%
Baker City 6.8 25%
Winnemucca 10.6 28%
Seattle 3.6 31%
Portland 4.5 24%
Roseburg 6.5 26%
Reno 12.3 16%
Eureka 4.3 34%
Sacramento 15.2 24%
San Francisco 8.7 34%
Fresno 15.4 16%
Los Angeles 10.0 17%
San Diego 8.5 31%
Yuma 21.4 14%
Phoenix 14.6 20%

FIG. 2

to exist in the area being photographed while the sky was more or less overcast at the station where the observations were made.

Tables 3(*b*) and 4 should be studied together. The former shows the number of flights made and the available photographic days as reported by the Weather Bureau; and the latter discloses that only 54.8% of the total flights, accounting for 61.4% of the total photography, occurred on days of 0.1 or less overcast.

TABLE 3.—STUDY OF FLIGHT CONDITIONS FOR AERIAL PHOTOGRAPHY, BY PROJECT AREAS

No.	Description	East Central[a]	North Central	Southern	Western	Totals
	(*a*) DEFINITION OF PROJECT AREAS					
1	States included	New England New York Pennsylvania New Jersey Delaware Kentucky Maryland North Carolina Tennessee Virginia West Virginia	Illinois Indiana Iowa Michigan Minnesota Missouri Nebraska Ohio Wisconsin South Dakota	Alabama Arkansas Florida Georgia Louisiana Oklahoma South Carolina Texas Mississippi	Washington Oregon California Nevada Idaho Utah Oregon New Mexico Colorado Wyoming Montana North Dakota Kansas	....
	(*b*) NUMBER OF AVAILABLE PHOTOGRAPHIC DAYS (SKY, 0.1 OR LESS OVERCAST)					
2	Number of flights....	659	1,080	621	517	2,877
3	Clear days[b]........	762	776	787	648	2,973
	(*c*) TIME REQUIRED, TOTAL AND AVERAGE					
	Hours:					
4	En route..........	1,325	2,173	1,159	1,222	5,879
5	Photography......	1,383	2,636	1,408	1,185	6,612
6	Total...........	2,708	4,809	2,567	2,407	12,491
	Hours per Flight:					
7	En route..........	2.01	2.02	1.87	2.36	2.04
8	Photography......	2.10	2.44	2.27	2.29	2.31
9	Total...........	4.11	4.46	4.14	4.65	4.35
	(*d*) AREA COVERAGE					
10	Square miles[c].......	137,236	261,784	121,997	113,441	634,458
	Square Miles per:					
11	Flight............	204	242	197	219	220
12	Photographic hour..	99.1	99.4	86.7	95.7	96.1
	(*e*) NET SQUARE MILES PER EXPOSURE					
13	Number of exposures.	53,396	111,302	84,460	53,392	....
	Square Miles:					
14	Total photographs..	85,939	149,857	85,138	68,400	....
15	Per exposure.......	1.61	1.35	1.01	1.28	....

[a] Includes the northeastern United States. [b] Reported days, 0.1 or less overcast. [c] Reported total coverage.

Two factors are important in appraising the value of the reported cloudiness: (1) The method of averaging 7 a.m., noon, and 7 p.m. observations, as used by the Weather Bureau; and (2) the location of the Weather Bureau observer, who might have been at considerable distance from the area being photographed. Thus, it is perfectly possible that, at the 7 a.m. observation, it was raining or that the sky was completely overcast, clear by 10 a.m., and 0.2 overcast at 7 p.m. Such a day would be reported as 0.4 overcast although it would be excellent photographic weather. Similarly, the cloud conditions over the area as a whole may have been 0.1 or less overcast and 0.4 overcast where the observer was located. Moreover, with the present availability of highly sensitive photographic film (which is several times "faster" than that produced a few years ago) complete cloudlessness is no longer an absolute requirement; on the contrary, sufficiently high cirrus clouds sometimes have a beneficial effect on the quality of the photographs by softening the light and reducing the intensity of the shadows.

TABLE 4.—NUMBER OF FLIGHTS AND THE AREA COVERED, AS RELATED TO CLOUD CONDITIONS

(As Reported by the Weather Bureau from the Project Area)

Proportion of sky overcast[a]	Number of flights	Square miles covered[b]	PERCENTAGE OF TOTAL:	
			Flights	Coverage
0.1[c]	1,576	389,334	54.8	61.4
0.2	383	82,011	13.4	12.9
0.3	309	58,953	10.7	9.3
0.4	221	44,212	7.7	7.0
0.5	143	22,994	5.0	3.6
0.6	117	16,062	4.1	2.5
0.7	59	9,555	2.0	1.5
0.8	25	2,991	0.8	0.5
0.9	27	4,462	0.9	0.7
1.0	17	3,884	0.6	0.6
Total	2,877	634,458	....	....

[a] Average cloudiness. [b] Reported total coverage. [c] 0.1 of sky, or less, overcast.

A study of the maximum 2-day performance of each contractor, totaling one hundred and one flights, showed an average coverage of 630 sq miles and 5.06 photographic hours per day as compared with 220 sq miles and 2.31 hours for all flights. These hundred and one flights (or 3.5% of all flights) accounted for 63,501 sq miles, or better than 10% of the total coverage. The reported cloud conditions of the 101 days when these records were made showed 84 days, that were 0.1 or less overcast and 11 days that were 0.2 overcast. All factors considered, it seems evident that for all practical purposes the index (namely, the day of 0.1 clouds or less) has substantial validity as a photographic day.

Effect of Sunlight.—Thus far no mention has been made of the effect of season, or of varying hours of sunlight, upon the time available for photography. In general, photography in the northern latitudes ceases about November 15 (when snow may be expected or when the photographic time is too short to continue operations economically), and is rarely resumed before April 1.

In the southern states, between latitudes 27° to 36° north, where only a comparatively few days of snow coverage is normal, the length of photographic daylight is of primary importance. There is no period in these latitudes between the dates of January 21 and November 22 when there is not at least three full hours of sunlight (10:30 a.m. to 1:30 p.m.) and when the sun is not less than 30° above the horizon—a requirement that is generally specified in order to avoid objectionable shadows. There is a 2-hr period (11:00 a.m. to 1:00 p.m.) between December 4 and January 7 when the sun has a vertical angle of not less than 30° above the horizon. At latitude 30° north, the sun is 30° or more above the horizon for at least 3 hours during the winter months, which is more than the average time of photography (2.27 hours) in Table 3(*c*) for the southern region.

Little is known of the effect of winter sun on the quality of photography; but it is well known that when the sun is low the rays are obscured by considerable haze and other poor atmospheric conditions. The shadows cast by the winter sun are long, and they obscure photographic detail. The best photographic time is the midday period of a clear day following precipitation, and the best season is as nearly June 21 (the summer solstice) as clearing of snow from the ground and the leafing of the trees will permit.

Tables of solar altitudes and Weather Bureau reports should be consulted when planning aerial photographic projects extending into the winter months.

PHOTOGRAPHIC COVERAGE

General Considerations.—To prepare a program and to estimate an aerial photographic project properly, it is necessary to know what the plane and the crew can do. The area that a crew can photograph depends on many factors, some of which are: Speed and power of plane, experience and ability of crew, scale of photograph, elevation of plane required to obtain the necessary scale, topography of area (mountainous or flat) and its mean elevation, whether or not the area is marked by the outline of public land surveys of the United States General Land Office, availability of good flight maps, favorable weather conditions, and distances from flying base to area to be photographed.

In general, the faster a plane can climb to its position and the speedier it is, the more time can be spent on photography, and more photographs can thus be obtained in a given length of time. Better advantage can also be taken of favorable weather conditions.

The most important single factor is the ability of the crew. Good pilots for aerial vertical photography are not numerous because of the severe combination of qualities required of them. Only a small percentage of trained pilots seem to have the patience and mapping intuition of line and direction necessary for success in aerial photography. The photographer and pilot must work as one unit, which adds to the difficulty of obtaining many successful combinations. Consequently, the area that can be photographed in a given time depends very largely on the skill of the crew because it involves not only the original photography but also the additional reflying necessary to fill in gaps or to replace unacceptable work.

Areas that are sectionized by the Public Land Survey System are more easily photographed from the air because the pilot can use section lines to guide him. In general, the states that are not sectionized, in addition to the thirteen original states, are West Virginia, Kentucky, Tennessee, Vermont, Maine, and parts of Texas and Louisiana.

Good flight maps enable the members of the crew to plot the flight lines, and to select landmarks as guides in the air. If good maps are available, they should be supplied to the contractors. When rephotography of an area is to be undertaken, probably the best flight maps would be the "photo-index" sheets obtained from previous projects. These should be supplied to the crew.

Flying Time.—Of first consideration is the flying time of a single photographic expedition. Table 3(*c*) indicates that, on the average, it takes 2.04 hours for a plane to leave the ground, fly into photographic position, and return to the ground after completing its mission. For brevity, this interval may be called "en route time." Approximately 2.31 hours are required for photography, making a total of 4.35 hours for the complete flight.

It is rather interesting to note that, in the western division, the en route time required was appreciably above the average (2.36 hours), probably because of the high mean elevation of the terrain photographed and the greater distances between landing fields.

Area Coverage.—Table 3(*d*) indicates the estimated area coverage obtained on the various projects. The effect of power of the plane and the higher productivity obtained when flying over sectionized country are illustrated in the following:

Description	Average horsepower: 180 hp	350 hp
Coverage per flight (sq miles):		
Over sectionized areas	212	248
Over nonsectionized areas	195	164
General average	205	233

In nonsectionized areas a heavy plane seems to be at a greater disadvantage than a lighter plane, although the advantage is reversed in sectionized areas. No information could be obtained to explain the poorer performance of heavier planes in nonsectionized areas.

Coverage Per Exposure.—The cameras in use for vertical aerial photography produce photographs 7 in. by 9 in. and 9 in. by 9 in. in size. The areas of the photographs are therefore 63 sq in. and 81 sq in., respectively, and show a ground area (scale of 1 to 20,000) of 6.28 sq miles and 8.07 sq miles, respectively. If an average overlap and sidelap of 60% and 30%, respectively, are required, and the plane can maintain the necessary elevation, the net progressive area per exposure is 1.75 sq miles for the 7-in. by 9-in. photographs, and 2.24 sq miles for the 9-in. by 9-in. photographs. Apparently, however, the net average per exposure is considerably less as Table 3(*e*) indicates.

The SCS reported that a study of a project of about 68,000 sq miles indicates that each photograph (7 in. by 9 in.) covers a net area of approximately 1.26 sq miles.

Table 3(*e*) shows that there is some difference, even as between divisions, in the coverage that can be expected per exposure. Some divisions have had a higher proportion of 9-in. by 9-in. cameras on their projects, which accounts for the greater coverage per exposure. It appears that 1.25 sq miles and 1.75 sq miles per exposure of 7-in. by 9-in. and 9-in. by 9-in. cameras, respectively, may be used for estimating purposes. The difference between the theoretical and the actual area per exposure is accounted for largely by the fact that the specifications require the contractor to photograph more than the actual area of the project.. Most contracts required that each county be flown as a separate subproject. Certain requirements were met—as, for example, photographing several miles beyond the county boundaries which increases the number of exposures normally necessary for the area of the county. Consequently, the area of photography is greater than the area of the county and can be calculated by plotting flight lines on the map.

Reflights.—Upon completion of the initial photography of a project, the contractor may find that some of the work does not meet the requirements of the specification; that is, there may be gaps, insufficient overlap or sidelap, or other defects. Reflights are then necessary to obtain complete coverage. Upon inspection by the contracting officer, some of the material may also be found unsatisfactory and additional reflights are necessary.

From the available material, the following appears to be the average reflights by AAA divisions:

Division	Percentage of reflights
East Central	20.6
Northeast	29.3
North Central	29.0
Southern	28.7
Western	23.4
Average	26.3

The allowance for reflying is difficult to estimate since it is largely a matter of efficiency on the part of the contractor and of inspection on the part of the contracting officer. Past performance of the contractor and of the contracting officer, where known, should be used in determining the average for the estimate.

Unit Costs

General Considerations.—Determination of unit costs in the field of aerial photography is a hazardous venture. Only the contractor knows the initial and operating costs of his plant and equipment; the performance and efficiency of his planes and personnel; and other pertinent costs such as insurance, taxes, etc. There are numerous competitive differences between contractors such as prevailing scales of pay, quality and speed of planes and equipment, ability to use plant and equipment on other work, and location of main laboratory from project area—all of which make it difficult to estimate the cost of a project with precision. Nevertheless, it is possible to approximate the reasonable cost of a particular project.

Value of Plane and Equipment.—A study of the planes used in the AAA 1938 aerial photographic program considering value, power, and age of planes is summarized in Table 5(*a*). Because the medium and heavy planes are often purchased secondhand and also because very often contractors use these planes on other work, a cost of $7,500 seemed reasonable for the plane. Similarly, studies of camera and other equipment indicated $1,500 as a more than reasonable estimate. Many of the cameras had been in service since World War I. Assuming a 10-yr period of depreciation for both plane and equipment and 6% interest on the investment, a charge of $100 per month should be made. Hangar rentals in 1937 varied from $30 to $50 a month—$40 per month probably being a fair charge. Obviously, it is not generally feasible to keep planes working all year. The charge to projects should be on the basis of 10-month operation per year. The yearly charge for the plane and yearly hangar rental totals $1,680; and, if overhead is assumed at 15%, the total yearly charge is $1,932 or $193 per month on a 10-month operating basis.

TABLE 5.—COST OF AERIAL PHOTOGRAPHY

No. of planes (1)	Average horsepower (2)	(a) AVERAGE CAPITAL COSTS			(b) OPERATING COSTS (DOLLARS PER HOUR)					
			Probable Cost				Insurance			
		Age (years) (3)	New, in dollars (4)	Dollars per horsepower (5)	Fuel (6)	Overhaul (7)	Liability (8)	Compensation (9)	Overhead (10)	Total operating cost (11)
10	145	1.3	4,950	33.00	2.18	1.09	1.50	1.25	0.90	6.92
20	300	5.8	8,860	29.50	4.50	2.25	2.00	1.25	1.50	11.50
27	435	8.5	13,100	30.10	6.56	3.27	2.50	1.25	2.04	15.62

Operation and Maintenance of Planes.—From such sparse information as could be assembled it seemed that, assuming gasoline to cost 25¢ per gal and oil to cost 35¢ per quart, a rule-of-the-thumb formula of 1.5¢ per hp-hr could be derived. A similar formula for estimating the cost of overhauling and inspecting equal to 0.75¢ per hp-hr was derived. This value may be used in estimating despite the fact that aerial photographic pilots do much of the work themselves during the large amount of waiting time. Insurance on plane and equipment and public liability is quite high; and, although it is not known how much contractors carry, studies indicate that a reasonable charge is $1.50 per flying hr for light planes, $2 per flying hr for medium planes, and $2.50 per flying hr for heavy planes. In addition, one must add compensation insurance while flying, which varies from state to state. For estimating purposes, $1.25 per flying hr is assumed (Washington, D. C., rate in 1938). The cost of operating a plane is summarized in Table 5(*b*), assuming overhead at 15%. The hourly operating costs shown in this table explain the tendency of contractors to purchase new light planes in recent years.

Salaries of Pilot and Photographer.—The salaries of flyers and aerial photographers are subject to larger fluctuation. Some companies, for example, may pay entirely upon coverage completed and accepted; some may pay a base salary plus so much per square mile of accepted photography; and others may pay on a straight salary basis. The sums to be used in estimating should range between $500 and $600 per crew of two per month. An average of $550 per month may be used for estimating purposes.

Although compensation insurance for employees, while flying, was included in the operating cost of the plane (see Table 5(*b*)), there is an additional charge for employees while on the ground. This charge is also subject to large variations. Using 1938 rates for Washington, D. C., compensation insurance would amount to $20 a month for the crew. The Social Security insurance rate in 1938 was 4% of the salary, or a total of $22 per crew. Accordingly, $600 a month for the salary of the crew may be used in estimating; and to this sum should be added overhead, assumed at 15%. The total allowance for this item will be $690 per month.

Cost of Flying.—The cost of flying cannot be reduced to a square mile basis until the project area is known and the number of days available determined. For estimating, the time is divided into two parts—waiting time and flying time. The waiting-time cost per month is the sum of salaries for the crew of $690 and the charges for the plane and hangar rental of $193, a total of, say, $885 for estimating purposes. Flying time is computed on a flight basis. In the North Central States, for example, the average time of flight is 4.46 hours, or 4.5 hours for all practical purposes. For a medium-size plane, the cost per flight will be about $51.80 or, say, $52 for estimating purposes.

Because of the vagaries of the weather, the contractor must allow for some contingency. The method suggested in this analysis is to add to the waiting-time cost the percentage of variation shown in Fig. 2 for the area under consideration.

Cost of Photography.—Efficient organization plays a large part in the cost of photography. Volume production is another important element of cost. The following analysis is based on continuous production of large quantities and is not applicable to intermittent or low-volume production.

TABLE 6.—COST OF PHOTOGRAPHIC MATERIAL

Material	Quantity	Man-hours	Dollars per Stated Quantity			
			Labor	Material	Overhead	Total
Film	One roll	4	2.52	27.50	10.00	40.02
Contact Prints, Single Weight:						
Commercial stock	100, 8 in. by 10 in.	4	2.52	2.70	1.74	6.96
Waterproof stock	100, 8 in. by 10 in.	4	2.52	10.00	4.17	16.69
Negatives	100, 20 in. by 24 in.	55	34.70	141.00	58.57	234.27
Contact prints	100, 20 in. by 24 in.	9.5	6.00	19.70	8.57	34.27

A survey by the U. S. Department of Labor of twenty-five firms engaged in aerial photography indicated that, in 1938, the median wage paid to workers was from 52½¢ per hr to 57½¢ per hr. Firms on the East Coast and West Coast paid about 75¢ per hr, in Texas about 40¢ per hr, and in the Middle West about 60¢ per hr. For the purpose of this study, 60¢ is used which, with compensation and Social Security insurances (1938), would amount to 63¢ per hr. Table 6 contains the cost of the usual photographic material, when the camera used takes a 7-in. by 9-in. photograph. Overhead is estimated at 33⅓% of the cost of labor and materials.

A necessary requirement of aerial photography is a photo-index sheet which is a composite photograph of all the pictures taken, after all reflights have been made. Such a sheet makes identification simple and quick when a photograph of a particular area is required. To make such an index, contact prints are stapled on boards in the order and position that the photographs were taken in the air and are then copied. The coverage on one of these sheets, from actual tests, averages 240 sq miles. Assuming 1.25 sq miles per contact print, 192 prints will be required per sheet. Moreover, the contractor will be required to make a contact print for each exposure he makes in the air and, consequently, reflights of 30% must be included. Laboratory wastage of about 5% is an additional necessary allowance. Accordingly, the number of contact prints to be made for each sheet is 262 at \$6.96 per hundred—or a total cost of \$18.25. Prints may be stapled at about 25 per man-hr at a cost, for 192 prints, of \$4.85. Assuming \$6.00 for board, staples, and overhead, each board will cost about \$29.10 before it is photographed. The negative will cost about \$2.35 and the contact print will cost about 35¢—or a total of \$31.80 per index sheet.

The usual contract stipulates the delivery, by the contractor, of the aerial photographic negatives, one set of waterproof contact prints, one set each of photo-index negatives and photo-index sheets, and one set of single-weight prints which had been used in making the photo-index sheets. The photographic cost per square mile may be estimated by assuming a 1,000 sq mile area for simplicity of calculation. A roll of film will cover 137.5 sq miles (1.25 sq miles per exposure times 110 exposures per roll). Therefore, 1,000 sq miles will require 7.27 rolls of film, or 9.5 rolls allowing for 30% reflights. The number of waterproof prints required will be $\frac{1,000}{1.25}$ or 800 net. Allowing 5% waste, 840 prints will be printed. Reflying is not included in this item as these prints are made last, after all reflights have been finished. The number of index sheets is obtained by dividing 1,000 sq miles by 240 sq miles or 4.17 sheets. The cost in dollars per 1,000 sq miles (\$650) may be itemized as shown in Table 7.

TABLE 7.—COST OF MATERIALS FOR 1,000 SQUARE MILES BY PHOTOGRAPHY

Item	Quantity	Unit cost[a]	Total cost
Rolls of film..........	9.5 each	\$40.02	\$380.19
Waterproof prints.....	840 each	0.167	140.28
Photo-index sheets....	4.17 each	31.80	129.43
Total cost (dollars)...	1,000 sq miles	0.65	\$649.90

[a] Dollars per unit of quantity listed.

In 1938, some contractors were using cameras taking 9-in. by 9-in. photographs, a size now generally required. The cost of photography is somewhat less. This size of film comes in rolls of 150 ft and longer, instead of 75 ft for the 7-in. by 9-in. camera. For 150 ft the man hours required for developing is about 50% more than that shown in Table 6. The material cost is 100% more. Including overhead, the cost per roll is \$78.37. Printing productivity for 9-in. by 9-in. contact prints is the same as that shown in Table 6. The material cost is 20% higher—that is, in proportion to the areas of the photographic papers. A 7-in. by 9-in. print is made on an 8-in. by 10-in. paper, whereas a 9-in. by 9-in. print is made on a 10-in. by 10-in. paper.

The number of exposures may be computed by dividing the total length of the film in inches by 10 in. For a 150-ft roll, 180 exposures, each covering

1.75 sq miles, will cover 315 sq miles. Consequently the cost of film per 1,000 sq miles, including 30% reflights, will be $321.32. Similarly, allowing 5% waste, only 600 waterproof prints will be required per 1,000 sq miles at a cost of $116.10. The coverage of a photo-index map of 240 sq miles remains unchanged except that, instead of 262 prints of the 7-in. by 9-in. size, 187 of the 9-in. by 9-in. size will be required. The cost per index sheet is $26.56, computed as shown herein. Consequently, the cost per 1,000 sq miles is $547.42, or on a square-mile basis, about 10¢ less than the cost of photography using a 7-in. by 9-in. camera (see Table 7).

Other Cost Considerations.—Studies indicated that the size of project was an important element in the total cost. Every effort was made to block out contiguous areas of fairly large sizes for incorporation into single projects. In certain regions, areas averaged from 15,000 sq miles to 20,000 sq miles per project, and in others no more than 5,000 sq miles or 6,000 sq miles were possible. Adequate competition was another important factor. This could be controlled by releasing projects when more than enough equipment seemed to be available. Uniform specifications were absolutely necessary if costs were to be kept down; and this was done as nearly as practicable. Finally, efforts were made to schedule work on a year-round basis for as many planes as possible so as to spread the costs of equipment and overhead over more units of work. For this reason, contracts for aerial photography in the south stipulated that planes could take off on, or about, June 1 but had to be returned by, or before, December 1. This provided additional equipment for projects in areas where summer conditions provided greater utilization. Because of the known effect on costs of these variables, aerial photographic projects were cleared through the various bureaus of the U. S. Department of Agriculture and often with some of the state governments so that needs were all consolidated and programmed. Moreover, when bids were received and evaluated, they were subject to rejection if they exceeded, appreciably, the engineer's estimate of the project. The efficacy of the administrative policy implementing research conclusions was demonstrated by the reduction in costs obtained.

PROGRAMMING AND ESTIMATING PROJECTS

With such information, it is possible to program and estimate aerial photographic projects on, say, a scale of 1 to 20,000. Two examples will be used (both in the Middle West) to show the effect on the costs by reason of weather conditions. Both projects are to begin on June 1 and end before December 1. Project A (17,000 sq miles) is in northern Missouri, extending across the state, and project B (10,400 sq miles) is in southern Indiana. Project A cuts across weather regions 7 and 8, and project B is all in region 7.

Time Required.—By inspection, in Table 2, the average number of days in project A that are 0.1 or less overcast is about 7.5 per month, with a variation of 25%. As the area is about equally divided in regions 7 and 8, the monthly percentages will be averaged in project A. In project B, the probable average number of days per month is about 5.6, with a variation of about 27%. The time available for both projects using one-plane crews is computed as shown in Table 8. (For method of computing clear days available see Table 1 and supporting text.)

Although the average coverage per flight for the Middle West is 242 sq miles, the heavier plane in sectionized areas can cover 248 sq miles. For estimating purposes assume 240 sq miles per flight. The number of flights, including reflights of 30% for project A will be 92 and for project B, 56. The time required for project A is 6 months and for project B, say, 5 months, two planes and crews being used for each project. The waiting time in project A is about 5⅓ months, but because of the size of project the delays caused by inspection of material would justify the employment of the crew for the full six months. Extra flights—probably 10% of the total would be equitable—for reconnaissance and getting on and off the projects at the start and finish of the work must be added to the number of flights. Thus, project A would require one hundred and one flights and project B, sixty-two flights. The estimated costs of the projects are as listed in Table 9.

TABLE 8.—EXAMPLES OF ALLOCATING AVAILABLE TIME

Month	(a) PROJECT A; 17,000 SQ MILES IN NORTHERN MISSOURI			(b) PROJECT B; 10,400 SQ MILES IN SOUTHERN INDIANA		
	Percent of average month	Clear Days Available		Percent of average month	Clear Days Available	
		Number	Cumulative[a]		Number	Cumulative[a]
June.........	84	6.3	6.3	80	4.4	4.4
July.........	110	8.2	14.5	93	5.0	9.4
August.......	112	8.4	22.9	100	5.6	15.0
September....	125	9.4	32.3	125	7.0	22.0
October.......	148	11.1	43.4	162	9.1	31.1
November.....	109	8.2	51.6	118	6.6	37.7

[a] Cumulative days available to a one-plane crew. For a two-plane crew, multiply these values by 2.

TABLE 9.—COMPARATIVE ESTIMATES OF COSTS

No.	Item	PROJECT A		PROJECT B	
		Cost	Remarks	Cost	Remarks
1	Waiting time.........	$10,620	Six months[a]	$ 8,850	Five months[a]
2	Contingency.........	2,660	25% of item 1	2,390	27% of item 1
3	Flying time..........	5,250	101 flights[b]	3,220	62 flights[b]
4	Photography.........	11,050	17,000 sq miles[c]	6,760	10,400 sq miles[c]
5	Subtotal...........	$29,580		$21,220	
6	Profit..............	4,420	15%	3,180	15%
	Estimated Costs:				
7	Total..............	$34,000		$24,400	
8	Unit...............	$2.00	Per square mile	$2.35	Per square mile

[a] Two planes at $885 each. [b] At $52 each. [c] At 65¢ per sq mile, assuming camera takes 7-in. by 9-in. photographs.

Depending on the camera used in 1938, project A may cost from $1.90 to $2.00 per sq mile, whereas project B may cost from $2.25 to $2.35 per sq mile.

Sufficient information has been given to estimate costs for future programs. The average values of plane and equipment will probably not change appreciably. The costs of operating and maintaining a plane and of photographic material can be adjusted in accordance with the prevailing salary and wage rates and the costs of materials. The productivity of labor, discussed in the section, "Cost of Photography," has not been materially affected by new developments and may be used for current estimates. Since the size of the aerial photograph now generally used (1946) is 9 in. by 9 in., the cost of photographic materials should be estimated on that basis. For aerial photography of a different scale (other than 1 to 20,000) or which uses a camera with a longer or shorter focal length than 8¼ in., the foregoing information could be used but this would require considerable manipulation.

RESULTS OBTAINED

Analyses of a number of projects led to the belief that a properly administered program should result in average costs of from \$2 to \$2.50 per sq mile. In fact, an established year-round program might well cost less than \$2 per sq mile. This is important to consider, especially since aerial photography has such widespread usefulness even outside the field of engineering. The campaign to reduce the cost of aerial photography (the studies for which began late in 1937 and began to be implemented in the spring of 1938) was successful because of the increasing familiarity with the work to be done on the part of both contractors and contracting officers. Instructions were issued to all the technicians in the department acquainting them with the elements of programming and of costs so that the maximum economy could be obtained. The following paragraph is quoted from the instructions:

TABLE 10.—REDUCTION IN COSTS OF AERIAL PHOTOGRAPHY BY CAREFUL PLANNING AND PROGRAMMING

Description	1926–1937[a]	1938	1939	1940	1941
Area covered, in square miles	970,434	635,562	445,844	376,436	381,083
Total cost, in dollars	4,110,423	1,968,668	926,273	765,281	892,320
Unit cost, in dollars per square mile	4.24	3.09	2.07	2.03	2.34

[a] Dates are inclusive, and all dates are for the calendar year.

> "While quality of performance is most important, the cost of aerial photographic projects must be kept to a minimum. Proper programming is essential to economy of operations. Insofar as possible, programs should be planned on a 12-month basis, taking every possible advantage of the probable weather conditions. Areas should be large enough to permit economy of operation. Changes in specifications should be examined to determine how costs may be effected. And, lastly inspections of materials should be made promptly and fairly to reduce idle time and waste of materials."

The results obtained more than justified the efforts and expenditures in making the studies. The reduction in costs is shown in Table 10 which was prepared from data submitted by the U. S. Department of Agriculture.

ACKNOWLEDGMENT

Although the work was done under the direction of the writer, it could not have been accomplished without the invaluable assistance and cooperation of the technicians in the various divisions of the Agricultural Adjustment Administration, the Soil Conservation Service, the Forest Service, and other bureaus, and of some of the contractors who were generous of their time and ideas. The writer is particularly grateful for the aid which he received from Marshall S. Wright, Office of the Secretary; Louis A. Woodward, SCS; Grover M. Plew and J. Marvin Cultice, AAA, all of the U. S. Department of Agriculture.

CONCLUSIONS

The studies reported in this paper need greater testing and refinement, because the usefulness of aerial photography is just beginning to be understood and appreciated. Cultural changes in some parts of the United States were so rapid, even before the beginning of World War II, that it is more economical to refly the areas than to use old photography. For proper planning, many areas will require reflying every 5 years, on an average. When the use of aerial photography becomes widespread (as it may in the not-too-distant future) flying and photographing of from 400,000 to 500,000 sq miles per yr may be a normal operation. When that time comes more precise information may need to be developed.

DISCUSSION

T. W. Norcross,[3] M. ASCE.—The U. S. Department of Agriculture (USDA) has been using aerial photographs since approximately 1926. Until the Soil Conservation Service (SCS) started a comparatively large photographic program, little attempt was made to prepare an engineering estimate of the cost of such work. The size of the SCS program, and the personnel available, did not warrant the type of study later made under Mr. Sette's direction, utilizing data collected under the combined programs of all USDA bureaus. The rougher estimates made by the SCS took cognizance of terrain in a manner different from that presented in the paper.

Mr. Sette's approach to this problem is sound and logical. It is based on scale, lens, weather conditions, cost of equipment and materials, cost of operation plus a fair profit, and coverage per photographic exposure. No known tangible item has been omitted. The author concludes correctly that the method needs greater testing and refinement. Certain intangibles not previously included are believed worthy of consideration in refining the method employed in estimating costs.

Table 3(*d*) shows that 634,458 sq miles of photography were used as the basis of computations. This area (or the smaller areas covered in 1939, 1940, and 1941) can be considered as offering an annual photographic program of sufficient size to plan for continuous operation—an attraction to commercial concerns conducive to active competition. A smaller annual program can easily result in the operation of a smaller number of concerns, less competition, and resulting higher unit costs.

Aerial photographs are being sought more and more by a larger number of users as their value is appreciated. Photographic concerns in the United States are no longer dependent almost entirely on the federal government for large contracts and are engaging in photographic operations in foreign countries. Some of these new customers do not operate on the same basis as the federal government and are willing to pay the price asked to secure the desired material. Competition of this kind can reach such proportions that, excluding the increased costs of operations, either the bid price received will be higher than heretofore; or new concerns must be encouraged to restore competition. Increased competition may result in lower bid prices, but it also raises administrative costs of the contract and delays delivery of satisfactory materials.

In the paper, photography of different types of terrain was taken into consideration only to a limited extent—the effect of power of plane and the higher productivity obtained when flying over sectionized and nonsectionized areas were cited. In the data compiled, the preponderance of areas photographed was agricultural. In this type of terrain, land lines are easily recognizable in sectionized country. Using a heavy plane in nonsectionized country causes difficulties. In photographing mountainous timbered areas, even though in sectionized country, a similar lower productivity would be expected; and, in mountainous country, the heavier plane might easily be necessary to obtain the necessary flying height. When lack of competition is coupled with difficulty of terrain, particularly mountainous terrain, the bidder probably increases the unit cost out of proportion to the difficulty of operation. He knows that few concerns will have the type of equipment required and he will have a better opportunity to secure a price over and above that which would normally result. Necessary flying altitude and amount of available equipment for working at varying flight heights should be considered in the preparation of estimates. In the past many estimates have been made in conformity with the Sette method, with the result that review of the estimate, in conjunction with terrain conditions and required flight height, shows that, because the unit price was considered too low, it has been altered upward to take these difficulties into account.

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[3] Chf., Div. of Eng., U. S. Forest Service, U. S. Dept. of Agriculture, Washington, D. C.

Photographs are made for a variety of purposes. Generally those secured by USDA are for general land use studies which require the characteristics outlined by Mr. Sette. For mapping purposes photographs of entirely different characteristics may be, and usually are, required. To undertake all kinds of work, the contractors must be equipped to meet the varying requirements. This condition existed at the time the study was made, and the data in the paper were based on the assumption that equipment was adequate. Also, of the volume of work performed at that time, the largest part was for the USDA. Comparative costs to the USDA and mapping agencies would doubtless show a higher unit cost for others because of the smaller volume of work. During World War II, however, many concerns made all their equipment available to the war effort and must re-equip. Many are uncertain of the kind of equipment to purchase. Unit costs will reflect the cost of this equipment, and only through assurances of a large, well-planned program can a high rate of depreciation be avoided.

James M. Cultice,[4] Esq.—Aerial photographic activities were largely suspended for civilian purposes during World War II; and, with the preparation of new bid invitations, the usefulness of such data can again be apparent to contractors and purchasing officials and their engineers. Numerous requests have been received for the clear weather zone map (see Fig. 2).

The validity of the 0.1 cloudiness as an index for available photographic days should be emphasized. The paper indicates 2,973 clear days in contract areas, according to this specification, and 2,877 contract flights within the same period. The difference of 4% is very small and indicates that the probable[5] time necessary to complete an item can be estimated reliably from the standpoint of weather conditions. Although these comparisons are pertinent, it was not intended to convey the impression that flights are made regularly on days of 0.1 cloudiness, and this fact is adequately developed in the paper.

It would be of interest to note the actual award prices for the project areas estimated in Mr. Sette's study. The writer checked the contracts in the southern Indiana section and found a unit price in the successful bid of $2.24 per sq mile on an item of 10,421 sq miles, against an estimated $2.35. In northern Missouri where the weather pattern is more favorable, the contract price of

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[4] Chf., Eng. Section, Aerial Photographic Laboratory, Field Service Branch, Production and Marketing Administration, U. S. Dept. of Agriculture, Washington, D. C.

[5] "Days Available for Aerial Photography," by J. M. Cultice, *Photogrammetric Engineering*, Vol. 6, No. 1, p. 32.

$1.70 for an item of 12,203 sq miles was less than Mr. Sette's lower suggested value of $1.90. These prices were both for 1940 coverage and, thus far, rephotography has not been purchased in these areas. The open, well-developed, and well-sectionized country was of course, conducive to flying economies.

Wartime developments in new aerial photographic planes, cameras, and camera equipment, new flight line control instruments, and better and faster films should make the cost of aerial surveys comparable to the cost of those presented in the paper for areas similar in terrain to the agricultural lands for which the estimating study was originally developed. Existing photo indexes will facilitate the planning of flight lines and should aid the pilot to make better use of flight time through easier recognition of ground detail.

The widespread wartime use of aerial photography and the extension of its usefulness in all fields have greatly increased the demand for aerial coverage for all purposes. The resultant advantages of plane placement and the better utilization of weather conditions in smaller localized areas by concurrent projects will increase efficiency and lower costs. Furthermore, the helpfulness of the cost estimating system is equally effective regardless of price levels. Adequate corrections can be easily applied; and new methods, better equipment, and supplies can be evaluated and their effect on the total final estimate can be computed with some measure of real dependency.

Those concerned with the engineering and administrative phases of the acquisition of aerial photography in the United States Department of Agriculture have frequently noted the value of this study in effecting saving for industry and thereby producing saving for the government and others.

MARSHALL S. WRIGHT,[6] ESQ.—Since 1936 the U. S. Department of Agriculture has followed a procedure, whereby all proposed aerial photographic projects are first submitted to the Office of the Secretary for approval before photographic operations are initiated either directly or by contract. During the period from 1938 to December, 1941 (Pearl Harbor), all applications to initiate work had to be accompanied by an "Engineer's Estimate" of cost, in the manner the author has indicated. During World War II the requirement that an "Engineer's Estimate" be submitted to the Secretary's Office along with all applications to initiate aerial photographic operations was discontinued, due mostly to the fact that aerial photographic operations were greatly curtailed at the request of the War and Navy departments. Furthermore, it was practically impossible to prepare cost estimates, because of delays and unforeseen restrictions on normal operations.

A resumption of aerial photographic activities of the magnitude in effect during the period from 1936 to 1941, whereby practically two thirds of the area of the United States was photographed to rigid federal specifications by commercial contractors, would necessitate the continuance of a thorough check of estimated costs as balanced against contract prices to assure the government a commensurate return on its investment; and, what is equally important, would require a constant study to assure that the most efficient methods are employed. This would also necessitate, on the part of the contracting officer, a thorough knowledge of all improvements in photographic and flying equipment, field and laboratory technique, and an intimate knowledge of the facilities of all contractors.

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[6] Technical Asst. to the Chf., Office of Plant and Operations, U. S. Dept. of Agriculture, Washington, D. C.

In the inauguration of the cost-analysis method outlined by the author, possibly the two greatest concrete benefits that were derived were (1) the preparation and subsequent placing in the hands of every potential commercial contractor the indicated map (Fig. 2) of the United States showing, by regions, areas of comparable weather condition expectancies; and (2) as a consequence of benefit (1) the over-all planning of seasonal work to permit a contractor to be awarded areas in the northern part of the United States during the summer months. The map (benefit (1)), the cost of which probably could not be justified by any one single contractor, was a major contribution to the entire aerial photographic industry. Admittedly, it did "do something about the weather"; and, most directly, it was probably the greatest contributing factor toward an indicated reduction of approximately $1.00 per sq mile (compare 1938 with 1939 in Table 10).

Because of benefit (2), the contractor was enabled to discontinue this work in the fall, and move to the southern states and continue photographic operations in other contracted areas. The result was evident in lower bid prices because the contractor could utilize his personnel and equipment on a year-long basis.

One phase that the author has not mentioned is that the Congress of the United States included in the Agricultural Appropriation Act for 1938 the following provisions which permit the sale, to the public, of photographic reproductions:

> "The Secretary may furnish reproductions of such aerial or other photographs, mosaics, and maps as have been obtained in connection with the authorized work of the Department to farmers and governmental agencies at the estimated cost of furnishing such reproductions, and to persons other than farmers at such prices (not less than estimated cost of furnishing such reproductions) as the Secretary may determine, the money received from such sales to be deposited in the Treasury to the credit of the appropriations charged with the cost of making such reproductions. This section shall not affect the power of the Secretary to make other disposition of such or similar materials under any other provisions of existing law."

As evidence of the great usefulness of aerial photography in the solution of all land-use problems, the Department has sold approximately $850,000 worth of aerial photographic reproductions to states, public utility corporations, and individuals, excluding the great use for them by other federal agencies.

ROBERT H. RANDALL,[7] M. ASCE.—Both the user and the producer of aerial photography will profit by this paper. The studies described were undertaken at a time when the aerial photographic program of the United States Department of Agriculture (USDA) was in its early stages. The procedures that developed from them unquestionably saved considerable money in the first year's operation. Commercial organizations working on contracts with the USDA also undoubtedly profited by these studies. Originally undertaken for the use of the USDA in administering its own photographic program, the results of this pioneer work continue to be of use not only to the USDA but to all users of aerial photography in the federal government and to all concerns supplying aerial photographs.

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[7] Chf. Examiner, Surveying and Mapping, U. S. Bureau of the Budget, Washington D. C.

The studies reported in this paper constituted a program of research which had never been attempted on such a scale, by aerial photographers, individually or collectively. Reduced to usable and convenient form, they furnished statistics on weather conditions by which, for the first time, the USDA could plan its photographic requirements in different areas in such a way as to take advantage of the best weather conditions. This development was in itself an economy to the government. Furthermore, the photographic contractors were enabled to forecast conditions under which they would be required to operate, thus lowering their bid prices. During the first year that this analysis of weather conditions was available to the contractors, it is estimated that the USDA saved about $500,000.

In addition to the direct economies to the government and to the contractors by the availability of information on weather conditions, the studies described in this paper produced a second material, although less direct, benefit: The USDA was able to inaugurate a practice of making detailed estimates for each photographic job requested by any of its bureaus or agencies. This procedure of preparing estimates, and of checking the bids received against them, constituted an improvement in administrative practice, which, although no longer followed by the USDA in the degree of refinement that was observed in the earlier days of the photographic program, is still followed in general not only by the USDA but also by other federal departments and establishments.

It is impossible, of course, to determine the extent to which this administrative practice of USDA influenced other federal agencies outside the department. It is a fact, nevertheless, that all agencies procuring aerial photographs for the purpose of making maps or charts now recognize the common-sense value of careful advance estimating. In 1941, when these agencies (under the suggestion of the Bureau of the Budget in the Executive Office of the President) developed a uniform system for accounting and reporting their performance and costs, they included advance estimating for aerial photographic projects. In this system the planning of aerial photography can be separated into five steps: (1) Determination of limits of project; (2) consultation of records, to ascertain what maps and photographs are already available; (3) determination of scale or flying height; (4) preparation of estimates of time and cost; and (5) administrative and technical clearance to conduct the work—that is, to determine the availability of funds, obtain final authorization for expenditures, etc.

Although the aerial photographic requirements of the USDA and other federal agencies were many, at the time that Mr. Sette's studies were made, the aerial photographic interests of the United States are now even greater. During World War II the federal government photographed some 15,000,000 sq miles in various parts of the world; and, with this and other available cartographic material, compiled and published aeronautical charts covering all the land areas of the planet. Besides aeronautical charts, the armed forces also produced topographic maps on various scales in many parts of the world. To revise and improve these as the national interest may indicate, and to keep them up to date, implies a continued national interest in aerial photography in respect to not only the continental United States but the remainder of the world as well. The sensible procedures described in this paper should continue to contribute to the efficiency and economy with which this work may be done.

F. J. SETTE,[8] M. ASCE.—In asserting that the size of the annual photographic program is conducive to active competition and to lower prices, Mr. Norcross is quite correct, because it permits contractors to plan for continuous operation. It should be realized, however, that the size of the program by itself will not result in lower prices unless projects are scheduled in accordance with equipment available and the prevailing climatic conditions. The greater use of aerial photography for topographic mapping purposes and the need for rephotography because of cultural changes may well require a substantial, annual, aerial photographic program. This being so, planning, coordination, and integration, as now performed in the United States Department of Agriculture (USDA) for its own bureaus and agencies, should also be performed at a higher level, presumably in the Executive Office of the President (Bureau of the Budget) for the various departments of the federal government.

Large commercial concerns have been developing markets in foreign countries for some time, even before the war; and to the extent that they are successful and stay out of the domestic market, competition may be reduced. No American concern can safely stay out of the domestic market entirely.

In difficult terrain such as mountainous timbered areas, aerial photography may be considered more costly because of operational requirements. For example, the use of heavy planes is costly, narrowing the area of competition so that competitors may take advantage of this special circumstance. If an estimator knows the location of commercial concerns with reference to project areas, and the type of equipment at their disposal, he can determine a reasonable allowance for these circumstances.

Although the federal government is the largest user of aerial photography, the increasing use by others of the government's aerial photography, as reported by Mr. Wright, is gratifying. The writer feels that World War II interfered with the full utilization of the government's available aerial photography, by private interests. There should be an even greater demand in the future, especially if the photography is kept current with cultural changes. Mr. Wright states that one of the benefits outlined in the paper was that of enabling the contractor to utilize his personnel and equipment throughout the year by permitting him to move south in the fall. In the aerial photographic program of the summer of 1937, it apparently was the disappointing performance of a score of crews in the south that led the writer to an analysis of the monthly weather conditions. After 1937, large-scale immobilization of plane crews in the southern states in the summer and fall was no longer permitted.

Mr. Randall reports that the procedure of preparing estimates and of checking the bids against them is still followed, in general, not only by the

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[8] Deputy Director, Bureau of Constr. and Field Operations, Civilian Production Administration, Office of Temporary Controls, Washington, D. C.

USDA but also by other federal departments and establishments. If this procedure is continued in the future and if programs of various departments of the government are coordinated and integrated, extensive economies can be made by commercial concerns in planning their operations and in utilizing their equipment. Such economies, in turn, can be passed on to the government in the form of reduced bids.

The writer is particularly pleased that Mr. Cultice was able to give actual contract prices for aerial photography to compare with estimates developed in the two examples shown in the paper. Project A located in northern Missouri was estimated to cost from $1.90 to $2.00 per sq mile on the basis of 1938 prices. Mr. Cultice states that the contract price in 1940 for aerial photography in northern Missouri was $1.70 per sq mile. On September 19, 1946, bids were opened by the USDA, for aerial photography involving 13,484 sq miles in northwestern Missouri, southeastern Nebraska, and south-central Iowa. The low bid averaged $2.38 per sq mile. Project B was estimated to cost from $2.25 to $2.35 per sq mile. Mr. Cultice reports a contract price of $2.24 per sq mile in 1940. In 1946, the low bid for 8,430 sq miles (mostly in Indiana) was $2.42. One of the parcels in this area of 4,647 sq miles was bid at $2.35 per sq mile. On September 19, 1946, bids for a total coverage of 39,086 sq miles were requested, the low bid for which averaged $2.40 per sq mile. The project areas were located in Minnesota, Wisconsin, Iowa, Missouri, Nebraska, Illinois, Indiana, and Pennsylvania.

On December 30, 1946, bids were requested for a coverage of 17,178 sq miles in South Carolina, Georgia, and Florida; and the low bid averaged $2.53 per sq mile. For the September and December bid openings, the average of all the low bids was $2.44 per sq mile for a total coverage of 56,264 sq miles. Considering the scatter of the projects and the increases in cost of labor, materials, and supplies, the 1946 bids certainly compare favorably with the 1941 contract prices, which averaged $2.34 per sq mile for a coverage of 381,083 sq miles. It seems apparent that the gains made in the past are being maintained. Should wages and salaries keep increasing and the prices of supplies and materials keep pace, government technicians must give greater attention to each element of cost that enters into the contractor's bid in order to keep the costs of aerial photography as low as possible.

The writer is grateful to Messrs. Norcross, Cultice, Wright, and Randall for their contributions to his paper and for their favorable comments. The paper contained a number of assumptions, the validity of which could be upheld only by adequate cost records in the possession of commercial aerial photographers. Since the assumptions have not been challenged, the writer must conclude that these may not be too far "out of line" with actual costs. Contract awards for successive years, 1939 through 1941, and bids submitted in 1946, lend further support to the general reliability of the various assumptions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2308

CRITICAL STRESSES IN A CIRCULAR RING

By E. A. Ripperger,[1] Esq., and N. Davids,[2] Esq.

With Discussion by Messrs. Robert R. Philippe and Frank M. Mellinger, E. P. Popov, and E. A. Ripperger and N. Davids.

Synopsis

Many times, in investigations of the physical properties of foundation rock samples, tensile strength tests are required. Usually, the sample consists of one or more short cylindrical prisms of the type taken by core drilling, and then the conventional tension test (in which axial forces are applied to pull the specimen apart) is absolutely inadequate. As a suitable test for use under such circumstances and with such material the Soils Laboratory, U. S. Engineer Office, Pittsburgh, Pa. (later, the Soils Engineering Section, Cincinnati, Ohio, Testing Laboratory), devised and investigated the "ring test" described in this paper. As the name implies, specimens for this test are rings formed by cutting disks about 1 in. thick from cylindrical cores and by drilling a small hole in the centers of the disks. These rings are then placed in a standard testing machine and loaded in compression until failure occurs. The results of the tests have been interpreted in the light of certain unpublished data which Max Frocht[3] obtained using photoelastic methods.

Introduction

As part of an investigation of the feasibility of using the "ring test" in obtaining a measure of the tensile strength of concrete, the writers, in 1942–1943, made a mathematical investigation (the results of which are presented herein) of the type and distribution of the stresses normal to the section in the plane of the concentrated load—the so-called "critical section." Numerical values of these stresses were then computed for rings with diameter ratios ranging from 0.0 to 0.5. The last case (diameter ratio 0.5) has been analyzed by S. Timoshenko,[4,5] but by a somewhat different method.

Note.—Published in February, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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[3] "Photoelasticity," by Max Frocht, John Wiley & Sons, Inc., New York, N. Y., 1941, pp. 245–247.

[4] "On the Distribution of Stresses in a Circular Ring Compressed by Two Forces Acting Along a Diameter," by S. Timoshenko, *The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science*, Vol. 44, No. 259, July, 1922, p. 1014.

[5] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1934, pp. 114–116 and 119.

The exact general solution for the stresses in a ring with any type of loading was first obtained by A. Timpe[6] in 1905, and in 1924 was fully discussed by L. N. G. Filon.[7,8] In Mr. Filon's presentation the stress distributions are given in the form of infinite series whose coefficients are functions of the Fourier expansion coefficients which represent the stresses of the applied boundary forces. A difficulty in the present problem is the lack of a convergent Fourier development for the boundary stresses, since these become infinite under the loads. Nevertheless, the coefficients taken from this divergent series may legitimately be used to form a convergent series representing the stress distribution in the interior of the ring.

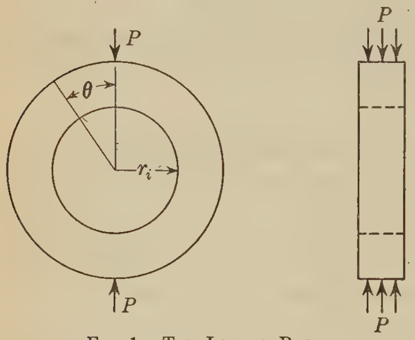

FIG. 1.—THE LOADED RING

NOTATION

The letter symbols in this paper are defined where they first appear, in the text or by line diagrams, and are assembled for convenience of reference in the Appendix.

ANALYSIS OF THE STRESSES

An arbitrary, symmetrical, normal loading of the outer boundary of the circular ring (see Fig. 1) may be described by the boundary conditions: For $r = r_o$—

$$\sigma_r = \frac{a_o}{2} + \sum_{n=1,2,3,\ldots}^{\infty} a_n \cos(n\,\theta) \qquad \text{(1a)}$$

for $r = r_i$—

$$\sigma_r = 0 \qquad \text{(1b)}$$

and, for $r = r_i$ and $r = r_o$—

$$\tau_{r\theta} = 0 \qquad \text{(1c)}$$

in which r_i is the inner radius of ring; r_o is the outer radius of ring; r and θ are plane polar coordinates; σ_r denotes the stress normal to the boundary (radial stress); and $\tau_{r\theta}$ is the shearing (tangential) stress. The compressive loading shown in Fig. 1 must be considered negative. Eq. 1a is the Fourier expansion for the stress distribution of the load, in which[9,10]

$$a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} \sigma_r \, d\theta \qquad \text{(2a)}$$

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[6] "Probleme der Spannungsverteilung in ebenen Systemen, einfach gelost mit hilfe der Airyschen Funktion," by A. Timpe, *Zeitschrift für Mathematik und Physik*, Vol. 52, 1905, pp. 348–383.

[7] "The Stresses in a Circular Ring," by L. N. G. Filon, *Selected Papers*, Inst. C. E., London, 1924.

[8] "A Treatise on Photoelasticity," by E. G. Coker and L. N. G. Filon, Univ. Press, Cambridge, England, 1931, pp. 375–378.

[9] "Fourier Series and Boundary Value Problems," by R. V. Churchill, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1941.

[10] "An Elementary Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics," by William E. Byerly, Ginn & Co., Boston, Mass., 1893

and

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \sigma_r \cos n\,\theta\, d\theta \ldots\ldots\ldots\ldots (2b)$$

R. V. Churchill[9] has published a mathematical proof of these relations as well as a general treatment of the Fourier series. The presence of concentrated, normal, line loads of magnitude P at $\theta = 0$ and $\theta = \pi$, in Eqs. 2, is characterized by the boundary stress functions: For $\theta = 0$ and $\theta = \pi$—

$$\sigma_r = \infty \ldots\ldots\ldots\ldots (3a)$$

and, for $\theta \neq 0$ and $\theta \neq \pi$—

$$\sigma_r = 0 \ldots\ldots\ldots\ldots (3b)$$

such that, for $-\frac{\pi}{2} \leqq \theta \leqq 0$—

$$r_o \int_{-0.5\pi}^{\theta} \sigma_r\, d\theta = 0 \ldots\ldots\ldots\ldots (4a)$$

and, for $0 < \theta \leqq \frac{\pi}{2}$—

$$r_o \int_{-0.5\pi}^{\theta} \sigma_r\, d\theta = P \ldots\ldots\ldots\ldots (4b)$$

The Fourier coefficients a_n of σ_r as defined by Eqs. 2 are found by integrating these equations by parts, using Eqs. 3 and 4, after substituting $\theta = 2\,\phi$, because of the symmetry:

$$a_n = \frac{2\,P}{\pi\, r_o}; \quad n = 0, 2, 4, \cdots \ldots\ldots\ldots\ldots (5a)$$

and

$$a_n = 0; \quad n = 1, 3, 5, \cdots \ldots\ldots\ldots\ldots (5b)$$

However, the Fourier series formed with these coefficients is divergent, since the coefficients a_n do not tend to become zero as n approaches infinity.

The general stress potential $\phi(r,\theta)$ for a circular ring under symmetrical loading may be written in the form:[7,8,11]

$$\phi = r^2_o \left[A_o \left(\frac{r}{r_o}\right)^2 + B_o \log\left(\frac{r}{r_o}\right)\right] \frac{a_o}{2} + r^2_o \sum_{n=2}^{\infty} \left[A_n \left(\frac{r}{r_o}\right)^{n+2} + B_n \left(\frac{r}{r_o}\right)^{n} + C_n \left(\frac{r_i}{r}\right)^{n} + D_n \left(\frac{r_i}{r}\right)^{n-2} \right] n\, a_n \cos\,(n\,\theta) \ldots\ldots\ldots\ldots (6)$$

The arbitrary constants A, B, C, and D appearing in this expression are evaluated by determining the stresses from the relations:[12]

$$\sigma_r = \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} \ldots\ldots\ldots\ldots (7a)$$

$$\sigma_\theta = \frac{\partial^2\phi}{\partial^2 r} \ldots\ldots\ldots\ldots (7b)$$

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[11] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London 1st Ed., 1934, p. 114.

[12] *Ibid.*, p. 53.

and

$$\tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right) \tag{7c}$$

—applying the boundary conditions Eqs. 1, solving the resultant simultaneous equations, and substituting the diameter ratio, $\frac{r_i}{r_o} = \bar{r}$. The constants thus derived are:

$$A_o = \frac{1}{2\,(1 - \bar{r}^2)} \tag{8a}$$

and

$$B_o = -\frac{\bar{r}^2}{1 - \bar{r}^2} \tag{8b}$$

Also,

$$A_n = \frac{1}{2\,R_n}\left(\frac{1 - \bar{r}^{2n}}{n} - \frac{1 - \bar{r}^{2n-2}}{n+1}\right) \tag{9a}$$

$$B_n = \frac{1}{2\,R_n}\left(\frac{1 - \bar{r}^{2n}}{n} - \frac{1 - \bar{r}^{2n+2}}{n-1}\right) \tag{9b}$$

$$C_n = \frac{\bar{r}^n}{2\,R_n}\left(-\frac{1 - \bar{r}^{2n}}{n} + \frac{1 - \bar{r}^{2n-2}}{n+1}\,\bar{r}^2\right) \tag{9c}$$

$$D_n = \frac{\bar{r}^n}{2\,R_n}\left(-\frac{1 - \bar{r}^{2n}}{n}\,\bar{r}^2 + \frac{1 - \bar{r}^{2n+2}}{n-1}\right) \tag{9d}$$

In Eqs. 8 and 9,

$$R_n = (1 - \bar{r}^{2n})^2 - n^2\bar{r}^{2n-2}\,(1 - \bar{r}^2)^2 \tag{10}$$

The series, Eq. 6, and hence the expressions for the stresses, Eqs. 7, require explicitly the knowledge of the Fourier coefficients a_n of the boundary stresses. Substituting Eqs. 5 in Eq. 6, and differentiating Eqs. 7, the following expression is derived for the stress σ_θ normal to the critical section $\theta = 0$:

$$\sigma_\theta = \left.\frac{\partial^2\phi}{\partial r^2}\right]_{\theta=0} = \frac{P\,K}{\pi\,r_o} \tag{11}$$

in which

$$K = 2\,(A_o + 4\,B_2) - B_o\left(\frac{r_o}{r}\right)^2 + \sum_{n=2,4,\ldots}^{\infty}\left[P_n\left(\frac{r}{r_o}\right)^n + Q_n\left(\frac{r_i}{r}\right)^{n+2}\right] \tag{12}$$

In Eq. 12, the coefficients P_n and Q_n are defined as:

$$P_n = 2\,n\,(n+1)\,(n+2)\,A_n + 2\,(n+1)\,(n+2)^2\,B_{n+2};\ n = 2, 4, \cdots \tag{13a}$$

and

$$Q_n = \left(\frac{2}{\bar{r}^2}\right)[n^2\,(n+1)\,C_n + n\,(n+1)\,(n+2)\,D_{n+2}];\ n = 2, 4, \cdots \tag{13b}$$

The constants $A_n, \cdots, D_n$ are defined by Eqs. 9. The power series, Eq. 12, however, converges for all values of r such that $r_i < r < r_o$ because P_n and Q_n

remain less than some fixed upper limit b as n approaches infinity; hence,

$$\sum \left[P_n \left(\frac{r}{r_o} \right)^n + Q_n \left(\frac{r_i}{r} \right)^{n+2} \right] \leqq b \sum \left(\frac{r}{r_o} \right)^n + b \sum \left(\frac{r_i}{r} \right)^{n+2} \quad \ldots .(14)$$

Since the right-hand member of Eq. 14 represents a geometric series with a ratio less than unity, it converges, as does the left-hand member by the comparison test. (For a discussion of the properties of an infinite series refer to the work of I. S. Sokolnikoff and E. S. Sokolnikoff.[13])

Even though Eq. 14 was derived from a nonconverging Fourier development, it represents, correctly, the stress in the interior of the ring because the "singular" stress function σ_r, as given by Eqs. 3, can be approximated by an arbitrarily close continuous distribution $\sigma^k{}_r$ (and hence a distribution whose Fourier expansion converges), with Fourier coefficients $a^k{}_n$. (For example,

$$\sigma^k{}_r = \sum_n \frac{2\,P}{\pi} \frac{\sin \dfrac{n}{k}}{\dfrac{n}{k}} \cos (n\,\theta); \quad n = 2, 4, \cdots \ldots\ldots\ldots\ldots (15)$$

which approaches σ_r as k approaches ∞, and as $a^k{}_n$ approaches $\dfrac{2\,P}{\pi}$.) The representation in the interior (Eqs. 10 and 11) for this distribution (denoted by $\sigma^k{}_\theta$) will be legitimate—that is, symbolically,

$$\sigma^k{}_\theta\,(r) = F[a^k{}_n] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which the brackets denote simply that the stress varies in some explicit manner with $a^k{}_n$. In any closed subinterval $(r_i + \epsilon) \leqq r \leqq (r_o - \epsilon)$, for arbitrarily small ϵ, the power series is uniformly convergent and hence varies continuously with its coefficients,[13] in other words, as $a^k{}_n$ approaches a_n—

$$\underset{k \to \infty}{\text{Limit}}\; \sigma^k{}_\theta = \underset{k \to \infty}{\text{limit}}\; F\,[a^k{}_n] = F\,[\text{limit}\; a^k{}_n] = F\,[a_n] = \sigma_\theta \ldots\ldots (17)$$

which is a restatement of Eqs. 11 and 12.

TABLE 1.—CONCENTRATION FACTOR K; $\left(\sigma_\theta = \dfrac{P\,K}{\pi\, r_o} \right)$

$\bar{r} = \frac{r_i}{r_o}$	VALUES OF r/r_o									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	−6.3847	−1.1957	−1.0136	− 0.9936	− 0.9800	−0.9602	−0.9335	−0.9003	−0.8612	−0.8165
0.2	....	−7.5984	−2.1215	− 1.3083	− 1.0555	−0.9004	−0.7546	−0.5961	−0.4181	−0.2185
0.3	....	....	−9.8565	− 3.4642	− 1.8118	−1.0751	−0.5572	−0.0762	+0.4273	+0.9764
0.4	....	....	....	−13.6820	− 5.3312	−2.3541	−0.6877	+0.6248	+1.8859	+3.2145
0.5	....	....	....	....	−20.3107	−7.9905	−2.4851	+1.1668	+4.3350	+7.5288

The condition that r shall remain in the interior of the ring is an essential requirement in the foregoing proof. If $r = r_o$ is substituted in Eq. 12, the

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[13] "Higher Mathematics for Engineers and Physicists," by Ivan S. Sokolnikoff and Elizabeth S. Sokolnikoff, McGraw-Hill Book Co., Inc., New York and London, 2d Ed., 1941.

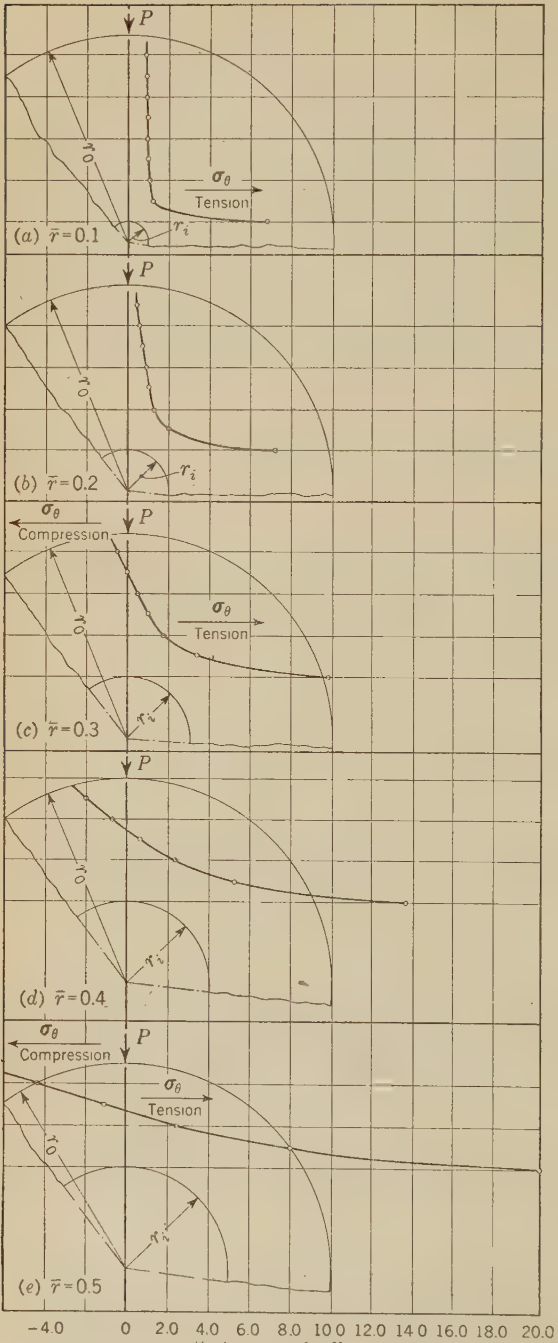

FIG. 2.—STRESSES NORMAL TO SECTIONS AT $\theta = 0$ AND $\theta = \pi$

series remains convergent, nevertheless. The value so obtained, however, is not the actual stress under the load (this stress is infinite, as discussed subsequently herein, under "Discussion of Wedge Action"[14]) but should be interpreted as the limiting value for $r = r_o - \epsilon$ as ϵ tends to become zero.

Numerical values or curves of the critical stress $\sigma_\theta]_{\theta=0}$ are most conveniently expressed in terms of the "concentration factor" K, defined by Eq. 12, from which the critical stress is obtained by using Eq. 11. Curves of K expressed as a function of r for various values of r_i/r_o have been plotted in Fig. 2. The segments of a loaded ring on which the stress distributions are shown for each of the five values of $\bar{r} = r_i/r_o$ are intended to show precisely how the stress varies along the critical plane, the edge of which appears as the vertical axis of the plot. The typical ring diagram in Fig. 3 depicts the infinitesimal "stress rectangle" showing the directions of the principal stresses. The numerical values of K in Table 1 were computed to five places, carefully checked by duplication and by using various relations, and then rounded to four places. In order to use the tables, or the curves, to obtain the stress at a distance r from the center of a given ring, it is necessary first to select the proper value of K in Table 1, and then to multiply by a constant as given by Eq. 11.

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[14] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1934, p. 96.

Thus, as an example, suppose the ring test is applied to a concrete specimen 2 in. thick having the outer radius $r_o = 10$ in. and the inner radius $r_i = 1$ in. If the applied load is 1,000 lb, what is the value of the stress σ_θ along the critical section 4 in. from the center? In this example, $\bar{r} = 0.1$ and $r/r_o = 0.4$, and Table 1 then gives $K = -0.99$ approximately. Since the load per unit thickness, P, equals -500 lb, Eq. 11 gives $\sigma_\theta = \dfrac{-500 \times -0.99}{3.14 \times 10} = 16$ lb per sq in.

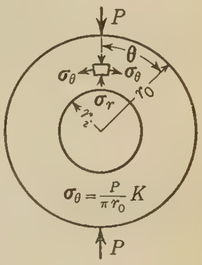

Fig. 3

Examination of the curves of Fig. 2 or of the data in Table 1 shows that the maximum tensile stress in the critical section occurs at the inner boundary $r = r_i$. This fact is most interesting because at this point failure is to be expected in brittle materials such as rocks or concrete, which are strong in compression and relatively weak in tension. Consequently, Fig. 4, showing the concentration factor K at the point $r = r_i$ as a function of $\bar{r}$, has been prepared to permit finding K for values of $\bar{r}$ other than multiples of 0.1. By reading this factor from the curve it is then possible to compute the "failure" stress of rings with any ratio of diameters between 0.0 and 0.5 by substituting the ultimate failure load per unit thickness, for P in Eq. 11. Thus, in the foregoing example, K for $r = r_i$ is -6.4; and, if failure of the specimen occurs at 6,000 lb, the failure stress in tension is

$$\sigma_\theta = \frac{3{,}000 \times -6.4}{3.14 \times 10} = 610 \text{ lb per sq in.}$$

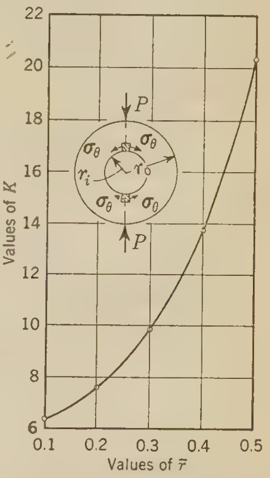

Fig. 4.—Stress Concentration Factor K at $r = r_i$, Shown as a Function of $\bar{r}$ (P Is the Load per Unit Thickness of Ring)

Discussion of Wedge Action

It is certainly apparent from inspection of the stress distribution curves of Fig. 2 that the resultant force over the critical section is not zero as should be required for equilibrium. As a matter of fact, the resultant force is P/π for each diameter ratio. This can be proved from the relation,

$$F = \int_{r_i}^{r_o} \sigma_\theta \, dr = \frac{\partial \phi}{\partial r}\bigg]_r^{r_o} = \frac{P}{\pi} \quad \ldots\ldots (18)$$

in which ϕ is the stress potential given by Eq. 6. This force is canceled, however, by an equal and opposite tangential force arising from the infinite stress, previously mentioned, directly under the load. The effect of a concentrated load in producing a concentrated force at right angles has been discussed by various writers[14] and is usually referred to as "wedge action."

Also, as the diameter ratio $\bar{r}$ $(= r_i/r_o)$ of the ring increases, the stress distribution more nearly approximates that expected from the linear bending theory of curved bars. On the other hand, for the smaller values of r_i, the stress is practically uniform over the cross section except for the singularity at the load and the high local stress concentration at the inner boundary.

It is of interest to consider the limiting case in which the inner radius, r_i, of the ring is allowed to approach zero, giving in effect a circular disk. If $r = r_i$ in Eq. 12 and then if r_i approaches zero:

$$\lim_{r_i \to 0} \sigma_\theta \Big]_{r = r_i} = \frac{6\,P}{\pi\, r_o} \qquad \text{(19)}$$

On the other hand, if $r_i = 0$ is used directly in Eq. 12, the stress distribution across the critical section of a circular disk under diametral loads is obtained directly. The result is a uniform tension with a magnitude of

$$\sigma_\theta = \frac{P}{\pi\, r_o} \qquad \text{(20)}$$

This result agrees with previous studies of this problem; and, if an infinitesimally small hole is made at the center of a circular disk of radius r_o under concentrated diametral loads, the stress at the hole normal to the load axis passes discontinuously from the value $\dfrac{P}{\pi\, r_o}$ to $\dfrac{6\,P}{\pi\, r_o}$.

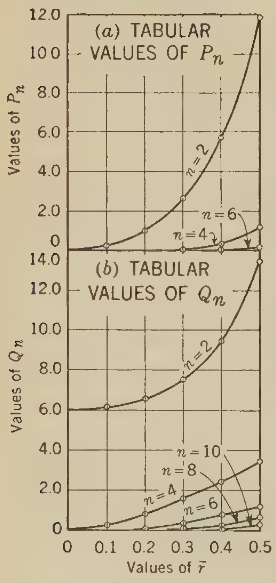

FIG. 5.—CURVES OF EQS. 12

Fig. 5 shows the coefficients P_n and Q_n, Eq. 12, plotted as functions of $\bar{r}$ and r. These curves illustrate the rapidity with which these functions converge and also provide a convenient method for extending the stress computations to intermediate values of $\bar{r}$. For more accurate work, the values in Table 2 may be used. These values were computed only to five places.

The singularity under the load quite naturally raises the question why, in spite of the infinite value of the stress at that point, a finite stress at $r = r_i$ should cause failure. One explanation that is advanced is that a slight plastic deformation occurs under the load which causes such a redistribution of load that the infinite stress over an infinitesimal area is changed to a finite stress over a finite area. If the material is as strong in compression as, say, concrete, such a redistribution would not materially affect the stress at an appreciable distance from the load. In actual tests, almost invariably, the failure begins at $r = r_i$ in the plane of the load and progresses along the line of the load to the outer boundary. To show that failure begins at the inner boundary a number of tests were stopped and the load was removed in time to halt the progression of failure, leaving the ring cracked from the inner boundary to a point near the outer boundary but still in one piece.

The ring test should be used with caution for concrete specimens because of the tendency of concrete to creep under tensile stress. Creep or plastic flow in the region of the high tensile stress concentration at the inner boundary may cause an appreciable redistribution of stress which, in turn, will make these methods of interpreting results erroneous. If the test is used for concrete specimens, the inner diameter should be large relative to the size of the aggregate, to reduce the effects of the local conditions of the concrete at the inner boundary.

TABLE 2.—TABULAR VALUES OF COEFFICIENTS P_n AND Q_n, EQS. 12

n	(a) COEFFICIENTS P_n					(b) COEFFICIENTS Q_n				
	$\bar{r}=0.1$	$\bar{r}=0.2$	$\bar{r}=0.3$	$\bar{r}=0.4$	$\bar{r}=0.5$	$\bar{r}=0.1$	$\bar{r}=0.2$	$\bar{r}=0.3$	$\bar{r}=0.4$	$\bar{r}=0.5$
2	0.24605	1.05981	2.70099	5.78192	11.85873	6.12487	6.56143	7.52293	9.48041	13.48380
4	0.00012	0.00713	0.07391	0.36549	1.21914	0.19602	0.73801	1.50703	2.39535	3.48053
6	....	0.00003	0.00159	0.02474	0.18093	0.00412	0.06193	0.28178	0.76115	1.51782
8	....	....	0.00003	0.00140	0.02361	0.00007	0.00425	0.04347	0.20812	0.63471
10	....	....	....	0.00007	0.00271	....	0.00026	0.00598	0.05087	0.24177
12	....	....	....	....	0.00027	....	0.00001	0.00076	0.01154	0.08570
14	....	....	....	....	....	....	....	0.00009	0.00249	0.02884
16	....	....	....	....	....	....	....	....	0.00052	0.00934
18	....	....	....	....	....	....	....	....	0.00010	0.00294
20	....	....	....	....	....	....	....	....	....	0.00090
22	....	....	....	....	....	....	....	....	....	0.00027

The plane disk may have an advantage in that the stress on the section through the plane of the load is uniform. Nevertheless, the plane disk is not satisfactory as a test specimen because there is no point in the critical plane where one or the other of the principal stresses becomes zero, as is desirable in pure tension tests.

APPENDIX. NOTATION

The following symbols, used in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = an arbitrary mathematical coefficient, with appropriate subscript, as required (also B, C, and D);
a = a coefficient in the Fourier series; a_o and a_n in Eqs. 2;
b = a fixed upper limit for the coefficients P_n and Q_n in Eq. 12 (see Eq. 14);
F = resultant force;
K = a concentration factor;

k = a parameter, or index, in Eq. 15;
n = a number;
P = a concentrated ultimate failure load, per unit of ring thickness;
P_n = coefficient in Eq. 12;
Q_n = coefficient in Eq. 12;
R = a substitution constant (see Eq. 10);
r = radius; a polar coordinate:
    r_i = inner radius of ring;
    r_o = outer radius of ring;
    $\bar{r} = r_i/r_o$ = diameter ratio;
ϵ = increment of radius;
θ = angular distance; a polar coordinate;
σ = unit stress:
    σ_r = radial stress;
    σ_θ = tangential stress;
τ = shear stress; $\tau_{r\theta}$ = tangential shear stress; and
ϕ = a function of r and θ (see Eq. 6); stress potential.

DISCUSSION

Robert R. Philippe[15] and Frank M. Mellinger,[16] Assoc. Members, ASCE.—The development, by Messrs. Ripperger and Davids, of L. N. G. Filon's formal solution to evaluate the critical stresses in circular rings is an excellent contribution, particularly in the testing of brittle materials. The original application of the "ring test" is attributed to Max M. Frocht of the Carnegie Institute of Technology, Pittsburgh, Pa., who had used this type of test to measure the tensile strength of bakelite. Inasmuch as the shape of specimen required is easily cut and drilled from cylindrical cores, the test was adapted to the testing of concrete and foundation rock in connection with the design of concrete dams. More recently the test has been applied with promising results to the evaluation and correlation of aggregate quarry cores. However, this discussion will be limited to the application of the ring test in determining the physical properties of the more massive types of foundation rocks.

If the tensile strength of a brittle material is determined by a ring test and its compressive strength is measured by an unconfined compression test, these two points will serve to define O. Mohr's[17] envelope of rupture for perfectly brittle material (Fig. 6). This envelope can be expressed as Coulomb's equation

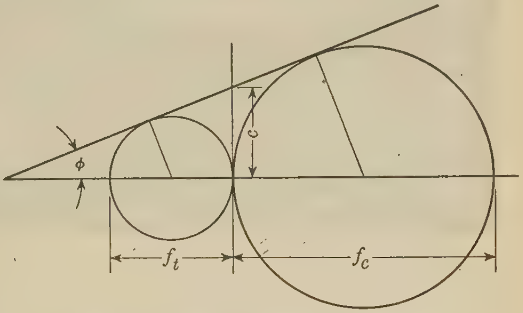

Fig. 6.—Mohr's Envelope of Rupture

$$\tau = c + p \tan \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which τ is the unit shearing strength; c is the apparent unit cohesion; p is the unit normal load; and ϕ is the angle of internal friction.

By examination, the following relationship of ϕ and c results in terms of the tensile strength f_t and unconfined compressive strength f_c.

$$c = \frac{f_t}{2 \cos \phi} (1 + \sin \phi) \ldots\ldots\ldots\ldots\ldots\ldots (22a)$$

and

$$\phi = \text{arc} \sin \frac{f_c - f_t}{f_c + f_t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

The application of the ring test for measuring the tensile strength of rock does make use of the assumption (upon which the authors' computations are based) that the tested material is truly elastic. Within the limitations of this assumption a ring specimen prepared and tested as described in this paper will yield the tensile strength of a brittle material by applying an adaptation of Eq. 20:

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[15] Prin. Engr., Ohio River Div. Laboratories, Mariemont, Ohio.

[16] Senior Engr., Ohio River Div. Laboratories, Mariemont, Ohio.

[17] "Abkandlungen aus dem Gebiete der Technischen Mechanik," by O. Mohr, W. Ernst, Berlin, 1914.

$$f_t = \sigma_\theta = \frac{P}{\pi r_0} K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Photographs of nominal rock specimens 3 in. and 6 in. in diameter which have been so tested are shown in Fig. 7. The regularity of the vertical break; the uniformity of results from two specimens, each cut from the ends of a compression specimen; the sensitive result; and the plausibility of over-all results—at least make this test useful for comparative purposes.

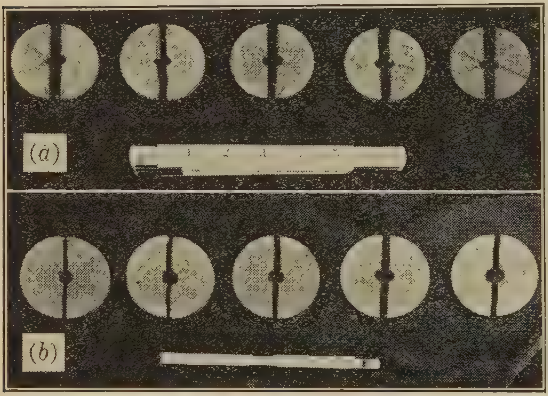

FIG. 7

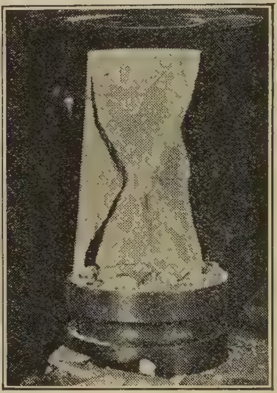

FIG. 8

The compressive strength f_c is measured by a simple, unconfined, compressive test on the rock. A specimen is prepared by cutting a rock core into a length equal to about twice the diameter, then lapping the end surfaces until they are truly perpendicular to the axis of the core. Extreme care in preparation is needed for good results, an outstanding example of which is shown in Fig. 8. The sharpness of the break shown on this photograph suggests another means of measuring the angle of internal friction. The angles of rupture (θ) according to J. B. Johnson[18] should bear the following relationship to the angle of internal friction (ϕ):

$$\phi = 2\,(\theta - 45°) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Generally the angles found in this manner are from 2° to 5° greater than those found by the measurement of f_t and f_c. In the case of the Sutton sandstone, illustrated in Fig. 8, the angle of fracture varied between 68° and 70°. By Eq. 24 the angle of internal friction is computed to be between 46° and 50°, as against the value of 46° determined by the measurement of f_t and f_c.

Table 3 lists some dam sites at which foundation investigations have yielded 3-in. and 6-in. rock cores that have been tested by the measurement of f_t using the ring test and f_c. The results are encouraging in so far as they are consistent with the classification and the relative appearance of these rocks. In addition there is a degree of correlation between the type of rock, its void ratio, and its physical properties as revealed by this method.

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[18] "The Materials of Construction," by J. B. Johnson, John Wiley & Sons, Inc., New York, N. Y., 1898, p. 25.

TABLE 3.—TENSILE STRENGTH, WITH COMPUTED VALUES OF THE ANGLE OF INTERNAL FRICTION ϕ AND THE COHESION COEFFICIENT c, FOUNDATION ROCK FROM SELECTED DAM SITES

Dam site	Type of foundation rock	Void ratio[a]	POUNDS PER SQUARE INCH			Internal friction ϕ (degrees)
			σ_e	σ_t	c	
Mining City, Ky.	Sandstone	0.258	3,050	762	762	37
Mining City, Ky.	Sandstone	0.135	6,150	1,770	1,665	34
Sutton, W. Va.	Hard gray sandstone	0.077	9,470	1,537	1,907	46
Berlin, Ohio	Brown, fine-grained sandstone	0.184	7,280	1,244	1,500	45
Berlin, Ohio	Gray, medium-grained, friable sandstone	0.232	5,720	1,116	1,268	42
Center Hill, Tenn.	Hard, fossiliferous limestone	0.019	8,326	3,540	2,715	23
Wolf Creek, Ky.	Hard, gray limestone	0.013	13,100	5,090	4,082	26
Falmouth, Ky.	Medium, crystalline, very fossiliferous limestone	0.019	15,200	4,280	4,030	34
Buggs Island, Va.	Gray, granatoid gneiss	....	18,607	5,950	5,271	31
Conemaugh, Pa.	Sandy, medium hard siltstone	0.036	5,920	2,215	1,811	27
Allatoona, Ga.	Dolomite	0.005	25,400	5,530	5,930	40
Allatoona, Ga.	Quartzite	0.007	21,300	5,410	5,366	37

[a] Ratio of the volume of voids to the volume of the solids.

Further means of checking this procedure can be obtained by conducting triaxial tests on cylindrical specimens. Steps in that direction are already being taken by the staff of the Ohio River Division Laboratories. However, the triaxial test requires apparatus and technique far too complicated for normal use; it is anticipated that its greatest value will be in obtaining measurements of stress and strain on the more critical specimens revealed by the ring and compression test results.

The need for a dam in some general locality is not often fixed by the quality of its foundation. The increasing size of dams, the complication of types of concrete sections, and the price competition of earth dams are making it more necessary to take advantage of all there is to know of the physical properties of the materials of construction. It is believed that the ring test for tensile strength determinations will be a very useful means of judging these properties.

E. P. POPOV,[19] JUN. ASCE.—The problem of determining the critical stresses in a circular ring, subjected to two concentrated opposing radial forces on the outer boundary, is not a new one. The solution is of practical importance in certain bearing and machine design problems as well as in the case cited by Messrs. Ripperger and Davids.

Since the series used by the authors to obtain a solution is shown to be nonconvergent in expressing the radial boundary stresses under the concentrated loads, the procedure may appear questionable. The use of this nonconvergent series can be avoided entirely by a method first suggested by S. Timoshenko.[4,20] Such a method was further recommended by L. N. G. Filon.[7] The Timoshenko method utilizes a known finite solution for two equal and opposite concentrated radial forces applied on the outer boundary of a solid cylinder. Then a concentric hole, equal to the bore of the hollow cylinder, is cut out of such a solid cylinder. Normal and shearing forces exist as distributed forces around the edge of the hole to maintain the state of stress in the remaining hollow cylinder equivalent to that in the original solid one. These edge forces can be expressed by convergent series. Application of equal and opposite forces around the edge of the hole can then be used for the determination of the constants of integration in the stress function[21] in terms of these boundary stresses. The resulting stresses are superposed on the stresses obtained from the finite solution of a solid cylinder to obtain true stress in a hollow cylinder. Displacements can be treated in a similar manner.

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[19] Asst. Prof., Civ. Eng., Univ. of California, Berkeley, Calif.

The special case of this problem by the foregoing method for $\bar{r} = 0.5$, as noted, was solved by Professor Timoshenko,[4,20] and extended further by his students. The case of $\bar{r} = \frac{1}{3}$ was solved by V. Billevicz,[22] and a general solution for any value of $\bar{r}$ was obtained by J. Maulbetsch, as reported by C. W. Nelson.[23]

The foregoing method involves the use of certain series. The radial, tangential, and shear stresses for a solid circular cylinder in the series form are shown[23] to be:

$$\sigma_r = \frac{2P}{\pi r_o}\left[-\tfrac{1}{2} + \cos 2\theta - \left(2\frac{r^2}{r^2_o} - \frac{r^4}{r^4_o}\right)\cos 4\theta + \left(3\frac{r^4}{r^4_o} - 2\frac{r^6}{r^6_o}\right)\cos 6\theta \cdots\right] \quad (25a)$$

$$\sigma_\theta = \frac{2P}{\pi r_o}\left[-\tfrac{1}{2} + \left(2\frac{r^2}{r^2_o} - 1\right)\cos 2\theta - \left(3\frac{r^4}{r^4_o} - 2\frac{r^2}{r^2_o}\right)\cos 4\theta \cdots\right] \quad (25b)$$

and

$$\tau_{r\theta} = -\frac{2P}{\pi r_o}\left[\left(1 - \frac{r^2}{r^2_o}\right)\sin 2\theta - 2\left(\frac{r^2}{r^2_o} - \frac{r^4}{r^4_o}\right)\sin 4\theta + 3\left(\frac{r^4}{r^4_o} - \frac{r^6}{r^6_o}\right)\sin 6\theta \cdots\right] \quad (25c)$$

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[20] "Stresses in a Ring Compressed by Two Forces Acting Along a Diameter," by S. Timoshenko, *Bulletin*, Kiev Polytechnic Inst., 1910 (in Russian).

[21] "On the Direct Determination of Stress in an Elastic Solid, with Application to the Theory of Plates," by J. H. Michell, *Proceedings*, London Mathematical Soc., Vol. 31, 1899, p. 111.

[22] "Analysis of Stresses in Circular Rings," by V. Billevicz, thesis presented to the Univ. of Michigan at Ann Arbor in 1931 in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[23] "Stresses and Displacements in a Hollow Circular Cylinder," by C. W. Nelson, thesis presented to the Univ. of Michigan at Ann Arbor in 1939 in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

These series are convergent and with a desired value of radius r can be used as the boundary stresses at the bore. In these series the angle θ is measured from a horizontal axis to the right of the origin. The loads are applied vertically.

For the derivation of Eqs. 25, and of other formulas pertaining to the analytical solution of the hollow circular cylinder, the readers are referred to Mr. Nelson's dissertation,[23] in which the problem is fully treated. However, for the purposes of comparison with the paper by Messrs. Ripperger and Davids, certain critical values compiled by Mr. Nelson are quoted. Perhaps the most significant points of stress on the hollow cylinder are those marked 1, 2, 3, and 4 in Fig. 9. The stress concentration factors for the tangential stresses corresponding to these points are shown in Table 4 for the different diameter ratios. The factors of stress concentration for the maximum tensile stresses are given through and beyond the diameter ratios presented by the authors in Table 1. Table 4 also shows some other values for additional points that may be of interest. The comparable points of stress are in complete agreement with the results obtained by Messrs. Ripperger and Davids, which is reassuring. In connection with Table 4, it may be worthy to note that, in these elastic analyses, only small deformations are considered, and thus a solution for a case such as $\bar{r} = 0.9$ should be questioned.

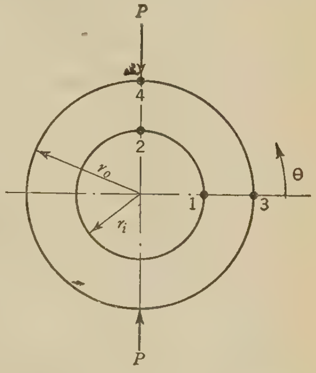

FIG. 9.—THE LOADED CIRCULAR CYLINDER

TABLE 4.—CONCENTRATION FACTOR K; $\left(\sigma_\theta = \frac{P\,K}{\pi\,r_o}\right)$

$\bar{r} = \frac{r_i}{r_o}$	Points (See Fig. 9)			
	1	2	3	4
0.9	353.80	−581.59	−299.42	512.22
0.8	92.07	−140.72	− 64.65	106.73
0.7	42.99	− 60.45	− 24.42	38.25
0.6	25.720	− 32.838	− 11.411	16.536
0.5	17.798	− 20.311	− 5.857	7.529
0.4	13.702	− 13.682	− 3.063	3.214
1/3	12.121	− 10.917	− 1.931	1.573
0.3	11.563	− 9.856	− 1.503	0.976
0.2	10.531	− 7.598	− 0.607	−0.218
0.1	10.108	− 6.385	− 0.143	−0.816

The solution devoelped by the authors, as well as the one used for obtaining the results shown herein, treats the problem of a hollow cylinder as a two-dimensional problem. In practice, including the case of foundation rock samples, the cylinder is of finite length, compressed between the flat surfaces of two larger bodies. The latter is not a two-dimensional problem. Actually the load distribution along the length of the load is not uniform.

Theoretically, considering the pressures between two bodies in contact, Mr. Nelson[23] shows that the load distribution along the length of the cylinder is approximately uniform except near the ends of the areas of contact, provided the width of actual contact is small compared to the length of the cylinder.[24] Since such requirements are usually met, these solutions are adequate.

The theoretical results are further verified by photoelastic tests briefly reported by O. J. Horger.[25] More details pertaining to these tests may be of interest and some are cited,[23] for comparison, in Table 5. The agreement between the theoretical and experimental results is seen to be excellent.

TABLE 5—COMPARISON OF STRESSES FOR POINT 2 IN FIG. 9

DIAMETERS, IN INCHES		Ratio of Col. 1 / Col. 2	Load P (lb per in. of length)	TANGENTIAL STRESS (Lb per Sq In.)		Ratio of Col. 5 / Col. 6
$2r_i$	$2r_o$			Photoelastic	Computed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.496	2.374	0.104	892	1,554	1,530	1.015
0.496	2.374	0.104	1,135	1,954	1,950	1.00
0.500	2.374	0.210	515	1,080	1,100	0.99
0.505	2.374	0.212	892	1,820	1,870	0.97
0.505	2.374	0.212	1,135	2,330	2,380	0.98
0.752	2.375	0.316	515	1,465	1,435	1.02
0.752	2.375	0.316	892	2,510	2,490	1.01
0.752	2.375	0.316	1,135	3,220	3,170	1.01
1.465	2.375	0.617	204	1,940	1,905	1.02
1.465	2.375	0.617	354	3,430	3,305	1.04
1.465	2.375	0.617	515	4,830	4,800	1.01

The writer is happy to offer these comments from a source not generally available as a verification and extension of the results presented by the authors.

E. A. RIPPERGER,[26] ESQ., AND N. DAVIDS,[27] ESQ.—Messrs. Philippe and Mellinger have described one practical application involving the use of the theoretical calculations contained in this paper, and, as Mr. Popov has suggested, there are no doubt many others. The wide variations in the nature of the stress distribution at the so-called critical section, as illustrated in Fig. 2, make it necessary, however, to use a great deal of caution in applying these theoretical results to the calculation of failure stresses for even relatively brittle materials. To illustrate and emphasize this point the data in Table 6 have been compiled. These computed failure stresses for concrete rings having the dimensions shown were calculated through the use of Eq. 11.

Specimens were moist cured for 28 days before testing and the maximum size of the aggregate was approximately $\frac{3}{4}$ in. The apparent increase in tensile strength as the value of $\bar{r}$ decreases is believed to be due chiefly to the decrease in linearity of the stress distribution at the critical section as $\bar{r}$ becomes small. A small plastic yielding of the concrete at the critical point enables the ring with $\bar{r} = 0.1$ to withstand a much larger increase in the load (P) required to cause failure, than a similar deformation would cause in a ring with, say, $\bar{r} = 0.3$. It will also be noted that variations of calculated strength from specimen to specimen are greater for $\bar{r} = 0.1$ than they are for larger values of $\bar{r}$. This may be attributed to the fact that, for the case where $\bar{r} = 0.1$, the hole in the ring represents a discontinuity of the same order of magnitude as the discontinuities introduced by the larger aggregate particles.

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[24] "Elastische Beruehrung zweier Halbraeume," by G. Lundberg, *Forschung auf dem Gebiete des Ingenieurwesens*, September-October, 1939, pp. 201-211.

[25] "Photoelastic Analysis Practically Applied to Design Problems," by O. J. Horger, *Journal of Applied Physics*, July, 1938, pp. 457-465.

[26] Dept. of Eng. Mechanics, Univ. of Texas, Austin, Tex.

[27] Math. Dept., Johns Hopkins Univ., Baltimore, Md.

TABLE 6.—Computed Failure Stresses for Concrete Rings[a]

Dimensions, In.			Failure Stress, Pounds per Square Inch				
r_i (1)	r_o (2)	t[b] (3)	Specimen 1 (4)	Specimen 2 (5)	Specimen 3 (6)	Specimen 4 (7)	Average (8)
2	6	4	1,230	1,390	1,290	1,240	1,287
1	5	4	1,780	1,790	1,750	1,820	1,785
$\frac{1}{2}$	3	1	2,480	2,390	2,610	2,160	2,410
Compressive strength[c]			6,720	6,680	6,630	7,290	6,830

[a] Unpublished results of an investigation by I. Narrow, Ohio River Division Laboratories, Mariemont, Ohio.
[b] Thickness of ring. [c] Compressive strength, in pounds per square inch, of 6-in. by 12-in. cylinder.

Mr. Popov implies that the use of coefficients taken from the nonconvergent boundary stress series in the development of a convergent series to represent interior stresses is a questionable procedure which should have been avoided by the use of Professor Timoshenko's method. Although it is certainly true that nonconvergent series must be used with much caution in mathematical processes, the procedure followed in this paper is shown to be quite correct (see the discussion of that point and the continuity proof, Eqs. 14, 15, 16, and 17) and at least a part of any value this paper may have lies in that demonstration. Despite the general mistrust of nonconvergent series by mathematicians, several problems in mathematical physics have been correctly solved by such series.[28] The ring problem is another example.

The question as to whether or not a loaded circular ring actually represents a case of plane stress does not arise in theoretical treatment of the problem, for it is assumed that the load is distributed in such a way as to give a plane-stress condition. The question does arise, however, when any practical application of the calculated results is contemplated, and Mr. Popov is to be commended for having posed the question and then answered it.

The authors are indebted to Mr. Popov for having brought to their attention some excellent work done on this problem, of which they were unaware at the time of their own investigation.

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[28] "An Introduction to the Theory of Infinite Series," by T. J. I. Bromwich, Macmillan Co., London, p. 317.

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EXPRESS HIGHWAY PLANNING IN METROPOLITAN AREAS

By Joseph Barnett,[1] M. ASCE

With Discussion by Messrs. Harry W. Lochner, Fred Lavis, Homer M. Hadley, Donald M. Baker, W. J. Van London, Merrill D. Knight, Jr., R. H. Baldock, Jacob Feld, Harold M. Lewis, Theodore T. McCrosky, Spencer A. Snook, Lawrence S. Waterbury, Bernard L. Weiner, George H. Herrold, Ralph R. Leffler, M. Hirschthal, and Joseph Barnett.

Synopsis

Perhaps, the need for free-flowing highway facilities to relieve traffic congestion in cities is evident, but attention is focused on this need by the superior facilities available on arterial routes in rural areas as compared with those in urban areas. The factors that influence the locations of arterial routes in cities and their effect on the city plan are discussed. The need for obtaining factual data is emphasized and the pattern of arterial routes developed in representative cities is shown. Brief comment is included on the subjects of by-passes versus radial routes, the need for flexibility, the use of existing streets, and the terminal problem. Relative merits of different types of expressways are presented. The need for, and description of, preliminary engineering reports to unite the numerous interested agencies are detailed, and an economic analysis of each project is suggested.

Introduction

Modern development of highway transportation has resulted in large differences between rural and urban roads, expecially in regard to the capacity of such roads to serve the traffic using them. Many cities are approached by numerous good roads of adequate capacity in rural and suburban areas; but in cities themselves, where the traffic reaches its highest volumes and where, therefore, there is the greatest justification for facilities permitting uninterrupted flow, the stops necessitated by street intersections increase in frequency, the average speed of travel decreases, and inconvenience and cost of motor

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vehicle operation rise sharply. The streets affected have undergone a progressive change in function from predominantly serving land to similarly predominantly serving new and mounting arterial traffic flows. Such improvements as have been made have generally been designed to combine in one facility the service of both classes of traffic, the local and the arterial. As yet there have been few instances in which the desirable separate provision for arterial traffic has been attempted.

With the increase in motor vehicle travel, it was inevitable that this unbalanced provision of highway facilities in urban and rural areas would contribute to the decentralization of cities. Suburban housing developments naturally were near main radial routes out of the cities. The typical distribution of city population is created by many factors, and it is not the writer's intention to delve into them except to emphasize that the improvement of radial routes in the adjacent suburban and rural areas, without comparable improvements within the cities, was one of these factors. Most cities present the pattern of a central business or commercial district, an adjacent ring of dilapidated old buildings, generally old residences (either converted to commercial use or allowed to run down to substandard housing), a surrounding area of medium-grade or high-class housing of more than average density, and a creeping or fingerlike development—new, heterogeneous, and uncontrolled—along the radial routes leading out of the city.

In most instances the radial routes present a picture that is far from pretty. Unrestricted development along the roadsides has made entrances to cities via highways as unattractive as entrances via railroads. Instead of the industrial development that borders the railroads, the highways are fringed by a conglomeration of roadside businesses which, with their varied and blatant signs, vie with one another for the attention of travelers, detract from the sightliness of the city's gateway, and constitute a positive detriment to efficient highway transportation. To the credit of the railroads it may be stated that, except for grade crossings, which are gradually being improved in operation or eliminated, the roadbeds are as useful today for the movement of trains as when they were built. Contrariwise, on the average highway approaching a city, each new roadside business contributes its measure of interference, and the accumulation of such interferences steadily decreases the capacity of the road and its effectiveness in serving moving vehicles. A creeping obsolescence develops which is no fault of the highway engineer; it is rather the fault of antiquated laws and the pressure of real estate interests which make it difficult and, in some cases, impracticable to protect the traffic facility from roadside interference.

BY-PASSES

By-passes have frequently been constructed around cities on the theory that the adequacy of existing city streets would be improved by rerouting the cars of travelers who do not wish to stop in the city. Another justification for such by-passes, frequently cited, is the removal of heavy commerical vehicles from the main street, along which the through route usually is located. This expedient might offer not only the advantage of traffic relief but also the elimination of the noise and odor common to such commercial carriers. In many cases by-passes have proved to be useful adjuncts; and, indeed, they should be considered for many municipalities where traffic data indicate that the need exists and where physical conditions permit. On the other hand, the fallacy of planning by-passes as a general solution of the city traffic problem has been glaringly revealed by the state-wide planning surveys, and other studies, which have shown that the bulk of the traffic approaching the larger cities is destined for points within the city itself and, for that reason, cannot be by-passed. Commonly, less than 10% of the traffic approaching cities of more than 300,000 population is destined for points beyond the urban area. By-passable traffic is about 20% in cities with populations of from 10,000 to 300,000, and only in communities of 2,500 or less does the by-passable traffic normally reach 50% of the total traffic.

Radial Highway Arteries Are the Greatest Need

Numerous fact-finding agencies have brought many highway transportation problems into focus. As a result, it has become increasingly clear that so much of the total traffic is urban in character that the need for allocation of funds for the improvement of highway facilities in metropolitan areas can no longer be ignored or relegated to a position of secondary importance. It has become clear, also, that traffic congestion in cities will be solved to the greatest extent feasible by the construction of facilities that permit the uninterrupted flow of traffic into and through the cities. The appalling number of traffic accidents is continually in the minds of highway planners; and, since most accidents occur at intersections in cities and a frequent type involves pedestrians, it is inevitable that the accident rate will be reduced by providing express highways on which crossings at grade are eliminated and pedestrians are not in close proximity to moving vehicles. As a consequence, highway planners have been giving preferential attention to plans for express highway projects that will provide solutions to the serious problems of city traffic congestion.

The mounting interest in urban arterial roads has been due to three general factors—(1) Availability of factual data; (2) necessity for accumulation of plans; and (3) the consciousness that planning for congestion relief has been inadequate.

(*1*) *Factual Data.*—Factual data regarding the movement and the origin and destination of motor vehicles in and approaching cities have been collected by various organizations such as state highway departments, other state agencies, county and city engineering organizations, planning commissions, and consultants. As a rule, these data leave much to be desired. Many cities have fairly good flow maps of traffic. Many cities have origin and destination data taken at an external cordon of stations on the main radial approach roads. Few cities have comprehensive origin and destination data that cover important intracity travel. However, regardless of the variation in the quantity or kind, such data almost always indicate that the most useful arterial road would be located radially with respect to the central business district. Since adequate traffic data reveal many more important needs, administrators supplied with such data have the comforting realization that they can proceed with plans with reasonable assurance that their judgment is based on, and backed by, facts.

Of all the varied data bearing on the planning of expressways in cities those showing the origin and destination of all traffic—through, suburban, and intracity—are most important in determining preliminary location of the expressway itself and in determining both the locations of access connections and the volumes of interchange traffic for which the expressways should be designed. The methods heretofore used in making comprehensive origin-destination studies are costly, laborious, and time consuming; and it is small wonder that few cities have undertaken to obtain data by such methods. The Public Roads Administration, Federal Works Agency, with the assistance and advice of experts of the Bureau of the Census, U. S. Department of Commerce, has developed a method for obtaining origin-destination data which promises to reduce cost, labor, and time sharply and at the same time enables the planner to arrive at more complete, accurate, and useful data. The method, roughly, follows the small-sample interview method used by public opinion polls except that the questions deal with travel facts such as "Where did you go yesterday, how, and for what purpose?" The questions are asked of occupants of a small percentage of selected sample dwellings in an area such as a census tract or part of tract, and the data obtained are expanded in proportion to the total dwellings of that area. Such questioning can be made by paid interviewers as rapidly as needed. The answers are transferred to coded punch cards from which the numerous factual data are summarized. A check on accuracy can be made by measuring traffic volumes at certain control points, such as river crossings and major intersections. As a result of the splendid cooperation of the officials of the cities and of the state highway departments, this method has been used, or is being used, in about thirty-eight metropolitan areas, and the number of cities is growing rapidly. It is felt that the "bugs" in the method have been eliminated.

Sometimes origin-destination data of traffic approaching a metropolitan area at a cordon of stations will be sufficient for locating an expressway. Such data will include by-passable traffic and traffic having a destination, or making a stop, in the city but the important intracity travel is missed. Origin-destination data at each station of a cordon are obtained by interviewing the drivers of a small percentage of vehicles leaving the city and expanding the results to the total traffic volume measured at the same time.

(*2*) *Shelf of Plans.*—No small measure of interest in urban arterials is due to the necessity for providing a shelf of plans for the construction of needed public works so that the construction industry could be prepared to provide jobs as soon as necessary. Quite naturally, industries are geared to city areas, and whenever there is unemployment, the necessity for jobs will be most severe in cities.

(*3*) *Relief of Congestion.*—The most important factor creating interest in urban arterial highways is the realization that only a small "dent" has been made in the solution of the traffic problems in cities of the United States. It is a law of nature (and in many cases a fortunate one) that people tend to forget past troubles. In many cities traffic conditions improved during the imposition of wartime driving limitations—to some extent because of the reduction in the number of vehicles. To a great extent, road users have forgotten the prewar miseries of congestion and will not be content to return to them. Highway engineers realize that there has been a huge unsatisfied travel demand; and they realize that it is imperative to meet that demand by planning the construction of free-flowing facilities that will solve the problem of congestion in urban areas. Widening, traffic-light control, one-way streets, rerouting, no parking in rush hours, and many other palliatives have been of great help, and will continue to be used; but the urban traffic problem will not be solved without generous provision for free-flowing facilities that will permit the uninterrupted flow of vehicles between points near origins and destinations.

Pattern of Arterial Routes

Discussions with state, county, and city officials regarding needed arterial highway facilities in urban areas inevitably invoke the statement that the particular city in question is different—it has special problems and cannot be considered a normal city. These statements generally are believed to be true; yet, when a comprehensive system of arterial routes in a city is laid out for long-term development, that city assumes a pattern which is unmistakably like those for almost all other cities of the same size. Adequate traffic data and the judgment of competent highway engineers inevitably lead to the conclusion that, to serve traffic best, arterial routes should be located so that the through routes outside the city are connected with the central business district. Practical considerations, such as the high cost of property in the central business district, require that the lines be relocated a little, perhaps several blocks, to the fringe areas that are so common in many cities. In the fringe areas, buildings have been allowed to deteriorate because no agency existed to control city development. These areas could not compete with the less expensive land surrounding them when additional housing was being constructed because their proximity to business made the land potentially valuable, although the time for conversion to business had not yet arrived. The houses in these areas could be rented as dwellings to low-income groups or to some types of businesses, and so could pay the carrying charges while the owners hoped that commercial expansion would absorb them. There usually has been no incentive to maintain the houses in good condition.

When the routes leading to the central business district are connected along these fringes, the pattern takes shape. In the larger cities, the procedure consists of the construction of a close-in circumferential route from which arterial roads to the outskirts of the city, and beyond, radiate in several directions. The pattern may twist, bulge, or be cut off on one or more sides; the inner circumferential route may be round, square, or elongated; the radial routes may be somewhat circuitous; and a large body of water or other topographic feature may block radial roads in some directions—nevertheless, the pattern is apparent. Fig. 1 shows the arterial highways, both existing and planned, in Cleveland, Ohio, one of the larger cities of the United States. The shaded area indicates the central business district.

In large metropolitan areas another characteristic part of the pattern is the outer circumferential route and sometimes an intermediate circumferential route. These routes are useful and, in some cities, very necessary facilities for traffic between the outskirts or suburban areas and between points near different radial routes. They are not first-order priority projects, however, although sometimes considered first because of the relative ease with which the right of way can be acquired.

The pattern of arterial routes just outlined, even though forced by the circumstance of available right of way, is close to ideal in many respects. One factor that must be emphasized in developing a system of arterial routes is flexibility—flexibility in choice of routes presented to a driver and flexibility in fitting future construction to future needs. When the central business district is surrounded by an inner circumferential route, a driver has wide latitude in choice of routes. An individual driver is not destined for the central business district or any other area. Only in the mind of the planner or designer is he considered one drop in the stream of traffic headed for an area. The individual driver is headed for a point in an area and he should be able to choose a route that will take him on a free-flowing facility as close to the point of destination as feasible. A flexible system of expressways will reduce his time of travel to a minimum and permit travel with ease and safety—the fundamental reasons for making the improvements and providing the service the road user expects. In addition, if the driver arrives as close to his destination as feasible on a free-flowing artery, he is not obligated to travel local streets any more than absolutely necessary, thus relieving congestion in the heart of the city where relief is needed most.

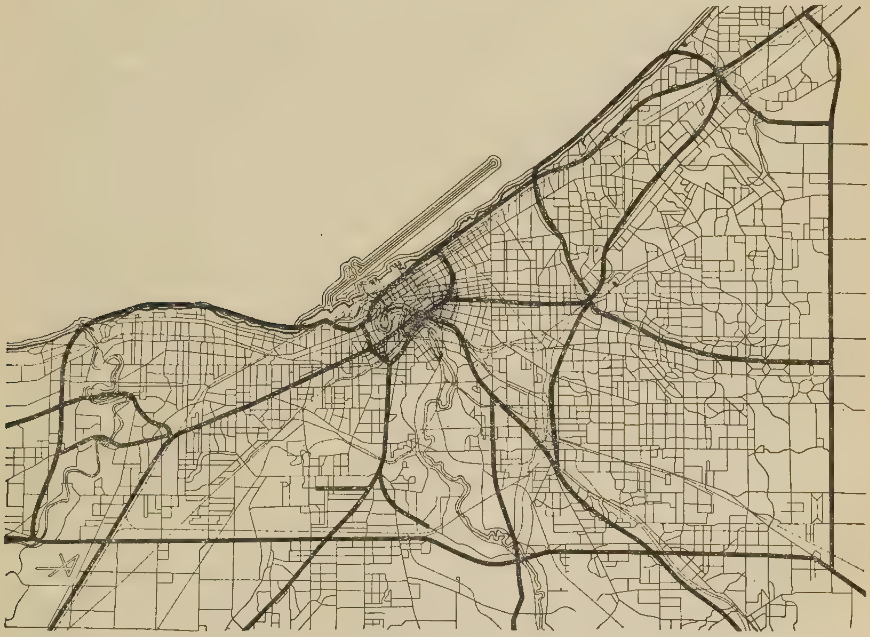

Fig. 1.—Pattern of Arterial Highways in a Large Metropolitan Area

USE OF EXISTING STREETS

The thoughts just expressed are the essence of the opposition to quite a different line of thinking by some planners. It has been proposed that arterial routes need not be developed as free-flowing facilities throughout their length, but rather that a type of design should be used to insure free movement on the radial routes entering a city where the expense of development is not significant. Such roads should be connected to one street, and preferably two or more existing surface streets, widened if feasible, through the central business district. On behalf of this plan, it is argued that the chosen route would not have to be on the edge of the central business district, but could be brought directly to the center of it and that, since such a large percentage of the traffic is destined for the central business district anyway, the purchase of expensive right of ways in or at the edge of the business district can be avoided and traffic given a wide choice of routes with normal street access at every block. One fallacy in such reasoning lies in the fact, just mentioned, that an individual vehicle is destined for a point in a district and use of surface streets should be reduced to a minimum. In addition, many drivers are not destined for the central business district but for an outlying district. They should not be encouraged to travel the local streets to go through the city or to reach another area in the city. Instead of relieving the area of most severe congestion, development along this plan will increase congestion because free-flowing facilities attract traffic from less desirable parallel routes and generate traffic that never existed before. To dump this traffic on to downtown surface streets already overcrowded would defeat a prime purpose of free-flowing facilities—the relief of congestion in the area of greatest congestion. In addition, there is no flexibility in such a plan for, if later observations indicate vital need for an expressway, the probability of being able to acquire sufficient right of way is nil and the door is closed to adequate development.

Surface streets are a necessary part of any comprehensive system of transportation for motor vehicles. They are the capillaries that carry the life blood of a city from the arteries to the limb extremities or final destinations, whether they are off-street or on-street parking areas, private or public garages, loading areas, or just street stops to drop off or pick up passengers or goods. Streets should be studied and regulated carefully to serve the area and abutting property in the desired manner. The interchanges between the free-flowing arterial road and local streets should be located and designed on the basis of traffic needs, effect on the arterial route, and effect on the surface distributors. The surface distributors may need adjustment to fit in with the over-all plan. These adjustments may consist of widening, conversion to one-way operation, changes in the traffic-light control cycles, or blocking cross streets to increase storage capacity—or any or all of these. Experience indicates that a complete plan is much more than "bulling" through a free-flowing type of arterial highway.

STAGE CONSTRUCTION

A complete system of free-flowing arterial highways cannot be built overnight or even in a few years. The cost per mile of such facilities is so great that a program of construction usually will have to be spread over several years to finance the improvement. To this end, routes to be improved should be divided into project sections that can be completed and connected to existing facilities to bring an immediate return in the form of road-user benefits on the investment made. Projects also can be constructed in progressive stages so that they can be used as facilities far superior to existing ones but not yet up to the ultimate design. For example, where cross streets do not carry large volumes of traffic, a section of expressway ultimately to be depressed can be constructed at grade. Other sections can be constructed with traffic on minor cross streets forbidden from crossing but permitted to make right turns to and from the expressway. Only on important cross streets would traffic be allowed to cross. Parallel frontage roads sometimes can be provided to serve as terminals for minor cross streets and as protection for through traffic even where through traffic lanes are subject to traffic-signal control at important cross streets. A facility that permits almost uninterrupted flow of traffic can be provided by separating the grades at heavily traveled cross streets, allowing less important ones to cross at grade, and terminating all minor cross streets. Additional grade separations can be constructed as required. Where frontage roads are to be provided or existing streets serve as frontage roads, these roads may be used as an interim stage for through traffic until it becomes feasible to construct the through traffic artery. If the right of way for the ultimate development is available, numerous methods of stage construction can be employed.

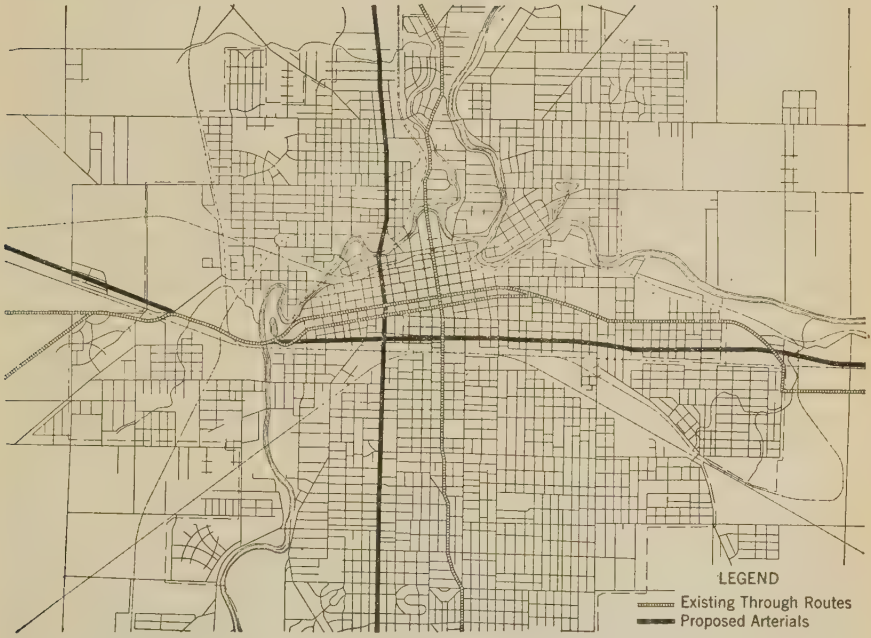

FIG. 2.—PATTERN OF ARTERIAL ROUTES IN A CITY OF MEDIUM SIZE

Long-range predictions of future traffic requirements probably cannot be made with a high degree of precision. It becomes incumbent to develop, as well as possible, a broad general plan by which it will be possible to make such changes as may be found necessary to serve future traffic. The lines of a good general plan are likely to bend and bulge here and there, the traffic volumes to be served may vary to some degree, and the requirements for interchange may vary in number and volume; but the salient features of a well-conceived bold plan will not change appreciably.

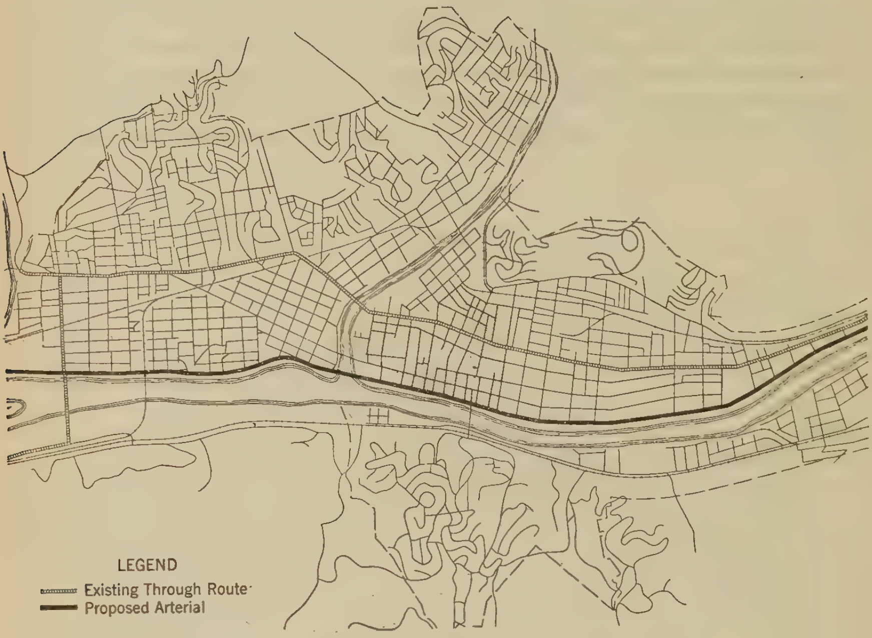

FIG. 3.—PATTERN OF AN ARTERIAL ROAD IN A SMALL CITY

The pattern of arterial routes just described, in which radial routes approach and are carried as close to the central business district as feasible, forming an inner ring, does not seem to materialize in cities of less than about 200,000. In such communities, two arterial roads at approximately right angles appear to be all that can be justified (see Fig. 2). This is a natural development since, generally, there are fewer radial routes and the central business district is smaller than in larger cities. With only two arterial roads in the smaller cities, the travel distance on surface streets to reach a destination may not be any greater than in larger cities with several arterial routes and an inner ring. In yet smaller cities, particularly where development is strung out along a railroad, a river, or an existing highway, one arterial road into and through the city generally is all that can be justified. The pattern in one such city is shown in Fig. 3. These are borderline cases and the origin and destination of local and through traffic should be examined to determine whether or not a by-pass is desirable.

TYPES OF EXPRESSWAYS

Many types of streets and highways can be considered free-flowing facilities, and there are just as many conflicting opinions regarding the desirability of each type. The engineer can only discuss their merits and deficiencies.

Streets or highways at grade can be constructed with several well-known expedients so that traffic moves well on them with only moderate interruption. The mere designation of a street as a preference street by "stop" signs on all cross streets will aid materially. At heavily traveled cross streets, however, signs indicating preference are inadequate. Traffic-signal control, or officer control, during hours of peak load or all day, is necessary. Isolated traffic-signal controls may not aid free movement; they may be detriments to free movement, but are required to minimize accidents and to regulate traffic. A series of signals progressively controlling traffic may be considered an aid in freeing the movement of vehicles.

Interference from the sides of the road is not reduced by these expedients; only conflicts with cross traffic at intersections are affected. To free through traffic from roadside interference, it has been common, where width permits, to provide separate side roads for abutting property and to segregate through traffic between islands or barrier strips separating it from the frontage roads. These are the "boulevards" found on major thoroughfares in many cities. They can be developed for high capacity by permitting crossings at grade only at important cross streets and terminating minor cross streets at the frontage roads. A facility of this type is shown in Fig. 4. Still greater capacity and freedom for through traffic can be developed by providing a divider between opposing traffic and by occasional grade separations.

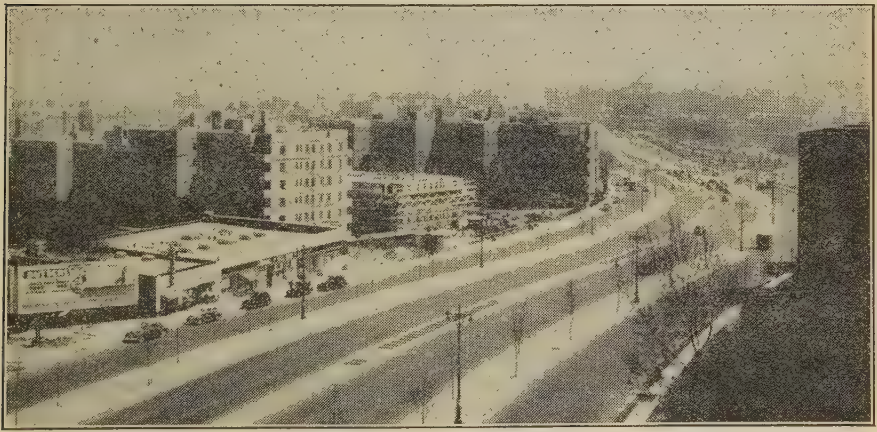

FIG. 4.—AN EXPRESSWAY AT GRADE

A surface expressway, although it is a tremendous improvement over any thoroughfare with all streets crossing at grade and is capable of handling large volumes of through traffic, is not truly a free-flowing facility. Traffic-signal controls are required for the occasional street crossing at grade and for the more frequent pedestrian crossings at grade. Structures that require pedestrians to walk up or down to bridges or underpasses are not used except under severe compulsion, can become nuisances, and often do not justify their cost. Pedestrian facilities will be used without strict control only when entrance to them is easier than crossing the expressway at grade—such as a footbridge at the general level of adjoining streets crossing over a depressed expressway.

The expressway type that has proved the most desirable in developed urban areas is that in which the express facility is at or below the general level of the surrounding area, no direct access with abutting property is provided, and all crossings at grade are eliminated. An example is shown in Fig. 5. In addition to providing free movement for all through traffic, the depressed type of expressway does not present a visual physical barrier between the areas adjacent to the expressway, and it is flexible as regards future requirements of cross traffic in that additional structures across the expressway can be provided readily, with little or no interference with through traffic during construction.

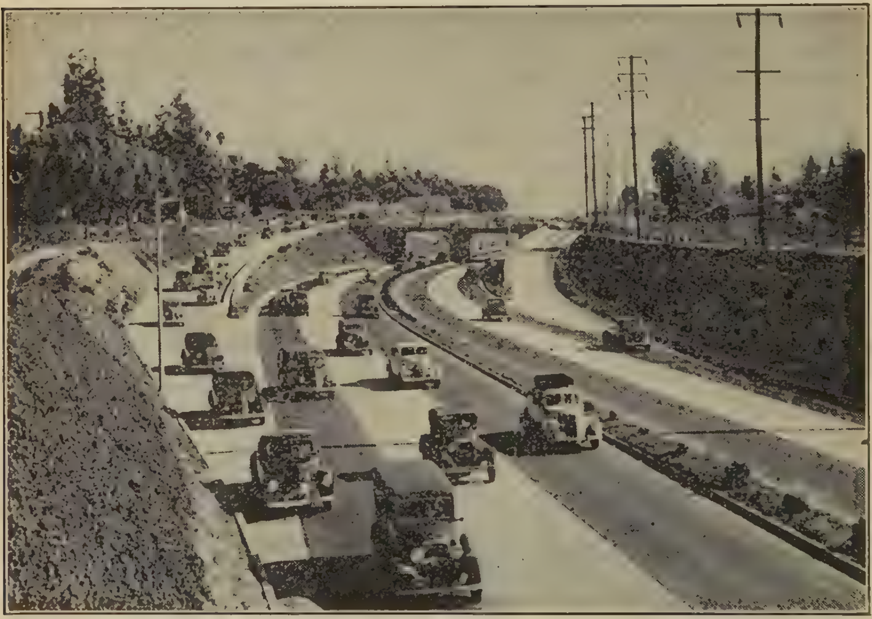

Fig. 5.—A Depressed Expressway

Control of Access

The control or limitation of access is necessary to the proper functioning of a free-flowing expressway. Sometimes (particularly on new facilities) this can be obtained when purchasing the right of way, provided the laws of the state permit. Controlling the right of access is not a simple matter. The rights of citizens to enter a public highway are rooted deep in fundamental common law and are not easily withdrawn.[2] The surest safeguards appear to be the purchase of these rights from the owners of adjacent property and the provision of frontage roads along both sides of the expressway to serve abutting property. Sometimes these lanes are newly constructed. Frequently, one or both are existing streets. Where the line of the expressway is generally parallel to an existing thoroughfare in a grid pattern of streets so typical of cities in the United States, it is preferable to acquire right of way between the streets rather than to widen one of them. Maintenance of, and changes in, public utilities in cities—particularly underground lines—are costly and annoying. Changes in utilities parallel to the expressway can be reduced to a minimum by locating the expressway between existing streets, which continue to act as frontage roads and as media for utilities. The maintenance of traffic in cities during construction also is important. By locating the proposed expressway between existing streets, the latter can function as before with interference resulting from construction operations limited largely to work on cross streets.

Elevated Structures

The elevated structure type of expressway (see Fig. 6) is the most costly of all, as regards construction. There is a prevailing impression that such expressways save right-of-way costs, and particularly that they can be located on existing streets without the acquisition of additional right of ways. This impression is correct to a limited extent only. Elevated structures can be constructed on right of ways barely wide enough for the structure proper, but this rarely is desirable. Existing streets have distinct functions in serving abutting property and in accommodating moving traffic, even though limited in capacity by the cross streets. Consequently, the placing of columns in the street for the support of an elevated structure seriously restricts the usefulness of the street. The structure itself blocks off light, creates a potential nuisance area below, and seriously damages property on both sides. It should be realized that, where elevated structures can be justified, the expected traffic volume usually is high enough to justify a four-lane or a six-lane facility which requires a width nearly that of the average city street.

Undesirable features of an elevated structure are avoided to some extent by increasing the width of right of way to accommodate surface frontage roads clear of the structure supports, thus releasing the area under the structure for parking or other businesses. The increased width generally is needed, in any case, at intervals, to provide space for ramps. The use of an area under an elevated structure for business purposes, say, retail or warehousing ventures, is desirable in that revenue is forthcoming and the policing and maintenance of the area is transferred to the owners of the establishments although the added fire hazard must be considered. By the time that the various refinements are worked out and a sufficient width is obtained to minimize the damage to abutting property, the required right of way probably is sufficient for the development of a depressed expressway.

There are two major advantages to elevated expressways aside from providing space below the structure for some useful purpose: (*a*) Advantage to local cross traffic; and (*b*) maintenance of utilities.

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[2] "Public Control of Highway Access and Roadside Development," by David R. Levin, Public Roads Administration, Washington, D. C., 1943.

(*a*) Nearly all streets can cross under the expressway without interference with through traffic—indeed, with less interference than existed prior to construction since the major traffic stream will use the overhead structure. An occasional street will be blocked off by a ramp.

(*b*) Utilities crossing the expressway are practically undisturbed by elevated structures. In some cities changes to utilities may be a major item of cost.

Sometimes topography will make elevated structures a necessity regardless of other considerations. If topography is rolling, a desirable control gradient of 3% or less may be feasible only where the expressway is in a depressed section crossing high ground and on an elevated structure crossing low ground. Low gradients are desirable, particularly for trucks which may slow down and reduce the capacity of the facility on even moderate gradients. On long steep, or even moderate, gradients, trucks are reduced to crawl speeds, seriously impeding following vehicles. This condition has been alleviated on existing two-lane roads by providing an additional distinctive lane for slow-moving vehicles upgrade. Truck drivers show a high degree of compliance. Where long gradients cannot be avoided on urban expressways, the use of such added lanes should be considered to avoid a reduction in capacity of the upgrade pavement.

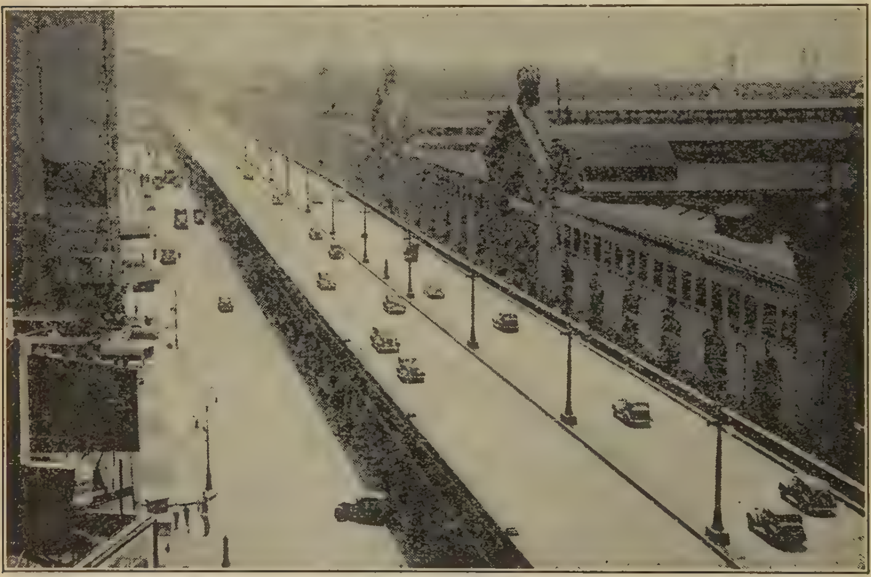

Fig. 6.—An Expressway on an Elevated Structure

Principles of Design

The design of an expressway, of each interchange and of each focal area such as a terminal, should be examined in the light of the principles that follow: (*a*)

The facility must fit traffic needs; (*b*) traffic operation on the completed facility must be free flowing and safe; (*c*) the path of the individual driver (at least that in a major traffic movement) should be as direct and simple as feasible and the required actions on his part be natural; (*d*) the operation should be flexible so that a driver will have a wide choice of destinations; (*e*) the facility should be flexible so that it can fit into any reasonable future change in the traffic pattern and so that it can be connected to important traffic facilities that may be provided in the future; (*f*) the facility should be integrated with the city plan; (*g*) there should be a wide distribution to the street system; (*h*) stage construction and development of short usable sections should be practicable; (*i*) it should be possible to maintain traffic adequately; (*j*) disruption of existing transit facilities, railroads, and utilities should be kept to a minimum; (*k*) important structures such as major bridges and buildings should be left intact; (*l*) the design should envision connecting the free-flowing facility to similar facilities beyond the immediate area of the problem; (*m*) the cost must not be out of line with the service rendered; (*n*) maintenance of the facility will be nominal; and (*o*) the completed facility should be pleasing in appearance.

The critical examination of interchanges is suggested because they are much more important in expressways in urban areas than in any other highway facility. Interchanges that are improperly located and designed can aggravate rather than relieve traffic congestion on city streets; they can affect their desired pattern adversely and can back up traffic on the expressway. The foregoing principles are not listed in the order of importance but to group those that are related to each other. For any one problem not all the listed principles may be applicable.

TERMINAL FACILITIES

Terminal facilities for both passenger and commercial vehicles are available in many cities in varying degrees and for widely varying charges. There are numerous ramifications of the important terminal problem; but its solution should go hand in hand with the development of expressways. Expressways in cities generate a considerable volume of traffic which tends to aggravate congestion in terminal areas if steps are not taken to provide for it. To solve the terminal problem, it is desirable that all interested agencies, particularly local public and private agencies, attack it broadly in the form of zoning control and the development of off-street parking and loading facilities by private and public interests. In many cities existing parking lots cannot be depended upon for future use. Their land value is so high that improvement by construction of commercial buildings should be expected as the city expands. An interesting development in some cities is the multiple use of such buildings for stores where retail frontage is valuable, for parking on the lower floors easily accessible to the streets by ramps, and for commercial businesses on the upper floors to which elevator service is convenient.

The problem of truck loading will be solved only by a bold approach. There appears to be no less justification for loading trucks inside business establishments than for loading freight cars on private sidings. The truck terminal, whether developed privately or as a union terminal, lessens street congestion in that it decreases the truck mileage in congested areas and may reduce the size of trucks operating therein. However, major improvement will require loading-dock space inside the building line of the ultimate destination of the goods carried. In some locations, the result of limiting or eliminating first-floor street frontage will not be an undue hardship; and, at other locations, passages to the rear of the property or ramps to upper floor docks will be justified. Zoning to require, or at least to encourage, development of dock space inside the building line appears justified.

Planning Agencies

Orderly procedure in express highway planning may change for different cities because of variations in the distribution of planning functions. The planning of express highways logically can be considered the function of the state highway department, the city engineer, the county engineer, the city plan commission, or the highway committee of the chamber of commerce—or any or all of these. Some cities have presented a spectacle of so much planning that what should have been a healthy rivalry resulted in a jam which prevented progress. In such cases the efforts of a coordinator become necessary to induce the various agencies to bury minority differences and to agree on a plan acceptable to all. These problems are the concern of all and it is gratifying to note, in many cities, a healthy cooperation between the several agencies interested in the cities' welfare.

Major highways in cities are the main arteries of transportation for people and goods; and, if they enable vehicles to travel only at low average speeds, the city suffers in both increased cost and time for all functions depending on transportation. If the major highways are developed as free-flowing expressways, the tempo of transportation in the city can be increased with favorable results for all. The writer cannot agree with visionary enthusiasts who believe that freeing transportation will of itself rehabilitate a city, although it can be of material aid. The forces of decentralization are too great to be stopped entirely, but the proper location and development of expressways can assist in the establishment of self-contained stable neighborhoods and in the stabilization of trade and values in the principal or central business district.

The self-contained neighborhood appears to be one objective on which most city planners agree. A neighborhood should be roughly from a half mile square to one mile square, and should contain, in addition to housing, all the essentials of living, such as schools, places of worship, community and recreation facilities, and the everyday stores—the butcher, the baker, and the modern version of the candlestick maker, the five-and-ten-cent store. The central business district would retain the larger establishments such as the department stores, clothing establishments, commercial and industrial offices, and banks. Properly located expressways can serve the triple function of acting as dividers between neighborhoods or between a neighborhood and an area of a different type, as transportation mediums for highway traffic between neighborhoods and the central business district, and as transportation arteries for through highway traffic. When travel to the central business district takes but a short time and the road users have convenient terminal locations for parking, loading, and unloading, the tendency for the larger establishments to move from the central area to the neighborhoods is likely to be retarded and the central business district stabilized. A detailed discussion of city planning as a whole is beyond the scope of this paper; but the highway engineer must appreciate that, in the location and design of urban arterial facilities, he is participating in city planning. Accordingly, it is essential that he cooperate intimately with all other agencies interested in the planning, development, operation, control, and general welfare of the city.

PRELIMINARY ENGINEERING REPORT

One of the most useful mediums to unite all interested agencies in the consummation and approval of a plan for an expressway is the preliminary engineering report. Such a report may cover one or several projects. A properly prepared report has several useful functions. First and foremost, it serves the designer himself by permitting him to examine the proposed solution objectively and to assure himself that the proposed facility will solve the transportation problem, that it fits in with the numerous related projects, that the design is feasible, and that the broad picture is complete. A designer may be so engrossed in detail that he loses the over-all picture—loose ends are not garnered and impracticable features creep in.

The preliminary engineering report helps to acquaint all agencies with the problem and its solution, to develop constructive criticism, to unearth errors in so far as it is possible to do so, to fit the project into the over-all city plan, and to focus the attention of fiscal authorities on funds needed for advancing the project from preliminary engineering status to right-of-way acquisition and final construction.

The cost of a preliminary engineering report almost always is manifoldly justified. Free-flowing express facilities through cities are complicated. Right of way is costly, numerous utilities are affected, and alternate locations or even slight adjustments in the line and the access connections might have serious consequences. Unlike those for rural highways, the construction plans for urban expressways may be materially affected by such adjustments and changes in the plans are costly and time consuming. It is desirable, therefore, that the salient geometric features of the design, arrangement of roads, alinement, width, accesses, walls, bridges, and other design features and standards be "tied down" before construction plans are begun. The preliminary engineering report can be used for such purpose. When the approval of all interested agencies is at hand, the detailed design can proceed with assurance that no radical changes will be made.

Preliminary engineering reports vary considerably with each project or group of projects and with different administrators. In some cases a line has previously been established definitely, and the report may show the results of location surveys and plans in considerable detail. In other cases the preliminary engineering report may present no more details than the general location and justification usually found in a city planning report. In such cases the report should be considered one of a series, the later reports going into greater detail before construction plans progress very far. In general a preliminary engineering report should include five broad phases: (I) Transportation, (II) Design loads, (III) Solution, (IV) Estimate of cost, and (V) Justification.

(I) The Transportation Problem.—The report should indicate the general transportation problem to be solved. To this end it should show the traffic and transportation situation. Where does traffic originate and where it it destined? What traffic flow exists as a result of traffic pressures and what traffic flow would exist under unrestricted conditions? What delays are encountered? How does traffic, as it flows, affect other functions of the city? How are industry, trade, and commerce influenced? How is mass transportation concerned and to what extent will the proposed facility serve it? How are pedestrians affected? What is done with vehicles when they reach their downtown destinations? All such questions should be answered in the preliminary engineering report. Existing conditions should be depicted by topographic maps and aerial views, both oblique and mosaic. Maps and graphs showing land use, social trends, and other data are useful in showing the problems that transportation in the city is facing. Land value and improvement maps are invaluable in analyzing the probable cost of right of ways in choosing locations. Utilities should be indicated, particularly those underground.

(II) Design Loads.—The report should indicate the traffic volumes for which the expressway is to be designed—either in that part of the report dealing with the problem or in that part showing the proposed solution; but traffic volumes should be shown somewhere. The design traffic volumes are the "loads" to be accommodated. There is no more logic in designing a highway facility without these loads than there is in designing a bridge structure without knowing the weights and distribution of the wheel loads it is to carry. All too frequently, the available traffic data are not sufficient to determine the probable traffic volumes that will move on the expressway and the probable number of vehicles that will make the several turns at each point of interchange. In such cases the data should first be obtained; but, where this is not feasible, the probable volumes should be determined by judgment (using the best data available) and thus recorded as a part of the preliminary design data. This procedure is superior to one in which only the number of traffic lanes is decided by the judgment of the same engineers.

(III) Solution.—The report should give the proposed solution, including a preliminary design plan and profile, which may be drawn to a scale of about 1 in. = 400 ft. They should show the edges of all pavements, including frontage roads, access connections, and adjacent streets. Structures and walls should be indicated. Landmarks and adjacent areas affected by the construction also should be shown. The alternate locations studied in arriving at the final location, as well as the alternate designs tried at each intersection, should be marked and an account given of the reasons for choosing the proposed design. The reports will pass through many hands, and a presentation of alternate ideas will help reviewing and affiliated agencies to avoid repetition of the same process of trial and elimination.

A preliminary engineering report should be prepared in a manner clearly understandable by the many officials and other individuals who are not engineers and to whom a blueprint is a mysterious document. Some parts of the design, particularly directional types of interchanges, are difficult to visualize in three dimensions. Such locations should be shown in perspective delineations which give clear pictures of what is intended. In some cases, models are made although they generally are expensive, require a long time to construct, and frequently do not show as large an area as one or more perspective delineations. These are made in an amazingly short time by men who generally are not engineers but who have the peculiar "knack" or gift required for that type of work. Aerial pictures, with the proposed facility drawn in, also have been advantageous. General pictures of what is intended are very useful when it becomes necessary to acquaint the public at large with the project. Newspapers welcome material of this character.

(*IV*) *Estimate of Cost.*—The preliminary engineering report should include a reasonably accurate estimate of the cost. Such cost should be broken down into the usual subdivisions of right of way, construction, engineering, and contingencies. In addition, it should be broken down further into sections which, preferably, would be usable upon completion. Expressways in cities take time to design, finance, and construct, and the administrator should have the cost estimate in such flexible form that it can be fitted into an ever-changing fiscal program. To this end the report should show the manner in which sections of the expressway can be constructed in stages and the cost of each stage.

(*V*) *Justification.*—The preliminary engineering report should describe fully the reasons justifying the expenditures proposed for the construction of the project—in particular the transportation problem, which it is intended to solve by the expressway, and how the solution will be accomplished. Frequently, reports discuss congestion and interference with industrial or commercial functions, and sometimes accident frequency, and follow such discussion by a description of the relief that will be afforded by the new facility and how it will be obtained. The effect of the expressway on the city plan and on the development and stabilization of important areas might well be included. These are worthwhile reasons for prosecuting the project, and the benefits sometimes are sufficiently evident to be enough justification; but, for a realistic businesslike approach, the report should include an economic analysis of the project.

An important part of an economic analysis is the balance sheet, showing on the one hand the annual cost and on the other the annual benefits. Since this paper deals with expressways, a treatment of benefits due to direct land service use can be omitted without appreciable error, in order to deal entirely with benefits to road users. The cost of an expressway in a city is a capital expenditure and should be sanctioned only after an analysis has been made to determine what benefits will accrue to road users. Road-user benefits are not the only returns, as has been noted. Intangible benefits, such as ease and comfort of driving and facility and convenience of travel, and indirect benefits, such as effects on city stabilization or development, may outweigh benefits to road users in some cases; but an economic analysis that compares road-user benefits with annual costs should be made. This relationship is the guide that administrators need in determining economic feasibility of a project or in comparing alternate solutions to a transportation problem, for it indicates in businessmen's terms the road service to be provided by the proposed expenditure of funds.

The annual cost of a project may be assumed to be the sum of the annual cost of financing plus the annual cost of maintenance and operation. The subject of finance is beyond the scope of this paper, but it is well to state that the annual cost of financing is the same for any one rate of interest and any one period of time of amortization, regardless of the method of financing adopted. The annual road-user benefits are the saving in the cost of vehicle operation, the saving in time, and the saving resulting from accident reduction. Of course, there are some differences in observations (and in their interpretation) for the cost of vehicle operation on various types of highway facilities. There are differences of opinion regarding the factors to be used in calculating the value of saving in time although there is general agreement that time is valuable even for pleasure-bent passenger vehicles. If factors are inaccurate to a great degree, the resulting analysis can be misleading; but, if available factors are reasonably accurate, the economic justification for a project should be developed and included in the preliminary engineering report. Such analyses are particularly useful in comparing two or more solutions for the same problem or in assigning priority ratings to several projects.

Considering the large number of vehicles involved, the savings in vehicle-operation costs effected by the elimination of "stop-and-go" driving and other speed changes, and the time saved in traveling steadily even if not at high speed, expressways in cities show high returns on the investment, well beyond comparable returns on most other types of highway and street improvements. The demonstration of this fact to the driving public will give administrative officials the courage to satisfy demands for expressways. Such demands are certain to increase in volume and intensity as the inevitable congestion in cities returns; as those responsible for the welfare of a city look for every aid in its rehabilitation, stabilization, or further progress; and as the driving public becomes acquainted with the many desirable features of travel on free-flowing protected highway facilities.

DISCUSSION

Harry W. Lochner,[3] M. ASCE.—In the development of an expressway plan for the average city, a sound and realistic viewpoint must be maintained. The plan should embody the ultimate improvement of the system with provision for building usable units as funds become available. Mr. Barnett mentions the past inadequacy of appropriations to develop the rural highway system. In most cases the development of a highway plan for urban areas involves the same problems.

The Federal Highway Act of 1944 authorized $1,500,000,000 for highway improvements during the first three years after the end of World War II. Compared to previous federal highway appropriations, that sum appears very large, but in reality it is small compared to the need in urban highway improvement. Federal funds for urban areas are allocated on a population basis, to be matched equally by local funds for construction, and on a one-third and two-thirds basis for right of way—the larger sum to be advanced by the state or local agencies.

As an example—the Federal Interstate Highway System, as proposed under the Act, calls for five interstate highways entering Louisville, Ky., a city of approximately 300,000. When federal funds have been matched by local appropriations, a three-year program involving the expenditure of approximately $3,500,000 will be possible. That sum is to be considered in relation to the average cost of expressways estimated at from $1,000,000 to $2,000,000 or to $2,500,000 per mile, in a city such as Louisville. Funds might be utilized in developing a short section of one route completely, or, more desirably, in developing a longer section partly with progressive improvement as additional funds become available. Such progressive construction gives motorists who are unacquainted with expressways the opportunity of appreciating their value.

Mr. Barnett has demonstrated the ease with which numerous methods of stage construction can be employed if the right of way for the development is available. Land acquisition is one of the important aspects of expressway development. Unfortunately, in most cases it is the most difficult to attain, and its realization is dependent to a large measure on the adequacy of the preliminary engineering report.

In the past, most states have required local agencies to obtain the right of way for highway improvements. It appears that in several states local agencies will still be required to obtain the right of way, with one-third aid from the federal government. In those cases, particularly, the plan must be sold to legislators to assure the appropriation of the relatively large amount of money required to secure the needed lands. One of the major functions of the preliminary engineering report is that of selling the plan—not only to the officials but to the public as well.

In most large cities where the cost of expressways is relatively high, the projects can be justified only by designing the route as an artery for all forms of transportation. Each artery should be planned to accommodate mass-transportation vehicles as well as private vehicles. The saving in time for the large number of mass-transportation riders which can be effected on expressways of itself will justify a very high proportion of the improvement cost.

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[3] Mgr. (H. W. Lochner & Co.), Chicago, Ill.

One of the intangible benefits to be derived from sound planning of expressways is the opportunity of developing adjacent public recreational facilities. In many instances fractional pieces of properties will remain after acquisition of land for highway right of way. This land can be secured for public use at a very small additional cost. Located along the expressway (which could be landscaped), such pieces of property can be improved as small parks or playgrounds for the benefit of the adjoining neighborhoods.

In planning expressways, traffic information is but one of the tools utilized. By the proper analysis of the data, factual information can be developed to substantiate the selected location. Traffic information is of major value in determining both volumes of traffic that will use the several access points and their probable required capacities. It is also particularly important in designing terminal facilities to accommodate traffic desiring to go to and from the downtown area. No traffic survey technique has been developed which gives complete accuracy and the fineness of information that might be desired. The results of every type of origin and destination traffic survey require careful analysis and application. Mr. Barnett mentions the "interview type" of origin-destination survey. Complete results of that type of survey are available for the City of Savannah (Ga.) which show that approximately 80% of the "to-work" traffic going to either industry or office was reported, together with a lesser percentage of the "to-shop" and "to-recreation" traffic. Fortunately the traffic to and from work consists of the larger part of traffic movement within cities, has the highest economic value, and accounts for most of the peak-hour traffic for which the expressway is designed.

Expressway planning, if it is to be of long-range benefit, must envisage the changes and expansion of the urban area. Mr. Barnett mentions the fringe of depreciated residental properties around downtown areas. In numerous instances those areas will be redeveloped for industrial purposes. In such cases an expressway might be located as a buffer between the industrial areas and the residential properties.

Expressways will play a major rôle in causing development and undevelopment. Access will be provided to new industrial, commercial, and residential areas. Each must fit into the most advantageous over-all plan of development. Each will influence estimated expressway traffic flows. Therefore, planning must of necessity be conceived on a broad basis and must be sufficiently flexible to accommodate future changes.

With the exception of New York City (N. Y.) and a few other isolated cities, motorists have not had the opportunity of using expressways. Therefore, it is difficult for them to comprehend the nature of such an improvement and its benefits. In Chicago (Ill.) the Outer Drive—13 miles in length—built in the park along the Lake Shore, follows general expressway standards. Even with the example, laymen in Chicago have difficulty in visualizing an expressway built through the developed areas of the city. In planning a system of expressways in the average city it is desirable, therefore, to make every effort to improve even a small area. Such an illustration will generate the public enthusiasm and desire for the completion of the entire system.

Fred Lavis,[4] M. ASCE.—The subject of this paper is timely and indicates the general nature of some of the problems. The paper, however, would be more useful if it were more specific.

To state (as the author does under the heading, "Introduction") that "* * * the roadbeds [of railroads] are as useful today for the movement of trains as when they were built" is a half truth which ignores the fact that roadbeds of railways, subgrade, ballast, ties, rails, and fastenings have been and are being continuously improved. They have little semblance to the roadbeds of even 25 years ago. The Pennsylvania Tunnel extension into New York, N. Y., was completed only about 35 years ago.

One may question the statement of the author that the elevated structure type of expressway is the most costly of all as regards construction and also the vague references to the value of operating cost of low rates of gradient. There should be some indication that a large volume of literature on the subject of trunk-line parkways, express highways, and freeways is available. Although it may be impractical to include a complete bibliography, it should be mentioned that there is much specific information on the subject. The following three papers by the writer are cited, for example: "The Money Value of a Car Minute,"[5] "Highways as Elements of Transportation,"[6] and "Safety and Speeds as Affecting Highway Design."[7]

Elaborate studies were made in Caracas, Venezuela, about 1940 or 1941, and the "Diagonales" were projected and partly built in the congested area of Buenos Aires, Argentina, before that.

The paper by Mr. Barnett is too vague and general to be of use to younger engineers, and it contains little that is new to engineers of knowledge and experience in this particular field. As a matter of fact, problems of this kind require the application of the greatest engineering experience and knowledge, and business judgment, to each specific case.

Homer M. Hadley,[8] Assoc. M. ASCE.—This is a truly admirable paper: Clear, lucid, and comprehensive. It covers the subject adequately in all major aspects without becoming involved in minor detail and trivialities, and the presentation is made so effectively and with such logic and reason that the stated best method of dealing with urban traffic congestion cannot fail to find acceptance in the minds of the great majority of readers.

In the development of detailed plans, arrangements for entering and leaving expressways and limited access highways offer the most troublesome problems. With two adjacent fast-moving streams of traffic in the same horizontal plane going in opposite directions, left-hand turns are neither permissible nor possible, and elaborate cloverleafs and grade separations may be necessary to deal with access and egress. This problem is not absent in elevated structures whose traffic, if all at one level, presents the same conflicts that ground traffic does.

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[4] Cons. Engr., New York, N. Y.

[5] *Annals*, American Academy of Political and Social Science, 1927.

[6] *Transactions*, ASCE, Vol. 95, 1931, p. 1020.

[7] *Proceedings*, Highway Research Board, National Research Council, Vol. 17, 1937, p. 79.

[8] Cons. Engr., Seattle, Wash.

In considering a specific case of this kind it appeared to the writer that these difficulties would largely disappear if a two-level structure were used with traffic separated—say, northbound on the upper level and southbound on the lower level. Under these conditions, access and egress are possible on either side; and, although special framing of the structure and special column positioning would be required at such places, these adjustments could readily be made. Moreover, a two-level structure actually would save right of way; for example, with a six-lane structure under the Design Standards for the National System of Interstate Highways,[9] a minimum width face to face of 83 ft of railings is required, whereas for a three-lane structure this dimension is 43 ft. A saving of a 40-ft right-of-way strip effects a very real saving in cost where land values are high.

How adversely such a two-level elevated structure would affect the use of a street beneath depends not a little upon the positions of columns. If they can be kept clear of the curb lines, the street traffic will not be affected at all; and, although the structure above would cause a certain loss of light, this loss would be far less than that beneath a structure of nearly double the width. If additional provision is made for a reasonable space between the viaduct structure and adjacent buildings and if the structure is given proper architectural treatment, the unfavorable effects would be slight indeed. Of course, there is the further consideration, stated by the author, that topography may make an elevated structure a necessity. When that is the case, the structure unquestionably can be made pleasing in appearance and any unfavorable aspects can be held to minor scale. When the Lake Washington Pontoon Bridge at Seattle, Wash., was first announced, its opponents declared not only that the beauty of the lake would be destroyed but that the value of the abutting property along the lake shores would also be destroyed. Nothing of the sort followed the construction of the bridge. The sole objectionable feature appears to be the noise caused by heavy trucks ascending the 5% grades on the elevated approaches. Passenger cars cause no annoyance whatsoever. In general, the bridge traffic is nonexistent, so far as ground level is concerned. True, this would not be the effect on tall buildings adjoining a viaduct. The traffic then would be at the level of upper floors. However, whether this coincidence would be objectionable or not depends on occupancy. In many cases it would be a matter of entire indifference.

The author makes no mention of vehicular tunnels in his paper. Obviously, they may be unavoidable at times and then they unquestionably are most acceptable. However, if their terminal points can be joined by other construction without too much loss of distance or grade, they appear to the writer to be the least desirable and most costly connections that can be considered. The author's views regarding vehicular tunnels from ½ mile to 1½ miles long are requested.

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[9] "Design Standards for the National System of Interstate Highways," A. A. S. H. O., adopted August 1, 1945.

Donald M. Baker,[10] M. ASCE.—Express highways are the third factor in the development of trafficways in metropolitan areas. Streets in these areas were not a great problem until the introduction of the motor vehicle. They served adjacent property and were financed and maintained by local taxes. Early motor vehicle traffic, traveling at speeds several times those of horse-drawn vehicles, required better roadway surfacing, which was paid for by local taxes or assessments, and in some instances by local bond issues.

Increasing speeds and traffic density during the decade 1920–1930 led to the development of arterial highways, created by the opening, widening, straightening, and paving of certain principal streets upon which a large part of such traffic concentrated. These arterials served primarily the motor vehicles using them, rather than abutting and adjacent land; but they were financed in most instances by assessments upon this land, on the theory that the land was benefited by the improvement to an extent which was at least equal to, and usually in excess of, the cost of the improvement. This theory proved to be sound in a few instances, but in widespread use was found to be a mistake, since the improvements benefited motor vehicle users far more than property owners, and the burden upon the latter was in many instances unsupportable.

Express highways should be planned and designed as mass transportation facilities to serve the people of a community; and to accommodate not only privately owned vehicles, but also public carriers, usually buses, although some situations will be found where they can accommodate rail facilities without too great an additional cost.

Until well into the 1920–1930 decade, mass transportation in urban and suburban areas was provided by rail lines. In most places these were located on the streets; but in the largest communities, where the capacity of surface lines was inadequate, they were constructed "offsurface." Offsurface rail facilities have greater passenger-carrying capacity per mile of track than do express highways per lane, and can serve more passengers per dollar invested in them than can express highways; but certain conditions have developed which, except under particular circumstances, make widespread extension of such lines very difficult of achievement.

The riding public does not appear to be willing to pay fares that will provide adequate service and yield operating expenses and debt service for offsurface rail facilities; this condition makes their financing primarily an obligation of the community itself. Demands upon public credit for the provision of other needed facilities, such as streets, schools, water and sewerage systems, parks and playgrounds, public buildings, etc., and the taxes necessary to pay for these facilities make the prospect of successful passage of public bond issues to pay for offsurface rail lines very discouraging.

Another drawback to their development arises from the decentralization of population in metropolitan areas, and the increasing ownership and utilization of motor vehicles. In the twenty-five years from 1916 to 1940, motor vehicle registration in the United States increased from 35 per 1,000 population to 236 per 1,000, or nearly seven times. With the revival of motor vehicle production, registration can shortly be expected to reach a density of one motor vehicle per family.

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[10] Partner, Ruscardon Engrs., Los Angeles, Calif.

Prior to 1920, new population followed the extension of rail lines into undeveloped areas surrounding the cities, and settled in close proximity to these lines. People soon found, however, that by owning an automobile they could live anywhere within the area and travel back and forth to their place of employment, or make other travel movements without dependence upon the rail system. Ownership of automobiles was difficult in localities of high population density; and, since most people desired to possess one, this encouraged them to move to areas of lesser density. Congestion due to the increasing number of motor vehicles made travel by rail slower and increased operating costs, with the resulting decrease in service and also in patronage. This spiral has continued to the present time.

The construction of rural and intercity highways was initially financed by local taxation, and later by bond issues, until the motor vehicle fuel and use taxes were developed. These taxes were initially collected and administered by state authorities, whose jurisdiction seldom extended into the corporate limits of cities; and more recently they have also been collected by the federal government for allocation to states. From the outset they have been devoted almost entirely to the construction and maintenance of roadways and highways outside of urban and suburban areas, although a high proportion was collected from motor vehicle owners and users operating largely within these areas. Beginning with the depression (1930–1940), improvement of streets and highways in heavily populated areas by bond issues and assessments upon the adjacent property proved to be infeasible; local pressure on state authorities forced the allocation of some of the proceeds of fuel and use taxes to cities and counties, although the amount so provided was seldom sufficient to more than care for maintenance and a nominal amount of arterial development.

At present, the network of rural and intercity highways is far more adequate to meet the needs of the traffic using them than are the highways and streets in metropolitan areas, with by far the greater vehicle mileage, at least in states with a high ratio of urban to rural population, occurring in cities and metropolitan areas. Tremendous financial losses resulting from deaths and accidents, delays due to traffic congestion, and losses of property values, occur in these areas. Gasoline rationing during World War II reduced the acuteness of the situation somewhat; but, even with the reduction in the number of motor vehicles since 1941, the elimination of gasoline rationing and the subsequent greatly increased use of automobiles are bringing about a serious situation. When renewed production of motor cars has brought registration back to prewar figures, this situation promises to be extremely critical.

Until recent years, the major interest of state and federal highway authorities, in so far as it applied to cities and metropolitan areas, appears to have been in providing facilities for by-passing traffic originating outside of these areas around their congested portions. Recently, however, there has developed a recognition of the needs of such areas for facilities which will reduce congestion within their boundaries.

The author is correct in his statement that the greatest need for express highway facilities is along routes radiating outward from the central business districts of metropolitan areas. A large volume of traffic does occur in a circumferential direction; but congestion comparable to that occurring on radial routes seldom occurs, since a far greater number of circumferential trafficways are usually available. Increasing congestion on radial routes as they approach central districts is due not only to the fewer number of trafficways existing as the central district is approached, but also to the steady contribution of traffic along these routes as they approach the central district which increases its total volume, and to the increasing interference by cross traffic at the more frequent number of intersections. In studies made as consultant to the Los Angeles Transportation Engineering Board in 1938–1939, the writer found that during the 12-hour period, 7:00 a.m. to 7:00 p.m., 35.7% of the total number of motor vehicles entering the central district of Los Angeles (Calif.) originated within a 2.5-mile radius of the central district, 53.1% originated within a 5-mile radius, and 63.7% within a 7.5-mile radius. Only 14.5% of those entering were registered from locations outside the metropolitan area.

The large use of motor vehicles, as a means of travel to the central district, is shown by the fact that 22.9% of the vehicles registered within the Los Angeles metropolitan area entered the central district during the aforementioned 12-hr period. Total persons entering the district during the 12-hr period by means of private automobiles equaled 15.6%, and by public transportation 10.7%, or a total of 26.3%, of the population of the Los Angeles metropolitan district. This may be compared with a figure found by the writer to be equal to 30% of the population of the metropolitan district entering the central district of Los Angeles in the year 1931. These percentages are consistent with the results of cordon counts around the central districts of ten large cities made during the late 1920's and the early 1930's, all such counts indicating that a number of persons equaling from 25% to 30% of the population of metropolitan areas enter the central district of such areas daily by various modes of transportation.

The need of by-passing traffic around central districts to relieve congestion in the approaches to and also within these districts, which was stressed by the author, is likewise acute. The suggested inner and outer by-passes are frequently desirable as an aid in the reduction of congestion in approaches to the district. In the foregoing studies, made in 1938–1939, it was found that from 25% to 40% of the vehicles entering the central district of Los Angeles were destined to points beyond it, and passed through it rather than followed existing but more circuitious by-pass routes. The elimination of this large number of vehicles from the streets within the area and from those approaching it would have done much to relieve congestion.

During the economic depression of the 1930's, many studies were made of traffic habits and characteristics within metropolitan areas, and many plans for relief of congestion were formulated. Most of these are still merely lines on paper and printed pages of text in reports. Difficulties in activating such plans always lie in finding ways and means of financing the improvements which they recommend. It is the writer's considered opinion that plans for express highways, or in fact for any other improvements, are too often wasted effort unless they include, in addition to the physical plan, a feasible one for financing the improvements recommended.

The "pay as you go" plan which contemplated the use of proceeds of fuel and use taxes derived from motor vehicles for financing highways was far sounder than that of using proceeds of 40-year bond issues to build roads and highways which had a 20-year life, or of assessing adjacent property to pay improvement costs; but city and regional planning, and highway engineering have made great advances since World War I. New highways can now be planned and constructed with the assurance that, given adequate maintenance, they will not be worn out or obsolete for at least half a century. Moreover, the "pay as you go" plan, even if fuel and use taxes are substantially increased, will not provide funds for express highway facilities now needed in metropolitan areas at a rate sufficient to catch up with increasing needs. These needs have been unsatisfied and accumulating, not since the date of Pearl Harbor, but since the beginning of the economic depression in 1929, and are increasing at an almost geometric rate.

Financial losses, due to inadequate travel facilities in metropolitan districts and the resulting congestion, are of vast proportions. In 1933, in a report made to the Central Business District Association of Los Angeles upon a rail rapid transit system for the Los Angeles area, the writer estimated that monetary losses in time to drivers of automobiles, and operating losses to public carriers, because of congestion and traffic delays in traveling to and from the central business district of Los Angeles, amounted to $15,000,000 annually. These were intangible losses. More tangible ones, due to reduction in property values, lost sales in retail stores in the area, traffic deaths and accidents, were not included in this estimate.

If adequate relief is to be achieved, expenditures of a very large order must be made, and made within the next few years, rather than over the next several decades. Because fuel and use taxes, even in increased sums, will be inadequate, the alternative would appear to be the issuance of bonds and the use of the increased tax revenue for debt service on the bonds.

Present interest rates are extremely low, and will be held so until the federal debt is greatly reduced. At current rates an annual revenue of one dollar over a period of from 40 to 50 years will pay interest and repay principal on from $25 to $30 in bonds, and allow an adequate sum for maintenance and operation of express highway facilities.

The cost of increased fuel taxes to a motor vehicle owner would be nominal. The average passenger automobile uses from 500 gal to 700 gal of gasoline per year. An increased gasoline tax of 2¢ per gal would cost the average owner from $10 to $14 per year, and would support from $250 to $420 in bonds for express highway purposes in metropolitan districts. Although savings in gasoline and tire mileage, and in the wear and tear on motor vehicles, may not entirely offset these costs, other and additional savings of a more intangible character, such as reduction in deaths and accidents, stabilization of property values, and reduction in travel time, will undoubtedly do more than offset the increase. The adoption of such a policy in the next decade, together with allocations from present state and federal revenues should fairly well satisfy the most pressing traffic needs of the metropolitan areas of the United States.

It is but fair and proper that motor vehicle owners outside of metropolitan areas contribute to the cost of express highway systems within these areas. In the past a substantial proportion of outside highways has been constructed and maintained by contributions from vehicle users operating within metropolitan areas. These contributions have been far out of proportion to the use made of outside highways by the urban and suburban contributors. Outside vehicle users will probably make as much use of metropolitan express highways as do the metropolitan users of the outside highways built through their financial aid.

If planners and highway engineers will give the same attention to financing improvements that they do to planning and designing them, the people will probably be able to enjoy them within a reasonably adequate time.

W. J. VAN LONDON,[11] ESQ.—The purpose of urban expressways is to provide fast and safe mass transportation for people and commodities within and across metropolitan areas. Mr. Barnett has presented a comprehensive discussion of this subject. Such transportation facilities are as much the responsibility of the state and federal governments as are any other arterial highways. The Post-War Highway Bill stipulates that a certain amount of the funds provided by the bill must be used in the development of expressways in urban areas. In Texas the amount is about $27,000,000 when matched equally with state funds—this provision being mandatory.

The Texas Highway Department is responsible for design and construction of urban expressways. All costs are paid from state and federal highway funds except the cost of right of way, which is paid by the respective cities. The general design is that adopted by the American Association of State Highway Officials and is the highest design standard ever adopted by that organization.

The thought seems to prevail that expressway highways are for the purpose of handling through highway traffic. This is far from true. A national survey of all towns in the United States with a population of more than 10,000 shows that only 10% of the traffic on highway routes in cities is through traffic. The remaining 90% is local to what might be termed the metropolitan area. The percentage of through and local traffic varies with the size of the city and the nature of city's enterprises. An interview survey made in 1939 of 35,000 vehicles conducted simultaneously on ten highways leading out of Houston disclosed that only 6% of all traffic was through highway traffic. An analysis of the first expressway proposed for Houston indicates that of 45,000 vehicles per day anticipated in 1948 about 1,000, or 2.2%, will be through highway traffic. Therefore, it appears that urban expressways are what the name implies—transportation facilities for urban traffic.

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[11] Engr.-Mgr., Houston Urban Expressways, Texas Highway Dept., Houston, Tex.

It is apparent that so-called by-pass or belt highways cannot be justified for through highway traffic alone. Also, the more directly routed expressways will more effectively handle the small volume of through highway traffic than would a by-pass highway.

Urban expressways do not generally follow existing heavily traveled streets or highway routes, principally because of high right-of-way cost through the developed areas adjacent to these improved streets. Also, these streets usually lead through the congested business area which it has been found best to by-pass a few blocks to avoid congesting business streets with traffic passing from one side of the metropolitan area to the other. This type of traffic is about one third of the total.

In general, the expressway projects will begin at a connection with one or more state highways at the edge of the urban area and pass near, or through, the central business area to a similar connection at the opposite edge of the urban area. From the beginning to a point near the business area they will generally consist of four or six divided freeway lanes, with property service roads as required. All railroads will be passed under or over. Minor streets will be closed and major streets will pass under or over, with adequate access roads to provide for interchange of traffic. Design speed on the freeway lanes is 50 miles per hr. Near the central business area the traffic will be spread to several existing streets which will be used for a distribution system. Probably two of these streets leading directly to the expressway section beyond the business area will be developed as one-way streets.

The value of freeways is often questioned, the contention being that proper traffic light control at street intersection is almost as good as a grade separation. A perfect light-controlled intersection will handle 42% of the traffic that a grade separation will handle, which is probably a sufficient answer to the question.

The construction cost of the expressways contemplated will range from \$750,000 to \$1,000,000 per mile. The right of way will cost from \$50,000 to \$100,000 per mile. Immediately the question arises—are the expressways worth the money? The only justification for a public improvement of this nature is its economic value. A time-travel analysis on an existing route and one proposed project in Houston which will carry 25,000 vehicles per day shows a time saving of 12 min between two common points. Assuming 15 miles per gal for gasoline consumption and a cost of 20¢ per gal, the fuel cost is 1¢ per min at 45 miles per hr—the anticipated expressway speed. Computing 365 days at 25,000 vehicles gives 9,125,000 vehicles per yr. A saving of 12¢ per vehicle would be \$1,095,000. The section of expressway analyzed will cost between four and five million dollars. The saving in fuel cost alone will pay for the improvement in less than 5 years. If other cost factors are considered, such as drivers' and passengers' time, reduced wear on equipment, ownership cost, wages to operators of commercial vehicles, earning value of commercial vehicles, etc., and an estimate of 3¢ per min is applied, the saving to the public would pay for the improvement in less than 2 years.

The planning of an urban expressway system, the over-all design, and the detail design features to provide fast, safe, and economical mass transportation for persons and commodities require very careful consideration of many factors. The first step in expressway development is to determine where and what the traffic is now, and where and what it will probably be 25 years hence. A route is then selected and a more detailed study made as to the present and probable future development of the affected areas. From these studies the location of street-grade separations, traffic interchanges, the number of traffic lanes required, etc., are determined. Little time and effort is required to make this statement but many months of tabulation and study of reports and data obtained from the various utility companies and bus companies; census bureau reports; city planning commission reports; chamber of commerce reports; past, present, and probable future population; and business and industrial trends; etc., as well as actual traffic counts and interviews are necessary to determine the "load for which the expressway structure should be designed." From these data, traffic flow maps are developed to determine where and what kind of traffic facilities are required. The proposed expressways which are expected to be completed by 1949 are designed to handle 1957 traffic loads.

The structural design features will also be based on the load to be carried. Bridge designs provide for the maximum anticipated loads. Earth and pavement structures will be designed for the maximum anticipated loads with a safety factor of about 1.5. The maximum permanent bearing value of the respective soils will be developed and the maximum anticipated traffic loads will be properly distributed over the soil foundation structures through various pavement structures. The types and designs of pavement structures will be governed largely by economic considerations. The cost of the earth structures, and of pavement structures, will be much higher than that for the usual structures of this kind; however, they will more nearly approach permanency and the cost will be justified by their long and uninterrupted service to large volumes of traffic.

To a considerable extent the foregoing is a specific application of the general principles set forth in this paper. The traffic analyses and the economic evaluation of specific cases in Houston afford conclusive evidence of the soundness of Mr. Barnett's presentation of these matters.

Merrill D. Knight, Jr.,[12] Assoc. M. ASCE.—A natural and rational approach to the problem of express highway planning is presented in this paper. One element in the location and the design of expressways not sufficiently covered is their relation to land use. Too often in the past the zoning plan for a community has not been given the consideration which its importance in the over-all development would seem to justify.

Although origin and destination surveys have become recognized as prime tools in expressway design, these reflect only an existing condition and must be correlated with present and future land use before the analysis will achieve its greatest value in the planning process. In the Savannah, Ga., study, origin and destination surveys were used with due consideration for industrial expansion. However, the procedure still requires some refinements. It is suggested that further investigations are desirable to determine if specific land uses, when located in respect to other specific land uses, may be used as guides to changes which may be anticipated in traffic flow as the metropolitan area develops.

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[12] Highway Engr., Public Roads Administration, Washington, D. C.

In the use of zoning maps in a location study, it is believed inadvisable to give too much importance to the consideration of expressways as barriers between zones of incompatible uses. Such a factor as an influence in design has been suggested by a number of engineers, notably, by Jac L. Gubbels.[13] Although expressways may assist in stabilizing values through the segregation of areas with comparable restrictions, this is certainly not their primary function. The primary function of moving traffic between areas of major generators immediately suggests that there should be a very real relationship between the highway plan and the land-use plan. To present a complete analysis of this relationship would require setting values on classified land uses as potential traffic generators.

Probably the greatest obstacle to such an analysis lies in the basis on which many zoning ordinances have been created. A typical city will undertake zoning primarily to protect property values. The first map to be drawn will be one showing existing uses. If retail businesses are widely scattered, the value of adjoining properties has already been depreciated for residential purposes. To avoid hardships resulting in the creation of districts containing too many nonconforming uses, the tendency is to allow larger areas with fewer restrictions. This variation from the optimum allocation of land to classified uses admittedly interposes difficulties in estimating future traffic to be generated by areas presumably planned for specified uses. Such difficulties would be inherent in any city plan possessing sufficient flexibility to permit future growth. To make a plan effective all components should be developed toward a common objective.

Frank H. Malley[14] states that "* * * as cities grow the downtown area remains relatively constant in size as compared with the urban area." He believes that "* * * the proper location and design of Urban Freeways is the greatest single element in the cure of cities' ills and in the directing of their proper and adequate future growth." The determination of the pattern for future urban growth constitutes the most complex phase of selecting locations for proposed expressways.

The Regional Planning Board for the County of Los Angeles (Calif.) in a discussion of the "Master Plan for Highways, 1941"[15] explored, on an over-all basis, the highway requirements as related to the various land uses within the entire area. To do this a pattern of land use was developed, toward which it was believed the ultimate growth of the region should be directed. However, the pattern fixed as a goal for Los Angeles was based on a desire to avoid an overcentralization which has already become a fact in other large metropolitan areas. Hence, it cannot be used as a pattern for many other places. In the

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[13] "Planning for City Highway Traffic," by Jac L. Gubbels, *Better Roads*, October, 1945, p. 29.

[14] "Location and Function of Urban Freeways," by Frank H. Malley, Postwar Patterns of City Growth, Am. Transit Assn., p. 18.

[15] "Master Plan for Highways, 1941," Regional Planning Board, County of Los Angeles, Los Angeles, Calif., 1941.

1941 plan, Los Angeles expressways were proposed to serve more as intercommunity highways than as a means of moving people from outlying areas to one or two congested centers. Other cities can benefit from a study of the Los Angeles approach, even though it is not entirely applicable to their problems.

Keeping in mind the necessity for the transportation of workers between their homes and places of employment, it is important to stress that, in assembling basic data, an analysis of land use, both existing and potential, when properly correlated with origin and destination surveys, can be of the highest value in selecting the route which will best meet present and prospective needs. To increase the significance of such analyses, however, further study is required before making any attempt to assign empirical values to specific land uses as traffic generators.

R. H. BALDOCK,[16] ESQ.—A remarkably clear explanation of the present need for urban traffic facilities permitting uninterrupted flow and a proper relating of these facilities to local service streets and terminal facilities are given in this paper. Mr. Barnett mentions the ever-increasing realization that intelligent planning must rely on factual information in regard to the needs and desires of the populace. These factual bases can best be obtained through the medium of comprehensive origin and destination studies, the techniques of which are outlined briefly. Indeed, the data assembled can be used in an infinite number of ways to solve the many and varied traffic problems in a metropolitan city. Mr. Barnett makes an interesting statement to the effect that the development of a pattern of arterial routes results in a remarkably uniform solution for cities of comparable size.

The writer is in full accord with the thoughts expressed in the paper. However, he takes the occasion to stress the following conditions:

1. The current pressing need for adequate terminal facilities to provide storage for varying periods of time, particularly in and near the central business district.
2. The need for improved and adequate mass transportation facilities to attract a greater proportion of people.
3. The job of selling a bold and forthright plan of express highways and terminal facilities to the local citizenry through detailing and publicizing the economic benefits that will accrue.

The writer has stressed the matter of terminal facilities not because this phase should necessarily carry a disproportionate weight in an over-all program but rather because deliberations and planning with respect to express highways have far outstripped planning for terminal facilities. Notwithstanding the fact that there is still much to be done in the matter of planning arterial expressways, it would seem that more detailed study should be accorded the problem of terminal facilities to bring it abreast, so to speak, of progress in the matter of arterial development. The construction of expressways will divert and consolidate a large part of traffic on parallel streets between major points of origin and destination. In addition, it will generate a new and additional increment of traffic by the very reason of the improved facility. This latter increment of traffic will operate to complicate the present acute parking situation even further.

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[16] State Highway Engr., Salem, Ore.

Moreover, an extended period of time will be required to build all the express highways and circumferential routes in a given metropolitan area and the present critical parking problem cannot wait. The terminal parking problem needs to be solved as part of the completed plan as far as feasible. It is further obvious that private enterprise cannot very well conduct the requisite preliminary studies for an intelligent appraisal of the parking problem.

In general, parking locations should be widely dispersed in and around the business district to prevent congestion on adjacent streets and to serve the public better. It may be found necessary for the city to acquire property for development of off-street parking facilities through the power of condemnation, if necessary, and to operate such facilities through leases to private individuals. Real estate values, and the economic requisite of realizing a fair return on the investment, may necessitate building a one-story or two-story structure to increase the parking space. The upper floor or floors can be made accessible by ramps and, where topography permits, by entrances from two different streets at different levels. In other cases, it may be desirable either to build or to give a long-period lease for the erection of a modern office building—the interior of which is utilized either through ramps or through elevators for the parking of motor vehicles. Soundproof and fireproof construction can be used; and, if the architecture is attractive, the utilization of the core of the building for car storage should not detract from the value of the office space around the exterior of the building. To accomplish these improvements, it may be necessary, by state legislation and city ordinances, to establish a parking authority and to provide funds for the capital investment and the acquisition of sites to be amortized through rentals over a period of years.

Mr. Barnett's statement that the truck-loading problem will be solved only through a bold approach is correct. Truck-terminal and loading-dock space inside the building line of the ultimate destination of goods will eliminate first-floor street frontage for truck unloading—an important observation. The incorporation in city ordinances of off-street parking and loading facilities as a condition precedent to new building construction is still in the pioneering stage; however, such a measure appears to be the only permanent solution of this aggravating problem.

Turning to mass transportation and its bearing on the problem, Mr. Barnett states (under the heading, "Planning Agencies"):

> "If the major highways are developed as free-flowing expressways, the tempo of transportation in the city can be increased with favorable results for all. The writer cannot agree with visionary enthusiasts who believe that freeing transportation will of itself rehabilitate a city, although it can be of material aid. The forces of decentralization are too great to be stopped entirely, but the proper location and development of expressways can assist in the establishment of self-contained stable neighborhoods and in the stabilization of trade and values in the principal or central business district."

The increase in the use of automobiles by people working or trading in the central business district cannot proceed to the point where every movement takes place through the use of a personal conveyance. It may be impossible to build the requisite expressways, and certainly it is impossible to construct the required terminal facilities to care for a greatly expanded use of private automobiles—nor is such use economical.

There appears to be a demand for greater use of mass-transportation facilities. The building of expressways and properly located bus terminals will facilitate this trend; and it will give to the user greater safety and almost equivalent speed and comfort of movement. Express and skip-stop bus service can be inaugurated on expressways from suburban districts into the heart of the city. The modern city planner should give careful consideration to this phase of transportation inasmuch as it appears to offer an excellent solution to the pressing problem of traffic congestion in regard to both movement and vehicle storage. If a proper solution is reached, many citizens using their own personal automobiles between the suburbs and the central business district will travel by mass transportation, for convenience and economy. It may, of course, be necessary to use the automobiles from the home to a suburban bus terminal. The lower land values, in such cases, should make possible car storage at reasonable rates.

In the selection of the type of mass-transportation vehicle, the street car running on rails cannot be used because of its inability to conform to the patterns of the traffic stream. Likewise, although the trolley bus is more flexible than the street car, its range of movement is definitely limited, and at times it becomes an obstruction to the free flow of traffic. The expense of erecting and maintaining the overhead wiring system is material and this type of construction detracts from the beauty of a well-designed expressway. With adequately powered motor buses, discomfort can be materially lessened while the freedom of movement makes for ideal operation in a stream of other motor vehicles.

Turning to the job of "selling" the local citizenry on the economic and social benefits accruing from building express highways and providing for adequate terminal facilities, Mr. Barnett has stated that the preliminary engineering report should describe fully the reasons for the expenditures proposed. Although many of the benefits accruing to the public by reason of the building of adequate express highways and terminal facilities are intangible and difficult to evaluate, in most instances it is possible to show that the yearly monetary benefits are much greater than the annual cost of amortizing and maintaining the facilities.

The annual road-user benefits, represented by the saving in cost of motor vehicle operation, the saving in time, and the saving by accident reduction, can be determined. When expressways have been planned carefully, based on origin and destination studies, the amount of these benefits accruing to a large volume of traffic is surprising.

As Mr. Barnett has stated, the direct road-user benefits are not the only returns. Intangible benefits which reduce driving strain in many cases far outweigh in the public mind the monetary values which accrue. An economic analysis properly presented to progressive civic groups, indicating the direct returns on the investment as well as the many indirect benefits which materialize, should sell such a program even to the more skeptical. In the matter of "selling" the program to a civic group, it is probable that engineers working on the technical phases are somewhat reticent to dramatize supporting data, and thereby fall somewhat short of making a case that will appeal to, and interest, a lay group.

JACOB FELD,[17] M. ASCE.—By assembling and organizing the pertinent data as well as the differences of opinion covering all phases of the problems involved in the planning of express highways through congested areas, the author has prepared a valuable contribution. There is considerable published literature on individual phases of the problem and even more on the designs prepared for special locations. However, a paper of this type which looks upon the problem as a whole, without any special reference to a definite locality, provides a framework for the use of the designer working on any phase of this problem.

Proper planning and execution of express highways will reduce congested traffic and expedite the flow of vehicles through urban areas. However, it would be uneconomical to attempt a complete elimination of congestion in large cities. As a matter of fact, the very basis of city existence is congestion and an efficient city cannot exist without large concentrations of vehicles. The popular expression for an inefficient city, where concentration has been reduced to the point of elimination of congestion, is "an overgrown village." People expect congestion when they enter cities and to some extent would be disappointed if they found none—for instance, the tendency of crowds to congregate on certain streets in every city, most of the sight-seers having no particular business in those streets.

An attempt to design through arteries for every possible concentration of traffic (such as that occurring for a few hours each week end during vacation months in a comparatively few roads leading to every large city) would be a tremendous waste of the facilities for practically 99% of the time. Fig. 7 shows the traffic congestion along the West Side Highway in Manhattan (New York, N. Y.) during the afternoon of Navy Day in 1945. Practically the same degree of congestion exists for about two hours on this highway every Sunday afternoon when the weather is favorable. Possibly a doubling of the capacity might eliminate this congestion for these short periods but it would not be economical to provide such additional facilities.

One method to reduce incidental peak concentrations would be to revise the method of marking through routes. From personal experience, it has also seemed to the writer that it is a mistake to mark certain streets going through a city with a definite route number—especially where a number of routes converge into a city center and then diverge on the opposite side of the center. It would help traffic if the general direction routing was provided for the motorist rather than a definite route number. For instance, a number of streets leading from the outskirts on the west side of the city could be marked east traffic and a very simple sign in the form of an "E" with an arrow could be easily identified. Traffic would then split through a number of streets rather than concentrate on a single route. Such marking would be advisable even where a through artery is provided to take the overflow during the congested periods.

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[17] Cons. Engr., New York, N. Y.

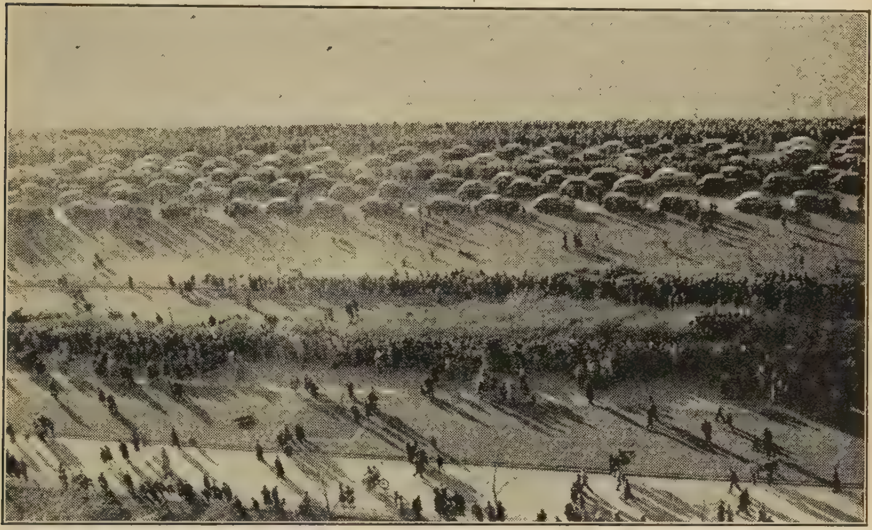

Fig. 7

The writer has found that much better time and less nerve-wracking driving results from by-passing marked routes. For instance, in the City of Waterbury (Conn.), all routes intersect at the center of the city. The provision of a "throughway" for all traffic, converging on the center from practically eight different directions would be uneconomical. A circular or square connecting highway through the outskirts of the city would help the city congestion very little (as the author states), because most of the traffic is going to different parts of the city. However, provision of a throughway going east and west and one going north and south, of minor traffic capacity, with roughly parallel streets marked with direction would solve the problem. This solution is similar to a design of a main pipe line with parallel by-passes for surplus or surge flow. A similar analogy can be found in multiple electrical connections in a grid system.

Urban traffic concentrations have been studied by traffic divisions of the New York City Police Department for many years. Much information can be obtained on peculiarities of traffic congestion from those sources. For instance, in 1936 the writer was organizing construction of a part of the Sixth Avenue

Subway in Manhattan, and among other things desired to find which days would have the least traffic concentration. The street had to be closed for short periods at a time and to avoid too much interference with the local business people it was thought advisable to plan the work so as to close the street on the lightest traffic days. Of course, streets are very often closed for repaving and for other construction, and traffic somehow finds its way around and takes care of itself. On the other hand, the experience of the police indicated that in retail areas like that part of Sixth Avenue, vehicular traffic was heaviest on Fridays.

The question was important enough to warrant the expenditure of some time, and traffic counts were made from April to December, 1936. Actual count of the number of vehicles passing the northerly intersection of 41st Street and the northerly intersection of 45th Street, on Sixth Avenue, were made for a period of 5 min each hour from 8 a.m. to 6 p.m. on six successive days (except Sunday) starting on the fifteenth of each month. Fig. 8 shows the variation of traffic averages for various months. At the time of the count, the roadway width of Sixth Avenue was 60 ft with an elevated structure in the middle of the street. The clear distance between elevated columns was 22 ft with unused trolley tracks in this part. Each side road was 17 ft in clear width. Theoretically, parking was prohibited, but actually delivery trucks and some private cars were almost always at each curb at each block. The results of the traffic count were used to schedule the main closure of the streets for the summer months when traffic concentration was least and also to avoid closing the streets on Fridays as much as possible, even though the increase in traffic counts for Fridays was not very much above the other days of the week.

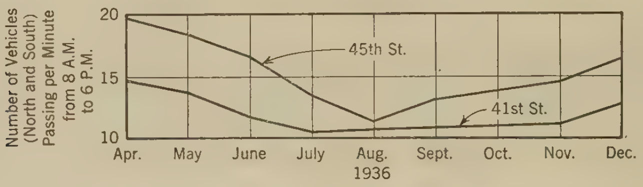

Fig. 8.—Traffic Count on Sixth Avenue, Manhattan

This example is cited to emphasize the difference in traffic concentration when commercial traffic is being considered rather than pleasure vehicles. The fact that truck concentration is not at a peak during week ends is often forgotten because of individual experiences of being tied up in vehicular traffic on Sundays.

There are two schools of thought on the location of interchanges between an express highway and intersecting main streets. One group feels that the interchanges should lead directly into the main street, to avoid addition of traffic in adjacent and usual residential areas. The opposing thought is to connect the interchange into subsidiary streets to allow the traffic leading or entering the expressway to filter through supplementary streets and thereby reduce congestion on the main streets, which usually already carry considerable traffic. The decision is difficult to make, especially where the main street is bounded on each side by residential land use and often by heavily concentrated populations. The meandering of traffic entering or leaving expressway streets serving apartment houses adds seriously to the traffic accident toll. From the safety point of view, it might be better to force all traffic on to business streets even though some congestion results.

The author must be complimented for the very clear manner in which he has analyzed this entire problem and for the open-minded discussion of different solutions for each phase. The design and construction of express highways is becoming a considerable part of the engineering and construction fields. The crystallization of thought and opinion at this time is therefore most useful.

Harold M. Lewis,[18] M. ASCE.—The need of coordinating main arterial highways with general city plans, and of planning suitable off-street terminals for passenger cars and trucks in urban centers, is clearly demonstrated by this important paper. The writer has found that many highway engineers are interested in city planning, but more of them should be; in turn, city planners should give more attention to the economic justification of the highways they propose. A master plan for major urban thoroughfares should do more than create a pattern of loops and radials; these latter should be designed to meet local demands in location, character, and capacity.

The initial development of expressways in the United States started with parkways designed to carry heavy passenger-car traffic through suburban areas. In a few cases such special routes may extend far enough to become interurban; but, essentially, the parkway has remained a feature of metropolitan development. Postwar highway plans, such as those in New York and California, will place more emphasis on interurban expressways for all types of traffic—passenger cars, buses, and trucks.

Most expressway problems still requiring engineering research arise in urban centers, where, as Mr. Barnett states, the proper location and design of "ons" and "offs" are important. These can be closer together on the edges of the business district than in suburban areas where their location is fixed primarily by that of major intersecting highways along the express route. An excellent example of the downtown loop, typical in a pattern for large cities, will be provided in Los Angeles, Calif., under its expressway program—partly completed, with other sections under construction, but mostly still in the planning stage.

At a "transportation clinic" held at Los Angeles in December, 1945, the writer presented an analysis of the relative proportions of through (by-pass) traffic that might use these loop expressways as compared with traffic that would use them for entering (or leaving) the central business district. It was recommended that entrances and exits be provided at twelve points. Fig. 9 indicates that the heaviest by-pass traffic would be along the north and south

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[18] Cons. Engr., New York, N. Y.

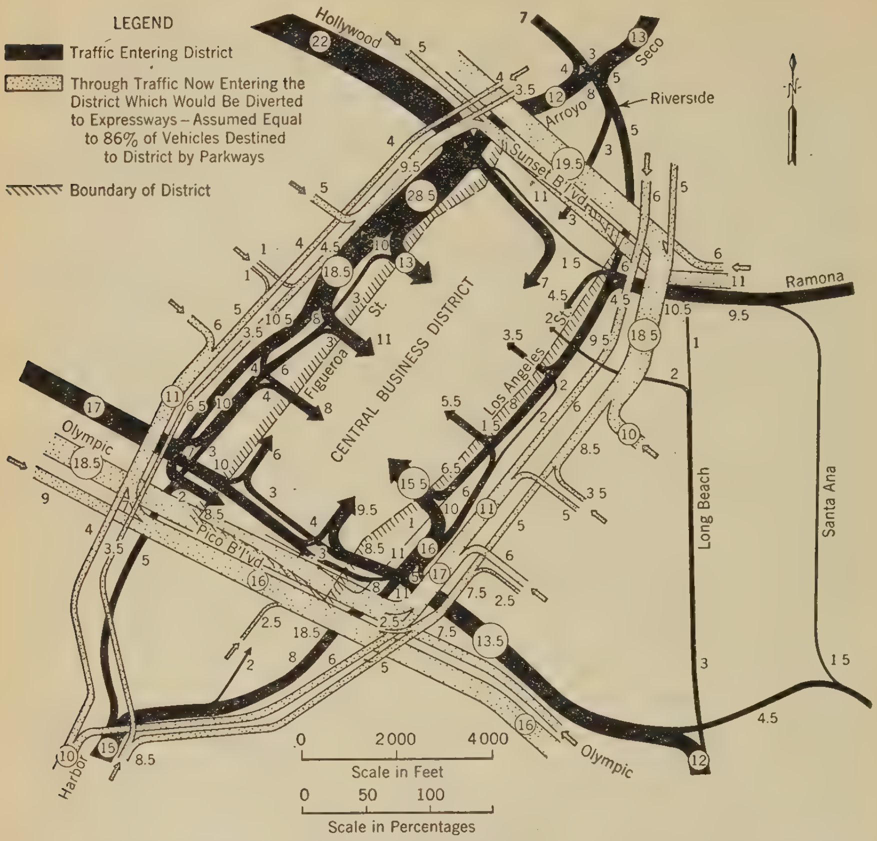
LEGEND
Traffic Entering District
Through Traffic Now Entering the District Which Would Be Diverted to Expressways – Assumed Equal to 86% of Vehicles Destined to District by Parkways
Boundary of District
Hollywood
Arroyo
Seco
Riverside
Sunset B'l'vd
Ramona
Figueroa St.
CENTRAL BUSINESS DISTRICT
Los Angeles St.
Olympic
Pico B'l'vd
Long Beach
Santa Ana
Harbor
N
0 2000 4000
Scale in Feet
0 50 100
Scale in Percentages

Fig. 9.—Estimated Distribution of Passenger Vehicles Entering and By-Passing the Central Business District Shown as Percentages of Total Entering by Parkways with Present Distribution of Population

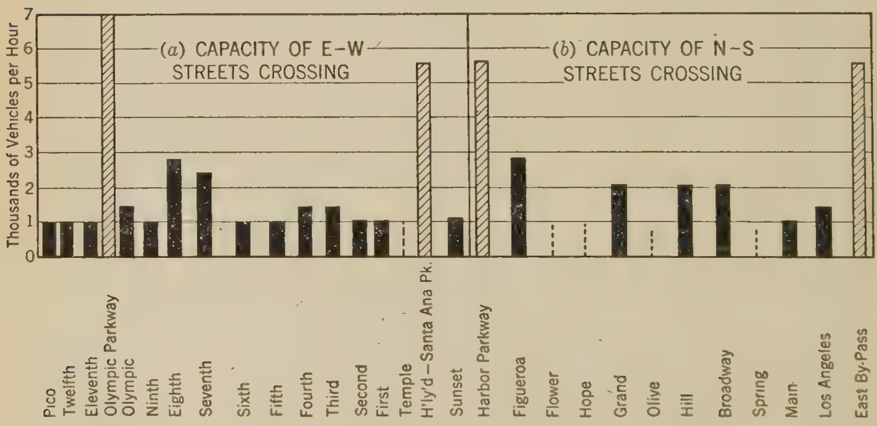

Fig. 10

sides of the loop and that the easterly section, known as the "East Bypass," would carry more through traffic than the westerly section, a part of the "Harbor Parkway." Of the through traffic now entering the district, the part destined to be diverted to expressways was assumed equal to 86% of all the vehicles entering the district from parkways. This assumption and the distribution of vehicles among the quadrants of the area were based on origin and cordon counts made in 1939. For the location of "on and off" points it was assumed that equal traffic would move in reverse directions.

It was proposed that the East Bypass be used for all types of traffic—passenger cars, buses, and trucks—but that other sections of the loop be limited to passenger cars and buses.

Fig. 10 indicates the increase that the loop expressways would provide in the capacity for traffic moving across or along the edges of the central business district. The solid bars show the hourly load that could be carried over existing, continuous streets crossing this district at their points of minimum capacity. The hatched bars show the additional capacity that would be provided by the expressways. Assuming that the Olympic Parkway (the southern section of the loop) were to be built with eight traffic lanes and the other elements of the loop had six traffic lanes, the number of lanes available for through east-west movement would be increased by 39% and their capacity by 63%. The number of lanes available for through north-south movement (now limited by the fact that four existing streets extend only part way through the district, however) would be increased by 60% and their capacity by 100%.

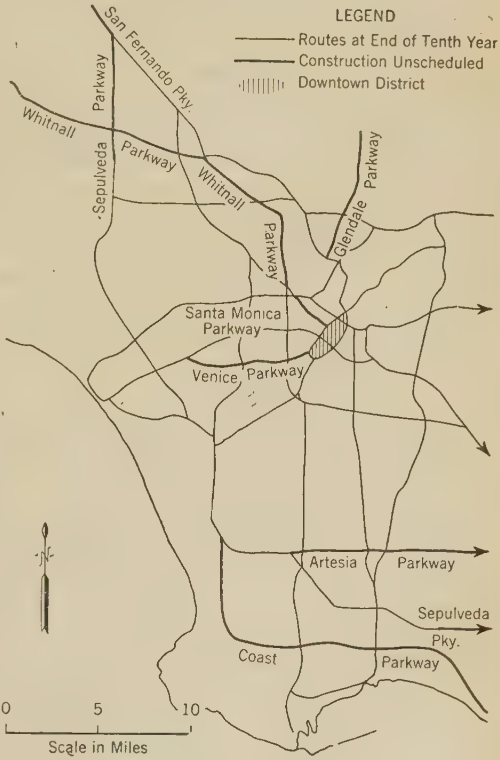
LEGEND
Routes at End of Tenth Year
Construction Unscheduled
Downtown District
San Fernando Pky.
Parkway
Whitnall
Parkway
Sepulveda
Whitnall
Glendale
Parkway
Parkway
Santa Monica
Parkway
Venice Parkway
Artesia
Parkway
Sepulveda
Pky.
Coast
Parkway
0
5
10
Scale in Miles

Fig. 11.—Expressway Program of the City of Los Angeles

In addition to about ten miles of existing routes, the complete system of expressways for Los Angeles city and county involves 358 route miles and a total cost of $582,000,000; of this, about 220 miles costing $400,000,000 will be within the city. Of the proposed system, 287 miles have been scheduled under a ten-year program; the remainder are unscheduled. The total of existing and proposed routes, differentiating between those scheduled and unscheduled, is shown in Fig. 11.

The panel of consultants taking part in the "clinic" agreed that about 54 miles of the expressway system should have set aside, as a part of the cross section of the routes, separate rights of way for mass transportation vehicles. The writer proposed that a little more than one third of the 297 miles existing and scheduled should have roadways available to passenger cars, trucks, and buses, and that an approximately equal mileage should be reserved for passenger vehicles. Of the remainder, both private passenger cars and a limited number of express buses might be permitted to use the roadways.

A check on Mr. Barnett's statement that the "By-passable traffic is about 20% in cities with populations of from 10,000 to 300,000" is provided by a survey made on a Saturday afternoon in April, 1946, at Glens Falls, N. Y. (1940 population 18,720). For a 2-hour period license numbers were recorded of all vehicles entering and leaving by the four main highways. Within this period it was found that 19.8% of the vehicles checked crossed the city and left again in a different observed direction. About half of this through traffic passed through without stopping and half stopped for a business or other errand. On a typical summer week end, when Glens Falls is traversed by through traffic to or from the Adirondacks, it is expected that the proportion of through traffic would be considerably higher and a second and longer count was made on July thirteenth to determine what the result would be.

THEODORE T. McCROSKY,[19] M. ASCE.—This paper contains a very able and complete statement of the need for express arteries through large cities and metropolitan regions, rather than up to their boundaries or by-passing them. The United States Public Roads Administration is empowered to make grants-in-aid for urban arteries, thus enabling many more states and cities to undertake this type of construction, with its heavy but necessary costs, than was possible when federal aid was restricted to highways outside city limits. It will be recalled, as a direct parallel, that many states did not traditionally contribute to the cost of highways within incorporated municipal jurisdictions. The change of state and federal financial policy now makes it possible to construct the highest type of traffic routes progressively outward from the urban center. Previously they generally had to be built inward from open country, with the center itself (where traffic is most congested) too often left with no arteries of modern free-flowing design As the author has stated, traffic load diminishes rapidly with distance from principal cities, so that the policy of building outward rather than inward is manifestly correct from a technical standpoint.

The proposed central artery through downtown Boston, Mass., is a case in point. It will receive the heavy streams of entering surface traffic, converging from the northwest and south toward its extremities, and lead them to, or through, the heart of the city. This project is sponsored by the city planning board, and the 1946 session of the state legislature authorized the Post-War

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[19] Executive Director, Greater Boston Development Committee, Inc., Boston, Mass.

Highway Commission to draw upon the highway fund (gasoline tax revenue) for preliminary surveys and engineering. It is planned to provide express connections with the Sumner vehicular tunnel leading to East Boston, and with the twin tube of this tunnel when constructed. Extension of the Central Artery outward beyond its currently planned termini might be a matter for future consideration, but is by no means as urgent as the construction of the downtown section.

The proposed Boston Central Artery also furnishes an excellent example of selecting right of way through a blighted "fringe area," as defined by the author, which is only a few blocks from principal central destinations.

The writer wishes to stress the importance of careful traffic counts and estimates for peak 15-min and 30-min periods, as a necessary basis for the sound determination of the correct number of vehicle lanes to provide for express highways. General, 24-hr counts are important in estimating total flow, particularly for toll facilities; but they do not answer the question of critical traffic loads that may result in congestion.

When serving as executive director of the Chicago (Ill.) Plan Commission in 1941 and 1942, the writer was one of the exponents of an "inner belt" expressway to distribute traffic from all radials. This inner belt utilized the existing Wacker Drive and Outer Drive and provided for completing the circuit. "The Preliminary Comprehensive Master Plan"[20] retains this important feature of the over-all future highway pattern.

Downtown off-street parking facilities for passenger cars are coming to be recognized as intrinsic parts, not only of comprehensive city planning, but of specific actual highway planning, design, and construction. When an express artery skirts close to the business center, parking spaces or garages should be close to the artery. Sometimes, they may be physically connected with it, so that cars can leave the highway and enter a garage without utilizing local surface streets. This arrangement is possible in cases where an elevated expressway (rather than the preferable depressed expressway) is required for compelling reasons.

With the inevitable traffic increases that every city must provide for, over the next 10 to 20 years, it is self-evident that maximum utilization must be made of the existing investment in surface streets, whether or not free-flowing arteries are constructed. Thus, city roadways must be free of parked cars to release curb-side lanes for moving cars; and in the daytime trucks must eventually be loaded and unloaded within building lines.

The writer considers that the great cost of express highways in downtown areas is better justified if these modern arteries are made available for commercial traffic, than if they are reserved exclusively for passenger cars. Commercial traffic includes both trucks and public transportation buses. It is argued sometimes that placing passenger cars on the new expressway will eliminate congestion on the old major surface street and make it entirely adequate for trucks. This argument neglects consideration of the delays at traffic lights and the fact that, to the truck operator, time lost is actually money "out-of-pocket"; whereas to the passenger-car driver, it is a case of time saved possibly being money "in-the-pocket."

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[20] "The Preliminary Comprehensive Master Plan," Chicago Plan Comm., Chicago, Ill., January, 1946.

A cogent reason for permitting buses on urban expressways is that no city (to the writer's knowledge, at least) has enough high-speed public transportation. A new expressway should thus be conceived as an addition to the public transportation network, and not as a competing facility that will encourage car owners to engage in needless driving, in lieu of utilizing the more economical mass transportation system. In appropriate cases the right of way should also include rapid-transit tracks.

If, for local reasons, an interim decision is made prohibiting commercial traffic on a planned express highway, there should nevertheless be no feature of the design that will later preclude a change of policy. Thus grades, curvature, clearances, and structural strength should all be designed on the basis of eventual use by heavy trucks and buses.

Because of the high capital cost of free-flowing express highways, even under favorable conditions of low land price, careful consideration should be given the policy of defraying at least a part of the debt service by tolls. In downtown districts where trips are short and ramps occur frequently, this will not generally be feasible from an operating standpoint. As the average trip lengthens, and with the wider spacing of ramps that is indicated for the outskirts of metropolitan areas, the feasibility of tolls increases. On highways, the driver who objects to a toll will not be required to pay it. He can always penalize himself, if he so desires, by using the slower local thoroughfare.

The author has stressed the great value of economic analysis as justification for proposed projects. Time saved, multiplied by traffic load, multiplied by value of time gives dollars saved. For the majority of passenger-car users, it is believed that the simple saving of time is what will determine their views—or vote—on a particular planned facility. In 1940, the New York City (N. Y.) Planning Commission prepared a city-wide plan of express highways, so located that no important origin or destination center was more than 1½ miles from one of these express routes. At that time Mr. Barnett made many valuable and constructive suggestions. The writer calculated that a trip of 12 miles would mean 1½ miles at each end on surface streets, and 9 miles by express highway. The 3 miles on local streets would take 15 min at an average speed of 12 miles per hr; and the 9 miles would take 15 min at an average speed of 36 miles per hr. Without express highways, one would have to go the entire distance on local streets at about 12 miles per hr—requiring 60 min. With the express highways, the same trip would take only 30 min, thus saving half of the previous running time.

The author has made a noteworthy contribution to the literature of urban highway planning. The translation into construction of the principles he has stated will go far to relieve the critical traffic congestion that faces every major American metropolitan region.

Spencer A. Snook,[21] M. ASCE.—It is an opportune time to emphasize the need of arterial highways in cities. The detailed requirements of existing data have been stated in papers by Fred Lavis, M. ASCE, and others,[22] and have been in use for a number of years, especially in New Jersey and within the New York metropolitan area. Nevertheless, it seems that arterial highways in most cities have been deliberately avoided, primarily because of cost; and so, by-passes have been built. These served the purpose for the through travel but totally neglected, in most cases, transportation in larger cities of urban commercial and commuter traffic. This problem should be concentrated upon, as usually the cities cannot afford to handle it. Most of the motor taxes come from these areas although the least money has been spent there.

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[21] Asst. Engr., N.Y.C.R.R., Cleveland, Ohio.

Obviously, congestion of traffic is the lack of free-flowing traffic; and arterial highways, with access at frequent points, will reduce the cost and time for this urban traffic. There are still plenty of useless, time-consuming traffic lights that could be of the actuated type for cross streets.

Elevated highways with proper side roadways do not make a bad appearance, if architecturally treated. They compare favorably in costs with the depressed highway depending upon local conditions. The traffic noise is not as great and the elevation assists the driver in knowing where he is going.

It is always of utmost importance to have the controls of the structural features definitely set, before the detailed plans and designs are started. Frequently plans that have been started are discarded because some controls are changed, and hence the total cost is increased.

The number of public planning agencies, by whatever name, having jurisdiction in one city should be as few as possible as too many cause "jams" in progress because of inability to agree.

LAWRENCE S. WATERBURY,[23] M. ASCE.—As Mr. Barnett has stated, in the rural areas there is a fairly good system of highways which, despite certain inadequacies, has served enormous traffic movements. However, it has been impossible, in many instances, to maintain these highways properly during the war period, and many of them have been quite heavily overloaded. As a result, it will be necessary to rebuild some of these, to construct additional links in the rural system of highways, and to improve many of them in accordance with modern design standards.

Between most of the large centers of population there are not too many serious delays encountered by traffic traveling over these highways. The cities, however, present quite a different situation. Traffic congestion and suggestions for its relief are in the minds of many people in the urban areas. Everyone is affected by congested traffic conditions—either as pedestrian, driver of a motor vehicle, or as a passenger using the mass-transportation facilities. Traffic congestion is something the average person encounters, in one way or another, causing him delay or inconvenience. He is naturally thinking of improvements and may have suggestions for relief of any intolerable conditions. He brings pressure on various civic groups, with the result that traffic committees in many of these civic organizations do some very constructive and helpful work in preparing plans for the relief of congestion.

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[22] "Highway Economies," by S. Johannesson, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

[23] Associate Engr., Parsons, Brinckerhoff, Hogan & Macdonald, New York, N. Y.

As has been stated, congested traffic conditions probably have been a factor in the deterioration of parts of the cities in the United States.

The need for relief in the urban areas has been recognized in the Federal Highway Act of 1944, which earmarked certain funds for use by the state highway departments in the development of modern highway facilities in the urban areas.

A free-flowing artery in the thickly settled city areas costs a considerable sum of money to construct, and the property damage costs are high. Naturally, it is impossible to construct these needed facilities overnight; it is necessary to construct them in stages, spreading the cost over a period of years. The sections that require the greatest relief should have first priority, and in the majority of cases these seem to be the most expensive in cost per mile of highway. In any business in which vast sums of money are to be invested, the investor would first investigate the economic justification before embarking on the venture. Since it is important for highway planners to be certain that they are proceeding in the right direction, they must collect and analyze, carefully, all the data bearing on the problem, and test all the possible solutions.

Mr. Barnett has emphasized the usefulness of a preliminary engineering report for testing these solutions and for the purpose of uniting all interested agencies in the consummation and approval of a plan for an expressway. This certainly should be the first step in express highway planning, since certain general principles of location and design would be covered in such a report.

Prior to any consideration of expressway location, a thorough study should be made of traffic characteristics of the urban areas. This will include not only a determination of the volume of traffic using the present facilities, but also a complete analysis of the origin and destination of this traffic. It will also be necessary to predict what the future pattern of traffic may be; and, when the most suitable location is decided upon, the volume of traffic that may be expected to use the new facility, must be estimated.

The existing land use of the urban area must be studied, and consideration given to future changes that may be expected as a matter of natural development, or desired as a part of the city improvement plans. These changes are certain to affect the future traffic pattern. A thorough analysis of land use in its relation to traffic is required, so that expressway locations will aid in the future favorable development of the urban area and serve the traffic needs adequately. Population studies, present and future, must be made at the same time as land uses are analyzed, as this too will be reflected in the future traffic pattern.

Mr. Barnett has reminded his readers that the factual data regarding the movement and the origin and destination of motor vehicles leave much to be desired and give the highway planner few clues regarding intracity traffic patterns. However, he must utilize whatever data are available, then supplement these with additional traffic surveys and research. The author has mentioned one method of securing these data, by home interviews, which is being given some rather severe tests for adequacy and accuracy. There are other methods of making urban traffic surveys which will also yield sufficient data to make forecasts.

It is impossible to suggest a standard method of treatment in the location of expressways which will have a common application in all cities. Each city constitutes a special problem which must be analyzed individually. However, there are certain general characteristics common to most cities. For the most part, cities have grown with no planned development. The street patterns were not laid out or designed for the large volumes of motor vehicle traffic that wish to use them and traffic congestion has been the result.

The nucleus of the city is the central business district. Extending from this area the city has expanded, and people have moved farther and farther out as transportation facilities of various types have been provided. As congestion increased in the central business district and on the various routes leading to it, decentralization of the city began, resulting in smaller business areas being scattered in the outlying regions. This has caused serious reductions in real estate values in the central business district and has resulted in physical deterioration of many properties.

Development of the expressway provides a facility for safe and rapid movement of a larger number of motor vehicles in and out of the business district, and over the routes leading to it. It has the same function as the development of rapid-transit systems in the solution of mass-transportation problems. The location of the expressway should be coordinated with the service provided by the mass-transportation system of the city and in this way will provide rapid transit for those persons using the mass-transportation facilities.

The traffic pattern of the average city generally consists of a flow of vehicles over several routes converging on the central business district. As they approach the center of the city, the traffic volume is increased by the influx of vehicles from the suburban areas along the routes. Congestion is generally at a maximum in the central district but there may be, and usually are, critical points as highways merge, or as cross traffic is encountered.

The expressway should be so located as to serve not only traffic from outside the city but also traffic originating within the city limits. In this way, the maximum benefit may be obtained from the improvement through provision of express service for local residents of the city. An analysis of traffic destination on main arteries approaching most cities shows that a relatively small proportion is through traffic. The expressway, therefore, should be located to provide service for the greatest number of users—namely, those destined for the central districts of the city and originating in near-by surburban and fringe areas. As the expressway passes through the outer regions of the city, it should be located so that the residents of those areas may have convenient access to enable them to use this new facility for express service from points reasonably close to their homes to the central business district.

By so locating the expressway in these outlying residential areas, buses can also use the expressway, and this will result in an improved mass-transportation service. This is an important factor, since mass-transportation facilities operating in urban and suburban areas provide a sizable share (especially in the larger cities) of the daily travel needs of persons to and from work, schools, stores, and places of worship or recreation.

The ratio of daily transit riders to the urban population shows the extensive use of mass transit by persons in the urban areas in the United States. In the population class of from 100,000 to 250,000, in the years prior to 1940, this ratio was 38%. In 1943 it was 105%, or more than doubled. In the population class of 500,000 to 1,000,000, the ratio was 72% for 1940, and for 1943 it was 145%.

In some cities, the expressway may be parallel to a rapid-transit or streetcar line; it may then be desirable to develop the expressway and improve the mass-transportation facilities as a joint project.

The expressway should penetrate the downtown or the central business district close enough to enable easy distribution to all parts of the area. Naturally, the location through, or adjoining, the central district will require a detailed study of property damage and construction costs. It will also involve an analysis of the effect on the business and property values in the area. Consideration must be given as well to the problem of providing parking space within short distances of the interchange locations on the expressway. Stopping areas will also be required to accommodate the buses using the expressway.

This is a general policy which will apply to most cities. There are certain instances, however, particularly in the case of some of the smaller cities located between two large cities, where the proportion of traffic passing through and beyond the city is far greater than that destined for the center of the city. In such cases, the expressway should skirt the more congested area or possibly by-pass the city completely.

In cities that have rivers flowing through or alongside the central business district, the expressway may be located advantageously along the river. This will usually result in lower construction cost and will simplify the intersection problem. Where such routes can be developed, full advantage should be taken of these natural barriers to cross-traffic movement but, of course, other things being equal, the most direct route is preferable to a circuitous one.

Although Mr. Barnett did not attempt to discuss structural features of the expressway, a few words regarding the location with reference to existing street patterns may be in order. The width of right of way for an express highway will generally require a full city block. In such cases, the present streets on both sides of the city blocks will remain as service roads. The center section will be used for the expressway and any land, not otherwise required, may be used for parking spaces or appropriate landscaping. In the outlying sections, it is preferable to follow the natural ground level as closely as possible. In the event that there are frequent cross streets to be "grade separated," it will often be desirable to introduce a rolling grade, with the expressway successively passing over and under the existing streets to provide grade separation. As the expressway route approaches the more developed sections of the city, it will frequently be necessary to carry the expressway continuously at a separate grade from the existing streets. In any case, elevated structures should be avoided where possible; however, it is a matter for individual treatment in each city. Although, in industrial sections, particularly where there are a series of railroad tracks at the street surface, it may be preferable to elevate the expressway.

At appropriate points along the expressway, provisions must be made for vehicles to enter or leave. To locate these interchange points properly, a thorough analysis of traffic movement is again essential. In outlying regions, it is not necessary or desirable to provide access at every crossroad. If possible, they should not be closer than from one half to one mile apart, depending on the location of the intersecting main arteries and their relative importance as feeder routes to the expressway.

As the expressway penetrates the central business district, the spacing of interchanges will probably be less frequent than in the outlying regions; they should be provided to give access to the most important streets in the central district.

Interchanges should be designed with adequate deceleration and acceleration lanes so that there is no interference with the main traffic stream by vehicles leaving or entering the expressway. The most satisfactory type of interchange is one in which the vehicles leave or enter the expressway by a ramp running as nearly parallel to the expressway as practicable. These ramps should connect with the service drive rather than make direct contact with a main cross street. In this way, there will be ample opportunity for the speed of the vehicle to be materially reduced from the speed at which it was traveling on the expressway. As a general policy, vehicles would leave the expressway at the interchange nearest their destination; thus, the motorist would drive a minimum distance on city streets to reach his destination after leaving the expressway. If the vehicle is to be parked in or near the central business district, the motorist should be encouraged to select a parking space that is nearest the point at which he leaves the expressway. The parking space preferably should be within reasonable walking distance or about 1,000 ft from the motorist's ultimate destination.

When the expressway location is properly chosen, it will serve bus and truck traffic as well as private motor vehicle traffic. It will provide a route that may be convenient for local city buses and also one which will serve intercity buses. Upon leaving the expressway, the bus should proceed to a terminal located, in so far as is practicable, at a point most convenient for the majority of the patrons. This terminal should be reached by a minimum of travel on the city streets after the bus leaves the expressway.

Consideration should be given to the possibility of developing bus stops along those expressways that are to serve local mass transportation. Under certain conditions, these may be located safely at the interchanges by providing a separate bus-stop lane, but this arrangement must not interfere with the exit or entrance to the expressway or the main traffic stream. By locating these bus stops along the expressway, it would not be necessary for the buses to leave the vicinity of the expressway until they approached the terminal, and much better local service would be provided.

Street and highway systems are public ways, and, as such, should be planned with a view to the accommodation of all types of highway travel or vehicles.

New facilities should be planned to incorporate, to the maximum extent practicable, provision for public transit facilities whenever warranted in the over-all plan based on population and land-use studies. In this way mass transportation will be made more safe, convenient, speedy, and economical for all the public.

The expressway, located so as to serve the freight distribution and industrial areas, should have interchanges at places where trucks may leave the expressway close to the truck terminals, requiring a minimum of travel on the surface streets.

Parking space, including parkway lots and garages for motor vehicles, should be located so as to be most convenient to the final destinations. Consideration should be given to providing parking space that is essentially a part of the expressway, with entrance and exit directly to it, but not so as to interfere with the through stream of traffic. Parking spaces so located would be reached without any travel on the surface streets of the central district.

The location of the expressway is intended primarily to serve the needs of high-speed traffic originating outside the city and passing to and through the congested central business district. In addition, it will serve the traffic originating in the outlying areas within the city limits and provide this traffic with a high-speed facility. Important consideration should be given to circulation within the central district and it is desirable to provide a facility that will assist in good circulation but not at the expense of retarding the expressway movement to and through the district. Careful consideration must be given to the development of the service drives and surface streets of the central business districts which are to serve as the distributors of the traffic after it leaves the expressway.

As stated, it is not possible to prescribe a standard method of treatment for the location of expressways which will provide a solution for all cities. However, the general principles outlined herein should provide sufficient guidance for the proper location and treatment of expressways as the characteristics of each city are analyzed. The facility can then be designed to serve the particular urban area under consideration best.

These principles would be the guide to be used in the preparation of the preliminary engineering report as recommended by Mr. Barnett. As a result of the various investigations to put into practice these general principles, planners would be in a position to make the recommendations for highway facilities which will adequately serve the present and future traffic needs and aid in the favorable development of urban area and region.

Bernard L. Weiner,[24] M. ASCE.—A rather complete general description of express highway planning is given in this paper. Considering the purely technical aspects of the problem, there is very little to be added. The paper is timely; one need be only a casual reader of the newspapers to known that the traffic problem has once more reached extremely serious proportions. It can scarcely be claimed, however, that any really courageous effort is being made toward its solution. Although the responsibility for this lack of effort lies in many quarters, the highway engineer cannot escape his share of the blame.

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[24] With Feld and Timoney, Cons. Engrs., New York, N. Y.

The fact that the automobile designer has advanced beyond the highway engineer is not surprising; and, as is usual in such cases, chaos is the result. The automobile industry has indeed made great progress. Changes in fundamental concepts take place exceedingly slowly and it only required thirty years for the manufacturers to realize that they had something which needed a new approach. Real progress in automobile design began when the industry realized that the concept of the "horseless carriage" was obsolete and led only to trouble. At that moment, an entirely new vehicle was born—the present-day automobile—which has changed very little except for "differences in the shape of the tin." Apparently, planners of highways and of other facilities still do not take into account the probability that the motor vehicle is here to stay.

Mr. Barnett writes (under the heading, "Terminal Facilities") that the truck-loading problem will be solved only by a bold approach. It might be added with equal justification that the "bold approach" is needed for all the problems that plague the world. Wherever mankind has made progress, it is a certainty that bold thinking has been done by someone or other; wherever mankind has failed, it is equally certain that bold thinking was lacking. Even worse, it has often been to someone's real or imagined interest to prevent such thinking—history is full of examples.

At that moment, the newspaper editors are "yelling their heads off" that the traffic congestion problem must be solved. They write "Something must be done"; yet, at the same time, they "point with horror" to the rising cost of local and national government and call for a reduction in "normal" expenditures. The problem can be solved, of course, but it will require more than the mere waving of a wand or the passing of a punitive law. It will not be a "costless" solution, but no investment is expensive if it insures adequate dividends. In cities of all sizes, the losses resulting from traffic congestion are so large and so apparent that there is very little doubt that large expenditures are justified to eliminate it.

It would be easy to show that no real attempt has ever been made to solve most problems of any importance that have been insoluble over long periods of time and have reached an acute stage. Characteristically, there has been nothing but tinkering; there has been little effort to reduce the problem to basic fundamentals. In attacking any problem, nothing should be considered so obvious that it is accepted without question.

Admittedly, large costs would be involved if all buildings were required to provide off-street loading areas, but it is also costly to provide elevator and other services. The street floor, however, need not necessarily be used for loading; the lower levels could be utilized. It is not at all beyond the range of possibility to provide elevators large enough to lower and raise entire trucks 10 ft or 20 ft below the street level, at locations where ramps are not possible for lack of space.

There has been altogether too much tendency to think of the express highway as a "study in motion," almost as if it were built for mere aimless motion.

There are three parts to a trip taken by a vehicle: The trip begins with the vehicle at rest at the starting point where it loads; it then proceeds to its destination over local streets and over highways; and it finally comes to rest again at its destination, where it unloads. Unless provision is made for the two end conditions, the express highways may just as well not be built. It is of little value to move a truck or passenger vehicle speedily from one point to another if all the time saved, and more, is lost because it cannot come to a stop quickly.

Traffic control—as distinct from mere law enforcement—is now largely in the hands of the police. It just happened that way. This control is so complete that the various police forces of the United States virtually have legislative powers. The police's point of view is punitive and only in rare instances is it constructive. "Hounding" the motorist and fining him, as much as fifteen dollars for parking in New York, N. Y., may achieve the result of keeping cars out of the congested areas; but it is no solution (see heading, "Radial Highway Arteries Are the Greatest Need: (3) Relief of Congestion"). The people do not receive the benefit of the use of their cars and, in a city like New York, the curse of centralization has made the public transportation system unfit for human occupation.

Better results could possibly be obtained if the police were restricted entirely to the enforcement of the law as interpreted and amplified (by the necessary administrative regulations) by an entirely independent body. Such a body should be made up of representatives of the various interests involved and should be under the general supervision of competent traffic and highway design engineers. It should be well financed, and the personnel should give its full time to the job of traffic control.

The purpose of this body would be to study ways and means of facilitating the movement of traffic in all its phases. Such studies would include major improvements, but comparatively minor changes would not be neglected either. Most important of all, since such a body would be made up of civilians, it would be more likely to keep in mind that the purpose of all regulations is not the punishment of violators but keeping traffic moving smoothly—and to permit it to stop where it legitimately should stop.

As stated, the police's point of view is rarely constructive. When the costly approach to a major bridge in New York City was opened to traffic, it was found that a combination exit to, and entrance from, a service road created a traffic hazard. As it happened, the exit was important, for it permitted traffic from a main thoroughfare to reach an existing bridge across the Harlem River, directly and quickly. The police "solved" the problem by closing the exit, thus vitiating an important part of a costly improvement. As it happens, a rather inexpensive piece of reconstruction would have eliminated the hazard and also saved the exit. Although it is quite a few years since the exit was closed, nothing has been done about it. This example is typical, however, of the police methods of controlling traffic.

Many traffic lights could well be eliminated; others can and should be synchronized. Also, more traffic controlled lights at minor intersections could be used to good advantage. The motorist is also too familiar with police signs reading, "No Parking Between Signs." It is a common experience to find such signs around a bank or main post office in a large city. Common sense would indicate that a reasonable parking period—from ten to fifteen minutes—be permitted to allow for the average business transactions at such places. Although this is done in small towns, the contrary is true in large cities—even in residential districts. The motorist would be more inclined to cooperate if it were evident to him that an attempt was being made to give him every possible "break." As it is now, rules and regulations are ignored and those who have legitimate and necessary business in restricted areas charge parking tickets to operating expense—when they are caught.

In his discussion of the deteriorated areas along the approaches to cities, Mr. Barnett decries the fact that obsolete laws and pressure groups prevent the protection of traffic facilities—a condition which, he states, is not the fault of the highway engineer. Issue might be taken with the last part of this statement. Not only in this particular case but also in the more general social-political-economic problems of the nation, the engineer is altogether too prone to consider his professional responsibilities from too narrow a point of view—that of a mere technician. It seems to the writer that it is as much the job of the engineer to educate the public, study the financial aspects of public improvements, etc., as it is his job to perform the actual technical duties.

The civil engineer has too long considered himself, and has also allowed others to think of him, as a spender rather than as a producer. It would be a good idea, in fact, to adopt or invent a new word for "taxes" as applied to public works—for the word "tax" originated in the days when people paid tribute for mere physical protection. In the matter of highways, the taxpayer should be made to realize that, just as he pays for his car and its maintenance, he must also pay for the highways, which are essential parts of his motor vehicle transportation system. Industry is becoming more and more dependent on public relations; why should not the professions likewise become dependent?

In passing, it should also be stated (see heading, "Radial Highway Arteries Are the Greatest Need: (2) Shelf of Plans") that the sooner the idea of using public works "to provide jobs" is abandoned, the better for the profession and for the nation. Public works should be built only because there is a need for them in their own right. Public works cannot cure unemployment, and any attempt to make them do so leads, sooner or later, to "make work" projects at starvation wages—which does no one any good.

Like Mr. Barnett, the writer does not wish to raise the subject of financing; but one observation may be made. Although the author expresses the "orthodox" and accepted "financial habits" in use today, the present method of financing by borrowing is an anachronism. The bold approach might well be applied to the financing phase of all projects—public works and other construction. It is well known that public works financed by borrowing cost eventually two and one-half dollars for every dollar originally borrowed. With a bold approach to this problem, this excessive cost could be largely eliminated.

It is an accepted truism that a bridge destroys the value of property whereas a tunnel improves it. This maxim could well be applied to the express highway. The viaduct should never be used except as a last resort. Furthermore, sound engineering takes all factors into account—including human nature. Man—in spite of his vaunted civilization—still likes the good earth, and being forced to ride above it is not pleasant. Also, from a more practical point of view, viaducts are never wide enough to provide for the inevitable disabled car. A single accident can, and does, block traffic for a mile and more back. From these and other points of view, the viaduct is undesirable.

Finally, in relation to expressways and parkways in general, there is another item to be mentioned. The landscaped and separated expressway tends to approach perfection—and, oddly enough, for this very reason tends to become monotonous even for comparatively short distances. For long distances—50 miles or 100 miles or more—the "perfect" highway does become deadly monotonous. The answer to this problem is difficult to define; but, nevertheless, highway engineers should give it their attention. The customer, after all, must be pleased.

George H. Herrold,[25] M. ASCE.—The difference between rural and urban routes and their capacity to serve the traffic using them is caused by the difference in population density in rural and urban areas, and by the fact that the states have large sums of money to expend freely in rural areas whereas such funds have not been available to the cities. As an example, the ninth Federal Reserve District (comprising Minnesota, North Dakota, South Dakota, and part of Wisconsin and Montana) has an average density of population of 13.5 persons per sq mile, compared with the cities of St. Paul and Minneapolis, Minn., which have an average density of 8,000 people per sq mile.

Before World War II, the United States was greatly exercised over traffic congestion and the decentralization of business districts. This decentralization or disintegration was caused by hazards created in the central business district by motor vehicles, by high speeds, and by poor law enforcement. Careful and timid people gradually stopped coming to the central business district unless they could not avoid it. Instead of being an attractive place to visit, the central business district became an undesirable place, with a corresponding reduction of business income and a lowering of taxable values.

During the war period people discovered that the transit industry was geared to handle the people who had to move from one place to another. Motor vehicle transportation decreased because of cars that broke down and because of restrictions in their use; and mass transportation increased by leaps and bounds. However, to a large extent, the congestion problem—the congestion that occurs at regular times and definite places—was solved. In St. Paul the number of people that used automobiles to take them to work decreased 47% whereas the number of people traveling on street cars and buses, with the aid of staggered working hours, increased accordingly.

When state trunk highways were first designated, the routes selected were existing highways leading to the city and its business district. These routes were chosen because they best represented the travel habits of farmers going to market or people from small towns going to the city. As a rule, the highways were improved only to the city limits, leaving to the city the problem of handling this traffic, which was augmented by the traffic generated in the city. Traffic from these trunk highways used the city street system to reach destinations—various "points" in many areas including the central business district.

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[25] Planning Engr., City Planning Board, St. Paul, Minn.

A city is laid out in blocks bounded by streets designed to make the blocks accessible and usable. On these blocks are buildings and people, and each block has its quota of motor vehicles. Traffic finds its way on cross streets to a trunk or arterial highway, and there it merges with the traffic from outside the city and proceeds toward destination areas and the central business district. As the traffic from outside the city moves toward the city's center, it merges with the traffic from the city's cross streets which are predominately for serving property owners.

Under the heading, "Radial Highway Arteries Are the Greatest Need," the author states:

> "It has become clear, also, that traffic congestion in cities will be solved to the greatest extent feasible by the construction of facilities that permit the uninterrupted flow of traffic into and through the cities."

Mr. Barnett cites three types of expressways: The expressway at grade—that is, built on the surface of the ground; a depressed expressway, and an elevated expressway. No matter which type is adopted, the expressway will be used by the same or increased volume of traffic, and the traffic will reach the central business district at the same time that it now arrives there. Instead of the city-generated traffic entering the expressway at every street intersection, a number of streets would be closed off and the traffic would enter the expressway every mile or so. Instead of one vehicle per minute entering the expressway at each street intersection there would be, say, sixteen vehicles per minute entering at designated access points. On an expressway these vehicles could travel at greater speed than on other routes. The driver could leave home 5 min or 7 min later, but would arrive at the central business district at the same time he arrives there now. Although an expressway does not solve any congestion problem, it increases speed and removes some frictions; and it moves people faster to the areas of congestion.

Fig. 12 shows the origin and destination study made by the Minnesota Highway Planning Survey in the St. Paul-Minneapolis metropolitan area. It shows the traffic flow in cities of St. Paul. (hatched) and Minneapolis (stippled) that originates in the satellite areas of the two cities. Each city has its distinct following and the flow from one to the other is negligible. The cordon where the interviews with motorists took place is about 1.5 miles outside the twin-city limits. In Fig. 12, the numbers within the hatched and stippled areas represent the average daily traffic observed between 6:00 a.m. and 10:00 p.m. in July and August, 1941. A traffic flow of less than 400 vehicles per day is shown by a single line and a number.

It is extremely important that "trading behavior"—the social habits and the economic reasons behind them—be studied in determining the location of an expressway and its access connections and exchanges. The intracity traffic flow presents a different picture. In St. Paul the wide traffic flow bands run

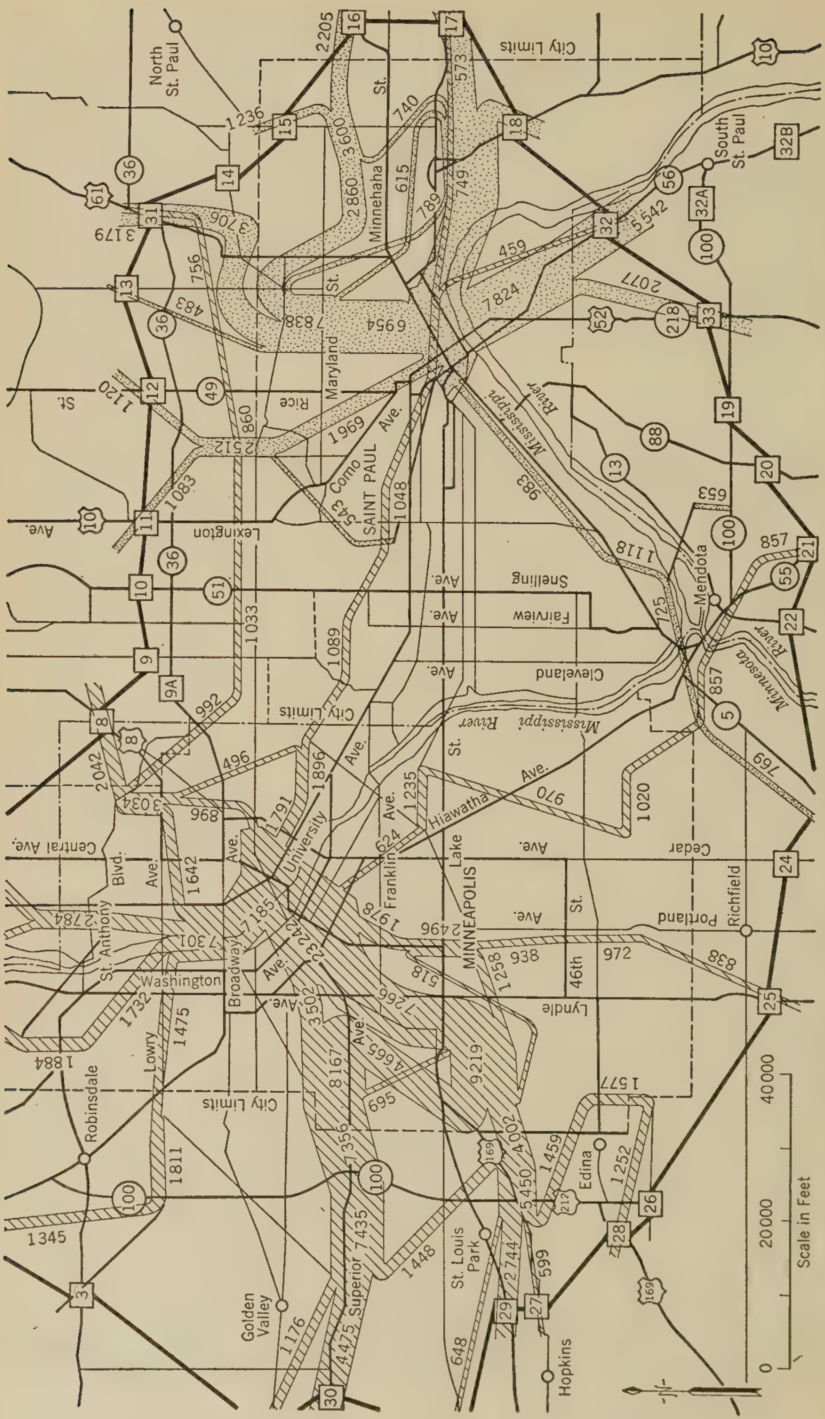
North St. Paul
City Limits
South St. Paul
SAINT PAUL
Minnehaha St.
Maryland St.
Rice St.
Como Ave.
Lexington Ave.
Snelling Ave.
Fairview Ave.
Cleveland Ave.
Mississippi River
Minnesota River
Mendota
MINNEAPOLIS
University Ave.
Hiawatha Ave.
Franklin Ave.
Lake St.
Central Ave.
St. Anthony Blvd.
Broadway Ave.
Washington Ave.
Lowry Ave.
Cedar Ave.
Portland Ave.
46th St.
Lyndale Ave.
Superior
Robinsdale
Golden Valley
St. Louis Park
Hopkins
Edina
Richfield
3179
3706
1236
2205
3600
2860
740
615
789
749
573
459
7824
5542
2077
6954
7838
756
483
1120
860
2512
1969
1083
543
1048
983
1118
653
857
125
1033
1089
992
496
1896
1791
1235
970
1020
3034
968
2042
1642
2784
7301
7185
2242
1978
2496
518
1258
938
972
838
1732
1884
1475
3502
1266
8167
4665
9219
695
1577
4002
1459
1252
1356
5450
1811
1345
7435
1448
2744
599
648
1176
4475
624
769
Scale in Feet
0
20000
40000

FIG. 12.—FLOW PATHS OF TRAFFIC THAT ORIGINATES IN THE SATELLITE AREAS OF ST. PAUL AND MINNEAPOLIS, MINN.

east and west. In Minneapolis they run north, south, and southwest, illustrating that the intracity motor traffic is predominantly for going from home to work (or vice versa), distributing commodities, shopping, and visiting—either for business or for pleasure. The wider bands on certain streets indicate a traffic excitor—people moving two, three, or ten blocks to contact one another. There is only an occasional through movement. The buildings along the streets and the people in them cause these motor vehicle movements.

The writer is apprehensive as to the effect of arguments or inferences that expressways solve congestion problems. The true problem that confronts all cities is one of mass transportation—to devise a well-balanced movement of people and goods.

It makes no difference (except as to the cost) whether the traffic is taken to congested areas on a limited-access surface street, a depressed limited-access highway, or an elevated expressway. Where a block of land plus two streets is to be acquired for construction, there are great advantages in the elevated highway—economies as to construction, economies as to the use of the land, economies in utilities, and economies in maintenance.

Mr. Barnett is correct in stressing the advantages of a close-in circumferential route and also (see "Use of Existing Streets") in stating that a complete plan is much more than "bulling" through a free-flowing type of highway. Fig. 3, showing the moving of an arterial road from the central business district to the riverbank, illustrates good location. In a number of places the author makes use of expressions such as "a flexible system of expressways," "flexible routes for the driver to choose from," etc. In a city things are fixed—buildings are erected, utilities constructed, and street systems are fixed in relation to permanent transit systems. The wealth of a city has been built up around these fixed things. The word "flexible" is extremely misleading.

Mr. Barnett writes (under the heading, "Planning Agencies") of "* * * a spectacle of so much planning that what should have been a healthy rivalry resulted in a jam which prevented progress." Possibly some one was stubborn or had a limited technique. However, cities are full of things called "progress" that were put there before all the facts were known. There are many activities in a city other than moving around in an automobile.

In the "Introduction," the author states that "As yet there have been few instances in which the desirable separate provision for arterial traffic has been attempted." The business transacted in a city, the visiting, and the contacts made are not from arterial traffic comparatively speaking. The converging people from 5,000 blocks of 50 sq miles of city make business; and, if these people come by automobile, congestion results.

It seems confusing to state (see "Use of Existing Streets") that:

> "One fallacy in such reasoning lies in the fact, just mentioned, that an individual vehicle is destined for a point in a district and use of surface streets should be reduced to a minimum."

Each vehicle is destined to a point and every point is a different location that can be reached only by surface streets. The existing major streets must accommodate the bulk of the movement of people and goods.

An express highway through a city requires land, approximately 400 ft wide, with ramps, service roads, roadways, and accelerating and decelerating lanes. Except where stragetically located, such a structure introduces a disrupting force to all the factors of good living. If a school building is on one side of this expressway, the children who live on the other side must go to one of the arterial crossings and backtrack to their school building instead of going the usual route directly from home to school; or there must be a new arrangement of school districts and new school buildings. Because they must carry all transportation of the closed streets in between, these arterial crossings become greater hazard factors than before. People going to church are subjected to the same inconvenience, and neighbors cannot visit neighbors as before. They cannot shop at their favorite neighborhood store or use their favorite neighborhood service station. For the city as a whole—school districts, church districts, social districts, and neighborhoods that may be cut in two by an express highway—the effect on the city tax structure may be an injury that cannot be compensated.

Mr. Barnett's paper concerns the Interregional or Interstate Highway System as extolled in *House Document No. 379* (2d session, 78th Congress). This system is a network of highways connecting the main cities of the nation, apparently designed for military routes. However, it is to be used in peacetime for all-purpose traffic, trucks, trailers, semitrailers, buses, and passenger automobiles. The standards of construction are to be the highest standards used in highway construction. The highways are to be designed for a speed as great as 70 miles per hr and will be expected to care for all motor traffic that is likely to be developed in the succeeding 20 years. The system will enable buses and trucks to compete effectively with railroads. It will bring these buses and trucks directly to the heart of the city, through the city, and beyond the city. As compensation to a city, it is stated that these express routes running through the business center, and through the city, will serve the population of the city between home and business. This conclusion is extremely doubtful.

The writers of *House Document No. 379* maintain that the shortest route may not always be desirable and that the most direct route should give way to locations which would not be disrupting—riverbanks, waterways, and railroad rights of way (where the street system has already been broken and where all development of the city in past years has been based on this situation). An express highway, if one is needed through a city, should be located where it will disrupt the cross street system the least. Surface streets are a necessary part of any comprehensive system of transportation. The traffic picture in a city is changed continually by the location of new buildings, new factories, new manufacturing districts, and new residential districts. Therefore, a plan must be adopted in which the express highway does not cut across streets that have been developed, to the fullest extent through the years, to serve the abutting property. In this case, location is far more important than engineering design, and it must not be thought that, if the location destroyed a few old buildings, it is necessarily a good location.

The problem confronting all cities is one of transporting people. An express highway is built for the use of passenger automobiles and all other motor vehicles—motor buses, motor trucks, and tractor-trailer and semitrailer combinations. It is not for the much needed segregation of passenger automobiles and the rapidly growing truck traffic.

The great spread of destinations of passengers from all parts of the city through the entire central commercial area, the manufacturing area, the wholesale area, the railroad terminal area, and the waterfront area requires further exploration. A check on where people alight from street cars and buses furnishes a good sample of this phenomenon. The people's habits and behavior, as they move from place to place, involve a surprising number of tangible and intangible factors, all of a variable but interrelated nature.

To expend from three to five million dollars per mile for an express highway into and through a city is a major operation, and location is all important. Large cities are prohibiting parking of cars in the central business district, establishing one-way streets, and exploring methods of "steady flow traffic" to make greater use of their existing street system. All these studies and experiments are valuable. Possibly the "close-in circumferential route" should be located and built first. The traffic flow into it would help locate radials and interchanges.

No passenger cars should be allowed inside the circumferential route. People would transfer from automobile to shuttle buses and taxicabs, or they would walk to their destination. From an economic standpoint it would be far better to improve mass transportation to the highest standards possible, and then later to follow up with motor vehicle express routes. Building express routes first will increase congestion to a self-limiting status. Building up mass transportation will move the people where they need to go and may stop disintegration of central districts.

Ralph R. Leffler,[26] Esq.—Pavement-minded engineers, paving bureaus, and paving and earth-moving interests have derived much profit and power from the decentralizing of the big city. These interests have almost complete control (officially and otherwise) of planning for recentralizing (rehabilitating) the big city for the "general welfare of all." It is not surprising that puny planning, thin thinking, and verbose vaporings characterize the offerings of such of these interests as are typically selfish and bureaucratic. Mr. Barnett's paper makes determined and fairly successful efforts to avoid the selfish and bureaucratic.

Nevertheless, Mr. Barnett's paper is unintentionally quite "pavement minded." It lacks sufficient "structural mindedness"—giving scant consideration to the merits of elevated highways (on columns) and none at all to the overwhelming advantages of two-deck elevated highways (on columns) in big cities. Elevated highways, particularly two-deck elevated highways, are the product of structurally-minded as distinguished from pavement-minded engineers.

The paper treats "Parking in the Central Business District" quite cautiously, briefly, and very inconclusively under the heading, "Terminal Facilities." Extended exploration of the merits of elevated highways, especially of two-deck elevated highways (on columns), would probably have led Mr. Barnett to much more conclusive and structurally-minded ideas on "Parking in the Central Business District" on the second, third, fourth, and fifth floors of existing and future buildings and on the "setback" roof of future buildings.

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[26] Engr. of Structural Design, The San. Dist. of Chicago, Chicago, Ill.

The features and merits of two-deck elevated highways (on columns) have been discussed by the writer elsewhere.[27,28] Thorough study will convince almost anyone, except a paving bureaucrat, that the structurally-minded engineer can be very helpful in recentralizing the decadent big city—if given the opportunity.

M. Hirschthal,[29] M. ASCE.—The profession is greatly indebted to Mr. Barnett for his timely and substantial contribution, which focuses attention on a great need—a thorough discussion of means of coordinating express highway traffic through cities and suburbs of a metropolis so as to produce a free flow throughout.

Development of the highway and its design have naturally followed closely upon the development of the automobile. When the automobile first made its appearance, a 16-ft width of roadway constituted a two-lane highway. This was shortly increased to 18 ft with an allowance of 9 ft for a width of lane; a four-lane highway was made 36 ft wide. At that time the general practice was to use 60% of the street width or right of way as roadway, thus providing a 36-ft road width for a 60-ft city street or highway. Most city streets have been laid out on this ratio basis, even to the 100-ft-wide express streets in which the roadway has a width of 60 ft. Lane widths in state highway design specifications were set at 9 ft as standard, and remained so until about 1941.

In recent years this lane width has been gradually increased to 12 ft (particularly on express highways), with a provision of 5 ft or 6 ft (in some instances, as much as 12 ft) for a "parkway" separation between the two sets of lanes for traffic in opposite directions, and with additional provisions for gutters and shoulders on either side. (Incidentally, in all this express highway planning, including New York and New Jersey state highways, nowhere has the writer seen any provision for the pedestrian—he is the forgotten man.)

Despite the widening of the lanes, perhaps because of such widening, it is not at all unusual to find cars straddling two lanes. Accidents continue to increase by reason of cars moving from one lane to another, or by passing another car and miscalculating the space available for that purpose, especially if a bus or a truck is one of the vehicles involved. The only way to avoid these all too frequent occurrences is to provide barriers between the lanes of traffic in the same direction; to prevent such crossovers except at definite locations, say, at one-half mile (or perhaps one-quarter mile) intervals, with proper safeguards similar to those for the many points of merging traffic from roads entering into the express highway. The writer anticipates electronic control in the future.

Such barriers could be provided, without the necessity of additional right of way, by making 9 ft the clear lane width, the actual requirement for a vehicle. The remaining 3 ft of the 12-ft lane widths could be used for barriers or curbs of sufficient height (15 in. minimum) to eliminate the possibility of vehicles mounting these curbs and at the same time to discourage the motorist from driving too close to them. Furthermore, this would simplify the assignment of a definite lane for trucks and other slow moving vehicles if desired. Hazards

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[27] "Superhighways in Widely Built-Up Areas," by Ralph R. Leffler, *Bulletin No. 74*, Am. Road Builders Assn., 1941, p. 5.

[28] "Two-Deck Elevated Highways," by Ralph R. Leffler, *Illinois Automobile Club Magazine*, Spring, 1945, p. 3.

[29] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

of sideswiping and similar accidents would be reduced to a minimum. Another advantage is attained in the economy of the cost of grade eliminations across the highway, both for railroads and other highway overcrossings and undercrossings, which would more than compensate for the extra cost of the barriers and the provision of drainage.

Provision of the 3-ft-wide barrier between the traffic lanes in the same direction permits locating in that space the columns or piers required for a bridge in the event of grade crossing elimination over the express highway (a railroad overcrossing) and eliminates the necessity of long-span construction, which is evidently far more costly than that for short spans. Also, where soil conditions are poor, it may obviate the necessity of expensive foundations by the substitution of a spread footing. This may be all that is required to carry the light reactions resulting from short-span design, while heavy foundations are likely to be required for piers and abutments that carry the loads from long spans. Moreover, the great majority of crossings between highways and railroads are at angles other than right angles, resulting in skew bridge structures of increased span length, thus accentuating the foregoing condition.

To cite an actual example: At Fox Hill, Mountain Lakes, N. J., state highway route No. 6 crosses the tracks of the Delaware, Lackawanna and Western Railroad (D.L. and W.R.R.) at an angle of 26° 19′ (a skew of 63° 41′), requiring a center-to-center span length of girder of 103 ft to span the required lanes in one direction; whereas each lane would require only one quarter of such span length had it been possible to provide these barriers and hence space for the location of columns or piers between each lane. It is quite evident that 103 lin ft of girders for four 26-ft spans would weigh a fraction of that for one 103-ft span, particularly for railroad loading, and that the sum of the reactions of the small spans would be similarly reduced for the footing design.

Where the express highway is the overcrossing in a grade elimination, the provision of this space for barriers will make available shallower floor depths because the floor system need span only the width of one lane with girders located in the barrier space, and the 15-in. or more available depth above the road surface may prove sufficient for a depth of girder of moderate span length, so that the girder itself (or a pair of girders) may act as the required barriers.

Similarly there can be cited an example of this case: New Jersey State Highway route No. S3 crosses the D.L. and W.R.R. tracks at Clifton, N. J., on a proposed bridge over the four railroad tracks. The plans provide for the girders to be 54 ft apart on each side of the 16-ft center "parkway" or "island" to accommodate the lanes in each direction. For this condition a 4½-ft floor depth is required. It is evident that a drastic reduction in this floor depth would result if the girders spanning the tracks were only 12 ft apart (in the barrier space) with additional resulting economy in the length of approaches required for this reduced floor depth. Numerous cases could be cited, but these have been selected because of their emphasis.

Approximately 25 years ago the writer proposed[30] to the New York City (N. Y.) authorities a remedy for the traffic congestion on the avenues running north and south, in the form of elevated sidewalks providing two additional traffic lanes, instead of the 25-ft sidewalk space, in each direction for local traffic, with pre-cast bridges spanning the intersecting street roadway openings. It was then emphasized that the shops would benefit thereby in having two stories of display to advertise their merchandise. A a matter of fact, this remedy could be applied as well to all the crosstown streets now so congested in midtown Manhattan.

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[30] "Elevated Sidewalks of Reinforced Concrete for New York City," by M. Hirschthal, *Engineering News-Record,* May 31, 1923, p. 974.

The writer hopes these comments will receive the attention of express highway planners in metropolitan as well as other areas.

Joseph Barnett,[31] M. ASCE.—Although the sixteen discussions are chiefly confirmations of the principles of express highway planning as given in the paper, some are critical. Mr. Lavis suggests that the profession would derive more benefit if the paper were more specific. It is agreed that each city and each project in a city needs detailed study; yet the solutions of specific problems show a definite sameness that leads to a set of principles. This is time-honored engineering procedure. The paper is intended to show these principles.

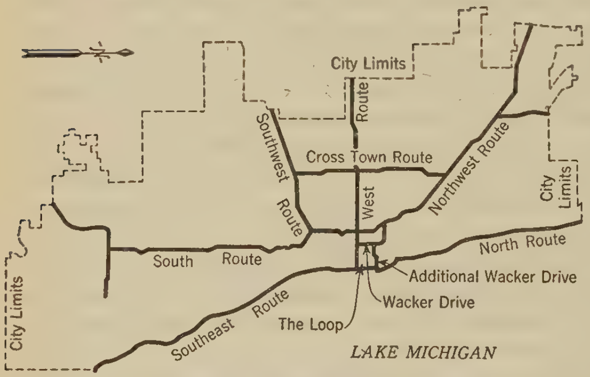

Fig. 13.—Comprehensive Superhighway System for City of Chicago, Ill.

As a matter of fact Figs. 1, 2, and 3, showing patterns of arterials, are actual cases although the names of the cities are not given. Figs. 4, 5, and 6 illustrate types of expressways which have been in operation for some years. An excellent example of the pattern of arterials described in the paper (namely, radial routes and an inner circumferential, with an intermediate or outer circumferential for the larger cities) is presented in Fig. 13, which shows the arterials for Chicago, Ill. The locations were determined over a period of years after factual origin-destination data were obtained and analyzed and the other factors outlined in the paper had been given serious consideration. A few of the arterials, such as the Lake Shore Drive, are in operation and the West Expressway is under construction.

The fact that this pattern is ideal in most respects is confirmed by the discussions of Messrs. Baker and Van London, who cite specific cases in which a large percentage of the traffic traversing the central business district is destined beyond it. An analysis of seven cities from this standpoint is shown in Fig. 14. By carrying the large portion of traffic destined beyond the central business district on free-flowing arteries, congestion of the downtown streets will be relieved and traffic necessarily using them will be better accommodated. On this factual basis Mr. Herrold's criticism is invalid. When paralleled by solutions to other problems, such as the terminal problem, there can be little doubt that relief of downtown traffic congestion will result. These facts are overwhelming arguments against the policy of carrying free-flowing arteries to the edges of central business districts and expecting existing streets to act both as distributors and as arteries for through traffic. These streets generally already are used to capacity and superimposing upon them the traffic induced by improving radial routes will further congest the downtown streets.

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[31] Chf., Urban Road Div., Public Roads Administration, Federal Works Agency, Washington, D. C.

A careful examination of the effects of a proposed solution on the downtown streets was discussed by Messrs. Baldock and Lewis and more attention to the terminal problem was suggested by Messrs. Baldock and Weiner. Both problems are related. No analysis of a city motor transport problem is complete without an analysis of the effects of heavily traveled free-flowing arteries on existing streets, an analysis of the terminal problem, and what should and can be done about both. Without such analyses a city may well find that, as Mr. Herrold stated, although the arteries will enable drivers to reach the central business district sooner, greater congestion may well occur. However, the opposite will result if a properly coordinated plan, based on factual data and not just on opinions, is carried through. In Atlanta, Ga., for example, the parking desires and the prospective parking facilities had a profound effect on the choice to locate an important arterial on one, rather than on the opposite, edge of the central business district. The latter location would have resulted in vehicles destined for the parking areas using streets through the central business district whereas with the former location vehicles destined for parking areas will travel short distances on existing streets not used as much as the downtown streets.

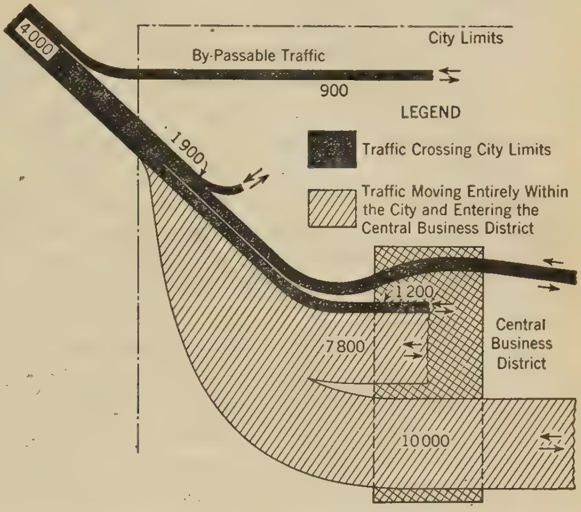
4000
City Limits
By-Passable Traffic
900
LEGEND
1900
Traffic Crossing City Limits
Traffic Moving Entirely Within the City and Entering the Central Business District
1200
7800
Central Business District
10000

Fig. 14.—Origins and Destinations of Traffic on an Interstate Route through a Typical City with a Population of from 50,000 to 100,000, As Based on Data from Seven Cities

Proper locations of arterials, traffic and parking desires, and effects of proposed solutions on existing streets cannot be safely determined by measurements of volumes on existing thoroughfares and the judgment of planners, even when backed by the customary data collected by city planners. These are valuable assets but they cannot be relied upon without factual data regarding origin and destination, parking and loading, and the many important needs revealed by the obtainment and analysis of such data. Expenditures for improvements of arterial routes in cities are large. Not only are initial expenditures high, but a location once accepted and agreed upon commits public officials to a long-range program of improvement involving comparatively large sums of money. The foundation for such a program should be sound justification based upon factual data; benefits to traffic, both arterial and other traffic affected thereby; and benefits to the city as a whole.

Factual origin-destination and parking data are not difficult to obtain and analyze and are not costly in the light of the improvement expenditures involved. The comprehensive home interview type of origin-destination survey has been undertaken in nearly sixty cities and the "bugs" have nearly all been eliminated. The cost is about 10¢ per person in large cities of 1,000,000 population or more. Cost per person increases as population decreases, reaching about 17¢ for the cities in the population group of from 50,000 to 100,000. A parking survey in the downtown area, generally needed to supplement the origin-destination survey, costs about $20,000 additional when made in cities in the larger group.

In not all cities are the cost, time, and effort of a comprehensive home interview type of survey justified. Although this method has been used successfully in cities with a population as small as 15,000, reliable origin-destination data adequate to establish the locations of arterial routes and to indicate the general type of design can be obtained in small cities at less cost by other methods. By one method, drivers of vehicles are interviewed as they are parked. Data are thus obtained not only as to parking habits and desires but also as to the origin and destination of the trip and the purpose thereof. Such surveys cost from $4,000 to $8,000. In another method origin-destination data are obtained from vehicles passing two or more cordons of survey stations. One cordon generally is placed around the edges of the urban area and another, around the central business district. The data are best procured by interviews, although postcard questionnaires are sometimes used. This method is successful if knowledge regarding trips between different sections of the city outside the central business district does not have an important bearing on the problem.

The value of adequate and reasonably accurate traffic data lies beyond the determination of desirable locations of arterial expressways. In many cases locations of arterials can be determined with very meager traffic data; but an analysis of the effects upon the existing street system, previously discussed, and a determination of the accesses and the traffic loads for which they should be designed cannot be made properly without such data.

The matter of terminals, both for passenger vehicles and trucks, is a vast subject and the statement of principles in the paper is condensed indeed. The problem has been excellently restated by Nathan Cherniack, M.ASCE, and by discussers of his paper.[32]

In regard to design types, Messrs. Hadley and Leffler bring up the matter of elevated structures, a two-level structure being one type suggested. It may be of interest that the elevated highway shown in Fig. 6 (which was designed many years ago) provides a second level, but this idea has been abandoned. The two-level elevated has the advantage of narrowness and economy as compared with a one-level elevated, but the ramps leading to the upper level present added problems and damage to adjacent land may be great. The advantages and disadvantages of elevateds as given in the paper are valid regardless of the number of levels. Several designs that began by considering the elevated type as obviously superior finally were designed as depressed expressways. This was not because the engineers were "pavement minded" as accused by Mr. Leffler. If they were, the elevated would not have been thought of initially. Very few elevated highways have been constructed; on these, wide right of way was available or was acquired.

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[32] "A Statement of the Parking Problem," by Nathan Cherniack, *Proceedings*, Highway Research Board, National Research Council, Vol. 25, 1945, p. 250.

Messrs. McCrosky, Hadley, and Waterbury suggest the possibility of direct connections between expressways, particularly elevateds, and parking garages. This possibility has been discussed widely because it appears to have merit. Traffic using such garages would not be delayed at terminals and would not use existing streets, thus reducing the volume of traffic thereon. However, a word of caution is in order. Assuming that all the practical and financial difficulties of such a development can be overcome, consideration must be given to the effect of operation on expressway traffic. The fundamental concept of an expressway is free movement and infrequent points of access. The rate at which vehicles can be moved into parking areas or garages is not great at best, so that numerous such points of ingress and egress would have to be provided unless an extremely large storage plaza is available. The plan does not appear to have the flexibility of operation that is desirable to insure against interference with traffic on the expressway. A more flexible plan, and one which appears readily attainable, is one in which the parking facilities are near the expressway but are not a part of it.

Mr. Lewis and Mr. McCrosky discuss the desirability of expressways for segregated traffic. It is agreed that there is fallacy in the theory that the construction of expressways or parkways for the exclusive use of passenger vehicles will benefit commercial vehicles sufficiently by releasing the existing streets to them; yet the segregation of traffic is desirable because of the smooth operation, high capacity, and great benefits to passenger vehicles and occupants including passengers in buses (where use by buses is feasible). In the larger cities separate expressways for passenger vehicles and for mixed traffic appear to be justified.

Several discussers take up the matter of transit. Improvement of transit is one of the important items in the broad attack necessary to relieve congestion in the cities. It is unrealistic to expect most of the home-to-work, home-to-shop, and home-to-business trips to be made in private vehicles, particularly in the larger cities. When properly designed, operated, and controlled, mass transportation can do much to relieve traffic congestion simply by reducing the number of vehicles on the streets and in the parking places. However, the position of express highways in relation to transit is not as simple as the discussers indicate.

In the larger cities rail transit might be included in the same right of way with expressways with some economy resulting to both—but such a case is certain to be exceptional. Use of expressways by buses is the more likely pattern into which transit will fit, particularly for rush hour express type of transit operation between outlying sections and downtown areas. Bus operation with frequent stops does not appear practical on expressways. Such stops would have to be designed with separate stopping areas and speed change lanes for entering and leaving without interfering with through movements. They logically should be at important cross streets where transfers to crossing transit lines would be desirable. Provisions for separate stops at such locations are costly since they involve widening or lengthening grade separation structures, and the speed change lanes hamper the design and operation of ramps if interchange is also to be provided. In addition, a poor service is rendered patrons when they are required to climb stairs or ramps.

Where interchange ramps are provided, bus operation and stopping problems are solved economically and superior service is given patrons, if the buses use the interchange ramps, stop at turnouts at the level of the cross streets, then use ramps again for returning to the expressway. Where cross traffic is not heavy, buses can cross the intersecting street. Good designs have been made in which buses U-turn on to ramps leading back to the expressway after loading passengers, thus avoiding the crossing of the intersecting street. This is not possible, of course, where single one-way ramps are provided in each quadrant of an intersection. Most bus companies are not partial to operation on expressways where stops necessarily are few and far between, since bus operation is profitable and patrons are served best where bus stops are frequent and near the destinations of patrons.

Mr. Weiner cites the need for better use of existing facilities. This need is obvious for such existing facilities as streets, intersections, and control devices, but the need also exists for better operation of expressways. In large metropolitan areas existing expressways frequently become clogged during peak load hours when freedom of movement is needed most. Sometimes, because of inadequacies of design, such as lack of a shoulder, a stopped vehicle on a through lane can jam traffic in the same direction on all lanes. Sometimes congestion is due to simple overload. Whatever the cause, slowing down of traffic to 25 miles per hr or less results in use of the expensive facility for volumes less than capacity. For example, an expressway loaded bumper to bumper, with all vehicles stopped, is carrying no traffic in an operational sense. At such times all vehicles would benefit by denying the expressway to vehicles to the amount of the overload. Where jamming occurs regularly, it can be anticipated and police controls arranged for at the critical entering ramps.

Mr. Feld rightly suggests that it is not feasible to design expressways for peak loads. However, his submission of a photograph of the Hendrick Hudson Parkway in New York, N. Y., on Navy Day, although interesting, is academic only. No one would be expected to design for such an extraordinary peak. The American Association of State Highway Officials considers the thirtieth highest hour of the year to be the appropriate hourly volume for use in design, the year to be that for which the facility is being designed. For certain locations the thirtieth highest hour is as little as 8% and for other locations as much as 20% of the average daily traffic.

The writer has not attempted to give closing statements on all matters taken up in the discussions. For some, comments can be found in the paper; others, although worthwhile, are not related to the principles of express highway planning in metropolitan areas. All the discussions, however, have added much to the worth of the paper and evoke the appreciation of the writer.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2310

FUTURE COSTS AND THEIR EFFECTS ON ENGINEERING BUDGETS

(PRESENTED BEFORE THE SANITARY ENGINEERING DIVISION OF THE ASCE ON JANUARY 17, 1946)

By Louis R. Howson,[1] M. ASCE

With Discussion by Messrs. Isador W. Mendelsohn, and Louis R. Howson.

Synopsis

In periods of rapidly changing construction costs many essential projects fail to go ahead because budgets are insufficient to cover the cost. This is particularly applicable to the postwar period. Appropriations made in 1940, after which other than war-related construction was practically suspended, are totally inadequate to cover costs in 1946 when restrictions were lifted and work could proceed. Budgets prepared in 1946 will likewise be inadequate unless they are made after consideration of the impact of wage adjustments, increased material costs, and government policies.

This paper presents a study of the effect of other wars upon construction costs, indicates some of the factors inherent in the present situation not experienced earlier, and forecasts the level of general construction costs until 1956 in relation to costs experienced in 1940.

1. Introduction

More briefly, perhaps, the subject of this paper may be stated as "What's Ahead for the Construction Industry?" The studies here considered were prompted by considerations of developments in sanitary engineering; to a very large extent, however, the future costs of sanitary engineering works are affected by the same fundamental conditions as the cost of the construction industry in general. The effect of such increased costs on budgets and financing of what are essentially public works offers some complications which are not inherent in private work.

Note.—Published in March, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., Alvord, Burdick and Howson, Chicago, Ill.

The spring of 1946 is a difficult time in which to prepare a discussion on this subject. V–E and V–J Days are so recent, and there is so much turmoil in labor relations, such delays in reconversion, and such serious maladjustments arising from the war, that accurate forecasting is impossible. As is well known it is the universal experience that there is a price "spree" after all wars. The picture in the present instance, however, is somewhat further obscured by the fact the psychological "spree" started nearly ten years before the war, and by the simultaneous adoption by the federal government of the conflicting economic policies during the war, consisting of price curbs or ceilings on commodities, combined with incentive payments to war workers.

With the war over, there is a vast reservoir of construction available. If it proceeds normally it will provide direct employment for at least 3,000,000 men, and indirectly affect many more. However, if costs rise too high, the public will refuse to pay the prevailing prices and construction will come to a dead stop. It matters little what the hourly wage of labor may be if none is employed.

Even with the critical shortage of housing, the most optimistic forecast is for the completion of 400,000 units in 1946, a figure 200,000 less than the number completed in 1940, and 300,000 less than the 10-yr average from 1921 to 1930, inclusive. The conflicting economic policies of price controls for materials and finished structures, and of higher hourly rates for labor, are found to prove difficult to reconcile.

That the volume of construction will be influenced by its cost is illustrated by a survey made for *Architectural Forum* by Crossley, Inc., which concluded "Prospects are thinking realistically and precisely about what they can afford with relation to savings and income—sizable proportions of people are inclined to postpone building or buying if the house they want is going to cost more than the price they now have in mind."

That statement is equally applicable to sanitary engineering works. It is therefore important to consider the conditions likely to affect costs and volume of sanitary works in the postwar years.

In starting consideration of this subject, reference may be made to a well-known member of the legal profession who in addressing a group of engineers stated that he always liked to be associated with engineers, for in all of his experience: "The engineers and lawyers stood back to back with the engineer facing forward." That is a brief but picturesque statement of one essential difference between the two professions. The legal profession relies almost wholly upon precedent. The engineering profession, being a creative one, necessarily makes its designs for the future. While the lessons and experience of the past are valuable guides to the engineer, construction and the period in which that construction must serve necessarily lie in the future. Every engineering undertaking is a forecast—even the appraisal of utilities already constructed and in service—for value itself is a measure of future usefulness.

Someone has defined a forecast as the art of drawing a mathematically precise line from an unwarranted assumption to a foregone conclusion. The forecast of this paper will not be of that type, for an engineer certainly appreciates that any sound opinion must be built upon a foundation of information about the forces with which that opinion is concerned.

2. Necessity for Appraisal of Future Conditions

Two major elements affecting construction projects are those of time and cost. Construction projects cannot be created and put into service without incurring expenditures of money and without the lapse of time. The more extensive the project the more important becomes the factor of time.

From a long experience with municipal projects, it has become almost axiomatic that ten years ahead is the "present" and any municipal utility that is not kept that far ahead of its projected requirements is almost habitually unable to meet them as they mature. Public projects such as water supply, sewerage, and sewage treatment, therefore, are peculiarly dependent for their success upon adequate future financing as well as the development of sound engineering design and execution. In periods of rapidly rising prices scores of sound public projects fall by the wayside because the funds appropriated for their execution have proved insufficient to cover the cost of construction at the time bids are taken. This invariably results in delay, misunderstandings, criticism, and, frequently, either restricting the project to a serious extent, or abandoning it for another decade or more. Adequate consideration of the factors affecting future costs and provision for them in estimates, even in such difficult times, will serve to avert some of these disappointments.

3. Construction Volume and Costs

The two important factors of construction volume and costs are closely interrelated. Construction volume is peculiarly sensitive to economic influences, the volume maxima usually occurring during periods of normal prosperity or that induced by war, and the minima normally being undertaken in times of depression when materials and labor prices are usually most favorable.

In diagrammatic form Fig. 1 shows new construction activity in the continental United States as reported in the industry report, construction and construction materials, of the Department of Commerce for the years 1929 to 1944, inclusive. Referring to the upper graph, Fig. 1, the points for 1945 and as forecasted for 1946 are from an address on "What's Ahead for 1946?" by William H. Shaw, chief of the construction statistics unit, of the construction division, Bureau of Foreign and Domestic Commerce, Washington, D. C. The figure of $15,000,000,000 for 1950 is taken from various government forecasts and construction industry goals; it will be noted that this is nearly 50% higher than the boom year of 1929, and a billion and a half dollars higher than construction expenditures during the peak war year of 1942 when new industrial plants, cantonments, and other war-related construction were under way. This is indeed an ambitious goal when expressed in terms of dollars and compared to past performance.

It is believed, however, that this upper curve by itself may be misleading, representing as it does dollars expenditure rather than the physical construction which results from that expenditure. With that thought in mind the dollars expenditure has been adjusted (lower graph of Fig. 1) to reflect the changes in construction costs, by dividing these costs by the *Engineering News-Record* construction cost index (1913 = 100) for that year. The last four years are based upon the writer's prediction as to construction cost indexes during that period.

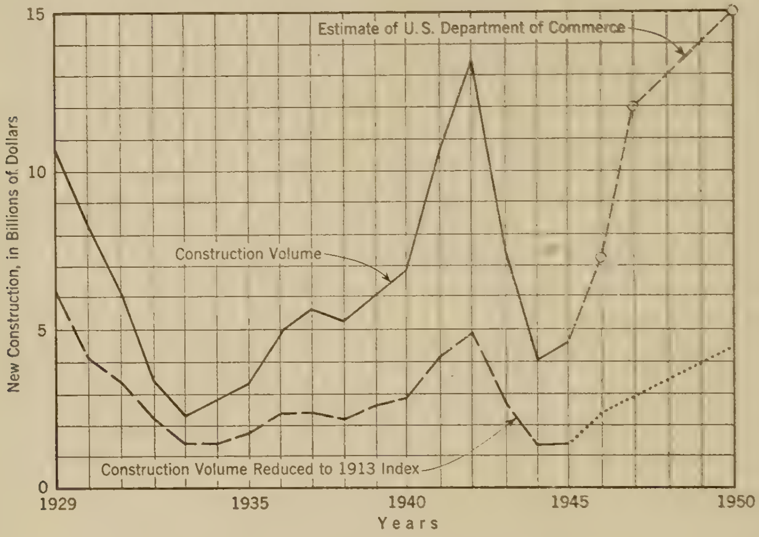

FIG. 1.—TOTAL NEW CONSTRUCTION IN UNITED STATES

It will be noticed from this lower line, which reflects costs in terms of structures built rather than of dollars spent, that even the war peak year of 1942 failed by approximately 20% to reach the construction volume of 1929; and that the forecast of 1950 will be approximately 30% below the construction of 1929 and approximately 10% below the construction of 1942. In other words, although the billions of dollars which it is hoped will flow into the construction industry in 1950 will be approximately 50% greater than those of the boom year of 1929, the actual construction resulting from that huge expenditure will be about 30% less.

Public Vs. Private Construction.—During periods of normal prosperity private construction in the United States usually leads public construction in the proportion of three or four to one.

The relation between public and private construction for the seventeen years from 1929 to 1945, inclusive, is shown in Fig. 2 together with the forecast made by Mr. Shaw, for 1946. It will be noticed that in only two years, other than during the war, did expenditures for public works exceed those made by private owners—and then by only a very small amount during the worst depression years, 1932 and 1933. The forecast of the construction division of the U. S. Department of Commerce for 1946 is that 70% of the construction will be privately financed and 30% financed by the public.

In his discussion, Mr. Shaw predicted that by the end of 1946 "we can and should be building at an annual rate of better than $9,000,000,000 with a goal of at least $12,000,000,000 in 1947." He continues with the statement, "Our forecast assumes reasonable costs—close to present levels." In appraising the possibilities therefore, it becomes pertinent to inquire as to whether costs will be maintained "close to present (1945) levels."

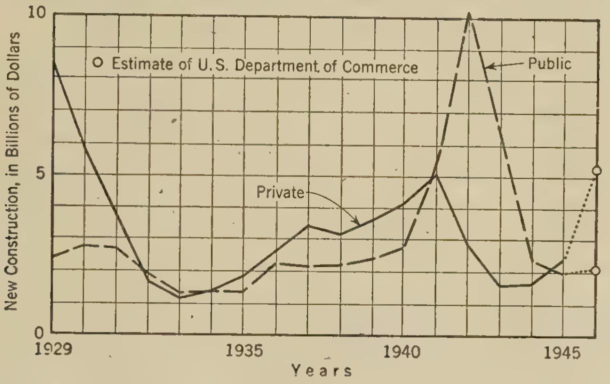

Fig. 2.—Public Vs. Private Expenditures for New Construction

4. Effect of Wars on Price Levels

Every war of lengthy duration has exerted a marked immediate effect on price levels but it has had a lasting effect as well. This is borne out by a study of recent wars and those within a period of centuries. The more nearly the present time is approached, the more accurate are the bases for the study.

The close of World War I left engineers confronted with the same type of problem that is being discussed here. At that time the late John W. Alvord, Hon. M. ASCE, was engaged on a number of public utility appraisals, one of which, the Bluefield Water Works and Improvement case, eventually found its way to the U. S. Supreme Court. The Bluefield decision was one of the two first decisions of that court holding that regulatory bodies must recognize the change in price levels resulting from the war.

A study by Mr. Alvord in 1921 at a time when construction costs of both labor and materials were at the all time peaks up to that date gave the data plotted in Fig. 3, showing the experience before, during, and after the Civil War as reflected in wages; and before, during, and up to that date as affected by World War I. Referring to the corresponding numbers on the diagram, the following notations apply:

(1) Wage statistics for period from 1840 to 1891 obtained from the Aldrich Report (Report of Committee on Finance of the U. S. Senate on Wholesale Prices, Wages and Transportation, 52d Congress, 2d Session). For period from 1892 to 1907 from the reports of the United States Bureau of Labor (*Bulletins Nos. 77 and 259*). From 1908 to 1920 from monthly *Labor Review*, March, 1921.

(2) Wage trend based on conditions preceding Civil War, ignoring effect of war.

(3) Average wage trend from 1879 to 1899; increase between curves (3) and (2) probably all due to effect of war.

(4) Wage trend based on conditions preceding World War I, ignoring effect of war.

(5) Actual wage conditions for twelve years after Civil War applied to present (1921) conditions to show probable wage curve after the war.

(6) Probable wage trend following World War I based on wage conditions following Civil War. Increase over curve (4) due to war.

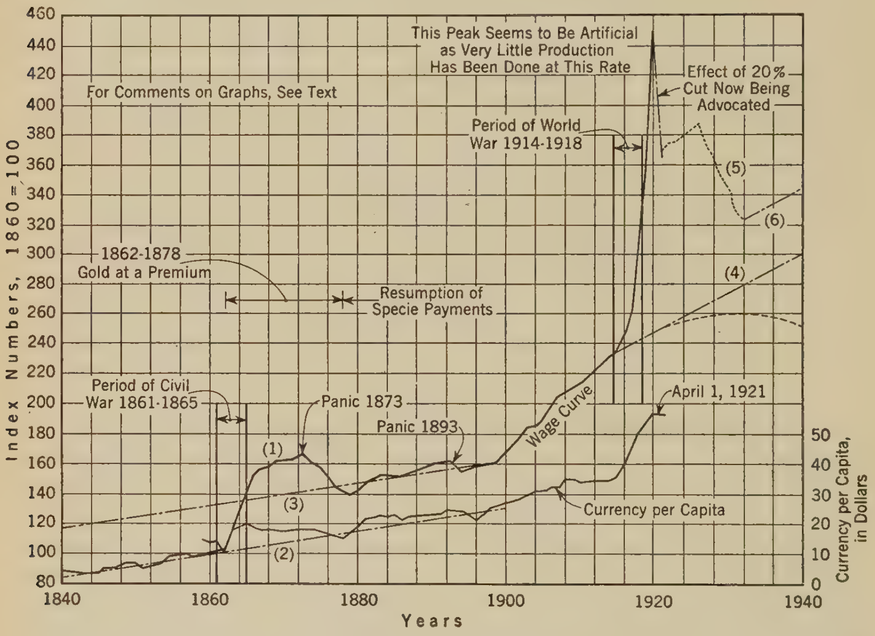

Fig. 3.—Relative Wages in United States, 1840–1920

The remarkable feature about Fig. 3 is that it forecasted very accurately the future wage levels, indicating a recession from 1921 to 1932 after which it predicted the upward trend would be again resumed. The low point on this diagram (1932) was approximately 145 as related to 100 for 1914, the beginning of the war. It is interesting twenty-four years after the estimate to check this forecast for the "trough" with the various construction cost indexes actually recorded for that same date. By way of comparison, some of the more important indexes for 1932 to the same base were as follows:

Engineering News-Record (E.N.–R.) construction cost index......	157
American Appraisal Company..................................	155
Associated General Contractors................................	171

In view of its historical significance, this exhibit (Fig. 3) is included herein just as it was presented with testimony.

In Fig. 4 there are three diagrams, each starting with the beginning of an important war in which the United States was engaged. The ordinates represent indexes with the first year of the war in each case being assumed as 100, as follows:

(*A*) World War I—E.N.-R. construction cost index (1913 = 100).
(*B*) Wages before, during, and after the Civil War (1840–1891) from Aldrich Report and from 1890 to 1919 from U. S. Bureau *Bulletins Nos. 77 and 259* and monthly *Labor Review*, March, 1921.
(*C*) World War II—E.N.-R. construction cost index (adjusted to 1939 = 100).

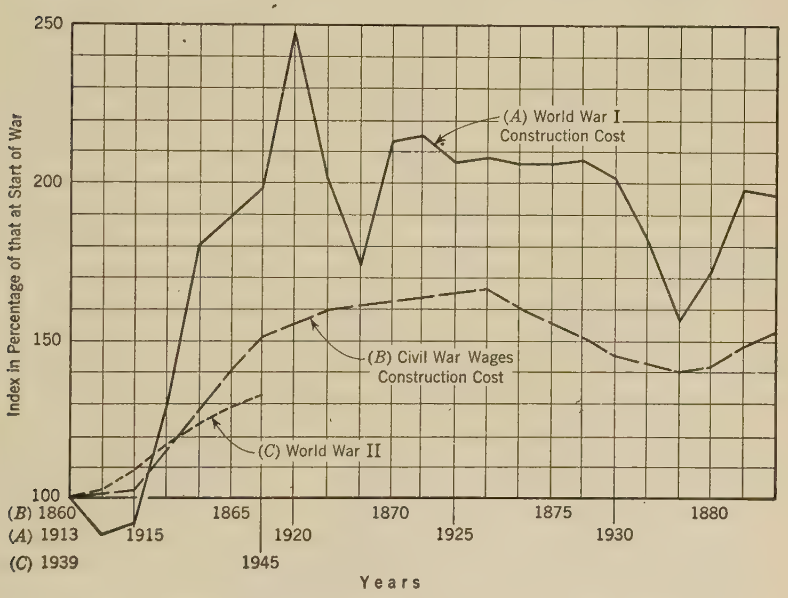

FIG. 4.—EFFECT OF WAR ON CONSTRUCTION COST AND WAGE INDEXES

The relationship and similarity during the three periods is apparent. In the Civil War and World War I aside from the extreme peak in 1920, the indexes continued to rise or maintain their level until nearly ten years after the close of the war. If it is logical to predict the future from the past, the E.N.-R. construction cost index, which as of January, 1946, was approximately 310 (1913 = 100), will rise to the 350 to 360 range by 1951 with nothing more than short time reactions in the interim.

To show the relation between wage rates and construction costs, Fig. 5 contains two diagrams, one (*A*) showing the union hourly wage rate of all building trades as reported by the U. S. Department of Labor and the other (*B*), E.N.-R. construction cost index. Just a casual glance at these diagrams, both platted with respect to the same time and index scales, will serve to show their close relationship. Construction costs have been somewhat more volatile and there would appear to be a lag in wage rates reflecting changing economic conditions, but aside from this minor difference, the relationship is remarkably close. This is as would be expected, for in the ordinary construction project

40% more or less of the total cost of project is spent for labor on the site, and a very substantial part of the remainder is expended for labor in manufacturing materials entering into the construction.

It should be noted that Fig. 5 does not reflect the result of post World War II strikes, wage increases, and cost increases. These might well raise graph (*B*) to 350 or 360, as compared to its 1945 figure of 310, instead of requiring until 1950 for prices to reach that level under a continuation of the average rate of increase on construction costs since 1932.

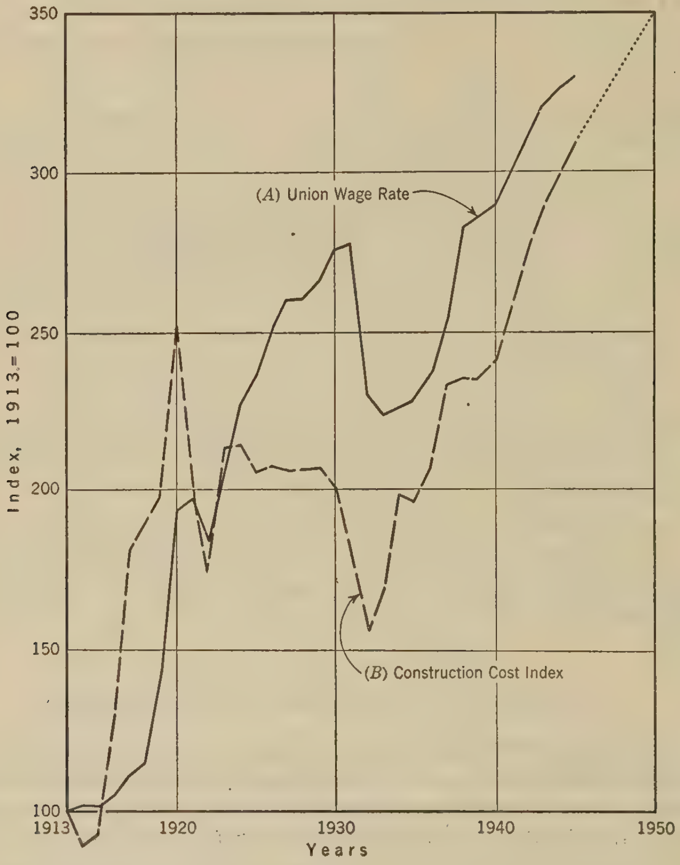

FIG. 5.—RELATION BETWEEN WAGE RATES AND CONSTRUCTION COST

5. CONSTRUCTION COST INDEX OF PAST AND FUTURE

In Fig. 5 there has also been platted the projection of E.N.-R. construction cost index to 1950. This graph discloses that:

(*a*) The only time since 1913 that the United States has had a substantially level construction cost index was during the period of prosperity extending through the twenties.

(*b*) The inclination of the construction cost index from 1932 to 1940, which is generally considered to have been a period of depression, was substantially as steep as during World War I, and World War II. This is believed to be a reflection of many factors, including general deterioration in labor output and efficiency; the growing effort of some labor union executives to discourage production incentives and piecework; the demoralizing effect of certain types of made work upon labor efficiency and morale; and various governmental efforts to keep prices up rather than permit them to react to ordinary economic laws.

It seems incongruous that, during the period covered by one of the worst depression periods ever experienced in this country, construction costs should rise more than 50%. With the prices of such basic commodities as corn and hogs depressed to 20% of normal and with 10,000,000 men out of work, construction costs increased at the rate of 10% per year. Contractors went out of business and sanitary engineering construction work was so revolutionized that for a period of nearly ten years not a foot of sewer was laid by contract in the City of Chicago (Ill.) The Work Projects Administration (W.P.A.) with its demoralizing effect on labor efficiency had effectively supplanted contract work. Public works were largely subsidized by federal funds and minimum wages were instituted as a condition for the subsidy. During the nine years from 1932 to 1940, inclusive, public expenditures for construction averaged 75% as much as private. This compares to a normal of from 30% to 40%.

In 1945 Harry E. Jordan, Affiliate ASCE, executive secretary of the American Water Works Association, in an effort to secure a cross section of opinion as to why the $2,000,000,000 municipal public works program was not progressing more rapidly to the blueprint and construction stages, sent a questionnaire to 150 consulting engineers, to engineers of the various state departments of health, and to others interested in water works and sewerage construction. The replies were prompt and comprehensive. In general, the reasons for the delay fell into three rather well-defined channels:

1. Rising costs of construction.
2. Uncertainty as to whether there would be federal participation in public works after the war.
3. Congestion in designing engineers' offices.

The first reason, that of rising costs, is fully discussed herein. The second reason, resulting from the uncertainty as to federal policies, can only be removed by prompt action in Washington and of that there seems to be little likelihood. The third reason, congestion in designing engineers' offices, is gradually ironing itself out.

Uncertainties Affect Contract Costs.—It was almost the universal experience during 1945 that such sanitary engineering work as went out to contractors for bids attracted few bidders and their prices were materially higher relative to prewar amounts than would be indicated by the increase in the E.N.–R. construction cost index. This reflects the uncertainty and fear inherent in the present picture. Uncertainty costs money. Contractors have their four fears: (1) Rising material prices; (2) inability to get materials when needed; (3) rising labor wage rates; and (4) decreased efficiency of labor.

Availability and efficiency of labor will probably improve somewhat after the unemployment benefits to war industry workers are stopped. It is believed that the only important recession from present high construction contract costs will be brought about by stabilizing wages and thus stabilizing prices of materials but that the general construction cost tendency thereafter will be upward for the next few years.

6. Conclusions

From the foregoing discussion it is believed certain general conclusions may be drawn:

1. It is apparent that hesitancy and uncertainty, particularly in the labor picture, both as to cost and efficiency, justify the reluctance of contractors to bid on work of any magnitude requiring any considerable length of time, unless a very liberal "contingency" factor is included. Washington's "no policy" labor attitude is believed to be largely responsible.

2. It is believed that clarifying the labor picture will greatly improve the possibility of securing fair bids on construction work; that wage stabilization even at a higher level will affect contractors' bids less than the previous uncertainty.

3. Engineers should be realistic. Postwar construction costs are bound to be materially higher than prewar. In most lines of general construction the increase above 1940 will probably approximate 50% by 1950, and in certain classes of construction such as pipe lines and sewers involving a large proportion of common labor, the increase will be materially higher.

4. Aside from the removal of the contingent item by the clarification of the labor situation, there is nothing in the picture of prospects for 1946–1956 to indicate that construction costs will be materially lowered. There will probably be a continued rise, possibly accelerated by the further rise in labor wage scales.

5. Rising construction costs particularly to an extent in excess of 50% above 1940 will materially retard construction and many public works will be deferred or abandoned.

6. One of the important deterrents toward normal procedure in public works construction is the failure of Washington to declare itself with respect to federal aid. Aside from a few of the very large cities most American cities do not want federal aid for their projects. However, many public works will be deferred awaiting a definite position relative to federal aid for public projects. Public officials do not want to be in the position of having their city proceed on its own when had it waited a few months it could have had a government subsidy which would either have reduced that city's cost or enabled it to do more work.

7. So far as sanitary engineering works are concerned, about $2,000,000,000 worth of water works and sewerage construction was in the development stage as of January 1, 1946, of which something less than 10% was in the blueprint stage. Cash was in hand for much of the water works improvements. Sewerage, however, was not so fortunate; much of this work was bound to be delayed as it was projected on the basis of 1940 costs which had already been exceeded by from 30% to 60%.

DISCUSSION

Isador W. Mendelsohn,[2] M. ASCE.—The subject of this timely paper concerns every engineer intimately. Construction costs are rising and are in a stage of unpredictable flux. As to the phenomena responsible for increasing construction costs, it is necessary to study the entire construction industry critically and to evaluate the numerous factors and their interplay. Anything short of such analysis will provide only an unbalanced judgment. When, in four of his seven conclusions, Mr. Howson stresses the relationship of wage rates to high construction costs and in none of the conclusions specifies definitely any other influences contributing to such high costs, it is a moot question as to whether his analysis of factors of construction costs is complete.

Characteristics of the Construction Industry.—The construction program in the United States for the period from 1915 to 1944 comprises new, work relief, and maintenance construction. In past years the cost of new work has varied from $2,350,000,000 in 1933 to $13,498,000,000 in 1942,[3] averaging $6,700,000,000 annually. Maintenance construction has averaged $3,000,000,000 annually, whereas work relief construction for the period from 1933 to 1943 has averaged $636,000,000. New construction includes private and public residential and nonresidential buildings, farms, public utilities, military and naval work, pipe lines, highways, sewage disposal, water supply, conservation, and development. In hearings before the Temporary National Economic Committee on Investigation of Concentration of Economic Power, Isador Lubin testified[4] that the construction industry employed 5.5% of nonagricultural workers in 1929 and consumed about 15% of the commodities produced in the United States for the period from 1919 to 1935. Willard Thorp stated[4] the construction industry has certain definite characteristics: (1) It produces a product which is not uniform in type, design, size, and location, and operates on a made-to-order basis; (2) the products are far from simple, requiring thousands of materials and a variety of skills; and (3) the demand is irregular, essentially local, and highly seasonal. Considering the third point, Beardsley Ruml has aptly stated that the circumstances which result in a profitable outlook for the building of a factory by one company will simultaneously affect thousands of companies.[5]

According to the Social Security Board, there were, in 1938, 22,000 general building contractors, 8,000 other general contractors, and nearly 67,000 special trade contractors, involving a total of more than 825,000 employees. To these should be added about 50,000[4] who operate "on their own" in the construction industry.

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[2] San. Engr., Washington, D. C.

[3] "Total Construction Activity in the Continental United States," Constr. Statistics Section, Constr. Div., Bureau of Foreign and Domestic Commerce, U. S. Dept. of Commerce, Washington, D. C., January, 1946.

[4] "Construction Industry," Temporary National Economic Committee, Pt. 11 of "Investigation of Concentration of Economic Power," June 27–July 14, 1939 (*No. Y4.T.24:Ec. 7/Pt. 11*, Supt. of Public Documents, Washington, D. C.).

[5] "Public Works and Construction After the War," by Beardsley Ruml, address before New York Building Cong., New York, N. Y., April 24, 1945.

In comparison with other industries, Mr. Ruml finds that the construction industry is costly and wasteful.[5] Pertinent factors[4] are: (1) Difficulty of organizing and planning an efficient work program, bringing many parts together in varying ways, places, and on various types of operations; (2) expensive purchasing, in small lots; (3) expensive selling practices; (4) uneven flow of work and employment; (5) little research, and resistance to new methods and materials; (6) inadequate and unstable financing; (7) bitter competitive practices leading to agreements, collusions, price controls, use of building codes, and union restrictions; (8) a tendency for each part of the construction industry to increase its own charges rather than to reduce them; (9) high transportation costs of construction materials (in 1936, 57% of the value of gravel and sand at destination, 28% of that of common brick, and 21% of that of lumber, shingles, and laths, being absorbed by freight charges); and (10) high degree of concentration of construction material production, showing relatively few producers for many important items, four leading companies[4] cutting 22.8% of the Douglas fir; 7% of the southern pine; and 49.3% of the structural steel shapes.

According to Mr. Ruml:[5]

> "Architects, building contractors, building supply companies, labor, finance and mortgage companies, all found it necessary—each in its own way—to establish and to hold a price structure high enough so that the days and hours of activity would pay for the time when there was little or nothing to do."

Unreasonable Restraints of Trade in the Construction Industry.—In the opinion of Thurman Arnold,[4] unreasonable restraints of trade are the most conspicuous reasons for high construction costs. Restraint practices developed in government proceedings involve producers and distributors of building materials, contractors, and labor. Producers of building materials have fixed prices by private arrangement.

Distributors of building materials try to raise the price of their services by establishing a fixed markup between the price they pay the manufacturer and the price at which they sell. Many groups of contractors set up little closed markets from which they exclude outside contractors or new types of services.

The building-trades unions have frequently been used as the "strong-arm squads" for collusive agreements among contractors, refusing to supply labor where the contractors' ring wishes labor withheld. In other cases the unions themselves have refused to permit the use of new products or new processes.

Restraints of trade in building have: (*a*) Harassed, boycotted, and eliminated competitors able and willing to reduce prices; (*b*) handicapped the use of prefabricated materials and thwarted the development of methods of mass production in the industry; and (*c*) prevented experiment in housing design, materials, and methods of construction.[4]

United States Department of Justice Action Against Restraints of Trade.—Since the passage of the Clayton Act in 1914, the Department of Justice has instituted approximately 674 cases,[6] of which 160 have involved some parts of the construction industry. In 1939 the Anti-Trust Division of the Department launched a broad attack on restraints in the building industry. Preliminary surveys were made in twenty-six cities. Grand juries were assembled in eleven cities. By September, 1940, ninety-eight indictments involving 1,281 defendants had been returned. Among the defendants were associations of manufacturers, distributors, contractors, and labor; and individual officers of trade associations, business enterprises, and labor unions. In addition to the criminal cases, nineteen equity proceedings were instituted, involving a total of 367 defendants (as stated by Thurman Arnold in an unpublished address before the Illinois Association of Real Estate Boards at Chicago, Ill., on September 20, 1940).

From this campaign economic results appeared more quickly than did legal results. In May, 1939, the Pittsburgh city engineer[7] drew up an estimate of the cost of the electrical work in a new municipal hospital being built with funds of the Public Works Administration (P.W.A.). His estimate was $105,000; the city advertised for bids, opened them, and found that the lowest was $154,000. Specifications were revised and the city re-advertised. The lowest bid was then $148,000, which was rejected. The third set of bids brought a low offer of $152,000.

About that time a team of eight men from the United States Department of Justice reached Pittsburgh and began its investigation. This team advised rejection of the latest bids. On November 3, a federal jury indicted twelve electrical contractors, a trade association, and forty-five individuals, charging a conspiracy to defraud through collusive bidding. A few days later the city received a new set of bids for this hospital electrical work—with a low, this time, of $117,000.

A little later the city[7] opened bids for the purchase of sand and gravel. For the first time in years, the sand and gravel bids received were not identical, and the quoted prices dropped from the prevailing level of $2.25 a ton to from $1.65 to $1.80 a ton—a saving of $17,000 on sand and gravel for the first quarter of 1940.

In another case a low-cost housing project in Pittsburgh was planned in two units, bids on the first being opened before the Anti-Trust Division began investigating and those on the second after the federal grand jury had been sitting about three months. The investigation resulted in bids lowering the electrical contract by 23.5%, the heating contract 27%, the plumbing contract 26%, and the general contract 12%. The over-all reduction in the cost per family on the second unit was $148 per room, or 17% of the total cost.

A federal grand jury in New Orleans returned an indictment on February 21, 1940, against three organizations representing lumber interests. These groups were charged with conspiracy to fix prices, to curtail output, to enforce an agreed policy of distribution, and to restrain trade by abuse of grading, grade marking, and inspection. All these practices were subsequently enjoined.

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[6] "Anti-Trust Cases in the Construction Industry," U. S. Dept. of Justice, Washington, D. C., July 1, 1946.

[7] "War and Prices," Temporary National Economic Committee, Pt. 21 of "Investigation of Concentration of Economic Power," December 4–8, 1939 (*No. Y4.T.24:Ec. 7/Pt. 21*, Supt. of Public Documents, Washington, D. C.).

Federal Trade Commission (F.T.C.) Action Against Restraints of Trade.—Illustrative of many unlawful monopolistic cases in restraint of trade concerning construction activity processed by the F.T.C. are the following:

1. Approximately thirty-one corporations engaged in the business of manufacturing water gate valves, hydrants, fittings, and similar products,[8] and in the sale of such products to cities and state and federal governments, comprised substantially all the manufacturers of such products used for water supply systems. They were located in seventeen different states. The monopolistic practices in this case consisted of agreements by these corporations to fix and maintain prices. The commission found that these practices had actually hindered price competition and had created in the members of this group a monopoly unlawfully restraining interstate commerce.
2. In 1937[8] the commission issued findings and an order against an organization of building material dealers with a membership of more than 150. The materials in which the organization dealt included cement, brick, tile, clay products, sewer pipe, sand, gravel, stone, lime, lumber, roofing, and other materials ordinarily used in the construction industry. The main objective of these associations and their members was to confine retail distribution in building materials and supplies to recognized dealers and to prevent the direct sale by manufacturers to all others.

In 1935 the United States, through the Procurement Division for the Relief Administration,[8] sent to manufacturers in one state an invitation for bids on 100,000 bbl of cement. As a result of action by the state association of builders' supplies, no cement company would quote prices. When the same invitation was mailed to manufacturers outside of that state, again no direct bids were received.

The commission found that interstate commerce in the sale and distribution of building materials was thus restrained because the organized collusion eliminated the so-called irregular dealers and manufacturers and producers selling to such dealers. Competition in the sale of all building materials was substantially lessened. Competitors of members were unable to obtain interstate shipments of their requirements. Costs to the consuming public were increased.

Identical Delivered Price Systems in the Construction Industry.—A paramount economic factor in unfair pricing in the construction industry is the identical delivered price systems, such as the basing-point, the freight-equalization, and the zone systems. According to the Procurement Division of the United States Treasury Department, during the period from December, 1937, to November, 1938, 76,000 bid openings, or 24.1% of 332,000 bid openings during that period, involved the receipt of identical bids from different sellers.

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[8] "Federal Trade Commission Report on Monopolistic Practices in Industries," Temporary National Economic Committee, Pt. 5-A of "Investigation of Concentration of Economic Power," March 2, 1939 (*No. Y4.T.24:Ec. 7/Pt. 5-A*, Supt. of Public Documents, Washington, D. C.).

This covered industries including all the major categories from which the government makes purchases.[9]

Concerning the basing-point system,[8] the federal government was in need of steel sheet piling for public jobs at Morehead City, N. C., at Miami, Fla., and for the Triborough Bridge in New York, N. Y., in 1933. The government received bids from only the four companies in the United States making this sheet piling, and those bids were alike for each one of the three jobs delivered at the three sites. The bids were determined by the basing-point system—for example, on the Miami project, freight from Pittsburgh to Miami plus the base price of the Pittsburgh manufacturer. Whichever one of these freight-plus-base price arrangements to Miami was lowest—that was the price charged by all four companies.

The price was identical for the unwelded piling per 100 lb, for piling corners, piling T-sections, fabrication, the extra in welding in pairs, copper content, and all-rail freight factor.

The president of a large corporation testified[8] regarding steel:

> "We generally make the prices.* * * If anyone in the industry makes a lower base price, then that price is taken * * * and becomes a part of the basing-point formula, and all delivered prices are identical again."

Among the industries[8] with basing-point systems are basic industries—steel, pig iron, cement, and lumber—and other industries—builders' supplies, road machinery, vitrified clay sewer pipe, construction machinery, reinforcing materials, valves and fittings, and cast-iron pressure pipe.

Identical delivered prices undermine incentives to economical production and distribution and enable obsolete mills to continue long after they would have gone out of business if price competition had prevailed. Above all, they deprive the public of the benefit of price competition.

In testimony on the basing-point procedure, Harold L. Ickes stated that[8] in purchases by the federal government between June, 1935, and March, 1936, under his administration as Secretary of the Interior, identical bids occurred at least 257 times involving a gross expenditure of $2,866,252.97. Identical bids were received for structural steel, steel tanks, valve boxes, well drilling, fire hydrants, pumps, plumbing and heating specialties, aerators (sewage), cast-iron pipe, water meters, and filter equipment.

Building Codes.—Another of the numerous aspects of the construction industry are the building codes. Such codes[10] are important and necessary in construction, primarily to protect safety and health and next to allow for all the efficiency that industry and enterprise can provide. A survey of the National Bureau of Standards indicates that 25% of the codes have not had a thorough overhauling for more than 20 years, whereas another 23% have not had one for from 16 to 20 years. An extensive survey[11] of building legislation by the John B. Pierce Foundation indicates that a complete revision of the Chicago code is desirable because of its obsolescence, restrictiveness of requirements, and too much variation from generally recognized material standards.

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[9] "Cartels," Temporary National Economic Committee, Pt. 25 of "Investigation of Concentration of Economic Power," January 15–19, 1940, p. 13,330 (*No. Y4.T.24:Ec. 7/Pt. 25*, Supt. of Public Documents, Washington, D. C.).

[10] "Your Building Code," by Miles L. Colean, National Committee on Housing, New York, N. Y., February, 1946.

[11] *Engineering News-Record*, April 18, 1946, pp. 84–87, 108, and 113.

Wide divergence of building code requirements among cities increases costs. Allowable working stresses in concrete, for instance, vary from 500 lb per sq in. to 1,000 lb per sq in.; minimum thicknesses of brick basement walls vary from 8 in. to 16 in. for the same height and load; live-load requirements for dwellings vary from 25 lb per sq ft to 80 lb per sq ft; minimum permitted floor areas for the same type of room vary from 60 sq ft to 120 sq ft; and minimum ceiling heights vary from 7 ft to 9 ft.

Analysis of a random selection of codes showed that the city requiring the lowest amount of pipe (by size and weight) in a one-story house saved 30% of the total amount of cast iron required by the city with maximum requirements. The costs under the minimum code were 80% of the costs under the maximum code for metals other than cast iron[10]—a saving of 20%.

Construction Cost Index, Engineering News-Record (E.N.-R.)—It is noted that the E.N.-R. construction cost index is referred to especially by Mr. Howson in developing his thesis of close relationship between wage rates and construction costs. It has been stated[11] that the E.N.-R. construction cost index:

> "* * * measures wage rate and material price trends. It is not adjusted for labor efficiency, competitive conditions, management, mechanization or other intangibles affecting construction costs."

—nor does it reflect quality of materials. It is well to note that none of the indexes listed[11] by the E.N.-R. construction cost index contain most of the mentioned factors and but one includes productivity of labor and efficiency of plant and management.

According to a letter dated June 28, 1946, the components for the original E.N.-R. construction cost index were chosen by selecting an imaginary cube of construction that would be composed of 2,500 lb of structural steel, 6 bbl of cement, 600 fbm of 12-in. by 12-in. long-leaf yellow pine, and 200 hours of common labor. This formula produced a value of 100 for 1913. Now the index has four components[12]—(1) structural steel shapes, Pittsburgh base price, multiplied by 25 cwt; (2) cement price at Chicago, exclusive of bags, multiplied by 6 bbl; (3) lumber (which until 1935 was 12-in. by 12-in. long-leaf yellow pine wholesale at New York and since 1935 2-in. by 4-in. S4S pine and fir in carload lots), E.N.-R. twenty-city average price, multiplied by 1,088 fbm; and (4) common labor, E.N.-R. twenty-city average of wage rates in force, multiplied by 200 hours. Analysis indicates that, as a reliable measure of construction cost trends (magnitude involved or competitive prices), the four components are representative neither of construction materials nor of labor.

Before 1921, the labor component represented less than 50% of the total material and labor weight,[11] whereas since 1921 with the exception of 1923, the labor component has been more than 50% and is now (1946) approximately 60% (see Table 1). Actually, labor on nonbuilding construction (see Table 2) approximates from 20% to 45% of labor and material costs.[13] According to a tabulation of labor and material estimates for various types of federal construction projects during 1945 prepared by the Bureau of Labor Statistics (B.L.S.),[11] the percentage of on-site labor varied from 40.0% to 47.6% for building and nonbuilding construction. Furthermore (see Table 3), common labor on nonbuilding construction approximates 40% of the total labor used (data supplied to the writer by Edward M. Gordon from the records of the B.L.S.). The common labor component of the E.N.-R. construction cost index does not truly reflect the skilled, semiskilled, and supervisory labor comprising the larger part of such construction.

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[12] "Survey of Current Business," U. S. Dept. of Commerce, 1942 Supplement, p. 183.

TABLE 1.—FOUR-COMPONENT PERCENTAGES OF E.N.-R. CONSTRUCTION COST INDEX (1913–1945)

Year	Index	Weighted Percentages of Construction Cost Index			
		Labor	Cement	Lumber	Steel
1913.....	100	38.0	7.1	17.4	37.5
1917.....	181.24	31.0	5.8	13.2	50.0
1919.....	198.42	47.0	8.0	13.3	31.7
1921.....	201.81	53.6	8.4	13.2	24.8
1929.....	207.02	52.8	6.5	17.6	23.1
1932.....	156.97	54.4	6.2	14.6	24.8
1935.....	196.44	53.7	6.6	16.8	22.9
1939.....	235.51	58.0	5.6	14.1	22.3
1940.....	241.96	57.7	5.5	15.1	21.7
1945.....	307.74	59.2	4.8	18.9	17.1

Table 2 indicates that the three material components of the E.N.-R. construction cost index represent generally minor portions of the total materials incorporated in the projects.

"Structural Steel Shapes" Component.—The classification, "structural steel shapes," is not representative, in quantity and price, of various forms of steel used in nonbuilding construction. From an extensive study[14] of consumers' prices of steel products made by the B.L.S. in 1943 for the period from January, 1939, to April, 1942, the following conclusions are presented:

a. Actual delivered prices paid by steel consumers deviate frequently from published delivered prices. By published price is meant the sum of published base price at the basing point nearest the consumer, plus published extras applicable to the given specification, plus rail freight from the basing point to the consumer's plant.

b. The relative importance of the three components of the delivered price is approximately: Base price, 79%; extras, 14%; and freight, 7%.

c. Base prices alone are neither good measures of the level of prices of steel nor adequate indicators of the comparative prices of different steel products.

d. The delivered price of steel was increased as much as 50% in some instances, because of higher freight costs resulting from changes in basing points.

e. Actual prices varied from 50% to 135% of the April, 1942, published delivered prices during the period covered, whereas published prices remained stable.

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[13] "Construction Studies of Public Works Administration Projects," by George D. Babcock, Director, Eng. Management, Federal Works Agency, Washington, D. C., 1943.

[14] "Labor Department Examines Consumers' Prices of Steel Products," *Iron Age*, April 25, 1946.

f. For many products a very small proportion of purchases is made at the base price.

g. Extras applicable to structural shapes include size and section, special cutting, length, United States Navy and American Society for Testing Materials specifications, cambering, chemical requirement, milling, splitting beams, painting, special marking, protected shipment, surface finish, federal specifications, quantity, test requirement, and restricted physical test.

TABLE 2.—PERCENTAGES OF MATERIAL, LABOR, AND OTHER COSTS FOR GROUPS OF PUBLIC WORKS ADMINISTRATION PROJECTS (1934–1939)

Item	Type of project	PERCENTAGE OF CONSTRUCTION COST (100%)			WEIGHTED AVERAGE, CONSTRUCTION PROJECTS		PERCENTAGE OF TOTAL MATERIAL COST				RANGE OF COSTS (THOUSANDS OF DOLLARS)	
		Material	Labor	Other	Date	No.	Cement	Lumber	Steel bars	Structural steel shapes	From	To
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Water Works:											
1	Surface supply.....	59.0	23.9	17.1	October, 1935	8	5.8	5.2	4.3	6.6	51	610
2	Distribution only..	67.0	17.8	15.2	January, 1936	8	0.5	0.2	0.1	3.5	43	268
	Power Plants:											
3	Steam and diesel...	75.0	14.1	10.9	October, 1936	8	2.2	1.6	0.6	4.3	71	661
	Dams:											
4	Concrete..........	48.0	24.8	27.2	August, 1937	2	36.9	8.2	0.6	0.2	491	543
	Electric Power:											
5	Transmission......	74.0	18.4	7.6	December, 1936	3	...	19.7	...	...	108	300
6	Distribution.......	72.2	14.2	13.6	May, 1938	6	...	17.2	...	...	65	256
	Bridges:											
7	Truss and girder...	61.6	23.7	14.7	November, 1936	6	10.3	4.0	2.8	62.6	549	2,089
	Sewage Disposal:											
8	Activated sludge...	62.6	24.0	13.4	July, 1936	6	12.2	7.4	7.8	2.9	95	334
9	Primary treatment.	57.9	30.2	11.9	November, 1936	8	8.2	6.1	6.7	6.6	77	402
10	Trickling filters....	62.5	20.0	17.5	December, 1936	6	9.6	5.4	3.2	2.0	46	379
	Sewers:											
11	Sanitary..........	44.0	36.1	19.9	October, 1936	10	4.8	2.5	...	...	57	498
12	Storm............	49.7	26.4	23.9	June, 1935	5	25.2	14.9	3.3	0.1	72	308
	Pavements:											
13	Bituminous.......	58.4	24.2	17.4	October, 1935	7	8.1	...	0.1	...	32	90
14	Concrete.........	59.5	22.5	18.0	November, 1935	5	42.7	...	9.1	...	23	110
	Hospitals (Fireproof):											
15	Concrete frame....	59.6	28.9	11.5	August, 1935	6	5.3	10.1	4.8	2.8	75	190
16	Steel frame.......	65.2	26.5	8.3	April, 1936	4	3.4	5.6	1.9	8.4	133	1,200
	Schools (Fireproof):											
17	Concrete frame....	56.2	33.0	10.8	January, 1937	7	8.1	12.3	6.7	6.9	94	220
18	Steel frame.......	59.1	31.4	9.5	June, 1936	5	4.5	10.3	2.8	9.3	89	1,700

TABLE 3.—PERCENTAGES OF VARIOUS CLASSES OF LABOR ON FEDERAL PROJECTS IN 1940

Type	Superintendent	Foreman	Clerks	Skilled	Semiskilled	Common	Other Unskilled
Highway[a]...........	1.9	8.8	1.5	15.5	32.0	39.6	0.7
Sewer and water[a].....	3.6	9.8	1.5	28.5	14.9	40.2	1.5
Civil aviation[a].......	1.8	7.6	2.0	24.1	25.1	38.5	0.9
Sanitary sewers[b]......		22.3[c]		35.9[d]		41.8[e]	

[a] Two hundred Public Works Administration and other federal projects. [b] Three projects at Greece, N. Y., totaling $327,000. [c] Overhead—that is, superintendents, foremen, and clerks. [d] Skilled and semiskilled labor. [e] Common and other classes of unskilled labor.

Lumber Component.—Lumber used as one of the four components of the E.N.-R. construction cost index does not portray competitive price trends. According to the Federal Trade Commission (F.T.C.),[15] of the total lumber cut in the United States (24,975,000,000 fbm) in 1939, nearly 83% was produced in four Pacific states and eleven southern states. About 58% of the total cut in 1935 was for building construction. A statement of the Interstate Commerce Commission for 1939 shows that out of each dollar at destination the cost of transportation amounted to 20.16¢ for lumber, shingles, and laths and 27.79¢ for cement.

In 1940 the B.L.S. index of lumber prices rose steadily from August 3 to September 7, for a total of more than six points. There was no shortage of lumber, actual or prospective.

The rise in lumber prices was used to justify an increase in the price of low-cost houses of more than 7%. Complaints reaching the Department of Justice suggested that some of these price increases were due to collusive price fixing and restriction of output. Accordingly, federal grand juries investigated the production and distribution of construction lumber. In February and April, 1941, consent decrees were entered in two cases which enjoined lumber associations against restraint in trade practices.[6]

According to the E.N.-R.,[16] current prices for 2-in. by 4-in. lumber delivered in the New York area have reached a new high of from $85 to $95 for southern pine and from $91 to $121 for Douglas fir. The price structure per 1,000 fbm of Douglas fir consists of: Mill price, $37; freight to New York, $25; dealer handling, $5; 30% markup, $20; and a truck delivery charge of from $3.55 to $32.

Another news announcement in the E.N.-R.[17] states that, in federal courts in California, Washington, Oregon, and Arizona, the Office of Price Administration (O.P.A.) filed forty suits against lumber firms, seeking a total of about $9,000,000 in treble damages on charges of illegal activities involved in handling 65,000,000 ft of lumber. The O.P.A. charged that shippers had diverted lumber by sending it to themselves at dummy addresses and holding it there for bargaining; and that mills were producing out-size timbers in order to charge premium prices.

Another aspect of the lumber trade that deserves attention is quality of product delivered. The E.N.-R.[18] carries the news item that the St. Louis (Mo.) area is being flooded with black market lumber, some of it so green that the "* * * sap practically is running out of it * * *." As one O.P.A. official stated:

> "The great tragedy of this 'racket' is not so much the inflated cost of this housing material, but the loss of investment that will come when this green wood starts to buckle * * *."

Cement Component.—There were 154 active cement manufacturing mills in the United States in 1941 with an estimated annual capacity of about 247,000,000 bbl. About 55% of the total United States Portland cement producing capacity is concentrated in ten companies.[15]

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[15] "Report of the Federal Trade Commission on Distribution Methods and Costs," Pt. III, "Building Materials—Lumber, Paints and Varnishes and Portland Cement," F.T.C., Washington, D. C., 1944.

[16] *Engineering News-Record,* June 27, 1946, p. 51.

[17] *Ibid.,* July 4, 1946, p. 1.

[18] *Ibid.,* June 6, 1946, p. 1.

Since about 1916, the delivered prices of cement have been identical at any particular destination in the United States.[6] For example, the United States Navy Department opened bids for cement on November 17, 1943, for eighteen delivery points along the seaboard from Portland, Me., to New Orleans, La. At Boston, Mass., sixteen companies bid an identical price of $2.52 per bbl. At Philadelphia, Pa., fifteen companies bid identically at $1.98 per bbl. The price in Washington, D. C., for sixteen companies was $2.11.

The F.T.C. has had a pending case against the basing-point system in the cement industry for a number of years and the Department of Justice brought a suit against a leading trade organization with about ninety member cement companies in June, 1945. The F.T.C. reported to the United States Senate in 1933 that, through the use of the basing-point system, many commercial and private purchasers have been forced to pay higher prices for cement. The basing-point system has encouraged crosshauling with resultant aggregate increases in freight.[8] On the basis of available data, the F.T.C. estimated that in 1927 there was an average unnecessary burden of 24.3¢ per bbl which, applied to the entire production of that year, made a total of about $42,000,000.

Labor Productivity.—Although the B.L.S. has been making studies on labor productivity since before 1898, it has not as yet developed an adequate measure based upon fundamental concepts in construction activity. Of productivity of labor in industry, W. D. Evans of B.L.S. has stated that,[19] in the period between the wars, most industries achieved large increases in man-hour output. Better processes, machines, handling of materials, organization of jobs—all contributed to improvement of productive efficiency. During World War II, unit labor requirements for liberty ship construction decreased almost 60% from the date the first ship was delivered in December, 1941, to December, 1943.[20] The decline in unit labor requirements reflects the economies of mass-production methods.

Apropos of labor productivity in building construction are some news items that appeared in 1946.[21,22] An Ohio contractor, charged by the Wage Stabilization Board with paying workmen illegal (too high) wages in constructing homes for war veterans, told how he "trimmed" construction costs so that he was able to sell five-room homes with an O.P.A. ceiling price of $7,250 for $5,850 and make money. By using his own superintendents and the best men obtainable, together with labor-saving equipment, and eliminating all subcontractors, this contractor reduced the cost of excavation from $75 to $14 per house; reduced the brickwork cost from $325 to $130 per house; sewer digging and pipe laying costs $100; cement basement floors and steps costs from $220 to $90; rough-in carpentry costs from $300 to $120; plumbing costs from $650 to $350; painting costs $250; electrical work from $140 to $60; and grading costs from $200 to $20 per unit.

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[19] "Recent Productivity Trends and Their Implications," by W. D. Evans, B.L.S., Washington, D. C., January 25, 1946.

[20] "Productivity Trends in American Industry," B.L.S., Washington, D. C., January, 1946.

[21] *Engineering News-Record*, May 23, 1946, p. 3.

[22] *Ibid.*, July 4, 1946, p. 2.

Efficiency of Management in Construction Activity.—Lives there an engineer who in the course of his professional practice has not witnessed examples of construction abandoned after little or no use which could have been forestalled by proper planning and management?

In the initial stages of P.W.A. several water distribution systems were installed before suitable water supplies were developed. Some of these supplies did not prove adequate. In the case of a sewage treatment plant, a garbage grinding unit was built but was only tested and has never been operated, because the city street department (which collects the garbage and is not under the authority of the sewage treatment plant) never changed the routes of the garbage collection and now disposes of the garbage by sanitary fill.

According to the E.N.-R.[23] an agreement was reached in 1946 between the District of Columbia and the Washington Suburban Sanitary Commission whereby the sewage from Montgomery and Prince Georges counties, Maryland, will be discharged into the Washington sewerage system for treatment at the District of Columbia plant at Blue Plains. It was learned that the Maryland sanitary commission will abandon the use of its 7.5-mgd primary treatment plant when the connecting sewers are built. This plant, costing about $800,000, was first put in operation in November, 1940.

In the opinion of Miles L. Colean[24] the excessive street and sanitary improvements in the outlying areas of many large cities during the economic boom of the 1920's are unwarranted public improvements due to political pressure.

Even in these days of high construction costs, low prices can be obtained by good planning. The previously mentioned case of the Ohio contractor is one example. In another, a low unit cost of $2.09[25] per sq ft was achieved for a one-story frame factory building of 27,000 sq ft erected in Los Angeles County, California, as a result of attention given to economy in preliminary studies of five schemes considered by the consulting engineers and architects.

As to the dependability of E.N.-R. construction cost index for measuring labor and material price trends, it is noted Mr. Howson reported elsewhere,[26] in 1946, that it is practically the universal experience of those attempting to let work by contract that the prices bid exceed greatly those estimated by adjusting prior construction costs according to the construction cost indexes, including E.N.-R.

Furthermore, E.N.-R. also considers that[27] uncertainties about material and labor supply, spotty conditions on labor efficiencies, and low productivity on some jobs provide the "invisible" costs that push many bids up higher than would be indicated by the cost indexes that measure material prices and labor wage trends.

In his paper Mr. Howson states that the steep rise of the E.N.-R. construction cost index from 1932 to 1940 is a reflection of many factors, including general deterioration in labor output and efficiency. In this view Mr. Howson seems to be at variance with the E.N.-R. which states[28] that the weighted indexes (including E.N.-R. construction cost index) measure only hourly wage rate and material price trends, whereas contractor indexes such as the Turner index reflect low labor productivity and attendant factors.

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[23] "District Will Treat Maryland Sewage," *Engineering News-Record*, June 27, 1946, p. 54.

[24] "Stabilizing the Construction Industry," by Miles L. Colean, National Planning Assn., February, 1945.

[25] "Cost Study of One-Story Factory," *Engineering News-Record*, April 18, 1946, pp. 102–103.

[26] "Construction Costs and Water Rates," by Louis R. Howson, *Journal*, A.W.W.A., June, 1946 p. 747.

[27] "Highlights of Construction Costs—1946," *Engineering News-Record*, April 18, 1946, p. 107.

An interesting comparison can be made of Mr. Howson's statement with that of Thurman Arnold on the cause of high construction costs in the period from 1932 to 1940. The latter states:

> "Building costs soared in 1937 and were largely responsible for reversing the upward trend of business throughout the country, not only in construction, but in all industries. A major factor in high building costs has been a series of concerted efforts to fix prices and to perpetuate antiquated methods."

Louis R. Howson,[29] M. ASCE.—The author is in agreement with the first premise of Mr. Mendelsohn's discussion, namely, that construction costs are rising. He cannot agree with the second statement that they are in a "stage of unpredictable flux." All engineering construction costs are necessarily a matter of prediction. Nothing can be built in the past and therefore the engineer is directly concerned with future construction costs. The past serves only as a basis for his estimate of the costs and conditions under which work must be executed in the future.

That it is practicable to predict with reasonable accuracy the effect of wars upon construction costs is well indicated by the forecast made by the late John W. Alvord, Hon. M. ASCE, reproduced as Fig. 3. In this forecast, made in 1920, Mr. Alvord predicted not only the lowest level which prices would reach following World War I, but also predicted that the low point would be reached 12 years after the date of his forecast, or in 1932.

Mr. Mendelsohn indicates that the author's analysis of factors of construction cost is inadequate in that the relationship of wage rates to high construction costs is believed unduly stressed. Reply to this suggestion is offered by reference to Fig. 5, which shows the relation between wage rates and construction cost from 1913 through 1945. The fact that Fig. 5 discloses the generally close relationship cannot be disputed either during a period of rising or a period of declining costs. For illustration, in the 33-yr period from 1913 to January, 1946, the *Engineering News-Record* (E.N.-R.) construction cost index increased from 100 to 316, while the average hourly wage rate of skilled and common construction labor increased from 100 to 353 in the same period.

In the shorter upswing during World War II—that is, from 1939 to April, 1946—the E.N.-R. construction cost index increased 42%, whereas the average of common and skilled labor increased 34½%. During 1946, as reported in *Engineering News-Record* for February 20, 1947, the E.N.-R.construction cost index rose 12.3% while skilled construction labor wages rose 10.8% and common labor wages rose 15.7%.

Similarly, at the beginning of the depression from 1930 to 1932, the E.N.-R. construction cost index decreased 23.1% while during the same period the average reduction in common and skilled labor rates was 25.2%. On the long upswing from 1932 to April, 1946, the E.N.-R. construction cost index rose 114.5% as compared to the average increase in common and skilled labor hourly wage rates of 103.5%.

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[28] "Construction Trends," *Engineering News-Record*, August 8, 1946, p. 63.

[29] Cons. Engr. (Alvord, Burdick and Howson), Chicago, Ill.

In this connection it was stated in the paper under the heading, "4. Effect of Wars on Price Levels:"

> "It should be noted that Fig. 5 [relation between wage rates and construction cost] does not reflect the result of post World War II strikes, wage increases, and cost increases. These might well raise graph B [E.N.-R. construction cost index] to 350 or 360 as compared to its 1945 figure of 310 * * *."

It is significant that as of January 1, 1947, one year after the paper was presented to the Society, the E.N.-R. construction cost index was 381. This short term prediction of the effect of strikes, wage increases, and cost increases also indicates that construction costs are not wholly in a stage of "unpredictable flux."

Mr. Mendelsohn quotes from Beardsley Ruml, Thurman Arnold, Harold L. Ickes, and others not experienced in engineering construction to develop what is apparently his premise that there are vested interests among contractors and materials' suppliers operating in restraint of trade and that the result is higher costs in the building industry than would otherwise be necessary. The author would call attention to the fact that not the least of the "vested interests" is organized labor under the control of some labor leaders.

Since Mr. Mendelsohn has based much of his discussion upon statements of those not in the construction industry, the author presumes to reply in kind from an editorial from the *Chicago Daily News* entitled "No Building." The following is quoted:

> "In the face of these demands, Northwestern University [Evanston, Ill.] has been forced to abandon a $3,000,000 building program—including dormitories, classrooms and hospital facilites—because building costs have increased 75% during the last year. Loyola University [Chicago, Ill.] is considering a two year moratorium on building. The University of Illinois [Urbana] is in process of curtailing a huge building program to the irreducible minimum.
>
> "There is another example of how high material prices and high wages in the building trades, and monopolistic control by union labor works against the public interest. The 75% increase in building costs is due in part to the increase in building wages; in part to the increased cost of materials. The increased cost of materials, in turn, is largely due to the increased cost of the labor that goes into their production.
>
> "Because the building trades unions have a tight monopoly over all building operations, new materials and new construction methods that could save time and money are banned. Because organized labor has a monopoly, it is able to keep the number of building tradesmen far below the demand, thus maintaining an artificial scarcity of labor.
>
> "Because of those conditions, construction necessary to the public welfare cannot go ahead. Monopolies in restraint of trade are forbidden by law; union labor operates in restraint of essential building. But union labor is exempt from regulation as a monopoly."

Mr. Mendelsohn calls attention to the effect of building codes upon construction costs. Certainly there is much room for improvement through the modernization of building codes.

Mr. Mendelsohn discusses the E.N.-R. construction cost index at considerable length. This index obviously does not attempt to reflect the prices resulting from the iniquitous black market practices in the sale of construction materials which are referred to by Mr. Mendelsohn as existing during the period of the late Office of Price Administration.

From Mr. Mendelsohn's statement with respect to the dependability of the E.N.-R. construction cost index in which he refers to a paper by the author reported elsewhere[26] it might be inferred that the author does not consider the E.N.-R. index reliable. That is far from the fact. To clarify the author's position the entire paragraph referred to by Mr. Mendelsohn is reproduced as follows:

> **"Construction Costs vs. Construction Cost Indexes**
>
> "In normal times, the *Engineering News-Record* Construction Cost Index is a valuable and reliable aid in translating costs of one period into estimates for another. However, 1946 is not a normal year. The construction industry is beset by fear—fear of strikes, uncertainty about labor rates and efficiency, inability to secure materials when needed and to contract for materials at a fixed price before bidding, and fear of government- and union-imposed restrictions affecting performance and costs. These all operate to restrict bidding and increase the allowance for contingencies and the total cost. These hazards are not reflected in construction cost indexes. Accordingly, in 1946 it is practically the universal experience of those attempting to let work by contract that the prices bid much exceed those estimated by adjusting prior construction costs according to the construction cost indexes."

Comparison of labor performance in building construction and in mass production industries cannot be fairly made. Buildings are, and to a major extent always will be, individual units built at scattered locations and each is essentially "custom built." There is no production line in general building construction. Production line methods do not apply to building construction and, particularly with respect to dwellings, standardization appears not to be desired by those who need living accommodations.

The author agrees with Mr. Mendelsohn that good planning results in lower costs but good planning is essential at all times and cannot balance increased labor wage rates and material costs. In studying construction cost trends it must be presumed that every engineering project has had the benefit of economic study and development.

The author appreciates the analysis of construction cost factors and their development in Mr. Mendelsohn's discussion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2311

DESIGN LIVE LOADS IN BUILDINGS

By John W. Dunham,[1] M. ASCE

With Discussion by Messrs. D. Lee Narver, N. N. Freeman, and John W. Dunham.

Synopsis

Based upon surveys of the actual live loads in office buildings and warehouses, and upon a consideration of probable actual live loads in apartment buildings and places of assembly, this paper develops and proposes a system for the reduction of basic live loads in the design of building members.

1. Introduction

General.—It is common practice to reduce basic live loads for use in the design of a particular member depending in some fashion on the size of the area supported. For columns this reduction has commonly been a function of the number of floors carried. For beams, when permitted, the reduction has been a function of the tributary area. Thus the general principle is well recognized.

Quantitative information that defines the relation between the probable average live load and the size of the loaded area in the various occupancies is difficult to find. Some pertinent information for office buildings has been published in the form of live-load surveys.[2] Such data are valuable but cover only limited parts of buildings. They do suggest, however, that larger reductions of live load than those ordinarily used may be permissible.

In order to obtain factual information bearing upon this relationship, the U. S. Public Buildings Administration surveyed the live loads in the southeast quarter of the Internal Revenue Building and the main portion of the Veterans Administration Building, both in Washington, D. C., and in eight private warehouses. The surveys were made under the general direction of the writer.

This paper is based upon a report, "Live Loads in Office Buildings and Warehouses," prepared by the writer in the structural engineering division of the U. S. Public Buildings Administration. In addition, it treats the question of live-load reduction in apartment buildings and assembly occupancies and proposes a general system of live-load reduction.

Note.—Published in April, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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[2] *Engineering News-Record*, March 29, 1923, p. 584.

Definitions.—For the purposes of this paper the following terms will be assumed to have the meanings indicated:

Basic Live Load.—The live load per square foot of floor area for which the floor is designed or may be posted.

Unreduced Live Load.—The product of the loaded area under consideration and the basic live load.

Actual Live Load.—The total of all the live load found in a given area.

Average Actual Live Load.—The total live load found in a given area divided by that area.

Design Live Load.—The product of the area under consideration and the basic live load for that area reduced as permitted by a design specification. This is the load that a member of a structure is designed to carry.

2. Design Live Loads in Office Buildings

Method of Conducting Surveys.—The average live load for each panel of floor including that in the corridors was obtained and recorded on a set of plans. The live load included furniture and its contents, movable property generally, and the people found in each space at the time it was surveyed. Weights were determined as follows: (*a*) Representative pieces of furniture, and their normal contents, were weighed with spring scales—the weights thus determined being used for all corresponding items found in the survey of the building; (*b*) persons found in a given panel at the time it was surveyed were counted and assumed to weigh 150 lb each; (*c*) special furniture and equipment not corresponding to the typical items were weighed with spring scales in the space in which they were found. In some cases the pieces were too large to weigh in their entirety and in these instances parts of the equipment, such as separate drawers, were weighed and the weight of the entire unit computed; and (*d*) weights of miscellaneous items such as rugs, pictures, and papers on top of filing cabinets were estimated.

Internal Revenue Building.—A survey was made of the live loads in the southeast section of the Internal Revenue Building (identified herein as I R Building), Washington, D. C. This part of the building is approximately 226 ft by 210 ft in plan and seven stories high, plus an unfinished attic story which is omitted from this analysis. There is a central court about 135 ft by 110 ft in plan extending through all stories. The average usable area of each floor in this portion of the building is approximately 26,000 sq ft. There are seventy columns in this area. The basic live load used in the design of the building is 100 lb per sq ft. No reduction was used for the floor slabs or beams. The design live load on the columns was obtained by considering the seventh floor to be the top and using the following percentages of the basic live load: 100% on the seventh floor; 90% on the sixth floor; 80% on the fifth floor; 70% on the fourth floor; 60% on the third floor; and 50% on each of the remaining floors.

Veterans Administration Building.—A survey was made of the live loads in the main part of this building (V A Building), which is L-shaped, 60 ft wide, and approximately 462 ft in developed length. There is also an attached wing 44 ft by 51 ft. It is eleven stories high above grade and there are 144 columns extending through all stories. The area of each floor in this part of the building including the wing is about 29,000 sq ft.

This building was not designed by the federal government but was purchased for governmental use. There is some doubt regarding the design live loads; however, for purposes of comparison, a basic live load of 70 lb per sq ft has been assumed. An assumed design live load on the columns was obtained by using the following percentages of the basic live load: 100% on the eleventh floor; 90% on the tenth floor; 80% on the ninth floor; 70% on the eighth floor; 60% on the seventh floor; and 50% on the remaining floors.

Statistical Distribution of Live Load.—The total areas occupied by average, actual live loads of various ranges are given in Table 1, both for individual floors and for each building as a whole. For instance, on the first floor of the I R Building 5,229 sq ft are occupied by average, actual live loads varying from 20 lb per sq ft to 40 lb per sq ft. For the building as a whole, 32,709 sq ft are occupied by actual live loads in this range. Of the total floor area of the building 88% supported an average, actual live load of 40 lb per sq ft, or less. Of the total floor area of the V A Building 99.5% supported an actual, average live load of 60 lb per sq ft or less, and 97.75% supported an actual, average live load of 40 lb per sq ft or less.

TABLE 1.—DISTRIBUTION OF ACTUAL UNIT LIVE LOADS, BY AREAS

Floor No.	LIVE LOAD,[a] IN POUNDS PER SQUARE FOOT:												
	0 to 20		20 to 40		40 to 60		60 to 80		80 to 100		100 to 110		Total
	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft	%	Sq ft
(a) INTERNAL REVENUE BUILDING													
1	19,814	74	5,229	20	890	3	790	3	....	....	....	....	26,723
2	23,999	90	1,988	7	....	....	....	....	....	....	825	3	26,812
3	18,449	70	5,040	19	1,030	4	1,100	4	770	3	....	....	26,389
4	21,367	81	4,965	19	....	....	....	....	....	....	....	....	26,332
5	25,480	97	845	3	....	....	....	....	....	....	....	....	26,325
6	7,850	30	9,966	39	6,830	26	1,290	5	....	....	....	....	25,936
7	10,290	45	4,676	21	6,412	28	1,263	6	....	....	....	....	22,641
Total	127,249	(70)	32,709	(18)	15,162	(8.5)	4,443	(2.5)	770	(0.5)	825	(0.5)	181,158
(b) VETERANS ADMINISTRATION BUILDING													
1	27,992	97.5	625	2.5	....	....	....	....	....	....			28,617
2	27,996	94.5	896	3	625	2.5	....	....	....	....			29,517
3	22,123	75	3,380	11	4,014	14	....	....	....	....			29,517
4	29,517	100	....	....	....	....	....	....	....	....			29,517
5	27,621	93.5	1,896	6.5	....	....	....	....	....	....			29,517
6	28,997	98	260	1	260	1	....	....	....	....			29,517
7	29,005	98	512	2	....	....	....	....	....	....			29,517
8	28,587	96.5	....	....	930	3.5	....	....	....	....			29,517
9	29,517	100	....	....	....	....	....	....	....	....			29,517
10	27,365	92.5	976	3.5	....	....	....	....	1,176	4			29,517
11	23,188	98	509	2	....	....	....	....	....	....			23,697
Total	301,908	(95)	9,054	(2.75)	5,829	(1.75)	....	....	1,176	(0.5)			317,967

[a] The percentages opposite each floor refer to the indicated total area of that floor. Those in parentheses, opposite to "Total" line, refer to the total area of the building.

Variation of Live Load on Floors.—The maximum, average, actual, unit live load found in the I R Building occurred on 825 sq ft of the second floor (in the V A Building, 1,176 sq ft of the tenth floor) and amounted to 106 lb per sq ft (90 lb per sq ft in the V A Building). To show how the average, actual live load varied with respect to area in the most heavily loaded regions, the continuous areas of 800, 1,000, 1,200, and 1,400 sq ft carrying the greatest average, actual live load were determined for each floor. The results have been plotted for each building in Fig. 1.

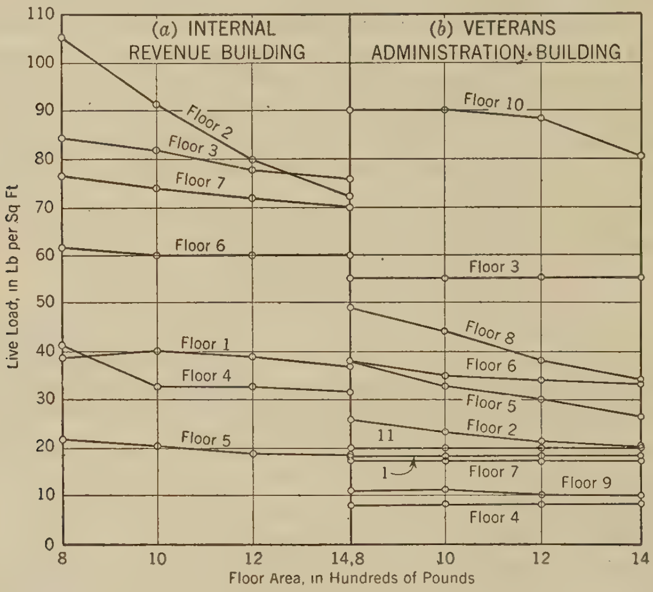
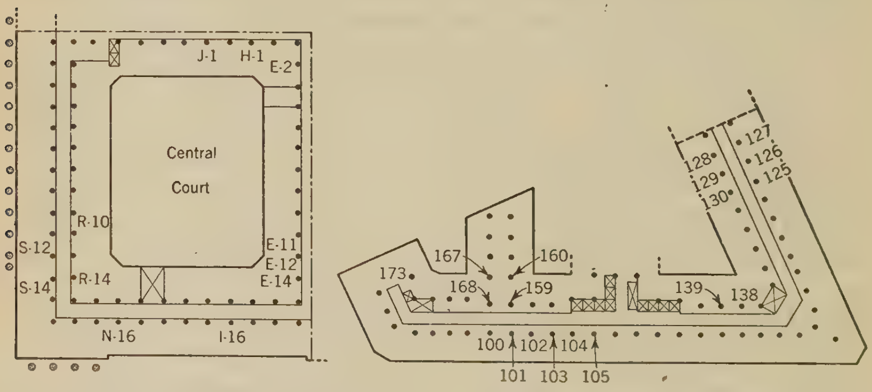


FIG. 1.—AVERAGE ACTUAL UNIT LIVE LOAD ON THE MOST HEAVILY LOADED FLOOR AREAS


J-1
H-1
E-2
Central
Court
R-10
S-12
S-14
R-14
E-11
E-12
E-14
N-16
I-16
127
126
125
128
129
130
167
160
173
168
159
139
138
100
102
104
101
103
105

(a) INTERNAL REVENUE BUILDING, SOUTHEAST QUARTER

(b) VETERANS ADMINISTRATION BUILDING, MAIN PORTION

FIG. 2.—LOCATION OF BUILDING COLUMNS

Variation of Live Load on Columns.—To show how the actual column live loads in the I R Building varied in relation to accumulated tributary area, the twelve columns (nineteen columns in the V A Building) which, from inspection, seemed to be the most heavily loaded were selected (see Fig. 2). These columns constitute more than one sixth of the columns in the part of the I R Building involved. The actual live load supported by each of these columns at each floor and its accumulation in each story were computed. The results for the critical columns in both buildings are given in Cols. 8 and 9 of Table 2. The

TABLE 2.—VARIATION OF TOTAL, ACTUAL, LIVE LOAD ON COLUMNS
(See Fig. 2 for Location of Columns)

Floor No.	TRIBUTARY AREA (SQ FT)		UNREDUCED LIVE LOAD, IN KIPS		ACTUAL LIVE LOAD						DESIGN LOAD	
	One floor	Cumulative	One floor	Cumulative	Percentage of:		One floor	Cumulative	Percentage of:		One floor	Cumulative
					Col. 4	Col. 5			Col. 12	Col. 13		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(*a*) INTERNAL REVENUE BUILDING, COLUMN E-2												
7	292	292	29.2	29.2	56	56	16.4	16.4	56	56	29.2	29.2
6	292	584	29.2	58.4	30	42	8.1	24.5	31	45	26.3	55.5
5	292	876	29.2	87.6	14	32	4.0	28.5	17	36	23.3	78.8
4	292	1,168	29.2	116.8	29	32	8.5	37.0	42	37	20.4	99.2
3	292	1,460	29.2	146.0	75	40	21.9	58.9	125	50	17.5	116.7
2	292	1,752	29.2	175.2	7	35	2.1	61.0	14	46	14.6	131.3
1	292	2,044	29.2	204.4	23	23	6.6	67.6	45	46	14.6	145.9
(*b*) INTERNAL REVENUE BUILDING, COLUMN H-1												
7	236	236	23.6	23.6	59	59	13.8	13.8	58	59	23.6	23.6
6	236	472	23.6	47.2	23	41	5.7	19.5	27	43	21.2	44.8
5	236	708	23.6	70.8	11	31	2.6	22.1	14	35	19.0	63.8
4	236	944	23.6	94.4	36	32	8.5	30.6	51	38	16.5	80.3
3	236	1,180	23.6	118.0	63	39	14.9	45.5	105	48	14.2	94.5
2	236	1,416	23.6	141.6	5	33	1.2	46.7	10	44	11.8	106.3
1	236	1,652	23.6	165.2	30	33	7.2	53.9	61	45	11.8	118.1
(*c*) INTERNAL REVENUE BUILDING, COLUMN S-14												
7	214	214	21.4	21.4	47	47	10.0	10.0	47	47	21.4	21.4
6	274	488	27.4	48.8	66	58	18.1	28.1	73	61	24.7	46.1
5	274	762	27.4	76.2	13	42	3.7	31.8	17	47	22.0	68.1
4	274	1,036	27.4	103.6	19	36	5.3	37.1	28	42	19.2	87.3
3	274	1,310	27.4	131.0	6	30	1.6	38.7	10	37	16.5	103.8
2	274	1,584	27.4	158.4	8	26	2.2	40.9	16	35	13.7	117.5
1	274	1,858	27.4	185.8	19	25	5.3	46.2	39	35	13.7	131.2
(*d*) VETERANS ADMINISTRATION BUILDING, COLUMN 138												
11	274	274	19.18	19.18	17	17	3.20	3.20	17	17	19.18	19.18
10	274	548	19.18	38.36	129	73	24.70	27.90	143	77	17.30	36.48
9	274	822	19.18	57.54	9	52	1.71	29.61	11	57	15.40	51.88
8	274	1,096	19.18	76.72	19	43	3.56	33.17	26	51	13.50	65.38
7	274	1,370	19.18	95.90	21	39	4.07	37.24	35	48	11.50	76.88
6	274	1,644	19.18	115.70	20	35	3.92	41.16	41	48	9.60	86.48
5	274	1,918	19.18	134.88	34	35	6.46	47.62	67	49	9.60	96.08
4	274	2,192	19.18	154.06	14	33	2.62	50.24	26	48	9.60	105.68
3	274	2,466	19.18	173.24	14	30	2.73	52.97	28	46	9.60	115.28
2	274	2,740	19.18	192.42	22	30	4.31	57.28	45	46	9.60	124.88
1	274	3,014	19.18	211.60	2	27	0.41	57.69	4	43	9.60	134.48

three columns of the I R Building that supported the highest, average, actual live loads (each for part of its length) are reported in Tables 2(*a*), 2(*b*), and 2(*c*). Table 2(*d*) includes the corresponding data for that column of the V A Building of the nineteen investigated, which supported the greatest, average, actual live load. Using a basic live load of 100 lb per sq ft (70 ft per sq ft in the V A Building), the unreduced live loads for each of these columns were computed and are recorded in Cols. 4 and 5. The actual live loads expressed as percentages of the corresponding unreduced live loads are given in Cols. 6 and 7.

In order to show, graphically, the relation between the average, actual live load supported by each story of the column and the accumulated area supported by that story the corresponding values for several representative columns of both buildings were plotted in Fig. 3. It will be noted in the I R Building that

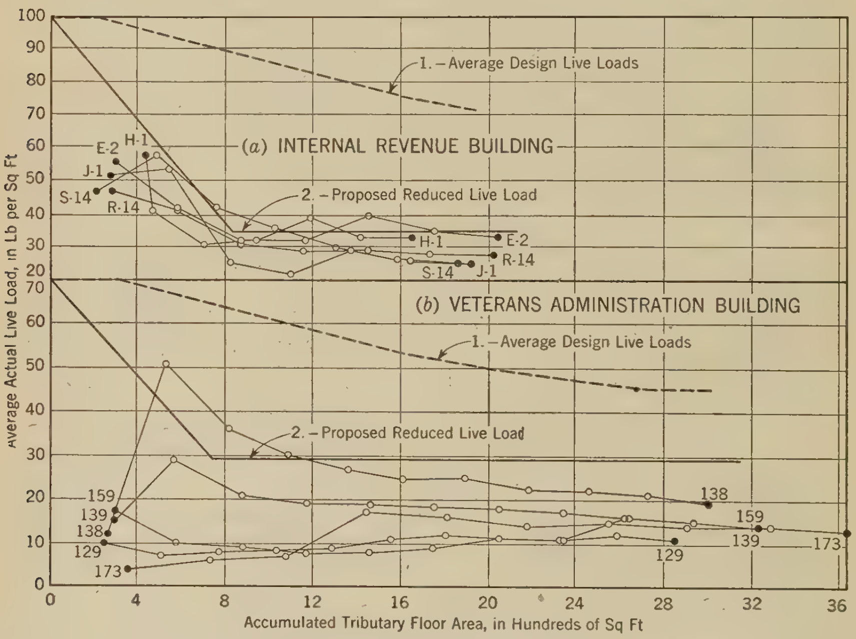


Fig. 3.—Variation of Live Load on the Columns of Buildings
(For Identity of Columns See Fig. 2)

no one column is critical but that different columns are critical for various areas. It appears that the intensity of live load diminishes faster on columns as the area increases than it does on floors.

Significance of Survey Data.—A consideration of the loadings found in these surveys and their comparison with those previously reported[2] indicates that the I R Building is representative of a rather heavy occupancy and that the V A Building is representative of an average occupancy, although the maximum load found in the latter is higher than appears usual.

The surveys show clearly that, as the tributary area of a member increases, the chances of its being fully loaded decrease. The decrease in actual live load was more marked in the case of columns than in that of floors. It appears to be a fact that the probability of several contiguous floor panels being fully loaded is much greater than that several successive stories of a column will be fully loaded.

One of the common ways of reducing the live loads on columns heretofore has been to use the following percentages of the basic live load: 100% on the top floor; 90% on the first floor below the top; 80% on the second floor below the top; 70% on the third floor below the top; 60% on the fourth floor below the top; and 50% on the other floors.

In order to compare the results obtained by this method with the loads that actually occurred in the buildings surveyed, the heavy dotted lines (curves 1, Fig. 3) were plotted in Figs. 3(*a*) and 3(*b*) to show the average design loads with respect to area, using the foregoing method of reduction, as applied to columns carrying 275 sq ft of floor. For this purpose basic live loads of 100 lb per sq ft and 70 lb per sq ft for the I R Building and the V A Building, respectively, have been used. It is clear from a comparison of the heavy dotted lines and the loads actually found that the correspondence is not good and that the shape of the line is not that of the actual loading.

It is noteworthy that the maximum live loads found in each case occurred in only one part of the building and that this part was less than 1% of the floor area of the building.

There is an obvious advantage, as regards flexibility of use, to design the building so that these maximum loads can be permitted anywhere. There is just as obviously a lack of economy in providing strength throughout the structure that will not be used in 99% of the building.

Therefore, it seems logical, if possible, to design an office building in accordance with a calculated risk of occasional overstress in the members, limited so as to insure against the possibility of serious overstress. If this can be done the building can be posted for its basic live load with the assurance that, even if any member is fully loaded, disaster is impossible.

Basic Live Loads and Design Live Loads.—Such a system of basic and design live loads is possible. It should conform to the following essentials: (*a*) A basic live load equal to the maximum to be allowed on any panel; (*b*) a rapid reduction to a minimum load based upon the relation of live and dead loads in such a manner that overstress on any member, due to full basic load throughout the structure, cannot exceed a predetermined value; and (*c*) the system should be such that only a very small portion of the structure will be overstressed by the probable actual live loads.

It is proposed that: (1) The basic live load shall be the maximum average live load that it is intended to allow on any panel; (2) the basic live load may be reduced for the design of any member at the rate of $\Delta w_L = 0.08\%$ per sq ft of tributary area; and (3) the maximum reduction shall not exceed Δw_L as determined by the following formula:

$$\Delta w_L = \frac{w_D + w_L}{4.33\, w_L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which w_D is the dead load per square foot of tributary area; and w_L is the basic live load. To express the reduction as a percentage, Eq. 1 is multiplied by 100. The maximum reduction Δw_L is such that, if a member has a tributary area large enough to allow the maximum reduction, and if the member is loaded with the full basic live load, the overstress will not exceed 30%. In other words,

$$w_L + w_D = 1.30\,[w_D + (1 - \Delta w_L)\,w_L] = 1.3\,w_D + 1.3\,w_L - 1.3\,w_L\,\Delta w_L \quad (2)$$

Simplifying and solving for Δw_L in Eq. 2 yields Eq. 1 as a proportion; multiplying by 100 expresses the reduction as a percentage.

The proposed system, then, conforms to two of the aforementioned essentials. The question as to whether it conforms to the third, "that only a very small portion of the structure will be overstressed by the probable actual live loads," can be best determined by comparison with the loads found in the surveys. The proposed rule is illustrated graphically by the heavy, solid line curve 2, in Fig. 3(*a*).

As an example, the rule can be applied to the design of the I R Building. The typical floor beam in this building has a tributary area of 335 sq ft. The live load reduction will be (335 × 0.08% = 26.80%), say, 27%. The design live load then is 73 lb per sq ft (plus 180 lb per sq ft dead load). Reference to Table 1(*a*) shows that not more than 3.5% of all the beams in the building would be overstressed when designed for this load. The most severe load reported was 106 lb per sq ft on an area of 825 sq ft. The most severely loaded beam, therefore, would be supporting (106 + 180 =) 286 lb per sq ft of tributary area. Since it has been assumed to be designed for (73 + 180 =) 253 lb per sq ft, the overstress will be about 13%.

So far as the beams in the I R Building are concerned, from 1% to 3½% of the beams would be overstressed in some degree. Only ½% would have the maximum overstress of 13%.

An inspection of Fig. 3(*a*) indicates that the columns would be overstressed in four locations. Since there are seventy columns and since there are seven stories of each column under consideration, this represents less than 1% of the column shafts in the building, considering each story length separately. The greatest overstress will occur in Col. E-2, Fig. 2(*a*). Considering the actual live loads and dead loads on the floors, this story of the column will be required to carry (40 + 180 =) 220 lb per sq ft although designed for (35 + 180 =) 215 lb per sq ft. The overstress obviously cannot be greater than 2.4%.

The proposed system is also illustrated graphically by the heavy, solid line, curve 2, in Fig. 3(*b*). The typical floor beam in the V A Building has a tributary area of 300 sq ft. The live-load reduction will be (300 × 0.08% =) 24%. The design live load then is (0.76 × 70 =) 53.2 lb per sq ft. The maximum live load found in the survey was 90 lb per sq ft (dead load 110 lb per sq ft). The most severely loaded beam would be supporting (90 + 110 =) 200 lb per sq ft. Since it has been assumed to be designed for (53.2 + 110 =) 163.2 lb per sq ft, the overstress would be about 22½%.

The live load of 90 lb per sq ft occurred on 1,176 sq ft. Since this is less than ½% of the area of the building as a whole it is apparent that less than ½% of the floor beams would be overstressed this amount. Table 1(*b*) indicates that the remainder of the beams would have no overstress.

Curve 2, Fig. 3(*b*), indicates that the maximum overstress on any story of any column would amount to about 6% in the ninth and tenth stories of column 138, Fig. 2(*b*). Since eleven stories of 144 columns are under consideration this means that only two of the story lengths out of 1,584 would be overstressed. It appears that the proposed system conforms to the third essential required of it for office occupancies.

The proposed system of basic live loads and design live loads will permit office buildings to be posted and policed for their basic live loads without: (*a*) The incorporation of so much unused strength as heretofore; (*b*) significant overstress in members from probable loadings; and (*c*) danger of structural failure in the remote contingency of full basic live loading on the entire tributary area of any member.

Proposed System Versus Lower Basic Live Loads.—It seems apparent that the questions of proper basic live loads and of proper reductions are inseparable. Another approach to the problem of obtaining as much economy as is justified is to lower the basic live loads and to allow smaller reductions than those proposed herein. For instance the report of the Building Code Committee of the U. S. Department of Commerce[3] recommended a minimum live load on floors, for office purposes, of 50 lb per sq ft. It also recommended that a percentage reduction in total column live loads be permitted as follows:

Floors supported	%
One	0
Two	10
Three	20
Four	30
Five	40
Six	45
Seven or more	50

It is pertinent, therefore, to compare these two general recommendations to determine their relative merits. The loads found in the survey of the V A Building, which averaged 11.6 lb per sq ft for the entire structure, are of the same order as those found in the surveys referred to in Section 1 under the heading "general." It will be enlightening therefore to compare the writer's proposal as it applies to that building, with the system of loads recommended by the Building Code Committee.

In Fig. 4(*a*), the heavy line represents the design loads according to the proposed system, assuming a basic live load of 70 lb per sq ft and a dead load of 110 lb per sq ft. The dotted lines represent the design loads according to the recommendations of the Building Code Committee, assuming a basic live load of 50 lb per sq ft and that the columns each carry an area of 275 sq ft per floor.

Here are two buildings, one designed for a 70-lb live load under the proposed system, and the other designed for a 50-lb live load under recommendation 2.

The slabs of the first will be designed for a higher live load. The beams of both will be designed for practically the same live loads. The upper story and the fourth, third, second, and first story columns in the first building will be designed for a pound or two more of live load per square foot than those in the second building. The ninth, eighth, seventh, and sixth story columns will be designed for a somewhat lower live load in the first building than in the second. The slabs in the first building will cost approximately 1½¢ more per sq ft than those of the second. The beams and columns will cost about the same in both buildings.

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[3] "Minimum Live Loads Allowable for Use in Design of Buildings, Report of Building Code Committee," U. S. Dept. of Commerce, November 1, 1924.

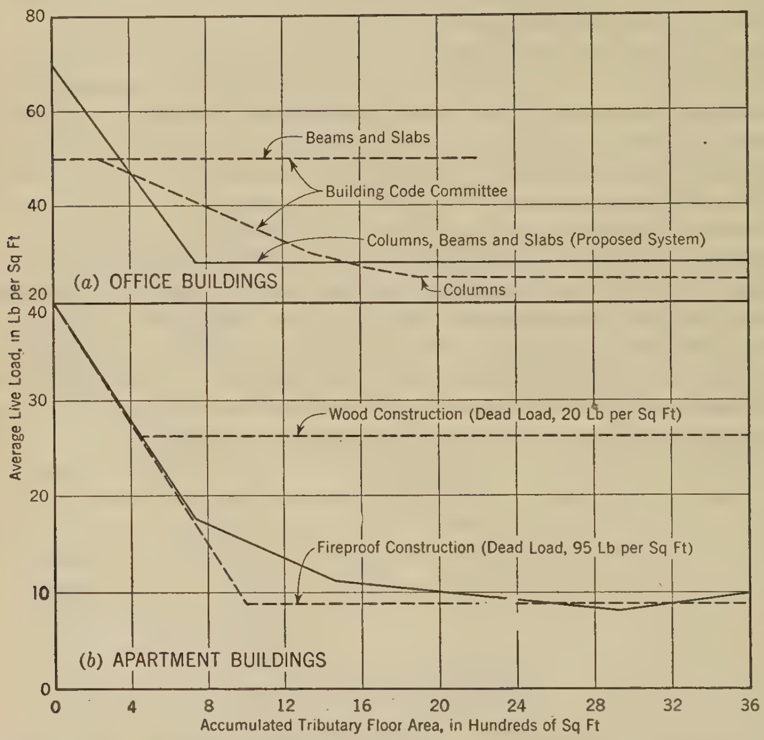

Fig. 4.—Comparison of Live-Load Reductions

However, although the difference in cost is slight, the difference in usability is significant. The first building may be posted for 70 lb per sq ft of live load. The second should be posted for 50 lb per sq ft of live load if both are to fulfil the requirement that the members should not be substantially overstressed by the probable maximum live loads. The first building will be much more flexible in its use without sacrifice of safety.

It appears from the foregoing comparison that a system having a high basic live load but permitting quick reductions relative to tributary area is preferable to one using lower live loads and permitting only nominal reductions.

3. Design Live Loads in Apartment Buildings

Probable Live Loads.—The live load in apartments is made up of movable property such as furniture and furnishings, and persons. The live load created by furniture and furnishings varies from 0 to 10 lb per sq ft, with an average of about 4 lb per sq ft.[3,4] The live load attributable to persons will vary from 0 to 30 or 40 lb per sq ft.[3] These loads will be very small except in the case of special gatherings such as parties, club meetings, and funerals. A living room containing one person for every 6 sq ft is crowded, and, although denser crowds are possible, they are very unlikely because of considerations of comfort and ventilation. A crowd with one person to every 6 sq ft will average about 20 to 25 lb per sq ft. Such a crowd is very unlikely to fill more than the living room, or living room and dining room together, at any one time. It is also probable that such a crowd will occur simultaneously in comparatively few of the apartments in a building.

It will be instructive to consider an average apartment building and to discuss how the average live load may be expected to vary in relation to the area of the building under consideration.

The Bureau of Standards report cited previously[4] contains a survey made of thirteen representative apartments ranging in area from 431 sq ft to 796 sq ft. The average area was 628 sq ft. The areas of the dining room and living room in these apartments averaged about 40% of the areas of the entire apartment in each case.

To consider an example, assume that: (1) Each apartment in an apartment building has an area of 650 sq ft; (2) for each apartment there are 75 sq ft of corridor; (3) the area in each apartment subject to crowding at any one time is 260 sq ft; (4) one apartment in five will contain a crowd at the same time; (5) the crowded spaces will be loaded with 40 lb per sq ft; and (6) the remaining spaces will be loaded with 5 lb per sq ft.

TABLE 3.—Examples of Probable Live Load

Area	w_L	One Apartment		Two Apartments		Five Apartments	
		Area (sq ft)	Load (lb)	Area (sq ft)	Load (lb)	Area (sq ft)	Load (lb)
1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Area subject to crowding..	40	260	10,400	260	10,400	520	20,800
Remaining areas.........	5	650 + 75 − 260	2,325	1,300 + 150 − 260	5,950	3,250 + 375 − 520	15,525
Total...............	..	725	12,725	1,450	16,350	3,625	36,325

In a single apartment and its proportionate amount of corridor, the total load will be 12,725 lb (see Table 3, Col. 4). Since the total area under consideration is 725 sq ft, the average live load is $\frac{12{,}725}{725} = 17.6$ lb per sq ft.

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[4] "Fire Resistance Classifications of Building Construction, BMS92," Bureau of Standards Rept, 1942.

In two apartments and their 150 sq ft of corridor, the total load will be 16,350 lb (Table 3, Col. 6). The average live load will be 11.3 lb per sq ft. Similarly for four apartments and the adjoining corridor, the total load will be 23,600 lb; the area, 2,900 sq ft; and the average live load, 8.1 lb per sq ft. In five apartments there will be 520 sq ft heavily loaded so that the total load will be (Table 3, Cols. 7 and 8) 36,325 lb. The area is 3,625 sq ft and the average live load is 10.0 lb per sq ft.

Basic Live Loads and Design Live Loads.—The foregoing average live loads are plotted against the corresponding areas in Fig. 4(*b*). The line connecting the points rises slightly (as would be expected at the area representing five apartments) and if continued would fall again until an area representing ten apartments were reached.

In order to test the validity of the proposal for reducing live loads in office buildings to this type of loading, the dotted lines were drawn to show the design live loads obtained by that proposal for wood construction and fireproof construction. The relationship of the average live load to the design live load obtained by applying the proposed reduction formula to a basic live load of 40 lb per sq ft is good.

There would be no overstress in the case of the wood construction, whereas in the case of the fireproof construction the maximum probable overstress indicated is 6%. Since the maximum possible overstress with full basic live load is limited to 30% by the method it is believed that the proposed method of live-load reduction is applicable also to this type of occupancy.

4. Design Live Loads in Assembly Occupancies

Spaces in buildings to be used for assembly purposes are definitely intended to be occupied fully at times. Moreover, there is a distinct probability of impact because occasionally the people present will move simultaneously; for example, they will rise at the playing of the national anthem. It is logical, therefore, that the basic live loads for such spaces should not be reduced for the design of supporting members.

5. Design Live Loads in Warehouses

General.—Most codes do not permit the use of reduced live loads for either the columns or the beams in warehouses. As far as floor members are concerned this practice seems sound. The tributary areas are not likely to be large and it is well-known that several adjacent floor panels are frequently fully loaded.

Accordingly, the investigation of variation in live-load intensities in warehouses was limited to column loads. The investigation was conducted to obtain data bearing on the relation of average, actual live loads on columns to the accumulated tributary areas carried by the columns. Surveys were made by the Public Buildings Administration, under the writer's direction, of eight representative, general-purpose warehouses in New York, N. Y., Philadelphia, Pa., Baltimore, Md., and Washington, D. C. Two buildings were surveyed in each city. The buildings varied in height between six and eleven stories.

Method of Conducting Surveys.—An engineer visited each building and selected from among the columns the one that, for its full height, had the largest percentage of its entire tributary area most heavily loaded. The rated or official live load, the actual live load on each floor, and the tributary area were obtained.

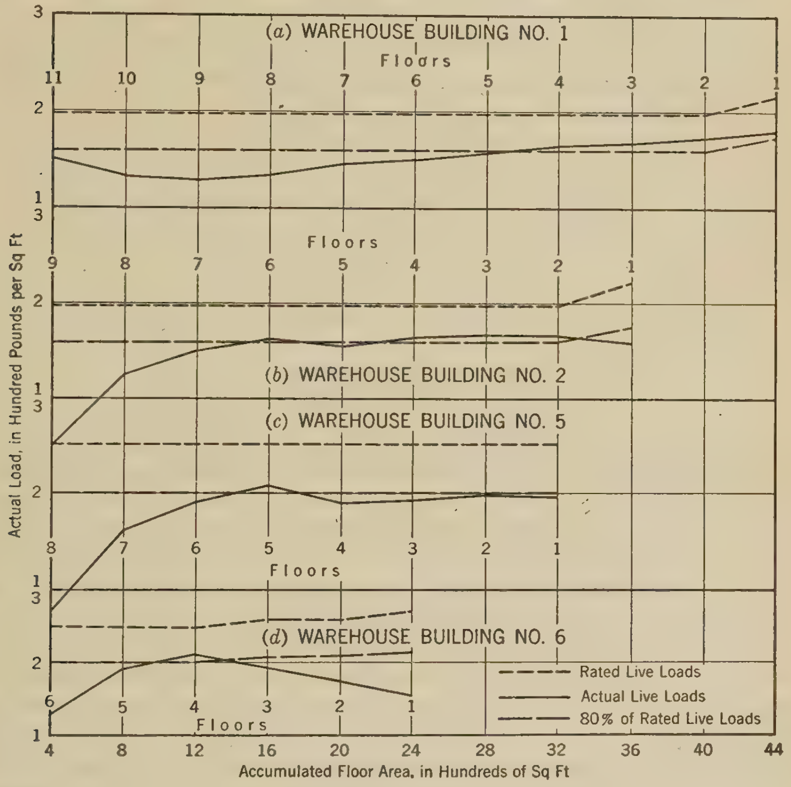

FIG. 5.—LIVE LOADS ON WAREHOUSE COLUMNS

This information for the four cases in which the actual live load was highest relative to the rated live load has been plotted in Fig. 5. The abscissas are cumulative tributary areas by floors. The average, actual live loads corresponding to each accumulation of area are indicated in each case by the full lines. The average, rated live loads corresponding to each accumulation of area are indicated by the dotted lines. For instance, the sixth story of the column in building No. 2, Fig. 5(*b*), was carrying a tributary area of 1,200 sq ft. The average, actual live load upon it was 150 lb per sq ft and the average, rated live load for that area is 200 lb per sq ft.

The most heavily loaded stories measured from the top down are summarized in Fig. 6. The heavy line in this figure indicates for each story the heaviest average, actual live load expressed as a percentage of the average, rated live load. For instance, the maximum ratio of average, actual live load to average, rated live load on the story below the top floor in any of the buildings was 0.75 and occurred on the column surveyed in warehouse building No. 1 (Fig. 5(*a*)).

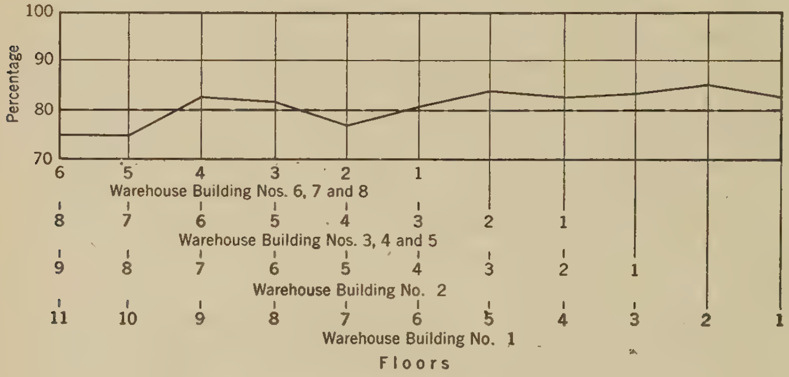

FIG. 6.—SUMMARY OF THE MOST HEAVILY LOADED STORIES

Significance of Survey Data.—The data show that, contrary to the trend found in office buildings, there is a tendency on the most heavily loaded column of storage buildings for the average, actual live load to increase as the accumulation of tributary areas increases. This arises from a general practice of placing comparatively light loads on the top floors. The data also indicate that rarely is any story loaded with an average, actual live load of more than 80% of the average, rated live load. A dashed line has been added to each part of Fig. 5 indicating 80% of the average, rated live load.

Basic Live Loads and Design Live Loads.—As in the case of office buildings it appears that the basic live load for storage buildings should be the maximum average floor load that it is desired to permit in any bay. Furthermore, the basic live load should be used unreduced for the floor and beam design. In the light of the data presented herein the basic live load for storage buildings should be reduced for storage buildings as for office buildings with a maximum reduction of 20% for the design of columns. In the columns surveyed, such a reduction would result in a maximum excess of average, actual live load over design live load of 6¼%. The percentage overstress resulting from this condition would be less than 6¼% depending upon the weight of the construction.

6. CONCLUSIONS

The foregoing facts and reasoning seem to the writer to justify the conclusion that it is sound to derive design live loads from basic live loads as follows:

A. The basic live load shall be the maximum average live load which it is intended shall be allowed on any panel of a floor of a building.

B. When the basic live load is 100 lb per sq ft or less, for any occupancy except assembly occupancy, the basic live load may be reduced for the design of any member at the rate of 0.08% per sq ft of tributary area; but the maximum reduction shall not exceed Δw_L as determined by Eq. 1.

C. In assembly occupancies the basic live load shall not be reduced for the design of any member.

D. When the basic live load is more than 100 lb per sq ft, it shall not be reduced for the design of floors, beams, girders, or trusses. It may be reduced at the rate of 0.08% per sq ft of tributary area for the design of columns but the maximum reduction shall not exceed 20%.

7. ACKNOWLEDGMENT

The field work in connection with the surveys of the Internal Revenue Building and the Veterans Administration Building was performed by P. H. Heimer and J. W. McLure, and by J. H. Feehan of the Office of the Supervising Engineer, U. S. Public Buildings Administration. The field work in connection with the surveys of the warehouses was performed by H. R. Williams of the Office of the Supervising Architect, U. S. Public Buildings Administration. Mr. Williams also prepared the figures and tables and reviewed the text of the original report. For permission to publish the survey data the writer is indebted to W. E. Reynolds, M. ASCE, Commissioner of Public Buildings.

For their cooperation and helpful suggestions in connection with the surveys of warehouses, thanks are also due to Warren T. Justice, president, The Pennsylvania Warehouse and Safe Deposit Company, Philadelphia, Pa., and to the following members of the Merchandise Division, American Warehousemen's Association: Charles E. Nichols, M. ASCE, manager, Washington, D. C.; F. T. Leahy, executive vice-president, Port of New York, New York, N. Y.; L. E. Naumann, president, and F. L. Malsey, vice-president, Baker and Williams, New York; Thomas E. Witters, president and general manager, Baltimore Fidelity Warehouse Company, Baltimore, Md.; W. E. Edgar, superintendent, Terminal Storage Company, Washington, D. C.; and J. P. Johnson, vice-president, Terminal Refrigeration and Warehouse Corporation, Washington, D. C.

The original manuscript from which this paper was prepared, containing unpublished tabular data, has been placed on file for reference at the Engineering Societies Library.

DISCUSSION

D. LEE NARVER,[5] M. ASCE.—It should be of interest to know that a live-load reduction formula for the design of buildings, developed to accomplish the results described by the author, was proposed to the Committee on Live Loads of the American Standards Association in October, 1939. The proposal went through the "crucible" of criticism of that committee and, after changes, was adopted in the form presented by the author. The proposal was first published in pamphlet form—"Minimum Design Loads in Buildings and Other Structures"—on May 22, 1945, by the American Standards Association (ASA—58.1—1945) and sponsored by the National Bureau of Standards. On October 19, 1945, the material was issued by the U. S. Department of Commerce in pamphlet form.[6]

The object of that committee's work, together with that of committees on other phases of building construction, was to formulate principles that can be used by cities throughout the United States in the development of uniformity of building codes.

In the sentence preceding Eq. 2, the author states that the formula limiting the "design live load" is so developed that, even if the "basic live load" is attained over large areas, design stresses will not be exceeded by more than 30% of their value. It is the writer's thought that this does not tell the full story. Consider the load values given for the Internal Revenue Building (I R Building): Since the maximum $\Delta W_L = \frac{100 + 180}{4.33 \times 100} = 0.645$, the minimum "design live load" is 35.5 lb per sq ft, and the minimum total design load $= 180 + 35.5 = 215.5$ lb per sq ft. This loading will develop the full design stress. The "basic live load" plus dead load equals $100 + 180 = 280$ lb per sq ft, which will produce 130% of the design stress.

If steel is the structural material being used, either as rolled shapes or in reinforced concrete, specifications A-7 of the American Society for Testing Materials establish a minimum yield point of 33,000 lb per sq in., or 165% of the allowable design stress of 20,000 lb per sq in. The net effect is $165\% - 130\% = 35\%$ of the design load of 215.5 lb per sq ft, or 75 lb per sq ft, as the load that must be added to the "basic live load" to develop the minimum assumed yield point. This means an overload capacity of the structure of $\frac{75}{100}$ or 75% of the "basic live load."

The I R Building is an example of office building loading on the heavy side of average; but consider an office building on the light side, whose dead load is 130 lb per sq ft and whose "basic live load" is 50 lb per sq ft.

By the proposed formula (Eq. 1): Maximum $\Delta W_L = 0.83$, or a minimum "design live load" equals 8.5 lb per sq ft. The design load equals $130 + 8.5 = 138.5$ lb per sq ft.

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[5] (Holmes & Narver), Los Angeles, Calif.

[6] "American Standards Building Code Requirements for Minimum Design Loads in Buildings and Other Structures," *Miscellaneous Publication No. M-179*, U. S. Bureau of Standards, U. S. Dept. of Commerce, 1945.

Under the same stress assumptions as before, there is 35% of the design load, or 48.5 lb per sq ft that must be added to the "basic live load" to develop the minimum assumed yield point. In this case, the overload capacity of the structure amounts to $\frac{48.5}{50} = 97\%$ of the "basic live load."

Both of these examples show a substantial overload capacity. The yield point of steel used in the foregoing calculations is a required minimum. For several years job tests have been showing the yield point of structural steel to be greater than 40,000 in tension; and in bending it is still higher. Assuming in practice that the yield point will not be less than 40,000 lb per sq in., Eq. 1 will result in an overload capacity of the structure of 150% over the "basic live load" for the I R Building and one of 194% for the example of a building having a dead load of 130 lb per sq ft and a "basic live load" of 50 lb per sq ft.

It is to be noticed that a limitation has been placed on the proposed live-load reduction formulas. It is recommended that assembly occupancies have no live-load reduction. This is a sign of warning that there may be other classes, or particular cases, of loading for which the formulas should not be followed. It is difficult to set up terms of a building code that will always be applicable, and the engineer using the proposed system of reduction should consider carefully whether or not his particular problem falls within the usable limits of the formulas. For example, a specialized storage building, in which full "basic live load" capacity will be attained during known seasonal periods, should have no reduction of live loads.

It appears that the proposed system of arriving at loads for design purposes is closer to giving a true result for the normal run of buildings than the systems now in common use. However, as in the case of all design, the closer the design approaches true conditions, the more important good engineering judgment becomes. It is a step in the right direction toward economic design, and economy is as much a consideration for the engineer as is a safe design.

The proposed system may involve more engineering work than the present methods, but this is a business problem and should not be an obstacle in the way of adoption if the system improves engineering practice and results in economies of construction.

N. N. Freeman,[7] Esq.—In bringing this important subject to the renewed interest of the engineering profession, this admirable, clearly written paper marks a forward step.

The surveys conducted under the author's supervision seem to have been limited in scope, however, and were not quite correct in procedure. The number of observations was obviously small; and the recurrent use of the expression "* * * at the time it was surveyed * * *"—indicates that most of the data are the result of only one observation made at a certain time and date. It is agreed that the live loads representing furniture and movable property may be fairly constant during a prolonged observation period. Persons, on the other hand, represent a very active live load, and figures may change considerably. There may be rush hours, queues, or conferences, and it would have been advisable if more extensive observations could have been made to determine the maximum actual live loads imposed by people in certain areas of the entire building. Such maximum figures, together with the more constant live loads, would then represent the actual live loads for such a structure.

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[7] Civ. Engr., Haifa, Palestine.

For the aforementioned reasons, data shown in Tables 1 and 2 represent only actual live loads at a certain time and date, and do not express critical live loads for the buildings in question.

Even if the loads given represent maximum actual live loads, data found in the survey of only two structures cannot be applied as general rules to all office buildings. Only extensive and prolonged investigations into many different types of such structures can supply a basis for an estimate of reasonably correct general design loads which represent actual conditions. Such investigations doubtless require considerable time, but zeal and money invested would be repaid many times over by savings in the design of new structures.

The author's proposal for determining amended design loads rests upon a basic assumption which has not been supported by any experimental evidence. He proposes to allow an occasional overstress in certain parts of the structure. Then he assumes that if this overstress is limited to 30% it is not dangerous and may be ignored in the design.

Whether such an assumption can be accepted without the support of extensive and complete survey data is certainly open to discussion. Such an overstress may reach more than a purely theoretical 30% and could cause a serious reduction in the factor of safety of the entire building. There are certain well-known influences and stresses, however, which only rarely can be included in designs, such as secondary stresses due to stiffness of joints in welded steel frames and in reinforced concrete constructions, uneven settlement of foundations, sudden gusts of wind, and earthquakes. In a proper design such stresses are allowed for by an ample factor of safety, the rate of which is fixed according to local conditions and experience. Even small overstresses at single points may be dangerous, because weakness is introduced into vital construction members and failure may occur in the case of any emergency.

Another obscure point is the assumption of a reduction rate of 0.08% per sq ft of tributary area. No information is given in the paper as to how this value was determined; and there is no clue indicating any special argument in the survey data.

It should be the purpose of further surveys to collect ample actual live-load data in many types of buildings over prolonged periods. Only then will it be possible to conduct a proper statistical investigation that would form the basis of recommendations for a new design system approaching actual conditions.

Before such data are available it seems reasonable to make certain reductions in the basic live loads, however, and the value of 50 lb per sq ft, with a corresponding reduction depending on the number of floors supported, should give safe design values. Especially important, also, is the recommendation by the Building Code Committee of the National Bureau of Standards, U. S. Department of Commerce[8] that a load of 2,000 lb be placed on any 2.5 ft of floor space wherever this load on an otherwise unloaded floor would produce greater stresses than the 50 lb per sq ft of distributed load.

With reference to apartment buildings the author reveals no new information and his computations are based upon assumed values, which may, or may not, approach actual conditions. In the design of apartment buildings, also, it would be advisable to collect proper and ample experimental data before any attempt is made to introduce a new method of analysis.

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[8] "Steel Construction," A.I.S.C., New York, N. Y., 1930, p. 52.

The chapter on warehouses includes some valuable observations; but a limited survey of only eight buildings cannot form a sufficiently broad basis for any general conclusions to be applicable to all warehouses. Additional information showing types of stores involved, movements of ingoing and outgoing consignments, and loading and transport facilities, is needed for a proper classification of data. Certain reductions in design loads seem to be feasible but further surveys are needed before any such assumptions can be made.

Conclusions.—On the basis of the foregoing, three conclusions are possible in a fair appraisal of this paper:

1. Survey data given are incomplete and insufficient to form a reliable basis for any new proposals to reduce design live loads.
2. Overstresses during design can only be permitted if it is desired to reduce the factor of safety accordingly. It should be proved that a limit of 30% is not dangerous, and the author should explain further how the reduction rate of 0.08% per sq ft of tributary floor area was determined.
3. Further extensive surveys should be made to collect sufficient reliable information as a basis for selecting new reduced live loads which approach actual conditions.

JOHN W. DUNHAM,[9] M. ASCE.—The reasoning of the paper is carried a step forward by Mr. Narver, who compares the resulting overstresses in two cases with the yield point of structural steel. He also adds a note of warning that the proposed reductions must be applied with good judgment. This latter is true of design rules generally.

Mr. Freeman states that the surveys presented in the paper seem to have been limited in scope and that the procedure was not quite correct inasmuch as only one observation was made on any one panel. He questions the source of the maximum overstress of 30% and the reduction rate of 0.08% per sq ft. Mr. Freeman also indicates that the use of a load of 50 lb per sq ft with a corresponding reduction, depending on the number of floors supported, should give safe values. It is assumed that this comment applies to office buildings, and that he has in mind the loads and the reductions recommended by the Building Code Committee.[3,8] These points will be discussed in order.

Any number of surveys will be limited in scope. The writer is not aware of a criterion of sufficiency in this respect, and feels that it is a matter of judgment. More surveys would be helpful but it is doubtful that they would change the picture appreciably.

If the offices in the Equitable Building (surveys of which have been reported[2]) may be assumed to average 500 sq ft in area, the total area surveyed in the five office buildings was 210,290 sq ft. The surveys reported in the paper included over 2,500 panels of office building floors, which had a total area of nearly 430,000 sq ft. There were about 140 columns which appeared to be loaded in such a manner as to be significant.

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[9] Asst. Chf. Structural Engr., Public Bldgs. Administration, Washington, D. C.

The arrangement of the load on each of the elements at the time it was surveyed indicates the arrangement that might be found on any of the others at another time. The writer feels that the arrangement of loads in space gave a valid indication of what the distribution on any one element would be in time.

Effort was directed toward determining the actual distributions and combinations in a large number of instances so as to obtain an indication of the probable maximum for a given class of element. It was known in advance that more severe loadings were possible. It was desired to find out what does happen, not what might happen.

The probability of overloading an office building with people is remote. The areas in which the very heavy loads occur represent a small percentage of the total areas surveyed and they are not spaces in which people congregate.

Although the suggested reductions well represent the worst conditions found in the surveys and estimated for the apartments, it was recognized that more severe conditions are possible, however remote their probability. It was this consideration that led to the formula which sets a limit on the proposed reductions. In the writer's opinion the possibility of dangerous overstress is thereby practically eliminated.

The proposed reduction of 0.08% per sq ft is empirical and resulted from the writer's study of the survey data. It gives values that start at no reduction for zero area and correspond well with the average live loads found on the columns carrying the heaviest live load at about 800 sq ft of tributary area (see Fig. 3).

The maximum reduction corresponding to 30% overstress was set to keep the overstress conservatively within the elastic limit of steel and well below the ultimate strengths of concrete and wood. It should be noted that nowhere in the buildings surveyed would the overstress have equaled 30%.

Adoption of the reduction rate and the limiting formula by the American Standards Association seems to indicate that qualified structural engineers concur with the writer's judgment on these two points.

In view of the surveys reported in this paper, and elsewhere,[2] it is contended that the use of 50 lb per sq ft as a live load in office buildings amounts to using an initially reduced live load. The report on the Equitable Building indicated a live load of 87 lb per sq ft in one office. The survey of the Veterans Administration Building, which had about the same average load as the buildings reported on previously,[2] indicated a maximum live load of 90 lb per sq ft. The writer believes that the live load of 50 lb per sq ft was established with the knowledge that a very small part of the building might receive higher loads, as indicated in the surveys, but with the feeling that they would not be enough higher to cause serious overstress.

The comparison indicated in Fig. 4(*a*) and the accompanying text supports the writer's opinion that a higher basic live load, together with the proposed system of reduction, will give a better balanced design for a given building than would result from the use of the recommendations of the Building Code Committee.[3]

In conclusion, the writer wishes to express his gratitude to the discussers of the paper for the additional argument and the new viewpoints they have presented.

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SOME THOUGHTS ON ENGINEERING EDUCATION

By Donald M. Baker,[1] M. ASCE

With Discussion by Messrs. E. S. Boalich, Russell C. Brinker, I. Oesterblom, Samuel T. Carpenter, Lynn Perry, L. E. Grinter, Robert O, Thomas, Clement C. Williams, N. W. Dougherty, H. A. Wagner M. E. McIver, Scott B. Lilly, William A. Conwell, A. Amirikian, C. A. Dykstra, Louis Balog, Henry B. Lynch, Alfred R. Golzé, and Donald M. Baker.

Synopsis

The views of a practicing engineer on engineering education as provided by engineering schools and colleges in the prewar period are presented in this paper; and some suggestions are offered for changes and modifications in academic courses for the postwar era. Supported by appropriate data, the paper discusses the organization of the engineering profession, and the general conditions encountered in the practice of engineering. It argues in support of the conclusions that postwar engineering education should emphasize fundamental principles in subjects throughout the broad field of engineering; and that postwar engineering education should also include courses to prepare a graduate for a career involving administrative, executive, and managerial activity, both technical and nontechnical in character. This paper holds that specialization, or the acquisition of knowledge of the advanced theories and their application to design, operation, and processes, should be primarily left to the years after graduation and that engineering schools should provide such positive and formal guidance and assistance to their alumni in this postgraduate period as will encourage and even urge them to pursue systematic studies toward such specialization.

The paper also urges that engineering educational institutions and engineering societies conduct research concerning the engineering profession, the practice of engineering, and the market for engineering services, in the postwar period—in other words, that engineers as well as engineering be given adequate attention.

Note.—Published in April, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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Introduction

Engineering education, as offered by schools and colleges for the several decades prior to World War II, has kept pace with changes and advances that have occurred in technology, through modification of existing course content and by the addition of new courses to the curriculum. It has not, however, made comparable progress in keeping abreast of changed conditions that have developed during that period in the pattern and structure of the engineering profession, in the practice of engineering, or in the market for engineering services.

Prewar engineering courses (at least in the opinion of graduates, if not from the viewpoint of subject content) tended more and more toward specialization in one of the major branches of engineering, or even in one of the subspecialities of engineering. Emphasis was laid on depth rather than on breadth of instruction, and on imparting skills and knowledge of practice, when experience shows a strong probability that the graduate through changing employment conditions would encounter little opportunity to utilize such skills, or would not have the responsibility for directing such practice, for a long period following his graduation. Little provision was made for furnishing directed guidance, organized assistance, or specific encouragement to those students who, following graduation, desired to continue their studies along specialized lines or in other fields of engineering. The product of formal prewar engineering educational courses presents a picture of a trained rather than of an educated person.

The impact of World War II on the educational system of the United States, and particularly on engineering education, has been such that in many schools prewar engineering courses practically ceased to exist. In many places, short courses in specialized subjects, and accelerated courses of longer duration designed to meet the needs of the armed forces, were substituted. With the end of the war, prewar courses are being reestablished.

Engineering faculties were disrupted during war years, most of their younger members being in uniform, whereas many older ones left, temporarily or permanently, for service to government or industry. Likewise, the source of supply of new instructors, normally drawn from the ranks of recent graduates, disappeared. As a result of the specialized training given many in the armed forces and in industry, however, much has been learned about the technique of instruction, and this knowledge will prove valuable in postwar courses.

It will require considerable time for engineering schools to readjust themselves to normal peacetime functioning. Many students who left prewar courses for the armed services, others who have attended short or accelerated courses, and many young men in service who may have enrolled in courses proposed to be given them during the interval between the end of hostilities and demobilization are seeking to return and secure degrees. These men are more mature and purposeful, and likewise more impatient to secure their degrees, than were the usual prewar upper classmen, and the problem of handling them will be somewhat difficult; but this situation will not last for more than a few years.

On the other hand, there are the usual crop of high school graduates seeking to enrol in normal postwar engineering courses. Once the general pattern and structure of postwar engineering education has become set, extreme difficulty will be encountered in making changes. If these latter are indicated, it is time to discuss them, and they should be discussed not alone by engineering educators, but by practicing engineers and employers of engineers—laymen and professionals at all levels of responsibility—who constitute the consumers or users of the product in the market for engineering services.

This paper presents the views and opinions, not of an engineering educator, but of a practicing engineer who has been employed by engineers, and also has employed engineers, and who has had a rather unique opportunity to observe the engineering profession and the practice of engineering in an objective manner. The conclusions advanced are not susceptible of definite proof from available data, rather they are primarily matters of opinion based upon observation and contact with large numbers of engineers over a long period of time.

The major purpose of the paper is not so much to argue in support of such conclusions as it is to evoke discussion of the subject of engineering education by the users of its product.

It is not believed that prewar engineering education was a failure. Accomplishments of engineers during the past half century have proved its worth. Rather, it is believed that it has done a creditable job, but that a time has arrived when it should adjust itself to current and future conditions, and should endeavor to educate students not solely to become engineers, but readily to enter engineering or other fields where their education will be of equal value. Four years is too short a time for a student to acquire a knowledge of more than the basic fundamentals of engineering and their simple application. If he so desires, he should be afforded assistance and guidance in his further search for knowledge. Every engineering student must work hard to graduate. This probably was the most valuable single thing that he learned in his course.

Prewar Engineering Education

The general objective of most engineering courses offered during the fifteen to twenty years prior to World War II appeared too often to produce a graduate well versed in the theory of, and with an academic knowledge of, practice in one particular branch, and in many instances in one specialty in the field of engineering, thus equipping the graduate to enter such field or specialty on graduation and encouraging him to do so, irrespective of what the market for services in such field might be in five or ten years.

The standard length of course was four years. Attempts to lengthen it (as had been done in the case of preparatory courses in some of the other professions) have not met with any particular success.

Because of the magnitude and complexity of the field of engineering, and the limited time available for formal professional education, those responsible for the general design and planning of engineering courses have been faced with the following alternatives:

I. To develop a broad general course, in which the student was given a thorough foundation in the basic fundamentals underlying engineering; or

II. To develop a course in which less time was given to the basic fundamentals of alternative I and more time was devoted to advanced applications to, and practices in, a single branch or specialty, with limited instruction in nontechnical subjects.

A course such as alternative I would include mathematics and the physical sciences, with their application to engineering subjects such as simple structures, mechanics of materials, thermodynamics, electrical circuits, electrical machines, and hydraulics. In addition to instruction in these general, basic practices in the field of general engineering, the broad course would include instruction in nontechnical subjects that would be useful to the engineer in his subsequent career, whether that career led to engineering, commerce, or industry. The acquisition of particular knowledge in the various specialities would be left for the period following graduation.

In the prewar period too many engineering courses were the result of a choice of alternative II. During the first two years the student was instructed in mathematics through calculus, and was given one year each of chemistry and physics. He also had a certain amount of work in the shop, field, and drafting room, with possibly a course or so in one or more elemental specialized subjects and a few courses in nontechnical subjects. Beginning in his junior year, at the latest, courses dealt with the application of the subjects studied in lower division courses to the more specialized field that he had been required to select, with further instruction in theory and practice in such specialized field. He analyzed stresses and made designs of large bridges and dams, studied the economics of power plants or systems of high-tension transmission lines, or considered problems of layout or management of industrial plants. He grappled with problems of a character that he would never meet, or at least would not have to solve, on his own responsibility for from at least twenty to twenty-five years after graduation.

Degrees were usually granted in the major branch of engineering represented by the course followed as an undergraduate. If the student returned for postgraduate study, his course too often consisted of more intensive specialization or of research.

The result of this general pattern and structure of engineering education was that the graduate had a good theoretical knowledge of the field or specialty that he had followed, an academic knowledge of practice in this field or specialty, and a slight training in some of the skills that he might need in later years. Although he might be called upon to utilize these skills, in such activities as drafting, computing, use of machines, or use of precise instruments shortly after graduation, not until a very considerable period following that date would he have acquired sufficient maturity and experience to allow him to use independent judgment or to assume any substantial responsibility in the exercise of his knowledge of theory and practice.

As a graduate he seldom had the ability to express himself adequately or a knowledge of how to acquire this ability. He had little knowledge of business or of business principles or practices, of the behavior of people, or of human institutions; and he had little or no information as to where or how to acquire this knowledge. He was primarily a trained, rather than an educated person. If he happened, through force of circumstances or through choice, to engage in some other branch or speciality of engineering, or if he desired to pursue further the specialty that he followed in his undergraduate years, he was left largely to his own devices and judgment for guidance in pursuing his further education. Few signposts existed to guide him; and there were no goals toward which he could strive in his search for knowledge as he strove for his bachelor's degree.

The prevalent specialization or study of the more advanced applications of basic fundamentals in engineering courses cannot have been due to any single cause or condition, but rather to a considerable number of them. Courses were planned and designed by engineering educators, who were, in most instances, themselves authorities in some specialty. It is a natural tendency for specialists to predominate on engineering faculties. Names of outstanding specialists give prestige and standing. Younger faculty members can best achieve eminence and likewise advancement through acquiring a reputation as an authority in some specialty.

A further cause of this prevalent specialization lies in the popular demand for courses in specialized subjects when a particular specialty has reached a stage which indicates that it might currently offer a considerable amount of employment. For example, interest in irrigation during the early years of the twentieth century developed widespread demand in western schools for courses in irrigation engineering. Rapid development of the petroleum industry during the decade from 1920 to 1930 caused many schools to offer courses in petroleum engineering or geology. At the present time (1946) the increasing development of the aeroplane is causing many schools to offer courses in aeronautical engineering, even though the best informed persons in the industry forecast a postwar reduction in production to from 10% to 20% of present volume. When one school offers courses in a popular specialty, others follow, so as not to be accused of falling behind in the march of progress. The usual result of this practice is that a surplus of men soon develops who were trained in these specialized fields, and many graduates who entered them following commencement are forced to seek other fields or specialities. The specialized courses continue, however, long after the demand for them has subsided, since there is a strong tendency to build curricula around the particular qualifications of individual faculty members.

In considering the progress of engineering education, as it is possible to encompass it within the limited time that the student spends in school, one might liken it to a basic industrial progress in which raw materials are subjected to certain operations and processes, from which an initial product results. This product undergoes subsequent processing of many types, each depending upon the ultimate product desired.

A steel plant, taking iron ore, coke, and limestone, produces first pig iron and then steel billets. From the latter a wide variety of products are made, differing greatly in character, quality, and composition. Seldom if ever does the steel plant attempt to process the billets beyond rolling them into shapes, or to produce the ultimate product, which may range from automobile bodies to pipe wrenches and watch springs. It produces a basic product which, however, is of such a character, composition, and quality that a more finished product can be made, through further processing, by plants which specialize in further processing for the ultimate markets.

Managers of the steel plants watch the ultimate market closely, forecast demands, and keep abreast of these demands both in volume and character of product. The engineering school that produces graduates who have majored in such specialities as structural engineering, power plant engineering, or design of gas engines may be likened to a steel plant that sought to produce not only steel billets, but also such finished products as automobile bodies or watch springs.

Specialty graduates are sometimes sought by employers to maintain the lower grades of responsibilities on their engineer staffs. In some few instances they are in demand when employers hope to groom a future manager or president; and they are always in demand, of course, when the economic requirements for engineers are greater than the available supply. In such cases, it is but natural for engineering faculties to feel satisfied that the type and character of the courses they offer meet the requirements of the market for engineering services.

Faculties have little contact with the market for engineers with from five to ten years or more of experience. If their members are engaged in outside practice, it is usually as specialists or in a consulting capacity where they have little contact with the rank and file of engineers who constitute the bulk of the profession. Employers seeking to engage older engineers secure them through sources other than alumni rosters and collegiate personnel placement offices.

THE ENGINEERING PROFESSION

The engineering profession consists of those persons, almost entirely men (few women are found in engineering), who practice engineering. In general, it may be considered as a heterogeneous rather than a homogeneous profession, with very nebulous boundaries. The field which the practice of engineering covers is so broad and so diversified, the professional educational background and training of those individuals engaged in it covers such wide extremes, and its members so frequently shift their employment, character of activity, and functions—that very little homogeneity exists. Furthermore, practice of en-

TABLE 1.—NUMBER OF TECHNICAL ENGINEERS LISTED IN

Line	Engineers	(a) TOTAL NUMBER LISTED				(b) NUMBER PER	
		1910	1920	1930	1940	1910	1920
1	Civil (and surveyors)	52,033	64,660	102,086	99,437	56.6	61.2
2	Electrical	15,278	27,077	57,837	54,895	16.6	25.6
3	Mechanical (and others)	14,514	36,689	54,356	106,514	15.8	34.7
4	Mining	6,930	6,695	11,970	9,236	7.5	6.2
5	Totals (averages in parentheses)	88,755	135,121	226,249	270,082	96.5	127.7

gineering involves activity in so many other lines that it is extremely difficult in many instances to determine whether or not a particular activity falls within or without such practice.

General Traits and Characteristics of Engineers.—In spite of this general heterogeneity of the profession and its more or less nebulous boundaries, engineers as a group of individuals do have certain more or less common traits and characteristics which they have developed as a result of their education and the type and character of their professional experience. For the same causes, they are also lacking in certain attributes.

On the credit side it may be stated that engineers in general are resourceful, hardworking, ingenious, and persistent, and have a great devotion to their work. On the debit side they are extremely narrow in their interests and outlook, and highly individualistic. An engineer is the original "man from Missouri" who "has to be shown" everything.

Engineers cannot express themselves well—too few graduate engineers can write a good report or make out a salesmanlike application for a job, even after they have been out of college from ten to twenty years. Many a graduate, capable of bigger things, never has an opportunity to do them because of his inability to "sell" his capabilities to those who might desire to utilize them. Engineers lack a "dollar" sense, seldom appreciate the relation between the cost, value, or profit. Their habit of basing their work, opinions, and conclusions on factual data, although it is an excellent one, tends to cause them to be too strongly influenced by the details or tangible aspects of a problem and to ignore the intangible phases. They fail to see the broad general features. Engineering educators should recognize the existence of these items on the debit side more than they have done and make a stronger attempt to reduce this general debit balance.

Number of Engineers.—Since the very fundamentals of engineering rest on the systematic collection, analysis, and interpretation of factual data, the paucity of available data pertaining to engineers and to the engineering profession is remarkable. Even data concerning the number of persons practicing engineering are of poor quality. For several decades the Federal Census of Occupations has listed the number of "technical engineers" gainfully employed in the United States, and Table 1 shows these figures for the past four decennial censuses. Line 3, Table 1, listing mechanical engineers "and others" includes

THE FEDERAL CENSUSES OF 1910 TO 1940, INCLUSIVE

100,000 POPULATION		(c) PERCENTAGE OF TOTAL IN EACH BRANCH				(d) PERCENTAGE INCREASE OVER 1910				Line
1930	1940	1910	1920	1930	1940	1910	1920	1930	1940	
83.2	75.5	58.6	47.9	45.1	36.8	100.0	124.3	196.3	191.0	1
47.2	41.7	17.2	20.0	25.6	20.3	100.0	177.1	378.5	359.1	2
44.2	81.0	16.4	27.1	24.0	39.5	100.0	252.8	374.4	734.0	3
9.8	7.0	7.8	5.0	5.3	3.4	100.0	96.6	172.7	133.3	4
184.4	205.2	100.0	100.0	100.0	100.0	(100.0)	(152.2)	(255.5)	(304.2)	5

engineers in all categories not otherwise mentioned in the table. The 1940 data include engineers seeking work but not those employed in emergency work. The total number of persons in each category listed (except mining engineers) has increased at a rate greater than the increase of national population, mechanical and other engineers showing the greatest rate of increase. Proportionately, the number of civil engineers and surveyors, listed together until 1940, decreased more than one third and that of mining engineers more than one half. Electrical engineers increased slightly, and mechanical and other engineers increased two and one-half times.

The 270,000 technical engineers listed in the 1940 Census cannot be considered as including all persons embraced in the practice of engineering in the United States. There are large numbers of persons listed under other occupational classifications who, without question, are following the practice of engineering. For example, the 1940 Census lists 9,236 mining engineers. Table 2 shows that in 1940 there were 297,375 persons employed in occupations either closely, or generally, allied with mining engineering, or with other branches of engineering. Without question there were a considerable number of mining engineers, among these 297,375 persons, and also large numbers of engineers in other classifications. Similarly there are many other occupational groups more or less closely associated with technological activities, such as employees of public agencies, railroads, power companies, contractors, and industrial plants, among whom will be found many engineers. It is quite probable that the actual number of persons who could have been properly enumerated as practicing engineers in 1940 was several times the number listed as technical engineers in the census of that year.

TABLE 2.—Number of Persons Listed in Occupations Among Which May Be Found a Certain Proportion of Technical Engineers (Censuses of 1910 to 1940, Inclusive)

Item	Description	1910	1920	1930	1940
1	Chemists, assayers, and metallurgists...	16,273	32,941	47,068	59,333
2	Designers, draftsmen, and inventors....	47,449	70,651	102,730	107,286
3	Mine foremen and inspectors..........	23,338	36,931	34,286	33,457
4	Mine operators, officials, and managers..	34,325	25,234	30,896	31,672
5	College presidents and professors.......	15,668	33,407	61,905	65,627
6	Totals...........................	137,053	199,164	276,885	297,375

This uncertainty as to the number of engineers in the United States is not in any way the responsibility of the Census Bureau. The enumerators of this bureau are not supposed to be skilled in personnel classification; they must accept the occupational classification given them, and in most instances they secure information as to the working members of a family from the housewife, whose knowledge of the occupation of her menfolk is usually ascertained from the title of the position which they hold, such as foreman, inspector, draftsman, designer, surveyor, or engineer.

An amusing incident illustrative of the difficulty of listing properly the occupational status in the census enumeration came to the writer's attention a few years ago. A friend of his, an engineer, was collecting occupational data in connection with a market survey in a large city, and was surprised to find some two hundred civil engineers listed as residing in a certain area of that city largely occupied by colored persons. Closer investigation disclosed the fact that a large number of colored drivers of garbage collection trucks and their assistants, who lived in the area, had given their occupations to the enumerator as sanitary engineers. They were employed by the Bureau of Sanitary Engineering of the City Engineering Department and, since sanitary engineering was a subclassification of civil engineering, they had been enumerated under the latter heading.

Composition of the Engineering Profession.—The diversity of activities, responsibilities, and functions in engineering practice is great. To describe, properly, the exact place that a person occupies in the practice of engineering, his duties must be classified under each of the following:

A. Horizontal Classification.—The position must be segregated as to the major branch of engineering under which it falls, and then under the particular specialty or subspecialty of the major branch.

B. Vertical Classification.—The relative importance, difficulty, and responsibility of the duties performed, which may range from subprofessional activity upward through the most complicated theory and practice, and the training and experience required to perform them must be determined.

C. Functional Classification.—The type and character of activity followed, such as design, construction, research, operation, sales, consultation, or administration, must be established.

Civil service and personnel administrators have made considerable progress in this type of "job specification," but the resulting number of individual classifications has reached vast proportions.

Seldom does an engineer remain in a given classification for any great length of time. The nature of his activity is one of constant change, such change frequently occurring in each of the three groups of classifications.

The 1940 Federal Census, for the first time, presented certain data relative to education, age, and income (from salaries and wages) for certain occupational groups, including technical engineers. Although there is no way of determining whether the data pertaining to this group are representative of those for the entire engineering profession (including engineers who were listed as following other occupations), as given, they do develop some interesting facts pertaining to a group who compose a substantial segment of the engineering profession. The census data are presented for that purpose, rather than as representative of the entire profession. Data for certain other occupational groups are also presented for purposes of comparison.

Education.—Table 3(a) contains data pertaining to educational qualifications of certain categories listed in the 1940 Census report under "Technical

TABLE 3.—PERCENTAGE ANALYSIS OF THE TOTAL NUMBER OF PERSONS IN THE 1940 CENSUS WHO WERE ENGAGED IN CERTAIN TECHNICAL AND OTHER OCCUPATIONS

Item	Description	TECHNICAL ENGINEERS					OTHER PROFESSIONAL AND SUBPROFESSIONAL					
		Civil engineers	Electrical engineers	Mechanical engineers	Other engineers[a]	All technical engineers	Chemists, assayers, and metallurgists	Draftsmen and designers	Architects	Lawyers and judges	Physicians and surgeons	Total professional and subprofessional
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	(a) EDUCATIONAL QUALIFICATIONS											
	College Course Completed:											
1	Four years or more	58.6	60.7	59.6	71.1	61.3	58.9	15.5	56.0	80.2	93.5	55.8
2	Three years	16.6	11.3	12.4	11.7	13.5	13.8	18.7	16.5	10.0	3.1	14.8
	High School Course Completed:											
3	Four years	12.8	12.2	12.7	8.6	12.1	14.0	35.7	15.3	4.8	1.2	13.8
4	Three years	4.7	5.6	5.7	3.9	5.1	6.3	15.5	4.5	1.6	0.4	6.6
5	No years	7.0	9.4	9.2	4.7	7.6	6.6	14.0	7.3	2.8	1.3	3.6
6	Not reported	0.3	0.8	0.4	0.0	0.4	0.4	0.6	0.4	0.6	0.5	5.4
	(b) AGE GROUPS											
7	Median age, in years	41.9	40.3	41.3	37.2	40.8	33.6	34.4	42.0	44.1	43.1	38.7
	Percentage Less Than:											
8	Twenty years	0.0	0.0	0.0	0.1	0.1	0.6	1.9	0.2	0.0	0.0	0.6
9	Twenty-five years	3.4	4.5	5.2	9.4	5.1	13.9	16.2	3.6	1.6	1.0	8.0
10	Thirty-five years	27.6	30.0	30.0	43.8	31.1	56.1	52.1	28.6	31.1	28.1	40.4
11	Forty-five years	60.0	68.0	62.0	71.9	63.8	80.3	77.7	55.4	58.1	52.1	66.4
12	Fifty-five years	84.8	89.5	86.2	90.0	86.9	93.2	92.3	81.5	77.2	71.3	84.4
13	Sixty-five years	97.0	98.5	97.3	98.0	97.6	98.5	98.5	94.2	90.8	88.3	95.0
14	Seventy-five years	99.7	99.9	99.7	99.8	99.8	99.8	99.9	99.3	98.2	97.9	99.3
15	Older than seventy five	0.3	0.1	0.3	0.2	0.2	0.2	0.1	0.7	1.8	2.1	0.7
	(c) SALARY GROUPS[b]											
	Salary Range[b] (Dollars)											
16	Less than 100	6.7	4.5	7.0	8.8	6.6	5.9	5.4	37.6	56.9	65.1	. . .
17	100 to 600	1.9	2.2	2.3	3.4	2.3	5.0	5.8	2.6	1.4	4.1	. . .
18	600 to 1,200	4.9	4.5	4.8	5.8	4.9	10.8	14.6	5.1	2.5	2.6	. . .
19	1,200 to 1,600	7.6	7.8	6.8	7.2	7.3	15.6	17.3	5.1	2.7	1.5	. . .
20	1,600 to 2,000	12.2	11.2	9.8	11.9	11.2	15.3	16.8	5.3	2.1	1.1	. . .
21	2,000 to 2,500	21.3	16.8	17.4	18.1	18.6	15.8	19.3	9.8	3.7	1.9	. . .
22	2,500 to 3,000	13.1	12.1	11.8	10.0	12.1	8.4	9.4	7.9	2.5	1.3	. . .
23	3,000 to 5,000	24.0	28.5	26.1	21.3	25.3	14.9	8.2	14.9	8.2	5.6	. . .
24	More than 5,000	5.8	9.5	10.9	10.7	8.9	5.6	1.1	5.0	9.6	6.0	. . .
25	Not reported	2.5	2.9	3.1	2.8	2.8	2.7	2.1	6.7	10.4	10.8	. . .

[a] Including mining, industrial, and chemical engineers. [b] Percentage of the total number reported for the one year 1939 whose income from wages and salaries was within the salaries ranges indicated. This does not include income from professional fees and partnerships, or from other kinds of work or sources of income.

Engineers," and to certain other occupational classifications. Technical engineers are somewhat better educated than the average of the professional and semiprofessional group, but they fall behind the members of the medical and legal professions. Three out of four technical engineers have completed three years of college, and three out of five have completed four years or more. As a member of the California State Board of Registration for Civil Engineers, the writer reviewed the professional biographies on about 6,000 applications for registration. Many of these applicants had received degrees from colleges everywhere in the United States and many of them were engaged in branches of engineering other than civil. From this experience, the writer is inclined to believe that the ratios in Table 3(*a*) would be somewhat reduced if applied to the entire number of engineers in the United States.

Age.—Table 3(*b*) indicates that technical engineers are somewhat older than the average of the professional and semiprofessional group, but slightly younger than the members of the medical and legal professions. A significant fact revealed in this table is that one third of the total number of technical engineers were within the range of from thirty-five to forty-five years old, whereas only one quarter of the members of the medical and legal professions were within this age bracket. Furthermore, although 86.9% of the technical engineers were younger than fifty-five years, only 77.2% of the lawyers and judges, and 71.3% of the physicians and surgeons were less than this age. This would indicate that a substantial number of technical engineers leave the strict practice of engineering during their forties, either to assume more or less nontechnical administrative duties in engineering or industrial organizations, or to leave the field of engineering completely. A much higher proportion of the members of the medical and legal professions remain in their respective professions throughout their active life.

Income.—Table 3(*c*) gives the relative proportion of those employed as technical engineers and in certain other occupational classifications in various income brackets, such income being derived solely from wages and salaries. The census report states that very few of those listed as receiving incomes of less than $100 per year from these sources reported any income whatsoever from wages and salaries, indicating that they undoubtedly were engaged in private practice and received their income almost entirely from professional fees. This would indicate that about one out of fifteen technical engineers was engaged in private practice as against six out of sixteen architects, five out of eleven lawyers, and two out of three physicians and surgeons. It might reasonably be assumed that a substantial number of those listed as receiving an annual income of less than $1,600 from wages and salaries were engaged primarily in private practice.

Effect of Professional Registration.—According to the latest reports, there are about 70,000 registered professional engineers in the United States, registration statutes being effective in all but three states. This number is only one quarter of the total number of technical engineers listed in the 1940 Census, and probably less than from 10% to 15% of the actual total number of engineers in the United States. Professional registration has been effective and accepted by other professions such as law, medicine, and dentistry for many years, and has served to identify in the mind of the public at large what constitutes the practice of these professions. For this reason basic data pertaining to them are far more comprehensive, conclusive, and definite than are data pertaining to engineers and engineering. The attitude of the profession toward engineering registration (or at least that of its leaders) was one of antagonism at first, but later developed into one of more or less indifference (or at least of only mild interest). This is more or less typical of professional attitudes toward other social and economic problems confronting the engineering profession.

The necessity of admitting to practice (when registration becomes effective) many persons of "borderline" qualifications, the wide variation in the standards maintained, and the wide variation in the diligence of enforcement of registration by the boards of the various states, combined with this professional indifference, has caused the results of registration to fall far short of those anticipated by its early proponents. It is a long-range program, however, and its full effectiveness will not be felt for several decades. In general, engineering educators have supported the movement, but not so actively that one of the strongest ambitions of every engineering graduate (recent or otherwise) is to receive, through it, official public certification of his technical qualifications as a member of his profession.

Organization and Structure of the Engineering Profession.—Definitions of engineering are legion, from the short ones found in dictionaries to the more complicated one encountered in engineering registration statutes. In essence, however, engineering is applied science—the application of scientific knowledge, pertaining to the properties of matter and the sources of power, to the creation of structures, machines, and manufactured products. Half a century ago, these products could be created by a small organization, few members of which were

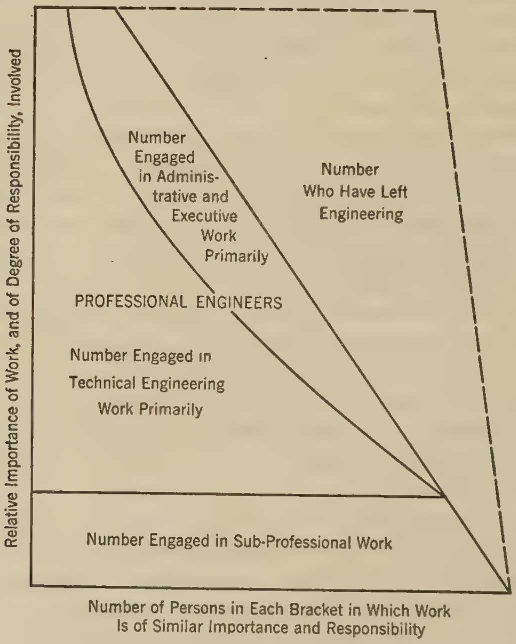
Relative Importance of Work, and of Degree of Responsibility, Involved
Number Engaged in Administrative and Executive Work Primarily
Number Who Have Left Engineering
PROFESSIONAL ENGINEERS
Number Engaged in Technical Engineering Work Primarily
Number Engaged in Sub-Professional Work
Number of Persons in Each Bracket in Which Work Is of Similar Importance and Responsibility

Fig. 1.—Cross Section of the Engineering Profession (Not to Scale)

considered specialists. Today the field of engineering knowledge and likewise the products created have increased to such an extent in size, complexity, and cost that a large group or organization (including subprofessionals, professionals, specialists, and administrators) is required to perform such a function. The current practice of engineering, except by those who serve in advisory or consulting capacities, is a mass-production activity, in which each individual serves as a cog in a machine and performs his specific duties as a unit of a much larger group. It is far more similar to the practice of banking or insurance than it is to that of medicine or law.

A cross section of the engineering profession, as shown in Fig. 1, is typical of those of other organization activities, being triangular in shape. At the bottom are large numbers of persons in positions that involve limited technical responsibility or knowledge, with few in the top brackets, most of the latter being engaged primarily in executive or administrative duties. It is more or less peculiar to the engineering profession that, as the importance and responsibility of the duties of the positions held by its members increase, the proportion of time spent in strictly technical activities decreases. This is not true of such professions as law, medicine, or dentistry.

TABLE 4.—SPECIALIZATION OF CORPORATE MEMBERS, AMERICAN SOCIETY OF CIVIL ENGINEERS (ASCE) AND AMERICAN INSTITUTE OF MINING AND METALLURGICAL ENGINEERS (AIME), EXPRESSED AS PERCENTAGES

Society	Engaged in Engineering			Not engaged in engineering
	Their own specialty (1)	Another specialty (2)	Total (3)	(4)
ASCE...	52	5	57	43
AIME...	36	6	42	58

Furthermore, an increasing number of engineers leave the profession as they advance in it. Table 4, based upon a 5% sample of the corporate membership of the American Society of Civil Engineers and the American Institute of Mining and Metallurgical Engineers, uniformly distributed throughout the 1937 membership roster of these societies, indicates this. When it is realized that all members listed as not following engineering had followed it for from at least six to eight years prior to their admission to the corporate grade in their society and had had at least one year in responsible charge of engineering work (which fact indicates that at the time they entered such grade they must have intended to follow engineering as a career), the results shown in Table 4 are rather significant.

THE PRACTICE OF ENGINEERING

Subject Matter.—Whereas a person engaged in the professions of law, medicine, banking, teaching, art, or drama deals primarily with the subject matter of his profession throughout his practice, a person engaged in the practice of engineering usually finds himself involved more and more with subjects in other fields as he advances in his profession. These subjects may be grouped under three headings, matter, men, and money—the three M's. Under the heading of "matter" are included technical subjects—problems of an engineering character. The heading, "men," would include subjects pertaining to human beings—activities, traits, habits, characteristics, and institutions. The heading of "money" would embrace subjects in the fields of economics, finance, and business.

A young engineer, just graduated, finds himself dealing with more or less routine, or mechanical, matters; but, as he acquires knowledge, experience, and maturity, he is assigned problems involving professional knowledge and judgment. Working as he usually does, as a member of an organization, he continues to advance professionally; but he also begins to assume administrative duties, at the same time depending on his subordinates more and more for technical results and opinions. He is then dealing with both "matter" and "men." Unless he develops into a specialist, he also begins to encounter problems in the field of money if and as he continues to advance. He becomes responsible not only for the technical correctness of his work and that of his subordinates and for the efficiency of their output, but he becomes responsible for the cost and economy of the thing or things produced. If he still continues to advance, he enters the field of "management," the fourth M, which embraces the other three—matter, men, and money. In this field whether he operates in a large or in a small organization, he becomes involved in such problems as the relation between volume of production and absorptive capacity of markets, and as the cost of production in relation to profits, working capital, financing, taxes, interest, and dividends. He must retain his broad knowledge and judgment in technical matters, even though he forgets technical details; but he must also exercise knowledge and judgment in other fields.

Not all engineers progress through these three fields to the fourth one, many remain strictly in fields concerned with the first M—matter; but the highest rewards, except in the case of outstanding specialists, come to those who attain positions in the field of management. In addition to those who follow the foregoing course, and retain their connection and contact with technical activities, many leave the field of engineering entirely, and become engaged in nontechnical activities in which their engineering education and training in the approach to and the solution of problems, the orderly collection and assembly of factual data, its analysis, interpretation, presentation, and use prove to be of extreme value. Engineering educational institutions should recognize the foregoing condition in the subject matter included in their curricula.

Employment Conditions in the Practice of Engineering.—The engineering graduate, on entering his profession, is faced with the strong probability that his specialty, his employer, and his place of employment will undergo a considerable number of changes during his career, if he continues to follow his profession. Table 5, although based on data concerning members of the American Society of Civil Engineers, is probably representative of the situation in other branches of engineering. This table represents changes in employment in the case of one hundred Juniors and one hundred Associate Members of the Society between 1930 and 1940. Intermediate changes during this ten-year period were not noted. The names selected were of members who had entered their respective grade during the five-year period, from 1926 to 1930, inclusive, who were picked at random, but who were uniformly distributed through the membership rosters for the selected decade. Specialities and employers were based upon titles given in the roster.

It is significant that nearly one half of the Juniors and one third of the Associate Members either had left engineering during this period or had changed the specialities they followed in 1930; that three out of five Juniors and one half of the Associate Members had changed employers; and that nearly three out of five Juniors and two out of five Associate Members had likewise changed the location of their employment. Undoubtedly a higher proportion of members than that shown in Table 5 had left the practice of engineering during the decade, as those enumerated were those who retained their membership for the period, and members who had left the Society during the period because they had left the practice of engineering were not included. There appeared to be somewhat greater stability in employment conditions among Associate Members than among Juniors. Whether this lack of stability among Juniors was accelerated because of conditions created by the economic depression during the period cannot be determined; but, from personal observation, the writer is not inclined to believe that this was so to any great degree.

TABLE 5.—CHANGE OF STATUS, AS ANALYZED BY THE RECORDS OF 100 JUNIORS (JUN.) AND 100 ASSOCIATE MEMBERS (ASSOC. M.), ASCE, BETWEEN 1930 AND 1940

Item	Description	Jun.	Assoc. M.
	(*a*) CHANGE IN SPECIALTY		
1	Abandoned engineering	11	5
	Remained in Engineering:		
2	Changed to another kind of engineering	36	29
3	Stayed in their original specialty	53	66
4	Total	100	100
	(*b*) CHANGE IN EMPLOYER		
5	No change in employer	40	49
6	Changed employer	60	51
	(*c*) CHANGE IN LOCATION OF EMPLOYMENT		
7	No change in location	43	58
8	Changed location	57	42
	(*d*) CHANGE IN SOCIETY STATUS		
	Advanced One Grade:		
9	Number advanced	100	36
10	Years required for advancement	7.0	6.8
	Advanced Two Grades:		
11	Number advanced	9	....
12	Years required for advancement	12.0	....

Rate of Advancement.—Another feature of interest revealed by Table 5 is the apparently slow rate of advancement by these members to more important positions. Slightly more than one third of those who were in the grade of Associate Member in 1930 had advanced to that of Member ten years later. The Member grade in the Society requires that an engineer shall have had five years of experience in responsible charge of important work—four years more than that required for the Associate Member grade—and shall be qualified to conceive and design engineering work. It likewise requires at least twelve years in the practice of engineering. It does not involve any financial outlay for transfer in grade, nor any increase in annual dues. It has been the writer's observation that a high proportion of Associate Members of the Society are extremely ambitious to advance to the Member grade, and the small proportion of those who had made such advance would seem to indicate a lack of opportunity for achieving the qualifications necessary for such advancement. Of greatest interest, however, is the relatively slow rate of advancement of Juniors. Of the one hundred Juniors considered—

Forty one required less than seven years,
Twenty nine required between seven and ten years, and
Thirty required more than ten years

—in which to advance to the grade of Associate Member. Theoretically, except for the age limit, a Junior of the Society, if he be a graduate of a recognized school, could attain the grade of Associate Member within a little more than two years after he is graduated. Few Juniors, if any, do so even upon reaching the minimum age (which is attained in from five to seven years after graduation). In the decade studied, advancement to the grade of Associate Member was optional until the Junior reached his thirty-third birthday, at which time he was dropped from membership if he had not advanced in grade.

During the three years from 1939 to 1941, inclusive, for every one hundred Juniors who did advance to the grade of Associate Member, there were forty three dropped for failing to advance on reaching the limiting age; seventy three were dropped for other reasons, and twenty one resigned—a total loss of one hundred and thirty seven. These figures indicate that the younger engineers, at least those who follow civil engineering and who have joined their own professional society, during this period at least, were making rather slow progress in their professions. This condition was recognized by the Society in 1942 when the limiting age for transfer from Junior to Associate Member was increased from thirty-three to thirty-five years.

In general, it is the writer's observation (based on a review of the professional biographies of from 6,000 to 7,000 engineers of all ages and lengths of experience who sought registration as civil engineers in California, or who sought admission to, or transfer in, membership grade in the Society) that the average engineer—at least if he follows civil engineering—does not attain a position in which he is called on to exercise independent engineering judgment or to assume engineering responsibility for anything except work of the simplest character, until at least from eight to ten years after he has graduated; and many do not attain such a position within that time. There are many exceptions to this statement, of course; but for all those who do make better progress there are a comparable number who require a longer period. During this period (which is at least from two to two and one-half times the period spent on the normal engineering course in college), a very substantial number of graduates will have changed specialties, employers, and location of employment, and a considerable number also will have left the practice of engineering to enter other fields. This period, during which the graduate has the opportunity to specialize in engineering, and likewise to acquire knowledge in other fields, should be recognized by engineering educators in designing their courses and curricula; and they should provide opportunity during this period for the guidance and assistance which graduates both desire and need.

Change in Character of Employment.—Another trend that became very significant during the decade from 1930 to 1940, at least in civil engineering, was the substantial increase in employment of civil engineers by public agencies. Table 6, based on a 5% sample uniformly distributed through the membership rosters of the Society for 1920, 1930, and 1940 indicates that, although there was little change in the relative proportion of Society membership employed either by private business or by public agencies, or engaged in private practice (or employed by engineers in private practice) between 1920 and 1930, in the following decade between 1930 and 1940 the number per 1,000 members employed by private concerns and engaged in private practice or employed by private practitioners, dropped almost one third. The number per 1,000 members employed by public agencies almost doubled. No doubt, some of this shift to employment by public agencies resulted from the economic depression of the 1930's but the outlook is for it to increase rather than to decline. The subject is worth further investigation.

TABLE 6.—TREND IN THE EMPLOYMENT OF ASCE MEMBERS

Item	Number, per 1,000 members, employed by:	1920[a]		1930[a]		1940[a]	
		No.	%	No.	%	No.	%
1	Private concerns	53.9	100.0	54.6	101.4	38.0	70.5
2	Public agencies[b]	23.9	100.0	22.6	94.5	45.8	191.8
3	Private practitioners[c]	22.2	100.0	22.8	100.7	16.2	72.9

[a] Under each year are listed the number (No.) in each 1,000 members who are employed as shown, and the percentage (%) of the 1920 value. [b] Excluding members in the armed forces. [c] Including employees of practitioners.

The relatively slow advancement of Juniors may be due to the relatively large numbers employed by public agencies, mostly under civil service, where advancement is more dependent on seniority and increase in size of organization than it is in private employment.

THE MARKET FOR ENGINEERING SERVICES

The foregoing discussion and statistics have been presented to furnish a concise picture of the engineering profession and of the practice of engineering, for which engineering education is presumed to furnish the preliminary preparation. The statistical data are not complete; nor are they conclusive; but the size of samples is sufficient to make the results at least indicative of conditions and trends. The data also indicate the further research which might be made from existing information. If engineering education is considered as only the initial processing of a raw material to meet the subsequent needs of a potential market (and it is assumed that such initial processing must be followed by much further processing), two alternatives exist in the planning and design of engineering courses:

1. A rather complete knowledge of the character and volume of the present and the future market should exist, if the initial processing is to be refined in character, or
2. The processing should be of a character that will allow the product to adapt itself to the existing and future market, if information as to the character and volume of the latter is lacking or deficient.

If data were available as to the number of persons who had been employed in occupations involving a greater or lesser degree of engineering knowledge and experience over the past few decades, it might be possible to forecast, with reasonable accuracy, the character and type of existing and future markets for engineering services. Few data of this type are available, however, and little has been done with those that do exist. The result has been that large numbers of engineering graduates have been produced in the United States, each year for many years, by plants representing substantial investments and operating costs; and these plants have had little or no information as to the capacity of the market to absorb or to utilize their product in its latter stages of processing. They knew that their product was absorbed after initial processing, but had little knowledge of its fate, or of the requirements of the market for its services, subsequent to a period of from three to five years following such initial processing. Although quantitative data pertaining to the general volume, type, and character of market for engineering services are almost completely lacking, some information is available as to the character and type of product sought by the consumers who constitute this market—the employers of engineers.

The American Association of Engineers in 1940[2] published an analysis of "job orders" or descriptions and specifications given by prospective employers seeking men through the association's employment department in Chicago, Ill., during the previous ten-year period. Positions to be filled from these orders numbered into the thousands; and annual compensation ranged from a few thousand dollars upward to five figures. Most requests were from employers in the Great Lakes area, but a considerable number were from various parts of the United States, and the basic data available were of sufficient magnitude and distribution to warrant the conclusion that they represented a good cross section of employer requirements throughout the nation. In addition to requiring that the employer specify the technical training and experience, salary offered, and duties involved in the position to be filled, he was requested to specify other characteristics and traits desired in the prospective employee. In most instances employers gave as much weight to nontechnical as to technical qualifications.

A summary of the required qualifications showed the following to be prominently stressed:

Item	Qualification
a	A soundness in engineering fundamentals that would enable the employee to translate experience gained in one enterprise to a different type of industry, but in the same general branch of engineering;

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[2] *Professional Engineer*, November, 1940.

b A habit of continuing study, not only of technical theory, discoveries, products, and processes as they are developed, but also of commercial developments;

c A cost consciousness that would allow the engineer to appreciate the cost of engineering and its results, in relation to the profit possibilities of the entire enterprise, and to understand the problems of the accounting department;

d An ability to handle men, which involves the abilities to secure the cooperation, confidence, and respect of subordinates, associates, and superiors; to inculcate in others the desire to do things for one and to follow one's leadership; and to lay out and plan future operations in such a manner that these operations, when undertaken, would achieve a desired result;

e An initiative and resourcefulness indicated by a desire to step out and solve problems without having to be urged by others to do so; and

f An ability to express ideas in written and spoken form, in a manner that would convey them not alone to other engineers but also to laymen.

The demand for technical graduates was not insistent, although there was an increasing desire throughout the period for some technical education. Where graduates were specified, stress was laid on good, rather than on high, scholastic records; and equal or greater emphasis was placed upon participation in extracurricular activities while in school—the latter being considered as sound training for leadership.

Although specialization in college courses was requested by some employers—where recent graduates were sought—most prospective employers stressed training in fundamentals, and broad application in the various fields of engineering, preferring to train their own employees in their respective specialities. Considerable opposition was voiced to scholastic specialization, on the basis that it tended to develop an inflexibility in the employee which did not readily allow his being transferred from one department to another in the course of his advancement. Some of the foregoing qualities sought by employers may be considered as being more or less a matter of inherent native traits or qualities; but all of them can be improved, and many of them developed by proper educational processes and instruction, which frequently will bring out potential qualities that otherwise would remain dormant.

Postwar Engineering Education

Advances in science and technology as a result of the war effort indicate a postwar market for engineering services greater in volume than that existing before World War II, with a demand for services of a more diversified, and in many instances of a more specialized, character. Other things can likewise be expected of this market—among them: (1) A longer period of study and training for engineers (because of the more complicated subjects involved in engineering practice); (2) the broader knowledge of a wide range of both technical and nontechnical subjects necessary to provide maturity of judgment; and (3) the greater demand in the increasingly industrialized postwar economy for executives and administrators who have had background and experience in engineering.

To meet the requirements of such postwar market, and to equip engineering graduates properly to cope with the situation adequately and to make the most effective contribution to the postwar economy, it would appear that the following changes in engineering education, as compared with those generally offered during the fifteen to twenty years prior to World War II, are indicated:

(*a*) Engineering courses should provide a broader scope of basic instruction in subjects that cover the entire field of engineering, should include in the curricula a greater proportion of nontechnical subjects of a character that will be of value to the engineer who may engage in executive or administrative activities, and should leave specialization primarily to the period following graduation.

(*b*) Provision should be made for the instruction—through night classes or correspondence courses, or in some other manner—of those students who desire to continue their studies in specialties or nontechnical subjects following graduation and who do not, or cannot, return for resident postgraduate study.

(*c*) Extensive research concerning the engineering profession, the practice of engineering, and the market for engineering services should be sponsored not only by engineering schools but also by engineering societies.

The first change suggested will unquestionably entail a drastic revision of curricula. Many courses in specialized subjects, given in the last two academic years, will have to be resident graduate courses, night courses, or correspondence courses. Many courses which are essentially training in skills will have to be discontinued, and likewise many nontechnical courses will have to be added. To the writer, it has always seemed rather fruitless for a student to take a highly specialized course in theory and practice in his senior year, and then wait for six, eight, or ten years following graduation before he has an opportunity to apply the theory or practice learned in such course in any capacity except that of a subordinate under immediate supervision. The excuse usually given for such courses is that, even if they are never utilized in later practice, they were good mind training. Granting that this be true, comparable training could be achieved through more courses in mathematics or science, the tools that the student will use in later life to aid him in increasing his technical knowledge. The specialized knowledge acquired in such courses, if needed, could be acquired during the six, eight, or ten years following graduation.

With the considerable changing from specialty to specialty that exists in engineering practice, it would seem that the student would be better equipped to make such changes if he had a broader knowledge of general engineering. He would not then have to go back so far to start acquiring basic knowledge of a new specialty when such a change was made.

Particularly in the first two years, many of the courses that seek primarily to impart certain skills, could well be eliminated; or the time spent upon them could be reduced, although the purpose for which they are offered could still be achieved. Such courses include surveying (essentially applied trigonometry and use of precise instruments), drafting, and certain shop courses. Ordinarily these are not of sufficient length to produce skilled craftsmen. They could well be supplanted at least in part by courses which would help to prepare the student to express himself clearly and well, which would teach him something of human nature, of institutions, of the laws of economics, of money, and of business practice; and how to figure costs and to determine values. If the student is to follow a career that will cause him to deal with "men" and "money," as well as with "matter" (and most engineers do follow such a career at least part way) he should be given some basic instruction in these subjects while in school and should not have to acquire a knowledge of their first principles following graduation.

The student should also receive a realistic picture of the profession which he has chosen for a life career—not the glorified aspect usually presented by older alumni or distinguished engineers who address upper classmen at meetings. He is entitled to know what he is "getting into"—something of the organization of his profession, of its ethics, and of its code of practice; of the kind of life he might expect to lead five, ten, twenty, or more years after graduation if he follows this or that branch, specialty, or type of employment; of the probable rate of increase in compensation to which he may look forward in later years; and of the probable opportunities in various fields of activity.

He should be furnished information as to opportunities in business and industry for a person with a technical education and some engineering experience. Too many graduates, after a few years of experience in engineering, find that they do not like it, but feel that, since their training and education have been along technical lines, there is no opportunity for them in other fields. They then remain as misfits in this profession when they might have made a success in some other line.

An engineering course that would prepare its graduates to enter the bottom strata in any branch of engineering, that would give them a foundation in those subjects which are fundamental to business and industry, and that would emphasize training in the engineering or scientific approach to the solution of problems will produce graduates who meet the requirements of the postwar market for engineering services far more adequately than a course which attempts to produce a graduate who has been prepared to enter one branch or specialty in engineering.

Every engineering graduate realizes—either on graduation or very soon thereafter—that he has just begun to accumulate the knowledge which he will need before he attains a position of responsibility in his profession. When he was graduated, he had the habit of study and a keen desire to continue to increase his knowledge. Too often this was soon lost for the lack of a properly supervised and directed course of study or reading that he could, and would, follow if he had proper guidance plus a goal toward which he could strive. Even if he desired to increase his knowledge of the work in which he became engaged on graduation, he was met with such extensive literature that he became confused, did not know where to start his further study, and too often contented himself with merely reading current articles in such technical journals as came to his attention.

Many engineering schools offer extension courses; but either these seldom lead to a specific objective or they are designed to provide complete instruction over a period of years in a given specialty. In addition, these courses do not set up any goal of accomplishment toward which the student can strive as he could for his engineering degree. The habit of study and the desire to increase knowledge is a very real asset in a graduate; it should be conserved and not allowed to dissipate, as too often happens. Only a small proportion of graduates can, or will, return for resident postgraduate study. Many of them are not able to attend night courses. They can enrol in correspondence courses, however, or pursue systematic reading courses which have been outlined under proper guidance.

If graduate instruction were planned and arranged so that the student, after completing a given course of study, taking an examination similar to those given by the better engineering registration boards, and submitting a thesis indicating his ability to perform original and responsible engineering work, was awarded a professional degree, many graduates of four-year courses would avail themselves of such opportunity, and the profession would benefit. Such courses would furnish preparation in specialized subjects offered in many undergraduate courses.

The foregoing suggestions would cause some changes in existing practices and habits of engineering faculty members; but they would probably result in an increase, rather than in a decrease, in the size of engineering faculties. Broad general courses covering the entire field of engineering would tend to increase the number of students returning for graduate work. Specialists who formerly gave instruction orally in their specialities would give more of it by correspondence and might have more time for research or possibly for outside practice. Because of the longer time over which they had contact with their students, the faculty would secure a better knowledge of the market for engineering services; and they would be able to exercise better control over courses and be better equipped to modify instruction to keep abreast of current needs.

Research concerning the engineering profession, the practice of engineering, and the market for engineering services—in brief, along the economic and sociological aspects of engineering—is sorely needed. As stated heretofore, this research is a responsibility not only of engineering educational institutions but likewise of engineering societies. Tools for this research have been developed during the past decade or two which allow a great amount of research to be done economically. The technique of scientific sampling has made great advances since 1935 enabling many basic data of a fully adequate quality for the purposes needed to be secured. Use of punch cards and tabulating machines has greatly reduced the cost of analyzing data.

Professional biographies of registered engineers and members of engineering societies, and records filed with civil service boards and commissions and with concerns employing large engineering staffs provide an untouched mine of information concerning the profession. New data are included in the 1940 Federal Census. The lack of such data in the past was probably due to the fact that engineers, engineering societies, and engineering instructors were far more interested in the technical aspects of engineering than in the economic or sociological aspects. Developments during the past few years indicate that engineering societies and schools must give thought and study to engineers as well as to engineering. Not only should the profession know something about itself and its members; but those who are preparing to enter it are entitled to this knowledge, and an obligation rests upon engineering schools and engineering societies to furnish them with it.

DISCUSSION

E. S. Boalich,[3] Esq.—Something has been radically wrong with engineering education as two groups of engineers will testify. In the language of the paper (see heading, "The Practice of Engineering") these groups are:

Group I—The wizards in matter, who "fell down" when they came to men, money, or management. Mr. Baker's recommendations should take care of most members of this group perfectly.

Group II—Those who were never too proficient in M-1, including some who might have been wizards in M-2, M-3, or M-4 (or in art or literature!).

An added suggestion is to go back just one step behind Mr. Baker's starting point and to do as the steel plant does when it analyzes carefully each batch of iron ore before processing it.

By intelligent screening in the first college year, of a five-year course, "ore" which would never produce "good steel," but which might be in demand for "ballast," could be properly diverted; and as a dividend an occasional "gold nugget" would turn up which might otherwise have been forever lost in "the fiery furnace."

Russell C. Brinker,[4] Assoc. M. ASCE.—To one who has returned to the teaching profession after some years in the armed forces where the results of different types of training could be observed, this paper presents many points of interest. Consideration of the most desirable content of engineering curricula is occupying the time of many deans and heads of engineering departments. Various engineering and educational societies have gone on record as favoring an increase in the time spent on humanities courses. This increase means either elimination of some technical courses, or expansion to a five-year program. The University of Minnesota at Minneapolis and Ohio State University at Columbus are representative of schools adopting the latter plan. Doubtless many others will follow, although a desire to permit returned veterans to graduate in as short a time as possible will retard this trend temporarily. In past years, fear of loss of students who could not afford the additional year delayed the change-over. Problems arising in connection with a change to the five-year program illustrate some of the difficulties to be overcome if Mr. Baker's recommendations are to be accepted.

Reduction in time, or elimination of some courses seeking primarily to impart skills, is suggested. Many graduates use these "skills" to begin or to continue their work. In normal times an estimated 60% of the civil engineering graduates of the University of Minnesota start their engineering careers in surveying or drafting. Without such skills, these men would be rodmen instead of instrumentmen; tracers instead of draftsmen. The self-confidence and additional compensation resulting from extended work in surveying and drafting might well keep the young graduate in his profession. It is not proposed that all these men remain in "skilled" jobs. Nevertheless, if advanced theory and specialty courses are to be deferred until some time after graduation then the engineering graduate should be well trained—if that properly describes his status—for the job that he will have a 60% chance of taking immediately after graduation.

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Drawing and surveying courses fill a definite need for some work in engineering during the first two years of college if student interest is to be maintained. This point becomes increasingly important if curricula are extended over five years and the professional courses held back a year. Pre-engineering requirements, like prelaw or premedical standards, might be desirable. It is undoubtedly true, though, that many men who apply themselves well in the later professional courses are concerned only with obtaining marks high enough in the preliminary work to insure entry to the professional schools.

Substitution of two-year programs of the trade school variety is not generally favored by the faculty or acceptable to a large percentage of students. The unlimited educational opportunities so characteristic of the United States inspire engineering students to try for the status of a four-year graduate or nothing.

It has been stated frequently that many engineers are unable to express themselves well, or to write good reports. One reason for these handicaps is probably the failure of most engineering faculty members to insist upon the continuation in all later subjects, of the standards set up in freshmen English courses. Professors and instructors are apparently not willing to take time to correct spelling and sentence construction and stress the need for improvement. Students, of course, feel that freshmen English courses are hurdles to get over or around, and that they are of no practical use in succeeding work, where "anything goes." Too often anything does go!

There is a tendency for faculty members to be specialists, perhaps engrossed in some isolated research problem that has little or no connection with class work. Teaching therefore becomes boring, and all interest is lost in the meticulous checking of problems and reports by students. National reputations for research or publications are not maintained by long hours of student consultations or paper grading. On the other hand, few professors in mechanical drawing, surveying, mathematics, and freshmen English courses (which require "plugging" at details) attain any great prominence; yet to be an inspiring teacher in these subjects—as compared with structural design courses, for example—really "takes something"!

Lack of the "dollar sense" is a highly valid criticism of new graduates in engineering—and of many experienced ones too. How many instructors or full professors touch even lightly on the mill price per pound of a steel beam or column that has been designed in class, and follow with statements on fabrication and erection costs? How many mention the cost of formwork and discuss its influence in selecting an economical design? Less than one hour's time spread over a semester would develop the ever-present student interest in "How much does it cost?" Probably not one surveying instructor in ten has discussed the cost of operating a survey party, including interest and depreciation on equipment, etc. Lowered professional standards are certain to result as shown by a recent case when a well-qualified senior student made a detailed land survey for \$5—his hourly wage probably being less than 25¢ per hr. Such practices may well be extended into succeeding work unless checked early by proper instruction.

Too few students know about the advantages and requirements for registration as a professional engineer. Medical students are indoctrinated early in the responsibilities imposed by the degree of Doctor of Medicine. Engineering students should begin hearing in the freshman year, and frequently thereafter, about professional registration so that it will become a goal secondary only to graduation. Unfortunately some faculty members are not registered and may hesitate to assume a "Do as I say, not as I do" attitude.

Engineering faculties should emphasize in junior and senior classes the need for continuing study after graduation. This may be done by mentioning in each course some of the important theories and problems omitted because of lack of time.

Except in large cities or college towns, postgraduate work must be conducted by correspondence courses since the absence of faculty and sufficiently large groups interested in a particular subject make the cost prohibitive. A "Center for Continuation Study" established at the University of Minnesota typifies the facilities needed in many cities to provide wide coverage. Correspondence study is commendable and the material covered "sticks" with the individual; but it does not have the advantage of personal contact with a teacher, and it is likely to be long drawn out and time consuming. Human nature being what it is, relatively few men will progress steadily unless a rigid schedule is set up. Class hours provide this schedule. Personal observation of the attitude of many men in the armed forces indicated a desire to take correspondence and classroom work but also a frequent lack of "push" to complete it.

Most important, a tangible goal must be extablished that a graduate can hope to reach within a reasonable and specific time. Several three-year or four-year "plans" could be set up. At the end of the first, a degree of Master of Civil Engineering might be the goal. Following a second plan the professional degree of "Civil Engineer" might be awarded. At the present time only two schools in the United States confer the "professional" degree (not in civil engineering) on a graduate at the end of four years. In other institutions, however, the requirements form a confused picture.[5] Some schools give it as an honorary degree, some for graduate work and a thesis, and some for a thesis only. Usually a minimum period of from four or five years after graduation is specified. On the basis of a broad outline to permit numerous options and substitutions, a definite program could be planned which would permit a graduate to study by correspondence and in the classroom, so that he would know how and when he might obtain a degree. Isolated courses are helpful and desirable but they are far from being a solution to the problem of continuation of study.

This program might be extended still further. The author mentions the progression through the three M's—matter, men, and money. At about the time that a man has finished work for the professional degree he should be interested in the second and third "M's." A third four-year plan could be laid out leading to a degree in business or administration. Education is not a matter of regimentation in thought. It is, however, a problem in regimentation of scholastic resources and an individual's time which planned programs will help to solve.

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[5] "Status of Engineering Degrees," by O. M. Leland, *Proceedings*, S.P.E.E., Vol. XLVI, 1938, p. 322.

Personal experience with men taking isolated evening courses indicates that a troubled conscience rather than a desire for knowledge impels some of them to register. A man feels he is not advancing, so he registers for a course, pays his fees, and sits back with the inner voice silenced. He may come to class to be amused or to acquire bits of information the easy way without studying. Little or no work is done outside class and absences are frequent. This type of individual will always increase evening enrolment in no-credit "occasional" courses, and perhaps he benefits enough to justify the tuition fee. A definite planned program, however, as opposed to isolated courses or lectures based on sales appeal, should reduce or eliminate the number of such men.

Engineering theory and computations are a difficult "grind" after a day spent on similar problems in field or office. It is a personal belief that, once past the fundamental humanities courses, a graduate would find it easier to begin and continue a program of study, either by night classes or by correspondence, in that field rather than in engineering. Schools contemplating preparation of long-range programs for graduates might consider this point when "pruning" engineering courses and "grafting" humanities subjects. Physicists have moved into many fields formerly claimed by engineers. This trend will continue if basic sciences and technical work are sacrificed to the humanities.

Many developments of the extensive service training programs should be introduced in high school and college teaching. The "Instructors Guide"[6] gives some hints that professors may long since have forgotten. A theory of service teaching is that students remember only 10% of what is heard in lectures. This is a point that "long-winded" faculty members might well take to heart. Emphasis on slides to reduce time and improve instruction is needed. Exposure to Japanese language instruction by the newer method of conversation first and grammar later was convincing proof of its advantages.

Faculty members probably know the needs of industry for new graduates but they do not know much about the market for older engineers. Graduating students are likely to consult with professors about that all-important first job but seldom request advice later when a change of position is contemplated. This may result from being too far from the campus, or from a desire to make one's own decisions. Possibly more recent contacts in the particular field are considered to carry greater weight. Also, a representative cross section of graduates does not return for consultations with the faculty. One of the compensations of teaching is having former students drop in for social calls or advice. Unfortunately, except for a few job hunters, only those in the higher scholastic brackets do so, and the problems of men who just "squeaked" through are not brought to light. During depression years, the school employment offices, run at least informally by most engineering schools, have many calls. Mainly, however, service to the new graduate is their primary function.

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[6] "Instructors Guide," Marine Corps Schools, Marine Barracks, Quantico, Va.

Mr. Baker's data show that many engineers leave their profession—more so than in the case of doctors, dentists, and lawyers. This is natural since an engineer is not handicapped by such a change. The training he has received fits him for many types of work. Doctors, dentists, and lawyers have a heavier investment in time and money for their highly specialized training, hence, there is a greater incentive to remain in their profession. Many college graduates in education, business, etc., also desert their spheres of specialized training for the "greener grass" of other fields. Men trained in engineering find it desirable to seek other employment because of the low salaries paid to engineers. This is a serious condition facing the profession. Fortunately, graduates of engineering schools prove capable of competing out of their own fields with personnel from business and arts colleges. This seems to indicate the type of training given in engineering schools, even if it develops only the habit of hard work and long hours, is reasonably satisfactory.

Mr. Baker's paper has undoubtedly crystallized the thoughts of many engineers and educators. The collection of supporting data is a worthy contribution.

I. Oesterblom,[7] M. ASCE.—"Thoughts on Education" is a very wide field, and it is pleasant that the author has strolled deliberately beyond its normal limits. Since the engineer has just proved to the world by his extraordinary performance in war that he is a giant in both mind and body, the education that lies beneath all his achievement is interesting. It must be good, and yet it might be better.

It must be good—for otherwise the performance would not have been so stupendous—and it might be better—for that is the spirit of the engineer. There is always a better way to do things. Were this not so there would be an end to progress.

A review by anybody with a keen and open mind must therefore be instructive, constructive, and of good purpose, and this review by Mr. Baker is especially so. The paper meets all these specifications and it also looks well into the future. However, something seems to be missing. There is a forgotten field which needs to be explored.

When the Hoogley River Bridge was built a few years ago from Calcutta to Howrah in India, it came into being through intense economic pressure which had accumulated over a number of years. As far as the bridge was concerned those years had not been idle ones. There was a temporary pontoon construction across the river which had been forced to serve for a very long time. Over the bridge had gone for years very heavy traffic between two large cities; and through the bridge there was also heavy river traffic. Each was increasing every year—dangerously so. Thus, the social and economic statesmen—and the engineers—had been set to work to plan a new bridge. These groups worked hard for many years because the problem was a difficult one. A huge structure was required, and a flexible one which would take care of the two antagonistic streams of traffic. Much space on land was needed, and all the land was occupied. The river was treacherous and "emotional," and it ran over a soil that was soft and shifting.

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[7] Engr., Carbide & Carbon Chemicals Corp., South Charleston, W. Va.

Thus, some official authorities and many consultants produced a number of bridge plans, all of which were excellent and beautiful and one of which was finally selected under the economic pressure of the conditions.

This case is selected as an illustration because it is so broadly typical and because substantially the same conditions exist for all the large engineering projects as they also do in a small way for the minor tasks. The economist—especially the contentedly Marxian—has a fine example of the economic pressure that governs the destinies of man—and he can see that only. However, there is something even more fundamental at work.

Thus one arrives at the forgotten field—to the creative urge in man; to the dreams of constructive work to be done; to the dreams of the engineer, which are born in his mind and which he sets down on paper in his waking hours. They are brilliant flashes, sometimes, but more often they came into being slowly and painfully, borne from experience and knowledge and from that curious urge in all men to create new things out of scattered stones.

The point that this commentator wishes to stress is that these dreams are the most important fact in engineering. Also that in both education and discussion this is officially a forgotten field. In fact, it is even worse than forgotten.

In any school or in any office "thou shalt" or "thou shalt not" predominates. The professor does his utmost to train a flexible mind into a rigid form; and, after school is finished, whatever is left of flexibility is hardened by office standards and specifications. It is not to be inferred that all these tokens of rigidity should not be and that they are not useful, but they should be accepted as servants of the mind and not as prisons to hold the mind captive or as urns in which the mind may be slowly petrified.

How is it then that, even under these conditions, so much is being created and that the performance in the war was so unusually and successfully creative?

This is a most interesting question. For in the welter of minds there are always a few that refuse to be held captive; or, if so held, that are ready to explode into action the moment they sense a call of urgency. They are just like the "Prodigal Judge" of Old Tennessee—besotted and contemptible as long as the soil seemed like a prison but magnificent and a supreme leader when there was work to do.

There is tragedy in the other side of the picture—and that is the prime justification for these comments. Fine minds with a fine education and high diplomas, frozen forever to the drafting boards—once minds ready for constructive dreams but condemned to the drudgery of "routineering." They are now mere slaves of formulas which the mind of a clerk would be happy to use and set down on paper and which he could do even better.

The world therefore is so much the poorer.

These minds have been destroyed by the authoritative attitudes of professors and chiefs, because so many of these have been fascinated by the exterior forms and have not yet seen the rich field of thoughts and dreams beyond. This, therefore, is an invitation to reorganize both material and methods accordingly. More imagination is needed in both school and office as is a better understanding that convention and rigid form should be aids and not masters and that each should be destroyed when it can no longer serve as an element of efficiency and progress.

Should there be special courses in creative imagination in the universities? Perhaps there should be in the future but not now—the time is not ripe. If such courses were started now, some professors would drop dead from fright at such sacrilegious interference with the worshipped forms and formulas. That result would be even more disastrous, for good conventions set the limits to the dreams and without them there would be, at best, beautiful chaos. By preference the thought of the constructive dream should always be present in everything, including seemingly dry calculus, that the teacher tries to convey to the student's mind and in all things the chief of an office attempts to create.

How much more attractive would be the work of each man if such inspiration were always present to pull the potential leader out of his weary armchair of humdrum formalism.

Samuel T. Carpenter,[8] Assoc. M. ASCE.—The clear thinking evidenced in this paper relative to the important problems faced by the engineering schools and engineering societies should be commended. It is to be hoped that in the future there will be more articles by men active within the profession, as their ideas are valuable in formulating the objectives of engineering curricula and standards for professional training.

The factual information in the paper represents important data and furthermore shows that a fundamental, rather than a highly specialized, training in engineering is desirable. It is interesting to note that the questionnaire study performed jointly by the Society's Committee on Engineering Education and the Cooperative Committee on Civil Engineering Education of the Society for the Promotion of Engineering Education verifies Mr. Baker's contentions. The survey indicates that, in general, engineering graduates are very weak in their ability to address a private group or public gathering, and very poor in writing a letter or report. They were also found to be practically deficient in their interest in public affairs. These deficiencies and other deficiencies noted by this survey can be largely corrected by a general improvement in engineering teaching staffs.

Mr. Baker appears to emphasize the part that engineering schools should take in the development of men after graduation. The writer heartily agrees with this, and believes that most engineering schools do attempt to do all they can for their graduates. However, the writer believes that the Society, as well as all other engineering societies, must take an increased interest and be a key factor in the professional development and postgraduate education of the young graduate. There are so many ways in which the engineering societies can and should assist in the professional development of the graduate that each and every one should have a full-time staff man to direct all its relations with engineering education and professional training. The Society should make a continuous survey of the needs of industry for graduates with a civil engineering education.

The Society should also sponsor a campaign directed toward employers of civil engineering graduates to encourage the treatment of the graduate as a prospective professional engineer. The Society should not approach engineering employers and ask them to be altruistic, but instead should sell them on the merits of the professionally trained man in such a way that employers will give the graduates an opportunity to rise from the subprofessional status in which many find themselves. Employers could be urged to adopt a training program which would indoctrinate the young man in all phases of work encountered in the engineering office. This is essential since, with the advent of the "Engineer-in-Training" status, the young man must be prepared to take his professional examination within a period of from five to ten years.

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[8] Chairman, Civ. Eng. Dept., Swarthmore College, Swarthmore, Pa.

The Society has recommended that a more fundamental education be given by civil engineering schools. It is believed that this places a responsibility upon the Society to use the Society organization more effectively as an instrument for professional development. Such a result could be brought about by a coordinated and unified effort on the part of the Society's Committees on Engineering Education, Juniors, and on Student Chapters; the civil engineering members of the Committee on Professional Training of the Engineers' Council for Professional Development; and other agencies of the Society. The over-all problem is so immense that space does not suffice to discuss it here. Accelerated trends in science and society call for education throughout a man's career. It is hoped that Mr. Baker's excellent article will stimulate the engineering societies, as well as the engineering schools, to action.

LYNN PERRY,[9] M. ASCE.—By the publication of this paper, the author has done much to help engineers see themselves as others see them. He has pointed out the usual personal shortcomings of many of the members of the profession. Then with grace and tact he has placed the blame squarely on the shoulders of the professors of engineering who are responsible for the curricula. Curricula considerations are very much more involved and far reaching than the engineer practicing his profession realizes. Perhaps some professors of engineering will follow the suggestion of T. R. Agg,[10] M. ASCE, and "talk back." Probably no more appropriate time could have been chosen to reopen this subject.

In the opening paragraphs, the author, after admitting technical competence and skills, frankly states that the colleges have been training rather than educating the student body. He is concerned about the pattern and structure of the engineering profession, the practice of engineering, and the marketing of engineering services. Without plunging into a deeper discussion by defining education, few professors will award any considerable educational value to such subject matter, by itself. The engineering societies have authorized student chapters for the very purposes indicated—that is, to hasten the student's conception of the breadth of his profession, to bring the activities of the profession and its members to his doorstep, and to place in his very hands the professional literature available. However, some students never attend a meeting of their student group just as some engineers never attend a meeting of their local section, some physicians never attend a meeting of the medical society, and some attorneys never attend a meeting of the bar association. Incidentally, such men are not likely to attend class reunions in great numbers. That is personality. Its roots are congenital, hereditary, or fed by earlier environment. The way to improve the professional personality is to nurture the promising and eliminate the hopeless. Do not imagine that a college professor can be influenced to change an engineering curriculum for the purpose of raising the personality standard of his student body.

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[9] Designing Engr., Dept. of Water and Sewers, Miami, Fla.

[10] "Freedom to 'Talk Back,'" by T. R. Agg, *Civil Engineering*, September, 1940, p. 549.

It has been stated that every man should be a salesman—that, if he has nothing else to sell, he has to sell himself. This is only partly true. Many of the most successful engineers never seek an engagement; engagements come to them. The same is true of the other professions. Being a broad and progressive profession, engineering has opened a field of technical salesmanship to introduce new techniques and new equipment in a conservative and stable society. Engineering salesmanship has been and will continue to be apparently more remunerative in the earlier years than design, construction, research, or production. Large firms have sales or contracting organizations, and technically trained young men who have a flare for that type of work seem to be in demand. Salesmanship, however, is almost as technical as surveying or thermodynamics and, instead of being founded on the immutable principles of mathematics, is based on human nature with all its vanity, superstition, and dogma. This field involves "how to make friends and influence people" and successful salesmanship requires much more study of human nature and self-discipline than most people realize. In the past, professional engagement has been based largely on record and recommendation, and has been continued because of performance. Agreeing that Alexander Pope was correct when he wrote "the proper study of mankind is man" and that popularity among one's acquaintances and geniality among strangers are desirable personal assets worthy of development, then, is or is not the engineering curriculum the proper place for this training (not education)? If "yes" is the answer to the question, it should be decided what subjects are to be removed from curricula to provide the necessary time and someone must accept the responsibility for the decision.

Mr. Baker states (under the heading, "Introduction") "Four years is too short a time," a fact not new to educators or engineers. From the purely professional standpoint, a longer course offers the solution to most of the problems. At the beginning of the century a number of universities offered longer courses; some still do, with a very small registration. Such courses are available, now, for the student who chooses them and, as they become more popular, the supply will probably meet the demand. The New York Section of the National Society of Professional Engineers has taken action on this subject and such action on the part of a few more such societies might force a five-year or six-year course. This trend may develop much faster than is realized. Thus, the effect on the student, the profession, and the university, somewhat interlaced, may be considered briefly.

The time and cost will increase and, presumably, result in fewer alumni. That is what happened in the medical profession. The next generation will have fewer engineers, but better trained ones. The demand for services will net the profession a larger slice of the fruits of the individual engineer's effort—an attractive prospect to the student who has the funds and is willing to devote the time required. It appears somewhat as though it would be a cog in a program to "make the rich richer and the poor poorer."

From the professional standpoint, most engineers are ready to endorse a program that promises to raise the standing of the profession. Those who have given time and thought to the subject feel that more time is the only solution and that the social problem can be met. Published data indicate that engineering is already a tolerably well-paid profession. It may be that engineers are already taking from society in a closer proportion to what they put in than is realized. Any movement that will result in the ability of the profession to contribute more to society will entitle the profession to a proportionally greater reward.

Engineering educators are likely to point to the first half of the twentieth century, with justifiable pride, as the era of most elaborate technical development in the history of human culture. This development has demanded and used both quality and quantity of effort. It has been predicted that, to maintain the present acceleration in development, society will require engineering talent of an increasingly higher quality in successive generations. On the other hand, it has not been predicted that it will require less; the reverse is more likely. The truly professional man would suffer from insomnia after participating in a plan to increase his emoluments by decreasing service. So it may be unwise to enter on a program likely to decrease the number of professional engineers without plans to supply the demand for a subprofessional group.

Many of the largest universities, supported by public funds, are enjoying mass educational programs developed on the basis of giving the greatest value to the greatest number. Hoards of students have crowded the classrooms and laboratories of these institutions for technical educations in four years although registration remains pitifully low at institutions offering longer courses. The general result has been that men with vision, foresight, energy, and determination have forged ahead into places of responsibility whereas those less abundantly endowed with personal qualities have remained in the subprofessional group longer. This program has placed engineering in its current position.

Now suppose the profession should force a six-year course. The student would probably register in the arts course, elect the first two years of an engineering curriculum during his last two years, stay two years longer, and be graduated. Such a procedure presents no unsurmountable curricular problem. There will be fewer alumni but the university budget could probably be balanced by an increase in tuition. The supply of talent will be better qualified but smaller. The quantity will have to come from elsewhere. The most likely sources are the technical high schools or a forced development of technical junior colleges. A number of these are functioning in the larger cities of the United States. The popular four-year course will be unjustified in its present form. Concurrent operation of both the longer course and the shorter course on the same campus has not proved satisfactory. Universities with adequate equipment and staff will likely follow the signpost to the longer course and adapt themselves. Industry, already short of technical talent, will have to endure through the transmission years. Will it? Can it? That is a personnel problem.

The 1945 report of the Society's Professional Committee on Engineering Education[11] indicates that the Society is likely to adhere to the four-year course in substantially its existing form for the present and suggests that at least 20% of the students' time be devoted to humanistic-cultural subjects. What can or should be placed in this 20%?

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[11] *Proceedings*, ASCE, March, 1946, p. 386.

The standard unit of credit, or "credit hour," is one hour of class per week per semester, requiring preparation—an average of two hours of preparation. Arts courses usually require 120 credit hours for graduation, 15 per semester or five courses meeting three times per week. Three hours of laboratory work per week is usually awarded one credit hour. Engineering courses are proverbially longer, requiring from 132 to 144 credit hours for graduation—an average of 16 to 18 per semester and five or sometimes six subjects. After World War I, most of the short courses in such subjects as masonry, tunneling, descriptive geometry, stereotomy, railroad location, railway track and trackwork, and modern languages were eliminated, grouped, or taught with some other subject. In this way room was made for electives in the humanistic-cultural field. At present, the required courses comprise about 80% of the time and electives about 20%; 20% of 132 hours is 26.4 credit hours, or just about one standard three-hour elective throughout the entire four years. Assume that a student has a patriotic urge and wants to elect a course in the Reserve Officers' Training Corps so that he can become a reserve officer in the army or navy upon graduation. That activity absorbs all his electives. The variety of subjects that a class of students desires to elect embraces almost all that can be offered. A fairly large number have visions of travel and elect a modern language, the subject which requires more of the student's time and from which he drives less benefit than any subject which is likely to be available. Nevertheless, if electives are offered, they should be free electives. Much can be stated in favor of a rigid curriculum; it is good training to form the habit of doing what has to be done.

A "letter to the editor" by Don Johnstone,[12] Assoc. M. ASCE, includes some diagrams that show very well how a curriculum is arranged with the fundamental subjects first, then the techniques based on those fundamentals, and finally a smattering of design. The present work requiring 80% of the four-year course will comprise 64% of a five-year course or 53% of a six-year course. The five-year course would permit time to cover some of the most glaring deficiencies that Mr. Baker has indicated but the six-year course would give much more latitude.

Many institutions have courses required for every student. Such courses are usually broadening and have the virtue of being taught by inspiring professors. Every course or subject should have two elements—education and training. Even purely engineering subjects are presented in such a manner that the student must derive some education from the course as a whole, even if it is primarily intended as a vocational subject. If the professor fails in this objective, he is a failure and his students are merely trained in a prospective vocation. Much pressure has been brought on the colleges to insert almost entirely informational courses. Most technical courses are sufficiently informational to keep the memory pliable. Faculties have resisted suggestions for subjects that can be learned by postgraduate reading and have insisted on subject matter that requires teaching rather than merely reading—factual and descriptive teaching. The former practice of making an assignment in a textbook and hearing the lesson is not nearly so prevalent as it was a generation ago.

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[12] *Civil Engineering*, June, 1946, p. 267.

For present purposes, subject matter may be considered as (1) distinctly intended for use in earning money (vocational); and (2) for the student to live with during his entire life (cultural). The educator has felt that it is fully as important to show the student the importance of living a full and fruitful life as it is to teach him how to earn some money. There is little time for such work in a standard four-year engineering course. At the same time, the professors who recommend their progeny to prospective employers, and the universities they represent, would feel it very keenly if one of their former students failed to meet the requirement on the first task to which he should be assigned by his employer. It is much more likely that the young engineer's first assignment will be routine in type rather than a monumental design or even an economic investigation. Therefore, professors take much care to assure themselves that, with little or no assistance, the student can make a presentable drawing, run a line of levels, make readable field notes, and follow specifications and directions. They will probably continue this practice even in the face of criticism.

Criticism has been directed at surveying and drawing. Most universities devote about 6% of the students' time to the former and 4% to the latter. Surveying is the first course to which the student is subjected in which he is shown how to apply mathematics. Prior to this, mathematics has been something that his elders have forced upon him and which had to be mastered, perfunctorily in too many cases. If he cannot develop a flare for applied mathematics, he learns it quickly and changes his course. No attempt is made to create a supply of expert witnesses in court cases but the adept student will form some conception of the relativity of measurements, the limitations of accurate physical measurements, the inaccuracy of the arithmetical mean, the value of neat and accurate records (dated and witnessed), and the shape and movement of the earth and some of the heavenly bodies, as well as the construction, use, and adjustment of some measuring equipment. Such subjects are by no means purely vocational for it is a known fact that few engineers practice surveying or geodesy as a vocation. Much the same may be true of drawing. Few students know how to ink a right-line pen when they report in their first class. Some lettering and draftsmanship must be taught. If a little freehand sketching is included, it may supply the basis for a refreshing avocation—Leonardo da Vinci did very well. The time allotted to these subjects has been cut to the very bone already.

English has been and is the most usual subject of criticism; it has been a problem for years. Engineering is taught by engineers, medicine by physicians, law, even business law, by attorneys; but English—well—few instructors have succeeded in creating any considerable enthusiam in the student body. It certainly is as important a tool as some technical subjects, and has educational and cultural value. The scholarly instructor who is a leader and who fires the imagination with the beauty of metaphor, harmony, and clear concise exposition, is only too rare—so the boys use a formula to secure a grade. Consequently, the diction, choice of words, and clarity of expression of a senior show little improvement over those of a freshman. He knows more to say; he has a broader interest but his language discloses his early environment. This condition is not a matter of curricula; few engineering faculties can secure the solution; it is an institutional shortcoming that can be corrected only by the institution.

Graduate "refresher" courses have not been supported with enthusiasm. The formal part of a person's education usually ends with the valedictory. It is doubtful if this will change considerably; certainly it will not change quickly.

A rather important aspect of education has not been mentioned by Mr. Baker—graduates who do not follow engineering. The writer entered college with sixty freshmen, twenty six of whom were graduated (in civil engineering). Five years after graduation, only five were practicing their profession. A contribution of five per year to the profession by a large university is not great. Some sixteen hundred were graduated at another institution with the writer's approval. Of these, probably not more than four hundred practiced engineering more than a few years. The others are in every walk of life from politics to the ministry; they are even newspaper reporters and authors. Their careers justify their training and the existence of their alma mater.

To assist the student in the solution of his financial problem, colleges have appropriated funds for scholarships up to one third or more of the student body. A point that has not been solved satisfactorily is the creation of a motive, on the part of the student—a motive sufficiently strong to drive him over the goal. The postwar student body will consist largely of former service men. The expenses of the men have been provided for and few of them will require artificial motivation. They will have a standard four-year course with its virtues and its shortcomings. Industry and the coming generation will extend to them a hearty welcome, subject them to the usual hardship, and life will be the better as the result of their training.

L. E. GRINTER,[13] M. ASCE.—The sincere appreciation of the colleges of engineering should be extended to Mr. Baker for his thoughts on engineering education. He is more complimentary than the men who are responsible for the present form of the engineering curriculum would be. The singular "curriculum" is used with justification because the variations between schools and departments has never been great and the band of variations has been narrowed by accrediting procedures. One question to be considered is whether the author's concept that specialization is bad and that a broad general curriculum is to be preferred might not lead to further standardization without adequate justification.

It is fairly clear from published reports that during the past two years nearly all colleges and universities have been reconsidering their objectives, their methods, and the results obtained. In another year, most engineering curricula will have been rewritten. Educators have been taking very seriously the common criticisms that their graduates are poorly qualified to communicate their thoughts to persons in other groups and that engineers are almost antisocial in their public lives. Certainly it is clear that engineering does not attract young orators and perhaps infrequently enlists anyone who could earn his living through writing. Furthermore, the study of facts, theories, and more facts does not stimulate the development of either the communicative arts or the ability to make friends or influence the actions of others.

Engineering educators want to broaden their students, but few are convinced that a course in public speaking and another course in English literature, technical writing, history, or public affairs will accomplish the desired result, They believe that a good long list of such courses under cultured stimulating teachers might produce results, but they fail to see how one can study such subjects in sufficient quantity and still end up as a trained engineer. The writer adds that the result would be a total lack of education unless the engineering student could be made to want to study these nontechnical subjects.

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[13] Vice-Pres. and Dean, Graduate School, Illinois Inst. of Technology, Chicago, Ill.

Whether educators like to believe it or not, it is true that the student himself has a considerable influence upon the engineering curriculum. By simply doing poorly in courses that do not attract his interest, the student can eventually change their content or even eliminate them from the curriculum. Educational administrators would prefer to maintain common studies for all engineers through the first two years of the curriculum. The student, by continually expressing his desire to start the study of some special branch of engineering immediately, has kept most institutions from adopting this eminently commonsense procedure. A uniform freshman course is as far as administrators have progressed against the desires of the student and of teacher specialists. Could it be that students and teachers are closer to the real purpose of education than the administrator plagued by schedules and budgets?

The question often arises as to whether the study of advanced technical problems in college has merit. In the words of the author (see heading "Prewar Engineering Education"), the student

> "* * * grappled with problems of a character that he would never meet, or at least would not have to solve, on his own responsibility for from at least twenty to twenty-five years after graduation."

Times are changing, or have already changed, to such extent that many laboratories are directed by engineers thirty years old or less. During World War II, young structural engineers carried grave responsibilities for airplane design work, and they will soon hold equivalent responsibilities in bridge and building design. There is no technical responsibility that a first-class engineer of thirty may not have an opportunity to assume. To this extent, the most advanced scientific training can be justified fully if it is not at the expense of a reasonably rounded education. The other objective of advanced scientific study is mental training in how to attack the analysis of such problems. It is not the particular problems that are important, but the experience of having solved difficult problems that develops self-confidence.

On the credit side of the engineer, the author rightly lists resourcefulness, persistence, and a willingness to accept hard work. Engineering education, as it has existed, must be credited with some of the development of these traits which are not common in liberal arts graduates. On the debit side are carelessness in the use of spoken and written English, lack of salesmanship, and an unawareness of the responsibility of the professional person for political and social relationships. The author wrongly criticizes the engineer for lack of a "dollar sense" which has long since become an inherent part of engineering education. In fact, no other group except business men shows so much interest in the practicality of its work.

The experience of counseling with young engineering students makes one skeptical of the practical results of attempting to force such individuals into the single pattern of broad general education of an engineering nature. The major institution that is famed for giving a single degree to all its engineering students still has been forced to provide considerable optional study despite tradition and the natural preselection of its students. It may be questioned, however, whether the number of vertical or professional subdivisions of the field that now exist are necessary or desirable. It is evident that the choice between mechanical and civil engineering, for example, should be made on the basis of interest rather than aptitude, and the same may be true for the various subdivisions of civil engineering. There are job classifications within the engineering field, however, which do require different kinds of aptitudes.

As an example, one may compare research and design on the one hand with operation and maintenance on the other. Another job classification clearly demanding a special group of aptitudes is management. As a contribution to the author's desire to educate some engineers broadly without missing the need that the writer sees for training others as specialists, the proposal of broad horizontal options through the entire group of engineering curricula deserves careful study.

One such option designed for the training of specialists might properly be in research, development, and design. Such an option would stress mathematics and its application to the solution of engineering problems. It would emphasize analysis and would also provide students with an opportunity for an internship of a few credit hours either in theoretical or laboratory research. At the opposite end of the educational spectrum would be the broad option in management which would provide for all the studies so aptly described by the author. In between would be professional engineering options, not essentially different from those in the present curricula, for students desiring to construct, operate, and maintain engineering works. These horizontal options are equally needed in all engineering fields.

It will become clear that the system of horizontal options provides an opportunity for the full use of the student's aptitudes within the field of engineering to which his interests lead him. He can still be a civil engineer while developing his talents for research, management, or practical engineering service. This is merely the theory of specialization applied in a new direction in engineering education. The result, if this concept is carried out at other institutions as it is planned at Illinois Institute in Chicago, will be to produce one limited group of highly trained specialists in research and the more scientific phases of design. Another limited group will be broadly educated with emphasis upon ultimate work in management within the field of civil engineering. No doubt this group will be more vocal—more interested in economics, politics, and social problems than have engineers of the past, but a larger group than either of these will still be trained for practical engineering work with considerable emphasis upon early competency for earning a living as civil engineers. If this is shortsighted, it is because practical young Americans will it so.

Robert O. Thomas,[14] Assoc. M. ASCE.—The parts of this paper that deal with the practice of engineering and the future market for engineering services are of especial interest. The subject should be given the widest possible consideration among all members of the profession. It should be kept continually alive by extended studies and reports in the technical press so that the profession as a whole may be made aware of the dangers confronting it and so that they may be informed of the measures which must be formulated and practiced in order to arrest the present apparent trend toward artisan rank.

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[14] Engr., Edward R. Bowen, Cons. Engr., Los Angeles, Calif.

For a number of years, the writer has been aware of trends in the practice of engineering which have been so well illustrated by Mr. Baker, and to which attention was called by the writer[15] in 1940.

A study of past annual reports of the Society has developed information of considerable interest relating to the growth, both of the Society itself and of the engineering profession in general. The disparity between the number of new members admitted each year since 1929 and the total gain in Society membership is very striking.

Table 7 illustrates vividly the fact that it is necessary for the Society to admit three new members to secure a permanent gain of only one member. Such a condition in a profession is one which invites extensive research into the causes underlying the movement of men into and out of the profession. Engineering is not a trade, nor is it a type of clerical work that can be taught quickly to an untrained candidate for a job. The 11,126 lost members represent at least four years of study per member—a huge investment in education (individual) and educational facilities (institutional). They represent a group which certainly, at one time, expected to follow the profession of engineering as a life career and prepared accordingly.

TABLE 7.—Membership Statistics, American Society of Civil Engineers, for the Year Preceding a Given Date

Date (Jan. 1)	Total members	New members	Losses	Net gain	Deaths
1929.....	13,315	1,066	558	508	135
1930.....	13,823	1,139	565	574	138
1931.....	14,397	1,055	524	531	140
1932.....	14,928	753	696	57	145
1933.....	14,985	534	580	−46	170
1934.....	14,939	757	786	−29	170
1935.....	14,910	923	764	159	165
1936.....	15,069	1,142	1,110	32	170
1937.....	15,101	1,123	765	358	150
1938.....	15,459	1,147	812	335	160
1939.....	15,794	1,240	1,034	206	177
1940.....	16,000	1,422	760	662	157
1941.....	16,662	1,399	623	776	179
1942.....	17,438	1,471	555	916	190
1943.....	18,354	1,576	500	1,076	190
1944.....	19,430	1,497	494	1,003	205
Total..	....	18,244	11,126	6,115	2,641

Over the sixteen-year period, the actual loss, not counting deaths, amounted to a total of 8,485 members. Each member should be regarded as an avoidable loss—either avoidable through the retention of the men in the profession of engineering and in the Society, or avoidable through more enlightened guidance in the precollege period to those seeking to follow the profession. It must also be remembered that this group of lost engineers represents only those who are able to number themselves among the group of former members of the Society; and it does not include the loss of the considerable number of engineers with which the Society has never come in contact. In considering the loss to the Society, several questions present themselves for discussion. What has turned these 8,485 members away from the Society? Have they left the practice of engineering or have they merely left the Society? If they are still following an engineering career, why did they leave the Society? How could the profession as a whole (or the Society as the representative of the profession) have saved these engineers, either for engineering or for the Society? Were they unsuited to practice the profession? Was some other line of endeavor more remunerative or more attractive? Were they given opportunity to advance in the profession they had chosen, or was their progress made difficult by the lack of cooperation, of fellowship, or of understanding and assistance given them by those who had already attained reputation and recognition in the profession? Whatever the causes, it is the writer's belief that they must be sought out and subjected to searching analysis, because it is only on the basis of truth that the profession can hope to progress.

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[15] "Engineering Trends in California," by R. O. Thomas, *Civil Engineering*, July, 1940, p. 466.

To secure an admittedly approximate answer to the question of the composition in membership of those leaving the Society, the writer made a study of the membership as given in the ASCE *Yearbook* for 1945. A random 10% sampling was made which resulted in determining that 35.3% of the members listed (a total of 7,250) had been members of the Society in 1929. The 1929 membership was 13,315, from which it may be seen that 6,065 of the members of the Society in that year were no longer connected with the Society. Inasmuch as there were only 2,043 Juniors in 1929, of which a considerable number must have advanced to corporate grade; and, since there have been only 2,641 deaths to date, including both those who were members in 1929 and those joining since, it is readily apparent that a major proportion of those lost to the Society must of necessity have been corporate members. Why have these men, who had followed the profession of engineering for an appreciable period and who had advanced to responsible positions in the profession, left the Society? The answer to this question is vital. Even if these men have left the profession, both professional and sentimental instincts should have kept their membership alive and retained their interests. If they have not left the profession, why are they no longer members of the Society?

To establish some verification for the foregoing data from an independent source, the writer had recourse to the admirable chart prepared by Arthur Richards,[16] M. ASCE, from which Table 8 was prepared. The chart included the results of case studies of 5,000 civil engineers by the Society for the Promotion of Engineering Education; 9,000 mechanical engineers by the American Society of Mechanical Engineers; and 15,000 highway engineers—a total of 29,000 case histories. The writer has applied the diagram to the theoretical histories of groups of 100 engineers beginning practice in successive five-year periods. The data show that of 633 engineers of various ages, active in their profession, there will be only 272 after a lapse of

TABLE 8.—MORTALITY STATISTICS IN THE ENGINEERING PROFESSION

Age	Number in the profession	Number remaining after fifteen years	Loss
25	100	97	3
30	135	66	69
35	131	44	87
40	97	30	67
45	66	20	46
50	44	10	34
55	30	5	25
60	20	..	20
65	10	..	10
Total	633	272	361

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[16] "Vocational Guidance in Engineering Lines," Am. Assn. of Engrs., 1933, p. 160.

fifteen years, the remaining 361 having either left the profession or died. This represents a loss, over a fifteen-year period, of 57% of the original total.

The number of engineers under the age of 35 is given in Table 8 as 366, or 58% of the total practicing engineers. The large percentage of younger men, contrasting with the figure of 14.9% for the Juniors in the American Society of Civil Engineers in 1929, seems to be explained by the preponderance of highway engineers in the basic data, highway engineering being particularly attractive to younger men because of the opportunities for outdoor work. Table 9 shows a comparison of data as developed for the Society for the sixteen-year period, 1929 to 1944, and as computed from the Richards curves for a fifteen-year period.

TABLE 9.—Comparison of ASCE and Richards Mortality Data

Item	ASCE	Richards
Number:		
Engaged in practice of engineering....	13,315	633
Remaining after fifteen years.........	7,250	272
Leaving during fifteen years..........	6,065	361
Percentage:		
Under the age of 35 (ASCE = 32 yr)..	14.9	58.0
Of the original number that has been lost..........................	45.6	57.0

Again taking into consideration the fact that a large percentage of the case histories studied by Mr. Richards applied to highway engineers (predominantly younger men), and the assumption that engineers, taken as a whole, who are the type desiring affiliation with a professional society, are somewhat more likely to remain in their profession than are those who do not so affiliate themselves—there still remains an excellent degree of correlation between the two studies. It can probably be stated as a general principle, that between 45% and 50% of the men who train for and enter the profession of engineering leave the profession for another line of endeavor prior to the termination of their working lives, and as a further corollary (assuming that practically all who leave after the age of 45 are lost by death) it can be stated that they leave the profession in the first fifteen years.

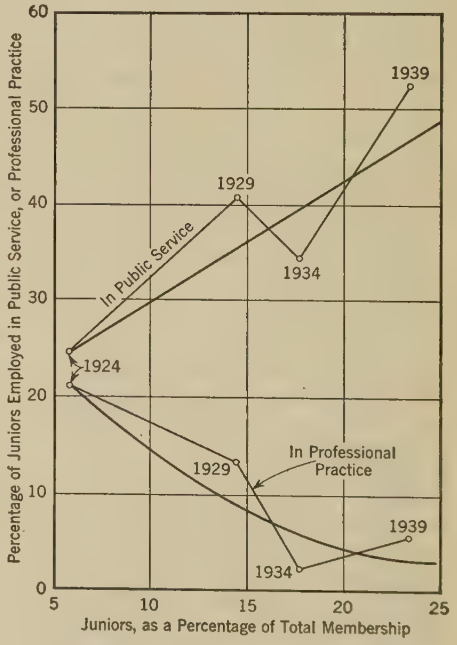

Fig. 2.—Growth of Percentage of Juniors, ASCE, in California, 1924–1939, and Concurrent Change in Employment Classification

This brings to the fore the subject of the loss of Juniors who find themselves unable to complete the requirements for transfer to the grade of Associate Member. This question has received deep and serious study by the Society's Board of Direction, and culminated in raising the age limit for Juniors to 35 in 1942. As time goes on, apparently, it is increasingly more difficult for Juniors to secure the necessary year in responsible charge of work, and this condition is reflected in the large number of Juniors dropped yearly because they have reached the age limit. This is explained by the large number of Juniors who enter the public service field, where the opportunities for promotion, and consequently for independent responsibility, are slow in coming. Fig. 2 shows graphically the trend of Juniors of the Society toward seeking employment with the large civil service engineering departments, and perhaps accounts in some measure for their failure to qualify for advancement in the Society. It may account, also, for a considerable number of the engineers lost to the profession, as the individual becomes disappointed in his chosen work when he finds the field to advancement and responsibility almost hopelessly blocked; and he turns to other and more attractive opportunities in other fields.

From the data presented herein the following questions are evident: Are young engineers being educated, or trained, properly? If so why are they (and also an appreciable portion of older engineers) leaving the profession, or the Society? If they are not being trained properly, what should be done to reorient engineering education? In the light of the demonstrated magnitude of the numbers of engineers who leave the profession, should the course be lengthened to include material from other major fields such as business administration, public administration, etc., to better prepare the candidate for an engineering degree not only to succeed in the practice of engineering but also in the pursuit of other occupations? Are opportunities in engineering, and the rewards concomitant with them, appropriate to the knowledge, ability, and responsibility involved? If not, what can be done to improve them? If they are, why is the nation losing the services of so many who were once engineers? The future of the civil engineering profession is acutely dependent on the solution to these and other interrelated questions.

CLEMENT C. WILLIAMS,[17] M. ASCE.—Gratitude is due the author for his able analysis of the character of the profession and for his collection of pertinent information concerning its composition. The paper discusses engineering education from the viewpoint of the practicing engineer, namely, a commodity purchasable on specifications. It is a viewpoint that widely prevailed prior to World War I. Many professors of civil engineering, with experience limited to T-square operations, pointed their students to the drafting rooms of the American Bridge Company; professors of electrical and mechanical engineering aimed their instruction chiefly toward the apprenticeships of the General Electric Company or the Westinghouse Company. The standard practices of those companies dominated the instruction of the senior year. Engineering then was confined to applied science and the accumulated art. With the expansion of research and of graduate preparation of faculty, the situation has changed; engineering has germinated its own investigative science.

Few educators of today (1946) consider the training of young men for the requirements of present practice as the gage of their objectives. The future careers are too fortuitous in detail to warrant planning with specificity. Engineering education is more a function of the native capacities of youth and of scientific discovery than of office requirements; instructional subject matter is derived more from researches than from practice. In this new era of education, engineering has made conspicuous advances.

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[17] Consultant in Eng. and Industrial Education, Madison, Wis.

Although all education has a social purpose, its social adaptation is not a primary concern of the college but is a matter for later determination. The author's analogy to steel billets and subsequent products is correct but somewhat remote. A closer, but still faulty, parallel would be that college attends to the growth of a tree so that it becomes strong and without defects, but leaves the question of what timbers may be cut from the log for later circumstances to decide. Engineering education consists in cultivating the "growth" of the faculties to observe, to think, and to express thoughts, and of the personal attributes essential to good citizenship and a full life. In mental discipline, it involves an acquaintance with the basic sciences and knowledge as thought tools and material, followed by exercises in thinking concerning supposititious engineering situations—all critically compared with the more mature and tested ideas of the teacher and the textbook. Exercises in thinking are as essential to mental development as physical exercises are to bodily improvement. The former are the crux of the learning process. The author seems to mistake these supposititious engineering exercises for an attempt to teach engineering practice. Of course, their value is enhanced by the closeness of their correspondence to actualities; and, if well chosen, they do impart a modicum of practical information. They also serve as a pedagogical device to stimulate interest, so indispensable to learning. They are of value in developing powers of ratiocination even if they relate to situations which never arise, or to situations entirely forgotten by the student, since these powers, if once ingrained, can be recalled from the subconscious mind although he may think them gone forever.

The author also seems to overestimate the maturity of student experience. What a student can absorb and assimilate depends on his previous experience. The concepts of a mature engineer cannot be grafted on to the tender experience of a youth who has had no observation of engineering work. A lecture beyond the student's clear grasp is futile, for "nothing learned, nothing taught" applies regardless of the profundity of the message.

Professions are not rigid arrangements of human services but are clusters of similar operations grouped more or less statistically according to the frequency of their occurrence. Mechanical skills fit into definite requirements, but engineering is expansible and adaptable to its social function. The sixty members of the writer's own class in civil engineering, as revealed by the annual class letter, have served society well; but relatively few of them at present are doing work closely related to the class instruction of the undergraduate years. Following the progress of hundreds of students who have passed under the writer's supervision brings the conviction that careers are affected by circumstances mainly coincidental, and that it is wholly conjectural whether any particular modification of their curricula would have improved their fortunes. Engineering education aims at a hypothetical service as a bull's-eye, but its results follow a scatter or probability pattern centered on the maximum of incidence.

It is not derogatory to engineering education, therefore, to find that those so prepared are distributed over a multitude of occupational needs in organized society. Indeed, since each particular activity cannot feasibly be accorded specialized training, this flexibility of engineering education is a social virtue. Education for the law presents a similar situation in that it prepares for a large number of ancillary vocations which contribute to successful society. In a city of 15,000, where the writer once lived, there were seventy-two lawyers—that is, persons who had been admitted to the bar—although not more than ten or a dozen practiced at court. The others were realtors, securities agents and abstracters; or were engaged in local politics, in business, or otherwise serving the community. The law touches human affairs at many points, just as applications of physical science do; hence, one may expect preparation for law, or for engineering, to be widely useful in organized society outside the strict confines of the respective professions. In medicine, by contrast, the preparation is more specific and is seldom applicable beyond health and hygiene; nevertheless, quite a few doctors become managers of business and civic leaders, as do many men who have had no higher education. From these circumstances, the inference may be drawn that specific training for unusual eventualities does not need to be included in the professional curriculum.

The author proposes that curricula provide a broader scope of basic instruction covering the entire field of engineering and leave advanced theory for graduate study. There were many protagonists of this idea a quarter of a century ago. A number of institutions, especially smaller colleges, adopted the plan of having three years for all departments in a common curriculum with departmental differentiation in the senior year. The University of Iowa, Iowa City, adopted this scheme. The writer became dean of engineering at that university in 1926 and witnessed the change back to the usual distribution of subject matter. Some of the reasons for the reversal were (1) the graduates did not make a favorable showing when they entered apprenticeship courses; (2) the classes could not use standard textbooks; and (3) most of the faculty were dissatisfied with the unstimulating elementary instruction involved. Some of the small colleges still continue the plan. Their alumni cannot enter graduate schools directly; nor are they so acceptable to industry; and their curricula are generally not accredited by the Engineers' Council for Professional Development (ECPD). Many engineering colleges have a "general engineering" curriculum which includes about equal parts of all branches, but it has not proved superior in vocational adjustment.

A longer period than four years in the engineering college may profitably be spent by many students, either (1) in pursuing the humanities or the social studies, (2) in the pure sciences and mathematics, or (3) in the technical subjects of engineering curricula other than the student's major. Many students so continued their education during the depression years when jobs were not available, and, generally, they found the additional study profitable. However, the length of an engineering education, like other engineering undertakings, has economic limitations, which, for the present scale of compensation, seems to be four years. The students of superior ability find postgraduate study, even through the doctorate, advantageous. For students of the lower half of the class, graduate study, so far as the writer has observed, seldom advances the engineer's professional success.

As a matter of fact, the preparation of engineering students is more general than most industrial personnel directors and practicing engineers are aware. Students have some instruction in all departments, and their training in nonmajor branches is preponderantly the same as if they pursued a common curriculum. The specialization is attained by shrinking slightly the time to each nonmajor branch and devoting the total increment thus gained to the major. Personnel directors overestimate the significance of this specialization. Seeking a mechanical engineer for quite general work in a factory, for example, they will not consider a civil, mining, or electrical engineering graduate even though he be in the upper percentiles of the class. The evils of departmental specialization lie more in the departmentalized recruiting practices of industry than in the educational results.

All educational programs should aim at the development of the whole man, whether the intellectual part of the curriculum pertains to engineering, law, medicine, social studies, or the humanities. In this respect, all education should be liberal—that is, directed to the whole free man. It should not be limited to the intellectual realm, but should include the emotional, the volitional, the social, the spiritual, and the physical. The latter qualities are little affected by the particular subject matter in the courses studied and should be given separate consideration. Narrow specialization relative to today's civilization is frequently found in the humanities as well as in the sciences. Any feasible inclusion of the humanities in the engineering curriculum will fall far short of the postcollege reading and reflection requisite of the cultured man. If that small inclusion initiates a taste and an inclination, it will be justified.

Although all of these capacities are educable, most college study is directed solely to the intellectual growth, with some attention paid to the physical. Effective pedagogic processes have not been devised for the other categories of attributes of the whole man. The older British universities do a somewhat better job than do the American; and, as a member of the Board of Visitors at the U. S. Naval Academy, the writer has been impressed with the accomplishments of that institution in the development of these personal qualities. To a considerable degree, the nurture of these qualities is subconscious, the product of environmental influences and institutional traditions. The neglect of these disciplines constitutes the chief defect, not only of engineering education, but of all American higher education.

As the author so well points out, postcollege engineering education should have attention. It occurs subsequent to the impressionability of youth and when the outlines of careers are becoming more discernible. It differs fundamentally from graduate study, which is aimed primarily at research methodology and investigation. Inasmuch as it pertains to the profession, perhaps the ECPD might undertake to formulate a suitable program.

The recent reorganization and amplification of the American Society for Engineering Education (formerly "Society for the Promotion of Engineering

Education") is significant of an enlarging grasp of engineering education. The appearance of a challenging paper by a practicing engineer, in the *Proceedings* of the oldest of the Founder Societies, is a portent in the same direction. Coupled with the expanding universe of engineering as a result of recent scientific advances, fresh attention to education may be epochally significant to the profession.

N. W. Dougherty,[18] M. ASCE.—From the practicing engineer's point of view the paper is a good presentation, but it overlooks many of the changes in engineering education that have taken place during the recent past. In the "Introduction" the author suggests that:

> "It [engineering education] has not, however, made comparable progress in keeping abreast of changed conditions that have developed during that period in the pattern and structure of the engineering profession, in the practice of engineering, or in the market for engineering services."

Some of the subjects omitted probably belong to the postgraduate years rather than to college courses. Education is an experience rather than an accumulation of information; it should be inspirational and continuing rather than complete in a four-year program.

All the colleges have been guilty of neglect in their continuing activity after graduation. On the other hand, if they do their in-college instruction in an inspirational way there will be less need of continuing instruction. Why should educated men continue to be spoon-fed after they enter the world of work and application? There are many things the colleges can do to help; but all they can do cannot make educated men if the students do not want to pay the price of education.

Again, quoting the last sentences of the "Introduction": "Every engineering student must work hard to graduate. This probably was the most valuable single thing that he learned in his course." Learning to work is valuable but learning to work effectively is more valuable. If the student has not learned this lesson he probably has not learned much—learned to work with books, the printed page, and rows of books; learned to glean a magazine page and sift the wheat from the chaff; learned to trace cause to effect; learned to respect law but not worship it; and learned to continue in his search for truth.

Prewar Engineering Education.—There is so much difference in the colleges that it is practically impossible to appraise what has, and what has not, been done. When the Engineers' Council for Professional Development (E.C.P.D.) has accredited fifteen different curricula and a host of options, breaking down the fifteen into more restricted fields, one can truthfully say that there has been too much specialization. The author sums up this practice by the following statement:

> "The general objective * * * appeared too often to produce a graduate well versed in theory of, and with an academic knowledge of, practice in one particular branch, and in many instances in one specialty in the field

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[18] Dean of Eng., The Univ. of Tennessee, Knoxville, Tenn.

> of engineering, thus equipping the graduate to enter such field or specialty on graduation and encouraging him to do so, irrespective of what the market for services in such field might be in five or ten years."

Whenever a student enters any program of study having a definite objective in mind he will try to find employment in that field on graduation although employment is scarce in the field. It is fallacious to believe, however, that the college training in engineering in any field restricts the trainee to employment in that special field. Too many who were trained as mechanical engineers are practicing electrical engineering, or electrical engineers are practicing mechanical engineering or civil engineering, to make the statement that training in a field ties the student to that field. The author draws the conclusion from his premises that either there must be a broad general course, or there must be applications to practice in a single branch or specialty. There are equal possibilities in a broad general course in civil engineering, for example, or specialization in the various fields of civil engineering.

Educators have a notion that they should give enough technology to transmit the attitude, method, and approach to the solution of problems which depend on broad fundamental laws of physics, chemistry, mechanics, and hydraulics. The method and approach can best be acquired by solving actual problems of engineering practice. True, they must be miniature and often from assumed data to illustrate a principle, but such problems do give an introduction to the engineering approach. To many educators the field of specialization is not so important as the method and the problems.

There is educational value, of course, in sequence courses. The emphasis of engineering schools is not necessarily on specialization but rather on the application of engineering theory to some engineering problem. If the problems come in an area of later employment there will be advantages; and should this not happen no harm has been done.

Except for the idea of "the field of general engineering," the author's course, as outlined, is in general agreement with the recommendations[19,20] of the American Society for Engineering Education—formerly, Society for the Promotion of Engineering Education (S.P.E.E.). Both have the same general objectives, namely, a well-rounded engineer rather than a skilled technician.

The author points to a well-known problem in engineering education:

> "He grappled with problems of a character that he would never meet, or at least would not have to solve, on his own responsibility for from at least twenty to twenty-five years after graduation."

This may be very true; but, if the teaching is to achieve method, attitude, approach, and confidence, no great harm has been done if the student never meets the problem again. For years engineering educators have recognized the problem of dissatisfaction and discouragement of students during the first five to eight years out of college. There is no question that much of the difficulty stems from the failure to use their advanced knowledge in immediate

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[19] "Aims and Scope of Engineering Curricula," *Proceedings*, S.P.E.E., Vol. XLVII, 1940, p. 555.

[20] "Report of the Committee on Engineering Education After the War," *ibid.*, Vol. LII, 1945, p. 36.

practice. Judgment should come with experience but not all experiences develop judgment. A part of the remedy is to produce understanding employers along with more understanding instruction.

The writer has attended many meetings with practicing engineers where attempts were made to discover "what is wrong with the colleges" and always has heard the statement: "As a graduate he seldom had the ability to express himself adequately or a knowledge of how to acquire this ability."

This seems to be a universal trait. Engineers are about the worst on the list, but it has been discovered in preachers, lawyers, doctors, dentists, and English professors. Good expression seems so easy that all are deceived by it. The practitioner, who cannot express himself, tends to blame some professor for his lack of proficiency. Good English is "a gift of the gods" and the result of real effort to acquire ability. Much of its lack is the result of having nothing to say or of having very hazy ideas. Good expression and clear thinking "go hand-in-hand."

The analogy of the steel mill is a little overdrawn but it does emphasize a well-known educational principle: Men are not materials, and educational experience will not make them mainsprings or auto bodies, but it may make it possible for them to be either. All men act as individuals; some of them are intelligent enough and self-reliant enough to succeed, whereas others are perfectly willing to follow the leader. No educational process has been developed to make all of them become leaders.

The Engineering Profession.—The author's study of the engineering profession is thought provoking; it points a way to a better understanding of the task of the colleges. More studies of this kind will be very helpful. What changes of definition have occurred during the forty years of census reporting? From where have the additions come? The author's description of the engineer is probably more nearly true to type than Saint Luke's definition of the Athenians: "That they spend their time in nothing except to hear and tell some new thing." Quoting the paper (see heading, "The Engineering Profession: General Traits and Characteristics of Engineers"):

> "On the credit side it may be stated that engineers in general are resourceful, hardworking, ingenious, and persistent, and have a great devotion to their work. On the debit side they are extremely narrow in their interests and outlook, and highly individualistic."

When engineering is considered "as a heterogeneous rather than a homogeneous profession" engineers are likely to fall into error when they attempt too much generalization regarding the characteristics of their profession; yet the writer believes that Mr. Baker has done well in the foregoing two sentences. By training, and no doubt by birth, the engineer is individualistic. He is not the "back patter" and the "glad hander" of some other professions. Part of that is due to his training and part to his very nature; his disposition caused him to study and begin the practice of engineering. Considerable change will be required to make over the introvert who goes into engineering into the extrovert salesman who disposes of the products of the assembly line. As a matter of fact both types are needed in industry and both types can profit by training in science and technology.

Table 3 is very interesting. A census reported in 1966 will probably raise the percentage of practitioners who enter engineering through the four-year college course. Engineering is now where law and medicine were at the beginning of the present century. Probably it should not aspire to the high percentage of college practitioners now required of law and medicine. The field is far more varied and gives much more opportunity for great variations in innate talent.

Fig. 1, giving a cross section of the engineering profession, is more than interesting. If it is at all to scale, it points to an evolution which is far more revolutionary than the preceding data suggest. The ordinate probably bears some relation to age or time spent in the profession. The author would do the profession a genuine service if he tried to make a similar diagram to scale, using professional age as the ordinate.

Postwar Engineering Education.—The changes suggested under this heading are those usually made by engineering educators as well as by those who practice engineering. Future changes will not be as drastic as those suggested because the curricula are rarely as specialized as the author assumes. More emphasis will be placed on the professional aspects of engineering—the broad bases of engineering practice—and less emphasis will be placed on specialized skills. For example, the following quotation expresses many of the ideas of the author as he describes his postwar engineering education:

> "For the scientific-technological stem, acquirement by the student of mastery of the basic principles, assumptions, empiricisms, and codes of practice which constitute the subject matter of engineering study, accompanied by acquisition of the ability to apply them to problems of practice which constitute the engineering method.
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> "For the humanistic-social stem, development of the ability to read, write, and speak the English language effectively; to understand, analyze, and express the essentials of an economic, social, or humanistic situation or problem; and to appreciate its implications and relationship to the life and work of an engineer. There is also a goal of development of an adequate concept of the duties of citizenship in a democratic society; and acquaintanceship with the enduring ideas and aspirations which men have evolved as guides to ethical and moral values; and an appreciation of cultural interests lying outside the field of engineering."[20]

The engineering profession owes a debt of gratitude to the author for this thought-provoking paper. For their part, the colleges have already stated what they think should be done in engineering education after the war.[20]

Members of the Society should see to it that civil engineering educators do not fall into some of the snares that have obstructed their paths in the early development of education for the civil engineer.

H. A. WAGNER,[21] ESQ.—Like Mr. Baker, the writer has become convinced that the entire plan of engineering education should be revamped to meet present problems. The schools should perform three important functions:

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[21] Cons. Mining and Metallurgical Engr., Chicago, Ill.

Train engineers in the scientific approach to the solution of problems; give them a foundation in subjects that are fundamental in business and industry; and provide a broader scope of instruction in subjects that cover the entire field of engineering, insuring a thorough foundation in the basic fundamentals underlying all branches of engineering.

These ends cannot be achieved by crowding additional subjects and disciplines into the present four-year course. The training recommended by Mr. Baker can be given only by eliminating from the curricula certain courses that develop elementary skills and proficiency in the use of scientific tools and instruments, and by deferring to postgraduate study those courses in advanced theory and practice, highly specialized, which now monopolize the time of students in their junior and senior years. The writer concurs in the recommendation that the schools develop strong, postgraduate extension courses that will enable engineers to specialize after they have discovered their own aptitudes and opportunities—after they have learned from experience the conditions that govern practice and the market for engineering services.

To the writer, as to Mr. Baker, this seems the auspicious time to consider drastic revision of engineering education, while curricula are still plastic due to the necessity of adapting engineering training to the exigencies of the war and its aftermath. Mr. Baker is not at all didactic in his specific recommendations for curriculum revision. He is definite and urgent in his plea that the schools and the societies conduct an extensive and thorough investigation into the composition and structure of the engineering profession, the conditions that govern and prevail in actual practice, and the market for engineering services—investigation that will provide authoritative guidance for those who may be charged with the task of revising curricula.

The very crux of the entire problem is graphically represented in Fig. 1, depicting the composition and structure of the profession. No one can appraise current curricula intelligently, much less undertake to revise them, until there is a clear understanding of these factors. A thorough investigation of the composition and structure of the engineering profession is an indispensable preliminary to curriculum revision.

After such an investigation has been made, engineering educators can properly evaluate Mr. Baker's specific recommendations for curriculum changes. It may be, however, that certain evolutionary processes, already apparent, will take the situation out of the hands of the profession and its educators. Certain developments are in evidence that already indicate a movement toward the three-pronged system of training which evolved in Germany before World War I.

This system includes three types of training. There is, first, the "Gewerbeschule," which corresponds roughly to the "technical high schools" in the United States. It equips students to use engineering tools and precision instruments, and develops rudimentary skills which are adjuncts to, rather than a part of, engineering. It is quite probable that these schools will be supplemented by union "trade schools" which, under the sponsorship of white collar unions, will serve the same purpose. They will offer only "pre-engineering" instruction.

Then there is the "Technische Mittelschule," which, in its curriculum and discipline, is equipped to prepare students to assume duties more closely related to, or actually a part of, professional engineering. This is done under the close supervision of those who are prepared to direct or perform creative work, requiring exercise of independent judgment. These intermediate schools will produce the indispensable "routineers."

A third type of school produces the fully professional engineer. This is the "Technische Hochschule," which confers state diplomas and doctors' degrees in engineering. Its graduates are masters of the fundamentals in all branches of engineering when they receive their diplomas; they are specialists, when they achieve a doctor's degree.

The present system of engineering education in America is expected to serve the needs of all these groups. Students who will spend their lives in the "subprofessional" bracket, performing only mechanical operations that are adjuncts to, rather than a part of, engineering, and those who will become indispensable routineers, pursue the same studies, submit to the same discipline that develops engineers capable of creative work, involving exercise of independent judgment. If all these groups are considered members of the engineering profession, the schools must continue to serve them all.

This is the problem posed by Fig. 1—a problem suggested, however, rather than stated. Mr. Baker has not the temerity to do what the engineering profession has neglected to do in the past fifty years—define its own scope.

There are four important segments in Fig. 1. It is fruitless to investigate the indicated composition and structure of the engineering profession, unless such an investigation leads to a clear decision, a resolution of the enigma, "What is the engineering profession?" and a question that has long been evaded: "What is its scope?" Eventually engineers must decide "Who are members of that profession?" What is learned from this investigation should enable the engineer to define and delimit the profession; to draw clear lines of demarcation between real professional engineering and the fringe of arts, crafts, and trades which now blend indistinguishably with it.

The profession must do this because it must decide how many of the segments represented in Fig. 1 require genuine engineering education. In accordance with that decision, the responsibility of the schools will be limited and curriculum revision must be controlled. Schools of engineering must know who shall be educated and for what capacities and functions students are to be "processed" before they can plan curricula intelligently.

Mr. Baker has stated plainly that the schools and the professional societies should investigate the composition and structure of the profession. He is not to be censured for withholding his own opinion as to the scope of professional engineering. He intimates that those who have "left engineering" are no longer members of the profession, by the device of a broken line, marking the outer boundary of that segment in Fig. 1. If he had felt strongly that those engaged in "subprofessional" work were not members of the profession, he might easily have suggested it by a similar device—a heavy line of separation or some other graphic representation of detachment. Mr. Baker did not allow himself to be diverted from his major thesis by a discussion of this highly controversial issue. He did unreservedly recommend clarification of the point, urging educators and societies to investigate the composition and structure of the profession.

There are many conceptions of "professional status" and of the "scope of the profession" and each has a substantial number of stout adherents. First, there are those who believe that a degree in engineering is unimpeachable evidence of professional status, of membership in the profession. Second, there are those who believe that, regardless of academic qualifications, performance of engineering duties is prima facie evidence of membership in the profession. Third, there is the contention of registered engineers that the engineering profession consists exclusively of those who, by registration, have a statutory right to use the title, "professional engineer." Another faction, more realistic, discounts the statutory restriction of the title as a legal technicality; and, in support of this attitude, cites the vast numbers of eminent engineers who, although qualified, have never chosen to register because the oddly constructed license laws in the United States make it unnecessary to do so. Many argue that by membership grades the engineering societies describe the extent of real professional engineering and designate "professional status." Most prevalent of all, perhaps, is the notion that professionalism is not an absolute, sharply definable state; it is held that there are gradations, and that the measure of "professionalism" is the degree of eminence.

Not one of these notions provides guidance for educators if they attempt to put in effect the curriculum changes recommended by Mr. Baker. The schools have attempted to reconcile all these ideas, and to serve the needs of a profession that is practically undefined.

The theory is advanced that those who eventually enter the segment in Fig. 1 inscribed as "Number engaged in technical engineering work primarily," and that designated "Number engaged in administrative and executive work primarily," must earn their spurs in "subprofessional work." If the profession were to embrace this theory, the schools, regarding the subprofessional group as members of the profession, must continue to offer courses that prepare potentially professional engineers for work involving tracing, drafting, and the use of precision instruments. Graduates in the subprofessional segment will work side by side with men whose training for such duties has been entirely practical, or has been obtained in technical high schools and trade schools.

If this group is not to be regarded as a part of the profession the time now allotted to such courses can be devoted to engineering fundamentals.

Other professions, sharply differentiating professional from nonprofessional practice, train technicians apart from future professionals. Bacteriologists, nurses, laboratory technicians, and other affiliates of medicine prepare for their jobs by taking courses much less rigorous than those required for the practice of medicine. It is impossible for bacteriologists, however brilliant their contribution to medical science, to qualify as physicians without taking the courses required for a doctor's degree in medicine. The line of demarcation between "professionals" and "nonprofessionals" is sharply drawn in licensure; the training of professionals and technicians is as plainly differentiated.

If and when the engineering profession sharply defines its scope, sets up standards of membership as clear and inflexible as those statutorily established by the profession of medicine, engineering education can follow medicine's example also in eliminating courses that merely develop skills used in preprofessional capacities. When this happens, the pre-engineering group, indicated in Fig. 1 as "subprofessional," will be quite definitely excluded from the profession, and it will no longer be the responsibility of the engineering schools. Other types of schools will prepare "subprofessionals" for pre-engineering functions. Whether this group will be excluded, or will secede, is a matter about which the writer will not make any predictions.

If this change comes, it may not be on the initiative of the profession. Professional status is now being defined by governmental agencies—not one, but many agencies. The Civil Service Commission, in its classification of nonprofessional, subprofessional, preprofessional, and professional employees is assuming, but not usurping, a prerogative which engineering has never exercised, drawing distinctions as it sees fit between professional and nonprofessional engineers. The United States Employment Service is preparing a series of booklets for the guidance of its placement counselors, describing the characteristics, qualifications, and duties of engineers serving in professional capacities. The Fair Labor Standards Act (F.L.S.A.) Administrator, after futile efforts to distinguish "professional status" by academic definition, finally resorted to the rule of thumb that, for the purposes of F.L.S.A., any engineer earning less than $200 a month in base pay is nonprofessional. Case by case, the National Labor Relations Board (N.L.R.B.) distinguishes "professional" from "nonprofessional" engineers. Nothing has as yet crystallized from its many decisions definite enough to be called a line of demarcation; but a pattern is slowly emerging. The Board's line of reasoning can be traced in the following interesting decisions: Aluminum Company of America (61 N.L.R.B. 180); Bethlehem Steel Company, Shipbuilding Division (Case No. 20-R-1256); Gielow, Incorporated (Case No. 2-R-5290); LaClede Steel Company (Case No. 14-R-1148); Neches Butane Products Company (Case No. 16-R-1159); Packard Motor Car Company (Case No. 7-R-1809); Stone and Webster Engineering Corporation (Case No. 1-R-2139); and Curtiss-Wright Corporation (Case No. 9-R-1738). The Bethlehem Steel Company case is particularly interesting.

Another and even more potent force is the appeal of the labor unions who argue that "professionalism" is a fetish, and who assert that unions can do more to improve the social and economic status of subprofessional (and professional) engineers than can any professional or technical society. They scoff at the tradition that engineers are individualists and cannot advance if any restraints are imposed on professional freedom. They insist that strong, collective action insures better pay, improved working conditions, and satisfactory recognition; and point to the relatively high wages of skilled and unskilled labor, in comparison with subprofessional, or even professional, salary schedules, to support the argument. They discount the theory that engineers advance more rapidly when they are free to exercise individual initiative, capitalize on native ability and aptitudes, and bargain directly with the employer.

In any investigation of the composition and structure of the engineering profession these conflicting theories should be tested. If engineering is thought of as a pyramidal structure (and Fig. 1 indicates that it is) there is less room at the top than there is in trades, which are more like cubes made up of a myriad of almost identical units. The engineering pyramid is irregularly stratified; and the individuals of which it is made differ greatly in ability, training, initiative, industry, and those temperamental qualities which so potently affect careers. There is also no inflexible relation between these good qualities and advancement from grade to grade. Opportunities—external factors—influence the progress of individuals from subprofessional to exalted technical rank. A large number of engineering graduates, regardless of their qualities, are destined to remain routineers simply because there is not enough room at the top for all of them.

Some engineers are beginning to be cynical about "professional status." Despite their degrees, graduates spend years in a kind of no-man's-land—not members of any trade, yet denied professional status by their own license laws. Appealing to engineers, disowned by the profession, the unions have convinced a great many of them that "professional status" is a myth, and "professional ethics" a fetish.

There are more practical objections which the unions find it harder to dismiss. Engineers have become identified with too many different unions. They change jobs frequently, and a new position may entail a complete change in union affiliation. It is not easy, therefore, to convince engineers that it is worthwhile to build a strong union—to pay substantial dues, to demand clauses in union contracts which strengthen the union itself and fight for these more resolutely than for clauses which insure direct and immediate benefits to the members. Although the craft and industrial unions are still absorbing subprofessional engineers (and even preprofessional and professional engineers), the major unions have recognized the disadvantage of this fragmentation, and they are building "white collar unions" in which, apparently, they hope to integrate clerical and "professional" workers.

Those who hope to retain these subprofessionals and routineers as members of the profession, must study the structure of the profession. If the chances of reaching the top are limited, some way must be discovered to offset this disadvantage and to insure pay for the subprofessionals commensurate with their training and services. Their salary schedules must bear such relation to wages of skilled and unskilled labor that they will not find it necessary to identify themselves with these groups in order to protect their interests.

On the other hand, if the profession were to renounce its claim to subprofessionals, eliminating from the engineering curriculum those courses which qualify students to perform subprofessional duties, it must be with full understanding of the effect of that decision.

First, the unions can be expected to set up "trade schools" and systems of apprenticeship such as they now maintain for operating engineers, for plumbers and decorators, and other trades. There will be a corresponding decrease in the number of students who enter engineering schools. Engineers cannot expect to continue half trade and half semiprofessional. If a large proportion of subprofessionals obtain their training in technical high schools and union trade schools, then by collective action establish high rates of pay, and through seniority clauses insure continuity of employment and automatic promotions, four-year courses leading to similar jobs will be less and less attractive.

Second, if the unions, rather than the professions, decide what constitutes "professional status"—if they draw the engineer's boundary lines for him—it may not be subprofessionals alone who will be absorbed. Just above the subprofessionals who perform mechanical operations subsidiary to, but not a part of, engineering, there are the "preprofessionals." These men, under close supervision, implement the plans of professional engineers. They need greater knowledge of engineering fundamentals; they participate intelligently in creative work; but they do not exercise independent judgment and, as engineering becomes more and more standardized, their jobs are correspondingly more and more routine. There is no assurance that these men will not also secede from the profession. If the unions gain control of "preprofessional" as well as subprofessional engineers, the usual routes by which graduates progress to "responsible charge" will be closed to those who are reluctant to join unions. Perhaps some institution, like the intermediate schools in Germany, will develop to equip "preprofessionals" with the knowledge of engineering fundamentals required in these routine jobs, like the short courses offered in the United States during World War II.

Eventually there will be left to the engineering schools only those students who have demonstrated a capacity for creative work or administrative service. Relatively few institutions of higher learning would be needed to "process" the number of engineers of this category that industry can absorb. It would be necessary for industry to accept the product of these schools as trainees, or understudies, instead of developing them gradually in subprofessional and preprofessional capacities.

The effect of this shrinkage upon engineering societies and upon employer-employee relations must be given careful consideration. If, by professional definition or by their own secession, a preponderance of subprofessional and preprofessional engineers are definitely separated from the profession, the structure of the societies will be changed as much as the structure of the profession. Engineers will not support both unions and societies. As long as they have a tenuous claim on professional status—indicated by such evasive designations as "preprofessional" engineers, or "engineers-in-training" or "engineering aides" —they may remain loyal. If through any act of the profession, or on their own volition, they become definitely recognized as nonmembers—outsiders—they can hardly be expected to remain in the societies.

Certainly the relation between engineer-employers and engineer-employees will be changed if a great majority of employee engineers in subprofessional and preprofessional ranks organize in the pattern of, and identify themselves with, skilled and unskilled labor. These organizations will be in a position to control the "processing" of men who will join their ranks just as they now control apprenticeship and training for the trades.

To the writer it seems Mr. Baker's plea for a thorough investigation, by engineering educators and technical societies, of the composition and structure of the engineering profession should be heeded. It should be acted upon before other agencies have decided, for the profession, that crucial question—"Who shall be members of the engineering profession?" This is the essence of professional freedom. It is a right, cherished by all other professions, which is given to a professional man in exchange for the service he renders to society, in order that he may not be impeded in such service. If it is not too late, the writer would strongly advise the profession to follow Mr. Baker's urgent recommendation. The present composition and structure of the engineering profession should be investigated thoroughly, and standards of qualification for membership should be redefined—new boundary lines for regulating entrance to the profession. Then let educators plan to "process" engineers so defined in engineering schools for utmost usefulness to industry and society.

M. E. McIver.[22]—Mr. Baker has urged that engineering educators and the technical societies examine carefully: (1) The composition and structure of the engineering profession, (2) the conditions encountered by engineering graduates in actual practice of engineering, and (3) the factors that govern the marketability of their services. He has recommended that the societies utilize the vast amount of authentic source material accumulated in their own files—the case histories of members, and the records of the employment service jointly sponsored by the societies.

Even now the Engineers Joint Council (EJC) is engaged in investigations along the lines suggested by Mr. Baker. The EJC, however, seems to be relying on the questionnaire method of obtaining data, instead of using the treasure-trove of facts stored in the files of the societies represented on the EJC.

American Association of Engineers (AAE) has tested the technique of investigation recommended by Mr. Baker and found it far more satisfactory than the questionnaire method of collecting data pertinent to the social and economic problems of the profession.

The disadvantages of the questionnaire method are:

1. Abuse of the device by commercial agencies has reduced its effectiveness. The promotional purpose of such agencies is thinly disguised by the pseudo-scientific veneer of their inquiries;
2. Employed engineers, unless they have been markedly successful, are reluctant to chronicle their careers for their societies and alma maters;
3. In answering questionnaires, employers and employees are inclined to report facts in a manner that reflects credit on themselves and their organizations; and
4. Sheer negligence and procrastination account, in part, for the necessity of repeated follow-ups to secure an adequate volume of replies.

For all these reasons, AAE has found the questionnaire method a slow, costly, energy-consuming process of collecting information. Its committees are inclined, moreover, to discount the accuracy of voluntary replies to inquiries that individuals may consider unwarrantably personal or intrusive. Whenever this device is necessary the questions are composed in such a way that the individual receiving the questionnaire will be intrigued or "needled" into replying.

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[22] National Secretary, Am. Assn. of Engrs., Chicago, Ill.

Sources of Authentic Data.—The AAE employment service is its most dependable source of information concerning the social and economic status of engineers, the conditions that prevail in actual practice, and the market for engineering services. It is also the most vital contact with employers of engineers. Case histories of applicants, placement records, job orders, etc., are assembled by the employment service as routine; and AAE committees thus have more accurate information than could be obtained by a questionnaire. All their energies can be expended on organization and interpretation of data, because this system eliminates the arduous preliminary effort of collecting information by questionnaire. Fundamental steps in the procedure recommended by Mr. Baker are demonstrated in an AAE study published in June, 1946.[23]

Corroboration of Evidence in Mr. Baker's Paper.—Nowhere in Mr. Baker's paper is there any suggestion of authoritarianism. The evidence he has introduced in support of his convictions for the revision of engineering curricula is offered, not as conclusive proof of these recommendations, but to demonstrate the need of a more thorough investigation of the conditions indicated by these data and to suggest desirable lines of investigation. Various studies made by AAE committees support Mr. Baker's basic recommendations and corroborate evidence that he has cited.[24,25,26,27] Like Mr. Baker, the writer is prepared to demonstrate the absolute truth of these propositions. Nevertheless, the findings from investigations, although limited in scope, are interestingly parallel to, or supplement, the facts and findings recorded by the author.

Overspecialization.—From engineers and from their employers, the writer has gained the impression that engineering education is overspecialized. Employees exhibit a certain timidity that is attributable to overspecialization. It engenders a reluctance to accept engineering assignments outside a very restricted technical area in which they have specialized.

Electrical engineers, for example, thoroughly grounded in the theory of fractional horsepower motors, consider themselves competent to handle only the specialized phase of design of household appliances. They refuse to accept responsibility in the production of these devices because they lack knowledge of mechanical engineering principles, or the metallurgy involved in a production assignment.

In their job orders, manufacturers of electrical appliances manifest the same skepticism; they demand "mechanical engineers with experience in electrical lines," instead of electrical engineers for production jobs. Employers have greater confidence in specialized experience than in specialized education.

These are impressions—some exploratory work has been done by AAE to test the theory, but, as yet, nothing is conclusive. A study of the effect of specialization on the careers of ex-service men reveals that they fall into four definable units.

First, there are the men whose technological education was derived exclusively from special courses given by some branch of the armed forces, and who expect to continue this line of work in civilian employment. Second, there are the men whose academic training was interrupted by induction into the armed forces. The third unit is made up of men who, although they had completed four-year courses and received degrees in some branch of engineering, were inducted before they had sufficient experience to qualify for a position in responsible charge of work. Finally, there is the group that, having attained full professional stature, entered the armed forces as technical specialists.

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[23] "Before and After VJ-Day," *Professional Engineer*, June, 1946, p. 5.

[24] *Professional Engineer*, November, 1940, p. 35.

[25] "The Engineer as His Employer Sees Him," *ibid.*, February, 1928, p. 5.

[26] "Nation-Wide Survey of Engineering Civic Status," *ibid.*, December, 1939, p. 2.

[27] "Survey of the Market for Engineering Services," *ibid.*, November, 1940, p. 3.

Taking care to secure evidence from all four types of ex-service men, AAE asks the subjects to answer a questionnaire, which supplements their experience records, and which is obviously designed to aid the placement counselors in their special problems. Unobtrusively, a series of questions is woven into this questionnaire which, considered in relation to each other and to the applicant's complete vocational history, will provide a body of interesting facts concerning the effect of academic specialization on engineering careers.

The questionnaire asks, not only about the degree of specialization, but about the circumstances that prompted the candidate to concentrate upon a special phase of engineering in college. The relation of his subsequent experience to this specialization is investigated, measuring the well-known effect of the economic depression on candidates for employment who during the 1930's took any available job, regardless of specialization. The aim is also to measure the effect of rapid technological change during the 1940's upon the employability of highly specialized graduates who have not been engaged in engineering during the war. Finally, the objective is to ascertain the effect of wartime experience on the career plans of men who, if the war had not interrupted those plans, would have followed a very narrow path of specialization.

This line of investigation is one that the EJC might profitably pursue, expanding the exploratory study begun by AAE into a thorough, full-scale investigation. For obvious reasons, ex-service men (representing all four groups) are the best possible subjects for a study of specialization. Questions that concern the subject personally are emphasized in the AAE questionnaire: Rapid changes in technology during his period of military service that may, in some degree, have outmoded his academic training or experience; "GI" rights to reemployment and educational subsidies; salary modifications that take into account his maturity and wartime experience. This illustrates the technique for overcoming the inertia, reluctance, or actual hostility that makes the questionnaire method generally unsatisfactory.

Some of the early replies to the AAE questionnaire support incidental findings of other committees. Several of the subjects have shown that they were strongly influenced by parents or faculty advisers in choosing specialties, and that they are about to "write off" that theoretical asset as a loss. They ascribe various reasons for changing career plans. Some of them find a particular line of specialization not as lucrative as they had hoped; some report the field overcrowded; others complain that the specialty entails confining work; a few have come to regard the line of work for which they prepared as "socially unimportant" in comparison with other projects with which they were associated in wartime.

The most casual examination of these documents reveals that these men confirm the opinions of employers of engineers that were reported by AAE in 1928.[25] Employers intimated that, under a system of academic specialization, young engineers have made this crucial choice when they lacked maturity to insure good judgment; they made it, moreover, without adequate knowledge of the conditions governing practice, or of the market for such services. Employers declared, rather than intimated, that the product of engineering schools was precocious in its knowledge of advanced engineering theory but deficient in appreciation of the business side of engineering, and that overspecialization was achieved at the cost of essential training in engineering economics.

At the same time, employers noted that a considerable proportion of students' time was spent in acquiring elementary skills and learning to use engineering or scientific tools and instruments; this, also, was at the expense of sound training in fundamentals throughout the broad field of engineering. This condition, employers reported, accounted for the low rates of compensation received by a large segment of engineering graduates, who, using only these elementary skills and tools, compete with subprofessionals whose competence is based entirely on practical rather than academic training. The 1940 "Survey of the Market for Engineering Services" to which Mr. Baker has referred,[24] reports evidence in "job orders" which corroborates the testimony of employers quoted in the 1928 report.[25]

It is only undergraduate specialization, obtained at the expense of broader training in engineering fundamentals, that is decried by employers. Engineers need these advanced courses after they have discovered their shortcomings in actual practice, or when opportunity for advancement depends on additional study. They are insatiable in their quest for scientific knowledge; it is this fact that explains the multiplicity of technical societies, and the sectional and cross-sectional pattern of organization within the societies. Mr. Baker might have argued that engineers, in this pattern of organization, demonstrate a potential responsiveness to systematic extension courses that may be offered by the schools. Undergraduate specialization is highly opportunistic. Postgraduate specialization is purposeful. If the schools offered well-planned, practical extension courses, engineers who have already embarked upon careers, who know the market for certain specialties, and who recognize their own limitations, should certainly be expected to avail themselves of such facilities quite as eagerly as they now seek advanced training in theory and practice from technical society meetings and technical society journals.

Mr. Baker's Analysis of the Engineering Profession.—No psychological tests and no scientific methods have been devised by AAE, that enable its committees to report authoritatively the mental quirks of engineers, or to trace tendencies toward an engineering education. The author has accurately reported certain general indications noted by AAE committees. A synthesis of employers' opinions, as expressed in job orders, presented in the 1940 survey[24] demonstrated that employers believe engineers' principal weaknesses are: Blindness to intangible phases of their work; inarticulateness; inability to sell their own capabilities; and an indifference to, or lack of, understanding of the "profit motive" involved in engineering projects. The same criticism was found in the earlier 1928 survey,[25] and to some extent in the 1939 survey.[26]

The 1946 study[23] leads one to conclude that, since 1941, there has been a marked increase in employer demand for engineers who can "count the cost."

It seems to be an outgrowth of the change-over from "cost-plus" systems of production of war goods to normal, competitive methods of producing civilian goods. There are other reasons.

Fearing inflation, and confronted by increased pay schedules for skilled and unskilled labor, business needs engineers who can get maximum returns from this more expensive labor force and so hold down unit costs of production, thereby averting an increase in prices of finished products. Dollar-conscious engineers are needed also to keep engineering costs in line with economy programs designed to offset high taxes.

Manufacturers need engineering executives who can understand and insure conformity with all manner of government regulations—for example, those of the Securities Exchange Commission, the Internal Revenue Department, and others which impose highly technical rules and require complex and meticulous records in which technological operations are a factor.

Finally, business needs expert engineering guidance in order to arrive at, and conform to, satisfactory collective bargaining contracts. Labor unions are represented by experts in negotiations for contracts and in grievance procedures; the employer must have business administration experts who understand engineering, or engineers who grasp the fundamentals of business administration in order to deal satisfactorily with the unions in setting up job classifications and specifications, wage and incentive systems, etc., and in living up to the very letter of agreements with the unions.

Mr. Baker omitted only one of the insistent demands made by employers in reporting the findings of the 1940 survey.[24] This was the growing demand for engineers of "pleasing appearance." Perhaps the author felt that this particular problem is the responsibility of the individual, not the schools. A number of AAE committees have suggested (and the point was made in the 1940 survey) that the schools may have rather overemphasized "ruggedness." Some misapprehension may have caused the students to confuse virility with untidiness. At one time, a committee requested the writer to check, casually, the rather prideful references published in engineering school journals to engineering students as "campus roughnecks." There was some corroboration of that theory, but it was not sufficient to warrant examining a great number of these journals.

For the 1940 survey, however, employer specifications were checked closely as to "appearance" of candidates. Of the job orders filed with AAE between 1927 and 1929, 1.6% stated that appearance would be a factor in evaluating candidates for jobs, and in 1939 and 1940, 13.6% of the orders made a similar requirement.

Census Bureau Statistics Quoted by Mr. Baker.—Mr. Baker has reproduced three interesting tables compiled by the U. S. Census Bureau, showing the distribution of technical engineers on the basis of educational attainments, age, and salaries. The author does not argue that these tables are necessarily representative of the entire profession. He does suggest that they are interesting, as they pertain to a substantial segment of cross section of the profession.

The 1940 survey included an analysis of data concerning 573 engineers who, at that time, made up the "active file" of registrants in the AAE employment service.[28] Segregating those qualified for responsible charge of work in each category, the distribution was as shown in Table 10. These groups were analyzed from the standpoint of educational qualifications, age, and salary ratings; but the AAE experiments are not exactly parallel to those made by the Census Bureau and reported by the author. For instance, AAE dealt exclusively with "employee" engineers, whereas the Census Bureau group included an unstated percentage of engineers engaged in private practice. In analyzing educational attainments and salary levels the AAE procedure was not precisely that of the Census Bureau. Certain findings are reported in Table 10 only because they bear a striking similarity to those of the Census Bureau.

For example, Table 3(*a*) can be compared with Table 10, which shows the distribution of the 573 engineers whose registration cards were the basis of the 1940 study.[29]

For civil and electrical engineers, the data in Table 10(*a*) are not greatly at variance with those in Table 3(*a*). A smaller percentage of the mechanical engineers, included in the Table 10(*a*) study, held degrees than did the corresponding group analyzed by the Census Bureau.

TABLE 10.—SIGNIFICANT FINDINGS IN THE 1940 SURVEY OF THE MARKET FOR ENGINEERING SERVICES

Description	Civil and construction	Electrical	Mechanical and industrial	Total
Number of records	249	88	236	573
Responsible charge of work (%)	59	60	65	62
(*a*) EDUCATIONAL ATTAINMENTS (PERCENTAGE OF TOTAL NUMBER)				
Degree in engineering	61.0	67.0	36.0	....
Some technical education	22.5	21.5	33.0	....
Trade and correspondence school	6.0	4.5	8.5	....
High school only	10.0	6.0	22.5	....
(*b*) AGE DISTRIBUTION (PERCENTAGE OF TOTAL NUMBER)				
Younger than 31 years	36.5	52.0	38.5	39.0
Younger than 40 years	69.5	81.0	69.0	70.0
Older than 45 years	18.5	12.5	19.0	18.0

Considering all registrants in Table 10 (573), and comparing their educational attainments with the Census Bureau data for "all technical engineers" (Table 3(*a*), Col. 6), the AAE group corresponds very closely to that studied by the Census Bureau. Of the 573 engineers, 52% were graduates, and an additional 26% had one or more years of technical school training (not including trade or correspondence school work). The total 78% practically tallies with the Census Bureau total of 78.8%, adding lines 1 and 2 of Col. 6 in Table 3(*a*).

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[28] *Professional Engineer*, November, 1940, p. 26.

[29] *Ibid.*, Group II, p. 29.

Table 3(*b*) indicates that 36.2% of "all technical engineers" (Col. 6, line 11) are older than 45. Only 18% of the engineers covered by the AAE study were in that age group. This is natural, because all the 573 engineers were applicants for employment, whereas, the Census Bureau studied both employee engineers and those engaged in private practice. Only 12% of the men placed by the AAE service in 1940 were older than 45.[30] In analyzing job orders it was found that one third of all employers filing such orders made a preliminary requirement as to age, and that only 1% of them expressed a willingness to hire men older than 45.

Although age restrictions were relaxed during World War II, the 1946 study[23] indicates that they have "tightened up" rapidly since the end of the war. The report of placements for 1939–1940 given in the 1940 survey,[30] shows an interesting resemblance to data quoted in Table 3(*c*) (Col. 6, lines 16 to 23). The annual salary ratings of engineers can be studied by segregating the entire sample into three groups of equal numbers; thus, one third of the engineers placed by AAE in 1939 and 1940 received annual salaries of: Less than $1,500; $1,500 to $2,400; and $2,400 to $4,200, respectively.

The 1940 study cites only the rate at which these men were placed, not the actual earnings for the year. The Census Bureau quoted actual and total earnings for 1939, which may have accrued from less than a year of employment, some of the men reported having worked only intermittently but at monthly rates higher than total earnings for the year seem to indicate. Although no real parallelism exists, a comparison of the AAE data with the Census Bureau's report that 21% of all technical engineers earned less than $1,600 in 1939, whereas another 29.8% earned between $1,600 and $2,500, produces no startling contradictions. In Col. 6, Table 3(*c*), adding lines 16 to 19, inclusive, indicates that 21.1% of all technical engineers earned less than $1,600.

Professional Registration.—Among the 573 engineers whose experience records were analyzed in Table 10, only forty four were registered as professional engineers, that is, 7½%. Of the 573, there were 355 who had experience in design or higher grades of practice that should have entitled them to register. Of this 355, only 12% were registered. Of twenty-two construction superintendents, only seven were registered. Only two of nineteen electrical engineers with experience in responsible charge were registered; among the mechanicals only one of ten who had served as chief engineers in industry was registered, and he was the only man in this category who had had no academic training. Employers' indifference to registration was evidenced by the fact that only one of several thousand job orders specified that registration was requisite. This is as true in 1946 as it was in 1940. In the 1946 study of job orders[23] only one employer stipulated that candidates must be registered architects, and not one employer asked for a registered engineer.

Summary.—The foregoing comment is presented on the personal responsibility of the writer and does not necessarily reflect the official views of the AAE. The intent has been to report data compiled by AAE that should be useful in the appraisal of the paper by Mr. Baker. As stated, some of the AAE committees have followed the very line of investigation recommended by the author and have used the technique that he suggests to the schools and professional societies. The findings reported by AAE bear a striking resemblance to those that resulted from Mr. Baker's independent investigations; and the inferences from the findings conform rather closely to Mr. Baker's recommendations that:

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[30] "Survey of the Market for Engineering Services," *Professional Engineer*, November, 1940, p. 48.

(1) Undergraduate education should be confined to engineering fundamentals and that it should cover the entire field of engineering rather than a single branch or specialty; and

(2) The schools should assume responsibility for specialization through systematic extension courses as well as postgraduate study in residence.

Scott B. Lilly,[31] M. ASCE.—This presentation of the views of a consulting engineer on engineering education is most timely. If changes in the curriculum are due, they should be made now while the effects of World War II are being felt; while every school is faced with changes in personnel; and while the first-hand experience with teaching skills and techniques gained from the instruction of great numbers of men during the war period is available. The writer agrees with the main thesis of the paper, that engineering education should emphasize fundamental principles, and that postgraduate education should be designed to prepare for a career involving administrative, executive, and managerial activities. There are certain phases of the problem of instruction, however, to which the writer would like to call particular attention.

Essentially, engineering is planning. It consists in meeting and solving problems involved in the design of a particular structure, machine, or business activity, before an investment is made. For its successful practice, the engineer must have a sound background, not only in science, but in experience. In fact, he must have the sound judgment based on experience if he is to create any enterprise that is successful. Under any type of government, investments must satisfy social needs if they are to be justified.

If, then, the great need of the successful engineer is judgment based on experience, why should a man go to college to study engineering? Why not revert to the old apprentice system? Certainly the details of practice can be learned more quickly and more thoroughly on the job than in the classroom. What can a college education give a man that he cannot get on the job? Just this: Unless a man has a theoretical background, he has no way of classifying and evaluating his experience. He has no knowledge of the limitations imposed by nature, no realization that, although a law may hold under certain conditions, abrupt changes may be encountered if its use is extended beyond certain definite limits.

If basic science is to provide this framework, it must be taught so that a greater proportion of students find it usable. Mathematics must be so thoroughly taught and learned that general principles expressed in symbols are readily understood; but these principles must be so thoroughly comprehended that the student realizes that they are useless unless he can get from them the answer to the concrete problem to be solved. The physical sciences must be so thoroughly mastered that a knowledge of theory will make the engineer able to predict the limit of what any approach can be expected to yield, together with a clear idea of the limits of the theory.

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The heart of this suggestion is that the amount of material taught should be limited to the ability of the students to comprehend clearly what is presented. Nothing is so useless as a knowledge of facts so poorly digested as to be incapable of use. The author states that too many engineers are "narrow." There is nothing inherent in engineering which makes education in that field "narrow." From the reports that are coming out of the liberal arts colleges, it appears that English, history, political science, and many others of the so-called humanities, are being taught so that the end product is narrow. Quoting from Alfred North Whitehead:[32]

> "In the teaching of science, the art of thought should be taught; namely, the art of forming clear conceptions applying to first-hand experience, the art of divining the general truths which apply, the art of testing divinations, and the art of utilizing general truths by reasoning to more particular cases of peculiar importance. Furthermore, a power of scientific exposition is necessary, so that the relevant issues, from a confused mass of ideas, can be stated clearly, with due emphasis on important points."

Engineering is being taught in this way in some engineering schools. A man who has been exposed to this discipline for four years cannot be called "narrow" or uneducated. Every thoughtful teacher in every course should be striving for the objectives stated in the quotation from Mr. Whitehead, whether the subject be mathematics, English, or history.

When he graduates, the student must realize that he has merely learned the vocabulary in college and that he must educate himself on the job. The interest that comes from immediate use, the joy of accomplishment that is experienced when, by reason of study the night before, a man is able to have the right answer at the right moment, gives the urge which may make such study a lifelong habit.

What has been said applies to the technical side of a man's life. Education should prepare a man for his leisure hours as well as his working hours. There is a real adventure involved in the meeting of great minds in books, in making friends with these authors, in the feeling that they are always there on the shelf, where they can be consulted. This type of companionship is waiting for those men who know how to read ideas and not words. It comes to those men who have been under the influence of teachers and friends whose joy is in introducing young men to these pleasures. The teachers who do this must be men endowed with enthusiasm, as well as with knowledge and taste. Is this too much to ask of those who present literature, psychology, and philosophy?

This is not an idle dream. Such teachers can be had if society wants them. Salaries can be made attractive enough to bring able, well-rounded men into teaching. However, men of the type desired crave recognition of the importance of their job even more than they crave money. They must feel that there is wide realization of the great difficulties involved in teaching; of the hours of study necessary; in general, that teaching demands the best that a man has, or can become, if he is to have real success; and that the ends sought justify such efforts.

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[32] "Aims of Education, and Other Essays," by Alfred North Whitehead, p. 81.

Such teachers would produce students with a professional attitude. They would be conscious of the fact that they have an obligation to extend the frontiers of knowledge in their own fields. More than that, they would have a knowledge of the impact of their activities on society. They would feel a responsibility toward society; with the background they had received from contact with active minds, with cultivated imaginations, they would be able and willing to engage in public affairs; and the problem of professional recognition for engineers would disappear.

With the help of industry, the teacher can do much to improve his condition. Let him spend some time each summer in industry; let him become familiar with current problems. If he knows these problems, the teacher's salary must rise or he will stay in industry. There is nothing like competition, not only to increase salaries, but to develop the teacher. Teachers must be alert, keen, broadminded, and well informed, if they are to be equal to today's needs.

There is no easy simple solution to this complicated problem. It is not one that can be solved by changing curricula, or studying in college five or six years instead of four. It is quality of instruction that is needed; it is the true professional attitude toward all activities that must be inculcated, that will finally win the battle; and quality in the student must come from quality in the faculty. What can be done to bring into teaching the best minds in the profession? That is the question to be solved. What can the members of the Society do to bring this about?

William A. Conwell,[33] M. ASCE.—The kind of material about which practicing engineers often talk but seldom write is contained in this paper. The topics, of course, have been treated at length by engineering educators in the rôle of scholars discussing their specialities. Such dissertations lack the freshness and exhilaration which are likely to characterize the probings of an intelligent man in a field in which, although he is not a specialist, he is highly interested. Such are the characteristics of this paper. The thoughts and musings are the type which might be heard in an informal discussion of the subject by engineers. In this instance, however, the ideas are supplemented by well-arranged data which give them force and weight ordinarily absent in informal discussion.

It is interesting to examine the proposed revisions to the methods of engineering education in the light of the functions of those concerned—the individual, the school, and the employer.

Under the system generally employed prior to World War II, the individual followed a course of instruction which was largely technical. He was left principally to his own resources in developing the nontechnical requirements of his profession, and of his cultural background. The school outlined courses and taught subjects which were technical or which barely skirted the fringes of the nontechnical requirements. Employers did one of two things, depending upon their needs. With the confidence that the logical thought processes induced by an engineering education would be of value to him whatever his business, one type of employer hired graduating engineers regardless of their specialties; acquainted them with his organization by a course of indoctrination; and then used them to solve his problems, whether in the field of engineering or not. It was this type of employer who voiced opposition to excessive specialization according to the 1940 analysis of the American Association of Engineers.[2] The other type of employer hired the graduate as a technologist and put him to work immediately on the solution of engineering problems. In the latter instance, the engineer's experience, gained in the solution of practice problems as a student, was indispensable.

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Under the plan of education proposed by the paper the individual would pursue a more general course and thus not be required to devote as much of his extracurricular activities to self-development. The school would offer the general course stressing fundamentals in technical subjects, eliminating practice problems, and including considerable nontechnical and probably cultural matter. Needless to say, the school would have to "sell" such a course to the embryo engineers who normally look askance at any subject that lacks the quality of being immediately and practically applicable in professional practice. The school would further expand opportunities for the employed graduate to continue his studies, a commendable proposal under any system of education, past, present, or future. If the employer is of the first type, interested in men trained as engineers regardless of branch, the suggested plan suits him admirably. All engineering graduates would be eligible for his indoctrination course. The second type of employer, who wants a technologist whom he can put to work on engineering problems, will find no graduate with the requisite experience in practical problem solution. He, too, must institute some kind of training. To borrow from Hardy Cross, M. ASCE, he must take the young engineer who has "hitched his wagon to a star" and induce him to get "his feet on the ground."[34]

In this comparison of the former and the proposed methods of education, one can discern the definite shift of responsibility from the individual to the school, and from the school to the employer. The school takes over the student's former responsibilities in the nontechnical field, and the employer, unless he confines his employees to men with advanced degrees, assumes the responsibility of the school for training in practice problems. This trend may be thought of as consistent with that in many other fields and as accompanying the higher degree of socialization which we shall continually find in our civilization; but it could hardly be said to enhance the engineer's individuality, at present one of his most precious possessions.

The writer is in hearty agreement with the author's premise that, in his undergraduate training, the student should be imbued with a sense of the importance of professional progress after graduation. Real progress is demonstrated, of course, by the engineer's performance of increasingly difficult work as his career develops; but this progress is often reflected in, and enhanced by, activities in three fields—advanced degrees, registration, and advanced grades of membership in engineering societies.

Appreciation of the value of the master's and the doctor's degrees is mounting as time goes on and there can be little doubt of their eventual attainment of the respect due them in the engineering world. The same cannot be said of the professional degree which is sought by relatively few graduates. The attitude toward this degree is surprising in view of the fact that its essence is recognition of experience, a most valuable asset of the practicing engineer.

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[34] *Transactions*, ASCE, Vol. 96, p. 140.

The trend in the field of registration for license to practice is "all to the good." The time should not be far off when all professional engineers will recognize the full value of registration.

Appreciation of membership and advanced grades in engineering societies is lacking in many engineers. As against the author's opinion, it is the writer's firm conviction that failure of an engineer to obtain membership in the engineering societies and to advance his grade consistently lies in his apathy rather than in his not having had opportunity to perform work which would entitle him to recognition.

It is the writer's belief, contrary to that of the author, that the economic depression of the 1930's had a profound effect upon the stability of Junior Members of that period.

There is some question as to whether an anomaly is not presented by the author's proposed revision of courses which excludes some subjects and adds others. One of the author's arguments for elimination of practice problems is that the graduate must wait eight or ten years before he has an opportunity to apply what he has learned (a premise with which the writer cannot wholly agree). At the same time the author recommends courses in "men," "money," and "management" for inclusion in undergraduate curricula; but in tracing the professional progress of the engineer, the author envisages his dealing first with "matter," next with "men," then with "money," and finally, at the height of his career, with "management." If the reason for eliminating practice problems is valid, it would seem that a similar reason would hold for eliminating the last three "M's" from the undergraduate work. These thoughts are presented in the belief that the author, in his closing discussion, might like to clarify an apparent inconsistency. They are not meant to imply that courses, particularly those dealing with "men," are out of place before graduation. Ability to get along with one's fellowmen is of paramount importance; and, if a student's home or early school training has failed to develop him in this direction, something should be done about it in college.

The writer questions how effective a five-year or ten-year forecast of the market for engineers can be. It is unlikely, for instance, that the great demand for radio engineers in the 1920's could have been predicted. On what basis could there have been foretold the need in the early 1940's for army engineers with training in "men" and "management" far beyond their years and experience? Flexibility in training, the second alternative suggested by the author, would seem to be the solution to this problem; but the goal of flexibility does not necessarily (in the opinion of the writer) require the elimination of practice problems in undergraduate work.

If the prewar graduate was "primarily a trained, rather than an educated person," his successor, under the proposed revisions, would probably continue to be so. Although certain technical training would be eliminated and more emphasis would be placed on purely educational subjects, the suggested courses in speaking, writing, business, etc., would be in the field of training rather than education.

A. AMIRIKIAN,[35] M. ASCE.—If the success of the present-day engineering education is to be gaged by the success of its product, as suggested by the author, the statistical data presented in the paper do not reflect a favorable condition in so far as the great majority of the members of the profession is concerned. This is unfortunately true whether success is measured by the rate of the young engineer's progress in the profession, his attained standing and prestige in the community, the recognition of his contribution, or the compensation for his services.

Causes contributory to this unhappy situation are many and complex. Although some of these causes may be traceable to certain deficiencies in engineering education and training, most of them in reality are related to engineering practice. As observed by the author, engineering is no longer practiced as a free profession, but is conducted along lines similar to a business enterprise. The change is not new; it has been taking place during the past several decades. As a result, the individual practitioner, the true representative of the free profession, is practically nonexistent today. He is forced either to seek employment with large engineering and architectural organizations or to join forces with other engineers to form an engineering firm, and thus become either an employer or an employee.

Various reasons are advanced for the evolution of engineering practice from a free profession to a competitive business enterprise. According to the opinion of some, this change is due to modern trends of industrial development with corresponding growth in engineering projects requiring the services of large engineering organizations for their execution. Others think it is due to a keen competition predicated on present-day business methods and practices. Regardless of what the real reasons may have been to cause this change, the important fact is that it has occurred. From an educational viewpoint, the only thing that can be done to improve the lot of the young engineer is to give him a training adaptable to the requirements of current practice.

An examination of engineering curricula will reveal that during the first half of the twentieth century no significant changes have been made in either programs of instruction or methods of training. The institutions are still engaged in the task of producing the finished product—the professional engineer. In this work, efforts are being made to equip the candidate with the necessary knowledge and training to practice engineering in the same way that students are being trained to practice medicine and law. At first approach, the program may appear as proper and its objective as logical; but a closer study of the means utilized in the program and the problem of professional practice of the finished product will bring out serious doubts concerning both the adequacy of the means as well as the propriety of the objective.

The first point of the argument needs little elaboration. It is generally admitted that a four-year engineering course is too short a period for the required training. The disparity between the set objective and the allocated time will be more apparent when comparison is made of the seven-year to eight-year training period prescribed by the legal and medical professions. The inadequacy of the means is further illustrated by the fact that there are few, indeed, if any, institutions which are equipped to give the specialized training required by present-day engineering practice.

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The other considerations concern the practical problem of placement of the finished product. Since the young engineer cannot open an office and practice his profession, as would a lawyer and a physician, after graduation there still remains the difficult task of securing a proper position. Strange as it may seem, his specialized training—even when assuming that he was able to obtain it—not only would be of little help in securing employment, but, in most cases, would prove a handicap. This is due to the fact that a young graduate is seldom able to find a position that will correspond to his particular specialization in school. Under normal circumstances, he has but little choice for employment, and usually he accepts the first job that comes along. Furthermore, because of his lack of experience, the organization that offers him employment considers him as a mere recruit who must go through the "grind"—regardless of his specialization or particular qualifications. Stated more frankly, his acceptance into the profession does not materially differ from that of an apprentice into a trade. The course of his later progress is equally discouraging—even more so than that of an apprentice. As a result, after surviving his initial disappointment, the young engineer keeps on changing his place of employment at frequent intervals, trying to adapt himself to the conditions, and, finally, he either becomes reconciled with the situation or he leaves the profession.

Unfortunately, little attention is focused on the latter phase of the problem. The statistical data presented in the paper concerning change in employment and abandonment of the profession reveal an alarming condition. According to the data, the turnover runs as high as 60%, and as many as 50% eventually leave the profession. Essentially, this is a condition suggestive of both maladjustment and malcontentment. Obviously, no changes in the educational system can appreciably affect the latter factor, since it is due entirely to current methods of engineering practice. The only thing that engineering institutions can do in this respect is to take the glamour out of engineering education and present a clear and realistic picture of the profession. Such a warning will, at least partly, obviate future disappointments and disillusionments on the part of the student. On the other hand, conditions of maladjustment can be greatly improved by introducing certain necessary changes in the educational system to provide the needed flexibility of adaptation. Briefly, these changes would consist of the following:

(*a*) *Undergraduate Work.*—Confine undergraduate instruction to courses in liberal arts and general engineering only. Assuming a four-year curriculum, the first two years would be devoted to liberal arts and the latter two years to engineering. The courses in the former period would consist primarily of social and cultural subjects; in the latter period, the main subjects would consist of three important "Ms" which form the backbone of any branch of engineering—

namely, mathematics, mechanics, and materials. There would be no specialized courses, and only simple and elementary applications of the fundamentals to a particular branch of engineering.

The advantages of such a combined undergraduate course are many. From the administrative standpoint, optimum use would be made of the institution's facilities and staff, and there would be little conflict or confusion in the arrangement of the curricula. From the point of view of the student, the derived benefits would be still more important: (1) His liberal education would provide a cultural background that would aid him in later life regardless of what line of endeavor he might eventually follow; (2) there would be ample time for a more considerate choice of a profession and an opportunity for a desired change at relatively later stages of instruction; and (3) the placement and adjustment problems after graduation would be greatly facilitated because of the general nature of the training.

(*b*) *Graduate Work.*—Confine postgraduate work to specialized courses in various branches and subbranches of engineering. The main purpose of these courses would be to provide up-to-date professional training required in current engineering practice. To be eligible for admission, the candidate should have completed undergraduate training and, in addition, should have had a minimum of five years of engineering experience. The importance of the latter requirement cannot be overemphasized. During such a trial period the young engineer would have had a good taste of the methods and practice of the profession for making his final decision; also, he probably would have located the particular line of engineering for which he is best suited.

The courses may be arranged on the basis of both resident and nonresident instruction. In most cases, however, the latter arrangement would prove to be the most satisfactory, since it would assure the practicability of the system. The duration of the period may vary from one to three years, in accordance with the requirements and the complexity of the specialization, and the training would culminate with the granting of a professional degree. As for the necessary instruction staff, the faculty would consist mostly of practicing engineers engaged on a part-time basis—leading specialists in the various fields of engineering. In this connection, it is to be noted that, unlike the medical and law schools, little or no attempt is made by engineering schools to utilize such talent.

(*c*) *Research Work.*—Supplement graduate courses with research work. There are many problems in the various fields of engineering which need elaborate research and experimentation. Most of these problems are directly related to the work of the practicing engineer. As a matter of fact, such problems constantly arise in devising new techniques to make more advantageous use of materials, or to improve the methods of applying an existing technique. If satisfactory arrangements could be made by the engineering schools, it would be possible for the research-minded engineer to bring his problems to the institution and work out a solution by utilizing its aid and facilities. Engineering research conducted on this basis would yield more practical and valuable results than are obtained from the present system of graduate work by which research is incidental and serves primarily as a vehicle for the inexperienced engineering graduate in his effort to obtain an advanced degree.

C. A. Dykstra,[36] Esq.—During the past few years, the writer has given some thought to engineering education, and has discussed the problem with many engineers and with members of engineering faculties. He agrees with the author that engineering training should be broad and fundamental and that it should place some emphasis on what is being called "general education." An engineer well grounded in mathematics and the necessary sciences who has some knowledge of the background of civilization, a reasonable mastery of the English language both spoken and written, and a speaking acquaintance with the fields of psychology and economics will have a better chance to advance in his profession than the one who too early acquaints himself with techniques and specialities in the field of engineering. These specialities can be mastered if and when the need for them arises. The mastery of a speciality will come with experience on the job, through special courses given by industry, and through the opportunities for advanced work either in residence or through extension courses in engineering schools.

Also there is need for the training of the imagination of engineers. Such education comes out of wide reading in the fields of literature and from the discussion of controversial issues; it comes from contact with other engineers and with those who practice other professions. It is time to begin to use the words, "engineering education," instead of the phrase, "engineering training." If the so-called training could be superimposed upon the education of the individual, wiser men would be developed in the profession. In modern technological civilization there is great need for the technicians called engineers, but, to master this mechanical civilization, technical men must be aware of the implications of modern life and they must feel at home in a changing world. A structural engineer should know more than how to build a bridge. He should be able to state whether a bridge is necessary, what will happen if it is built, and perhaps be able to suggest an alternate solution to what is traditionally thought of as a bridge problem.

Therefore, it is gratifying that the author is interested in the educational aspects of the engineering world and it is to be hoped he will continue to call attention to the fact that a study of engineers as well as a study of engineering is needed. After all, engineering schools are in the business of educating men, some of whom will be engineers for the remainder of their lives, and some of whom will find themselves with shifting and varying responsibilities which may be quite tangential to what is familiarly called the engineering profession. If, as it may happen, engineers are called much more frequently into management or into public life, the character and quality of their education becomes of real importance to the social structure.

Louis Balog,[37] Esq.—The major purpose of this inquiry as stated in the "Introduction" is: "* * * to evoke discussion of the subject of engineering education by the users of its product." The point of view of "the users of the product" does not necessarily coincide with public interest and with the professional interest of the individual. In fact the statistical data included in the paper indicate that these interests are quite divergent. The opinion expressed in the paper—that the assumption of administrative duties by an engineer constitutes professional advancement when he begins "depending on his subordiates more and more for technical results and opinions"—misrepresents the value of engineering science. It would appear reasonable that work involving technical science should be rated above all other activities, and that superior technical knowledge should enable the engineer to assign subordinates to the nontechnical duties of the managerial position. Educational institutions and professional organizations should not create men prepared to prosper on the knowledge of subordinate engineers; rather, they should produce engineers who will have reasonable hope of improving their conditions by professional, and not by business, efforts.

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A profession is a vocation that requires a learned education. A uniform educational standard, therefore, is a necessary requirement for creating a profession. This was well recognized in the 1920's by the American Medical Association in rating medical schools and in undertaking a relentless effort for the immediate closure of third-rate and, for the gradual elimination of all second-rate, institutions of medical instruction. As long as uniform educational requirements for the designation "engineer" are not established, the group classification of "engineering profession" remains meaningless. The divided authority of the university, the state registration board, and the engineering society in declaring individuals engineers, cannot result in a true professional status for the engineer, similar to that of medical men. The latter, upon complying with precisely defined scholastic requirements, are declared to be doctors of medicine on the strength of the authority of the university and become full-fledged members of their profession.

From statistical data, Mr. Baker concludes that the engineer at present cannot achieve professional status by education (see heading, "The Practice of Engineering: Rate of Advancement"):

> "* * * the average engineer—at least if he follows civil engineering—does not attain a position in which he is called on * * * to assume engineering responsibility for anything except work of the simplest character, until at least from eight to ten years after he has graduated; and many do not attain such a position within that time."

Under the heading, "Prewar Engineering Education," Mr. Baker states also that at the university the student: "* * * grappled with problems of a character that he would never meet, or at least would not have to solve, on his own responsibility for from at least twenty to twenty-five years after graduation." In striking contrast to the foregoing statements, Arthur J. Boase, M. ASCE, in reporting on the advanced stage of development of reinforced-concrete design in South American countries, observes that the leading designers of those countries are young men, the ages ranging from twenty-five to thirty-five years.[38] None of these highly educated, able designers (who have developed a reinforced-concrete design practice far superior to that of the level achieved in the United States) could satisfy Mr. Baker's criterion for engineering responsibility or be classified as able to design. This is an indication of something fundamentally wrong in the guiding principles that underlie his opinions concerning the purposes of civil engineering education and professional organization.

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[38] "South American Building Is Challenging," by Arthur J. Boase, *Engineering News-Record*, Vol. 133, 1944, p. 121.

Effecting that professional education and technical knowledge, so that it will be to the best advantage of the individual and the community, is the foremost duty of any group of men engaged in the same occupation. The engineering society has been ineffective in these professional objectives primarily because it is not a group of persons engaged in the same occupation and interest. According to Mr. Baker's statistics (Table 4), 43% of its corporate membership is not engaged in engineering; and a closer scrutiny of the actual activity of those in the grade of member, by Arthur Wardel Consoer, M. ASCE, reveals that "* * * only a pitifully small group * * * can really consider themselves to be practicing engineering as a profession."[39] In demonstrating similar facts (Fig. 1), the author arrives at the conclusion that engineering education should emphasize only the fundamental principles of engineering and should include executive and managerial training. At the convention of the American Society for Engineering Education in June, 1946, the recommendation was made that civil engineering education should emphasize fundamentals and that design "can be better learned in practice." Such recommendations are not to the best interests of the civil engineer. If South American engineers, credited with outstanding accomplishments at the age of twenty-five years[38] had received the education advocated by Mr. Baker and the American Society for Engineering Education, they could never have achieved real engineering prominence; a few may have become managers of some mass-production organization "twenty to twenty-five years after graduation." Why should the young men of the United States be denied an education that would equip them for independent accomplishment as civil engineers?

In discussing the "Organization and Structure of the Engineering Profession" under the heading, "The Engineering Profession," Mr. Baker states: "The current practice of engineering, except by those who serve in advisory or consulting capacities, is a mass-production activity * * *." Critical examination of this correct observation reveals, however, that this condition involves all the disadvantages and none of the advantages of mass production, and that it is detrimental to the professional interest of the civil engineer. Engineering structures are not produced by mass-production methods, and nature seldom provides the same conditions twice. A structure that fits perfectly into its location always represents saving to balance the insignificant cost of a good design. Mass-production principles manifest themselves in standardization, and standardized designs are the most effective means of hindering progress. They minimize the value of talent in design and hinder the useful employment of engineers. The application of mass production to design has created the highly valued executive, but it has degraded the designer, and eliminated competitive design and, with it, the proper recognition of individual accomplishments, which is the prime requisite for professional existence.

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[39] "The Professional Status of Civil Engineers," by Arthur Wardel Consoer, *Civil Engineering*, November, 1939, p. 673.

The management of engineering organizations, at present, may be a purely parasitic activity in which mere trader's cunning, or certain dexterity in the use of technical terms, is sufficient qualification for a position above the engineer. The slight technical knowledge that is required for this form of management is demonstrated by the fact that most heads of engineering organizations advanced to their positions through salesmanship or assumed executive duties. The emphasis placed on management and organization, at the expense of technical knowledge, has lowered the designer to the status of a laborer. The user of the product, whose advice Mr. Baker seeks in an effort to improve engineering education, paid the highly educated designer (the true representative of the civil engineering profession) less than the wages of unskilled labor, and terminated his employment immediately upon completion of the work. These practices have forced designers to use the methods of union labor in their struggle for existence, a disgrace that removed from the civil engineer even the semblance of belonging to a learned profession. Mr. Baker accepts this as an unalterable situation and recommends that the student of engineering be advised as to what he is "getting into." According to Mr. Baker (under the heading, "Postwar Engineering Education"):

> "The student should also receive a realistic picture of the profession which he has chosen for a life career—not the glorified aspect usually presented by older alumni or distinguished engineers who address upper classmen at meetings."

It is of even greater importance to the student to know that the distinguished engineers are purposely not telling the truth. The incredibly cynical admissions of this appalling fact, in private conversation, show the prevailing conditions in the civil engineering profession to be based on the betrayal of morality on which all human relations should rest.

The author's recommendations that the experience records of engineers be studied in order to discover needed educational improvements and the demand for engineering services can yield no useful results. Such records do not show the actual accomplishments or the capabilities of anyone; and the demand for engineering services is never revealed by such records.

Examination of engineering structures will indicate educational and professional conditions; it will indicate needed educational improvements and wasted opportunities for the employment of engineering services. The professional biographies of the engineers, enumerating their engagements, will indicate nothing. Anywhere and everywhere, structures can be observed which disclose the need for engineering services—instances where the use of expert engineering talent would have resulted in savings, improved appearance, and increased engineering employment.

Mr. Baker's analysis of the advancement of engineers in the qualification grades of the engineering society, therefore, cannot be accepted as a suitable guide for the improvement of engineering education or professional organization. On the contrary, the misleading conclusions that result indicate that the rejection of his basic principles is a necessary requirement for the achievement of actual improvements.

The classification of engineers by interested private individuals, as "juniors," "associates," and "members," that postpones recognition of design ability until after a person has passed the prime of life, [40,41] and that declares any one older than thirty-five years able to design, irrespective of his actual occupation, is incompatible with the conception of a learned profession. The growth of technical sciences has resulted in the evolution of educational institutions that actually eliminate the system of apprenticeship established by this antiquated and wholly unjust classification of engineers by the engineering society. If this were not an artificially created situation, the university as an institution for educating civil engineers could be regarded as a complete failure. The inadequacy of the university is a direct result of the influence exerted by the controlling ideas in the existing organization of the civil engineering profesision.

The engineer will attain professional status when a properly constituted university has become recognized as the sole authority for qualifying an individual, upon completion of specified studies and examinations. Since the rights of individuals are guaranteed by the authority of the state, the state should establish the educational requirements leading to those rights and should be represented by its properly constituted registration board in the final examinations of engineers at the university. Such an arrangement eliminates duplication of effort. It eliminates the system of dependence on personal recommendations; it guarantees that engineers will be able to begin practicing their profession at a reasonable age and succeed according to their ability.

A professional man is necessarily a lifelong student whose knowledge should increase steadily. The admission to a profession, however, should be based on a precisely defined, formal education which preferably should include the entire educational period. The scholastic level of high schools in the United States is lower than that of preparatory schools of other countries of the world by at least two years of study.[42,43] A more efficient secondary education is of great interest to the engineer. He should be given the possibility of acquiring a proper cultural and basic scientific education and the formation of mental habits that result in self-dependence and maturity, before entering the university at the age of eighteen. Experience in education indicates that this can be achieved and a professional organization of engineers should influence the establishment of such an educational procedure.

It is rational to spend the first two years at the university with the basic and advanced theories now given as graduate study, the following two years with the application of theories to design, and the fifth year with specialized technical studies and courses in economics and statistics. This university program of five years should be based on at least 40 hours each week in lectures, laboratory, and design work. Upon completion of this work, and the examina-

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[40] "Man's Most Creative Years," by Harvey C. Lehman, *The Scientific Monthly*, Vol. 59, 1944, pp. 384–393.

[41] "'Intellectual' Versus 'Physical' Peak Performance," by Harvey C. Lehman, *ibid.*, Vol. 61, 1945, pp. 127–137.

[42] "Rhodes Scholarships and American Scholars," by G. R. Parkin, *The Atlantic Monthly*, Vol. 124, pp. 365–375.

[43] "Secondary Education in the United States," by William A. Smith, The Macmillan Co., New York, N. Y., 1932, p. 108.

tions, the candidate should prepare a thesis and pass a written and oral examination in his three principal engineering subjects before a board comprising members of the university and federal registration authorities. This joint board should declare the successful candidate an engineer, and he should be acknowledged a full-fledged member of the profession at the age of twenty-four.

The academic prestige of the engineer's diploma should be the same as that of the doctor of medicine. Courses that do not require compulsory work and specific examinations by a duly formed combination of university and registration boards should be disqualified in appraising candidates for the degree of engineer. It is a truism that taking courses does not mean that the individual has any knowledge of the subjects involved.

The graduate engineer, upon presentation of a paper which is rated satisfactory by a special board of the university, and upon passing an oral examination, should be able to obtain the academic degree of doctor from the university. This degree, however, should not give rights in addition to those acquired with his diploma. The degrees of Bachelor of Science and Master of Science, in engineering, should not exist. All oral examinations should be public.

The sequence of studies described in this discussion is psychologically more sound for the development of the proper sense of values than the prevailing reversed procedure. Observation indicates that an elementary structural education, followed by the study of the theories of elasticity and stability, often results in an attitude that misjudges the rôle and value of mathematical procedures in creating structures. The layout determines the cost, quality, and appearance of a structure. The performance is ascertained by analysis only; but analysis can never improve upon a structure if the layout is faulty. All the knowledge contained in the curricula is necessary in structural design. The subjects of design, therefore, must follow the mastering of advanced theories.

Mr. Baker's idea of transforming engineers into managers by the study of such a miscellany of subjects as "human nature," "institutions," and "business practice" should be rejected. Only superior technical knowledge should be the qualification for leading positions in engineering. The only nontechnical subjects required of the engineer should be economics and statistics. The analysis of scientifically classified collections of fact relating to national and world economy is a suitable method of interpreting conditions and values by technically trained minds.

As executive training or experience does not make one a designer, teaching experience does not make one a professor. Thorough technical knowledge of the subjects involved is the fundamental requirement in both cases. A thousand students would rather listen to the lectures of an outstanding teacher than groups of fifty, to men of limited knowledge and faulty point of view. Personal contact between lecturer and students is not necessary. Self-dependence is essential for the development of the student. The professor should not be required to expend more time than that required for lectures and examinations.

During the first two years at the university, since the emphasis is on analysis, the engineer is trained by men who are primarily theorists. After the second year, the lectures should be given by practicing engineers. Lecturers should be required, concurrently, to engage in research or design, keep abreast of all progress in their respective lines, and modify their lectures every year. The compensation of lecturers and their assistants should be so high that everyone would strive to attain the knowledge that qualifies men for a position at the university.

The educational and qualifying procedures outlined in this discussion, and the establishment of free competition in design, are the only means for establishing the civil engineer in his proper professional status. A suggestion that, in order to succeed, the engineer should be a politician, an orator, an executive, a salesman, or anything but a person who applies technical science in his work, is an implicit admission that technical knowledge is of no value. This condition follows from the prevailing system of qualifying engineers through personal recommendations, by which means anyone can be made a great engineer or a lifelong slave. As long as it is possible to declare every technical function a matter of detail—as long as it is possible to accord credit for the work of others to anyone whose advancement is the personal interest of a user of the product—the existence of a civil engineering profession will remain a mere illusion.

HENRY B. LYNCH,[44] M. ASCE.—A well-considered, thoughtful statement of a problem of greatest importance to the engineering profession has been presented by Mr. Baker. In the past few years it has become increasingly necessary that the time devoted to preparation for an engineering career be scheduled with precision. More subjects should not be crowded into the college curriculum. In fact, there is criticism in some places that too much is attempted already. It has become necessary either to extend the time devoted to educating the engineer or to jettison some of the material formerly taught, to make room for new material that cannot be omitted. The author has suggested a good working basis for extending courses.

In appraising engineering education, in listing the subjects that can be discarded or curtailed, and in naming courses that could be given greater weight, the views and experiences of mature practicing engineers throw as much light as do those of engineering educators.

For many years it has seemed to the writer that the most important duty imposed upon the engineering college is that, first of all, the graduate be a broadly educated man. The aim of the curriculum should have been to impart judgment rather than skill, knowledge rather than training. He should have a broad knowledge—even though not a detailed knowledge—of history, English, and literature. He should be well grounded in mathematics and science and such foreign language as seems proper, without forgetting that he is being educated to become an engineer.

These objectives cannot all be accomplished unless there is a sharp curtailment in the time now devoted to subprofessional training. In an education leading to an engineering degree, much of the shopwork should be eliminated and such courses as surveying and mechanical drawing should be greatly shortened. Even so, provision should be made in some such manner as that proposed by the author, for extension courses preparing for further degrees.

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[44] Cons. Engr., Los Angeles, Calif.

These objectives cannot be attained through the college alone. Considerable groundwork must be done in the preparatory schools. For this purpose a much sharper distinction should be made between education that ends in the high schools and that which leads to a college degree. In preparing the engineering student, the high schools could well reduce greatly the elective subjects permitted, with a view to giving a thorough grounding in those subjects which a broadly educated person must have, so that the colleges may devote more time to the necessary engineering subjects.

Alfred R. Golzé,[45] Assoc. M. ASCE.—The experience of the Bureau of Reclamation shows a need for greater appreciation by the engineering schools of the fields of endeavor into which graduates will advance. This experience supports Mr. Baker's conclusions that engineering education should emphasize fundamental principles in subjects throughout the broad field of engineering and that a graduate engineer should be able to undertake a career leading to administrative, executive, or managerial responsibilities. The unusual conditions affecting educational institutions during and following World War II provide an unexcelled opportunity for possible curriculum adjustments to meet extreme weaknesses.

The Bureau of Reclamation is considered primarily an engineering organization. Its accomplishments in the fields of civil, electrical, and mechanical engineering are well known; yet of its present staff of approximately 15,000 employees less than 30% are engineers devoting their time exclusively to strictly engineering phases of planning, design, and operation processes. The other employees—legal, clerical, and skilled labor—are engaged in multitudinous tasks of administration. Since the work of planning, building, and operating reclamation projects is essentially engineering in character, it follows that men with engineering training would seem to be the most logical group to advance to administrative and supervisory positions. It has been the Bureau's experience, however, that engineers, regardless of the excellent technical education they receive in the colleges, are not as well equipped to move into supervisory and administrative positions where employees of all professions and trades come under their direction, as are men who have received their basic education in fields with broader perspectives. For instance, men in charge of operating reclamation projects must be versatile creatures. They must excel at public relations in dealing with both water users and their innumerable requests, and with labor, both organized and unorganized. They must be good accountants to recover the construction and operating costs assessed against a project; they must write an endless number of brief, concise reports of every description; and they must operate and maintain massive dams and miles of canals, laterals, and drains, and direct the operation of fleets of equipment.

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The Branch of Operation and Maintenance of the Bureau of Reclamation has the responsibility for the operation of completed irrigation facilities. There is a woeful lack of qualified men to fill developing vacancies in the field. On the older irrigation projects the officials in charge are reaching retirement age. New projects requiring complete staffs of competent men are being brought to completion. In both cases the Bureau has no source of trained material from which to draw to fill the key positions on operating projects. This condition is caused not only by the failure of the Bureau to appreciate its own needs and to provide a proper training program, but also by a feeling on the part of most men who elect to stay with the civil engineering profession that the field of construction is the one to follow.

With the college curriculums in civil engineering shaped as they are in most schools, it is only natural that the graduate should judge that the sole professional opportunities are in construction. The lack of interest in the operation of completed engineering facilities—which is only one example—seems to be due to two factors: (1) A failure on the graduate's part to appreciate the engineering importance of these facilities to the continued welfare of the community and of the United States; and (2) the mercenary aspects of low salary scales.

With respect to factor (1), if Mr. Baker's engineering course, alternative I, were adopted, it would be of great assistance. In regard to factor (2), the Bureau of Reclamation, together with other federal agencies, pays the same salaries for operation and maintenance duties as those given to construction and planning engineers, largely eliminating the salary differential as a reason for keeping young engineers out of this field.

Seeking to determine the situation in the western universities, the writer has recently completed a trip through California and the Southwest where he visited some of the major universities giving courses in engineering and irrigation agriculture. In nearly every case, although the faculties appreciated the need for emphasizing the part that engineers can play in western development, the colleges were making no particular effort in that direction. In other words, the standard four-year engineering courses are about the same as the courses given in the universities of the Middle West and the East. Most colleges consider four years academically too short a time to provide an engineering student with other than the technical tools with which to begin his career. Such activities as public speaking, administration, western resource development, and national economics are seldom considered.

Resorting again to the Bureau of Reclamation as an illustration, it has expended more than $1,000,000,000 in providing a high standard of living to secure agriculture for the people of the West. It has on its books nearly another billion dollars of authorized work and ahead are more years of billion dollar expenditures. In thirty years the men who are now in colleges will be in charge of this vast investment. It is self-evident that the universities, particularly those of the West, carry a major responsibility in properly training men for this difficult and important work.

A promising aspect of the situation at most western schools is the evident desire of the faculty members to widen the scope of the engineering training and to increase the consciousness of students to their future responsibilities in developing and administering the natural resources of their part of the United States. Shortcomings in public relations and report writing are well understood. Formerly several of the western schools were leaders in these fields and their staff members have become nationally known for their contributions to irrigation engineering. With the passage of years, however, there has been a slowing down of the leadership because of age and turnover so that institutions which formerly led the field are having difficulty in maintaining their rank. If other schools were coming to the front this would be a healthy development injecting new blood and new ideas in the field of engineering education, but there is little indication that a broader engineering education adapted to the needs of the consumer is becoming available.

Although the situation appears gloomy, the spark of self-improvement does exist in most of the schools visited by the writer, and presumably it exists elsewhere. Once the peak of the present high enrolment of returning veterans has been passed, a number of western schools should seek to motivate engineering curriculums to reflect the needs of the nation, and particularly those of the western states, by arousing in the neophyte engineer a consciousness of the responsibilities attending his profession in the broad fields of economics, public relations, and administration. If the users of engineers, as well as the producers of engineers, will accept Mr. Baker's challenge, this objective ultimately should be forthcoming.

DONALD M. BAKER,[46] M. ASCE.—In reading the discussions of this paper, one is reminded of the old Hindu story about the blind men who were taken to visit an elephant, and afterward asked to describe the animal. The man who felt the elephant's legs stated that an elephant was like a tree, the one who had felt his trunk described the animal as being like a huge snake, while the one who had felt his tail likened him to a thick rope.

Each discusser—practicing engineer, educator, or layman—discusses engineers, the practice of engineering, and engineering education from his contacts with them. Some have been wide and varied, have involved large numbers of engineers practicing many specialties in varied locations, while others, although extending over considerable periods of time, have been with limited groups and sometimes within limited areas.

Most of the engineering educators appear to have had in mind recent graduates, seeking employment following graduation or during the early years of their practice, or possibly to be thinking of the employers endeavoring to recruit young graduates to fill subprofessional or preprofessional positions in their organizations. Professor Brinker talks about maintaining student interest in courses and about preparing civil engineering graduates so that, following commencement, they may secure positions as instrumentmen rather than as rodmen, as draftsmen rather than as tracers. Professor Carpenter feels, and rightly so, that employers of young graduates should be convinced of the value of viewing such graduates as professionally trained men, and of affording them an opportunity to rise from subprofessional to professional status.

On the other hand, practicing engineers and others seem to have in mind the engineer in his more mature years, when he is practicing as a specialist or as a

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"routineer," an engineering executive or administrator, a business man, or merely as a contented and worthwhile citizen.

Mr. Balog, when he sees the "highly educated designer" as a true representative of the civil engineering profession, apparently has in mind those young South American structural designers—twenty-five to thirty-five years old—who, he states, have developed a reinforced concrete design practice far superior to that of the level achieved in this country. In this assertion he ignores the efforts of those civil engineers, young and old, whose endeavors, in many instances involving more or less routine design, have resulted in many huge dams; water supply, flood control, irrigation, and power projects; highways; railroads; airfields; and port facilities; that in turn have done so much to establish the high standard of living prevalent in the United States. Perhaps Mr. Balog has in mind a definition of "design" which involves merely the use of formulas and charts to determine the dimensions of various members of a structure. Should not this word "design" include the conception, planning, arrangement, location, dimensions, use, and general economics of a structure or utility.

Mr. Wagner and Miss McIver, from their long and intimate experience with engineers of all ages, kinds, and capacities, and with large numbers of employers of engineers, have probably achieved a broader and deeper contact with these than is the case with most of the other discussers. It is significant, in view of this, that Mr. Wagner poses these extremely pertinent questions: "What is engineering?" "What is the engineering profession?" and "What is its scope?" When the answers to these questions are known with a reasonable degree of certainty, engineering societies and engineering educators will have a far sounder basis for their activities than they now have.

The writer throughout his paper and this discussion has referred to engineering as a profession, although for some considerable time there has been a question in his mind as to whether the practice of engineering involves the practice of a profession, or whether it involves a business in which a professional man is engaged. No attempt is made herein to answer this question; it is one which merits serious consideration by engineers, engineering societies, and engineering educators.

The statistical material used in this paper to present the composition and structure of the engineering profession, and the conditions that govern and prevail in actual practice, embrace, on a nationwide scale, a very large and representative segment of the engineering profession. Where derived data are based on sampling, the samples were of a size adequate to produce results of a quality sufficient to meet the purposes of this paper. Conclusions reached were substantially in agreement with independent studies made by Mr. Thomas and Miss McIver. This does not mean, however, that further work of this character is unnecessary. On the other hand, it is urgently needed, and the author would be the first to admit the inadequacy of his basic data. He submits, however, that the statistical material presented all points in the same direction, and, furthermore, conclusions reached coincide with experience.

National engineering societies during recent years have given far more attention to engineers, as against engineering, than was given in the past. This may be due, in part, to the large additions of younger men to their membership and the demand by these younger engineers for a display of such interest. Activity along these lines can be expected to continue to increase, if for no other reason than that changed conditions growing out of wartime employment will require it. These activities, however, cannot be carried on with the highest degree of effectiveness until more knowledge is obtained concerning the answers to the three questions posed by Mr. Wagner. An expenditure by national engineering societies of the order of $500,000 in this field over a five-year period would yield handsome returns to their members, to industry, and to the nation at large.

The paper, although by its title pertaining to the broad field of engineering education, was limited primarily to a discussion of the engineering profession, the practice of engineering, educational curricula, and course content. The writer also made recommendations as to modification or changes in these last two for the purpose of producing young engineers who, as time goes on, can more adequately meet the demand for engineering services in the postwar market. Had the paper fully covered the subject, it should have included a discussion of engineering instructors, engineering instruction, and the problem, strongly and properly emphasized by Mr. Boalich, of how better to control the admission to engineering courses of students who possess those tempermental and other personal characteristics which fit them to follow engineering and to be content in its practice.

Mr. Lynch suggests that preparation for collegiate training in engineering should commence in high school. This suggestion has a great deal of merit, although it probably will not be adopted until such time as engineering schools, societies, and state registration boards have come to work more closely together toward a common objective. Although there still exists a widespread feeling among engineers that professional status may be obtained without a formal engineering education—due probably to the still large number of men claiming such status without holding a degree—the next generation of engineers will have a far smaller percentage of men within its ranks who have not had a formal engineering education. This situation has already occurred in medicine, dentistry, accounting, and law. The young man who makes up his mind in high school to follow professional engineering and begins then to shape his course and plans for the "long pull" will be the one who attains professional status in the highest sense of the words.

A number of discussers refer to the desirability of developing imagination in students. Mr. Oesterblom talks about the "creative urge," and very rightly decries the tendency to "train a flexible mind into a rigid form." Imagination and vision are personal traits. Some persons possess them in a greater degree than do others, but even in those who inherently lack them, they can be cultivated by inspirational instruction. This latter is extremely difficult to secure; and only too often those instructors who possess the ability to give it either leave teaching or are advanced rapidly to positions on the faculty where their contacts are with older students.

Professor Lilly has pointed out the crux of this problem in his statement, "* * * quality in the student must come from quality in the faculty."

It is customary in developing faculties in engineering, as well as in other schools of collegiate grade, to assign as instructors in freshman subjects recent graduates, if possible men who have achieved scholastic honors. If and when these younger instructors attain experience and seniority, they are assigned to sophomore courses and, ultimately, with greater seniority and possibly with increasing reputation derived from research and writing, may arrive at teaching junior and senior courses.

Freshman and sophomore students from eighteen to twenty years old are passing through the most impressionable and idealistic period of their adult life. Seldom do they know just why they enrolled in an engineering course, although many of them may think they do. During these formative first two years, their contact is almost entirely with immature and inexperienced instructors; and it is upon such instructors that the responsibility largely rests of providing inspired instruction at the time when it is most needed and most effective. Even after the student has reached his junior year, his contacts are seldom with instructors who have had experience in the practice of engineering in a responsible capacity and who are familiar with problems commonly met in engineering practice. With only such contacts as described, how is the student, during the first two years, to determine whether he is fitted to, or will want to follow, engineering as a career and be happy in so doing.

Perhaps the lack of student interest during the first two years of the engineering course referred to by Professor Brinker and others is due more to uninteresting instructors and unimpressive instruction than to course content. Inspirational instruction is as much a matter of the personality of the instructor as it is of his reputation in or knowledge of his subject. It is also more a matter of the manner in which he presents his subject than the content of his lectures. The writer recalls his course in descriptive geometry, a subject usually disliked by most students. The instructor had never taught the course before, and had but a casual knowledge of the subject. He managed, however, to keep several pages ahead of the class in text, so they did not discover the shallowness of his knowledge. He had an engaging personality, and managed so to arouse the interest of his students in the subject and in himself that every student acquired a deep interest in the course, and a thorough knowledge of this normally dry and uninteresting, although valuable, subject. He also recalls a course in hydraulic machinery given by an instructor who was an outstanding authority, a man who had done much pioneering work in the design and development of pumping machinery and turbines. This instructor, however, droned through his lecture and filled the blackboard with formulas and mathematical developments of theory. Few in this class acquired much knowledge of the subject by the end of the course.

Were it possible to develop a faculty that had as one fourth to one third of its members engineers who had from fifteen to twenty years of experience in the practice of engineering, at least one half of which time was in a fairly responsible capacity, and if freshman and sophomore students were provided the opportunity to sit for three or four hours per week under some of these instructors, and under some of the older instructors who had achieved eminence in engineering—men to whom they could look with respect and even awe—

this lack of interest would not exist. Furthermore, if freshman and sophomore students were told—and they should be told repeatedly—just how their course had been developed, why each subject had been included therein, the interrelationship between all subjects in the course, and the relationship of the structure and pattern of the course to their future practice, they might within this two-year period gain a picture of what they were doing, why they were doing it, and the direction in which they were headed. In addition, it is believed that when the deans or department heads commenced to justify to themselves the courses given, prior to presenting such justification to the younger students, the conclusion might be reached that some of these courses could not be justified and they might be eliminated. It might also be discovered that other courses could only be justified because Professor Soandso, who has been giving a certain course for years, objected to teaching any other subject.

Mr. Perry states that the need of salesmanship by every man is but a partly true statement. A number of discussers decry the need of salesmanship by engineers, and question whether it can be taught. Salesmanship involves a knowledge of the product sold, of people, and the faculty of clear and logical expression, verbal and written. One never completes his education in these subjects any more than he does in engineering subjects, but the student is entitled to be told, with emphasis, of the importance to him of salesmanship. He should, at a minimum, be able to make out an effective application for employment, although only one graduate out of ten can do this. He should be able to write a clear and effective one-page letter, and to meet a prospective employer without stammering and stuttering. Few graduates can do these things. Criticism, by instructors, of the manner in which students recite and of the quality of expression found in their papers and reports will often prove as effective as formal courses in literature and expression.

The writer believes it requires salesmanship for a graduate to secure his first engagement, and that salesmanship is required in order for him to advance to higher positions. He has known and worked with many so-called financially successful and eminent engineers, and with many engineers who were far less successful financially and far less eminent, but who possessed equal ability and capacity. Too often it has been found that the only difference between men in the two groups is the amount of salesmanship practiced by the more financially succesful and more eminent engineers.

It is not so readily possible to determine the quality of engineering service as it is the quality of other professional service. A doctor loses or cures a patient. A lawyer wins or loses a case. Their successes or failures are soon known, and as these accumulate their professional reputation or standing is enhanced or disappears. Floods that wash out dams, earthquakes that destroy buildings, loadings that cause bridges to fail may not occur until years after the death of the engineer responsible for their design or construction. One engineer may develop an ore deposit in one manner and another develop it differently. In either case, the success or failure of the mine is usually credited to the character of the deposit, or to the market, not to the engineer. One engineer may plan, design, and construct a water supply project costing millions of dollars. This aids in making him successful and eminent, even though twenty years after the project has been completed, it is found to have been greatly overdeveloped, and cannot meet interest on its bonds. Another engineer may develop a much smaller project costing thousands instead of millions, but which is very successful and able to pay off its bonds before maturity. This latter engineer is seldon heard of unless he possesses the quality of salesmanship.

It is admitted that engineers cannot advertise the value and worth of their services in the same manner as do makers of soap and laxatives, but the "better mouse trap" theory became obsolete with the advent of newspaper and periodical advertising. Some engineers have jobs seek them; but the writer is familiar with too many engineers and engineering organizations, public and private, large and small, who have their salesmen or contactmen, sometimes dignified by the title of public relations counsel, to believe that work well done is the only requirement for securing more and better work to perform. Engineers write papers and publish books for the purpose of giving the profession the benefit of their experience and knowledge, or for the purpose of advertising themselves, but usually for both purposes. They join organizations, civic, religious, fraternal, social, and professional, to make contacts. The better an engineer is known the greater the opportunity he will have to perform more interesting and more important work; and he can become known more quickly and effectively if he tells the world about himself in a dignified and ethical manner. Too many instructors fail to realize that it is a wasted effort to teach a student how to design a structure, a piece of machinery or equipment, if the student cannot convince a prospective employer or client that he is capable of doing such a job and, therefore, should be given an opportunity to do it.

There appears to be, on the part of most of the engineering educators who discussed this paper, a mistaken view of the writer's proposal that the current four-year engineering course be limited to fundamentals in the broad field of engineering, and broadened to include subjects in business, the humanities, and expression. This proposal constituted only a part of the program suggested, and was urged in view of prevailing opinion and experience over several decades that few engineering students will, for economic and other reasons, spend more than four years in residence at any engineering school to secure an engineering education. Attention is called to the statements under the heading, "Postwar Engineering Education," that the postwar market for engineering services can be expected to require:

> "(1) A longer period of study and training for engineers * * *; (2) the broader knowledge of a wide range of both technical and nontechnical subjects necessary to provide maturity of judgment; and (3) the greater demand * * * for executives and administrators who have had background and experience in engineering."

The requirements set forth under items (2) and (3) are pointedly emphasized in a survey released[47] in May 1947, which covered personnel placement bureaus in one hundred and one colleges and universities in the United States. This study finds a notable trend on the part of employers toward seeking college graduates with engineering degrees for positions in sales work, office administration, and other nontechnical lines. It also finds that employers, except when seeking men for research and design activities, lay far more stress upon a graduate's ability to "get on" with people, his personality, and his extracurricular activities while in school, than they do upon his scholastic record. It also states that most large concerns that have instituted training programs for entering employees who are graduates give little weight to any specialization which a graduate may have undertaken while in college, preferring to train these graduates themselves in specialized subjects. These findings confirm those in the study made by the American Association of Engineers in 1940, which were quoted by the writer.

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[47] "Twelfth Annual Survey of College Placement Experience," Northwestern National Life Insurance Co., Minneapolis, Minn., 1947.

To meet these requirements, a four-year course as outlined was suggested, to be followed by an opportunity for postgraduate study, either in residence or through extension or correspondence, in specialties or in nontechnical subjects, with the goal of a professional degree when a suitable amount of such study had been satisfactorily completed. These two items are the keys to the program suggested in the paper, and must be considered as being inseparable.

To effect such a program would not call for anything revolutionary. It would mean harder work for instructors during the time necessary to adapt existing courses to new conditions, and to develop new courses. It would also call for some education of, and cooperation and understanding by, personnel directors and employers in the realization that the graduates of the four-year courses they would then be employing would not be so highly trained in certain specialties, but would have a broader training in fundamentals, could be more flexibly used, and could be expected to secure their specialized training during the early years of their employment. A few years of experience with graduates of this type should soon convince employers of their increased value. The greatest burden would fall upon the graduate working for his professional degree at the same time he was earning a living, and also upon his wife and family if he acquires these during such period. These should realize, however, that the sacrifices called for will be compensated later.

The discussion by Mr. Wagner of the composition and structure of the engineering profession, and the information which he presents pertaining to German technical education prior to World War I are of extreme interest. Fig. 1 was presented to illustrate the statement that the practice of engineering is a mass activity—not mass production as mistakenly inferred by Mr. Balog—and that any product created by engineers is the work of many persons with varying kinds and degrees of technical training, experience, and ability. Mr. Wagner states that if engineering is considered as being pyramidal in shape, as against the cubical shape of other professions and trades, a large number of engineering graduates, regardless of their qualities or abilities, are doomed to remain in positions in the middle or lower sections of the pyramid, because there is not room for them at the top. This fact is true of any group activity of this character. An army is composed of a few generals, a substantially larger number of captains and lieutenants, a still larger number of noncommissioned officers, and a great number of privates. A private may advance to be a general, but only if he has outstanding ability and undergoes arduous training. All privates and sergeants receive specialized training, but not that of a character provided for potential generals.

It may be, as Mr. Wagner states, that other forces and organizations will solve the problem of fitting engineering education to conditions prevailing in practice, without the aid of engineering schools or societies, and that this may develop a pattern somewhat similar to that existing in Germany before World War I, adapted, however, to American conditions. This does not remove from engineering schools or societies the responsibility of endeavoring to solve the problem. Naturally, however, until far more information is available as to the number of engineers required to perform technical duties in the various branches of engineering in each functional bracket (from nonprofessional up through subprofessional, and preprofessional, to professional), and the number of so-called "routineers" among the professional workers, and until more information is available upon which opinion can be developed as to future trends in these numbers, little more than an approach can be made to fitting the general pattern of engineering education to the requirements of the future market.

Adoption of the educational program suggested in this paper would undoubtedly decrease the number of those who embark upon an engineering course with the expectation of achieving full professional status, and would increase the number of those who will be satisfied to seek an education which will qualify them to perform subprofessional or routine professional duties. Mr. Perry questions the wisdom of decreasing the number of professional engineers without, at the same time, making plans to supply adequately the demands for a subprofessional group. He is correct in this view.

In medicine, as Mr. Wagner states, those who do not expect to achieve professional status are only required to follow an educational program equipping them for subprofessional work. Laboratory technicians, nurses, and pharmacists are not secured from the ranks of those who embark upon, but for some reason fail to complete, the entire medical course, or who, having secured their medical degree, lack certain qualities or characteristics necessary for professional practice, and fall back upon subprofessional work. Every person seeking to practice medicine knows at the outset of his premedical course, and usually in high school, that he must complete this course, his medical course, and a year or more of internship, and then submit to an extremely difficult and searching state examination before he can be admitted to practice. He also knows that he can specialize only after further and prolonged study. One does not hear of the need of providing undergraduate courses in medicine in order to maintain the interest of the student.

Professor Grinter agrees that courses in public speaking, English literature, technical writing, history, and public affairs, when given under cultured and stimulating teachers, would produce results in broadening students; but he questions how a student can study such courses in sufficient quantity and still "end up as a trained engineer." This raises the question, "Can a trained engineer be produced by having a student successfully pursue four years of study at an engineering school, even at a school of the highest caliber?" The answer is manifestly in the negative. The author believes, however, that if a student should follow a four-year course, such as is outlined in this paper, supplemented by graduate study of sufficient scope, character, and time to attain the professional degree, he will more closely approach the status of a trained engineer than will the graduate of current four-year courses after he has practiced from six to ten years.

Mr. Williams states that engineering education consists of cultivating the "growth" of the faculties to observe, to think, and to express thoughts, and of the personal attributes essential to citizenship and a full life. This is a most worthy objective, but if the writer's contacts—with thousands of graduates of all ages, in all branches of the profession, and holding all degrees of responsibility—are a guide, such objectives have been attained in an extremely small percentage of engineering graduates.

Professor Dougherty suggests that the writer has overlooked many changes that have taken place in engineering education. A review of courses offered in a considerable number of engineering schools; discussions with younger graduates; and a reading of news items in technical journals pertaining to engineering faculty additions, to improvements in school facilities and equipment, and to changes in courses, give the impression that there has not been any material change in engineering education since the period prior to World War I, beyond that of keeping well abreast of technical developments and practices.

Professor Dougherty also suggests that Fig. 1 be presented with professional age as the ordinate. This might be accomplished by use of material presented in Table 3(*b*), but it would serve little purpose. The presentation of Fig. 1, which was not drawn to scale, was, as heretofore stated, to indicate the fact that, as the relative amount of engineering work performed and the degree of responsibility attached to such work increased, the number of engineers required materially decreased. There is, of course, some degree of correlation between age, on the one hand, and the importance of work performed and the responsibility assumed, on the other, but no data are readily available, by which such correlation could be developed. The research suggested by the writer would involve the collection of analysis and interpretation of this type of data; but, until such research has been conducted, a great deal of the activities carried on to improve the profession technically, socially, and economically will be performed in the dark, or at least in the twilight.

Professor Lilly with much truth states that "Education should prepare a man for his leisure hours as well as his working hours." Miss McIver has pointed out that it is not now the fashion to portray engineers as "roughnecks," and that more and more stress is being placed by employers upon appearance. If engineering education is to provide a vocabulary, as mentioned by Professor Lilly, and also to provide for leisure hours, it would seem that the students should have an opportunity to acquire a nontechnical vocabulary; for many of his contacts following graduation will include those who do not possess or understand a technical vocabulary and with whom communication must be had in nontechnical terms. Furthermore, if the student has never been given even a taste or a glimpse of good literature, of music, history, or art, how will he know that these exist, or where to turn to gain an acquaintance with them. Even a course offering contact with these things and involving one hour a week during four years, where no problem sets were assigned and no grades given except for attendance, would develop an interest on the part of many graduates which would greatly help them in preparing their leisure hours in later years.

Mr. Conwell indicates that the program suggested in this paper shifts the responsibility for developing nontechnical requirements of the engineering profession and for the cultural background of the student from the latter to the school, and shifts some of the responsibility for obtaining technical knowledge from the school to the employer. The writer agrees with this, and believes that it is proper. Under current practice, the student, an immature youth of seventeen or eighteen, presents himself to an engineering school and says "I want to be an engineer. Will you please make one out of me." The school proceeds to work on, and sometimes with, him for four years, filling him with technical theories and practice. At the end of the four years' time it takes those who have survived, the school gives each an engraved piece of parchment with his name on it, and tells him,

> "We have done what we could. We know that you can hold down the first job you are offered, and we hope that you can progress to higher and more responsible positions. We know that some of our graduates have done so, although we have not heard from most of the others."

The employer greets the graduate with open arms, and with high hopes that he has hired a future chief engineer, manager, or even president, and then, in most cases, promptly forgets him, although continuing to grouse about the inadequacies of engineering schools. The world seems very cold to the graduate, except for the opportunity sometimes provided once a month, if he resides in a fair-sized community, of attending the meeting of the local engineers' club or of the local section of a national engineering society. At such meetings he may sit next to his boss at dinner, talk about something else besides his daily tasks, and may also listen to some older fellow engineer "advertise" himself by telling about work he has recently performed. The graduate is on his own. His alma mater and his employer, as well as his profession, do have a responsibility toward him which they have, to a great degree, left unfulfilled.

Mr. Amirikian emphasizes the "practical problem of placement of the finished product," referring to graduates in law, medicine, and engineering. The engineering graduate is accepted by most employers as an apprentice in a trade, with the difference that he has acquired a familiarity with some of the tools which he will use in his trade, and is able to use a few of the simpler ones in minor operations. A further difference in this analogy, however, is that a plumber's apprentice can only become a plumber and not a bricklayer, whereas a civil engineering graduate may, and frequently is, apprenticed in mechanical, electrical, mining, or chemical engineering, and is then required to learn the use of a new type of tools.

The discussion by Mr. Thomas illustrates what can be developed concerning engineers and the practice of engineering from existing basic data. Table 7, showing that since 1929 the Society has taken three new members to gain one, serves to raise some extremely pertinent questions, among which is included, "How many engineers must be graduated from the current engineering courses to produce one professional engineer successfully practicing at the age of, say, forty seven?" An analysis of the relation of losses in Society membership to total membership, other than from death, shows that the high loss ratio was not due to the depression of the 1930's. Table 8 indicates that one half the number of engineers who were practicing their profession at the age of thirty had left their profession fifteen years later, or by the time they were forty five, and that such loss ratio increases for engineers over thirty years old. Although the basic data upon which these conclusions rest may not be a strictly representative sample, nevertheless, the conclusions are interesting and significant, and involve a sample of sufficient size to warrant their being given serious consideration. Mr. Thomas' study resulting in Fig. 2, which shows the increase of employment of younger civil engineers in public service, might well be extended to other branches of engineering. Fig. 2 shows the deterioration of private practice in civil engineering as a field of opportunity for younger engineers. Such trend commenced long before the depression, so it cannot be charged to conditions created thereby.

Miss McIver's discussion of the methods and source of material used by the American Association of Engineers in its studies of engineers and of the practice of engineering indicates what can be accomplished through the use of existing data. The writer is in thorough agreement with her conclusions as to the disadvantages of the use of questionnaires to which voluntary replies are requested, and as to the unreliability of results secured where basic data are collected by this method. The unreliability of these results was established in a dramatic manner by a poll of the 1936 presidential election. Such polls had been taken a number of times previously and showed a high degree of accuracy with election results, but the sampling in the 1936 poll was apparently very unrepresentative.

Past studies of engineering compensation, based on voluntary replies to questionnaires, bear no resemblance to the material presented in Table 3(*c*), which is based on the salaries of every technical engineer in the country. Those receiving income disproportionately low compared to their age, length of experience, or position held, either do not reply or give higher than actual figures, while recipients of high income always proudly answer. The results, therefore, are heavily weighted by replies from those receiving higher incomes. Even if sampling by this method is truly representative, there is no way of determining this fact. Polls compiled by George Gallup and by the magazine *Fortune*, based on samples carefully selected so as to be representative, although extremely small in number compared to the universe sampled, are statistically sound and have been proved to be highly accurate. Furthermore, sampling, whether it be on the basis of voluntary replies to questionnaires, or through personally interrogating the respective sample, only gives current views, attitudes, and facts. Trends can best be determined by utilization of existing basic data.

Experience records of engineers as a source of basic data in the study of the composition and structure of the engineering profession were suggested as the best available information. These records include the type and kind of work performed by engineers, the extent of responsibility which was placed upon them, the relative importance of the work which they were performing at various stages of their careers, the general rate of progress which they have made in their profession, the changes from one branch or specialty to another, and the changes from one location or employer to another. They were not suggested, as Mr. Balog has assumed, as a source of information as to the accomplishments or capabilities of engineers. The writer agrees with Mr. Balog in his statement that:

> "Anywhere and everywhere, structures can be observed which disclose the need for engineering services—instances where the use of expert engineering talent would have resulted in savings, improved appearance, and increased engineering employment."

If engineers were better salesmen this statement would not be true. Agreement is not possible, however, with Mr. Balog's statement that:

> "The management of engineering organizations, at present, may be a purely parasitic activity in which mere trader's cunning, or certain dexterity in the use of technical terms, is sufficient qualification for a position above the engineer."

The writer's statement—

> "The current practice of engineering, except by those who serve in advisory or consulting capacities, is a mass-production activity, in which each individual serves as a cog in a machine and performs his specific duties as a unit of a much larger group"

—was not quoted in its entirety by Mr. Balog. The creations of engineers, particularly when they involve structures or facilities, are not those of mass production, but are those of an organized group of technical men working with others. Before a large engineering structure can be designed and built, its location, site, income possibilities, and many other features must be studied. Alternate preliminary plans and designs must be prepared, the economics of each plan and design investigated. Final plans are then prepared and a cost estimate made. The structure then must be built, and working plans prepared, foundation conditions explored, and changes made in the design during construction.

Tracers, draftsmen, designers—structural, mechanical, electrical, and others—rodmen, instrumentmen, chiefs of party, laboratory technicians, concrete technologists, resident engineers, inspectors, and a multitude of other personnel are required, some performing perfunctory duties, some routine work, and others technical work requiring extremely high skill, knowledge, and judgment. The designer is but one member of the team—an important one it is true, but no more important than many others. At the top is the manager. He coordinates, plans, drives, and pushes, makes the important decisions, or confirms those made by others. If he devoted his time primarily to the engineering phases of the operation which he was managing and, as Mr. Balog suggests, "assigns subordinates to the nontechnical duties of the managerial position," one wonders if the job would ever be completed, and if so at what cost.

Again, the questions posed by Mr. Wagner arise. Was the chief engineer of the Golden Gate Bridge (San Francisco, Calif.) practicing engineering only during the times when he stopped in the drafting room to discuss with his chief designer the arrangement, size, and thickness of plates to be used in the design of the towers, and the unit stresses of steel to be allowed in the cables of this, the longest suspension bridge in the world? Or was he practicing engineering during the entire period from the time such bridge was conceived and determined in his mind to be feasible until the time it was completed and opened to traffic?

Mr. Golzé well expresses the viewpoint of the engineering employer when he speaks of the need of the U. S. Bureau of Reclamation, a large public engineering organization, which is passing into the stage of operating the engineering projects it has built, and clearly foresees the need for engineers who can administer and operate these projects, as well as design and build them.

Mr. Dykstra gives an excellent description of the type of engineer whose services will be required in coming years when he states,

> "In modern technological civilization there is great need for the technicians called engineers, but, to master this mechanical civilization, technical men must be aware of the implications of modern life and they must feel at home in a changing world. A structural engineer should know more than how to build a bridge. He should be able to state whether a bridge is necessary, what will happen if it is built, and, perhaps, be able to suggest an alternate solution for what is traditionally thought of as a bridge problem."

During the fall of 1946, the writer encountered an article[48] from which the following excerpts are quoted:

> "Because the engineering schools have always made it their chief aim to impart the technical information needed in industrial production, and because both scientific knowledge and industrial practice have grown so rapidly, the attention of technical schools has been focused chiefly upon keeping up to date in science and practice. The university emphasis on research in natural science has also tended to magnify the importance of technique and to minimize the importance of personality * * *. Therefor it seems necessary to consider the question whether this emphasis on technique is producing a new and higher type of engineer, or whether the engineering profession still stakes its faith on the fundamental thesis that personal character is, after all, the real foundation for achievement * * *."
>
> "Progress is being made towards the conception that there is really but one profession of engineering, in spite of its apparent division into the several well known branches. War conditions have not only hastened public recognition of the engineer as an expert in applied science and fostered solidarity in the profession, they have also opened to him new fields of activity * * *."
>
> "Hence there has arisen a pressing demand for men who can deal with labor and with business administration in the engineering spirit * * *."
>
> "These new opportunities for the engineer have been gradually developing for a number of years, but the profession as a whole has been slow to discern them. The war has focused attention on them and precipitated a general recognition of them. It is also evident that the mastery of these new activities depends in greater measure than does mastery of the traditional types of engineering on the personality of the man * * *. Therefor as engineering expands into new fields now opening before it, the conception that character, judgment, efficiency, and the understanding of men are no less necessary than technical knowledge and skill will become more and more impelling, and it will become more and more essential that schools of engineering pay greater attention to the effect of their work on the personal development of students * * *."

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[48] *Engineering News-Record*, October 24, 1918, p. 742.

> "The net result is that curricula and methods of instruction that were devised to supply the intellectual element in production by imparting knowledge of natural science must be reorganized to meet the new industrial demand for engineering administrators and the larger professional demand for men of strong personality * * *."

It should be noted that these excerpts appeared in an article published in October, 1918, and are quotations from the report on the three-year study of engineering education made by Charles R. Mann, under the joint auspices of the Carnegie Foundation for the Advancement of Teaching and the Joint Committee on Engineering Education of the national engineering societies.

The writer feels that Mr. Mann's statements, made less than a month prior to Armistice Day of World War I, are even more pertinent today.

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Paper No. 2313

MOMENT-STIFFNESS RELATIONS IN CONTINUOUS FRAMES WITH PRISMATIC MEMBERS

By George H. Dell,[1] Assoc. M. ASCE

With Discussion by Messrs. I. Oesterblom, William A. Conwell, and George H. Dell.

Synopsis

The design of a continuous frame usually involves a process of successive revisions and analyses, starting with an estimate of the stiffnesses of the various members. On the basis of the stresses obtained from an analysis of this structure, a preliminary design is made. The preliminary design is next analyzed, and it is usually found that some of the members are unnecessarily strong and some are deficient in strength. The designer then makes whatever changes seem appropriate in those members and reanalyzes the structure. This process may have to be repeated several times before the design and the analysis are in agreement.

When the stiffness of one or more members is changed, the moments in all the members are affected. Since the moment at a given section in a continuous structure is a function of the stiffnesses of the various members, the differential of the moment (or moment differential) is a function of the differential changes in the stiffnesses. The resulting relation between the differential of the moment and the differentials of the stiffnesses is designated a "moment-stiffness relation." In some types of structures composed of prismatic members, the analytical part of the design procedure may be facilitated by the use of such moment-stiffness relations. Moment-stiffness relations may also be used in connection with studies aimed at improving the economy of an existing design.

A method of determining the moment-stiffness relations, using moments created by unit couples applied at the several joints, and by unit horizontal forces at the joints in cases involving sidesway, is presented in this paper. The numerical values of the moments resulting from these unit couples and from unit horizontal forces are available for use in obtaining the stresses pro-

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duced in the original structure by the various combinations of dead load and live load.

The paper also includes a description and illustration of a procedure whereby the moment-stiffness relations can be used to bring the design from the preliminary stage to the final stage.

Since the shears, reactions, and deflections may be obtained from the moments, the differential changes in these quantities may likewise be calculated from the differential changes in the moments, if desired.

The types of structures discussed are those in which the members, assumed to be prismatic, are subjected to bending and shearing forces in a single plane and which may be analyzed from a consideration of the equilibrium of moments and horizontal forces at the various joints—for example, continuous beams, rectangular bents of one or more stories, and saw-tooth bents such as those shown subsequently in Figs. 1(*a*), 2(*a*), 3(*a*), and 5.

Experience indicates that moment-stiffness relations are a useful aid in the design of continuous framed structures composed of prismatic members, particularly in cases where the columns are subject to moments of considerable magnitude. These relations also provide a basis for the determination of possible improvements in an existing design.

As differential quantities are only approximate representations of the true changes, the relative stiffnesses upon which the moment-stiffness relations are based should be chosen with some care, or they should be determined from a preliminary design, and the final results should generally be checked by a direct analysis. It should be recognized that, in some instances, where large stiffness changes occur, the moment differentials will be subject to rather large errors and also that, where only minor changes in stiffness occur, the use of moment-stiffness relations is not particularly advantageous unless subsequent economy studies are to be made.

Conventions and Symbols

The sign convention used for moments is as follows: Tension on the upper side of a horizontal member corresponds to negative moment at the left end and positive moment at the right end; for vertical or inclined members, tension on the left face corresponds to negative moment at the lower end and positive moment at the upper end. Horizontal shears at the ends of vertical or inclined members are considered positive when acting toward the right. Letter symbols are defined where they first appear, and are assembled alphabetically, for convenience of reference, in Appendix 1.

Moment-Stiffness Relations

A general expression for the moment-stiffness relation (that is, the relation between the moment differential and the changes in the stiffnesses of the various members) may be derived by analogy with the analytical process used in the determination of the moments. With the general expression as a guide (see Eq. 7), the moment-stiffness relations for a particular structure may be evaluated most conveniently by moment-distribution procedures.

A brief description of the types of structures dealt with herein has been given in the "Synopsis." With regard to any typical member, AB (horizontal, vertical, or inclined), in such structures, the moment at point A is the sum of C_{AB}, the fixed-end moment due to loads between points A and B, and M'_{AB}, the elastic moment or "M'-moment." The latter (M'_{AB}) is equal to the flexural stiffness $\left(K = \frac{E\,I}{L}\right)$ of the member, times a linear function, ψ, of the elastic deformations. The fixed-end moments being constant, the elastic deformations are functions of the stiffnesses of the various members. The following expression is thus obtained for M_{AB}:

$$M_{AB} = C_{AB} + K_{AB}\,\psi_{AB} \qquad (1)$$

The corresponding moment differential is $dM_{AB} = \psi_{AB}\,dK_{AB} + K_{AB}\,d\psi_{AB}$ which, by putting ψ_{AB} equal to M'_{AB}/K_{AB} and dK_{AB} equal to $i_{AB}\,K_{AB}$, becomes

$$dM_{AB} = M'_{AB}\,i_{AB} + K_{AB}\,d\psi_{AB} \qquad (2)$$

In Eq. 2, i is the "stiffness-change ratio."

The horizontal shear at point G in a vertical or inclined member GH, denoted by H_{GH}, is the sum of the simple beam shear and the shear due to the end moments, M_{GH} and M_{HG}. The end moment is the sum of the fixed-end moment C and the M'-moment. Similarly, by defining the fixed-end shear, Q, as the sum of the simple beam shear and the shear created by the fixed-end moments, the horizontal end shear H may be defined as the sum of Q_{GH}, the fixed-end horizontal shear, and H'_{GH}, the horizontal shear due to the M'-moments. Shear H'_{GH} is then equal to the flexural stiffness, K_{GH}, times a linear function, ϕ_{GH}, of the elastic deformations. The following expression is thus obtained for H_{GH}:

$$H_{GH} = Q_{GH} + K_{GH}\,\phi_{GH} \qquad (3)$$

The corresponding shear differential is $dH_{GH} = \phi_{GH}\,dK_{GH} + K_{GH}\,d\phi_{GH}$, which, by making ϕ_{GH} equal to H'_{GH}/K_{GH} and dK_{GH} equal to $i_{GH}\,K_{GH}$, becomes

$$dH_{GH} = H'_{GH}\,i_{GH} + K_{GH}\,d\phi_{GH} \qquad (4)$$

An analysis of the moments by either the slope-deflection method or moment-distribution procedures is equivalent to making ΣM equal to zero at the various joints and Σ(horizontal forces) equal to zero for each condition of sidesway or horizontal joint displacement, and solving for the M'-moments, which are then added to the fixed-end moments. Thus, there are two separate classes of equations or operations, which, for convenience, will be designated as "moment equations" and "shear equations." In the case of a moment equation for a given joint J, the constant term, or unbalanced moment, is ΣC_{Jn}—the sum of the fixed-end moments at that joint. In the case of a shear equation for a given joint or section, the constant term, or unbalanced force, is ΣQ, the sum of the fixed-end horizontal shears, together with any external horizontal forces that may be acting on the joint or section in question.

The moments may be obtained by superposition. Let $m_{A,AB}$, $m_{B,AB}$, . . ., be the moments at joint A in the member AB due to unit couples applied at joints A, B, . . ., respectively; and let $m_{\Delta 1,AB}$, $m_{\Delta 2,AB}$, . . ., be the moments at joint A in the member AB due to unit horizontal shears, or joint forces, corresponding to conditions Nos. 1, 2, . . ., of sidesway or horizontal joint displacement. The following relations may then be stated:

A. The M'-moment at point A in member AB, due to an unbalanced moment of ΣC_{Jn} at joint J, is equal to $m_{J,AB}\,\Sigma C_{Jn}$.

B. The M'-moment at end A in member AB, due to an unbalanced horizontal force of ΣQ_1, is equal to $m_{\Delta 1,AB}\,\Sigma Q_1$.

The total value of M'_{AB} or $K_{AB}\,\psi_{AB}$, in Eq. 1, due to all the unbalanced moments and horizontal forces is thus given by the following expression:

$$M'_{AB} = m_{A,AB}\,\Sigma C_{An} + m_{B,AB}\,\Sigma C_{Bn} + \cdots + m_{\Delta 1,AB}\,\Sigma Q_1 + m_{\Delta 2,AB}\,\Sigma Q_2 + \cdots \quad (5a)$$

The moment M_{AB} may then be expressed as follows:

$$M_{AB} = m_{A,AB}\,\Sigma C_{An} + m_{B,AB}\,\Sigma C_{Bn} + \cdots + m_{\Delta 1,AB}\,\Sigma Q_1 + m_{\Delta 2,AB}\,\Sigma Q_2 + \cdots + C_{AB} \quad (5b)$$

The way is now paved for describing an analogous procedure of obtaining the moment differentials, or moment-stiffness relations. From a comparison of Eqs. 1 and 2, it is noted that the quantity $M'_{AB}\,i_{AB}$, in Eq. 2, corresponds to the fixed-end moment C_{AB}, in Eq. 1; and the quantity $K_{AB}\,d\psi_{AB}$, in Eq. 2, corresponds to the M'-moment, represented by the term $K_{AB}\,\psi_{AB}$, in Eq. 1. Likewise, from a comparison of Eqs. 3 and 4, it is seen that the quantity $H'_{GH}\,i_{GH}$, in Eq. 4, corresponds to the fixed-end shear Q_{GH}, in Eq. 3; and the quantity $K_{GH}\,d\phi_{GH}$, in Eq. 4, corresponds to H'_{GH}, or the term $K_{GH}\,\phi_{GH}$, in Eq. 3.

It can be shown that, just as the moments are obtainable by making ΣM equal to zero at the various joints, and Σ(horizontal forces) equal to zero for each condition of sidesway or horizontal joint displacement, the moment differentials are obtainable by making $\Sigma(dM)$ equal to zero and $\Sigma(dH)$ equal to zero for each moment equation and each shear equation, respectively. Corresponding to the moment equation for a given joint J, the unbalanced moment is $\Sigma(M'_{Jn}\,i_{Jn})$; and, corresponding to the shear equation for a given joint or section, the unbalanced shear is $\Sigma(H'\,i)$.

In connection with the determination of the moment differential dM_{AB} by the principle of superposition, the following relations may now be stated:

A′. The part of the quantity $K_{AB}\,d\psi_{AB}$ in Eq. 2, due to an unbalanced moment of $\Sigma(M'_{Jn}\,i_{Jn})$ at joint J, is equal to $m_{J,AB}\,\Sigma(M'_{Jn}\,i_{Jn})$.

B′. The part of the quantity $K_{AB}\,d\psi_{AB}$ in Eq. 2, due to an unbalanced horizontal shear of $\Sigma(H'_1\,i_1)$, is equal to $m_{\Delta 1,AB}\,\Sigma(H'_1\,i_1)$.

The total value of the quantity $K_{AB}\,d\psi_{AB}$ in Eq. 2, due to all the unbalanced differential moments and horizontal shears, is therefore:

$$K_{AB}\,d\psi_{AB} = m_{A,AB}\,\Sigma(M'_{An}\,i_{An}) + m_{B,AB}\,\Sigma(M'_{Bn}\,i_{Bn}) + \cdots + m_{\Delta 1,AB}\,\Sigma(H'_1\,i_1) + m_{\Delta 2,AB}\,\Sigma(H'_2\,i_2) + \cdots \ldots\ldots\ldots\ldots (6)$$

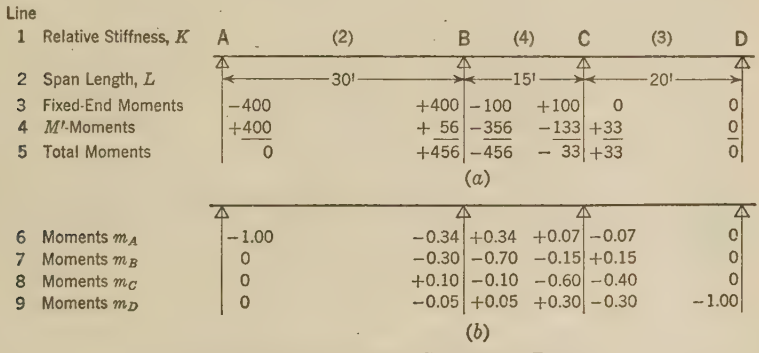
In accordance with Eq.2, the total moment differential, or moment-stiffness relation, for M_{AB} is then as follows:

$$dM_{AB} = m_{A,AB}\,\Sigma(M'_{An}\,i_{An}) + m_{B,AB}\,\Sigma(M'_{Bn}\,i_{Bn}) + \cdots + m_{\Delta 1,AB}\,\Sigma(H'_1\,i_1) + m_{\Delta 2,AB}\,\Sigma(H'_2\,i_2) + \cdots + M'_{AB}\,i_{AB} \ldots\ldots\ldots\ldots (7)$$

As a check, it should be noted that the sum of the coefficients of i in a complete moment-stiffness relation is theoretically equal to zero.

Referring to Eq. 7, the procedure for obtaining the moment-stiffness relation for a given monent M_{AB}, due to a particular arrangement of loading producing given M'-moments in the various members, will be clarified by the following rules:

1. Place a unit couple successively at joints A, B, . . ., N and distribute moments throughout the structure in each instance. This step provides the first class of m-moments for the ends of all members in the structure. In obtaining the moment-stiffness relation for M_{AB}, the particular values to be taken from this step are $m_{A,AB}$, $m_{B,AB}$, . . ., $m_{N,AB}$. If the structure is subject to sidesway, the correction for sidesway should be made in finding these m-moments.



Line		A		B		C	D
1	Relative Stiffness, K	(2)		(4)		(3)	
2	Span Length, L	30'		15'		20'	
3	Fixed-End Moments	−400	+400	−100	+100	0	0
4	M'-Moments	+400	+ 56	−356	−133	+33	0
5	Total Moments	0	+456	−456	− 33	+33	0

(a)

Line		A		B		C	D
6	Moments m_A	−1.00	−0.34	+0.34	+0.07	−0.07	0
7	Moments m_B	0	−0.30	−0.70	−0.15	+0.15	0
8	Moments m_C	0	+0.10	−0.10	−0.60	−0.40	0
9	Moments m_D	0	−0.05	+0.05	+0.30	−0.30	−1.00

(b)

FIG. 1.—SIMPLY-SUPPORTED CONTINUOUS BEAM

2. Place a unit horizontal force successively at appropriate joints of the structure, corresponding to the various conditions of sidesway or horizontal joint displacement to which the structure is subject, and find the moments thus produced throughout the structure in each instance, when the structure is in equilibrium. This step provides the second class of m-moments for the ends of all members in the structure. For obtaining the moment-stiffness relation for M_{AB}, the particular values to be taken from this step are $m_{\Delta 1,AB}$, $m_{\Delta 2,AB}$, etc.

3. At joint A take the product of $m_{A,AB}$ (see step 1) times the sum of the quantities $M'\,i$ for the various members connecting at that joint; similarly, take products for each joint of the structure that is required to be balanced.

4. Corresponding to each condition of sidesway or horizontal joint displacement to which the structure is subject, take the product of $m_{\Delta, AB}$ (see step 2) times the sum of the quantities $H' i$ for the various members entering into the given sidesway condition.

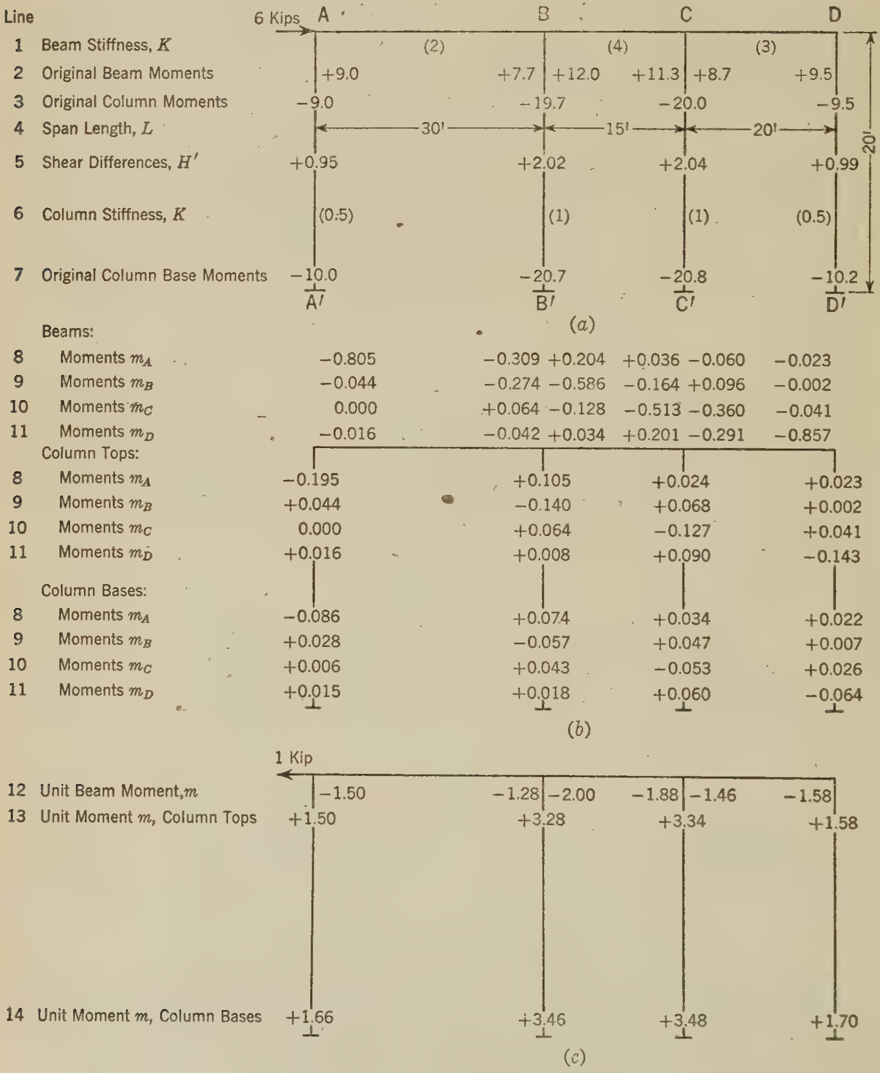

Line
6 Kips
A
B
C
D
1 Beam Stiffness, K
(2)
(4)
(3)
2 Original Beam Moments
+9.0
+7.7
+12.0
+11.3
+8.7
+9.5
3 Original Column Moments
−9.0
−19.7
−20.0
−9.5
4 Span Length, L
30'
15'
20'
20'
5 Shear Differences, H'
+0.95
+2.02
+2.04
+0.99
6 Column Stiffness, K
(0.5)
(1)
(1)
(0.5)
7 Original Column Base Moments
−10.0
−20.7
−20.8
−10.2
A'
B'
C'
D'
(a)
Beams:
8 Moments m_A
−0.805
−0.309 +0.204
+0.036 −0.060
−0.023
9 Moments m_B
−0.044
−0.274 −0.586
−0.164 +0.096
−0.002
10 Moments m_C
0.000
+0.064 −0.128
−0.513 −0.360
−0.041
11 Moments m_D
−0.016
−0.042 +0.034
+0.201 −0.291
−0.857
Column Tops:
8 Moments m_A
−0.195
+0.105
+0.024
+0.023
9 Moments m_B
+0.044
−0.140
+0.068
+0.002
10 Moments m_C
0.000
+0.064
−0.127
+0.041
11 Moments m_D
+0.016
+0.008
+0.090
−0.143
Column Bases:
8 Moments m_A
−0.086
+0.074
+0.034
+0.022
9 Moments m_B
+0.028
−0.057
+0.047
+0.007
10 Moments m_C
+0.006
+0.043
−0.053
+0.026
11 Moments m_D
+0.015
+0.018
+0.060
−0.064
(b)
1 Kip
12 Unit Beam Moment, m
−1.50
−1.28
−2.00
−1.88
−1.46
−1.58
13 Unit Moment m, Column Tops
+1.50
+3.28
+3.34
+1.58
14 Unit Moment m, Column Bases
+1.66
+3.46
+3.48
+1.70
(c)

FIG. 2.—RECTANGULAR RIGID FRAME

5. To the sum of all the terms in steps 3 and 4 add the term $M'_{AB}\, i_{AB}$, and place the resulting expression equal to dM_{AB}. A check is obtained by the sum of the coefficients of i, which is theoretically equal to zero.

The application of the foregoing steps to a variety of structures is explained in the following examples, after which the use of moment-stiffness relations in connection with design will be explained and illustrated.

Example 1. Simply-Supported Continuous Beam.—In this problem it is required to calculate the moment-stiffness relation for M_{BA} in Fig. 1(*a*) if the spans AB and BC carry a uniformly distributed load of 5$\frac{1}{3}$ kips per ft. Fig. 1(*a*) shows the fixed-end moments, the M'-moments, and the total moments. The relative stiffnesses are shown in parentheses (line 1, Fig. 1(*a*)). The m-moments in Fig. 1(*b*) were obtained by placing a unit counterclockwise couple at successive joints. In accordance with Eq. 7, the required moment-stiffness relation is: $dM_{BA} = -0.34\,(400\,i_{AB}) - 0.30\,(56\,i_{AB} - 356\,i_{BC}) + 0.10\,(-133\,i_{BC} + 33\,i_{CD}) - 0.05\,(0\,i_{CD}) + 56\,i_{AB}$; or $dM_{BA} = -96.8\,i_{AB} + 93.5\,i_{BC} + 3.3\,i_{CD}$. As a check: $-96.8 + 93.5 + 3.3 = 0$.

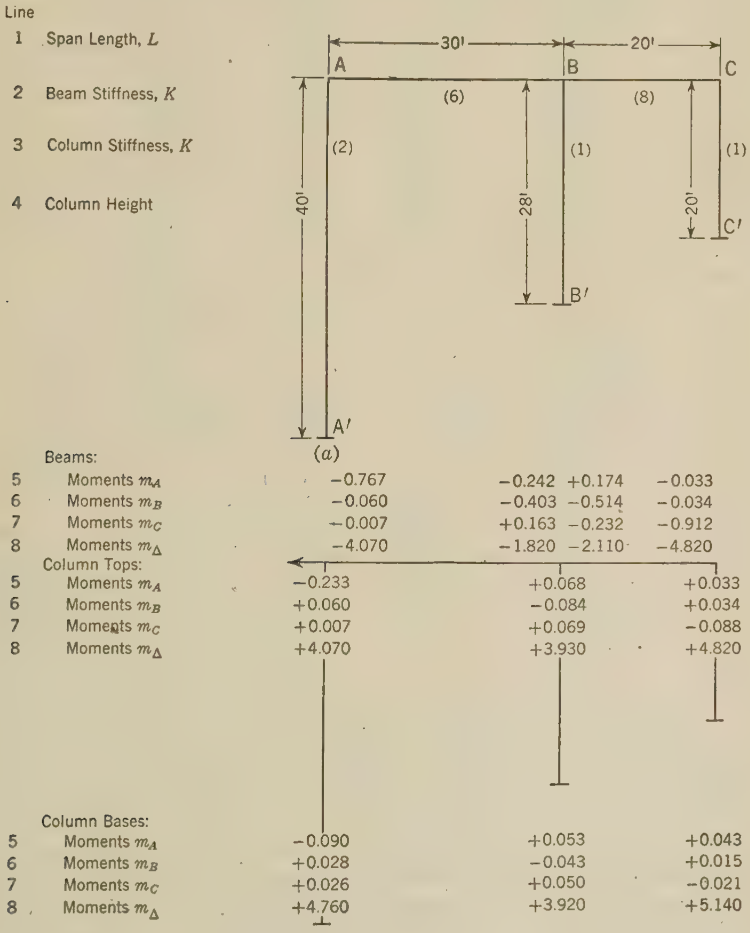

Line
1 Span Length, L
2 Beam Stiffness, K
3 Column Stiffness, K
4 Column Height
30'
20'
A
B
C
(6)
(8)
(2)
(1)
(1)
40'
28'
20'
C'
B'
A'
(*a*)
Beams:
5 Moments m_A
6 Moments m_B
7 Moments m_C
8 Moments m_Δ
−0.767
−0.060
−0.007
−4.070
−0.242 +0.174
−0.403 −0.514
+0.163 −0.232
−1.820 −2.110
−0.033
−0.034
−0.912
−4.820
Column Tops:
5 Moments m_A
6 Moments m_B
7 Moments m_C
8 Moments m_Δ
−0.233
+0.060
+0.007
+4.070
+0.068
−0.084
+0.069
+3.930
+0.033
+0.034
−0.088
+4.820
Column Bases:
5 Moments m_A
6 Moments m_B
7 Moments m_C
8 Moments m_Δ
−0.090
+0.028
+0.026
+4.760
+0.053
−0.043
+0.050
+3.920
+0.043
+0.015
−0.021
+5.140
(*b*)

FIG. 3.—RIGID FRAME WITH UNEQUAL LEGS

Example 2. Rectangular Rigid Frame.—This problem requires the computation of the moment-stiffness relations for M_{BC} and $M_{A'A}$ for the loading arrangement in Fig. 2(*a*). The moments produced in the original structure by the horizontal force of 6 kips are shown in Fig. 2(*a*). All the moments in

Fig. 2(*a*) are also M'-moments. The fixed-end moments C are all zero, since there are no intermediate loads. The values of H' at the tops of the columns are written in line 5, Fig. 2(*a*).

The m-moments due to a unit couple at joints A, B, C, and D, respectively, are shown in Fig. 2(*b*). These moments have been corrected for sidesway. Fig. 2(*c*) gives the m-moments due to a unit horizontal force at the top of column AA′.

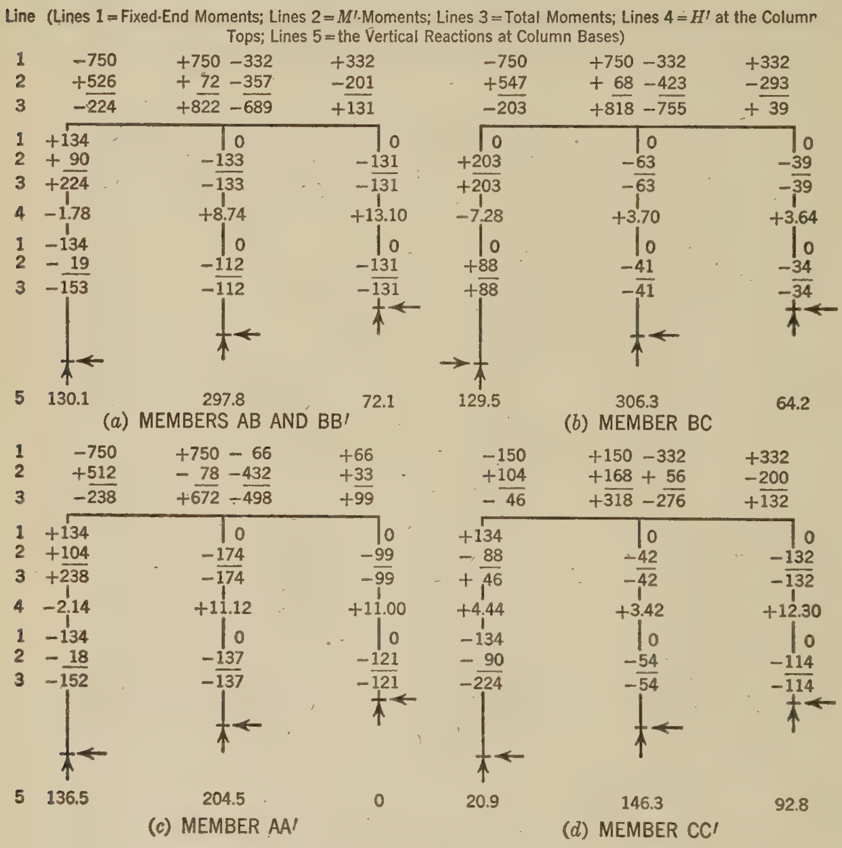

Line (Lines 1 = Fixed-End Moments; Lines 2 = M′-Moments; Lines 3 = Total Moments; Lines 4 = H′ at the Column Tops; Lines 5 = the Vertical Reactions at Column Bases)
1 −750 +750 −332 +332 −750 +750 −332 +332
2 +526 + 72 −357 −201 +547 + 68 −423 −293
3 −224 +822 −689 +131 −203 +818 −755 + 39
1 +134 0 0 0 0 0
2 + 90 −133 −131 +203 −63 −39
3 +224 −133 −131 +203 −63 −39
4 −1.78 +8.74 +13.10 −7.28 +3.70 +3.64
1 −134 0 0 0 0 0
2 − 19 −112 −131 +88 −41 −34
3 −153 −112 −131 +88 −41 −34
5 130.1 297.8 72.1 129.5 306.3 64.2
(a) MEMBERS AB AND BB′
(b) MEMBER BC
1 −750 +750 − 66 +66 −150 +150 −332 +332
2 +512 − 78 −432 +33 +104 +168 + 56 −200
3 −238 +672 −498 +99 − 46 +318 −276 +132
1 +134 0 0 +134 0 0
2 +104 −174 −99 − 88 −42 −132
3 +238 −174 −99 + 46 −42 −132
4 −2.14 +11.12 +11.00 +4.44 +3.42 +12.30
1 −134 0 0 −134 0 0
2 − 18 −137 −121 − 90 −54 −114
3 −152 −137 −121 −224 −54 −114
5 136.5 204.5 0 20.9 146.3 92.8
(c) MEMBER AA′
(d) MEMBER CC′

FIG. 4.—CONTROLLING ARRANGEMENT FOR THE ANALYSIS OF MEMBERS IN FIG. 3

In the following calculation of the required moment-stiffness relations (which is made in accordance with Eq. 7), the members AA′, AB, BB′, BC, CC′, CD, and DD′ are denoted by the subscripts 1 to 7, respectively: $dM_{BC} = +0.204(-9.0\,i_1 + 9.0\,i_2) - 0.586\,(7.7\,i_2 - 19.7\,i_3 + 12.0\,i_4) - 0.128\,(11.3\,i_4 - 20.0\,i_5 + 8.7\,i_6) + 0.034\,(9.5\,i_6 - 9.5\,i_7) - 2.00\,(0.95\,i_1 + 2.02\,i_3 + 2.04\,i_5 + 0.99\,i_7) + 12.0\,i_4$. Simplifying:

$$dM_{BC} = -3.7\,i_1 - 2.7\,i_2 + 7.5\,i_3 + 3.6\,i_4 - 1.5\,i_5 - 0.8\,i_6 - 2.3\,i_7 \quad \text{..(8a)}$$

As a check: $-3.7 - 2.7 + 7.5 + 3.6 - 1.5 - 0.8 - 2.3 = +0.1$. Similarly: $dM_{A'A} = -0.086(-9.0\,i_1 + 9.0\,i_2) + 0.028(7.7\,i_2 - 19.7\,i_3 + 12.0\,i_4) + 0.006(11.3\,i_4 - 20.0\,i_5 + 8.7\,i_6) + 0.015(9.5\,i_6 - 9.5\,i_7) + 1.66(0.95\,i_1 + 2.02\,i_3 + 2.04\,i_5 + 0.99\,i_7) - 10.0\,i_1$. Simplifying:

$$dM_{A'A} = -7.6\,i_1 - 0.6\,i_2 + 2.7\,i_3 + 0.4\,i_4 + 3.3\,i_5 + 0.2\,i_6 + 1.5\,i_7 \quad \text{..(8b)}$$

As a check to Eq. 8*b*: $-7.6 - 0.6 + 2.7 + 0.4 + 3.3 + 0.2 + 1.5 = -0.1$.

Example 3. Rigid Frame with Unequal Legs.—In Fig. 3(*a*), showing a bent with three legs of unequal length, it is required to calculate the moment-stiffness relation for M_{BA} when both spans carry a uniform dead load of 2 kips per ft and a uniform live load of 8 kips per ft, and the column AA′ (Fig. 3(*a*)) is subjected to uniform live load of 1 kip per ft.

Fig. 4(*a*) shows the fixed-end moments (lines 1), the M'-moments (lines 2), and the total moments (lines 3) and also the values of H' at the tops of the columns (lines 4). The m-moments, m_A, m_B, m_C, and m_Δ, are given in Fig. 3(*b*).

In the following calculation of the required moment-stiffness relation, by Eq. 7, the members AA′, AB, BB′, BC, and CC′ are denoted by the subscripts 1 to 5, respectively: $dM_{BA} = -0.242(90\,i_1 + 526\,i_2) - 0.403(72\,i_2 - 133\,i_3 - 357\,i_4) + 0.163(-201\,i_4 - 131\,i_5) - 1.82(-1.78\,i_1 + 8.74\,i_3 + 13.10\,i_6) + 72\,i_2$; which, simplified, becomes

$$dM_{BA} = -18.6\,i_1 - 84.4\,i_2 + 37.4\,i_3 + 110.8\,i_4 - 45.2\,i_5 \quad \text{........(9)}$$

As a check: $-18.6 - 84.4 + 37.4 + 110.8 - 45.2 = 0$.

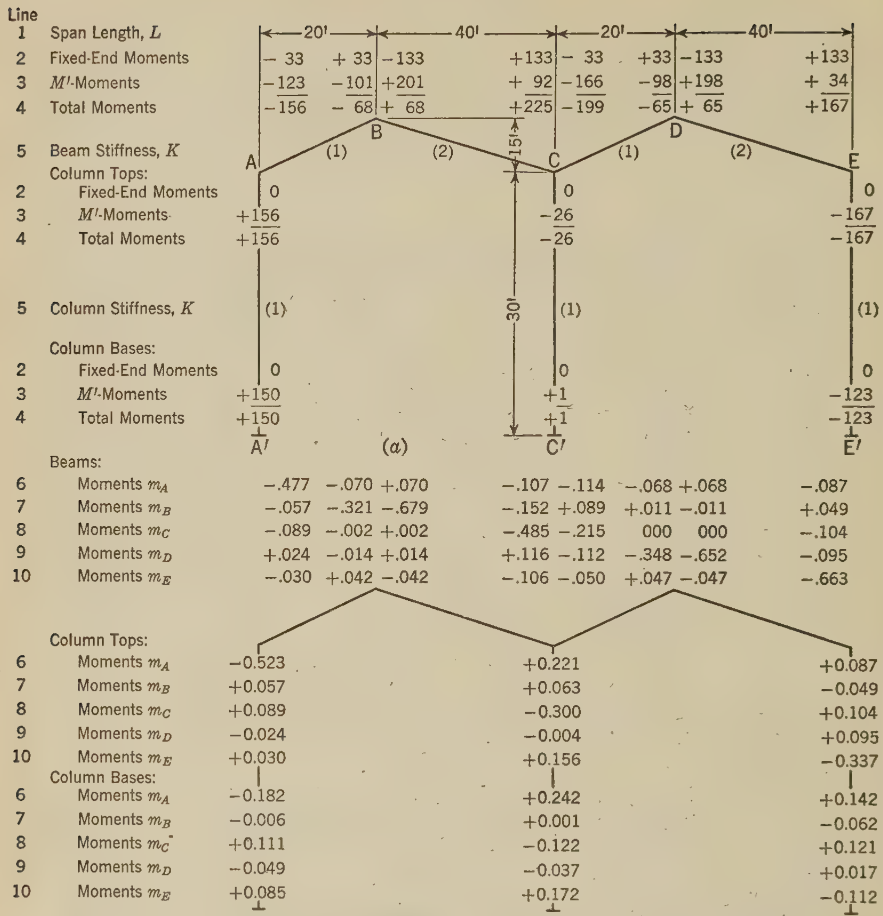
The moment-stiffness relation in Eq. 9 is shown in tabular form in the first line of Table 1.

Example 4. Two-Span, Saw-Tooth Bent.—Calculate the moment-stiffness relations for M_{AB} and $M_{CC'}$, in Fig. 5(*a*), when the spans carry a uniformly distributed load of 1 kip per ft of horizontal projection. The analyses shown in Figs. 5 were obtained by alternate distributions of moments and shears. The procedure is briefly described in Appendix 2. Fig. 5(*a*) shows the fixed-end moments (lines 2), the M'-moments (lines 3), and the total moments (lines 4). The m-moments due to unit couples at joints A, B, C, D, and E are shown in Fig. 5(*b*), lines 5, 6, 7, 8, and 9, respectively. Fig. 5(*c*) gives the m-moments due to unit horizontal forces at points A, C, and E, indicated by $m_{\Delta A}$, $m_{\Delta C}$, and $m_{\Delta E}$ in lines 10, 11, and 12, respectively.

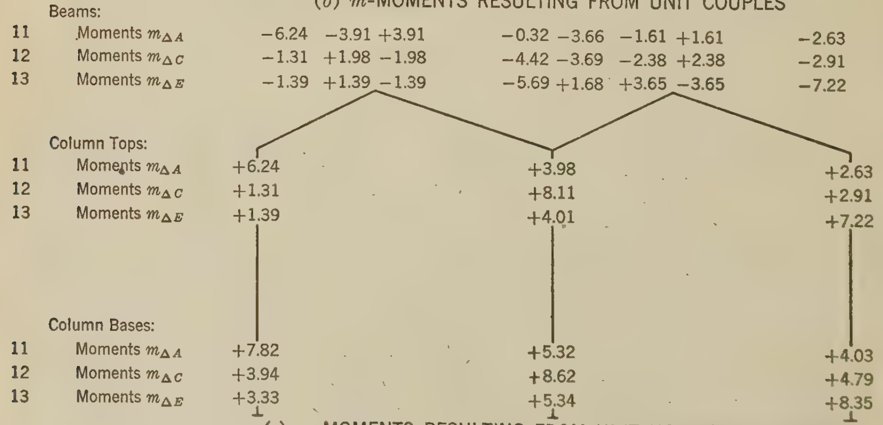
TABLE 1.—MOMENT-STIFFNESS RELATIONS

Moment	i_1	i_2	i_3	i_4	i_5
M_{BA}......	−18.6	− 84.4	+37.4	+110.8	−45.2
M_{BC}......	+50.8	+ 60.2	+24.6	−137.6	+ 1.4
$M_{AA'}$......	+71.0	−123.8	+34.8	− 25.8	+44.2
$M_{BB'}$......	− 0.8	+ 29.8	−87.4	+ 16.2	+42.4
$M_{CC'}$......	+18.4	+ 9.0	+15.0	+ 19.4	−61.2

The members AA′, AB, BC, CC′, CD, DE, and EE′ will be denoted by the subscripts 1 to 7, respectively. In the application of Eq. 7 to the structure of Fig. 5, the terms indicated by $\Sigma(H'_1\,i_1)\,\Sigma(H'_2\,i_2)$, etc., require particular attention. The horizontal end shears at joints A, C, and E must be kept in


Line
1 Span Length, L 20' 40' 20' 40'
2 Fixed-End Moments −33 +33 −133 +133 −33 +33 −133 +133
3 M′-Moments −123 −101 +201 +92 −166 −98 +198 +34
4 Total Moments −156 −68 +68 +225 −199 −65 +65 +167
A B C D E
5 Beam Stiffness, K (1) (2) (1) (2)
15'
Column Tops:
2 Fixed-End Moments 0 0 0
3 M′-Moments +156 −26 −167
4 Total Moments +156 −26 −167
5 Column Stiffness, K (1) (1) (1)
30'
Column Bases:
2 Fixed-End Moments 0 0 0
3 M′-Moments +150 +1 −123
4 Total Moments +150 +1 −123
A′ C′ E′
(a)
Beams:
6 Moments m_A −.477 −.070 +.070 −.107 −.114 −.068 +.068 −.087
7 Moments m_B −.057 −.321 −.679 −.152 +.089 +.011 −.011 +.049
8 Moments m_C −.089 −.002 +.002 −.485 −.215 000 000 −.104
9 Moments m_D +.024 −.014 +.014 +.116 −.112 −.348 −.652 −.095
10 Moments m_E −.030 +.042 −.042 −.106 −.050 +.047 −.047 −.663
Column Tops:
6 Moments m_A −0.523 +0.221 +0.087
7 Moments m_B +0.057 +0.063 −0.049
8 Moments m_C +0.089 −0.300 +0.104
9 Moments m_D −0.024 −0.004 +0.095
10 Moments m_E +0.030 +0.156 −0.337
Column Bases:
6 Moments m_A −0.182 +0.242 +0.142
7 Moments m_B −0.006 +0.001 −0.062
8 Moments m_C +0.111 −0.122 +0.121
9 Moments m_D −0.049 −0.037 +0.017
10 Moments m_E +0.085 +0.172 −0.112

(b) m-MOMENTS RESULTING FROM UNIT COUPLES


Beams:
11 Moments $m_{\Delta A}$ −6.24 −3.91 +3.91 −0.32 −3.66 −1.61 +1.61 −2.63
12 Moments $m_{\Delta C}$ −1.31 +1.98 −1.98 −4.42 −3.69 −2.38 +2.38 −2.91
13 Moments $m_{\Delta E}$ −1.39 +1.39 −1.39 −5.69 +1.68 +3.65 −3.65 −7.22
Column Tops:
11 Moments $m_{\Delta A}$ +6.24 +3.98 +2.63
12 Moments $m_{\Delta C}$ +1.31 +8.11 +2.91
13 Moments $m_{\Delta E}$ +1.39 +4.01 +7.22
Column Bases:
11 Moments $m_{\Delta A}$ +7.82 +5.32 +4.03
12 Moments $m_{\Delta C}$ +3.94 +8.62 +4.79
13 Moments $m_{\Delta E}$ +3.33 +5.34 +8.35

(c) m-MOMENTS RESULTING FROM UNIT HORIZONTAL FORCES

FIG. 5.—TWO-SPAN, RIGID FRAME WITH A UNIFORMLY DISTRIBUTED LOAD OF 1 KIP PER FOOT ON SPANS

equilibrium: hence, at point A—

$$\Sigma(H'\,i) = -\frac{M'_{AA'} + M'_{A'A}}{h_1}\,i_1 + \frac{M'_{AB} + M'_{BA}}{h'_2}\,i_2 - \frac{M'_{BC} + M'_{CB}}{h'_3}\,i_3 \quad \text{..(10a)}$$

at point C—

$$\Sigma(H'\,i) = -\frac{M'_{AB} + M'_{BA}}{h'_2}\,i_2 + \frac{M'_{BC} + M'_{CB}}{h'_3}\,i_3 - \frac{M'_{CC'} + M'_{C'C}}{h_4}\,i_4 + \frac{M'_{CD} + M'_{DC}}{h'_5}\,i_5 - \frac{M'_{DE} + M'_{ED}}{h'_6}\,i_6 \quad \text{............(10b)}$$

and, at point *E*—

$$\Sigma(H'\,i) = -\frac{M'_{CD} + M'_{DC}}{h'_5}\,i_5 + \frac{M'_{DE} + M'_{ED}}{h'_6}\,i_6 - \frac{M'_{EE'} + M'_{E'E}}{h_7}\,i_7 \quad \text{..(10c)}$$

In Eqs. 10, h_1, h_4, and h_7 are the heights of the columns AA′, CC′, and EE′, respectively. The meanings of h'_2 and h'_3 are illustrated in Fig. 6; h'_5 and h'_6 in the right-hand span are analogous to h'_2 and h'_3 in the left-hand span, respectively. In Fig. 5, $h'_2 = h'_5 = 22.5$ ft, and $h'_3 = h'_6 = 45$ ft. The required moment-stiffness relations are then as follows: $dM_{AB} = -0.477\,(156\,i_1 - 123\,i_2) - 0.057\,(-101\,i_2 + 201\,i_3) - 0.089\,(92\,i_3 - 26\,i_4 - 166\,i_5) + 0.024\,(-98\,i_5 + 198\,i_6) - 0.030\,(34\,i_6 - 167\,i_7) - 6.24\,(-10.2\,i_1 - 10.0\,i_2 - 6.50\,i_3) - 1.31\,(10.0\,i_2 + 6.50\,i_3 + 0.83\,i_4 - 11.7\,i_5 - 5.17\,i_6) - 1.39\,(11.7\,i_5 + 5.17\,i_6 + 9.67\,i_7) - 123\,i_2$. Simplifying:

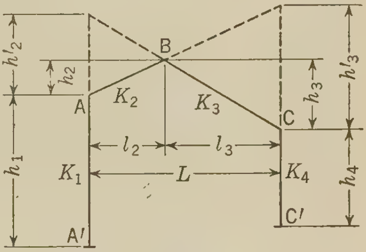


Fig. 6

$$dM_{AB} = -10.8\,i_1 - 9.3\,i_2 + 12.5\,i_3 + 1.2\,i_4 + 11.6\,i_5 + 3.3\,i_6 - 8.4\,i_7 \quad \text{..(11a)}$$

As a check: − 10.8 − 9.3 + 12.5 + 1.2 + 11.6 + 3.3 − 8.4 = + 0.1. Also, $dM_{CC'} = +0.221\,(156\,i_1 - 123\,i_2) + 0.063\,(-101\,i_2 + 201\,i_3) - 0.300\,(92\,i_3 - 26\,i_4 - 166\,i_5) - 0.004\,(-98\,i_5 + 198\,i_6) + 0.156\,(34\,i_6 - 167\,i_7) + 3.98\,(-10.2\,i_1 - 10.0\,i_2 - 6.50\,i_3) + 8.11\,(10.0\,i_2 + 6.50\,i_3 + 0.83\,i_4 - 11.7\,i_5 - 5.17\,i_6) + 4.01\,(11.7\,i_5 + 5.17\,i_6 + 9.67\,i_7) - 26\,i_4$. Again simplifying:

$$dM_{CC'} = -6.1\,i_1 + 7.7\,i_2 + 11.9\,i_3 - 11.5\,i_4 + 2.1\,i_5 - 16.8\,i_6 + 12.7\,i_7 \quad \text{.(11b)}$$

As a check: − 6.1 + 7.7 + 11.9 − 11.5 + 2.1 − 16.8 + 12.7 = 0.

Application to Design

The following outline, although subject to variation, should serve as a general guide in designing with the aid of moment-stiffness relations:

(1) The relative stiffnesses upon which the moment-stiffness relations are to be based are chosen carefully, or they are determined from a preliminary design. The structure thus obtained is called the "basic structure."

(2) The basic structure is analyzed to determine (*a*) the *m*-moments due to unit couples placed at successive joints; and (*b*) the *m*-moments due to unit

horizontal forces or shears corresponding to the various sidesway conditions existing in the structure. If sidesway conditions are present, correction for sidesway should be made in finding the former group of m-moments.

(3) Analyses of the basic structure are made to determine (*a*) the loading arrangements controlling the design of the various members; (*b*) the values of the corresponding controlling moments and axial forces; and (*c*) the M'-moments and values of H' needed for the evaluation of the moment-stiffness relations. The performance of these analyses is expedited by the use of the m-moments obtained in step (2).

(4) The members are designed for the controlling stresses obtained in step (3). The resulting stiffnesses of the members will be different, in general, from those of the basic structure. The stiffness-change ratio, i, or the change in stiffness divided by the stiffness assumed in the basic structure, is calculated for each member.

(5) Moment-stiffness relations corresponding to the various controlling moments are calculated, and are tabulated in such a way that the coefficients of i for a given member all appear in the same column, as shown in Table 1. (The changes in the axial forces in vertical or inclined compression members, due to subsequent changes in the stiffnesses of the various members, generally do not appreciably affect the design and therefore may usually be disregarded.)

(6) By reference to the tabulated moment-stiffness relations, the changes in all the controlling moments, due to a change of stiffness in a given member (from step (4)), are computed by a single setting of the slide rule. Similarly, the moment changes due to the changes in stiffness of the various other members are calculated. In the performance of this step the designer is rewarded for the labor of obtaining the moment-stiffness relations. With their aid, he is able not only to calculate, rapidly, the effects of any number of consecutive revisions in the design, but to form an advance estimate of the effects of contemplated revisions.

(7) The members are redesigned in accordance with the revised moments obtained in step (6), and the moment changes thus produced are again computed, as in step (6). This process is continued until no further changes in design are required. The stiffness-change ratios are always based upon the stiffnesses assumed in the basic structure.

Example 5.—The bent outlined in Fig. 3(*a*) is to be designed in steel in accordance with the 1936 specifications of the American Institute of Steel Construction (AISC) revised in 1941. The horizontal members carry a dead load of 2 kips per ft and a live load of 8 kips per ft. Provision is also to be made for a live load of 1 kip per ft applied uniformly to the side of column AA′. The weight of the columns will be neglected. The unsupported lengths of the columns are: AA′, 13½ ft; BB′, 14 ft; and CC′, 20 ft.

A description of the design procedure conforming to steps (1) to (7) is as follows:

(1) The basic structure and assumed stiffnesses are as shown in Fig. 3(*a*).

(2) The m-moments, m_A, m_B, m_C, and m_Δ are shown in Fig. 3(*b*).

(3*a*) Figs. 4 contain the analyses corresponding to the loading arrangements that were found to control the design of the members. In Fig. 4(*a*), the loading, which controls the design of members AB and BB′, consists of dead load plus live load on both spans, and lateral live load. In Fig. 4(*b*), the loading that controls the design of member BC consists of dead load plus live load on both spans, but no lateral live load. The loading in Fig. 4(*c*) consists of dead load plus live load on the left span, dead load on the right span, and lateral live load. This loading controls the design of member AA′. The loading in Fig. 4(*d*) consists of dead load on the left span, dead load plus live load on the right span, and lateral live load. This loading controls the design of member CC′.

(3*b*) and (3*c*) Figs. 4 show the fixed-end moments, the M'-moments, and the total moments, in the order named. These analyses were made with the aid of the m-moments shown in Fig. 3(*b*), utilizing relations A and B which were stated in connection with the development of Eq. 5*b*. The values of H' at the tops of the columns are entered in lines 4 in Figs. 4.

The controlling moments (M) and axial forces (F) are as follows: For member AB, $M = 822$ ft-kips (line 3); for member BB′, $M = 133$ ft-kips (line 3) and $F = 297.8$ kips (line 5), as shown in Fig. 4(*a*); for member BC, $M = 755$ ft-kips (line 3), as shown in Fig. 4(*b*); for member AA′, $M = 238$ ft-kips (line 3) and $F = 136.5$ kips (line 5), as shown in Fig. 4(*c*); and for member CC′, $M = 132$ ft-kips (line 3) and $F = 92.8$ kips (line 5), as shown in Fig. 4(*d*).

(4) The design calculations corresponding to step (4) are shown in the first cycle of Table 2. The data for the design of girders, Table 2(*a*), include: M, the required S, $K\ L$, I_o, I, i, the section selected, and the actual S. For member AB, the required S is $\frac{822 \times 12}{20} = 493$, which is provided by a 36-in. WF at 150 lb, with $S = 503$ and $I = 9{,}012$. For member BC, the required S is $\frac{755 \times 12}{20} = 453$, which is likewise provided by a 36-in. WF at 150 lb. For member AB, $I_o = I = 9{,}012$; hence, $i = 0$. The inertia moment I_o for the remaining members is found by multiplying this value of I_o by the ratio of the value of $K\ L$ for the member in question to the value of $K_{AB}\ L_{AB}$. Thus, for member BC, $I_o = \frac{160}{180}(9{,}012) = 8{,}011$; and therefore $i = \frac{9{,}012 - 8{,}011}{8{,}011} = +\,0.125$.

The data for the design of columns, Table 2(*b*), include F, M, $K\ L$, A_c, A_b, the total of $A_c + A_b$, A'_c, A'_b, the total of $A'_c + A'_b$, I_o, I, i, $L'/\bar{r}$, L'/b, the section selected, s_c, s_b, and the area provided. The columns are designed to resist, in addition to the axial force, (I) the moment at the top or bottom, or (II) 70% of the moment in case (I). In case (I), the required area is $A_c + A_b$. With allowable stresses of 17 kips per sq in. and 20 kips per sq in. for axial compression and bending, respectively, $A_c = \frac{F}{17}$ and $A_b = \frac{0.6\ M\ c}{\bar{r}^2_b}$. In case

(II), the required area is $A'_c + A'_b$, where $A'_c = \frac{F}{s_c}$ and $A'_b = \frac{14}{s_b} A_b$. Case (I) was found to control in each instance.

TABLE 2.—Design Calculations Corresponding to Step (4)

Line	(a) Girders			(b) Columns					
1 2 3 4	Member M Required S K L (1)	(I_o) I i (2)	Section S (3)	Member F M K L (4)	A_c A_b Total (5)	A'_c A'_b Total (6)	(I_o) I i (7)	L'/r L'/b Section (8)	s_c s_b Area (9)
First Cycle									
1	AB:			AA′:					
2	822	(9,012)	....	136.5	8.0	8.9	(4,006)	58.5	15.3
3	493	9,012	36WF150	238	21.8	15.3	1,852	<15	20
4	180	0	503	80	29.8	24.2	−0.54	18WF105	30.9
1	BC:			BB′:					
2	755	(8,011)	....	297.8	17.5	19.5	(1,400)	59	15.3
3	453	9,012	36WF150	133	12.2	8.5	1,852	<15	20
4	160	+0.125	503	28	29.7	28.0	+0.323	18WF105	30.9
1				CC′:					
2	....	....	....	92.8	5.5	7.5	(1,001)	97	12.4
3	....	....	....	132	15.0	12.4	797	24	17.0
4	....	....	....	20	20.5	19.9	−0.204	14WF74	21.7
Second Cycle									
1	AB:			AA′:					
2	867.1	(9,012)	....	136.5	8.0	8.9	(4,006)	59	15.3
3	520	9,739	36WF160	188.6	17.4	12.2	1,675	<15	20
4	180	+0.08	541	80	25.4	21.1	−0.044	18WF96	28.2
1	BC:			BB′:					
2	787.1	(8,011)	....	297.8	17.5	19.7	(1,400)	61	15.1
3	471	9,012	36WF150	165.0	15.1	10.6	2,034	<15	20
4	160	0	503	28	32.6	30.3	+0.13	18WF114	33.5
1				CC′:					
2	....	....	....	92.8	5.5	7.7	(1,001)	98	12.3
3	....	....	....	120.4	14.1	11.6	724	24	17.0
4	....	....	....	20	19.6	19.3	−0.073	14WF68	20.0
Third Cycle									
1	AB:			AA′:					
2	869.3	(9,012)	....	136.5	8.0	8.9	(4,006)	59	15.3
3	521	9,739	36WF160	186.8	17.2	12.0	1,675	<15	20
4	180	0	541	80	25.2	20.9	0	18WF96	28.2
1	BC:			BB′:					
2	786.2	(8,011)	....	297.8	17.5	19.7	(1,400)	61	15.1
3	471	9,012	36WF150	179.4	16.4	11.5	2,034	<15	20
4	160	0	503	28	33.9	31.2	0	18WF114	33.5
1				CC′:					
2	....	....	....	92.8	5.5	7.7	(1,001)	98	12.3
3	....	....	....	115.9	13.5	11.1	724	24	17.0
4	....	....	....	20	19.0	18.8	0	14WF68	20.0

In the design of the columns some effort was made to conform with the relative stiffnesses assumed in Fig. 3(*a*), but the immediate objective was to obtain a design in which the area agreed closely with that required. With reference to Table 2(*b*) (first cycle), for column AA′ the required area = 29.8 sq in.; $I_o = \frac{80}{180}$ (9,012) = 4,006; the area provided (18-in. WF at 105 lb) = 30.9 sq in.; I = 1,852; and $i = \frac{1{,}852 - 4{,}006}{4{,}006} = -0.54$. For column BB′, the required area = 29.7 sq in.; $I_o = \frac{28}{180}$ (9,012) = 1,400; the area provided (18-in. WF at 105 lb) = 30.9 sq in.; I = 1,852; and $i = \frac{1{,}852 - 1{,}400}{1{,}400} = +0.323$. For column CC′, the required area = 20.5 sq in.; $I_o = \frac{20}{180}$ (9,012) = 1,001; the area provided (14-in. WF at 74 lb) = 21.7 sq in.; I = 797; and $i = \frac{797 - 1{,}001}{1{,}001} = -0.204$.

If subscripts 1 to 5 are used to refer to members AA′, AB, BB′, BC, and CC′, respectively, the stiffness-change ratios resulting from the first cycle of design are as follows: $i_1 = -0.54$; $i_2 = 0$; $i_3 = +0.323$; $i_4 = +0.125$; and $i_5 = -0.204$.

(5) Moment-stiffness relations are required: For M_{BA} and $M_{BB'}$, using values of M' and H' in Fig. 4(*a*); for M_{BC}, using values of M' and H' in Fig. 4(*b*); for $M_{AA'}$, using values of M' and H' in Fig. 4(*c*); and, for $M_{CC'}$, using values of M' and H' in Fig. 4(*d*). The calculation of the moment-stiffness relation for M_{BA} has already been explained in detail (see Example 3). The moment-stiffness relations for M_{BC}, $M_{AA'}$, $M_{BB'}$, and $M_{CC'}$ are calculated in a similar manner as shown in Table 1, together with the moment-stiffness relation for M_{BA}.

TABLE 3.—MOMENT CHANGES

Symbol	Original moment (1)	$i_1 = -0.54$ (2)	$i_3 = +0.323$ (3)	$i_4 = +0.125$ (4)	$i_5 = -0.204$ (5)	Revised moment (6)	$i_2 = +0.08$ (7)
M_{BA}	+822	+10.0	+12.1	+13.8	+ 9.2	+867.1	− 6.8
M_{BC}	−755	−27.4	+ 7.9	−17.2	− 0.3	−792.0	+ 4.9
$M_{AA'}$	+238	−38.4	+11.2	− 3.2	− 9.0	+198.6	−10.0
$M_{BB'}$	−133	+ 0.4	−28.2	+ 2.0	− 8.6	−167.4	+ 2.4
$M_{CC'}$	−132	− 9.9	+ 4.8	+ 2.4	+12.5	−122.2	+ 0.7

TABLE 3.—(*Continued*)

Symbol	Revised moment (8)	$i_1 = 0.044$ (9)	Revised moment (10)	$i_3 = +0.13$ (11)	Revised moment (12)	$i_5 = -0.073$ (13)	Final moment (14)
M_{BA}	+860.3	+0.8	+861.1	+ 4.9	+866.0	+3.3	+869.3
M_{BC}	−787.1	−2.2	−789.3	+ 3.2	−786.1	−0.1	−786.2
$M_{AA'}$	+188.6	−3.1	+185.5	+ 4.5	+190.0	−3.2	+186.8
$M_{BB'}$	−165.0	0	−165.0	−11.3	−176.3	−3.1	−179.4
$M_{CC'}$	−121.5	−0.8	−122.3	+ 1.9	−120.4	+4.5	−115.9

(6) The calculated changes in the controlling moments are shown in Table 3. The original values of the moments are shown in Col. 1. Col. 2 contains the moment changes due to decreasing the stiffness of member AA′ by 54%

($i_1 = -0.54$). These moment changes were obtained by multiplying the coefficients of i_1, in Table 1, by -0.54. Cols. 3, 4, and 5, Table 3, which were calculated in a similar manner, show the remaining moment changes resulting from the first cycle of design, and Col. 6 shows the revised values of the moments. The values of F were assumed to remain essentially unchanged.

(7) In the second cycle of design, for which the calculations are shown in Table 2, the moments were revised after each change in section. For member AB, $M = 867.1$ ft-kips (Col. 6, Table 3), requiring a 36-in. WF at 160 lb, with $i_2 = +0.08$, which resulted in the moment changes and revised moments shown in Cols. 7 and 8, Table 3. For member BC, $M = 787.1$ ft-kips, and no change in design was required. For member AA′, $M = 188.6$ ft-kips, requiring an 18-in. WF at 96 lb, with $i_1 = -0.044$, which produced the moment changes and revised moments shown in Cols. 9 and 10, Table 3. For member BB′, $M = 165.0$ ft-kips, requiring an 18-in. WF at 114 lb, with $i_3 = +0.13$, and producing the moment changes and revised moments given in Cols. 11 and 12, Table 3. For member CC′, $M = 120.4$ ft-kips, requiring a 14-in. WF at 68 lb, with $i_5 = -0.073$, and producing the moment changes and revised moments given in Cols. 13 and 14 of Table 3.

Calculations for a third cycle of design, shown in Table 2, indicate that no additional changes were required.

For comparison with the values of F and M shown in the third cycle of Table 2, and in Col. 14, Table 3, the results of a direct final analysis were found to be as follows:

Member	F	M
AB	....	867
BC	....	796
AA′	133.7	184
BB′	302.7	176
CC′	93.6	113

Economy Studies

In the course of the foregoing design, alternate selections were available which, if adopted, would have resulted in a final design differing somewhat from that shown. Furthermore, it is to be expected that several designers, although adhering closely to the procedure herein described, but each starting with a different set of assumed stiffnesses, would obtain designs differing more or less from each other. In connection with an existing design there arises, therefore, the question of whether, by changing the stiffnesses of certain members, a saving can be effected in the total amount of main material. Such a saving may occur in two principal ways: First, a change in stiffness due to the substitution of a lighter member may be accompanied by a change in moment such that the resulting unit stresses in the lighter member may meet the requirements of the specifications, without producing overstress in the remaining members; and, second, it may be found possible to reduce the stresses in a given member (and thus to use a lighter member) by changing the stiffness of adjacent members, in addition to that of the member to be made lighter.

In the latter case, sometimes the weight per foot of a long member may be decreased with the aid of an increase in the weight per foot of an adjacent, shorter, member, with a resultant saving in total weight.

The writer's experience indicates that, in one or both of these ways, a saving of from approximately 3% to 6% of the total weight can frequently be obtained.

It does not appear practicable to formulate a fixed procedure for such investigations, except to state that it is essentially a matter of trial. Moment-stiffness relations are well suited to a rapid determination of the effects of various changes aimed at securing increased economy. In the foregoing example it is found, with the aid of the moment-stiffness relations in Table 1, that the member AA′ can be designed with a 24-in. WF section at 80 lb, with a saving of 640 lb, or about 4% of the total weight of the main members in the bent.

Conclusion

Experience indicates that moment-stiffness relations are useful aids in the design of continuous framed structures composed of prismatic members, particularly in cases where the columns are subject to moments of considerable magnitude. These relations also provide a basis for the determination of possible improvements in an existing design.

As differential quantities are only approximate representations of the true changes, the relative stiffnesses upon which the moment-stiffness relations are based should be chosen with some care, or determined from a preliminary design; and the final results should generally be checked by a direct analysis. It should be recognized that, in some instances, where large stiffness changes occur, the moment differentials will be subject to rather large errors; also, that, where only minor changes in stiffness occur, the use of moment-stiffness relations is not particularly advantageous unless subsequent economy studies are to be made.

APPENDIX 1. NOTATION

The following letter symbols, used in this paper and in its discussions, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

A = section area required for given design conditions:
  A_b = area for bending at a section with lateral support;
  A_c = area for axial compression at a section with lateral support (see Table 2);
  A'_b = area for bending at an unsupported section;
  A'_c = area for axial compression at an unsupported section;

b = width of a section;

C = fixed-end moment caused by loads between the ends of a member;

c = distance from the gravity axis of a member to the outer fiber in bending;

E = modulus of elasticity;

F = axial force that controls the design of a member;

H = horizontal shear at the end of a vertical or inclined member;

H' = the difference between H and the fixed-end shear Q (that is, $H' = H - Q$);

H'' = horizontal shear correction resulting from a distribution of shears;

h = column heights, the length of the vertical projection of an inclined member; h' = the vertical distance from the bottom of one inclined member to the prolongation of the other inclined member in the same span (see Fig. 6);

I = rectangular moment of inertia of a member as designed; I_s = the moment of inertia of a member corresponding to the stiffness assumed in the basic structure (that is, I_s is proportional to $K L$);

i = the stiffness-change ratio of a member = $\frac{\Delta K}{K}$;

K = the relative flexural stiffness, or the $\frac{E I}{L}$-value of a member, as assumed in the basic structure;

L = length of a given member;

L' = unsupported length of a member;

ΔL = relative horizontal displacement of the opposite ends of a given span;

l = length of the horizontal projection of an inclined member;

M = moment, moment at the end of a member that controls its design;

M' = difference between M and C (that is, $M' = M - C$);

M'' = the fixed-end moments created by horizontal displacements of column tops, visualized in balancing horizontal shears;

ΔM = a change in moment resulting from the distribution of moments;

m_x = moment created by a unit couple applied at a place whose location is given by a subscript; that is, m_A is the moment caused by a unit couple at joint A and m_H is the moment created by a unit horizontal force that produces sidesway;

P = concentrated loads applied to various members of a structure;

Q = fixed-end horizontal shear due to loads between the ends of a vertical or inclined member, or the sum of the simple beam shear and the shear created by fixed-end moments; ΣQ is the sum of the fixed-end horizontal shears on a given joint or section and any external horizontal forces that may be acting on it;

r = least radius of gyration; r_b = radius of gyration in the plane of bending;

S = section modulus, I/c;
s = allowable unit stress at an unsupported section:
  s_b = bending stress;
  s_c = compressive stress;
V = vertical shear;
x = lever arm of a vertical load P from left support; x' = lever arm from right support (see Fig. 8);
y = vertical distance to a horizontal load P;
γ = side-thrust stiffness;
Δ = sidesway or displacement produced by a unit horizontal force; horizontal displacement of the top of a column;
κ = average rate of convergence;
ϕ = a linear function of the elastic deformations, given by the relation $H' = K\phi$; and
ψ = a linear function of the elastic deformations, given by the relation $M' = K\psi$.

APPENDIX 2. ANALYSIS OF MULTIPLE-SPAN PEAKED BENTS

Multiple-span, peaked, or saw-tooth bents may be analyzed with a fair degree of accuracy and facility by alternate distributions of moments and horizontal shears. (The distribution of joint forces was suggested by Hardy Cross, M. ASCE, in 1926, in a mimeographed treatise entitled "Statically Indeterminate Structures.") The procedure involves the following steps:

1. Compute fixed-end moments at all joints and balance by moment distribution, assuming that the joints are not displaced.
2. Compute the horizontal shears at the tops of the columns and at the ends of the spans and balance them by distribution to the columns and adjoining spans in proportion to the side-thrust stiffness of these units; by addition, find the resulting shear corrections.
3. Compute the fixed-end moments due to the shear corrections and displacements occurring in the preceding distribution of horizontal shears and balance by moment distribution, as before.
4. From the changes in the moments resulting from the preceding distribution of moments, compute the parts of the horizontal shears remaining to be balanced and balance them by horizontal shear distribution, as before.
5. Repeat steps 3 and 4 until the solution converges. Horizontal displacements and horizontal end shears are considered positive to the right.

The side-thrust stiffness of a column is defined as the horizontal force or shear which, applied at the top of the column, will produce a unit horizontal displacement, without rotation, of the top of the column. The side-thrust stiffness of a span (consisting of two inclined members) is defined as the horizontal force or shear required to produce unit relative horizontal displacement of the ends of the span without rotation of the ends or of the peak joint. In other words, if the ends of the span are denoted by the letters A and C, and the peak joint by the letter B, the side-thrust stiffness of the span is the value of of one of the two equal and opposite horizontal forces or shears which, applied at points A and C, will produce a unit change in the length of the span, without rotation of the joints A, B, and C, and without any vertical movement of joints A and C.

The similarities between moment distribution and horizontal shear distribution as applied to multiple-span peaked bents are apparent from the following brief description of the two processes:

Item	Moment distribution	Horizontal shear distribution
1.	The joints are balanced consecutively with regard to moments.	The joints at the tops of the columns are balanced consecutively with regard to horizontal shears.
2.	In balancing a given joint the only joint movement permitted is the rotation of the joint in question.	In balancing the joints at the tops of columns the only joint movements permitted are a horizontal displacement of the joint in question, and a vertical and horizontal displacement of the peak joints in adjacent spans. The joints are not permitted to rotate.
3.	The unbalanced moment is distributed to the connecting members in proportion to their flexural stiffness or resistance to rotation.	The unbalanced horizontal shear is distributed to the column and adjacent spans in proportion to their side-thrust stiffness, or resistance to horizontal displacement.
4.	The balancing of the moments produces fixed-end moments, or carry-over moments, at the opposite ends of the members; hence, whenever the joints are all balanced individually, a new set of unbalanced moments arises from the carry-over moments.	The balancing of the horizontal shears produces horizontal shears, or carry-over shears, at the opposite ends of the adjoining spans; hence, whenever the joints at the tops of the columns are all balanced individually, a new set of unbalanced horizontal shears arises from the carry-over shears. (A shear correction applied at one end of a span produces an equal and opposite shear at the other end of the span.)
5.	To obtain a complete distribution the process of balancing and carrying over must be continued until the corrections become negligible.	To obtain a complete distribution the process of balancing and carrying over must be continued until the corrections become negligible. This process may be expedited by the summation of a series.

Item	Moment distribution	Horizontal shear distribution
6.	Before the second, or any subsequent, distribution of moments can be made it is necessary to compute the fixed-end moments produced by the displacements occurring in connection with a corresponding distribution of horizontal shears.	Before a distribution of horizontal shears can be made it is necessary to compute the horizontal shears, or parts of them, which are required to be balanced.
7.	These fixed-end moments (item 6) are computed by simple formulas obtained from a consideration of the elastic deformations (see Eqs. 19, 23*a*, and 23*b*, derived subsequently).	These quantities (item 6) are obtainable by statics (see Eqs. 12, 14, 15*a*, and 15*b*, derived subsequently).

In the computation of items 1 to 7, numerical subscripts are used to designate the various members; for example, in Fig. 6, the column AA′ is designated by the subscript 1; and members AB and BC, by the subscripts 2 and 3, respectively.

The horizontal shears required in the first distribution of shears are obtained (by statics) from the loads on the structure and from the moments resulting from the first distribution of moments. The horizontal shear at the top of a column AA′ is as follows (see Figs. 6 and 7):

$$H_{AA'} = -\frac{M_{AA'} + M_{A'A} + \Sigma(P\,y)}{h_1} \quad \ldots\ldots (12)$$

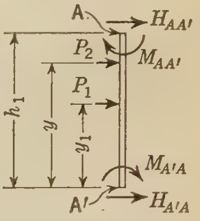


Fig. 7

Expressions for the horizontal shears at the lower ends of the inclined members are obtained by combining the equations of statics pertaining to the separate members and eliminating the unwanted horizontal and vertical forces. Thus, in Fig. 8 (involving vertical loads), by taking moments about points A and C:

$$M_{AB} + M_{BA} + V_B l_2 - H_B h_2 + \Sigma(P\,x) = 0 \quad ..(13a)$$

and

$$M_{BC} + M_{CB} + V_B l_3 + H_B h_3 - \Sigma(P\,x') = 0 \quad ..(13b)$$

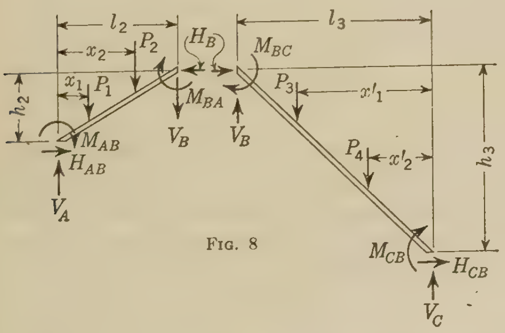


Fig. 8

Elimination of V_B and substitution from the relations $H_{AB} = -H_{CB} = H_B$ result in the following expression for the desired end shears:

$$H_{AB} = -H_{CB} = \frac{M_{AB} + M_{BA} + \Sigma(P\,x)}{h'_2} - \frac{M_{BC} + M_{CB} - \Sigma(P\,x')}{h'_3} \quad ..(14)$$

in which $h'_2 = h_2 + \frac{l_2}{l_3} h_3$ and $h'_3 = h_3 + \frac{l_3}{l_2} h_2$ (see Fig. 6).

Equations for the calculation of the parts of the horizontal shears required to be balanced in subsequent distributions of horizontal shears are obtained in a similar manner; then, however, the quantities $\Sigma(P\,y)$, $\Sigma(P\,x)$, and $\Sigma(P\,x')$ are zero. Therefore, the horizontal shears which require balancing at the end of the second, or of a subsequent, distribution of moments are given by:

$$H_{AA'} = -\frac{\delta M_{AA'} + \delta M_{A'A}}{h_1} \qquad (15a)$$

and

$$H_{AB} = -H_{CB} = \frac{\delta M_{AB} + \delta M_{BA}}{h'_2} - \frac{\delta M_{BC} + \delta M_{CB}}{h'_3} \qquad (15b)$$

in which $\delta M_{AA'}$, $\delta M_{A'A}$, δM_{AB}, etc., are the changes in moment resulting from the distribution of moments.

Consider now the side-thrust stiffnesses and the fixed-end moments produced by the displacements occurring in the distribution of shears. The fixed-end moments of the column in Fig. 7, produced by the displacement Δ_A, are

$$M''_{AA'} = M''_{A'A} = -\frac{6\,K_1}{h_1}\,\Delta_A \qquad (16)$$

The corresponding shear correction, denoted by $H''_{AA'}$, is obtained by substituting these values of $M''_{AA'}$ and $M''_{A'A}$ (Eq. 16) in place of $\delta M_{AA'}$ and $\delta M_{A'A}$ in Eq. 15a:

$$H''_{AA'} = \frac{12\,K_1}{h^2_1}\,\Delta_A \qquad (17)$$

The side-thrust stiffness of column AA′, found by placing $\Delta_A = 1$ in Eq. 17, is, therefore:

$$\gamma_{AA'} = \frac{12\,K_1}{h^2_1} \qquad (18)$$

The fixed-end moments accompanying the displacement Δ_A are obtained in terms of $H''_{AA'}$ by eliminating Δ_A from Eqs. 16 and 17, as follows:

$$M''_{AA'} = M''_{A'A} = -\frac{H''_{AA'}\,h_1}{2} \qquad (19)$$

By moment areas, or otherwise, it can be shown that, if the shear corrections, H''_{AB} and H''_{CB}, are applied at the ends of the span ABC, Fig. 6, in such a way as to produce a change in length, dL, of the span, L, without rotation of the joints A, B, C, and without any vertical movement of A and C, the following fixed-end moments will be obtained:

$$M''_{AB} = M''_{BA} = -\frac{6\,K_2}{h'_2}\,dL \qquad (20a)$$

and

$$M''_{BC} = M''_{CB} = +\frac{6\,K_3}{h'_3}\,dL \qquad (20b)$$

The corresponding shear corrections are obtained by substitution of these values of M'' (Eqs. 20) in place of the values of δM in Eq. 15*b*:

$$H''_{AB} = -12\left[\frac{K_2}{(h'_2)^2} + \frac{K_3}{(h'_3)^2}\right] dL \qquad (21a)$$

and

$$H''_{CB} = +12\left[\frac{K_2}{(h'_2)^2} + \frac{K_3}{(h'_3)^2}\right] dL \qquad (21b)$$

The side-thrust stiffness of span ABC may be obtained by placing $dL = -1$ in Eq. 21*a* or by placing $dL = +1$ in Eq. 21*b*, giving

$$\gamma_{AC} = 12\left[\frac{K_2}{(h'_2)^2} + \frac{K_3}{(h'_3)^2}\right] \qquad (22)$$

The fixed-end moments accompanying the displacement dL are obtained in terms of the shear corrections, in the case of the member AB, by eliminating dL from Eqs. 20*a* and 21*a*, and, in the case of the member BC, by eliminating dL from Eqs. 20*b* and 21*b*, as follows:

$$M''_{AB} = M''_{BA} = \frac{H''_{BA}}{2} \frac{\dfrac{K_2}{h'_2}}{\dfrac{K_2}{(h'_2)^2} + \dfrac{K_3}{(h'_3)^2}} \qquad (23a)$$

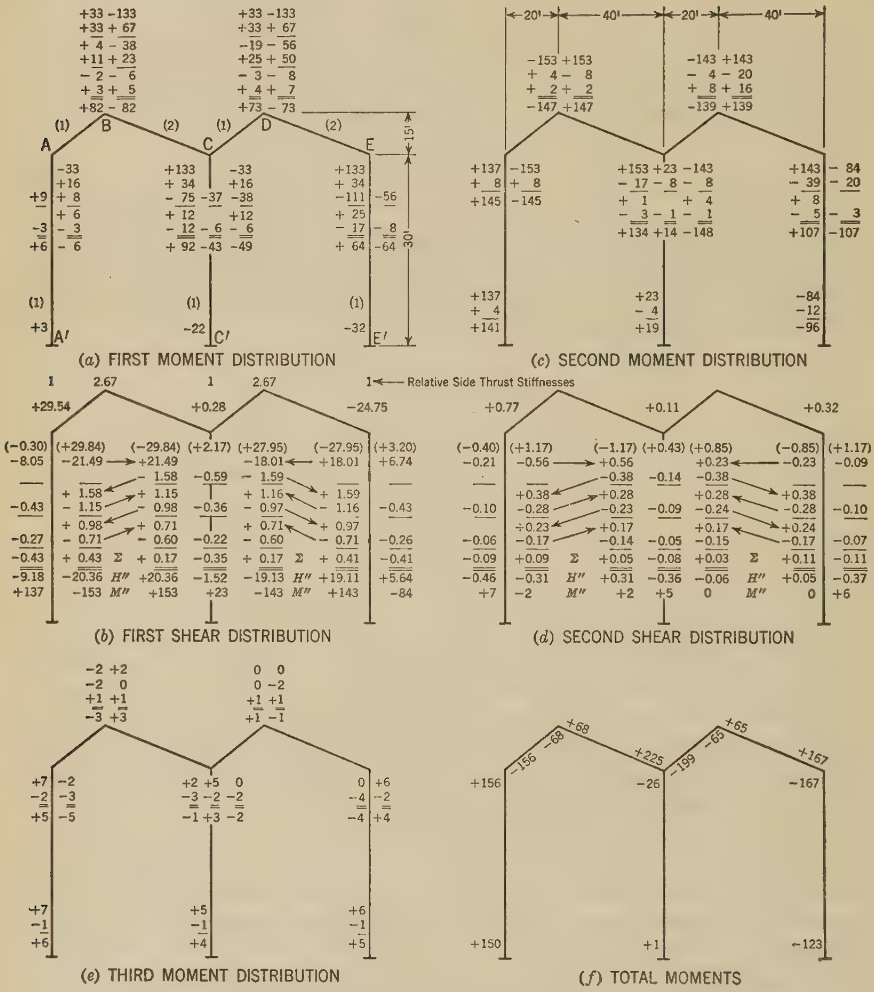
and

$$M''_{BC} = M''_{CB} = \frac{H''_{CB}}{2} \frac{\dfrac{K_3}{h'_3}}{\dfrac{K_2}{(h'_2)^2} + \dfrac{K_3}{(h'_3)^2}} \qquad (23b)$$

Example 6.—Determine the bending moments at the ends of the members of the structure illustrated in Figs. 5(*a*) and 9(*a*), the loading consisting of 1 kip per ft of horizontal projection, uniformly distributed over both spans.

The relative flexural stiffnesses are shown in Fig. 9(*a*), and the relative side-thrust stiffnesses of the three columns and two spans are shown in Fig. 9(*b*). The fixed-end moments of members AB, BC, CD, and DE, due to the given loading, are distributed in Fig. 9(*a*).

The horizontal shears acting at the tops of the columns and at the ends of the spans at the beginning of the first shear distribution are shown in parentheses in Fig. 9(*b*). They are calculated in accordance with Eqs. 12 and 14. The unbalanced horizontal shears, or the algebraic sums of the quantities in parentheses, are written above the various joints in Fig. 9(*b*). The joints were balanced in the order A, E, and C. In the first cycle, the unbalanced shear of +29.54 kips at joint A is distributed to the column AA′ and the span ABC in proportion to their side-thrust stiffnesses—namely, 1.00 and 2.67—the corrections being −8.05 and −21.49, respectively. The unbalanced shear of −24.75 kips at joint E is distributed to the column EE′ and the span EDC in a like manner, the corrections being +6.74 and +18.01, respectively. Corrections assigned at one end of a given span are carried over to the other end with a change in sign; hence, at joint C, the unbalanced shear is + 0.28 + 21.49 − 18.01 = + 3.76 kips. This quantity is distributed to column CC′ and the spans CBA and CDE in the proportions of 1.00, 2.67, and 2.67—the corrections being −0.59, −1.58, and −1.59, respectively. After the last two corrections are carried over to joints A and C, respectively, with change in sign, the second and third cycles of shear distribution are performed. The corrections


A
B
C
D
E
A′
C′
E′
(1)
(2)
15′
30′
20′
40′
1←— Relative Side Thrust Stiffnesses
1
2.67
+29.54
+0.28
−24.75
+0.77
+0.11
+0.32
Σ
H″
M″
+68
−68
−156
+225
−199
−65
+65
+167
+156
−26
−167
+150
+1
−123

(a) FIRST MOMENT DISTRIBUTION

(b) FIRST SHEAR DISTRIBUTION

(c) SECOND MOMENT DISTRIBUTION

(d) SECOND SHEAR DISTRIBUTION

(e) THIRD MOMENT DISTRIBUTION

(f) TOTAL MOMENTS

FIG. 9.—ANALYSIS OF MULTIPLE-SPAN PEAKED BENTS

now form geometric series, converging at approximately equal rates at the various joints. As the convergence is rather slow, a considerable saving in arithmetic may be made if the remainders are calculated.

The average rate of convergence, κ, may be found with sufficient precision by dividing the sum of the unbalanced shears (taken without regard to sign) in the third cycle by the sum of the unbalanced shears in the second cycle. Hence, in the example, $\kappa = \frac{098 + 1.42 + 0.97}{1.58 + 2.31 + 1.59} = 0.613$. The remainders are summed by multiplying the results of the third cycle of distributions by $\frac{\kappa}{1 - \kappa} = \frac{0.613}{0.387} = 1.58$, and are designated by Σ in Fig. 9(*b*).

The total horizontal shear corrections are next found, by addition of each column of figures, exclusive of the quantities in parentheses; they are designated by H'' in Fig. 9(*b*). As a check on the shear distribution, the sum of values of H'' at a given joint should be equal and opposite to the original unbalanced shear. The sums in question are -29.54, -0.29, and $+24.75$. The fixed-end moments arising from the balancing of the horizontal shears are marked M'' in Fig. 9(*b*); they are calculated in accordance with Eqs. 19, 23*a*, and 23*b*.

The M''-values obtained in Fig. 9(*b*) are written as fixed-end moments in Fig. 9(*c*), and the joints are again balanced by moment distribution. The unbalanced horizontal shears are then calculated from the changes in the moments of Fig. 9(*c*), in accordance with Eqs. 15*a* and 15*b*, and are written in parentheses in Fig. 9(*d*). The calculations are as follows:

$$H_{AA'} = \frac{-12}{30} = -0.40 \text{ kip}; \quad H_{CC'} = \frac{+13}{30} = +0.43 \text{ kip};$$

$$H_{EE'} = \frac{+35}{30} = +1.17 \text{ kips};$$

$$H_{AB} = \frac{8 + 6}{22.5} - \frac{-6 - 19}{45} = +1.17 \text{ kips}; \quad H_{CB} = -1.17 \text{ kips};$$

$$H_{CD} = \frac{-5 + 4}{22.5} - \frac{-4 - 36}{45} = +0.85 \text{ kip}; \quad H_{ED} = -0.85 \text{ kip}.$$

The second shear distribution is shown in Fig. 9(*d*); the computations were made in the same way as those for the first shear distribution. The resulting fixed-end moments, marked M'' in Fig. 9(*d*), are balanced by moment distribution in Fig. 9(*e*). The solution is summarized in Fig. 9(*f*), the moments shown being the sums of the quantities appearing in Figs. 9(*a*), 9(*c*), and 9(*e*).

DISCUSSION

I. Oesterblom,[2] M. ASCE.—The idea expressed in the title of this paper is excellent; and the author promises a development that is much needed for a quick adjustment of moment schedules of a framework to which changes are made—particularly, in industrial construction where loads so often must be shifted and sections changed. It remains to be demonstrated that the paper fulfils that promise. The third paragraph of the "Synopsis" and the examples that follow it suggest that the author may have a broader scope in mind: A universal method as a convenient substitute for the Hardy Cross relaxation method.

An important truth appears in the statement (second paragraph of the "Synopsis") that

> "Since the moment at a given section in a continuous structure is a function of the stiffnesses of the various members, the differential of the moment (or moment differential) is a function of the differential changes in the stiffnesses."

but this statement will not help very much unless the functional relation between the moment and all the sections (or the functional relations between one section and all the moments) can be given a very simple and true expression. In view of this fact, both the culminating Eq. 7 and the first sentence of paragraph three in the "Synopsis" seem incongruous. The complex appearance of Eq. 7 is most repellent; and to establish the moment-stiffness relation independently would mean the application of a few cycles of relaxation to the entire frame for each unit couple or unit force. How utterly impossible this task is—in a commercial sense—is not apparent from the paper, which demonstrates only primitive frames; and it is seldom that an engineer is given such simple tasks. This is true for all the examples except the bents in the "Appendix."

Not only is Eq. 7 unattractive, but the question may also be asked: Is it correct? Professor Dell declares cautiously (last paragraph of the "Synopsis") that the "* * * differential quantities are only approximate representations of the true changes." If it is correct, is it useful? Possibly it is useful for design but not for the correction of moments. The basic formula, Eq. 1, was differentiated to the form

$$dM_{AB} = \Psi_{AB}\, dK_{AB} + K_{AB}\, d\Psi_{AB} \dots\dots\dots\dots\dots (24)$$

—leading to Eq. 2. This differentiation is permissible only if the curves of the slowly changing values of K_{AB} and Ψ_{AB} can be given a functional expression in relation to one another or to a fourth variable. What Professor Dell has in mind, seemingly, is not this condition but a rather violent quantum variation of K_{AB}, and thus also of Ψ_{AB}; such a relation does not permit differentiation. It could be done, of course, if the functional relation had been expressed. The sequence of developments that follows, in the paper, is correspondingly suspect, and the superposition that leads to Eq. 7 is entirely out of order. Therefore,

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Eq. 7 represents mere approximation at best; and it may be seriously in error.

Assuming, however, that it is reasonably true; how useful is it? Can a moment at any point be found independently by taking advantage of the stiffness elements of the entire frame? Can moment corrections be applied at all points by the section adjustment at a few points? The profession will be grateful for Professor Dell's answer to this question, because it will have considerable economic significance.

An interesting challenge to test the paper would be its application to the example in Fig. 1, expanded to a three-span, seven-story frame with fixed foundations. Can the author write the moment-stiffness relations for all points of such a frame? This is the kind of frame an engineer encounters intermittently and quite often. Applying the Dell method, an analyst would have his choice (1) of computing all the nodal point moments (if this is what he needs in the design) or (2) of changing the size of three or more of the members in the frame (this is what often happens) and computing the changes to all the nodal point moments due to the change of size (if he prefers).

In the solution of any similar problem there is a fundamental difficulty. The moment at any point is not only an aggregate of superposed moment elements, but also—in a philosophical sense—a divergent series and a very unusual one, which is difficult to set up and quite impossible to integrate. It is fanlike and interwoven—fanlike, because it is a complex assortment of many subordinate series; and it is interwoven, because these subordinate series are intermittently associated with, and assisting, one another. At the same time each one converges separately toward zero.

Naturally, this concept is to be taken as a philosophical ideal only. It is doubtful if any one will ever be able to give definite mathematical expression to the accumulation of moments along their devious paths of approach; but the idea should be helpful nonetheless, both as a guide as to what still may be done and as a warning as to what should not be attempted. The history of kinetics is ample proof how true this is. The great pioneers—Clapeyron, Maxwell, Greene, Otto Mohr, Castigliano, Williot, Manderla, Fidler, Müller-Breslau, Ostenfeld—all made important contributions to theory and method; but they were limited in their applications to continuous beams or arches, and to simple frames.

A number of years ago, the writer had occasion to assign a similar problem to a colleague, distinguished for his grasp of all the advanced methods of structural analysis. The structure was a three-span, three-story, reinforced-concrete building with different spans and story heights and with concrete elements fully designed. There were different uniformly distributed floor loadings on each span, one load concentration on each of the three floors, and an applied wind load—in all possible combinations. The problem was to determine all the maximum dead-load and live-load moments at the nodal points and in the girders.

This colleague had the choice of any one of the classical methods available at that time (and he knew them all); but after three months of study he abandoned the problem as being beyond the reach of human effort.

It was only when the revolutionary method of relaxation was introduced by Hardy Cross,[3] M. ASCE, and R. V. Southwell[4] that one succeeded—with fair economy of labor and complete accuracy—in computing the moments in a really complicated modern frame. The reason why this method succeeded lies also in the underlying philosophy of the problem. By its progressive and alternate freezing and releasing of joints, the relaxation method superimposes a method that is synchronized to the difficulties of the problem; it is thus that the innumerable redundants, hidden in as many equations and set up in a hopeless matrix of determinants, are eliminated. There is only one of the earlier methods to which this does not apply: The graphical method by Fidler, later developed by Müller-Breslau for continuous beams and arches and by Strassner for continuous frames. This method also has given designers a device for mastering the difficulties, and it should be used more extensively.

The critical parts of this discussion are not directed to Professor Dell only; but to the many others who have tried to eclipse the Hardy Cross method by offering a better one. No doubt details of procedure can be improved upon; but until a definitely superior method is discovered, which fits the difficulties of the problem better than the relaxation method, it is mere waste of time and paper to try the impossible. Details of procedure can be improved: Many already have been offered and accepted by the profession. Those who aspire to be pioneers might be more successfully constructive if they worked along these lines and also with the comparatively unknown graphical solutions.

If Professor Dell merely intended to present a rough and approximate solution for simple frames the answer is that a one-cycle relaxation will do equally well; it will be just as rapid—if not more so. Two cycles will be required for heavy point loads only, especially if they are near the supports. Even the famous old equation offered by Clapeyron in 1857 would do as well; and in most such cases it would be better than Eq. 7 proposed by Professor Dell.

William A. Conwell,[5] M. ASCE.—The merit of this paper lies principally in the fact that it extends the theoretical horizon of moment distribution. Many engineers, faced with the prospect of repeated analyses of a problem by moment distribution, have cast about for a more direct method of obtaining results. Their search usually brought them into contact with a variety of equations which separated the designer's mind from the physical characteristics of the structure under consideration. Since "avoidance of such general equations was the object"[6] of moment distribution, the hunt usually ceased at the first encounter with algebra, and a quick retreat to moment-distribution fundamentals was effected. The author, however, was rewarded when he continued his probing after first discouragements. Although use of the moment-stiffness relations does involve formulas, these latter are intimately associated with the fundamental physical characteristic of the change in bending moments caused by a change in stiffness and, if only from this point of view, are worthy of study.

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[3] *Transactions*, Vol. 96, 1932, p. 1.

[4] "An Introduction to the Theory of Elasticity," by R. V. Southwell, Oxford Univ. Press, 1936, p. 91.

[5] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

[6] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 141.

An inspection of Table 1 is enlightening in this regard. It will be noted, for instance, that a stiffness-change ratio, i_2, of 10% in member AB produces a change of −8.44 in M_{BA}; but an equal 10% stiffness-change ratio, i_4, in member BC produces a greater change, +11.08, in M_{BA}. Similarly, a 10% value of i_1 in member AA′ produces a change of +7.10 in $M_{AA'}$, which is less than the −12.38 change produced by a similar 10% value of i_2 in member AB. It is but another step to observe that the effect of a change in stiffness on a bending moment does not necessarily increase with the proximity of the member in which the stiffness change occurs. For example: A 10% value of i_1 in member AA′, the one most distant from member CC′, produces a change of +1.84 in $M_{CC'}$, which is almost as great as the +1.94 produced by a 10% value of i_4 in the adjacent member, BC. From the foregoing it may be readily concluded than a change in stiffness in one member cannot be made without the possibility of a profound effect on even the most distant members of the frame.

With a view to comparing a moment-stiffness analysis with a direct analysis, as regards amount of labor, rapidity of convergence, and other items, the writer designed the rigid frame of Examples 3 and 5, employing direct methods only. The third direct analysis—that is, the second after the original—produced the values given by the author in step (7), Example 5, as the results of his final direct analysis used to check the moment-stiffness method. The only variance of any significance in those data was in the 132.0 kips obtained for F in member AA′ as against 133.7 kips. Accordingly, in Table 4, the author's values were used as the final correct figures, and the several stages of the two methods of analysis are compared with the final results. As may be seen, each successive cycle of the direct analysis steadily approaches the final figures. The second cycle of the moment-stiffness analysis compares favorably with that of the direct analysis; but the third cycle of the former is erratic, when compared to the complete conformance of the latter with the final results. The differences are, of course, small in magnitude, insufficient to produce changes in section, and easily explainable by the unusually great differences

TABLE 4.—Comparison of Results of Moment-Stiffness and Direct Analyses in Example 5

Type of Load	Member	First Cycle		Second Cycle				Third Cycle		
		Value	Error, %	Moment Stiffness		Direct		Moment Stiffness		Direct
				Value	Error, %	Value	Error, %	Value	Error, %	Value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Axial	AA′	136.5	+2.1	136.5	+2.1	132	−1.3	136.5	+2.1	133.7
	BB′	297.8	−1.6	297.8	−1.6	303	−0.1	297.8	−1.6	302.7
	CC′	92.8	−0.9	92.8	−0.9	94	+0.4	92.8	−0.9	93.6
End moment	AB	822	−5.2	867.1	0	865	−0.2	869.3	+0.3	867
	BC	755	−5.1	787.1	−1.1	798	+0.3	786.2	−1.2	796
	AA′	238	+29.3	188.6	+2.5	192	+4.3	186.8	+1.5	184
	BB′	133	−24.4	165.0	−6.2	166	−5.7	179.4	+1.9	176
	CC′	132	+16.8	120.4	+6.5	120	+6.2	115.9	+2.6	113

between the stiffness of certain members in the final design and those in the basic structure.

As to the comparison of the amount of work, the direct method required four analyses for each cycle—one each for uniform loads on members AA′, AB, and BC, and one for a horizontal load at the top of the frame—a total of twelve analyses. The moment-stiffness method initially requires four analyses, one each for moments m_A, m_B, m_C, and m_D. If the recommendation in the "Synopsis," that "the final results should generally be checked by direct analysis" is followed, four additional analyses will be required, making a total of eight. Between the initial and final analyses, the moment-stiffness method requires two series of substitutions in Eq. 7 for changes in the controlling moments in each member. A comparison of the methods in this instance would seem to indicate that the choice is between four direct analyses of the structure and ten substitutions in Eq. 7. Both methods require a certain amount of superposition and proportioning of analyses, but these would largely balance on the work ledger sheet. The decision as to method would probably be one of personal preference, since the labor involved appears to be about the same. Elimination of one condition of loading would throw the balance sharply in favor of the direct method as one less analysis would be required in each cycle, whereas the number of analyses for moment stiffness would remain the same. On the other hand, an additional condition of loading would not be as favorable to the moment-stiffness method since, in the direct method, use can be made of m_A, m_B, m_C, and m_D computed for each cycle and the number of analyses per cycle can be held to a maximum of four.

The second paragraph under step (4), Example 5, should be carefully followed in conjunction with Table 2 to be readily understood. However, the appreciation of designers is gained by calling attention to the check that the sum of coefficients of i in any expression for dM equals zero. Another valuable feature is the analysis of multiple-span peaked bents in "Appendix 2."

Finally, the author has exhibited a fine sense of balance in presenting the paper for what it is worth, allowing it to stand on its merits and making no claims that cannot be substantiated by a reading of the paper and practice in use of the methods.

George H. Dell,[7] Assoc. M. ASCE.—Contrary to Mr. Oesterblom's inference, the moment-stiffness relations were not intended to provide a new method of analysis. They are but a means of calculating changes in the moments as the design is progressively modified and, therefore, are merely an auxiliary tool for use in connection with the moments obtained by one of the standard methods of analysis. In describing the use of moment-stiffness relations, the writer, far from trying to eclipse the Hardy Cross method by offering a better one, recommended and used the Cross method exclusively in the calculation of the moment-stiffness relations. A method cannot possibly be a "convenient substitute" for the Cross method when it relies on that method for its results.

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[7] Asst. Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

The use of moment-stiffness relations places no restrictions on the designer's choice of methods for analyzing the structure or for designing the members, but it does offer a means of calculating the moment changes with rapidity and with a degree of accuracy such that the resulting design is unlikely to contain errors of any consequence.

One of the advantages of using moment-stiffness relations, not previously mentioned, is that it is not necessary to calculate the revised moments in all parts of the structure for a given arrangement of loads, but only at points where the moments control the design of members. Another important advantage worthy of re-emphasis is that, in redesigning a given member, the effects of a proposed change in stiffness are immediately apparent from the moment-stiffness relation, and one is thus able to anticipate and obviate the danger that after the member is revised it will be more seriously stressed than before.

The true moment change that occurs as a result of revisions in the stiffnesses of any or all the members can be expressed in the form of a Taylor series. The writer doubts whether such an expression is what Mr. Oesterblom had in mind in stating that "It is doubtful if any one will ever be able to give definite mathematical expression to the accumulation of moments along their devious paths of approach * * *." It is more likely that he was referring to the problem of determining the maximum positive and negative moments in a structure, such as the three-story building described by him. Inasmuch as the paper (except for "Appendix 2") was not concerned primarily with methods of analysis, the introduction of this problem is beyond the scope of the paper. However, it seems evident that if separate analyses are made to determine the moments throughout the structure, as a result of loads applied to one member at a time, the signs will indicate which members should be loaded in order that the maximum moment may be obtained at a given point. A more difficult and equally pertinent problem is that of finding the particular combination of axial forces and moments that will produce the greatest stresses in the columns.

Mr. Oesterblom's difficulty in following the derivation of Eq. 7 may be clarified by reviewing the statement (in the paragraph containing Eq. 1) that ψ is a linear function of the elastic deformations, which, in turn, are functions of the stiffnesses of the various members. The moments can be determined uniquely by a system of linearly independent "slope-deflection" equations, regardless of the values of the stiffnesses, and hence the moment changes may be legitimately represented by a Taylor series development,[8] provided the series converges. Furthermore, since the independent variables (the K-values) occur linearly in the slope-deflection equations, it is entirely proper to apply the principle of superposition in the determination of the first-order moment changes, resulting from a series of successive changes in stiffness in the same member, or in different members. In addition, since the slope-deflection equations are linear, the moment changes of second, and higher, orders may similarly be obtained by superposition; that is, the terms of a higher order may be found by applying, to the basic structure, certain fixed-end moments obtained in solving for the terms of the preceding order. Thus, Eq. 2 can be rewritten as follows:

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[8] "The Effects of Errors or Variations in the Arbitrary Constants of Simultaneous Equations," by G. H. Dell, *Bulletin No. 309*, Univ. of Illinois Eng. Experiment Station, Urbana, Dec. 6, 1938, Sections 6, 7, and 18, and Appendix A.

$$dM_{AB} = M'_{AB}\, i_{AB} + \delta M'_{AB} \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Eq. 25 expresses the fact that the first-order moment changes are found by solving for the balancing moments, $\delta M'$, resulting from the fixed-end moments, $M'\, i$. It can be shown[9] that the second-order changes are given by:

$$\delta^2 M_{AB} = \delta M'_{AB}\, i_{AB} + \delta^2 M'_{AB} \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Eq. 26 expresses the fact that the second-order moment changes $\delta^2 M$ may be found by using the quantities $\delta M'\, i$ as fixed-end moments.

It does not appear feasible to establish a general criterion for the convergence of the series, and even if such a proof were available, it is necessary that the series converge rapidly in order that the moment change, given by Eq. 7, be reasonably accurate. For the purpose of indicating the rate of convergence of the series for a particular structure, the writer has computed the changes in the controlling moments of Example 5, up to and including the third-order increments, as shown in Table 5. It should be noted, in comparing these results with those of the final analysis as given originally, that there is a small uncertainty in the third or fourth significant figure in both instances, as a result of slide rule inaccuracies, differences in procedure, and small discrepancies in the values of the stiffnesses.

TABLE 5.—MOMENT CHANGES OF HIGHER ORDER, EXAMPLE 5

Symbol	Original moment	MOMENT CHANGES			Total moment
		First term	Second term	Third term	
M_{BA} ..	+822	+47.6	−2.59	−0.60	+866.4
M_{BC} ..	−755	−31.3	−7.90	−1.68	−795.9
$M_{AA'}$ ..	+238	−50.9	−2.76	−0.49	+183.8
$M_{BB'}$ ..	−133	−46.5	+4.06	−0.60	−176.1
$M_{CC'}$ ..	−132	+16.4	+0.76	+0.12	−114.7

Mr. Oesterblom asks: "Can a moment at any point be found independently by taking advantage of the stiffness elements of the entire frame?" It has been shown that the process of determining the moment changes by Eq. 7 is equivalent to performing an analysis of the basic structure to determine the moments resulting from the use of the ($M'\, i$)-quantities as fixed-end moments. From this it follows that all joints of the structure are balanced; in addition, all sidesway conditions are satisfied, so that, theoretically, all the moments are solved, although, as previously stated, it is necessary to compute only the ones that control the design of given members.

The next question—"Can moment corrections be applied at all points by the section adjustment at a few points?"—seems to indicate that Mr. Oesterblom has in mind a method of finding the changes in all the end moments due to a single stiffness change, rather than a method of finding the change in a particular moment because of changes in all the stiffnesses. The writer has treated such a method elsewhere;[9] it consists in applying to the basic structure the M'-moments of the member whose stiffness is changed, and finding the resulting moments throughout the structure. The latter values give the coefficients of the partial moment changes in the moment-stiffness relation.

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[9] "Adjustments in the Design of Statically Indeterminate Structures," by G. H. Dell, thesis presented to the University of Illinois at Urbana, Ill., in May, 1943, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Time and space do not permit a detailed description of the method in this closure; however, as it does not involve the use of *m*-moments, the procedure might be advantageous in the design of the three-span, seven-story structure which Mr. Oesterblom mentions in connection with the question, "Can the author write the moment-stiffness relations for all points of such a frame?" In further reply to the latter question, it should be noted: First, that there is seldom any need for the moment change at more than one point in a particular member; and, second, that Eq. 7, being perfectly general, will enable one to write the moment-stiffness relation for any of the end moments if the required moments, resulting from unit couples and unit horizontal forces, are known for the basic structure. Furthermore, if moment-stiffness relations are required for some intermediate point on the span, they may be obtained by a suitable combination of the moment-stiffness relations for the ends of the member.

It is regrettable that Mr. Oesterblom finds Eq. 7 complex, but it is not possible to formulate a general statement of the first-order moment changes without including all the factors involved. As a matter of fact, the complexity is a function, not of the equation, but of the structure itself. Doubtless, in an extensive structure, the magnitudes of a great many of the *m*-moments would be so small as to permit their omission from a particular moment-stiffness relation.

There is probably no advantage in using moment-stiffness relations in the design of continuous beams; the principal field for their application is in frames in which the columns are subjected to considerable moments and, consequently, where sidesway plays an important part. Therefore, the procedures recommended by Mr. Oesterblom as being superior (the application of one cycle of moment distribution, or the use of the equation of three moments) would be extremely inappropriate and inaccurate. In Example 5, the final moments (Table 3, Col. 14), in foot-kips, as given by one cycle of distribution are as follows: $M_{BA} = 817$; $M_{BC} = 803$; $M_{AA'} = 205$; $M_{BB'} = 46$; and $M_{CC'} = 25$. The errors are 5.7%, 0.9%, 11.4%, 74%, and 78%, respectively. By applying three cycles of distribution, which would be closely equivalent to using the equation of three moments, the corresponding moments are 821, 794, 231, 57, and 31, with errors of 5.3%, 0.2%, 25%, 67%, and 72%, respectively.

It is true that in cases where only minor revisions are involved in the design of a frame there is no need for moment-stiffness relations; however, it is not always possible to foretell whether a particular problem will involve difficulties. Where difficult situations do occur, moment-stiffness relations will be advantageous.

Mr. Conwell notes that in the frame of Example 5 a given end moment may be appreciably influenced by a change in stiffness of a remote member. This tendency would doubtless diminish in proportion to the number of spans and stories, so that terms with very small *m*-moments could be omitted from the moment-stiffness relations. He also calls attention to the value of the check afforded by the sum of the coefficients. In this connection, it is desirable that a proof be given of the statement that the sum of the coefficients in a complete moment-stiffness relation should be equal to zero. By making all the stiffness-change ratios equal to a constant value, c, one obtains from Eq. 7:

$$dM_{AB} = c\,(m_{A,AB}\,\Sigma M'_{An} + m_{B,AB}\,\Sigma M'_{Bn} + \cdots$$
$$+ m_{\Delta 1,AB}\,\Sigma H'_1 + m_{\Delta 2,AB}\,\Sigma H'_2 + \cdots + M'_{AB}) \ldots\ldots (27a)$$

For equilibrium, at any point, J, $\Sigma C_{Jn} + \Sigma M'_{Jn} = 0$ and, for each sidesway condition, $\Sigma Q + \Sigma H' = 0$. The expression for dM_{AB} may then be written as follows:

$$dM_{AB} = -\,c\,(m_{A,AB}\,\Sigma C_{An} + m_{B,AB}\,\Sigma C_{Bn} + \cdots$$
$$+ m_{\Delta 1,AB}\,\Sigma Q_1 + m_{\Delta 2,AB}\,\Sigma Q_2 + \cdots - M'_{AB}) \ldots\ldots (27b)$$

By taking note of Eq. 5a it is seen that Eq. 27b thus reduces to:

$$dM_{AB} = -\,c\,(M'_{AB} - M'_{AB}) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (27c)$$

The investigation reported by Mr. Conwell in Table 4, dealing with a comparison of designing by successive revisions and direct analyses and by use of moment-stiffness relations, constitutes a sincere and unprejudiced trial, for which the writer is duly appreciative. It tends to confirm the writer's view that, if a final check analysis is necessary, a very considerable length of time is required for frame analysis, whether moment-stiffness relations are used or not. If the final check analysis is omitted, the use of moment-stiffness relations should result in a material saving of time. It is the writer's opinion that moments obtained by means of moment-stiffness relations will seldom be in error by more than 5%. On the average, half the errors would be on the unsafe side and half, on the side of safety.

In the case of a column, whose required area depends on the direct stress as well as on the moment, the resulting percentage of error in cross section would be only a part of the error in the moment. In using rolled sections, excess area is usually provided; whereas, in concrete structures the sizes of the members are such that there may be considerable error involved in the usual practice of using the moment of inertia of a member all the way to the intersection of center lines. In view of these, and other, considerations it would appear that the omission of the final check analysis would have no serious consequences. In this connection, it may not be irrelevant to refer to the common practice of basing the design of members of a truss on the primary stresses, although secondary stresses of considerable magnitude are generally present, which are always additive to the effects of direct stress.

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THRUST EXERTED BY EXPANDING ICE SHEET

By Edwin Rose,[1] Esq.

With Discussion by Messrs. Ernest Brown, B. R. McGrath, Robert E. Kennedy, and Edwin Rose.

Synopsis

Expansion of an ice sheet, as the result of a temperature rise after a cold wave, can develop considerable thrust against abutting structures. Design loads for this effect on dams and hydraulic structures have varied from no allowance to values of from 40 to 50 kips per lin ft. A study and analysis, conducted to obtain a rational procedure for estimating the magnitude of ice pressures, and a review of available material on the subject are presented in this paper.

By the application of experimentally determined ice temperature-pressure relationships, an analysis is made that yields the magnitude of pressures developed for the variables of ice thickness and the rate of rise of air temperature. The temperature rise that follows the absorption of solar radiation is considered in the study of pressures developed. The effect of conditions of restraint around the edges of the ice sheet is estimated. Thickness and buckling of ice sheets, extreme temperature variations occurring in nature, and field measurements of ice temperatures and pressures are discussed. An analysis is made, for comparison, of measured and computed pressures. Curves are given to provide a guide for the reduction in reservoir water elevation, allowing for the ice-thrust effect on existing dams subject to ice pressure but not designed for it.

It is concluded that design loads of from 40 to 50 kips per lin ft are too extreme and are not realized even under the most severe conditions. Results of analysis show pressures developed for the important factors of thickness, rate of increase in air temperature, lateral restraint, and solar energy absorption. These data are presented in graphical form to facilitate the selection and interpolation of values for design loads and indicate maximum probable ice thrusts for continental United States to range from 5 to 20 kips per lin ft.

Note.—Published in May, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] **Engr., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.**

1. Introduction

In latitudes where severe winters prevail, a solid covering of ice forms on quiet surfaces of rivers and lakes. When such an ice sheet is subjected to a rise in temperature, the expansion of the ice causes the development of pressure against abutting structures. Able investigators (see Appendix) (1)(2)(3)(4)(5)(6)(7)[2] have conducted numerous difficult experiments and have obtained important data pertaining to the physical properties of ice, but the complexity of the problem is such that the magnitude of the thrust exerted by an expanding ice sheet cannot be predicted accurately. The variable nature of the load deformation and of the thermal expansion properties of ice makes this a difficult problem to analyze.

In the design of dams the consideration of ice pressure has varied from no allowance to values as high as 50 kips per lin ft (1)(2)(8). (The kip is 1 "kilo-pound"—that is, 1,000 lb.) The high estimates have been based on a crushing pressure of ice of about 400 lb per sq in. Assuming that this crushing pressure was realized at the upper surface and that no pressure was exerted at the lower surface, the total force developed, assuming linear variation within the ice sheet, was computed as 2,400 lb per in. of ice thickness per linear foot. For an 18-in. ice sheet, the thrust would amount to 43,200 lb per lin ft.

However, the peculiar plastic properties of ice, and the effects of incomplete restraint and limited temperature variations, make it seem unlikely that such high pressures would be realized, even under the most severe conditions. Experiments conducted by E. Brown and G. C. Clarke, M. ASCE (3), indicate that, because ice flows when it is loaded, considerable pressure can develop only if the temperature rise is rapid.

The simultaneous occurrence of critical conditions of the many variables required to produce any considerable ice pressure seems rather improbable. Nevertheless, experience has shown that such pressures can, and occasionally do, develop. The designer is interested in the maximum probable pressure likely to occur within the life of a particular structure, and the present study was conducted from this viewpoint. It is proposed to present a review and study of the material available on this subject and, by application of experimental data, to obtain a rational procedure for determining the magnitude of ice pressures on dams. Properties of ice are so variable and experimental and field data are so scarce that the results can be only an approximation of actual ice pressures.

The effect of expansion during the time that water is freezing is not considered, since it is not likely to be of practical importance. The thrust exerted by large fields of moving ice also is not discussed.

2. Plastic Properties of Ice

Ice is not an elastic solid; it behaves as a viscous solid, flowing under applied load—the extent of flow varying with time and temperature. The rate of yielding also changes with the structure of the ice and with the direction of loads according to the optical axes of the ice crystals. In the formation of an ice sheet, the ice crystals are so oriented that the optical axes are normal to the freezing surface (4).

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

Values of the modulus of elasticity of ice have been determined by different investigators (1)(4)(6) and range from about 250 kips per sq in. to 1,400 kips per sq in. The variation in the time factor is the principal cause of the wide range of values. The maximum value cited (1,400) was determined by dynamic methods and is unaffected by progressive yielding (4). The modulus of elasticity is greater with lower temperatures and decreases as the applied load increases, and also as the loading time interval increases. Tests reported by the Joint Board of Engineers for the St. Lawrence Waterway (6) show plastic action under very small loads.

The applied load and the rate of applied load in the problem under consideration develop as a result of the rate of expansion of the ice sheet. Ice expands with increasing temperature and contracts with decreasing temperature. However, the coefficient of expansion of ice is also variable with different temperatures, increasing with increasing temperature. The mean linear coefficient of expansion is about 0.00003 per degree Fahrenheit.

The crushing strength of ice has generally been accepted as about 400 lb per sq in., but this also depends on the rate of loading and the temperature. The St. Lawrence Waterway tests (6) show a considerable increase in crushing strength as the temperature drops, values of 300, 693, and 811 lb per sq in. being recorded at temperatures of 28° F, 14° F, and 2° F, respectively.

3. Laboratory Data

In 1932 Messrs. Brown and Clarke presented (3) results of laboratory tests designed especially to secure data on pressures exerted by expanding ice. These were conducted on 3-in. cubical ice specimens placed between two loading blocks in a testing machine. The cube was oriented so that the pressures would act normal to the length, or optical axes, of crystals as under natural conditions. The specimens were subjected to controlled temperature changes and the force developed by the expanding ice was measured. From the results of the tests, Messrs. Brown and Clarke plotted a curve (3*a*) showing the nonlinear relation between the rate of change of temperature and the rate of change of pressure. Fig. 1 (mostly from data released by Messrs. Brown and Clarke (3)) shows that, as the rate of change of temperature increases, the rate of change of pressure increases at a greater than linear rate. The dotted part of the curve, indicating a slight extension beyond the Brown-Clarke experimental data, is based on results of tests on continuous

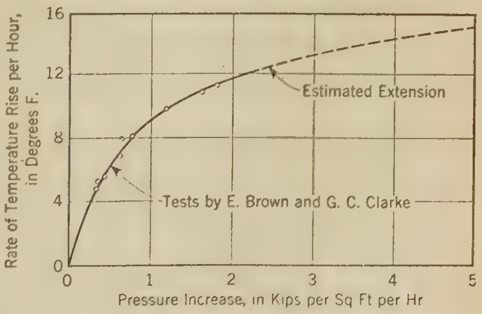


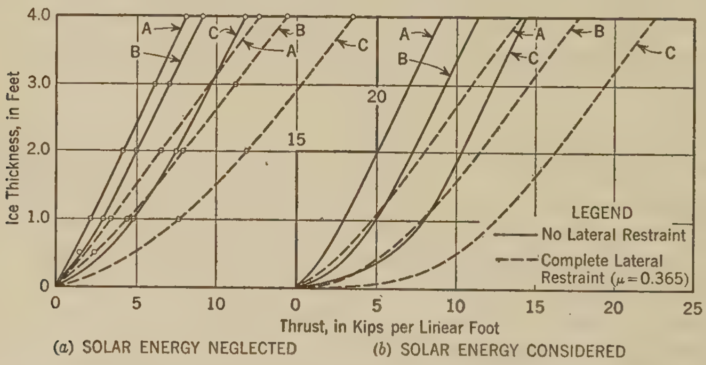
Fig. 1.—Pressure Change Resulting from Increases in the Temperature of Ice (Tests Conducted with No Lateral Restraint)

yielding of ice under sustained load given in the Report of the Joint Board of Engineers on St. Lawrence Waterway (6).

The importance of the time factor is indicated by these results—the greatest pressure change develops when the temperature rate of change is greatest. If temperature change in the ice is relatively rapid, the flow strain lags in relation to total strain, causing greater pressure to develop than occurs for the condition of slower temperature change. These experiments serve to verify previous tests and provide definite data on the magnitude of the forces occurring as a result of particular temperature changes.

4. Application of Experimental Data

As mentioned by Messrs. Brown and Clarke, the test conditions of their experiments are not entirely representative of field conditions of an expanding ice sheet: First because of the rate and extent to which a rise in air temperature will be felt through the ice sheet and, second, because of the effect of lateral restraint of the ice in a direction parallel to the dam. By estimating the maximum rate of air-temperature rise and the effect of lateral restraint, an analysis can be made that will give the maximum ice pressure for these conditions. Crack formation is assumed not to affect the continuity of the ice sheet since cracks generally form at night when the ice has contracted. The cracks become filled with water which freezes to form a solid sheet at daybreak when the normal conditions for expansion begin.



(a) SOLAR ENERGY NEGLECTED (b) SOLAR ENERGY CONSIDERED

Fig. 2.—Ice Thrusts for the Variables, Ice Thickness, Air-Temperature Rise, and Restraint

Since the rapidity of temperature rise is of great importance in the development of expansion pressure in an ice sheet, it appeared desirable to compute pressures for several different rates of air-temperature rise and various thicknesses of ice sheets. Consequently, ice pressures were calculated for air-temperature rises of 5° F, 10° F, and 15° F per hr, for ice thicknesses as great as 4 ft. The results are shown in Fig. 2(*a*).

The assumptions used in this analysis were as follows:

1. The initial air temperature is − 40° F;
2. The initial ice temperature varies linearly from − 40° F at the air surface to + 32° F at the lower surface;

3. The air temperature changes at rates of + 5° F (curve A), + 10° F (curve B), and + 15° F (curve C) per hr, from − 40° F to + 32° F; then it remains constant at + 32° F until the temperature throughout the ice sheet has risen to about + 32° F;

4. Emissivity effects are neglected—that is, the temperature of the ice at the air surface is assumed the same as the air temperature itself;

5. The thickness of the ice sheet remains constant;

6. The diffusivity constant of ice, h^2, in feet² per hour, equals 0.0434 (4);

7. Poisson's ratio, μ (4), equals 0.365; and

8. Absorption of solar energy is neglected, consideration of this factor being treated subsequently.

The temperature gradients within the ice sheet are computed by the approximate method developed by E. Schmidt (9)(10)(11)(12). The ice sheet is considered divided into a finite number N of slabs having equal thickness increments, ΔL equals L/N, normal to the surface. The time interval Δt in hours and the space intervals ΔL in feet are selected so that:

$$\frac{(\Delta L)^2}{2\, h^2\, \Delta t} = 1 \quad \text{.......................... (1)}$$

If θ_p, θ_q, and θ_r are temperatures at the beginning of the time interval Δt of three successive points, p, q, and r, distances ΔL apart, then

$$\theta'_q = \tfrac{1}{2}\,(\theta_p + \theta_r) \quad \text{......................... (2)}$$

in which θ'_q is the temperature of point q at the end of the time interval Δt.

From the temperature gradients in the ice sheet thus computed, the pressures at various depths developed are determined from Fig. 1. The total load per linear foot of structure can then be computed by summation across the depth of the ice sheet. Loads are plotted for various depths of ice sheets in Fig. 2(*a*). A maximum pressure of about 11.8 kips per lin ft is indicated for the 4-ft ice sheet for no lateral restraint. An independent analysis of ice pressure, by B. R. McGrath for an assumed air-temperature rise of 10° F per hr, gave a good comparison with the results shown in Fig. 2(*a*).

The effect of lateral restraint is estimated on the basis of the behavior of an elastic slab. Elastic behavior is approached only for the conditions of very rapid strain changes when flow is small. Poisson's ratio may not be applicable for flow conditions, but pressures based on elastic behavior for restraint conditions will give the most severe loading. The ratio of increase in stress for complete lateral restraint to the stress developed for no lateral restraint is given by $\frac{\mu}{1 - \mu}$. Assuming $\mu = 0.365$ (4), a factor of 0.575 is obtained for the increase in ice thrust resulting from complete restraint. The values computed for no lateral restraint are increased by this factor and are plotted as dashed curves in Fig. 2(*a*). These curves represent the upper limit or extreme pressures for the assumed conditions. The maximum pressure for complete lateral restraint for the 4-ft ice sheet is shown as approximately 18.5 kips per lin ft.

Although the curve in Fig. 1 cannot be applied for more rapid temperature rises, rough estimates of pressures were made for extreme cases with rises of 20° F and 72° F per hr. Thrusts per linear foot of ice sheet 1 ft thick amounted to about 8 kips and 15 kips, respectively, for no lateral restraint and to about 12.5 kips and 24 kips, respectively, for full lateral restraint for the 20° F and the 72° F rise. A 4-ft ice sheet is estimated to exert a thrust of from about 15 kips for no lateral restraint to 24 kips for full lateral restraint for the assumed 20° F per hr of air-temperature rise.

5. Solar Radiation Considerations

When the ice sheet is free of snow and exposed to sunlight, its temperature may have an added increment of increase by solar radiation, quite apart from the change caused by an increase in air temperature. Some increase in ice pressure may be anticipated because of absorption of solar energy.

The magnitudes of ice thrusts are computed to include the solar energy effects for the same conditions assumed in the preparation of the data presented in Section 4. The following additional assumptions were made for inclusion of solar radiation:

9. The latitude is 40°;

10. The vernal equinox is considered as the time limit for the formation of clear solid ice of considerable thickness;

11. An atmospheric transmission constant (13)(14) of 0.9 is based on a clear atmosphere of low humidity at a fairly high elevation for continental United States; and

12. A coefficient of heat transfer (9) of 2 Btu per sq ft per hr per degree Fahrenheit is assumed to allow for surface losses of the absorbed solar energy.

For these assumed conditions, the solar energy that reaches the surface of the ice sheet during midday is then about 250 Btu per sq ft per hr. N. E. Dorsey (4) states that, in the visible spectrum, the absorptivity of ice is small but that, beyond a wave length of one micron, it is great and analogous to water. Consequently, about 40% of the solar energy reaching the surface, essentially all that of wave length greater than one micron, is absorbed within 4 in. of the surface. For a clear ice sheet, 1 ft thick, an additional 15% of the energy is absorbed whereas, for a 4-ft thickness, this additional absorption amounts to about 25%. Most of the remainder of the energy is transmitted although a small percentage is reflected.

The results of this analysis, shown in Fig. 2(*b*), indicate that the absorption of solar energy may produce a considerable increase in thrust above that caused by the rise of the air temperature only. This analysis is believed based on severe assumptions, and therefore the results represent a reasonable estimate of the maximum value of ice thrust. A maximum pressure of about 14.2 kips per lin ft is shown in Fig. 2(*b*) for the 4-ft ice sheet with no lateral restraint and with an air-temperature rise of 15° F per hr. This value represents about a 20% increase in the corresponding pressure developed, neglecting solar energy. For the thinner ice sheets, the percentage of pressure increase to be expected with the inclusion of solar effects is higher, as shown by comparison of Figs. 2(*a*) and 2(*b*). Factors that influence the degree to which solar energy will affect the temperature and pressure changes in the ice sheet include: The sun's altitude, cloudiness, elevation above sea level, sky radiation, atmospheric transmission losses, clearness of ice, wind velocity, and losses due to convection, conduction, back radiation, and evaporation.

Records obtained at reservoirs controlled by the Bureau of Reclamation, United States Department of the Interior (USBR), show cases in which the ice sheet does not freeze solidly to the structure but is separated from the dam by a film of water or by unsound or rotten ice. Similar conditions often exist around the edges of a reservoir, in which case restraint is very limited. Factors which may contribute to this condition are exposure of the upstream face of a dam to direct sunlight and heat stored in a massive concrete structure. Under such conditions, when the ice adjacent to the dam is melted, rotten, or slushy, the thrust transmitted to the dam is insignificant.

If the upstream face of the dam is in the shade so that the ice sheet is likely to be well frozen to the dam and if a similar condition exists around the shore in the near vicinity of the dam, but a considerable area of the central part of the ice is exposed to the sun's radiation, then a condition probably exists favorable to the development of an increase in pressure against the dam as a result of solar energy absorption.

6. Field Data on Ice Pressures

The late William Cain, M. ASCE, cited a case (1) of the field measurement of ice pressure in which expanding ice between two bridge abutments 90 ft apart caused the surface to arch the ice 3 ft above its original level, developing a thrust of 21 kips for a 12-in. ice sheet. This value presumably represents thrust per linear foot although the reference (1) does not state specifically. Computation by the thin cylinder formula gives 20 kips per sq ft for this pressure. Another case of record (1) is that by Duncan MacPherson concerning a bridge pier subjected to ice pressure on one side only. The thrust of ice in this case was given as 18 kips per lin ft for a 12-in. ice sheet. At Keokuk, Iowa, a gate structure was broken by the thrust of an expanding ice sheet. This thrust, even though partly relieved by round-nose piers above the broken gate, exceeded 7.4 kips per lin ft (15).

Ice pressures and ice temperatures were measured at the Hastings Lock and Dam on the Mississippi River in Minnesota during the winter of 1932–1933. An account of these measurements (5) was given by H. M. Hill, M. ASCE. Simultaneous records of pressures and temperatures were obtained for a period of several days.

Measurements were taken at two different locations and by two different methods. In the first method, a simple beam of two 10-in. channel sections was placed in the plane of the ice sheet supported against the guide wall of the lock at the west side of the river. The beam was calibrated in a testing machine prior to its use, and the deflections were read by a micrometer with an electrical contact. This beam was set in place so as to measure the pressure exerted laterally—that is, normal to the direction of flow of the river.

Ice pressures were also determined in the upstream-downstream direction by strain measurements in the principal upper members of the Tainter gate near the central part of the dam. The pressure was determined directly from the stress-strain relation and the horizontal component of the strain measurement in the members. The gate framing is complicated, and a more exact determination of pressure would have necessitated strain measurements in all members of the gate. However, the strains in the top member provided guides to the intensity of ice pressure.

Ice temperatures at depths of 6 in., 12 in., and 15 in. in the 18-in. ice sheet were recorded at a point in the shade near the test beam. No ice temperatures were measured at the Tainter gate. There was a 2-in. cover of snow on the ice sheet during the period of measurement.

The maximum pressure measured at the beam was about 1.4 kips per lin ft and that at the Tainter gate was about 4 kips per lin ft. Factors that might have caused a larger pressure to exist at the gate are: (*a*) Increased thickness of ice sheet at the gate, (*b*) the downstream drag of the current on underside of the ice, and (*c*) the effect of arch action across those gates, which were open and through which water was discharging, causing a concentration of thrust at the gates that were frozen. The method of pressure measurement probably was not as accurate at the Tainter gate as at the beam.

Measured data at Hastings Dam indicate that pressure developed as the result of a rise in temperature in the subzero region will not be as great as that resulting from a similar temperature change above zero.

7. Comparison of Measured and Computed Values

The period of the most rapid temperature rise reported by Mr. Hill at the Hastings Dam (5) is selected for study. For the 23-hr period from midnight, January 17, to 11 p.m., January 18, the temperature rose from 10° F to 32.5° F. By applying the experimental temperature-pressure data of Messrs. Brown and Clarke to the temperature change actually recorded in the ice sheet during this period, pressure changes are computed to provide a check on the pressure changes determined from deflection and strain measurements over this same period.

A comparison of the measured ice-thrust changes (in pounds per linear foot) with those computed from the measured ice-temperature data is as follows:

Description	Thrust
Computed Thrust Change:	
No lateral restraint	695
Complete lateral restraint (Poisson's ratio assumed as 0.365)	904
Measured Thrust Change:	
At the test beam	890
At the Tainter gate	3,500

The record of temperature change indicated 1.538° F per hr for 13 hours, followed by 0.250° F per hr for 10 hours. Inspection of these values discloses that the computed change in ice thrust, with complete lateral restraint (904 lb per ft) provides a close check on the measured value at the beam (890 lb per ft). However, it is not likely that lateral restraint was complete at this location. Therefore, an intermediate value between no lateral restraint and complete lateral restraint may be a more logical one for comparison and would give a slight increase in measured value over the computed value.

The measured thrust at the Tainter gate (3,500 lb per ft) is very high compared with that computed for complete restraint, but this unusually large value may be charged, at least partly, to the factors in Section 6. The uncertainty of the effect of these various factors scarcely justifies a quantitative comparison of measured values at the Tainter gates. However, the actual pressures developed at this location may be higher than those computed from the temperature-pressure data in Fig. 1.

8. Extremes in Air-Temperature Rise

Since the development of high ice pressures is a function of the rapidity of air-temperature rise, some study was made of the literature to determine the extent of extreme variations occurring in nature. Rapid air-temperature rises are not uncommon in regions where chinook winds occur.

Some extremely rapid values of temperature rise from low temperatures have been recorded at Havre, Mont., including one case of a rise of 27° F in 5 min. One of the most remarkable temperature fluctuations on record (16) occurred in the Black Hills region of South Dakota in January, 1943, after a severe cold wave in which low temperatures of − 40° F were recorded. The temperature at Rapid City, N. Dak., was characterized by alternating rapid rises and drops, such as a rise of 32° F in 4 min, followed by a drop of 22° F in 3 min, and then by another rise of 36° F in 5 min. The interval between these periods of extreme changes was usually less than an hour. After several cycles of this type of variation, the temperature remained stationary above freezing. An extreme rise of 49° F from − 4° F to + 45° F was recorded in 2 min at Spearfish, S. Dak., during this same period. Earlier Black Hills records include a number of similar fluctuations but none of such phenomenal rapidity.

These records indicate that very rapid temperature rises do occur in nature. Although the cases cited are extremes, it is not unreasonable, in certain localities where chinooks are common, to expect rapid temperature rises from subzero temperature to above the freezing point at rates of the order of 5° F to 15° F per hr. A sustained temperature rise for several hours at a rate approaching 5° F per hr might reasonably occur in almost any region of subzero winter temperatures.

9. Consideration of Buckling of an Ice Sheet

In an elastic-slab analysis (17), G. Albenga indicates that an ice sheet would buckle long before the high pressures sometimes considered could develop. He assumes that the ice sheet is in a rectangular basin and considers it as an elastic slab, simply supported at the boundary, and computes the critical pressure applied at the edges at which buckling will occur. A critical value of about 16 kips per lin ft is obtained for an ice sheet about 20 in. thick in a square basin about 165 ft on a side.

In an unpublished memorandum on this subject, R. E. Glover states that Mr. Albenga has neglected the stabilizing effects of flotation forces on the slab. An analysis including these effects is then made and the results, when adjusted to the same modulus of elasticity used in Mr. Albenga's analysis, give a critical pressure of about 35.4 kips per lin ft.

At this point, the reefs or pressure ridges formed on northern lakes by the expansion and buckling of an ice sheet might be mentioned. Records indicate that reefs form in a warm period preceded by a very cold one and that generally the ice is free or nearly free of snow (7). The fact that pressure ridges usually form when there is little or no snow cover on the ice indicates that the lack of snow permits more rapid diffusion of surface heat throughout the ice with a consequent development of higher ice pressure. The thermal conductivity of snow, especially if it is fresh loose snow, is considerably lower than that of ice; therefore, it acts as an insulator. Pressure ridges seldom occur when the ice is thicker than 1 ft although they are known to have formed with the ice 20 in. thick. This phenomenon might be interpreted as indicating that, for ice sheets less than 1 ft thick, the critical buckling load is the limiting factor governing the ice pressure on a dam, whereas, for very thick ice sheets (about 2 ft or more), the crushing strength or pressure developed before appreciable flow would be the limiting factor.

10. Thickness of Ice Sheets

Although the air temperature is a factor that controls the rate of freezing of an ice sheet, actual thicknesses developed are dependent on local conditions of cloudiness, exposure to direct sunlight, snow cover, radiation, wind, water currents, latitude, and elevation.

Unpublished studies, by R. E. Kennedy, M. ASCE, of ice thicknesses on reservoirs of USBR projects indicate the occurrence of a maximum thickness of about 2.5 ft during the 4-yr period of record. Although these data are obtained for the latitude range of continental United States, greater thicknesses do occur in this country, and ice sheets more than 5 ft thick have been measured in Russia (18) and northern Canada (6).

The data recorded for the USBR projects also reveal that considerably greater ice thickness develops at places that are free of snow as compared to the snow-covered parts of the ice sheet a short distance away. This is in accord with expectations, but the extent of the variation in thickness is sometimes considerable. For example, on one reservoir, two measurements were taken approximately 60 ft apart, and the only factor that could be noted as contributing to the difference in ice thickness was a snow cover of 6 in. at the one point with no snow at the other. The ice thickness, relatively early in the winter period, was 9 in. at the former location, compared with 23 in. at the point of zero snow cover. Later, during the winter, the thicknesses at these two places approached uniformity as the result of continued cold weather and the drifting of snow. High winds sometimes keep the ice surface relatively free of snow, even after a considerable snowfall.

11. Reduction in Water Level to Compensate for Ice Thrust

At an existing dam, designed with no allowance for ice pressure, the following procedure may be used to provide safety against expanding ice. Generally all dams are designed for the full reservoir condition with the water surface at or very near the top of dam. It would be possible to compensate for the effect of an ice thrust by lowering the reservoir level during the season when ice pressures may exist, so that the total load at the critical section would not be larger than for the full reservoir condition.

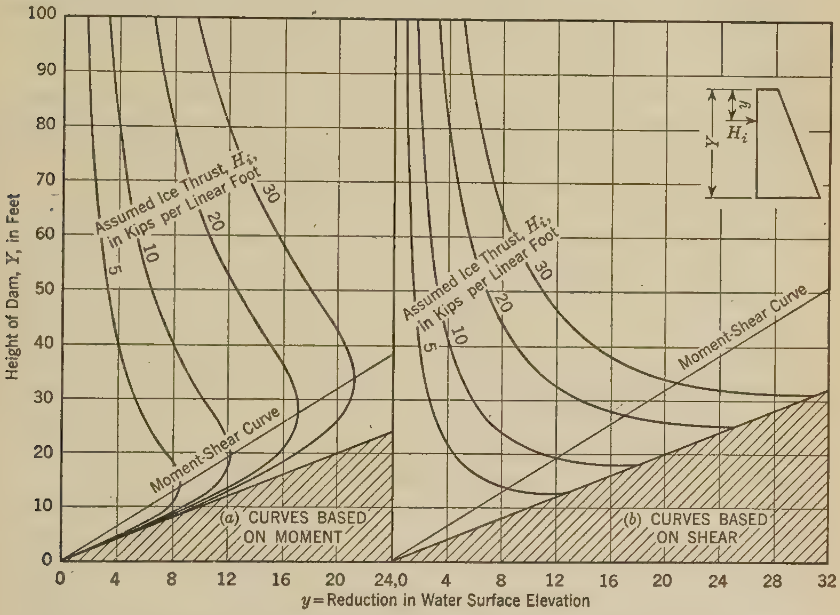


Fig. 3.—Reduction in Reservoir Water Level to Compensate for Ice Thrust

Fig. 3 shows the required lowering (y) of a reservoir to balance the effect of the indicated ice thrusts for variations in the heights (Y) of dams. The criterion used for the reduction in Fig. 3(*a*) is the moment at the base, whereas Fig. 3(*b*) shows similar results for the horizontal shear as the governing factor. Inspection of the curves in Figs. 3(*a*) and 3(*b*) discloses that the moment effect governs (that is, yields larger reductions in the water level) for all heights greater than those at the maximum points of curves in Fig. 3(*a*). The location above which moment governs and below which shear governs is shown by the indicated lines marked "moment-shear line" in Figs. 3(*a*) and 3(*b*). The base of a dam is ordinarily the critical section, but the curves can be applied to any other elevation by considering the height above the elevation in question. The curves are applicable to the gravity-type dam in which the load is transferred vertically downward to the foundation.

The curves indicate that, in the case of high dams, the ice thrust (H_i) is of relatively minor importance in its effect on forces at the base of the section, whereas for low dams it is of considerable importance. For high dams, it should be borne in mind that an elevation at, say, midheight, might give a condition more severe than the base elevation, even though the latter is the location of maximum stress for full reservoir. Complete consideration of this factor requires actual stress and stability analyses.

12. CONCLUSIONS

In regard to the magnitude of ice pressures against dams, design loads of from 40 to 50 kips per lin ft appear to be too severe. A general guide for the determination of design loads, based on analysis applying available laboratory test data, is established so that a reasonable estimate can be made by interpretation of local conditions of maximum thickness, restraint, and temperature variation.

A fair estimate of the maximum ice pressure exerted against a dam or hydraulic structure can be obtained from Fig. 2. To apply these data to a particular locality, it is necessary to establish (1) the maximum thickness to be expected, (2) the maximum likely rate of air-temperature rise, (3) the extent of restraint at the proposed structure, and (4) the extent of exposure to solar radiation. The first two items can be determined, at least approximately, by study of meteorological records and historical data for the region. The third can be estimated by study of the topography and of the location and type of structure to be built. The exposure of the ice to solar radiation during winter can be determined from the location of structure. With these variables known, the corresponding ice pressure developed can be obtained from Fig. 2. Although field measurements are too meager to be conclusive, they indicate that slightly higher pressures may exist than those determined by application of the experimental temperature-pressure data.

In consideration of restraint, the effect of sloping or of unstable banks is of primary importance. If the bank parallel to the axis of a straight dam is steep and the banks on the sides are also steep, and if the ice is solidly frozen to the restraining edges, then restraint is complete, or nearly so. If the opposite bank is steep but the banks on the sides are sloping or unstable, the condition of no lateral restraint is approached. If the edge of the ice sheet on all sides is free to slide up sloping banks, then a condition of no restraint exists; consequently, the pressure exerted against structure is minor. The curves for complete lateral restraint in Fig. 2 represent the extreme pressures based on the very severe condition of elastic restraint. As such these curves represent an outside limit for the air-temperature change and thickness considered.

Direct exposure of the ice sheet to the sun's rays results in an additional ice-temperature rise and the transmission of increased thrust to the dam. If sound ice is frozen solidly to the dam and a large part of the ice sheet is exposed to the sun's rays, an allowance for ice pressure can be determined from Fig. 2(*b*). Comparison of results shown in Figs. 2(*a*) and 2(*b*), considering no lateral restraint, reveals that inclusion of solar energy causes increases in ice pressures of from about 10% to 65%. This inclusion of solar energy is based on the severe assumptions of 40° latitude and the time of the vernal equinox.

Before making a definite selection of the value obtained by use of Fig. 2 as the design load, consideration should also be given the following factors which may dictate some modification or special study:

1. The drag of current under an ice sheet, especially in regard to river ice;
2. Snow cover on the ice sheet;
3. The action of wind;
4. Confinement of ice in a location where volume expansion during the freezing process may cause damage; and
5. Fluctuation in reservoir water-surface elevation.

The drag of the current beneath an ice sheet (factor 1) is particularly important in regard to river ice. If continuous snow cover can be assumed (factor 2), the insulating effect will reduce both the thickness of ice developed and the rate of ice-temperature rise. Wind will tend to keep snow cover from remaining on the ice (factor 3). A strong wind blowing toward the dam may cause an increase in thrust due to skin-friction action on the ice surface. If the ice is broken or rough, this action may be important. This is especially true during periods when the ice is breaking up and large fields of moving ice may be piling against structures and forcing through passages. This special problem is not considered herein, but is discussed elsewhere (8). Volume expansion during the freezing process creates extreme pressures at places where opportunities for expansion are restricted (factor 4); but such conditions are not likely to occur in rivers or lakes. Appreciable change in elevation of reservoir water surface (factor 5) would be likely to cause bending, cracking, or other rupture of continuity of the ice sheet near the structure, which, in general, would tend to reduce the danger of ice thrust.

Fig. 2 shows the thrusts developed for assumed rates of air-temperature rise of 5° F, 10° F, and 15° F per hr. Values of from 5 to 20 kips per lin ft are indicated for maximum probable ice thrusts for continental United States. For those limited geographical areas where extreme temperature fluctuations have occurred and might be anticipated in the future, the maximum pressure expected (even for conditions of severe restraint and thick ice with solar energy included) is estimated at about 30 kips per lin ft as an extreme value. Regardless of the rapidity of the rise in the air temperature, the rate of temperature rise in the ice sheet is a function of the diffusivity constant of ice and consequently occurs at a more limited rate at greater depths. At increasing depths this reduced rate of temperature rise is more pronounced so that a limit to the possible pressure is approached.

The critical buckling pressures obtained by considering the ice sheet as an elastic slab are probably not ordinarily reached, primarily because the plastic yielding effect cannot be accounted for. Records of reef formation indicate that arching and buckling of an ice sheet does occur but that there is a limiting thickness of about 1 ft beyond which buckling will rarely occur. This fact might indicate that in a heavy ice sheet the stability is greater and that the flow of ice under pressure relieves the critical pressures required for buckling. In cases where the ice sheet is closely restrained and the horizontal dimensions are relatively small, more pressure may develop than in the case of an ice sheet of larger horizontal dimensions in which buckling may occur to relieve extreme pressures, provided that the thickness is small enough.

For existing structures not designed for ice pressure, although such pressures may occasionally occur, it may be desirable to limit the elevation of the reservoir surface during the winter season so that ice pressure would not be acting coincident with the full water-load condition. Generally, all dams are designed for the full reservoir condition with the water level at or near the top of the dam. Fig. 3 presents the curves showing the required water-surface reduction from full reservoir condition during the season when ice thrust may occur so that the added effect of ice thrust is offset by the reduction in the water load. Curves are shown for several assumed values of ice thrusts for various heights of structure, based on moment and shear as governing criterions. The effect of ice thrust is of relatively minor importance at the base of high dams; but for low sections it represents a factor of considerable importance even for small ice loads. The data in Fig. 3 are based on the assumption that all the load is transferred vertically down to the foundation. For example, the effect of ice load on stresses in an arch dam could not be studied in this manner, but would require a complete stress analysis.

In general, the discussion has been applied to pressure against a vertical or nearly vertical face of a concrete structure. Since the adhering power of sound ice is great, considerable pressure could be exerted even though the surface of the structure is somewhat sloping. The slope of an earth dam would be very flat, however, and the extent of any damage would likely be limited to disturbance of riprap and surfacing material.

Because of the complex nature of the problem and the need for additional data, the ice-pressure problem was not analyzed more thoroughly and comprehensively at this time. It is desirable to secure test data to determine the effect of lateral restraint and especially to obtain more field measurements of both ice temperatures and pressures. When this information is available, it should be possible to prepare more definite design data. The results do present a reasonable basis for evaluating the magnitude of the ice pressures to be expected for the important factors of ice thickness, rapidity of air-temperature rise, lateral restraint, and solar radiation absorption.

13. Acknowledgments

The material presented herein is a result of work conducted in the Denver (Colo.) office, USBR. E. H. Larson assisted the writer in computing the data. Comments and suggestions by R. E. Glover were appreciated. The work was conducted in the Technical Engineering Division under the direction of Ivan E. Houk, M. ASCE. All designs and investigations were supervised by J. L. Savage, Hon. M. ASCE. All design and construction activities were under the general direction of Walker R. Young, M. ASCE. All activities of the USBR were under the direction of Harry W. Bashore, M. ASCE, commissioner, with headquarters in Washington, D. C.

APPENDIX. BIBLIOGRAPHY ON ICE PRESSURES

(1) "Ice Engineering," by H. T. Barnes, Renouf Publishing Co., Montreal, Que., Canada, 1928.

(2) "Results of Experiments on Strength of Ice," by George G. Bell, *Proceedings*, Maine Soc. of Civ. Engrs., Vol. 1, 1911, pp. 41–46.

(3) **"Ice Thrust in Connection with Hydro-Electric Plant Design,"** by E. Brown and G. C. Clarke, *The Engineering Journal*, January, 1932, pp. 18–25. (*a*) Fig. 8, p. 24. (*b*) p. 25.

(4) **"Properties of Ordinary Water Substance,"** by N. E. Dorsey, Reinhold Pub. Corp., New York, N. Y., 1940, pp. 395–510.

(5) **"Field Measurements of Ice Pressures at Hastings Lock and Dam,"** by H. M. Hill, *The Military Engineer*, March–April, 1935, pp. 119–122.

(6) **"Report of Joint Board of Engineers on the St. Lawrence Waterway,"** St. Lawrence Waterways Projects, Ottawa, Ont., Canada, 1927, Appendix E, Ice Formation on the St. Lawrence, and Appendix F, Experiments on the Strength of Ice.

(7) **"Observations and Experiments on Ice Expansion,"** by Walter H. Sawyer, *Proceedings*, Maine Soc. of Civ. Engrs., Vol. 1, 1911, pp. 27–40.

(8) **"Ice as Affecting Power Plants: Final Report of Committee of Power Division,"** *Transactions*, ASCE, Vol. 95, 1931, p. 1143.

(9) **"Heat Transmission,"** by W. H. McAdams, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, pp. 27–44.

(10) **"Resistance to Cracking of Surface Layer of Concrete Gravity Dams,"** by G. A. Nielaender, 2d Cong. on Large Dams, Vol. 3, Washington, D. C., 1936.

(11) **"A. Föppl's Festschrift,"** by E. Schmidt, J. Springer, Berlin, 1924, p. 179.

(12) **"Applied Heat Transmission,"** by Herman J. Stoever, McGraw-Hill Book Co., Inc., New York, N. Y., 1941, pp. 17–22.

(13) **"Smithsonian Physical Tables,"** by F. E. Fowle, Smithsonian Institution, 1934, pp. 608–614.

(14) **"Evaporation as a Function of Insolation,"** by Burt Richardson, *Transactions*, ASCE, Vol. 95, 1931, pp. 996–1019.

(15) *Engineering News-Record*, November 10, 1927, pp. 742–743.

(16) **"Remarkable Temperature Fluctuations in Black Hills Region, January, 1943,"** *Monthly Weather Review*, March, 1943.

(17) **"Thrust of Ice Against Dam,"** by G. Albenga, *Annali Lavori Pubblici*, Vol. 3, 1921, pp. 71–74.

(18) **"Use of Compressed Air for Combating Ice Troubles of Magnitostroy Dam,"** by N. L. Ioungkind, *Transactions*, Scientific Research Inst. of Hydrotechnics, Vol. XV, 1935, Leningrad, pp. 209–210.

(19) **"Ice Formation,"** by H. T. Barnes, John Wiley & Sons, Inc., New York, N. Y., 1906.

(20) **"The Control of Temperature in Massive Concrete Structures,"** by Douglas McHenry, Am. Inst. of Physics, 1941, Volume entitled, "Temperature; Its Measurement and Control in Science and Industry," Reinhold Publishing Corp., p. 912.

(21) **"Suitability of Ice for Aircraft Landings,"** by Robert P. Sharp, *Transactions*, Am. Geophysical Union, Vol. 28, No. 1, pp. 111–119.

(22) **"Safeguarding Hydro Plants Against the Ice Menace,"** by Paul E. Gisiger, *Civil Engineering*, January, 1947, p. 24.

DISCUSSION

Ernest Brown,[5] Esq.—A welcome addition to the knowledge of factors that determine the pressure exerted by an expanding sheet of ice is presented in this paper. The writer first became interested in the fundamental question of the strength of ice in 1927 when he conducted tests for the Joint Board of Engineers on the St. Lawrence Waterway, and prepared the Appendix to the report, dealing with the strength of ice under different conditions. Some few years later, when plans were being made for the building of a hydroelectric plant on the Churchill River, the question of selecting a well-founded value for the ice thrust against the dam became urgent. Values as high as 45 000 lb per lin ft were recorded as having been used. Such a large value would indicate extremely large ice-pressure moments as compared with water-pressure moments, since the head was about 50 ft. The plant, if constructed, would be at the most northerly site yet developed in Canada. Absence of roads made it necessary to bring all materials to the site over frozen lakes and rivers during the winter months, and the economic aspect of the project depended largely on the question of the value of the ice thrust under probable conditions at the site. There seemed to be good reason to doubt the existence of such extremely high pressures as had been used.

However, the data from the waterways tests could not be used to furnish a dependable value for thrust likely to develop from a prolonged and gradual rise of temperature, although they supported the opinion that such high pressures as that mentioned above would probably not arise. Therefore, the writer, in association with G. C. Clarke, M. ASCE, conducted laboratory tests under controlled conditions in an effort to simulate the conditions in nature. The results were reported in a joint paper to the Engineering Institute of Canada in 1932, and along with the waterways tests, are listed in the Bibliography of the paper (3)(6). Mr. Rose has used these results in preparing the curves shown in Fig. 2, which give estimates of the thrust for ice sheets of different thicknesses, considering different rates of air temperature rise and restraint. He has applied the principles of heat transmission to estimate the temperature gradients within the ice sheet under prescribed conditions, and has used the data found by the writer and Mr. Clarke in the Churchill River tests in conjunction with the computed gradients to give the total thrust, using a process of summation.

Each individual project presents its own special conditions. The designer must estimate the probable thickness of ice sheet, nature of snow cover, rate of air temperature rise, restraint at the site, etc., and then must decide on a value for the thrust to be used. He must consider the question of the depth to which temperature changes of the atmosphere may penetrate the ice sheet, since the temperature gradient is an important factor. He may not be able to satisfy his desire for precise data, and will have to use his best judgment after a careful study of all available information. This procedure was followed for the Churchill River project, and no attempts were made to compute temperature gradients in the ice sheet.

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It would be helpful if the author would give, in some detail, the calculations for a typical temperature gradient, and submit some selected curves showing the estimated gradients on which the thrusts in Fig. 2 are based. The applicability of the curves might be judged better if the engineer were enabled to consider the degree of agreement between the computed gradients for a few typical conditions, and the gradients that appear to be reasonably likely to occur when the available data for a particular project have been thoroughly studied. It is interesting to note that the curves in Fig. 2, although covering a very wide set of assumed conditions, do not indicate the likelihood of the existence of such high pressures as have been used in some designs. The writer believes that pressures as great as 50,000 lb per lin ft are imaginary and fantastic. Mr. Rose's data seem to be better founded.

The writer's experience in the testing of ice in a laboratory suggests that experiments on temperature gradients would be very difficult, whether under controlled conditions or in a natural sheet of ice. However, at some future date experiments will be made and will form another link in the chain of knowledge. In the meantime, some typical examples of the gradients used in preparing Fig. 2 would add greatly to the valuable study of the problem which Mr. Rose has presented so ably.

The writer regrets that it has not fallen to his lot to pursue the work on the Churchill River project further. It is gratifying to find that the results then obtained have been used so effectively in the broad general study made by Mr. Rose.

B. R. McGrath,[4] Esq.—The determination of the thrust exerted by an expanding ice sheet is a complex problem because of the plastic behavior of the ice when subjected to temperature increases, and because of various conditions of restraint. There are also a number of influencing factors in nature which affect the rate of temperature change and the degree of restraint encountered at a specific location. The paper by Mr. Rose offers an excellent approach to the solution of this problem. Estimated ice pressures obtained by this analysis are believed to be sufficiently conservative for good practice in the design of hydraulic structures.

As suggested by the author's statement (see paragraph following Eq. 2), on June 22, 1942, the writer, with the aid of valuable suggestions from Douglas McHenry (20), submitted a memorandum to the chief designing engineer of the Bureau of Reclamation setting forth a method for computing ice pressures based on:

(*a*) The experimental data of Messrs. Brown and Clarke on ice pressures due to temperature increases (3);

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[4] Engr., Bureau of Reclamation, Denver, Colo.

(*b*) An adaptation to ice sheets of the graphical method developed by E. Schmidt for obtaining temperature gradients within a mass for one-dimensional heat flow (11); and

(*c*) An evaluation of the effects of lateral restraint according to the theory of elasticity.

The preliminary nature of the work was emphasized in the memorandum, and it was recommended that additional studies be made to consider several factors entering into the problem. Shortly thereafter, the writer was called to active duty in the Civil Engineering Corps of the U. S. Navy and was unable to continue work on the problem.

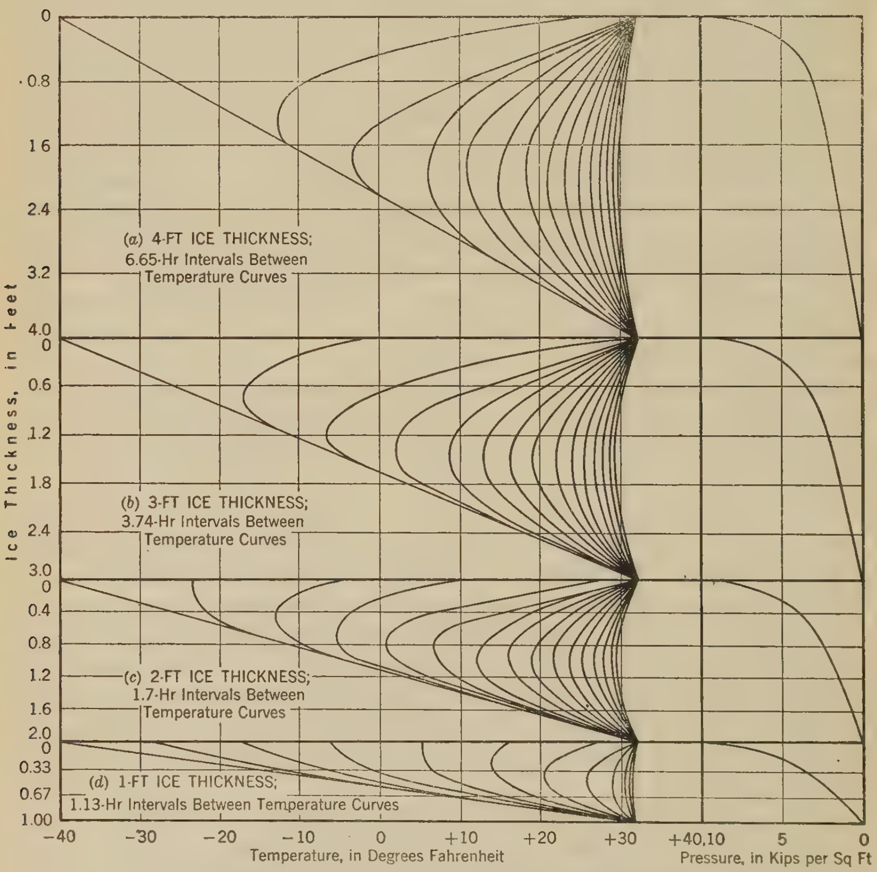

FIG. 4.—ICE TEMPERATURE CURVES AND RESULTING PRESSURE CURVES

The purpose of the study was to devise some simple usable curves for evaluating ice pressures against hydraulic structures. It was prepared from a further interpretation of results obtained by Messrs. Brown and Clarke as cited by Mr. Rose (3). These writers (3*b*) called attention to two important factors in the problem of determining ice pressure on a dam—the rate and extent to which a rise in air temperature will be transmitted through the ice cover, and the effect of restraining the ice in a direction parallel to the dam.

An approach to the solution of the first of these problems began with the first three of the assumptions mentioned by Mr. Rose in Section 4. Using a graphical method developed by E. Schmidt (11) and described by Douglas McHenry (20), and data presented by Messrs. Brown and Clarke (3), the writer prepared the pressure diagrams shown in Fig. 4. They were taken from several temperature gradients, at known time intervals, and within a normal range of ice thicknesses. The rate of temperature rise was determined at different depths within the ice mass for time intervals computed from Eq. 1, with a diffusivity constant h^2 for ice equal to 0.0481 $\frac{\text{ft}^2}{\text{hr}}$. Designating θ_p, θ_q, and θ_r as the temperatures at three successive space intervals on a gradient curve at a given time, the writer developed substantially the formula cited by Mr. Rose as Eq. 2.

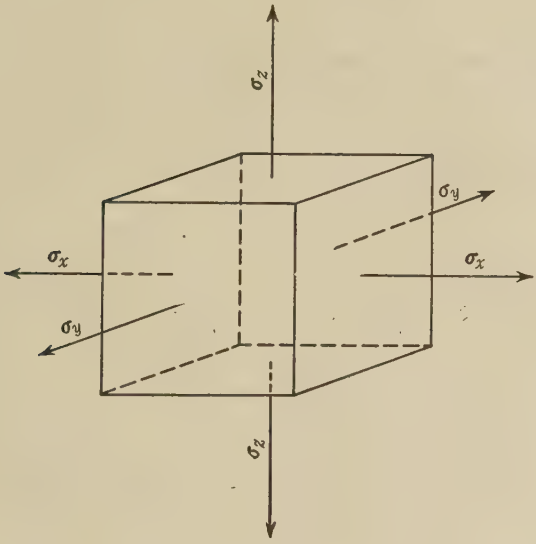

FIG. 5

With the help of Fig. 4 equivalent ice pressures σ were found at each space interval for each temperature time interval. The total pressure acting within the ice was summed at each space interval for the total time intervals involved, and pressure curves were plotted showing the distribution of total pressure within the ice sheet.

The effect of the lateral restraint of the ice in a direction parallel to the dam may be estimated by first assuming a differential element of ice such as that shown in Fig. 5.

For static equilibrium:

$$\xi_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y - \sigma_z)\right] + \alpha\,\Delta\theta \tag{3a}$$

$$\xi_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_x - \sigma_z)\right] + \alpha\,\Delta\theta \tag{3b}$$

and

$$\xi_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x - \sigma_y)\right] + \alpha\,\Delta\theta \tag{3c}$$

in which ξ_x, ξ_y, and ξ_z are unit strains and σ_x, σ_y, and σ_z are unit stresses in the x, y, and z directions, respectively. Furthermore, α is the coefficient of expansion; $\Delta\theta$ is the temperature change ; E is Young's modulus; and μ is Poisson's ratio.

For complete restraint in the x-direction and the y-direction and no restraint in the z-direction:

$$\xi_x = \xi_y = 0; \qquad \sigma_x = \sigma_y; \qquad \text{and } \sigma_z = 0 \tag{4}$$

From Eqs. 3*a* or 3*b*:

$$\sigma_y - \mu\,\sigma_y + E\,\alpha\,\Delta\theta = 0 \tag{5a}$$

or,

$$\sigma_y = -\frac{E\,\alpha\,\Delta\theta}{1 - \mu} \tag{5b}$$

Let σ'_y represent the stress due to restraint in the y-direction only; then

$$\sigma'_y = -E\,\alpha\,\Delta\theta \tag{6}$$

and the difference ($\Delta\sigma_y$) between σ_y and σ'_y is

$$\Delta\sigma_y = -E\,\alpha\,\Delta\theta\,\frac{1}{1 - \mu} + E\,\alpha\,\Delta\theta \tag{7a}$$

or,

$$\Delta\sigma_y = -E\,\alpha\,\Delta\theta\,\frac{\mu}{1 - \mu} \tag{7b}$$

The ratio of increase of stress for complete lateral restraint then becomes

$$\frac{\Delta\sigma_y}{\sigma'_y} = \frac{-E\,\alpha\,\Delta\theta\,\frac{\mu}{1 - \mu}}{-E\,\alpha\,\Delta\theta} = \frac{\mu}{1 - \mu} \tag{8}$$

If $\mu = 0.25$, then $\frac{\Delta\sigma_y}{\sigma'_y} = \frac{1}{3}$; and if $\mu = 0.5$, $\frac{\Delta\sigma_y}{\sigma'_y} = 1$. It is thought that the two assumed values of $\mu = 0.25$ and $\mu = 0.5$ may be taken as limits, and that the stress for a condition of complete lateral restraint should be increased over that of no lateral restraint by a ratio somewhere between the values of $\frac{1}{3}$ and 1.

Fig. 6 shows ice pressures for thicknesses that would be likely to occur in the reservoir. Areas of pressure curves in Fig. 4 were plotted as points when constructing the curve for the condition of no lateral restraint. These points were then increased by $\frac{1}{3}$ and by 1, and curves were drawn corresponding to the limits mentioned. Table 1 is a comparison of ice-pressure values obtained from Fig. 6 and those obtained from curve B, Fig. 2(*a*). The values check within $3\frac{1}{2}\%$ for a condition of no lateral restraint. For complete lateral restraint, the maximum difference between the average of the two limiting values of Poisson's ratio used by the writer and curve B, Fig. 2(*a*), is approximately 2%.

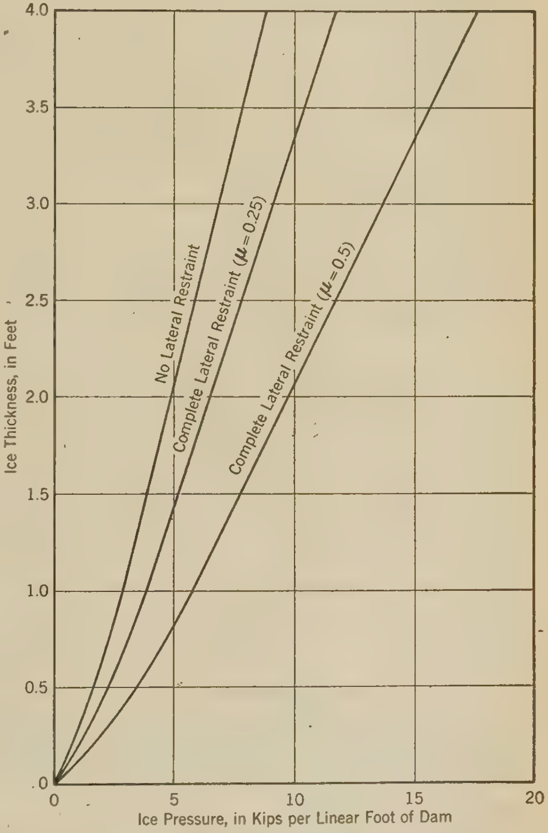

FIG. 6.—ICE PRESSURE CURVES FOR CONDITIONS OF NO LATERAL RESTRAINT AND COMPLETE LATERAL RESTRAINT, PLOTTED AGAINST ICE THICKNESS

Except in unusual cases, it is believed that ice pressures based on a temperature change of 5° F per hr would be satisfactory for design purposes especially for ice thicknesses greater than 2 ft. This is illustrated by a study of Fig. 4. After rising from −40° F to +32° F, the air temperature then remains constant during a considerable length of time while the temperature of the ice rises to this level. For an ice thickness of 2 ft this period is approximately 17 hours. Likewise, an ice thickness of 4 ft requires approximately 80 hours of constant air temperature at +32° F to permit the total pressure to develop.

It would seem very unlikely that such an assumption of constant temperature at this particular level would actually take place in nature immediately following a relatively short period of constant rise from −40° F. Some fluctuation of air temperature, either upward above freezing or downward, during this period would seem most likely with a consequent reduction in ice pressures over those obtained in the analysis.

Further study of the problem would probably reveal that fluctuations of temperature tend to affect the pressures developed within an ice sheet in a manner somewhat similar to a relatively slower rate of constant temperature changes.

TABLE 1.—COMPARISON OF ICE PRESSURES, IN THOUSANDS OF POUNDS (KIPS) PER LINEAR FOOT

Ice thickness (ft)	FIG. 6				CURVE B, FIG. 2(*a*)	
	No lateral restraint	Complete Lateral Restraint			No lateral restraint	Complete restraint
		$\mu = 0.25$	$\mu = 0.5$	Average $\mu = 0.375$		$\mu = 0.365$
4....	8.8	11.7	17.7	14.7	9.1	14.4
3....	6.9	9.1	13.7	11.4	7.1	11.2
2....	4.9	6.5	9.7	8.1	5.0	7.9
1....	2.9	3.9	5.8	4.85	2.9	4.4

Another important factor entering into the solution of the problem is the variation to be expected in Poisson's ratio for ice due to changes in its plasticity at different temperatures, the higher Poisson's ratio values occurring at higher temperatures. A search made of available publications on the subject of ice pressures reveals no practical data for the determination of this effect.

The writer desires to suggest, without detracting in any way from the valuable presentation by Mr. Rose, that much remains to be accomplished which might supplement the work already done. Valuable contributions could be made toward a more exact analysis and a better understanding of the ice pressures likely to occur at given reservoir locations through:

(1) Field measurements of ice pressures with their accompanying temperature records taken at various reservoirs, whereby the degree of restraint exerted on the ice sheet could be predicted for a design problem;

(2) Analyses based on climatological studies of various locations within the northern states; and

(3) Laboratory tests for the determination of Poisson's ratio variations with ice-temperature changes.

ROBERT E. KENNEDY,[5] M. ASCE.—In Section 10 of his paper the author mentions unpublished studies on ice thickness on reservoirs of Bureau of Reclamation projects. These studies were initiated in 1939 through the Office of the Chief Engineer. The readings were taken once a week in most cases and continued for three or four years. Since this is a limited period in which to determine any kind of maximum, another request was sent to the field offices in

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[5] Bureau of Reclamation, Boulder City, Nev.

TABLE 2.—Ice Thickness on Reservoirs and Lakes, Bureau of Reclamation

Reservoir and dam (1)	Project (2)	Latitude (north) (3)	Water surface[a] (ft) (4)	Winters of record (5)	Ice-cover period[b] (6)	Maximum Thickness In. (7)	Maximum Thickness Month (8)	Maximum Thickness Year (9)
California:								
Boca	Truckee storage	39° 20′	5,590	1939–1942	Dec.–Apr.	17	Feb.–Mar.	1942
Clear Lake	Klamath	41° 55′	4,530	1939–1942	Dec.–Mar.	10	Jan.	1942
Colorado:								
Grand Lake[c]	Colorado-Big Thompson	40° 15′	8,365	1920–1944	Dec.–May	25½	Mar.	1942
Mary's Lake[c]		40° 23′	8,022	1941–1942	Nov.–Apr.	18	Mar.	1942
Taylor Park	Uncompahgre	38° 50′	9,292	1939–1943	Nov.–May	26	Jan.–Apr.	1943
Idaho:								
American Falls	Minidoka	42° 50′	4,345	1939–1943	Dec.–Apr.	16	Mar.	1942
Arrowrock	Boise	43° 30′	3,150	1925–1945	Dec.–Apr.	25	....	....
Black Canyon	Boise	44° 0′	2,480	1931–1944	Oct.–Mar.	14	....	....
Deadwood	Boise	44° 15′	5,315	1939–1944	Dec.–Apr.	26	Mar.	1942
Island Park	Upper Snake River	44° 24′	6,295	1939–1942	Nov.–May	14	Jan.	1941
Lake Wolcott[d]	Minidoka	42° 40′	4,240	1939–1943	Dec.–Mar.	14	Feb.	1942
Montana:								
Bowdoin Lake[c]	Milk River	48° 25′	2,205	1939–1942	Oct.–Apr.	26	Feb. 1940	Mar. 1941
Fresno	Milk River	48° 35′	2,540	1939–1942	Nov.–Apr.	23	Mar.	1940
Gibson	Sun River	47° 35′	4,665	1939–1941	Nov.–Apr.	16	Feb.–Mar.	1940
Lower St. Mary's Lake	Milk River	48° 45′	4,600	1939–1942	Nov.–Apr.	21½	Mar.	1940
Nelson	Milk River	48° 30′	2,210	1939–1942	Nov.–Apr.	29	Feb.	1941
Sherburne Lakes	Milk River	48° 50′	4,750	1939–1942	Nov.–Apr.	22½	Mar.	1942
Nevada:								
Rye Patch	Humboldt	40° 30′	4,100	1936–1944	Dec.–Jan.	12	....	....
Oregon:								
Agency Valley	Vale	44° 0′	3,330	1939–1942 1945–1946	Dec.–Mar.	13	Feb. 1941	Feb. 1946
Gerber	Klamath	42° 10′	4,820	1939–1942	Dec.–Apr.	15	Mar.	1942
Owyhee[e]	Owyhee	43° 30′	2,600	1932–1944	Feb.	0	....	....
Thief Valley	Baker	45° 0′	3,100	1939–1942	Nov.–Mar.	14	Jan.	1942
Unity	Burnt River	44° 30′	3,800	1939–1942	Dec.–Apr.	14½	Feb.	1942
Upper Klamath Lake[f]	Klamath	42° 15′	4,140	1939–1942	Dec.–Mar.	10½	Jan.	1942
South Dakota:								
Belle Fourche	Belle Fourche	44° 40′	2,950	1924–1944	Nov.–Apr.	31	Jan.	1928
Utah:								
Deer Creek	Provo River	40° 30′	5,318	1941–1945	Dec.–Apr.	18	....	....
Hyrum	Hyrum	41° 40′	4,655	1939–1942 1945	Dec.–Apr.	20	Mar.	1942
Moon Lake	Moon Lake	40° 33′	8,115	1939–1943	Nov.–May	25	Mar.	1943
Pine View	Ogden River	41° 15′	4,845	1939–1942	Nov.–Apr.	8	Feb.	1942
Washington:								
Bumping Lake	Yakima	46° 50′	3,410	1939–1943	Dec.–Apr.	15	Feb.	1943
Lake CleElum[g]	Yakima	47° 15′	2,185	1929–1944	Jan.–Apr.	10	Feb.	1929
Lake Kachess	Yakima	47° 15′	2,232	1939–1943	Nov.–Apr.	12	Mar.	1942
Lake Keechelus	Yakima	47° 18′	2,480	1939–1943	Dec.–Apr.	14	Jan.–Apr.	1942
Tieton	Yakima	46° 40′	2,880	1939–1943	Dec.–Apr.	13	Mar.	1942
Wyoming:								
Alcova	Kendrick	42° 30′	5,420	1939–1942	Nov.–Apr.	16½	Feb.	1940
Bull Lake	Riverton	43° 15′	5,765	1919–1944	Dec.–Apr.	26	....	1928–1929
Diversion Dam[h]	Riverton	43° 15′	5,569	1919–1944	Nov.–Mar.	36	....	1928–1929
Grassy Lake	Upper Snake River	44° 8′	7,150	1939–1941	Oct.–May	16	Jan.	1941
Guernsey	North Platte	42° 15′	4,410	1939–1940 1941–1942	Dec.–Mar.	17	Feb.	1940
Jackson Lake	Minidoka	43° 50′	6,750	1939–1942	Dec.–May	30	Mar.	1941
Ocean Lake	Riverton	43° 15′	5,216	1928–1944	Nov.–Apr.	28	....	1928–1929
Pathfinder	North Platte	42° 30′	5,780	1940–1942	Nov.–Apr.	23½	Feb.	1941
Pilot Butte	Riverton	43° 15′	5,440	1925–1944	Nov.–Apr.	28 to 30	....	1928–1929
Seminoe	Kendrick	42° 10′	6,250	1939–1942	Nov.–Apr.	24½	Mar.	1942
Shoshone	Shoshone	44° 30′	5,280	1933–1944	Dec.–Apr.	22	....	1934 and 1937

[a] Average water-surface elevation, in feet. [b] Maximum range. [c] No dam. [d] Minidoka Dam. [e] No ice in the winter of 1940–1941. [f] Link River Dam. [g] Approximately 0.5 in. of ice in February, 1942. [h] Diversion Dam on Wind River.

December, 1944, asking for any other reliable records taken from memory or otherwise covering longer periods. All results are incorporated in Table 2.

A multiple correlation was made of observed maximum ice thickness with latitude and elevation. The correlation R of 0.479 with a standard error of 0.120 indicates a highly significant relationship. Significance is accorded a correlation of thirty or more items when R is at least twice its standard error, often called σ. High significance exists when the correlation is at least three times the value of σ.

The formula resulting from the correlation is

$$L_{max} = 1.15\,\phi + 2.58\,Z - 43.4 \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which L_{max} is estimated maximum thickness in inches, ϕ is latitude in degrees, and Z is elevation in thousands of feet. The error of the estimate is 6.7 in., meaning that if Eq. 9 were used to compute the thickness for comparison with the observed value, about one third of the time the computed thickness would be larger or smaller than the observed thickness, by about 7 in. or more. Actually, it proved to be outside the ±7-in. range in thirteen of the forty-five readings.

The author uses ice thickness in units of a foot, and areas of pressure in terms of square feet. Eq. 9 could be expected to predict thicknesses that would fall within about a half foot of the observed value about two thirds of the time. That prediction is limited to places within latitudes 39° N. to 49° N., and elevations from 2,000 ft to 9,000 ft in western United States west of the 105th meridian, and for such meteorological conditions as existed mainly between 1939 and 1943. Extrapolation beyond the limits of the original data is always uncertain. The author cites a thickness of 5 ft observed in northern Canada. To compute such a thickness one must go as far north as seems probable, say, latitude 65° N., and to an unlikely elevation of 10,000 ft.

Eq. 9 has additional limitations. Although the correlation coefficient of 0.48 is highly significant with respect to its standard error, it, itself, is not high. In fact, it is so low that the corresponding coefficient of alienation is uncomfortably high. It raises doubts as to the efficacy of the formula. The alienation coefficient is $(1 - R^2)^{0.5}$ and amounts to 0.88. It measures the effect of omitting pertinent factors that contribute to the formation of ice. It would appear that more effective factors are omitted than are included.

To illustrate some of the omissions, reference is made to the larger items of heat in the energy equation. Heat contributed to a water surface comes mainly from two sources, the infrared wave lengths of solar energy and the latent heat released when ice is formed. Heat lost is mainly the outgoing black body radiation which figures so evidently in the freezing of ice on clear cold nights.

Latitude and elevation enter directly into the energy equation only in connection with solar radiation. Even there cloudiness is an important factor and it is omitted from Eq. 9.

Latitude and elevation have only an indirect effect on outgoing black body radiation which is primarily a function of surface air temperature expressed in degrees absolute. Eq. 9 does not include air temperature.

Omitted from Eq. 9 is the effect of atmospheric moisture in gaseous state but not in clouds. This moisture radiates back to earth in amounts 30% to 60% of that of a black body. Again the presence of low or middle clouds reduces outgoing radiation also.

Other factors not related at all closely to latitude and elevation are loss of heat by sublimation, conduction, convection and wind movement, and the saving of heat by snow cover.

Even with these limitations, Eq. 9 may be adequate for the problem in hand. The formation and dissipation of ice by the energy equation is another subject.

Edwin Rose,[6] **Esq.**—Temperature conditions in an ice sheet are governed by the fundamental equation for the case of one-dimensional flow of heat:

$$\frac{\partial \theta}{\partial t} = h^2 \frac{\partial^2 \theta}{\partial x^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

E. Schmidt (11) gives an approximate method for determination of temperatures to satisfy Eq. 10 under various boundary conditions. Use of this method is illustrated by the following example. Temperatures are computed in an ice sheet 2 ft thick based on the assumptions given in Section 4 for curve B, Fig. 2(*a*). The 2-ft ice sheet is assumed divided into eight slabs of equal thickness, ΔL, equal to 0.25 ft. From Eq. 1, the time interval, Δt, equals 0.72 hour. By Eq. 2, the temperature gradients at time intervals Δt are computed as shown in Table 3.

TABLE 3.—TYPICAL COMPUTATION OF ICE TEMPERATURES, IN DEGREES FAHRENHEIT

ELAPSED TIME		DISTANCE FROM TOP SURFACE								
Intervals (1)	Hours (2)	0 (3)	$1\,\Delta L =$ 0.25 ft (4)	$2\,\Delta L =$ 0.50 ft (5)	$3\,\Delta L =$ 0.75 ft (6)	$4\,\Delta L =$ 1.00 ft (7)	$5\,\Delta L =$ 1.25 ft (8)	$6\,\Delta L =$ 1.50 ft (9)	$7\,\Delta L =$ 1.75 ft (10)	$8\,\Delta L =$ 2.00 ft (11)
0....	0	−40	−31	−22	−13	−4	5	14	23	32
1 Δt....	0.72	−32.8	−31	−22	−13	−4	5	14	23	32
2 Δt....	1.44	−25.6	−27.4	−22	−13	−4	5	14	23	32
3 Δt....	2.16	−18.4	−23.8	−20.2	−13	−4	5	14	23	32
4 Δt....	2.88	−11.2	−19.3	−18.4	−12.1	−4	5	14	23	32
5 Δt....	3.60	− 4.0	−14.8	−15.7	−11.2	−3.5	5	14	23	32
6 Δt....	4.32	3.2	− 9.9	−13.0	− 9.6	−3.1	5.2	14.0	23	32
7 Δt....	5.04	10.4	− 4.9	− 9.7	− 8.0	−2.2	5.4	14.1	23.0	32.0

Use of Eq. 2 in a typical computation is shown by selecting three consecutive points $p = 0$, $q = 1\,\Delta L$, and $r = 2\,\Delta L$, at time $4\,\Delta t$. Then the temperature θ'_q at time $5\,\Delta t$ equals $\frac{\theta_p + \theta_r}{2}$ or $\frac{(-11.2) + (-18.4)}{2} = -14.8$, as given in Table 3. The temperatures may be computed in this manner or by the graphical method indicated by Mr. McGrath. Determination of temperature changes in an ice sheet by solution of Eq. 10 for various exposure conditions has been obtained by F. W. Taylor.

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[6] Engr., Bureau of Reclamation, U. S. Dept of the Interior, Denver, Colo.

Figs. 7 and 8 show estimated temperature gradients and the corresponding pressure curves and total thrusts for several selected examples covering a fairly broad range of conditions. Fig. 7 gives curves and data for a 2-ft ice sheet with air temperature rises of 5° F, 10° F, and 15° F per hr (see curves A, B, and C, Fig. 2(*a*)), and also for the case of 5° F per hr with solar energy effects

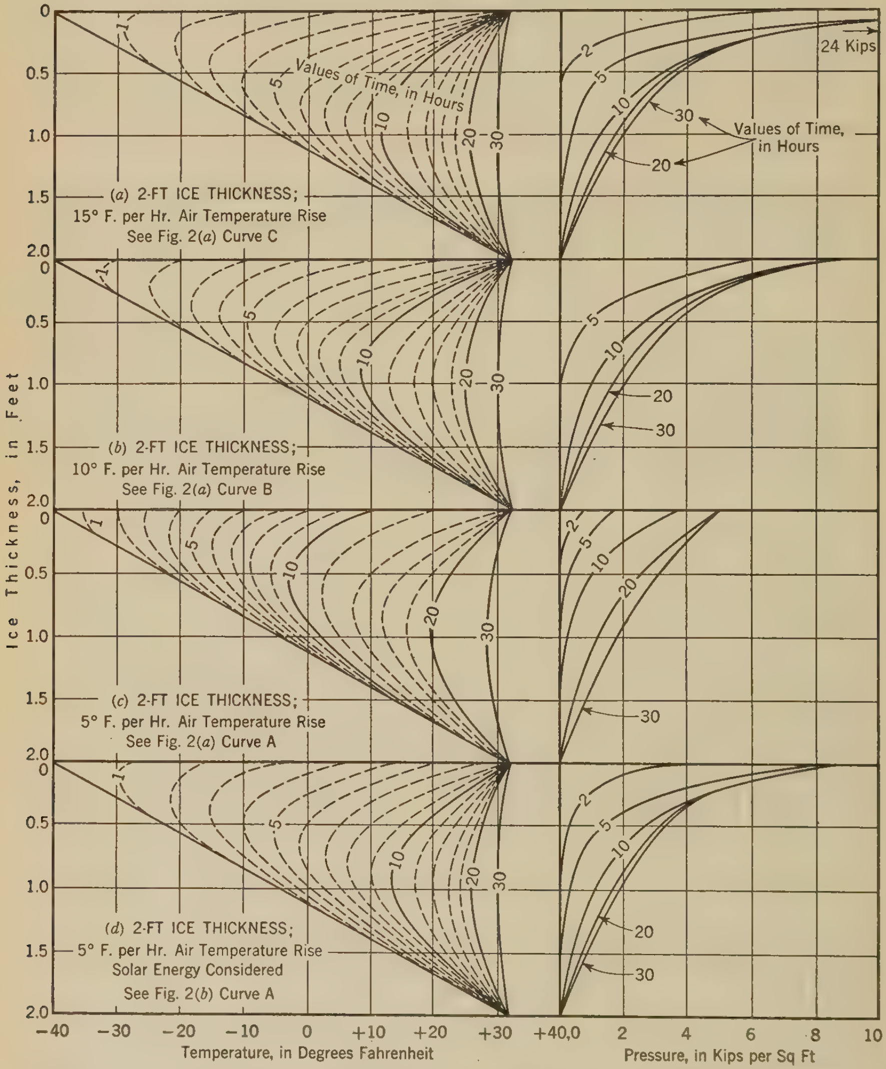

FIG. 7.—ICE TEMPERATURE CURVES AND RESULTING PRESSURE CURVES FOR 2-FT ICE SHEET

included as discussed in Section 5 (see curve A, Fig. 2(*b*)). Fig. 8 illustrates the temperature gradients and pressures developed for 1-ft and 4-ft ice sheets with an assumed air temperature rise of 10° F per hr (see curve B, Fig. 2(*a*)).

The temperature gradients shown in Figs. 7 and 8 are estimated for 1-hour time increments. From these several temperature gradients, the pressures at the selected depths in the ice sheet are determined by the data given in Fig. 1,

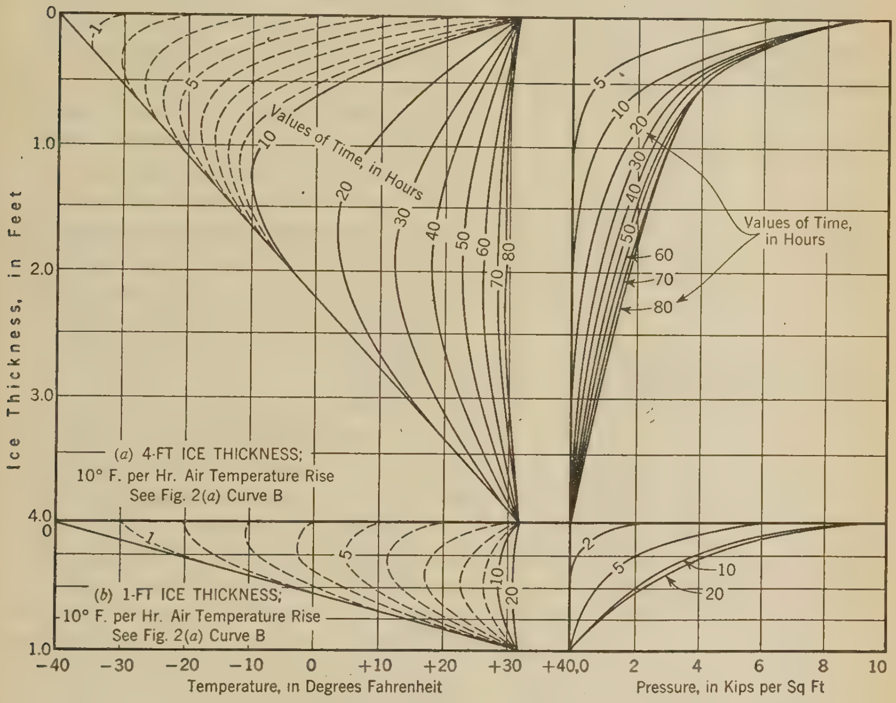

FIG. 8.—ICE TEMPERATURE CURVES AND RESULTING PRESSURE CURVES

and pressure curves are drawn for several periods of time. These pressure curves are shown in Figs. 7 and 8 for times corresponding to the temperature curves. By summation of the area under these curves, the total thrust developed per linear foot at the various times is given in Table 4. The last value for each condition in Table 4 corresponds to the value of thrust per linear foot plotted on the appropriate curve in Fig. 2.

Results of Mr. McGrath's analysis of ice pressures for an assumed air temperature rise of 10° F per hr, as given in Figs. 4 and 6 and Table 1, show a good comparison with data presented in Figs. 2(*a*), 7, and 8. Mr. McGrath presents the pertinent thought that pressures likely to develop in thick ice will probably be less than those obtained in the analysis. The writer agrees that some reduction in the pressures as shown in Figs. 2, 6, and 8 for ice sheets thicker than 2 ft would seem justified due to the unlikely occurrence of the assumed constant air temperature of 32° F for a period of from 30 hours to 70 hours following a rapid rise in temperature from − 40° F. Inspection of the pressure curves given in Fig. 8 for the 4-ft ice sheet reveals that the maximum pressure ordinate at considerable depth in the ice is not reached until about 80 hours after the beginning of the temperature rise. In addition, the high pressures at and near the top surface of the ice occur within the first 10 hours; therefore, because of the tendency of flow under load, this high pressure would very likely be reduced appreciably before the maximum pressures at the greater depths are reached some 70 hours later. This consideration serves to substantiate the belief that the curves in Fig. 2 are very conservative, particularly for thick ice sheets.

By estimating the pressures for lateral restraint with Poisson's ratios equal to 0.25 and 0.50, Mr. McGrath provides a considerable range of values, from which the designer by exercise of judgment may select design values to fit a particular case. The use of Poisson's ratio of 0.50 gives extremely severe restraint conditions. Ice pressures determined on the basis of Poisson's ratio of 0.365 and the assumption of elastic behavior are believed sufficiently conservative. Elastic behavior of ice, as assumed in computing lateral restraint effects, is approached only when the temperature change of the ice is very rapid and the flow strain is small—a condition which rarely occurs, particularly at any considerable depth in the ice sheet. In view of the uncertainty of the value of Poisson's ratio for ice, the use of a range of values may be desirable, but a lower top value is suggested.

TABLE 4.—TOTAL THRUST OF ICE SHEETS FOR VARIOUS CONDITIONS

Ice thickness (ft) (1)	Air temperature rise (degrees F per hr) (2)	Time (hr) (3)	Total thrust (kips per lin ft) (4)	Fig. No. (5)
1......	10	2 5 10 20	0.3 1.6 2.8 2.9	8(*b*)
2......	15	2 5 10 20 30	1.2 4.7 6.3 7.0 7.3	7(*a*)
2......	10	2 5 10 20 30	0.5 1.7 3.6 4.7 5.0	7(*b*)
2......	5	2 5 10 20 30	0.2 0.7 1.5 3.5 4.1	7(*c*)
2......	5[a]	2 5 10 20 30	0.8 2.6 4.0 4.7 5.0	7(*d*)
4......	10	5 10 20 30 40 50 60 70 80	1.8 3.8 5.3 6.5 7.4 8.1 8.5 8.9 9.1	8(*a*)

[a] Solar energy considered.

The tabulation of ice thicknesses recorded on reservoirs and lakes on various projects as listed by Mr. Kennedy in Table 2 is a worthy contribution of data. Credit is also due those men in the field who performed the difficult task of gathering the data. By a statistical analysis of the data in Table 2, Mr. Kennedy derives Eq. 9, which expresses the maximum ice thickness as a function of latitude and elevation. This equation, although limited in accuracy due to both the short period of record of data and the omission of pertinent factors cited, can be used to approximate ice thicknesses. It is believed that a more adequate expression might be derived if consideration were given to air temperatures along with the data in Table 2 as fundamental factors influencing ice thickness.

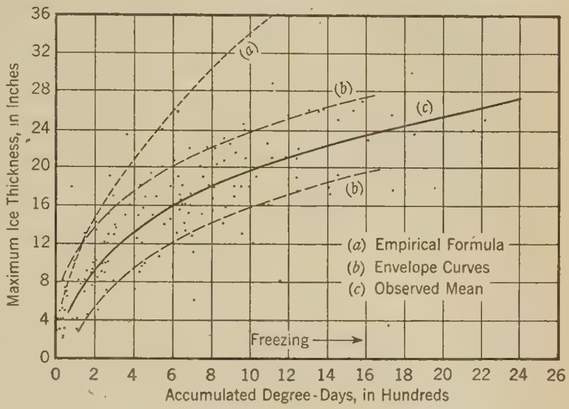

FIG. 9.—MAXIMUM ICE THICKNESS FOR FREEZING DEGREE-DAYS

Mr. Kennedy, in his study of the listed data, plotted the maximum recorded ice thicknesses against accumulated degree-days as shown in Fig. 9. Curve (c) is the mean line drawn by eye, with curves (b) enclosing roughly two thirds of the plotted points. Curve (a) represents the growth of an ice sheet as determined by the empirical formula given by N. E. Dorsey (4*a*) assuming quiet water and no snow. In a recent paper (21), Robert P. Sharp presents a similar curve, which falls approximately midway between curve (a) and curve (c) in Fig. 9. The wide range between the empirical curve and the recorded data in Fig. 9 is probably due to variation of such factors as wind, current, snow, cloudiness, etc. However, Fig. 9 may be used as a guide to predict the expected maximum ice thickness, if air temperatures are known or can be estimated.

In personal correspondence with S. P. Wing, M. ASCE, regarding allowance made for ice pressure in design of dams in Norway, Thor G. Brandzaeg states that, although no official Norwegian regulations are printed as yet:

> "* * * up to 1938 an ice pressure of 10 metric tons per meter (6720 lbs. per ft.) was used for design of dams; since then the ice load has been reduced to 8 tons per meter (5380 lbs. per ft.) for dams with vertical upstream faces and 5 tons per meter (3350 lbs. per ft.) for dams with sloping upstream faces."

In another paper (22), Paul E. Gisiger, M. ASCE, states that:

> "* * * static ice pressure, except in very confined areas or spaces, will, under the most severe climatic conditions, seldom exceed 10,000 lb per lin ft."

The writer agrees with Mr. McGrath that much remains to be accomplished to secure a better understanding of ice pressures. The Bureau of Reclamation under the immediate supervision of Douglas McHenry has undertaken a program to secure both field and laboratory data on ice pressures and related problems. No definite results are as yet available, but some field work has been in progress. As Professor Brown states, the engineer may not be able to satisfy his desire for precise data and will have to use his best judgment after careful consideration and study of available information. It is believed that, until more precise information is available, the material presented in the paper serves as a conservative basis for the designer and enables him to estimate the expected maximum ice thrust by the rational procedure indicated.

The writer wishes to express his appreciation to those who commented on this paper. It is hoped that interest has been stimulated in this subject to hasten the time when there will be a more definite basis for determination of ice thrusts, despite the large number of variables and the recognized complexity of the problem.

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Paper No. 2315

OBSERVATIONS ON THE BEHAVIOR OF ALUMINUM ALLOY TEST GIRDERS

By R. L. Moore,[1] Assoc. M. ASCE

Synopsis

The results of an experimental investigation of the behavior of eight built-up test girders of a standard aluminum alloy are presented in this paper. Data on web-buckling characteristics, stress distribution, and ultimate girder strengths are included. The object of these tests was to show to what extent elastic buckling theory could be used to predict buckling phenomena and to provide some data on the relation between first buckling and ultimate load-carrying capacities in aluminum alloy structures of this type.

I. Introduction

The tests described in this paper were made in 1937 and 1938 as part of a general program of research on the behavior of aluminum alloy structures. Although the tests were made originally to aid in the design of built-up girders simulating types common in bridge and building construction, the information obtained relative to strength, stiffness, and capability of withstanding large deformations without failure is of fundamental importance in all fields of aluminum alloy structural design.

One of the primary objects of these tests was to determine to what extent the theory of elastic stability could be used as a basis for the prediction of buckling phenomena.[2] Experimental data are presented to show the buckling characteristics of a number of web panels of different sizes under various combinations of shear and bending stress. A few data on the local buckling resistance of outstanding flanges are also included. Of equal or perhaps even greater significance from the standpoint of design are the results obtained pertaining to the ultimate strengths of the girders. Tests to failure constitute an important part of aircraft structural analysis, whereas the designer in the field of general structures seldom has an opportunity to appraise conventional design

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Note.—Published in June, 1946, *Proceedings*. The position and the title given are those in effect when the paper was received for publication.

[1] Research Structural Engr., Aluminum Research Laboratories, New Kensington, Pa.

[2] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 408.

procedures in the light of ultimate load-carrying capacities. The third phase of the investigation concerns measurements of strain and deflection made as a check on the ordinary flexure theory applied to built-up girders.

II. Specimens

Fig. 1 shows the structural details of the eight girders built for test. All material, including the rivets, was aluminum alloy 17S–T having a nominal composition of 4.0% copper, 0.5% manganese, 0.5% magnesium, and the remaining 95% aluminum.[3] The tensile properties of the web and flange material are given in Table 1. Standard specimens for sheet metals were used.[4]

TABLE 1.—Tensile Properties of Aluminum Alloy 17S–T Used in Test Girders

Property	Web Plate (18 In. by 108 In.)[a]								Flange Angles		
	$\frac{3}{16}$ In.		$\frac{1}{8}$ In.		$\frac{3}{32}$ In.		$\frac{1}{16}$ In.		5 in. by 3 in. by $\frac{3}{8}$ in.	4 in. by 3 in. by $\frac{5}{16}$ in.	3 in. by 3 in. by $\frac{1}{4}$ in.
	W	X	W	X	W	X	W	X			
Strength, in Kips per Sq In.:											
Yield (Offset = 0.2%)......	42.8	37.8	41.8	35.8	42.4	37.4	42.4	38.1	42.5	41.4	43.8
Ultimate..................	60.4	58.0	58.6	57.2	59.3	57.3	57.6	56.1	60.6	60.0	60.7
Percentage elongation in 2 in.....	20.7	19.7	20.2	19.7	18.5	20.0	17.5	17.5	18.0	17.8	18.0

[a] W = "with-grain" specimens and X = cross-grain specimens.

The over-all dimensions of all the specimens were the same—108-in. length by 18-in. depth—but, as shown in Table 2, a considerable range of proportions

TABLE 2.—Section Elements of Test Girders[a]

Specimen (see Fig. 1)	Web or Cover Plates		Four Flange Angles		A (in.^{2})	I (In.^{4})		S (In.^{3})	
	Key	t	Key	Section		Gross	Net[b]	Gross	Net[b]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1 and 2	A	$\frac{3}{16}$	B	5 in. by 3 in. by $\frac{3}{8}$ in.	3.38	886	809	98.4	89.9
3	A	$\frac{3}{16}$	B	5 in. by 3 in. by $\frac{3}{8}$ in.					
3[c]	C	($\frac{5}{16}$)	....	....	3.38	1,514	1,262	162.0	136.0
4, 5, and 6	E	$\frac{1}{8}$	F	4 in. by 3 in. by $\frac{5}{16}$ in.	2.25	638	596	70.9	66.2
7 and 7A	G	$\frac{3}{32}$	H	3 in. by 3 in. by $\frac{1}{4}$ in.	1.69	434	401	48.2	44.6
8 and 8A	I	$\frac{1}{16}$	H	3 in. by 3 in. by $\frac{1}{4}$ in.	1.12	418	387	46.4	43.0

[a] All web plates are 18 in. deep and 9 ft long. The thickness t, in inches, is given in Col. 3; and the key letters in Cols. 2 and 4 refer to corresponding plates and flanges in Fig. 1. In Cols. 6 to 10, A denotes "gross area of web," I denotes "moment of inertia," and S is the section modulus. [b] Rivet holes deducted from both top and bottom flanges. [c] Two cover plates, 12 in. wide.

was investigated. Webs having a clear depth between flanges of 12 in. and thicknesses of $\frac{1}{16}$, $\frac{3}{32}$, $\frac{1}{8}$, and $\frac{3}{16}$ in. were employed. Three sizes of flange angles,

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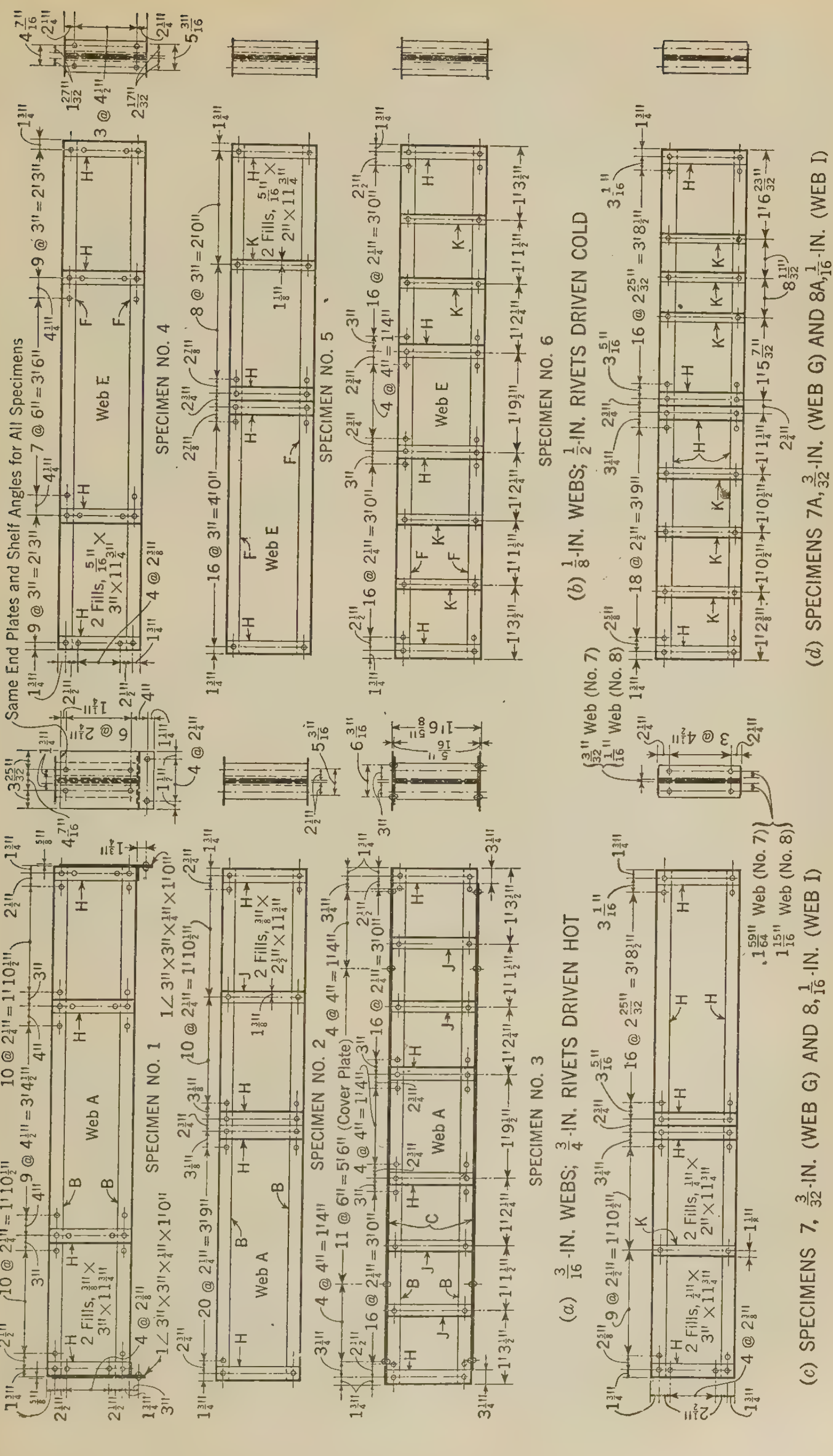
[3] *Transactions*, ASCE, Vol. 102, 1937, p. 1369.

[4] "Standard Methods of Tension Testing of Metallic Materials, A.S.T.M. Standards, E8–42, Pt. 1, p. 899, Fig. 2.


Same End Plates and Shelf Angles for All Specimens
SPECIMEN NO. 1
SPECIMEN NO. 2
SPECIMEN NO. 3
SPECIMEN NO. 4
SPECIMEN NO. 5
SPECIMEN NO. 6
Web A
Web E
2 Fills, 3/8"×3"×11 3/4"
2 Fills, 5/16"×3"×11 3/4"
2 Fills, 5/16"×2"×11 3/4"
2 Fills, 3/8"×2 1/2"×11 3/4"
2 Fills, 1/4"×3"×11 3/4"
2 Fills, 1/4"×2"×11 3/4"
1∠3"×3"×1/4"×1'0"
10 @ 2 1/4" = 1'10 1/2"
9 @ 4 1/2" = 3'4 1/2"
20 @ 2 1/4" = 3'9"
11 @ 6" = 5'6" (Cover Plate)
16 @ 2 1/4" = 3'0"
4 @ 4" = 1'4"
9 @ 3" = 2'3"
7 @ 6" = 3'6"
16 @ 3" = 4'0"
8 @ 3" = 2'0"
18 @ 2 1/2" = 3'9"
16 @ 2 25/32" = 3'8 1/2"
9 @ 2 1/2" = 1'10 1/2"
3/32" Web (No. 7)
1/16" Web (No. 8)
H
B
C
F
J
K

(a) 3/16-IN. WEBS; 3/4-IN. RIVETS DRIVEN HOT

(b) 1/8-IN. WEBS; 1/2-IN. RIVETS DRIVEN COLD

(c) SPECIMENS 7, 3/32-IN. (WEB G) AND 8, 1/16-IN. (WEB I)

(d) SPECIMENS 7A, 3/32-IN. (WEB G) AND 8A, 1/16-IN. (WEB I)

FIG. 1.—TEST GIRDERS, ALUMINUM ALLOY 17 S-T (FOR DIMENSIONS AND SECTION ELEMENTS, SEE TABLE 2)

3 by 3 by $\frac{1}{4}$ in., 4 by 3 by $\frac{5}{16}$ in., and 5 by 3 by $\frac{3}{8}$ in., were used, the last in one case with 12 by $\frac{5}{16}$-in. cover plates.

The key letters in Col. 4, Table 2, serve to identify corresponding section elements in Fig. 1, designated by letters A to H. In addition, Fig. 1 indicates the stiffeners J and K, with sections as follows:

Key	Section
J	$2\frac{1}{2}$ in. by $2\frac{1}{2}$ in. by $\frac{1}{4}$ in.
K	2 in. by 2 in. by $\frac{3}{16}$ in.

The $\frac{3}{4}$-in. diameter 17S–T rivets used in specimens 1, 2, and 3 were driven hot, whereas the $\frac{1}{2}$-in. diameter rivets in the remaining specimens were all driven cold. Cone-point driven heads were used throughout. Hole clearances of $\frac{1}{32}$ in. were allowed for both sizes. Intermediate web stiffeners were arranged to provide a number of different sizes of panel as well as panels of like proportions subjected to different combinations of shear and bending stresses. A few panels subjected to pure bending were also provided. No attempt was made to design the intermediate stiffeners other than to make reasonably certain that they were adequate to confine web buckling to the individual panels.

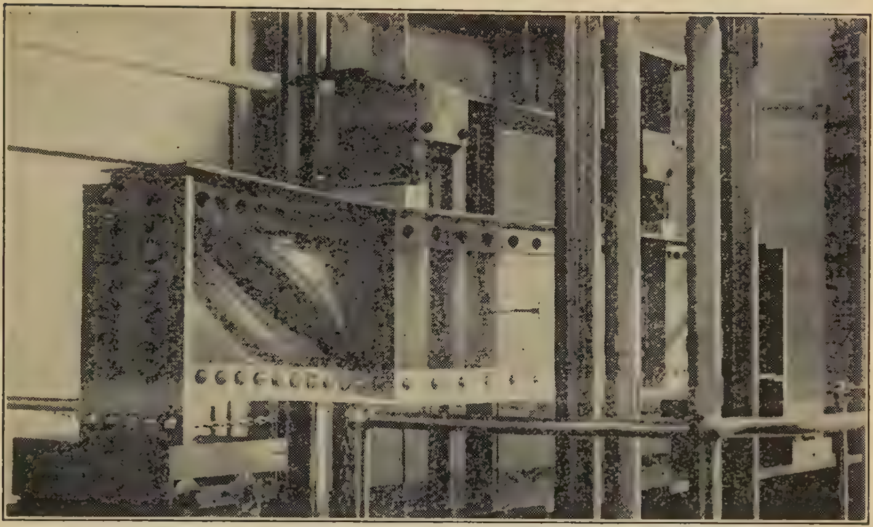
III. PROCEDURE

All girders were tested on a 9-ft span. Loads were applied centrally, or at two points symmetrical with respect to the center, in an hydraulic testing machine (capacity 300 kips, or 300,000 lb) as shown in Fig. 2. The pairs of vertical stiffeners marked H (3 by 3 by $\frac{1}{4}$ in.), in Fig. 1, were load-bearing stiffeners and indicate the position of the applied loads relative to the ends of the span.

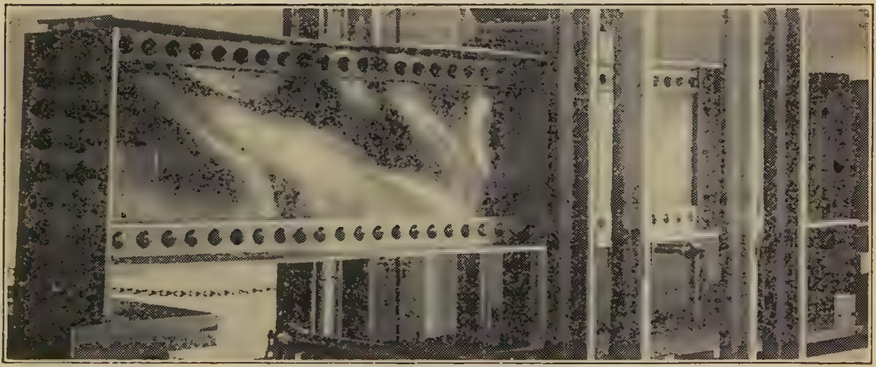
Measurements on the girders included determinations of lateral deflections of the webs, vertical deflections at the center of the span, and strain distribution in the webs and flanges. Lateral deflections (see Fig. 3) were measured with a dial indicator, graduated in 0.001-in. intervals and inserted between the webs and a reference bar placed against the girder flanges. Readings were taken at seven stations, lettered A to G, over the clear depth of the webs at a number of different sections, numbered 1, 2, 3, etc., along the length of the girders. Vertical deflections were determined by fine wires stretched between the ends of the girders and by mirrored scales attached on both sides of the webs at the center. Figs. 2(*a*) and 2(*c*) show this deflection apparatus. Strains were measured with 2-in. strain gages, supplemented in a few tests by tensometers on $\frac{1}{2}$-in. gage lengths. Strain rosettes, consisting of four intersecting gage lines 45° apart, as shown in Fig. 4, were used for strain determinations on the part of the webs between flanges. Single longitudinal gage lines were used for the measurement of flange strains. All girders were loaded until some part of the structure failed or until failure appeared imminent because of the large distortions produced.

IV. RESULTS AND DISCUSSION

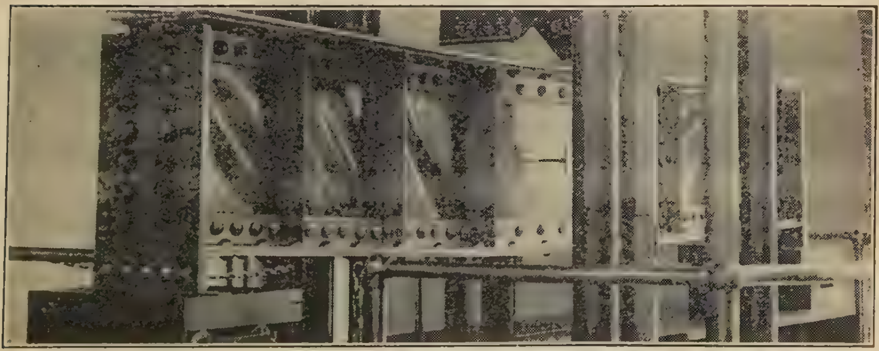
Figs. 3, 4, 5, and 6, in the form of load-deflection curves and stress-distribution diagrams, show the type of information obtained in these girder



(*a*) SPECIMEN 1; $P = 125$ KIPS



(*b*) SPECIMEN 2; $P = 100$ KIPS

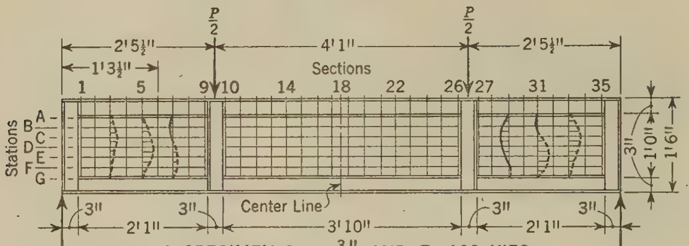


(*c*) SPECIMEN 3; $P = 150$ KIPS

FIG. 2.—SPECIMENS WITH $\frac{1}{16}$-IN. WEBS UNDER LOAD

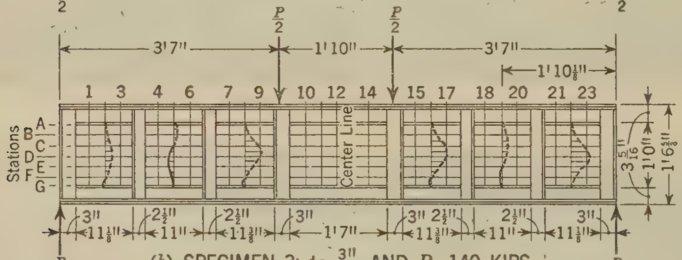
tests. Detailed results for all specimens, including data on ultimate strengths, are presented in Tables 3, 4, and 5.

(*A*) *Lateral Deflections of Webs.*—Generally, it is not possible to obtain well-defined experimental buckling loads for flat web panels subjected to various combinations of shear and bending stress. In only about one fourth of the cases considered in this investigation was an abrupt change in the load-versus-lateral deflection characteristics of the webs observed which might be interpreted as indicating definite buckling phenomena. Figs. 5(*c*) and 5(*d*) show



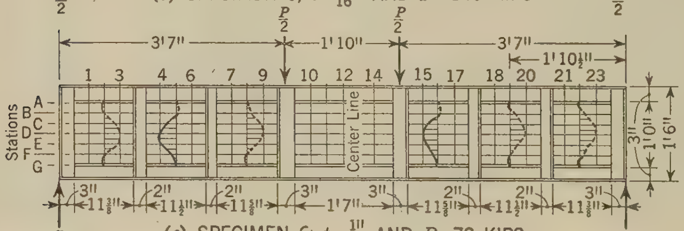
P/2
P/2
2'5½"
4'1"
2'5½"
1'3½"
Sections
1 5 9 10 14 18 22 26 27 31 35
Stations
A B C D E F G
3"
1'0"
1'6"
3" 3" Center Line 3" 3"
2'1" 3'10" 2'1"

(*a*) SPECIMEN 1; $t=\frac{3}{16}''$ AND $P=100$ KIPS



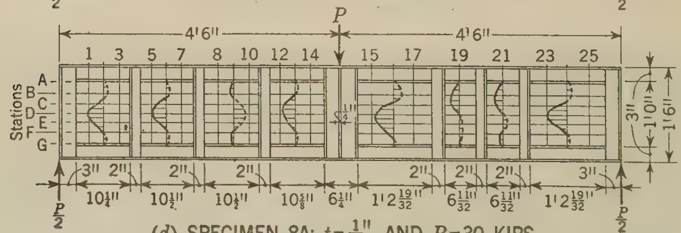
P/2
P/2
P/2
P/2
3'7"
1'10"
3'7"
1'10⅛"
1 3 4 6 7 9 10 12 14 15 17 18 20 21 23
Stations
A B C D E F G
Center Line
3 5/16"
1'0"
1'6⅝"
3" 2½" 2½" 3" 3" 2½" 2½" 3"
11⅛" 11" 11⅛" 1'7" 11⅛" 11" 11⅛"

(*b*) SPECIMEN 3; $t=\frac{3}{16}''$ AND $P=140$ KIPS



P/2
P/2
P/2
P/2
3'7"
1'10"
3'7"
1'10½"
1 3 4 6 7 9 10 12 14 15 17 18 20 21 23
Stations
A B C D E F G
Center Line
3"
1'0"
1'6"
3" 2" 2" 3" 3" 2" 2" 3"
11⅜" 11½" 11⅜" 1'7" 11⅜" 11½" 11⅜"

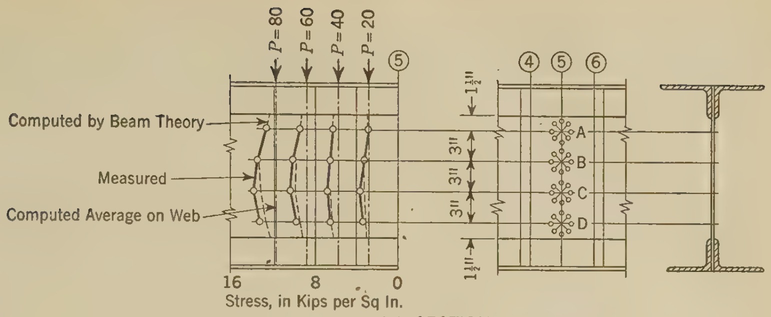
(*c*) SPECIMEN 6; $t=\frac{1}{8}''$ AND $P=72$ KIPS



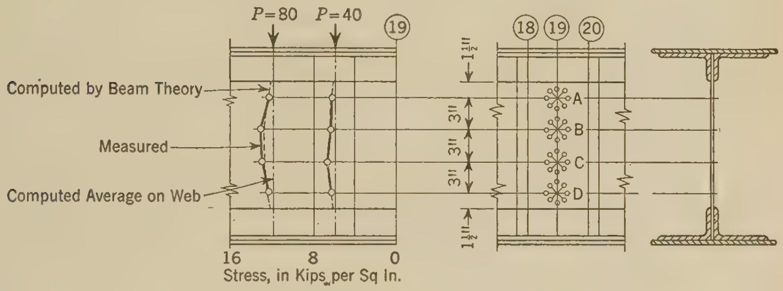
P
P/2
P/2
4'6"
4'6"
1 3 5 7 8 10 12 14 15 17 19 21 23 25
Stations
A B C D E F G
¼"
3"
1'0"
1'6"
3" 2" 2" 2" 2" 2" 2" 3"
10¼" 10½" 10½" 10⅝" 6¼" 1'2 19/32" 6 11/32" 6 11/32" 1'2 19/32"

(*d*) SPECIMEN 8A; $t=\frac{1}{16}''$ AND $P=30$ KIPS

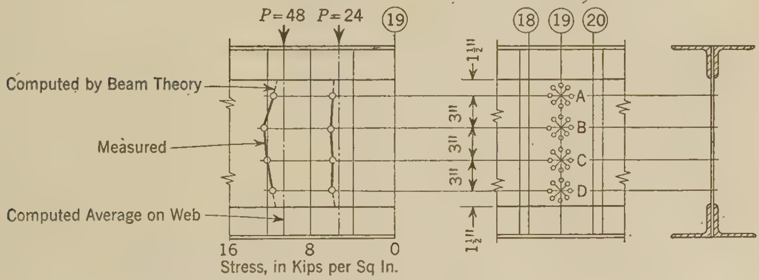
FIG. 3.—LATERAL DEFLECTIONS OF THE WEBS



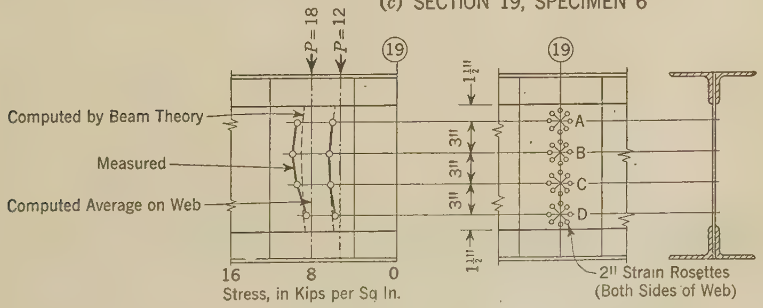
(a) SECTION 5, SPECIMEN 1



(b) SECTION 19, SPECIMEN 3



(c) SECTION 19, SPECIMEN 6



(d) SECTION 19, SPECIMEN 8A

FIG. 4.—COMPARISON OF MEASURED AND COMPUTED SHEAR STRESSES

some of the best examples of load-deflection curves having fairly pronounced breaks or changes in slope. Those in Fig. 5(*a*), which are typical of a large number obtained, are obviously of little help in determining critical buckling loads.

TABLE 3.—MAXIMUM MEASURED LATERAL DEFLECTIONS OF WEBS (CLEAR DEPTH OF A PANEL, 12 IN.) FOR THEORETICAL SHEAR-BUCKLING LOADS

Specimen (1)	Panel length[a] (in.) (2)	Stress[b] (kips per sq in.) (3)	Test load[c] (kips) (4)	Web deflection[d] (in.) (5)	Ratio, deflection to least panel width (6)
		(*a*) $\frac{3}{16}$-IN. WEB			
1	25	14.6	88.3	0.064	1 to 190
2	$47\frac{7}{8}$	13.1	79.2	0.021	1 to 570
2	$22\frac{3}{4}$	15.1	91.3	0.033	1 to 360
2	$22\frac{5}{8}$	15.1	91.3	0.037	1 to 320
3	$11\frac{1}{8}$	22.0	140.0	0.240	1 to 50
3	11	22.0	140.0	0.080	1 to 140
3	$11\frac{3}{8}$	22.0	140.0	0.190	1 to 60
		(*b*) $\frac{1}{8}$-IN. WEB			
4	24	6.6	26.6	0.010	1 to 1,200
5	$47\frac{7}{8}$	5.8	23.3	0.027	1 to 440
5	$21\frac{5}{8}$	6.8	27.4	0.018	1 to 670
5	$24\frac{1}{2}$	6.5	26.2	0.025	1 to 480
6	$11\frac{3}{8}$	10.4	41.9	0.010	1 to 1,140
6	$11\frac{1}{2}$	10.2	41.0	0.020	1 to 570
6	$11\frac{5}{8}$	10.1	40.6	0.018	1 to 650
		(*c*) $\frac{3}{32}$-IN. WEB			
7	$47\frac{7}{8}$	3.3	9.9	0.018	1 to 670
7	$23\frac{3}{4}$	3.8	11.4	0.031	1 to 390
7	$23\frac{1}{8}$	3.8	11.4	0.016	1 to 750
7A	$14\frac{19}{32}$	4.7	14.1	0.012	1 to 1,000
7A	$10\frac{1}{2}$	6.4	19.2	0.020	1 to 520
7A	$6\frac{11}{32}$	13.4	40.2	0.010	1 to 640
		(*d*) $\frac{1}{16}$-IN. WEB			
8	$47\frac{7}{8}$	1.5	3.0	0.029	1 to 410
8	$22\frac{3}{4}$	1.7	3.4	0.012	1 to 1,000
8	$23\frac{1}{8}$	1.7	3.4	0.004	1 to 3,000
8A	$14\frac{19}{32}$	2.1	4.2	0.000	....
8A	$10\frac{1}{2}$	2.8	5.6	0.010	1 to 1,050
8A	$6\frac{11}{32}$	6.0	12.1	0.008	1 to 790

[a] Clear distance between stiffeners. [b] Theoretical shear-buckling stress, computed by using the clear dimensions of the panels and assuming simply-supported edges. [c] Corresponding test loads, or the loads required to produce the stresses at the neutral axis of the webs, equal to the theoretical shear-buckling values in Col. 3. [d] Corresponding maximum measured web deflection.

Since it was not possible to select experimental buckling loads for all panels considered, it is of interest to consider what evidence of buckling was obtained for loads indicated by theory as critical. Figs. 4 and 6 and Table 3 provide data for a comparison between maximum measured lateral deflections of the webs and theoretical shear-buckling loads. The latter were computed as the loads required to produce shear stresses at the neutral axis of the webs (according to the flexure theory) equal to the theoretical shear-buckling values, based on the clear dimensions of the panels and the assumption of simply-supported edges.[2,5] With the exception of the panels of specimen 3, which were subjected to average shear stresses sufficiently high to produce plastic buckling, the maximum measured lateral deflections were generally quite small. For fifteen of the twenty-six sizes of panel listed in Table 3, for example, the maximum measured deflections were less than 1/500 the least width of panel. In only one case involving elastic action did the maximum deflection corresponding to the buckling load exceed one third of the web thickness. A number of the lateral deflection curves shown in Figs. 5(*c*) and 5(*d*) indicate that the webs remained essentially flat for loads considerably above the theoretical buckling values.

TABLE 4.—COMPARISON OF MEASURED AND COMPUTED VERTICAL DEFLECTIONS AND SHEARING STRESSES

Line	Description	Spec.[a] 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6	Spec. 7A	Spec. 8A
	(*a*) VERTICAL DEFLECTIONS, IN INCHES, AT THE CENTER OF THE SPAN								
1	Test load, in kips	85.0	80.0	80.0	45.0	32.0	60.0	32.0	20.0
2	Measured deflections	0.267	0.425	0.250[b]	0.176	0.212	0.365	0.308	0.235
3	Computed deflections[c]	0.275	0.395	0.258	0.203	0.227	0.374	0.320	0.246
4	Ratio, line 2 to line 3	0.97	1.08	0.97	0.87	0.93	0.98	0.96	0.96
	Percentages,[d] Caused by:								
5	Flexure	65	58	49	64	56	60	59	49
6	Shear	35	42	51	36	44	40	41	51
	(*b*) VERTICAL SHEARING STRESSES,[e] IN KIPS PER SQ INCH, IN WEBS								
7	Test load, in kips	80.0	80.0	80.0	50.0	40.0	48.0	40.0	18.0
	Computed Stresses:								
8	Average (gross area)	11.8	11.8	11.8	11.1	8.9	10.7	11.8	8.0
9	Rosettes A and D	12.8	12.8	12.3	12.0	9.6	11.5	12.9	8.7
10	Rosettes B and C	13.2	13.2	12.6	12.3	9.9	11.8	13.3	8.9
11	Average of rosettes	13.0	13.0	12.45	12.15	9.75	11.65	13.1	8.8
	Measured Stresses:								
12	Rosette A	12.8	12.5	12.2	11.4	9.5	11.4	13.3	9.2
13	Rosette B	13.6	13.0	13.1	11.7	10.6	12.3	13.8	9.6
14	Rosette C	14.0	13.2	12.9	12.3	9.9	12.1	13.3	9.2
15	Rosette D	13.4	13.1	12.2	12.4	8.8	11.6	12.0	8.4
16	Average	13.4	13.0	12.6	12.0	9.7	11.8	13.1	9.1
	Ratios,[f] Measured to Computed:								
17	Line 16 to line 8 (1.10)	1.14	1.10	1.07	1.08	1.09	1.10	1.11	1.14
18	Line 16 to line 11 (1.01)	1.03	1.00	1.01	0.99	0.99	1.01	1.00	1.03

[a] Specimen. [b] Second test. [c] Based on elements of gross sections. [d] Percentages for the computed deflections. [e] The typical location of the strain rosettes is shown in Fig. 4. The stresses were computed by the shear formula, based on the flexure theory. [f] Averages for each group of specimens are shown in parentheses in lines 17 and 18.

Theoretical shear-buckling loads were selected as the basis of comparison in Fig. 5 and in Table 3 because of the low ratio of bending to shearing stresses

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[5] "Alcoa Structural Handbook," Aluminum Co. of America, Pittsburgh, Pa., 1945, pp. 42 and 61.

computed for the panels considered. The load-versus-lateral deflection curves in Figs. 5(*c*) and 5(*d*) indicate that there was no consistent or significant difference between the behavior of the panels near the center of span, where the bending moments were a maximum, and the behavior of panels at the ends of the span subjected to the same shear but where the moments were relatively small.

It was concluded from these observations that, in general, shear-buckling loads computed in the manner described underestimate the point at which significant web buckling may occur and that such loads therefore provide a conservative basis for design. (Theoretical shear-buckling loads for fixed edges are about 65% greater than those for simply-supported edges.[6]) The web panels considered (clear distances) were not simply supported at the edges, as assumed, because of the restraining effects of the flange and stiffener angles. As far as the web deflections observed in these tests were concerned, however, accidental eccentricities of loading, resulting either from initial out-of-flatness of the webs or from the manner in which the loads were applied, tended to offset the edge-restraining effects. These compensating factors will be present in all girder webs.

TABLE 5.—COMPARISON OF MEASURED AND COMPUTED BENDING STRESSES (KIPS PER SQUARE INCH)

Specimen	Distance from end (in.)	Test load (kips)	Computed stress (gross area)	MEASURED STRESS				RATIOS, MEASURED TO COMPUTED STRESS			
				Flange		Web		Flange		Web	
				Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	15.5	100.0	7.9	6.6	9.5	10.5	10.0	0.84	1.20	1.33	1.27
1	54.0[a]	100.0	15.0	13.6	15.8	14.7	16.4	0.91	1.05	0.98	1.09
2	39.5	80.0	16.0	15.0	17.1	16.6	21.8	0.94	1.07	1.04	1.36
3	54.0[a]	120.0	15.9	....	....	13.5[b]	16.7[b]	....	....	0.85	1.05
3	54.0[a]	120.0	14.7	16.7[b]	15.7[b]	....	....	1.14	1.07	....	....
3	35.8	120.0	13.2	....	....	11.4[b]	14.7[b]	....	....	0.86	1.11
3	35.8	120.0	12.2	13.8[b]	13.8[b]	....	....	1.13	1.13	....	....
4	54.0[a]	90.0	18.1	16.2	17.7	16.5	18.3	0.90	0.98	0.91	1.01
5	38.8	32.0	8.8	8.6	8.0	8.3	9.4	0.98	0.91	0.94	1.07
6	54.0[a]	72.0	21.8	18.8	23.8	21.5	23.3	0.86	1.09	0.99	1.07
6	35.8	64.0	16.1	15.3	17.0	16.5	19.8	0.95	1.06	1.02	1.23
7A	43.8	40.0	18.1	18.9	18.8	20.3	17.6	1.04	1.04	1.12	0.97
8A	43.8	30.0	14.1	17.5	13.7	21.4	12.0	1.24	0.97	1.52	0.85
Average difference (%) between measured and computed stresses.....								10.6	7.7	13.6	13.1
Number of cases (out of 11) in which the measured stress is less than the computed value..								7	3	6	2

[a] At the center of the span. [b] For specimen 3, the flange stresses (Cols. 5 and 6) are the averages for the cover plates, since the web at the point indicated was not accessible. The web stresses (Cols. 7 and 8) are the averages for the underside of the flange angles (see Fig. 6(*c*)). All other values are extreme fiber bending stresses.

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[6] "Formulas for Stress and Strain," by R. J. Roark, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, p. 269.

The photographs in Figs. 2(*a*) and 2(*c*) and the load-versus-lateral deflection curves in Figs. 5(*a*), 5(*b*), and 5(*c*) show no evidence of web buckling in the panels at the center of the girders subjected to pure bending. Application of the buckling theory[2,5] to panels of this type indicates that the stresses required to produce buckling were appreciably higher than the maximum developed in these tests.

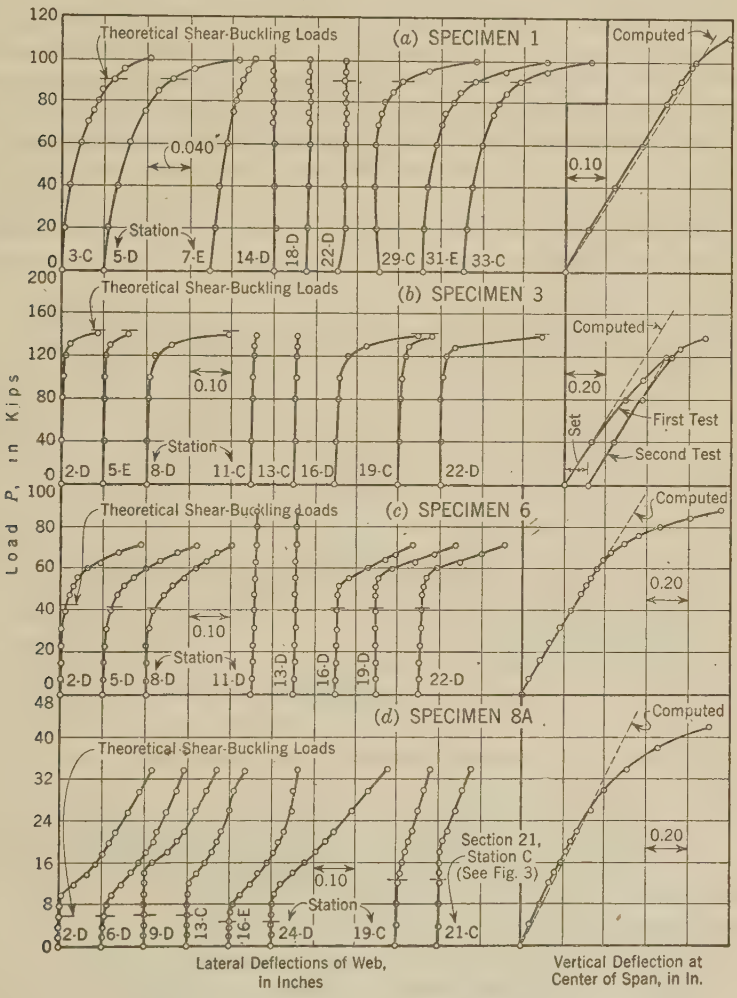

FIG. 5.—LOAD-DEFLECTION DIAGRAMS (FOR LATERAL DEFLECTIONS AT CORRESPONDING STATIONS AND SECTIONS, SEE FIG. 3)

(*B*) *Vertical Deflections.*—Fig. 5 also shows typical relations between loads and measured vertical deflections at the center of the span. A comparison of these with the lateral deflection data in Fig. 5 indicates that the vertical deflections of the girders were not sensitive to the first buckling action of the webs. Approximately linear relations between load and vertical deflection were obtained for loads considerably beyond the point at which buckling of the webs was readily apparent.

Table 4(*a*) affords a comparison between measured and computed vertical deflections for all girders. The test loads used for this comparison were the estimated beam proportional limits for all specimens except specimens 2 and 3. The computed deflections were obtained on the assumption of simple flexure (E = 10,300 kips per sq in.), with an allowance for deformations due to shear (G = 3,850 kips per sq in.), computed in the usual manner on the assumption of a uniform distribution of shear stress over the gross area of the webs. In general, the agreement between measured and computed deflections was quite satisfactory. Differences ranged from a maximum of 13% in one case to from 3% to 8% for all others. In only one case did the measured deflections exceed the computed values. From 35% to 51% of the total deflections was computed to be the result of shear—a higher ratio of shearing-to-bending deformations than is usually obtained in girder design. It is significant that the ordinary methods of computing deflections are applicable to specimens of these proportions. Since the maximum bending and shearing stresses corresponding to the test loads in Table 4(*a*) were considerably less than the yield strengths of the material, it appears that the first over-all yielding indicated by the vertical deflection curves can be attributed largely to the buckling action of the webs rather than to plastic deformations.

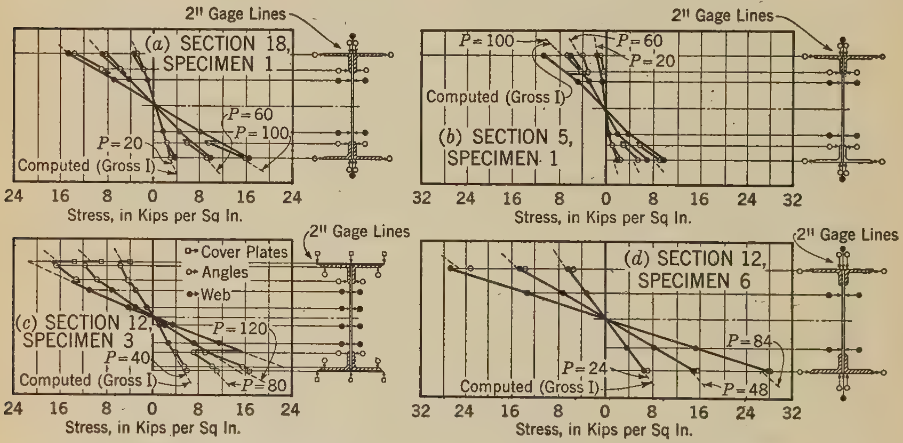

FIG. 6.—COMPARISON OF MEASURED AND COMPUTED BENDING STRESSES

(*C*) *Stress Distribution.*—The shear-buckling loads selected for the girders were those loads which, according to the flexure theory, were required to produce shearing stresses at the neutral axis of the webs equal to the theoretical shear-buckling stresses for the panels. If the buckling loads had been based on the assumption that the critical stresses in shear were uniformly distributed over the entire gross area of the webs (as is generally assumed in design), somewhat higher buckling loads would have been obtained. However, the shearing-stress distribution diagrams shown in Fig. 4, and the comparison between measured and computed values shown in Table 4(*b*) indicate that the theoretical distribution of shear stress between flanges was approached very closely in these tests. Differences between average measured and computed vertical shearing stresses for the four rosette locations on the webs were not more than about 3%, as compared to differences of about 10% between measured stresses and average values based on the gross area of the webs. The measured shearing stresses were determined as the differences between the strains measured on the 45° diagonal gage lines of the rosettes multiplied by the shear modulus of elasticity (3,850 kips per sq in.).[7]

Differences between measured and computed bending stresses in built-up girders apparently may result from the lack of integral action between the component parts of the girder, the reductions in area of section at rivet holes, and the effect of stress concentrations and unequal distribution of load between rivets. Complete integral action is usually assumed although the degree attainable would seem to depend on the relative proportions of webs and flanges and the efficiency of the connections between these parts. The differences between measured and computed vertical deflections in Table 4(*a*) did not indicate any significant lack of integral action in the test girders, although it is quite evident from the bending-stress-distribution diagrams in Figs. 6(*a*), 6(*b*), and 6(*c*) that all parts of specimens 1 and 3 were not stressed to the same degree. This effect was generally more pronounced on sections subjected to combined shear and bending than on sections subjected to pure bending.

Table 5 gives a comparison between measured and computed stresses in the webs and flanges of all girders. In most cases the stresses measured in the top flanges, in both webs and angles, were less than those observed at corresponding points on the bottom flanges. Extreme fiber stresses in the webs were generally higher than those measured at adjacent points in the flange angles—indicating some lack of integral action. The agreement between individual measured and computed bending stresses was definitely not as satisfactory as that shown for shearing stresses in Table 4(*b*), although it will be appreciated that the stress distribution in the flanges was probably considerably more complex than that produced in the webs. In addition to the lack of integral action, the length and position of the strain-gage lines relative to rivet holes, and the possibility of considerable variation in load on individual rivets, undoubtedly accounted for some of the variations in measured stresses observed. In view of the generally good agreement obtained between measured and computed vertical deflections, it seems reasonable to believe that the average bending stresses along the length were likewise in fair agreement with computed values.

General structural design practice is not consistent regarding the procedure to be followed in making allowances for rivet holes in computing section elements. The use of net rather than gross section elements for the tension flanges would have resulted in a generally better agreement between average measured and computed tensile stresses. The use of net section for the compression flanges, however, would have resulted in poorer agreement.

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[7] "The Determination of Stresses from Strains on Three Intersecting Gage Lines and Its Application to Actual Tests," by William R. Osgood and R. G. Sturm, *Research Paper No. 559*, National Bureau of Standards, 1933.

(*D*) *Ultimate Strengths of the Girders.*—Table 6 gives a summary of the maximum test loads, together with the corresponding computed maximum bending stresses or moduli of failure and the average shearing stresses on the gross areas of the webs. Average shear and bearing stresses in the flange and load-bearing stiffener rivets are also included. These stresses were all computed on the assumption that the structural action of the girders was the same as that obtained in the early stages of the tests before web buckling. The bending stresses in all cases were considerably less than the tensile yield strengths of the material given in Table 1. The average shear stresses exceeded the shear yield strength of 22 kips per sq in. estimated for the web material[5] in four of the eight cases considered and ranged from one to approximately twelve times the theoretical shear-buckling stresses for the largest panels of each girder. The latter comparisons emphasize how little effect the buckling action of the web may have on the ultimate strength of a girder.

TABLE 6.—COMPUTED STRESSES, IN KIPS PER SQUARE INCH, CORRESPONDING TO MAXIMUM TEST LOADS, IN KIPS

Line	Description	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6	Spec. 7A	Spec. 8A
1	Maximum test load...	155.0[a]	107.2[a,b]	170.0[a]	94.1[a]	70.8[a,b]	105.4[a,c]	67.3[a,c]	55.0[a,c,d]
2	Bending stresses[e].....	23.2	29.4	22.5	18.9	27.0	32.0	37.7	32.0
3	Web shear[f]..........	23.0	15.9	25.1	20.9	15.7	23.4	19.9	24.4
4	Rivet bearing........	75.3	52.1	82.5	116.0	87.4	114.8	102.3	130.5
5	Rivet shear..........	11.5	8.0	12.6	17.5	13.2	17.4	11.6	9.9

[a] Large shear distortions at failure. [b] Twisting of flange at failure. [c] Local buckling of flange at failure. [d] Web fractures in the rivet holes at failure. [e] The moduli of failure, or the extreme fiber stresses at the center of the span, are based on gross area. [f] Average web shear based on gross area.

Three types of failure were obtained. In specimens 1, 3, and 4, the shearing distortions produced in the webs became so great that it seemed advisable to stop the tests. It was quite evident, both from the condition of the girders and the marked increases in deflection obtained without appreciable increases in load, that a practical limit of load-carrying capacity had been reached. Figs. 2(*a*) and 2(*c*) show the nature of the web buckling produced. The average shearing stresses, corresponding to the maximum loads on these specimens, ranged from 20.9 to 25.1 kips per sq in., whereas the maximum computed bending stress was only 23.2 kips per sq in.

In specimens 2 and 5, failure occurred by a combination of web buckling and twisting of the compression flanges although, from the relatively low value of flange stress computed for the buckled region, it would appear that web buckling was the primary cause of failure. There was no evidence of flange twisting on the opposite sides of these same girders where intermediate web stiffeners were used and web buckling was not so pronounced. Figs. 2(*b*) and 7 illustrate the type of buckling failures obtained. The maximum test loads represent ultimate load-carrying capacities even though the average shearing stresses were slightly less than 16 kips per sq in.

In specimens 6, 7A, and 8A, the web distortions were accompanied by local buckling of the compression flanges. Secondary diagonal-tension fractures of the web were also obtained in specimen 8A as shown in Fig. 8. Fig. 7 illustrates the nature of the buckling failures. The bending stresses of from 32.0 to 37.7 kips per sq in., computed for these specimens, were the highest obtained. The average shear stresses ranged from 19.9 to 24.4 kips per sq in. From the standpoint of utilizing the strength of the web and flange material, the specimens of this group represent the most balanced designs.

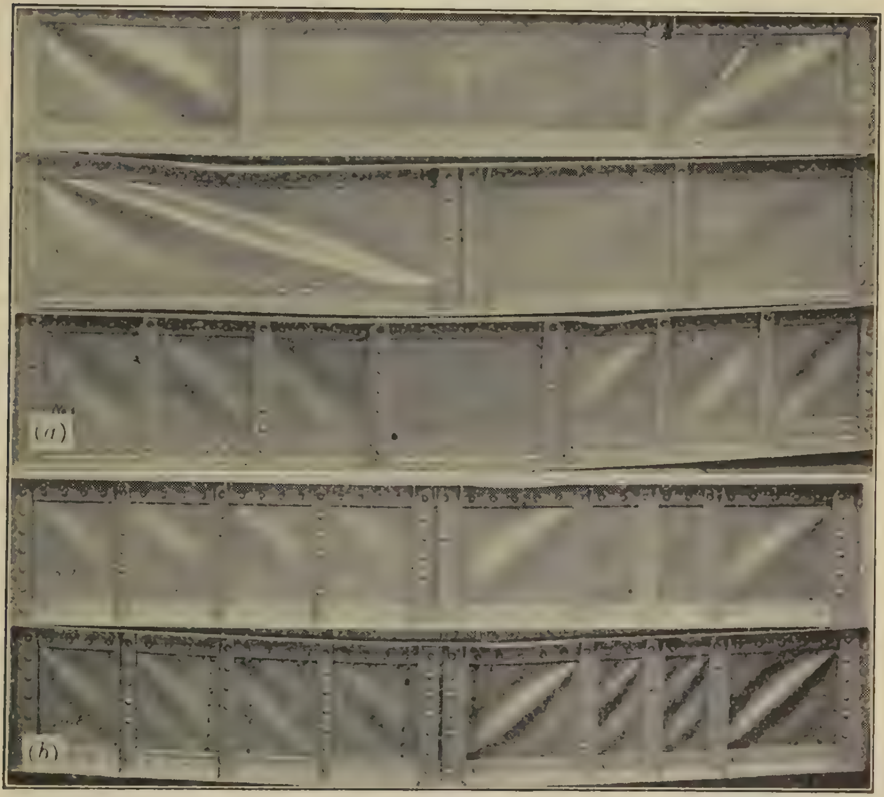

FIG. 7.—ALUMINUM ALLOY GIRDERS AFTER TEST TO FAILURE

(a) SPECIMENS 4, 5, AND 6 ($\frac{1}{8}$-IN. WEBS)
(b) SPECIMENS 7A AND 8A ($\frac{3}{32}$-IN. WEBS)

The data pertaining to local buckling of outstanding compression flanges on specimens 6, 7A, and 8A are probably not sufficient to establish a general criterion for predicting buckling of this type. The average flange stresses computed for the sections at which buckling actually occurred correspond to the buckling stresses computed for flat plates, subjected to edge compression, with one edge free and the other from 50% to 100% fixed.[5]

The average shearing stresses given in Table 6 for the flange and stiffener rivets indicate that there was a considerable margin of safety against failure in these connections, since the ultimate shear strength of aluminum alloy 17S–T is about 36,000 lb per sq in. The average computed bearing stresses, however, were higher in some cases than the ultimate bearing strengths generally accepted for this alloy. The variation in rivet-hole distortions shown in Fig. 8 indicates that the bearing stresses on some rivets, near the ends of the diagonal-tension buckles, must have been considerably higher than the averages computed. The maximum increases in hole diameter ranged from about 15% to 20%.

(*E*) *Tension-Field Action.*—As stated in Section IV(D), the initial buckling load for a shear-resistant girder web does not necessarily limit its ultimate load-carrying capacity. For example, specimen 8A supported a load approximately twelve times that which produced first buckling of the largest panels. The civil engineer has become accustomed to look on buckling as an undesirable phenomenon and avoids such action by the use of low design stresses and intermediate web stiffeners. The aeronautical engineer, on the other hand, recognizes that buckling of a girder web means a transition from shear-resistant action, where diagonal tensile and compressive forces are equally effective in resisting shear, to so-called tension-field action, where the shear in excess of the buckling value is carried largely by diagonal tension. The strength of a web considered on the latter basis depends on the properties of the web material and on the capacity of the flanges and stiffeners to resist failure until the web has reached its ultimate load-carrying capacity.

The original theory of tension-field action, in which the web was assumed to have no flexural stiffness and all the shear was carried by diagonal tension, was presented by Herbert Wagner.[8] The practical results of this analysis have been summarized by Paul Kuhn,[9] A. S. Niles, Assoc. M. ASCE, and J. S. Newell.[10] Since all webs are shear resistant to some extent, however, considerable experimental as well as analytical work has been done in an effort to modify the theory to make it more suitable for design. E. E. Sechler and L. G. Dunn have presented a summary[11] of the results of some of this work; and they give data on the yield and ultimate strengths of a number of tension-field girders of aircraft proportions. This approach to the shear problem in girder webs is emphasized because it represents an attempt in some fields of design to take advantage of useful girder strengths beyond first buckling values.

Although the tests in this report have provided considerable evidence of the factors associated with tension-field action, the proportions of the girders were not believed to justify a detailed tension-field analysis in view of the present state of the theory. A tension-field failure was obtained in only one girder (8A), and this action was accompanied by local buckling of the compression flange. Tension-field action presupposes that the web buckles into a large number of small folds and that the diagonal tensile forces are distributed fairly uniformly along the flanges and end stiffeners. The buckle patterns in Figs. 2(*a*) and 7 and the variation in hole elongations in Fig. 8 are scarcely consistent with this assumed action. Stiffener spacings relative to the depth of web are generally much closer in aircraft tension-field girders than was the case in these tests. Ratios of web depth to thickness, moreover, may range from 500 to 1,000 or more, whereas the highest ratio in these tests was only 192.

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[8] "Flat Sheet Metal Girders with Very Thin Metal Web," by Herbert Wagner, Pts. I, II, and III, *Technical Memorandums Nos. 604, 605, and 606*, National Advisory Council for Aeronautics, 1931.

[9] "A Summary of Design Formulas for Beams Having Thin Webs in Diagonal Tension," by Paul Kuhn, *Technical Note No. 469*, National Advisory Council for Aeronautics, 1933.

[10] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 1938, 2d Ed., Vol. I, p. 154.

[11] "Airplane Structural Analysis and Design," by E. E. Sechler and L. G. Dunn, John Wiley & Sons, Inc., New York, N. Y., 1942, p. 234.

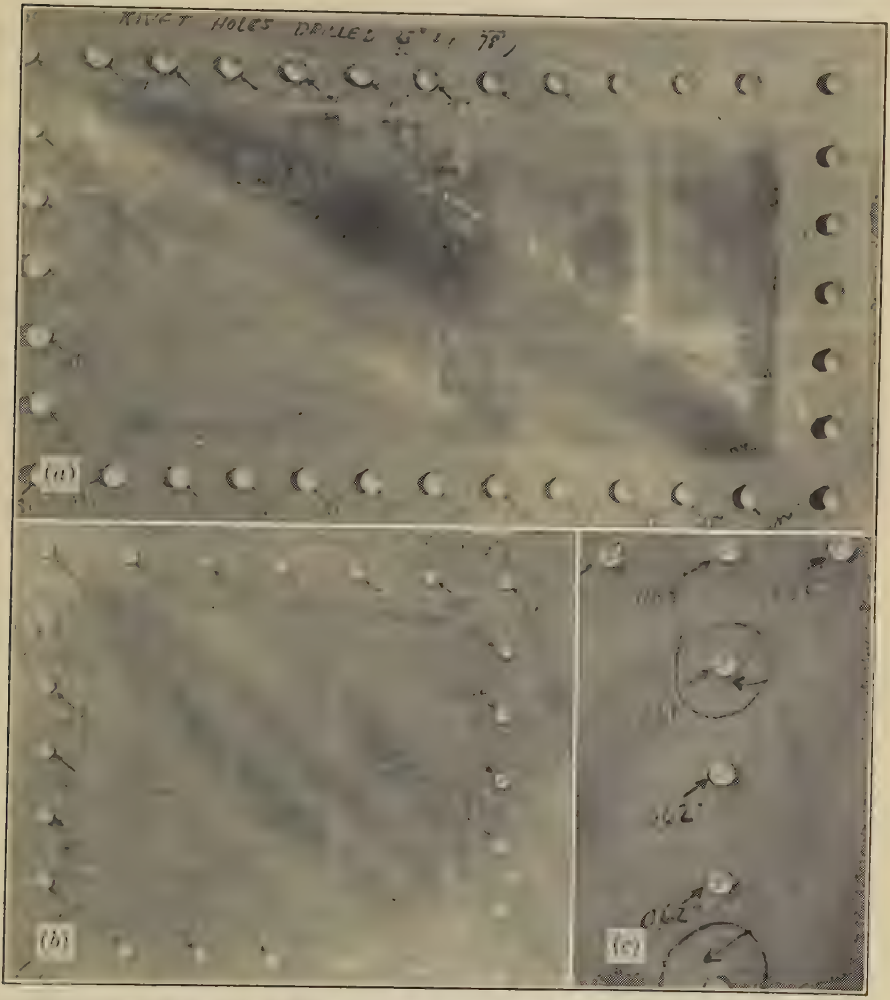


FIG. 8.—TEST GIRDERS WITH FLANGE AND STIFFENER ANGLES REMOVED TO SHOW HOLE DISTORTIONS (NUMERALS INDICATE ELONGATED HOLE DIAMETERS; UNITS ARE IN INCHES)

(*a*) End Panel, Specimen No. 1
(*b*) End Panel, Specimen 8A
(*c*) Intermediate Panel, Specimen 8A

The choice of buckling angle is an important factor in tension-field analysis, although the dependence of this factor on girder proportions is as yet unknown. Some evidence was obtained in these tests to show that the buckling angle does not remain constant with changing loads. In specimen 5, for example, the angle of buckle shifted from approximately 33° at a load of 40 kips to 15° for the maximum load of 70.8 kips (Fig. 7).

Fig. 9 shows a comparison of measured and computed principal stresses and maximum shears in the web of specimen 8A. The measured stresses were based on the average of strains on both sides of the webs.[7] The computed stresses were determined as the maximum values for combined bending and shear at the various rosette locations. The interesting observation to be made from this figure is that the diagonal compressions measured at the center of the largest panel (section 16) were approximately equal to the diagonal tensions for loads to 18.0 kips—or more than four times the theoretical buckling load for the panel. Strain measurements were not made for loads in the vicinity of the maximum test value, so it is not possible to state at which point tension-field action, which ultimately fractured the web, became significant. For girders of the proportions considered herein, the problem obviously requires further investigation.

V. Conclusions

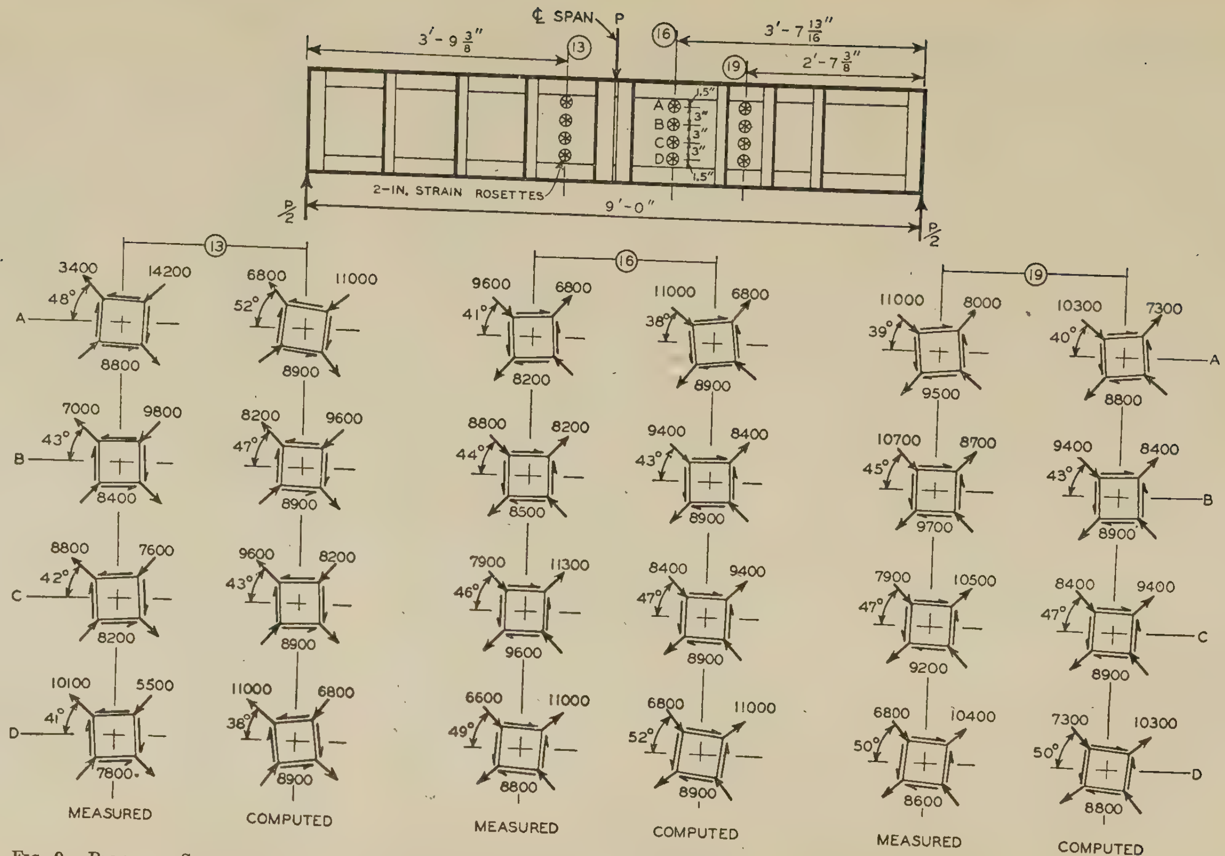
The results of this investigation of the structural behavior of a number of built-up test girders of aluminum alloy 17S–T are believed to warrant the following conclusions:

a. Well-defined buckling loads generally cannot be obtained experimentally for flat webs subjected to combined bending and shear because of unavoidable eccentricities of loading. The computation of buckling values for shear alone, using clear dimensions of panels and assuming simply-supported edges,[2,5] probably underestimates the buckling resistance of most panels but provides a simple, conservative basis for design. Edge-restraining effects, which tend to retard visible buckling, are offset to some extent by eccentricities of loading and the effects of bending superimposed upon those of shear.

b. The measured vertical deflections at the center of span in Table 4(*a*) averaged within about 5% of the computed values based on a consideration of bending and shear. Approximately linear relations between load and deflection were obtained for loads considerably greater than those which produced first web buckling, indicating that over-all stiffness was not particularly sensitive to the first buckling action of the webs.

c. The measured vertical shearing stresses shown in Table 4(*b*) for the part of the webs between flanges were in very close agreement with values computed according to the flexure theory, the maximum difference being about 3%. These stresses, which were the significant values from the standpoint of web buckling, averaged about 10% higher than those based on the gross area of the webs.

d. The measured bending stresses for the flanges in Table 5 were not in as close agreement with computed values as shown in Table 4(*b*) for shearing


℄ SPAN
P
3′-9 3/8″
3′-7 13/16″
2′-7 3/8″
13
16
19
A
B
C
D
1.5″
3″
3″
3″
1.5″
2-IN. STRAIN ROSETTES
9′-0″
P/2
P/2
3400
14200
48°
8800
7000
9800
43°
8400
8800
7600
42°
8200
10100
5500
41°
7800
MEASURED
6800
11000
52°
8900
8200
9600
47°
8900
9600
8200
43°
8900
11000
6800
38°
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COMPUTED
9600
6800
41°
8200
8800
8200
44°
8500
7900
11300
46°
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COMPUTED

FIG. 9.—PRINCIPAL STRESSES AND MAXIMUM SHEARS *F* OF A LOAD *P* OF 18 KIPS; SPECIMEN 8A (STRESSES, IN KIPS PER SQ IN., ARE THE AVERAGES FOR TWO SIDES OF THE WEB)

stresses. Lack of integral action between flanges and webs and the effects of variable load distribution between rivets and of stress concentrations at rivet holes are factors that complicate any attempt to obtain an experimental check on the computed bending stresses in built-up girders of this type.

e. Table 6 gives a summary of the maximum test loads with the corresponding computed maximum bending and shearing stresses. Although it is difficult to generalize regarding the significance of these data for the prediction of ultimate load-carrying capacities for built-up girders, the stresses given indicate the ranges in which various types of buckling may be expected. Figs. 2, 7, and 8 show the types of failure obtained.

f. It is quite apparent that shear buckling in a girder web may bear little relation to its useful or ultimate load-carrying capacity. The analysis of webs on the basis of tension-field action as well as on that of shear-resistant action perhaps merits more consideration by civil engineers.

g. On the basis of the local flange-buckling resistances observed for specimens 6, 7A, and 8A, buckling may be estimated by considering that a girder flange acts as a plate subjected to uniform edge compression, with one edge free and the other about 50% fixed.[5]

h. The rivet-bearing stresses actually developed in the thin webs were undoubtedly greater than the average computed values in Table 6 because of the unequal distribution of load occurring after web buckling. The maximum of 130.5 kips per sq in. computed for specimen 8A indicates the high bearing values attainable when lateral restraint is provided. Fig. 8 shows the rivet-hole distortions produced in two thicknesses of webs.

i. As indicated by the variation in rivet-hole distortions, the diagonal tensions developed after buckling were not uniformly distributed along the length of the flange and stiffener angles. Although no direct measure of their maximum intensity was obtained, the tensions corresponding to an average shearing stress of 24.4 kips per sq in. in specimen 8A were sufficient to produce fracture of the web at two rivet holes for an intermediate stiffener, as shown in Fig. 8, and also at the end rivet in the compression flange.

VI. Acknowledgments

The tests described in this paper were made at the Aluminum Research Laboratories in New Kensington, Pa., under the direction of R. L. Templin and E. C. Hartmann, Members, ASCE. The work has been sponsored for publication as a project of the Committee of the Structural Division on Design in Lightweight Structural Alloys.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2316

SPACE-RESECTION PROBLEMS IN PHOTOGRAMMETRY

By P. H. Underwood,[1] M. ASCE

With Discussion by Messrs. Earl Church, Ralph O. Anderson, G. H. Dell, and P. H. Underwood.

Synopsis

The paper gives a solution of the resection problem in space to find the position of a camera station and to obtain orientation data for a vertical photograph with a small amount of tilt by applying methods of mathematical calculation combined with some graphical work.

In solutions of this kind, approximate methods must be used for some of the steps. In this paper such approximations play an important part. The solution is based upon the use of two three-sided pyramids with a common vertex at the camera station, with sides in common planes, and with triangular bases—the one in the plane of the photograph and the other in the plane of the control points on the ground. The vertex angles of these pyramids are calculated by methods not very dissimilar to those commonly used. Then the lengths of the side edges of the ground pyramid are calculated, by use of the cosines of the base angles to expedite the work.

In solving next for the coordinates of the camera station, squared terms are introduced to make it easy to reach a precise solution by approximations. A method of "position circles" to obtain the elements of exterior orientation is developed and it is believed that this considerably facilitates this part of the solution. A method is also given for finding the tilt, swing, and azimuth of the principal line by mathematical calculations. A numerical example is added to illustrate the theoretical methods.

Theoretical Analysis

The determination of the elements of exterior orientation of a so-called vertical photograph is a problem that has received much consideration and has been treated in various ways. A useful list of references has been published

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Note.—Published in September, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof., Surveying, School of Civ. Eng., Cornell Univ., Ithaca, N. Y.

by R. Hugershoff and H. Cranz.[2] Earl Church,[3] Assoc. M. ASCE, has also done considerable work on this subject in the United States. The writer proposes to outline a method, certain steps of which he believes to have advantages over other methods in current use.

The letter symbols are defined where they first appear, in text or by illustration, and are assembled alphabetically, for convenience of reference, in the Appendix.

The problem may be outlined thus: A vertical photograph of the ordinary type with a certain small angle of tilt is being analyzed. On this photograph are three points of control which are suitably located for determining the elements of exterior orientation; that is, points not near the center of the photograph and not close to one another. The rectangular coordinates of these three points with respect to axes on the photograph are known. The y-axis is vertical and the x-axis horizontal, the origin being at the principal point of the picture. Let these three points be designated a, b, and c (Fig. 1) with coordinates x_a, y_a, x_b, y_b, x_c, and y_c, respectively. Let the corresponding points on the ground be A, B, and C, whose known coordinates with respect to a coordinate system on the ground are X_A, Y_A, Z_A, etc. The focal length of the camera f has supposedly been determined by calibration and is known. The problem is to find the tilt i, the swing s, or angle made by the principal line with the y-axis on the photograph, the coordinates of the camera station on the ground control system, and the azimuth of the principal line on the ground-control system.

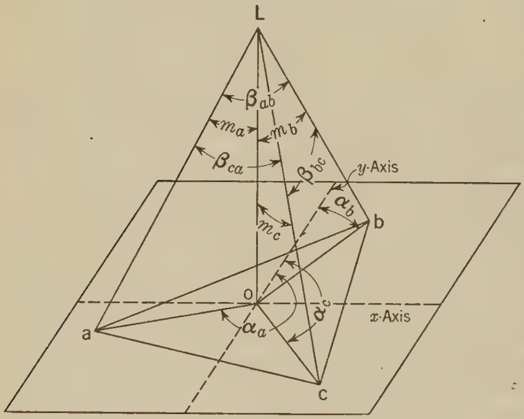

Fig. 1

The first step is to compute the azimuths on the plate α_a, α_b, and α_c, reckoned clockwise from the positive end of the y-axis, of the lines oa, ob, and oc (o being the principal point), the lengths of these lines, the lengths of the sides of the triangle abc, the lengths of the edges of the plate pyramid, La, Lb, and Lc (L, the vertex of the pyramid, is the perspective center, or lens center), and the angles aLo, bLo, and cLo which will be designated m_a, m_b, and m_c, respectively. Thus:

$$\tan \alpha_a = \frac{x_a}{y_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

$$\tan \alpha_b = \frac{x_b}{y_b} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

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[2] "Grundlagen der Photogrammetrie aus Luftfahrzeugen," by R. Hugershoff and H. Cranz, Konrad Wittwer, Stuttgart, 1919.

[3] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, 1944, Chapter XII.

and

$$\tan \alpha_c = \frac{x_c}{y_c} \quad \text{(1c)}$$

Also,

$$\overline{oa} = \sqrt{x^2_a + y^2_a} \quad \text{(2a)}$$

$$\overline{ob} = \sqrt{x^2_b + y^2_b} \quad \text{(2b)}$$

and

$$\overline{oc} = \sqrt{x^2_c + y^2_c} \quad \text{(2c)}$$

As a check, $\overline{oa} = \frac{x_a}{\sin \alpha_a} = \frac{y_a}{\cos \alpha_a}$, etc. Furthermore,

$$\overline{ab} = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \quad \text{(3a)}$$

$$\overline{bc} = \sqrt{(x_c - x_b)^2 + (y_c - y_b)^2} \quad \text{(3b)}$$

and

$$\overline{ac} = \sqrt{(x_a - x_c)^2 + (y_a - y_c)^2} \quad \text{(3c)}$$

Finally,

$$\overline{La} = \sqrt{\overline{ao}^2 + f^2} \quad \text{(4a)}$$

$$\overline{Lb} = \sqrt{\overline{bo}^2 + f^2} \quad \text{(4b)}$$

and

$$\overline{Lc} = \sqrt{\overline{co}^2 + f^2} \quad \text{(4c)}$$

The next step is to solve for the angles aLb, bLc, and cLa at the vertex of the pyramid, calling these angles for brevity β_{ab}, β_{bc}, and β_{ca}, respectively. These calculations are somewhat similar to those described by Professor Church.[4] Solve for these angles by applying the cosine formula for a plane triangle in succession to the triangles aLb, bLc, and cLa. Thus,

$$\cos \beta_{ab} = \frac{-\overline{ab}^2 + \overline{La}^2 + \overline{Lb}^2}{2\,\overline{La} \times \overline{Lb}} \quad \text{(5a)}$$

$$\cos \beta_{bc} = \frac{-\overline{bc}^2 + \overline{Lb}^2 + \overline{Lc}^2}{2\,\overline{Lb} \times \overline{Lc}} \quad \text{(5b)}$$

and

$$\cos \beta_{ca} = \frac{-\overline{ac}^2 + \overline{Lc}^2 + \overline{La}^2}{2\,\overline{Lc} \times \overline{La}} \quad \text{(5c)}$$

Also solve for m_a, m_b, and m_c by the formulas:

$$\tan m_a = \frac{\overline{ao}}{f} \quad \text{(6a)}$$

$$\tan m_b = \frac{\overline{bo}}{f} \quad \text{(6b)}$$

and

$$\tan m_c = \frac{\overline{co}}{f} \quad \text{(6c)}$$

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[4] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, 1944, Chapter XII, p. 537.

These calculations may be checked by the formulas:

$$\sin m_a = \frac{\overline{ao}}{\overline{La}}, \qquad \sin m_b = \frac{\overline{bo}}{\overline{Lb}}, \qquad \text{and} \qquad \sin m_c = \frac{\overline{co}}{\overline{Lc}};$$

or by

$$\cos m_a = \frac{f}{\overline{La}}, \qquad \cos m_b = \frac{f}{\overline{Lb}}, \qquad \text{and} \qquad \cos m_c = \frac{f}{\overline{Lc}}.$$

The various steps in the solution that is to follow necessitate finding the coordinates with respect to the ground-survey system of the camera station L. After finding the coordinates of L the remainder of the solution includes finding the tilt, the swing, and the azimuth with respect to the ground-survey system of the principal plane of the photograph. It is now proposed to outline the various steps required to determine the coordinates of point L.

When the camera station L is connected to the three control points on the ground by the lines LA, LB, and LC a triangular pyramid may be considered to be formed with the vertex at point L and with the triangle ABC forming its base. The base of this pyramid will generally be inclined to the horizontal since the three control points are likely to be at different elevations.

The inclined lengths of the sides of the base may be calculated by the formulas:

$$\overline{AB} = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2} \quad \text{(7a)}$$

$$\overline{BC} = \sqrt{(X_C - X_B)^2 + (Y_C - Y_B)^2 + (Z_C - Z_B)^2} \quad \text{(7b)}$$

and

$$\overline{CA} = \sqrt{(X_A - X_C)^2 + (Y_A - Y_C)^2 + (Z_A - Z_C)^2} \quad \text{(7c)}$$

For later use the horizontal lengths of these sides should also be computed by dropping the Z-terms from Eqs. 7. If the Z-differences are small, the calculations may be checked by the approximate formula,

$$(\text{Slope length}) - (\text{horizontal length}) = \frac{(Z\text{-difference})^2}{2\,(\text{horizontal, or slope, length})} \quad (8)$$

A graphical solution to find approximate lengths of the lines LA, LB, and LC should now be made. To do this the angles β (Fig. 1) are laid off with a protractor, or by the method of tangents, to give a development of the sides of the plate and ground pyramids, which have a common vertex and common side faces (see Fig. 2). The construction of the angles may be checked by

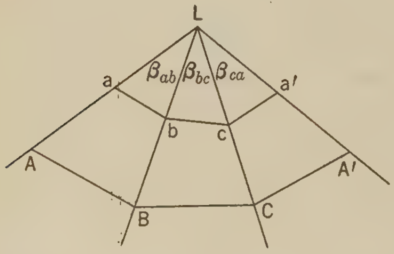

FIG. 2

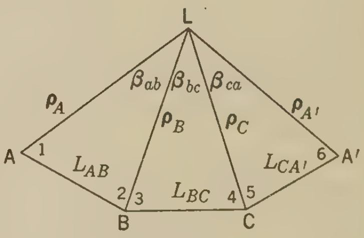

FIG. 3

scaling off on these lines the distances La, Lb, and Lc, respectively, and then scaling from the drawing the lengths of ab, bc, and ca, and comparing with the previously calculated values.

To draw the base of the developed ground pyramid and thus determine the lengths of LA, LB, and LC the computer makes use of two conditions: First, that the lengths of the sides of this pyramid included between the lines La, Lb, Lc, and La′ must be equal in order to the corresponding inclined lengths AB, BC, and CA and, second, that the length of LA must be equal to that of LA′ since, of course, each represents the same edge of the ground pyramid. Graphically this problem cannot be solved directly but must be done by a trial-and-error method. After drawing AB to scale, parallel or approximately parallel to ab, strike an arc with radius equal to BC and center at point B to cut Lc at point C, and another arc with radius of length to scale equal to line CA, to cut La′ at point A′. If LA′ is not equal to LA (and it probably will not be) or nearly so, another trial may be made, starting at a point near A which is judged to meet the conditions more nearly. (A graphical method of finding the new position of point A, which, however, involves considerable labor, will be outlined subsequently.) In this way approximate lengths of LA, LB, and LC may be obtained, the precision of which will depend upon the scale of the drawing, the care with which the drafting is done, and the degree to which the approximation is carried. As a matter of fact the analytical solution which is to follow, although facilitated by having closely approximate values of LA, LB, and LC at the start, may be made from no more accurate initial values than can be found from the approximate relations $\frac{\overline{\mathrm{LA}}}{\overline{\mathrm{La}}} = \frac{\overline{\mathrm{AB}}}{\overline{\mathrm{ab}}} = \frac{\overline{\mathrm{CA}}}{\overline{\mathrm{ca}}}$, to find the length LA, averaging to obtain a better result. The degree of approximation in the value of LA thus obtained is dependent, of course, upon the tilt of the picture and the differences in elevation between control points.

In Fig. 3 the radial lines are designated ρ and the angles at the base of the pyramid are numbered for simplicity. Beginning with the approximate value of LA found by one of the methods just outlined, ρ_A, ρ_B, ρ_C, and $\rho_{A'}$ are computed in turn from the known quantities, the slope lengths AB, BC, and CA or L_{AB}, L_{BC}, and $L_{CA'}$, and the known angles β at the vertex of the pyramid. Since it is desired also to determine values of the angles 1, 2, 3, 4, 5, and 6, Fig. 3, these calculations will be made by use of the sine law. Thus,

$$\sin \angle\ (2) = \frac{\rho_A \sin \beta_{ab}}{L_{AB}} \qquad \text{(9a)}$$

$$\angle\ (1) = 180° - [\beta_{ab} + \angle\ (2)] \qquad \text{(9b)}$$

and

$$\rho_B = \frac{\rho_A \sin \angle\ (1)}{\sin \angle\ (2)} = \frac{L_{AB} \sin \angle\ (1)}{\sin \beta_{ab}} \qquad \text{(9c)}$$

The sides ρ_C and $\rho_{A'}$ are then computed in turn, first solving for ρ_C from ρ_B and then for $\rho_A{}'$ from ρ_C by similar formulas.

Of course, the value of $\rho_{A'}$ should be equal to ρ_A but it almost certainly will not prove that way since the value of ρ_A is only approximate. This first solution may be considered a trial one from which the engineer may derive, by the method now to be developed, a more accurate one. If this second solution does not give a sufficiently close check between $\rho_{A'}$ and ρ_A, further trials, as needed, may be made to yield a final solution with any required degree of precision.

A relationship between small changes in ρ_A and corresponding changes in $\rho_{A'}$ may be established by the method of rates and differentials. In the triangle LAB, applying the cosine formula:

$$L^2_{AB} = \rho^2_A + \rho^2_B - 2\,\rho_A\,\rho_B \cos\beta_{ab} \ldots\ldots\ldots\ldots\ldots (10)$$

In Fig. 3, L_{AB} and β_{ab} may be considered constant and ρ_A and ρ_B, variables. Then, $0 = 2\,\rho_A\,d\rho_A + 2\,\rho_B\,d\rho_B - 2\,\rho_A \cos\beta_{ab}\,d\rho_B - 2\,\rho_B \cos\beta_{ab}\,d\rho_A$; whence $\rho_B\,d\rho_B - \rho_A \cos\beta_{ab}\,d\rho_B = -\rho_A\,d\rho_A + \rho_B \cos\beta_{ab}\,d\rho_A$. Consequently,

$$\frac{d\rho_B}{d\rho_A} = -\frac{\rho_A - \rho_B \cos\beta_{ab}}{\rho_B - \rho_A \cos\beta_{ab}} \ldots\ldots\ldots\ldots\ldots (11a)$$

Similarly, it can be shown that

$$\frac{d\rho_C}{d\rho_B} = -\frac{\rho_B - \rho_C \cos\beta_{bc}}{\rho_C - \rho_B \cos\beta_{bc}} \ldots\ldots\ldots\ldots\ldots (11b)$$

and

$$\frac{d\rho_{A'}}{d\rho_C} = -\frac{\rho_C - \rho_{A'} \cos\beta_{ca}}{\rho_{A'} - \rho_C \cos\beta_{ca}} \ldots\ldots\ldots\ldots\ldots (11c)$$

For simplicity, let the product of Eqs. 11 be

$$\frac{d\rho_B}{d\rho_A} \times \frac{d\rho_C}{d\rho_B} \times \frac{d\rho_{A'}}{d\rho_C} = \frac{d\rho_{A'}}{d\rho_A} = u \ldots\ldots\ldots\ldots\ldots (12)$$

The constant u may be found from Eqs. 11 directly or it may be found more simply from a relationship between functions of the known angles 1, 2, 3, 4, 5, and 6, Fig. 3. From the triangles formed by dropping perpendiculars from points A, B, C, and A′ upon the adjacent lines from L—that is, upon LA, LB, LC, and LA′, Fig. 3—the numerators in Eqs. 11 are found to be: $L_{AB} \cos \angle$ (1), $L_{BC} \cos \angle$ (3), and $L_{CA'} \cos \angle$ (5), respectively, whereas the corresponding denominators are $L_{AB} \cos \angle$ (2), $L_{BC} \cos \angle$ (4), and $L_{CA'} \cos \angle$ (6). Substituting these values in Eq. 12, the lengths L cancel out and

$$u = \frac{d\rho_{A'}}{d\rho_A} = -\frac{\cos\angle(1)}{\cos\angle(2)} \times \frac{\cos\angle(3)}{\cos\angle(4)} \times \frac{\cos\angle(5)}{\cos\angle(6)} \ldots\ldots (13)$$

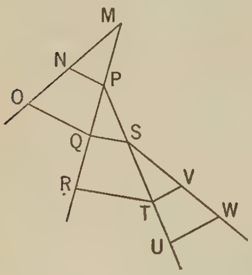

FIG. 4

This value of u may be found graphically as follows: After making a trial graphical solution as already described (to make La′ = La in Fig. 2) draw a line MO, Fig. 4, and on this line lay off to any convenient scale MN equal in length to $L_{AB} \cos \angle$ (1) and NO equal to $L_{AB} \cos \angle$ (2) and on an intersecting line MR lay off MP equal to $L_{BC} \cos \angle$ (4). Connect points N and P and draw line OQ parallel to line NP. Then by the well-known formula for proportionality of intercepts between parallel lines, $\overline{MN}/\overline{NO} = \overline{MP}/\overline{PQ}$. Now lay off QR equal to $L_{BC} \cos \angle$ (3), draw an intersecting line PU through point P, lay off PS equal to $L_{CA'} \cos \angle$ (5) and draw RT parallel to QS. Then $\overline{PS}/\overline{St} = \overline{PQ}/\overline{QR}$. By a similar construction lay off TU equal to $L_{CA'} \cos \angle$ (6), draw an intersecting line SW through point S and draw the parallel lines TV and UW. Then $\overline{ST}/\overline{TU} = \overline{SV}/\overline{VW}$.

By multiplying together the three sets of ratios just obtained, PQ and ST cancel out and $\dfrac{\overline{MN} \times \overline{PS}}{\overline{NO} \times \overline{TU}} = \dfrac{\overline{MP} \times \overline{SV}}{\overline{QR} \times \overline{VW}}$ or, rearranged,

$$\frac{\overline{SV}}{\overline{VW}} = \frac{\overline{MN} \times \overline{QR} \times \overline{PS}}{\overline{NO} \times \overline{MP} \times \overline{TU}} \qquad (14)$$

By construction, $\overline{MN} = L_{AB} \cos \angle$ (1), $\overline{QR} = L_{BC} \cos \angle$ (3), $\overline{PS} = L_{CA'} \cos \angle$ (5); $\overline{NO} = L_{AB} \cos \angle$ (2), $\overline{MP} = L_{BC} \cos \angle$ (4), and $\overline{TU} = L_{CA'} \cos \angle$ (6). Substituting these values in Eq. 14*a*, the ratio $\dfrac{\overline{SV}}{\overline{VW}}$ is found equal to u as expressed in Eq. 13. In other words, by dividing the scaled length of line SV by that of line VW in Fig. 4, the value of the ratio u is obtainable.

For infinitesimal changes in ρ_A and $\rho_{A'}$, $d\rho_{A'} = u\, d\rho_A$ and, for small changes of finite amount,

$$\Delta\rho_{A'} = u\, \Delta\rho_A \qquad (15a)$$

—approximately. To effect a solution of the given problem the two edges of the ground pyramid must be made equal—that is, in Fig. 3, LA′ must be made equal to LA. Thus,

$$\rho_A + \Delta\rho_A = \rho_{A'} + \Delta\rho_{A'} \qquad (15b)$$

In Eq. 15*b*, substituting for $\Delta\rho_{A'}$ its value from Eq. 15*a* and solving for $\Delta\rho_A$:

$$\Delta\rho_A = \frac{\rho_{A'} - \rho_A}{1 - u} \qquad (16)$$

It should be noted that usually when the control points have been properly chosen the angles, 1, 2, 3, 4, 5, and 6, will all be acute angles with positive cosines and thus u will be a negative quantity, and $\Delta\rho_A$ and $\Delta\rho_{A'}$ will be of opposite sign.

For use in graphical solutions (and, in fact, for analytical solutions except when a highly accurate solution is desired) $\Delta\rho_A$ and $\Delta\rho_{A'}$ may be found graphically in Fig. 4 by laying off SW equal in length to $\rho_{A'} - \rho_A$ before drawing the parallel lines UW and TV. Then line SV will be equal, to scale, to $\Delta\rho_{A'}$ and VW to $\Delta\rho_A$. The signs may be noted separately.

After finding the corrections to ρ_A and $\rho_{A'}$ as indicated, a corrected value of ρ_A may be found and a second solution for ρ_B, ρ_C, and $\rho_{A'}$ carried through in the same way as the first—that is, either graphically or analytically—to reach more accurate results than in the first. Unless a very precise solution is required, a third approximate solution will usually not be necessary. Any subsequent solution after the first may be checked approximately as ρ_B and ρ_C are found, because the corrections to the old values of these quantities to obtain the new ones may be found, from the value of $\Delta\rho_A$ used, by the relationships:

$$\Delta\rho_B = -\frac{\cos\angle(1)}{\cos\angle(2)}\Delta\rho_A \quad \text{(17a)}$$

and

$$\Delta\rho_C = -\frac{\cos\angle(3)}{\cos\angle(4)}\Delta\rho_B \quad \text{(17b)}$$

After two approximate solutions have been made a third one may be made, to give highly precise results, by using the values of u obtained in the first two solutions to compute an average value of this as ρ_A changes from its value obtained in the second solution to a predicted value to be obtained by a third solution. Thus u will change a certain amount as ρ_A changes from its first value to its second. Using the value of u from the second solution the value of ρ_A is calculated for a third solution. Assuming that, as ρ_A changes, u changes in proportion, a value of u may be calculated for the proposed third solution. Then the change in ρ_A may be recalculated by using a value of u which is an average of the value after the second solution and the value obtained in the manner mentioned for a third solution. Unless u is changing rapidly this value should lead to a value of ρ_A which is close to the true value.

Favorable and Unfavorable Positions of the Three Control Points.—As previously stated, control points should be well separated on the photograph, preferably located so as to form a nearly equilateral triangle when adjoining pairs are connected, with the principal point near the center of the triangle. Under these circumstances the angles 1 to 6, Fig. 3, will be acute angles whose cosines are of considerable size and not changing very rapidly. Furthermore, the value of u will be nearly unity and will not change rapidly as ρ_A is modified from one approximate solution to the next. If one of the angles 1 to 6 should be a right angle or nearly so, the solution would be much more unstable, because the cosine would be zero, or very small, and would be changing at a rapid rate with any modification of sides in effecting a solution. One side might be changed by a considerable amount without changing an adjoining one very appreciably. Thus small errors in the coordinates of the control points, or small errors in the solution, might result in large errors in the calculated lengths of the edges of the ground pyramid.

Coordinates of Station L.—Having determined the values of LA, LB, and LC, the next step in the solution is to obtain the coordinates, in space, of the camera station L. If X_L, Y_L, and Z_L represent these coordinates and if the formula for finding the distance between them is applied (see Eqs. 7):

$$(X_L - X_A)^2 + (Y_L - Y_A)^2 + (Z_L - Z_A)^2 - \overline{\text{LA}}^2 = 0 \quad \text{(18a)}$$

$$(X_L - X_B)^2 + (Y_L - Y_B)^2 + (Z_L - Z_B)^2 - \overline{\text{LB}}^2 = 0 \quad \text{(18b)}$$

and

$$(X_L - X_C)^2 + (Y_L - Y_C)^2 + (Z_L - Z_C)^2 - \overline{\text{LC}}^2 = 0 \quad \text{(18c)}$$

Since it is impractical to solve Eqs. 18 directly for the unknown X_L, Y_L, and Z_L, approximate methods must be used again. A graphical or analytical solution of the three-point problem may be used to obtain approximate values of X_L and Y_L. This solution utilizes the angles about the principal point on the picture as approximations of the corresponding angles in the map plane about the vertical point, or horizontal projection of the camera station. In the graphical method the three control points are plotted on a map by the use of their coordinates; after taking off the three lines oa, ob, and oc, from the photograph, on a piece of transparent paper, cloth, or celluloid, the transfer is slipped around on the map until the three lines pass through the corresponding mapped points. The principal point is then pricked through to give the approximate location, in plan, of the camera station from which to scale off the approximate values of X_L and Y_L. Those who have a calculating machine available and are familiar with the method of solving for the coordinates of a point by the three-point problem, working directly from the coordinates of the control points, very likely can make an analytical solution more quickly than they can the graphical one, unless the points have already been mapped. No very exact solution need be made (nor can it be made) in either case, since the horizontal angles are only approximately known.

A sufficiently accurate value of Z_L may be known from the height of flight or from that of an adjoining photograph in the same flight. If not thus known it may be calculated to a sufficient degree of accuracy in the following way by comparing distances on the map with corresponding ones on the photograph. The distances OA, OB, and OC from the camera station O, to each of the control points, in turn, are scaled on the map. Then, approximately,

$$Z_L - Z_A = \frac{\overline{\mathrm{OA}} \times f}{\overline{\mathrm{oa}}} \quad \text{(19a)}$$

$$Z_L - Z_B = \frac{\overline{\mathrm{OB}} \times f}{\overline{\mathrm{ob}}} \quad \text{(19b)}$$

and

$$Z_L - Z_C = \frac{\overline{\mathrm{OC}} \times f}{\overline{\mathrm{oc}}} \quad \text{(19c)}$$

Since the method is approximate there will be discrepancies in the values of Z_L obtained from Eqs. 19. An average may be taken.

Next, let $X_L = X_{LO} + \Delta X$, $Y_L = Y_{LO} + \Delta Y$, and $Z_L = Z_{LO} + \Delta Z$, in which X_{LO}, Y_{LO}, and Z_{LO} are the approximate values of the coordinates of the camera station just found and ΔX, ΔY, and ΔZ are corrections to these to obtain the true values X_L, Y_L, and Z_L. By substituting these values of X_L, Y_L, and Z_L in the three simultaneous equations already written and combining and simplifying numerical values three simultaneous equations in ΔX, ΔY, ΔZ are obtained, as follows:

$$a_1 \Delta X + b_1 \Delta Y + c_1 \Delta Z + \tfrac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2) + q_1 = 0 \quad \text{(20a)}$$

$$a_2 \Delta X + b_2 \Delta Y + c_2 \Delta Z + \tfrac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2) + q_2 = 0 \quad \text{(20b)}$$

and

$$a_3\,\Delta X + b_3\,\Delta Y + c_3\,\Delta Z + \tfrac{1}{2}\,(\Delta X^2 + \Delta Y^2 + \Delta Z^2) + q_3 = 0 \ldots \tag{20c}$$

In Eqs. 20,

$$a_1 = X_{LO} - X_A \ldots\ldots \tag{21a}$$

$$b_1 = Y_{LO} - Y_A \ldots\ldots \tag{21b}$$

$$c_1 = Z_{LO} - Z_A \ldots\ldots \tag{21c}$$

and

$$q_1 = \tfrac{1}{2}\,(a^2{}_1 + b^2{}_1 + z^2{}_1 - \overline{\mathrm{LA}}^2) \ldots\ldots \tag{21d}$$

Corresponding terms in Eqs. 20 are written similarly. If good approximations have been made to the true values of the coordinates of the camera station, ΔX, ΔY, and ΔZ will be relatively small quantities and the further solution will be simplified. However, by the approximate method about to be explained a solution for ΔX, ΔY, and ΔZ, accurate to a high degree of precision, may be made without excessive labor even though the approximation in coordinates is not close. To simplify Eqs. 20 assume that $\frac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2) = k$ and write

$$a_1\,\Delta X + b_1\,\Delta Y + c_1\,\Delta Z + k + q_1 = 0 \ldots\ldots \tag{22a}$$

$$a_2\,\Delta X + b_2\,\Delta Y + c_2\,\Delta Z + k + q_2 = 0 \ldots\ldots \tag{22b}$$

and

$$a_3\,\Delta X + b_3\,\Delta Y + c_3\,\Delta Z + k + q_3 = 0 \ldots\ldots \tag{22c}$$

Eqs. 22 are now to be solved for ΔX, ΔY, and ΔZ. The first step is to eliminate ΔZ between Eq. 22*a* and Eq. 22*b* and then between Eq. 22*a* and Eq. 22*c*, each time finding coefficients for the *k*-terms. This will result in two simultaneous equations in ΔX and ΔY in which there will be *k*-terms. Usually the coefficients of k will be small because the *c*-coefficients are likely to be nearly of the same magnitude, since Z_L is large compared with the differences in elevation between the control points. Next eliminate ΔY between the equations just found, again carrying through the coefficients of k. Neglecting k, solve for ΔX and then for ΔY and ΔZ by substituting back in the preceding equations. With these values of ΔX, ΔY, and ΔZ compute a value of k and then solve again, correcting this time for the value of k. This will result in a closer approximation to the values of ΔX, ΔY, and ΔZ. If considered necessary a new value of k can be found and the substitution made for a third solution, and similarly for other solutions, to approach values as precise as necessary. It will be found, however, that a rapid approach to true values is made by this method. The addition of these values to X_{LO}, Y_{LO}, and Z_{LO}, respectively, gives X_L, Y_L, and Z_L, which may be checked if desired by substituting back into Eqs. 18.

Let the angles between the vertical LV and each of the lines, LA, LB, and LC, in turn be designated M_A, M_B, and M_C, respectively. (Point V is the vertical intersection in the XY-plane). Then

$$\cos M_A = \frac{Z_L - Z_A}{\overline{\mathrm{LA}}} \ldots\ldots \tag{23a}$$

$$\cos M_B = \frac{Z_L - Z_B}{\overline{\mathrm{LB}}} \ldots\ldots \tag{23b}$$

and

$$\cos M_C = \frac{Z_L - Z_C}{\overline{\mathrm{LC}}} \quad \text{(23}c\text{)}$$

It will be necessary later to know the azimuths in the ground-survey system of the lines VA, VB, and VC, which may as well be found at this time. The azimuth of line VA, say α_A, may be found from the formula

$$\tan \alpha_A = \frac{X_A - X_V}{Y_A - Y_V} \quad \text{(24)}$$

and α_B and α_C, the azimuths of lines VB and VC, are found by similar formulas. As a check on the computed values of M_A, M_B, and M_C, find the length of VA, by the formula,

$$\overline{\mathrm{VA}} = \frac{X_A - XL_L}{\sin \alpha_A} = \frac{Y_A - Y_L}{\cos \alpha_A} \quad \text{(25)}$$

The lengths VB and VC are determined by similar formulas. Next compute M_A, M_B, and M_C by the formulas:

$$\tan M_A = \frac{\overline{\mathrm{VA}}}{Z_L - Z_A} \quad \text{(26}a\text{)}$$

$$\tan M_B = \frac{\overline{\mathrm{VB}}}{Z_L - Z_B} \quad \text{(26}b\text{)}$$

and

$$\tan M_C = \frac{\overline{\mathrm{VC}}}{Z_L - Z_C} \quad \text{(26}c\text{)}$$

Elements of Exterior Orientation.—It is now proposed to develop a method of finding the elements of exterior orientation by an application of position circles, thus finding the tilt and the swing of the photograph and the azimuth on the ground-survey system of the principal plane. Thus in Fig. 5, length Lo is the principal distance or (as it is usually called) the focal length *f*, whereas o is the principal point and v is the vertical point on the photograph. The principal line is ov. In the ground datum plane point V is the nadir point, or vertical point, and O is the intersection of the camera axis with the ground plane. The swing *s* is the plate azimuth of the principal line ov reckoned clockwise from the positive end of the *y*-axis on the photograph. Line VO is the intersection of the principal plane with the datum plane or ground plane and its azimuth (α_{VO}) reckoned, say, from the north, is an element of exterior orientation. One of the points of control, A, is photographed at point a and point A′ is the intersection of line LA with the datum plane. Similar points for the control points B and C are shown at b, c, B′, and C′. From calculations already made m_a, m_b; m_c, M_A, M_B, and M_C are known as well as the azimuth angles on the plate α_a, α_b, α_c and these on the ground α_A, α_B, and α_C. The solution of this problem requires the determination of the tilt of the photograph, *i*, which is equal to the angle oLv or OLV, the swing, *s*, and the azimuth of the principal plane, α_{VO}.

To facilitate this solution a sphere is taken with its center at the camera station L. Although the radius might be made any arbitrary length, in this case it is taken equal to the focal length. Hence, the plane of the picture will be tangent to the sphere at point o. The various lines and planes through point L may be considered to intersect the surface of the sphere and thus give points and arcs upon it. As yet the position of v is unknown because it has not been established with respect to point o; nor has the position of point O been established on the ground-control system. It is known, however, that on the surface of the sphere the distances in arc from point o to points a, b, and c as projected onto the sphere by the radial lines are equal, respectively, to m_a, m_b, and m_c and that point V on the sphere must lie at arc distances of M_A, M_B, and M_C from points a, b, and c, respectively. Therefore, if position circles are drawn on the sphere with points a, b, and c, respectively, as centers using in turn the arcs M_A, M_B, and M_C as radii, point V must lie at their intersection. To show these circles requires a projection similar to an azimuthal equidistant map projection with point o as the center, and with points a, b, and c placed in correct azimuthal relationships. The latter are in the plane of the picture and at distances from point o equal to their actual distances in arc from that point on the sphere. Fig. 6 is such a projection to be considered as showing relationships seen from the inside of the sphere. The lines oa, ob, and oc are drawn so as to reproduce their directions on the photograph and their lengths in arc (equal, respectively, to m_a, m_b, and m_c) are laid off to a certain linear scale, as for instance one millimeter equals one degree. Using the same scale, if arcs are struck with a, b, and c as centers and with M_A, M_B, and M_C,

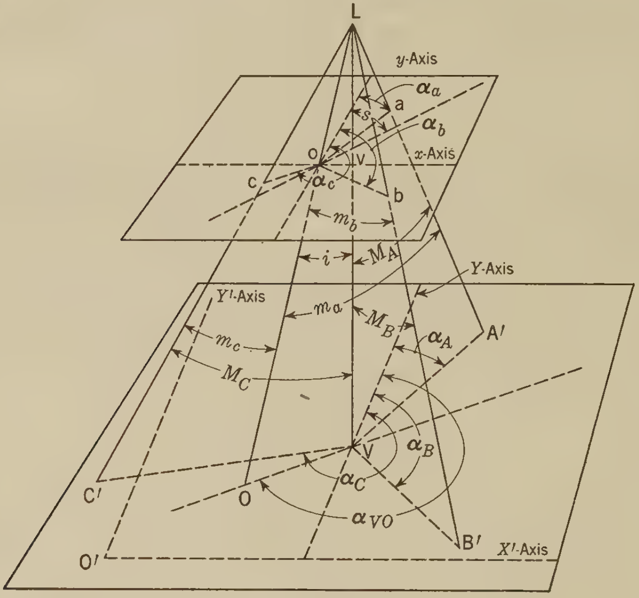

Fig. 5

respectively, as radii, these circles will be represented on the projection fairly accurately in the region of particular interest (the region in the vicinity of point o) and will thus fix, by their intersection, approximately the position of the vertical point v (see Fig. 6). However, for small values of the tilt such as are encountered in well-taken vertical photographs, the distance vo is short compared to lengths oa, ob, and oc, and neither its length nor its direction will be well established by this method of construction. For a better determination, the scale of the projection will be much enlarged to show only the area in the vicinity of points o and v and, therefore, with points a, b, and c, no longer available as centers of arcs. Thus in Fig. 7 by use of the coordinates of the points a, b, and c in the photograph, or in some other way, lay off the lines oa′, ob′, and oc′ to represent the lines radiating from point o to points a, b, and c. It is known that the position circle with its center at point a and with its radius of M_A cuts line oa′ at a distance equal to $m_a - M_A$ from point o. This intersection will be on the same side of point o as point a′ if $m_a - M_A$ is plus, but on the opposite side of point o from a′, and, therefore, on a′o extended, if $m_a - M_A$ is minus. Therefore (see Fig. 7) line od is laid off equal to $m_a - M_A$ to a large scale such as 1 mm = 1 min. In this illustrative example, od is laid off on line oa, since it is assumed that M_A is smaller than m_a; and at point d a perpendicular to line oa′ is erected. The perpendicular to od at d is the tangent to the position circle at that point and may be considered to represent approximately the position circle for a short distance on either side of this point. Similarly, line of is laid off along line ob′, and line og along line oc′ extended, equal in length, respectively, to $m_b - M_B$ and $m_c - M_C$. (If the difference such as $m_a - M_A$ is plus, it is laid off from point o in the direction of the point; whereas if it is minus, as in the case of line og, it is laid off in the opposite direction or on the line extended.) Perpendiculars to lines ob′ and oc′ are drawn at points f and g to represent the tangents to the position circles through those points. While the position circles, if they could be accurately drawn, would meet in a point, the perpendiculars will meet in pairs to give three points of intersection and what may be considered a triangle of error. By considering that the arcs would lie on the same side of the tangents that the control points

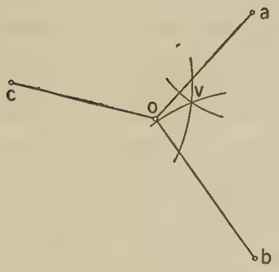

FIG. 6

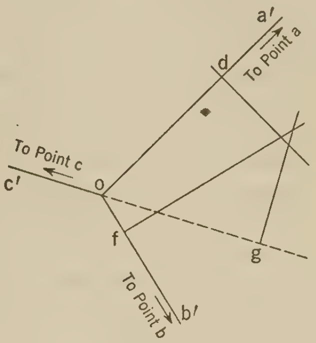

FIG. 7

do, one may spot the approximate location of point v. Then, by scaling off the length of line ov and its direction with respect to the y-axis, values of i and s may be obtained.

By a small amount of computation and further construction work, an accurate location of point v may be obtained. Thus it is known that for some little distance from the point of tangency of a circle the offset from the tangent to the circle varies closely as the square of the distance along the tangent; and is equal to $h^2/(2\,r)$, in which h is the distance from the point of tangency and r the radius of the circle. In this case the distance from the approximate location of point v to the tangent point may be scaled and the radius is M_A to the scale of the drawing, assumed herein to be 1 mm = 1 min. After the offsets have been computed, they may be laid off from their respective tangents in the vicinity of point v, care being taken to lay them off on the proper side of the tangent, and parallels to the tangents drawn through the extremities of the offsets. These should meet in a point or, if not, leave a very small triangle of error to fix point v in either case very closely. By scaling, the tilt should then be found to within a small fraction of a minute and the swing s to within a small fraction of a degree.

By considering the construction to be in the ground plane with the sphere tangent at the vertical point, a similar construction centering on point V gives a check solution for i and makes it possible to scale off the value of α_{VO}, which should be as accurate as the value previously found for s.

Mathematical Solution.—Of course graphical methods may be avoided in finding i, s, α_{ov}, and α_{VO} by using a mathematical method based on the application of spherical trigonometry. Consider the lines Lo, Lv, La, Lb, and Lc to be prolonged from the center of the sphere at point L, Fig. 5, until they intersect the surface of the sphere; and suppose these points to be connected with arcs of great circles to form the construction sketched in Fig. 8. The arcs ab, bc, and ac are equal, respectively, to β_{ab}, β_{bc}, and β_{ca}, the angles at the common vertex of the plate and the ground pyramid. Solving this triangle, formed by points a, b, and c, one can find its interior angles. Also arcs oa, ob, and oc are equal, respectively, to the known angles m_a, m_b, and m_c. Angles aob, boc, and aoc can be found from the plate azimuths. By the sine law one may then solve by spherical trigonometry to find the angles at points a, b, and c in the triangles aob, boc, and aoc. Similarly, knowing that arcs av, bv, and cv are equal,

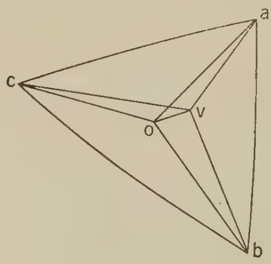

FIG. 8

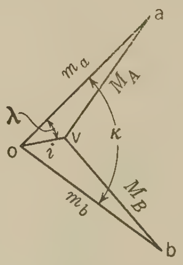

FIG. 9

respectively, to M_A, M_B, and M_C, and finding the angles avb, bvc, and avc from the azimuths on the ground system of lines VA, VB, and VC, one may solve for the angles at points a, b, and c, in the triangles avb, bvc, and avc. Then in the triangle avo, for instance, the sides av and ao are known; the angle vao is the difference between angles bav and bao. A solution may then be made for the length vo or i and for the angles at points v and o. From these angles the azimuth of line ov, or s, can be found by adding the angle aov to the plate azimuth of line oa; and the azimuth of line vo on the ground-survey system may be obtained by subtracting the angle avo from the azimuth of line VA on the ground system. The solution may be checked by similar use of the triangles bvo and ovc, one or both. These calculations are rather long and tedious and are not likely to check very well unless extended to a number of decimal places either by natural functions or by logarithms, since the angles at points a, b, and c in the triangles oav, obv, and ovc are small.

The following method may be adopted to check the values of i, s, and α_{VO} and to obtain them to a higher degree of precision. Consider two of the three triangles involving the line vo (say, aov and bov with the side vo or i, Fig. 9, in common). The sides ao, bo, av, and bv are known accurately to be equal to m_a, m_b, M_A, and M_B, respectively. The value of $\overline{\text{vo}}$ is known approximately from the graphical solution. For simplicity let the angle boa, which is known, be designated by κ and the angle aov, which is known only approximately, be designated by λ; then:

$$\cos M_A = \cos m_a \cos i + \sin m_a \sin i \cos \lambda \qquad (27a)$$

and

$$\cos M_B = \cos m_b \cos i + \sin m_b \sin i \cos (\kappa - \lambda) \qquad (27b)$$

By solving each equation for sin i and equating values,

$$\sin i = \frac{\cos M_A - \cos m_a \cos i}{\sin m_a \cos \lambda} = \frac{\cos M_B - \cos m_b \cos i}{\sin m_b \cos (\kappa - \lambda)} \qquad (28a)$$

which may be written

$$\frac{\cos (\lambda - \kappa)}{\cos \lambda} = \frac{\sin m_a (\cos M_B - \cos m_b \cos i)}{\sin m_b (\cos M_A - \cos m_a \cos i)} \qquad (28b)$$

Since $\cos (\lambda - \kappa) = \cos \lambda \cos \kappa + \sin \lambda \sin \kappa$:

$$\cos \kappa + \tan \lambda \sin \kappa = \frac{\sin m_a (\cos M_B - \cos m_b \cos i)}{\sin m_b (\cos M_A - \cos m_a \cos i)} \qquad (28c)$$

Solving for tan λ:

$$\tan \lambda = \left[\frac{\sin m_a (\cos M_B - \cos m_b \cos i)}{\sin m_b (\cos M_A - \cos m_a \cos i)} - \cos \kappa \right] \frac{1}{\sin \kappa} \qquad (29)$$

From the graphical solution, i is known approximately. Therefore, since i is small and the cosine changes but slowly for changes in the angle, cos i may be considered to be known closely. All other quantities involved in the right-hand member of Eq. 29 are known; and by substitution in this equation an approximate value of λ can be determined, a value accurate enough for all practical purposes. Eqs. 28 can then be solved for i from its sine, giving a close approximation to the true value of i. The swing s may be obtained by adding λ to the direction of line ao. By the sine law the angle avo may be found, and this value, subtracted from the azimuth of line VA, will give α_{VO}. Thus, i, s, and α_{VO} may be determined to a high degree of precision by computations supplementing the graphical solution.

Azimuths from Measurements of Photographs.—Another application of "position circles" is in finding azimuths from the vertical point on the ground as functions of directions from the principal point to points on the photograph. If this is done for a point showing in the overlapping part of two photographs whose elements of orientation have been found, the data may be used to compute the coordinates and elevation of the point. Thus, let position q be any point on a vertical photograph. Suppose that the coordinates of this point on the photograph are x_q and y_q. The direction of line q measured on the photograph may then be scaled, or computed from the relation that

$$\tan \alpha_q = \frac{x_q}{y_q} \qquad\qquad (30)$$

Solve also for the angle m_q at the vertex of the pyramid.

On the diagram from which line ov, or i, was originally determined, or on a new diagram such as Fig. 10, lay off the line oq′ in the direction of point q,

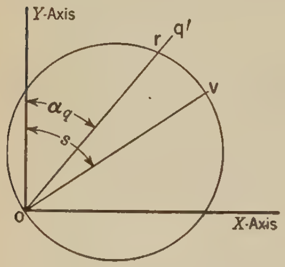

Fig. 10

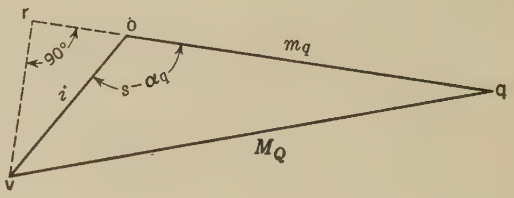

Fig. 11

making the angle between line oq′ and the y-axis equal to α_q. Also draw the line ov (if it has not already been drawn), laying off its length to the same scale that was formerly used in determining line ov or using a similar large scale such as 1 mm = 1 min or arc. If many points are to be dealt with, a circle should now be constructed on line ov as a diameter. The chord formed by the intersection of the line (prolonged if necessary) and the circle (line or in Fig. 10) is scaled, as is also the chord rv—say, in minutes of arc to the same scale as line ov. The distance ro gives the approximate difference between M_Q and m_q. A more accurate value of this difference may be found by correcting by an amount equal to $\frac{\overline{\text{vr}}^2}{2\,M_Q}$ in which $\overline{\text{vr}}$ and M_Q are both expressed in minutes of arc. The correction in this case should be added to $\overline{\text{ro}}$ to derive a more accurate value of the difference $M_Q - m_q$. By adding this difference to m_q the distance M_Q may be found. In the triangle voq, Fig. 11,

$$\frac{\sin \mathrm{v}}{\sin (s - \alpha_q)} = \frac{\sin m_q}{\sin M_Q} \qquad (31a)$$

and

$$\frac{\sin \mathrm{q}}{\sin (s - \alpha_q)} = \frac{\sin i}{\sin M_Q} \qquad (31b)$$

The angles v and q can be determined by Eqs. 31.

An alternative solution, which may be made with the slide rule or computing machine, is to consider the right triangle rvq, Fig. 11, and to use the formula—

$$\sin \mathrm{q} = \frac{\sin \overline{\mathrm{rv}}}{\sin M_Q} \qquad (32)$$

—to find the value of q. The spherical excess, in minutes, (e) of triangle voq may be found approximately by using the formula,

$$e = \frac{\pi}{21{,}600} \times \overline{\mathrm{rv}} \times \overline{\mathrm{mq}} \qquad (33)$$

in which $\overline{\mathrm{rv}}$ and m_q are to be expressed in minutes of arc. Then $<$ ovq is the difference between $(180 + e)$ and $(s - \alpha_q + <\mathrm{ovq})$. The azimuth of the arc Vq is obtained by adding the angle ovq to the azimuth α_{vo} of arc VO. If a map is under construction, after the vertical points have been plotted, radial lines to points such as q can be laid off with the protractor, or the coordinates of points suitably placed for intersections can be computed. By scaling or computing, the horizontal distance from the located points to the camera stations can be found and, by using angles such as M_Q, the elevation may be determined either graphically or by calculation. For the point q the difference in elevation between it and the camera station is equal to the horizontal distance times the cotangent of M_Q. This completes the determination of the coordinates of point q.

NUMERICAL EXAMPLE

To demonstrate the application of theory by a numerical example, a 7-in. by 9-in. photograph is selected for the purpose of determining the elements of orientation. This print designated ART-3-14 was one of a series taken for the Agricultural Adjustment Administration, U. S. Department of Agriculture, which covers an area well controlled by ground surveys. On this photograph three control points A, B, and C, well located for the purpose, were chosen; their coordinates, referred to axes through the principal point (center) of the photograph, were measured by a method that was fairly precise. These coordinates, x and y, were determined to hundredths of a millimeter. The ground coordinates came from triangulation and traverse surveys for the X and Y, and from spirit leveling for the Z. They are expressed in feet, with the Z-coordinates giving the elevations above sea level, as shown in Table 1. The lengths of AB, BC, and CA, and the horizontal and slope distances in Table 1, were computed by aid of a computing machine from the survey coordinates by use of the formulas in the first part of this paper (see Eqs. 7 and 8).

TABLE 1.—COORDINATES OF CONTROL POINTS; HORIZONTAL AND SLOPE DISTANCES

Description	Point A	Point B	Point C
Ground Coordinates, in Feet:			
X	35,900.2	27,420.5	36,541.0
Y	5,890.9	15,924.9	16,481.3
Z (elevations above sea level)	1,238.4	1,384.6	1,348.9
Map Coordinates, in Millimeters:			
x	−26.88	+102.69	−33.11
y	+74.57	− 70.70	−83.16
Horizontal distances, in feet	13,137.23	9,137.46	10,609.77
Slope distances, in feet	13,138.03	9,137.53	10,610.34

Calculations are now made from the coordinates of the points on the photograph and the focal length f, which is known to be 209.55 mm, to determine the azimuths (α in line 2, Table 2) from the principal point to the control points.

TABLE 2.—AZIMUTHS FROM PRINCIPAL POINT TO CONTROL POINTS; ANGLES AT VERTEX

Line	Quantity	Point a	Point b	Point c
1	Tan $\alpha = x/y$	0.36046668	1.45247525	0.39814815
2	Azimuth, α	340° 10′ 39″	124° 32′ 48″	201° 42′ 36″
3	$x^2 + y^2$	6,283.2193	15,543.7261	8,011.8577
4	Distance, $\sqrt{x^2 + y^2}$	79.267	124.675	89.509
5	$x_2 - x_1$	+129.57	−135.80	+6.23
6	$y_2 - y_1$	−145.27	− 12.46	+157.73
7	$l^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$	37,891.7578	18,596.8916	24,917.5658
8	Length, l	194.655	136.370	157.853
9	$x^2 + y^2 + f^2$	50,194.42	59,454.93	51,923.06
10	Slope distance ($\sqrt{\text{line 9}}$)	224.041	243.834	227.866
11	Sin m	0.353806	0.511311	0.392814
12	Tan m	0.378272	0.59465	0.427149
13	Angle m	20° 43′ 13″	30° 45′ 04″	23° 07′ 47″

The distances ab, bc, and ca on the picture, and the slope distances La, Lb, and Lc, are given in lines 8 and 10, Table 2. The angles between Lo and each of the lines La, Lb, and Lc, in turn, are given in line 13, Table 2. As a check, these angles are found from their tangents and checked by their sines.

The vertex angles aLb, bLc, and cLa, Fig. 2, are now computed by applying Eqs. 5, in turn, to each of the three triangles aLb, bLc, and cLa, the sides of which are known. The foregoing calculations do not differ greatly from those used by Professor Church[4]. Thus, by Eq. 5*a*:

$$\cos \beta_{ab} = \frac{-37{,}891.7578 + 50{,}194.4218 + 59{,}454.9286}{2 \times 224.041 \times 243.834}$$

$$= \frac{71{,}757.5926}{109.257.6264} = 0.65677422;$$

$\beta_{ab} = 48° 56' 44''.46$; and $\sin \beta_{ab} = 0.75408730$. Similarly, by Eq. 5*b*, $\cos \beta_{bc} = 0.83494086$; $\beta_{bc} = 33° 23' 25''.15$; and $\sin \beta_{bc} = 0.55033968$. Finally, by Eq. 5*c*, $\cos \beta_{ca} = 0.75610098$; $\beta_{ca} = 40° 52' 42''.00$; and $\sin \beta_{ca} = 0.65445494$.

The next step in the solution is to find values of LA, LB, and LC, and for this purpose it is necessary to have an approximate value of one of these, say, of LA $= \rho_A$, to use in the solution. This approximate value may be found in any one of several ways. A simple one is to consider the lines ab, bc, and ca to be parallel, respectively, to AB, BC, and CA, and to solve by proportion considering the face triangles to be similar. In this example a somewhat more precise method, partly graphical and partly by calculation, which will not be described, was used. The approximate value of LA found by this method was 15,234 ft. Using this value the calculations shown below were made. Referring to Fig. 3: AB $= L_{AB} = 13{,}138.03$; BC $= L_{BC} = 9{,}137.53$; CA $= L_{CA'} = 10{,}610.34$. Substituting these values of L and β in Eqs. 9: $\sin \angle (2) = 0.87439016$ and $\angle (2) = 60° \; 58' \; 22''.38$; $\angle (1) = 180° - \beta_{ab} - \angle (2) = 70° \; 04' \; 53''.16$; and $\sin \angle (1) = 0.94017777$. By Eq. 9*c*, $\rho_B = 16{,}380.18$. By similar adaptations of Eqs. 9 values of ρ_C and $\rho_{A'}$ are found to be 15,169.91 and 15,213.49, respectively. As this is done, angles 3, 4, 5, and 6 are found to be, respectively, 66° 00′ 58″.40, 80° 35′ 36″.45, 69° 46′ 52″.82, and 69° 20′ 25″.18. By substitution of values of the cosines of angles 1 to 6 in Eq. 13 it is possible to compute u as $-0.7021 \times 2.4873 \times 0.9795 = -1.7105$, which, by Eq. 15*a*, is equal to $\frac{\Delta\rho_A'}{\Delta\rho_A}$.

As stated in connection with Eq. 15*b*, $\Delta\rho_{A'}$ and $\Delta\rho_A$ are the corrections to be applied to $\rho_{A'}$ and ρ_A, respectively, to make them equal; thus, $15{,}234 + \Delta\rho_A = 15{,}213.49 + \Delta\rho_{A'}$ and $\Delta\rho_A = \Delta\rho_{A'} - 21.51$. As computed by Eq. 15*a*, $\Delta\rho_{A'} = -1.7105\,\Delta\rho_A$; and, thus, $2.7105\,\Delta\rho_A = -21.51$ and $\Delta\rho_A = -7.567$. Also $\Delta\rho_B = -0.7021\,\Delta\rho_A = +5.313$, $\Delta\rho_C = +1.7463\,\Delta\rho_A = -13.214$, and $\Delta\rho_{A'} = -1.7105\,\Delta\rho_A = +12.943$. Applying these corrections to the previously computed values of ρ, the following corrected values of these quantities are: $\rho_A = 15{,}226.43$, $\rho_B = 16{,}385.49$, $\rho_C = 15{,}156.70$, and $\rho_{A'} = 15{,}226.43$.

A recalculation of these quantities (which is not shown herein for lack of space) showed that these results were practically correct, the value of ρ_B checking exactly, that of ρ_C within a tenth, and that of $\rho_{A'}$ within two one hundredths. These values have been computed to more significant figures than would be warranted in practice in order to show that the solution may be extended to a high degree of precision by the method outlined.

To set up the equations for solving for the coordinates in space, X_L, Y_L, and Z_L of the camera station, it is necessary to have approximate values of these coordinates. A graphical solution of the three-point problem, under the assumption that the photograph is vertical, gives approximate values of X_L and Y_L. The third coordinate, Z_L, may be found by use of the *m*-angles, and values of ρ to obtain three values of Z_L, which are then averaged to derive the value to be used. In this case a somewhat more accurate method was used but will not be explained.

The approximate values used and the resulting equations and solution for finding the corrections to these are presented in Table 3(*a*).

TABLE 3.—ILLUSTRATIVE EXAMPLE; TABULATION OF CALCULATIONS

Line	Quantity	Point A	Point B	Point C
	(a) SOLUTION OF EQS. 18 (X_L = 34,456.0, Y_L = 11,249.5, AND Z_L = 15,419.8)			
1	$X_L - X$	− 1,444.2	+ 7,035.5	− 2,085.0
2	$Y_L - Y$	+ 5,358.6	− 4,675.4	− 5,231.8
3	$Z_L - Z$	+14,181.4	+14,035.2	+14,070.9
4	ρ	15,226.43	16,385.49	15,156.59
5	$(X_L - X)^2$	2,085,713	49,498,260	4,347,225
6	$(Y_L - Y)^2$	28,714,594	21,859,365	27,371,731
7	$(Z_L - Z)^2$	201,112,106	196,986,839	197,990,227
8	Σ	231,912,413	268,344,464	229,709,183
9	$\Sigma\rho^2$	231,844,170	268,484,282	229,722,220
10	Difference	+68,243	−139,818	−13,037
	(b) SOLUTION OF EQS. 23, USING CORRECTED COORDINATES, X_L = 34,463.2, Y_L = 11,245.2, Z_L = 15,419.7			
1	$X_L - X$	− 1,437.0	+ 7,042.7	− 2,077.8
2	$Y_L - Y$	+ 5,354.3	− 4,679.7	− 5,236.1
3	$Z_L - Z$	+14,181.3	+14,035.1	+14,070.8
4	ρ	15,226.43	16,385.49	15,156.59
11	Cos M	0.931361	0.856561	0.928362
12	Angles M	21° 21′ 07″	31° 04′ 03″	21° 49′ 09″
	(c) POSITION OF THE VERTICAL POINT v ON THE PHOTOGRAPH AND OF POINT V ON THE MAP			
	Angle m:			
13	Degrees	20° 43′ 12″	30° 45′ 04″	23° 07′ 47″
14	Minutes	1,243.2	1,845.1	1,387.8
	Angle M:			
15	Degrees	21° 21′ 07″	31° 04′ 03″	21° 49′ 09″
16	Minutes	1,281.1	1,864.0	1,309.1
	$m - M$:			
17	Degrees	−0° 37′ 55″	−0° 18′ 59″	+1° 18′ 38″
18	Minutes	−37.9	−19.0	+78.6
	Ground Coordinates:			
19	h (min)	79	85	34
20	Offset, $h^2/(2\ M)$	2.4	2.0	0.4
	Map Coordinates:			
21	h (min)	77	84	38
22	Offset, $h^2/(2\ m)$	2.4	1.9	0.5

TABLE 4.—COMPUTATION OF CORRECTIONS BY EQS. 22

Eq.	ΔX	ΔY	ΔZ	k	Constant
A	−1,444.2	+5,358.6	+14,181.4	+1	+34,122
B	+7,035.5	−4,675.4	+14,035.2	+1	−69,909
C	−2,085.0	−5,231.8	+14,070.9	+1	− 6,519
1	−0.10184	+0.37786	+1	+0.000071	+ 2.4061
2	+0.50128	−0.33312	+1	+0.000071	− 4.9810
3	−0.14818	−0.37182	+1	+0.000071	− 0.4633
4	+0.60312	−0.71098	0	0	− 7.3871
5	+0.64946	+0.03870	0	0	− 4.5177
6	+1	−1.17885	0	0	−12.2482
7	+1	+0.05958	0	0	− 6.9562
8	0	+1.23843	0	0	+ 5.2920

The equations A, B, and C, Table 4, are the original Eqs. 22 from which Eqs. 1, 2, and 3 are derived by dividing each in turn by its ΔZ-coefficient. Eq. 4, Table 4, is obtained by subtracting Eq. 1 from Eq. 2; and Eq. 5 is similarly obtained by subtracting 3 from 2. This eliminates ΔZ. A similar elimination of ΔX gives Eq. 8, which may be solved for ΔY. Substitutions in Eqs. 6 and 1 of Table 4 give ΔX and ΔZ; thus: $\Delta Y = \frac{-5.2920}{1.23843} = -4.273$; $\Delta X = +12.248 - 5.037 = +7.211$; and $\Delta Z = -2.406 + 1.615 + 0.734 = -0.057$.

Since k is equal to $\frac{1}{2}(\Delta X^2 + \Delta Y^2 + \Delta Z^2)$ these terms are negligible in this case, and have been neglected. By making a solution without them and then computing a value of k to be substituted, a fairly exact solution is possible even when ΔX, ΔY, and ΔZ are large.

By applying the corrections just found to the approximate values of the coordinates, the following final values are found for the camera station L: $X_L = 34{,}463.2$; $Y_L = 11{,}245.2$; and $Z_L = 15{,}419.7$.

The angles, M_A, M_B, and M_C, which the lines LA, LB, and LC make with the vertical, are computed by Eq. 23, as shown in Table 3(*b*). The values of X_L, Y_L, and Z_L may be checked as was done in this case by applying the formulas for computing distances from coordinate difference (Eqs. 7 and 18) to obtain values of LA, LB, and LC from the quantities in Table 3(*b*). These calculations are not shown.

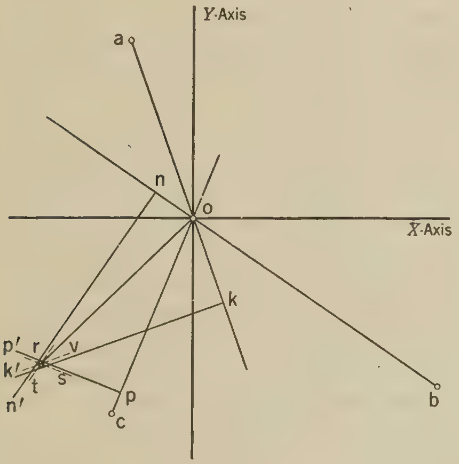

FIG. 12

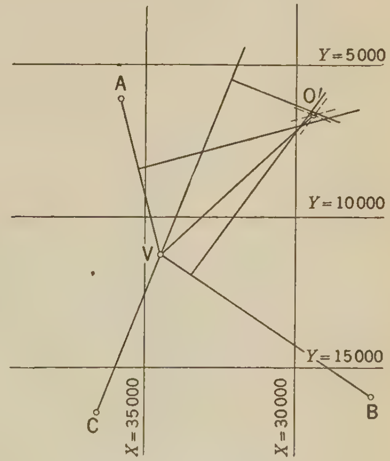

FIG. 13

The vertical point of the photograph is now found in Fig. 12, which is drawn similarly to Fig. 7. The distances ok, on, and op are laid off equal to $m_a - M_A$, $m_b - M_B$, and $m_c - M_C$, respectively, using a scale of 1 mm = 1 min of arc. In Fig. 12, the three perpendiculars kk′, nn′, and pp′ instead of meeting in a point form, by their intersections, a triangle rst that may be called a triangle of error or, more accurately, the "triangle of the tangents." It is known that v, the point sought, lies in or near this triangle in such a position that its distances from the perpendiculars kk′, nn′, and pp′ are proportional to the squares of the distances from the lines oa, ob, and oc, respectively. With this in mind a point was spotted as a trial position of v. (This is not shown in Fig. 12 but was near the position shown for v.) By the method previously explained these offsets are calculated and laid off from their respective lines to locate some parallel lines, shown dotted, which should meet at the required point, v. In this case a small triangle of error was found, within which the point v was located by estimation. The distance ov was scaled and found to be 86.8 min or 1° 26′.8, which is the tilt i. The direction of ov, clockwise from the y-axis, was found by scaling to be 225° 55′. This is the swing.

The azimuth of the principal plane is found by aid of Fig. 13, which represents a map upon which the positions of A, B, C, and V are plotted by their coordinates to a scale of 1 in. = 1,000 ft. By a construction similar to that in Fig. 12, the point O′ where the camera axis prolonged would meet the map plane is found. The scaled length O′V gives the tilt, and the azimuth of VO′ (from the south in this case) gives the azimuth of the principal plane. The value of the tilt was found to be 1° 26′.8 (checking that previously determined) and the azimuth of the principal plane was found to be 227° 40′.

As an indication of the precision of the method of Table 3(c) it may be stated that an analytical method gave the following values: i = 1° 26′ 47″; swing, s = 225° 59′ 35″; and the azimuth of the principal plane, α_{VO} = 227° 24′ 23″. Thus the tilt checked exactly, the swing within 5 min, and the azimuth of the principal plane within 15 min.

Eq. 29 will now be applied to find more exact values of the swing and of the azimuth of the principal plane, and Eq. 28a to obtain a more exact value of the tilt. The use of both of these equations affords a partial check on the numerical work.

TABLE 5.—DATA FOR THE SOLUTION OF EQ. 29

Symbol	Angle	Sine	Cosine
m_a	23° 07′ 47″	0.392814	0.919618
M_A	21° 49′ 09″	0.371678	0.928362
m_b	20° 43′ 13″	0.353806	0.935319
M_B	21° 21′ 07″	0.364096	0.931361
κ	138° 28′ 03″	0.663045	−0.748580
i	1° 26′ 48″[a]	. . . .	0.999681

[a] Approximately

It is known that line ov lies between lines oc and oa in this case; and, therefore, κ is the angle between the latter two lines, or is equal to 138° 28′ 03″, the difference between the plate azimuths of the lines oc and oa, previously calculated. The angle λ is measured clockwise from the line oc. In applying Eqs. 28a and 29 the subscripts a, b, A, B are to be construed as applying to the points that hitherto have been called, respectively, c, a, C, and A. Substituting, in Eq. 29, appropriate values from Table 5:

$$\tan \lambda = \left[\frac{0.392814\ (0.931362 - 0.935021)}{0.353806\ (0.928362 - 0.919325)} + 0.748580 \right] \frac{1}{0.663045} = 0.451161.$$

Therefore: λ = 24° 16′ 59″; $\sin \lambda = 0.411245$; $\cos \lambda = 0.911525$; $\kappa - \lambda$ = 114° 11′ 04″; and $\cos (\kappa - \lambda) = -0.409675$. Similarly, for the solution of Eq. 28a: By the first equality—

$$\sin i = \frac{0.009037}{0.358060} = 0.025239 \text{ and } i = 1° 26′ 47″$$

and, by the second equality—

$$\sin i = \frac{-0.003659}{-0.144945} = 0.025244$$

The latter value checks the former value, although the first of the two expressions in Eq. 28*a* is probably the more exact since it is derived from larger quantities. The swing is found by adding the angle λ to the plate azimuth of line oc and is thus found to be 225° 59′ 35″.

It remains to find the azimuth of the principal plane. This may be done by solving for the angle at point v in the triangle ocv by using the sine law and then subtracting this angle from the azimuth of line VC. The latter azimuth is found by the coordinate differences between the *X*-ordinate and *Y*-ordinate of the vertical point V and those of point C. These coordinates of point V are the same as those of the camera station L.

Thus, $\text{sine } \overline{\text{ovc}} = \frac{\sin \lambda \sin m_a}{\sin M_A} = 0.434631$ and $\angle \overline{\text{ovc}} = 154° 14' 17''$. Tan $\alpha_{VC} = \frac{X_C - X_V}{Y_C - Y_V} = \frac{+2{,}077.8}{+5{,}236.1} = 0.396822$ and α_{VC}, the azimuth of line VC, = 21° 38′ 40″. By subtracting the angle ovc from α_{VC} the azimuth of the principal plane is found to be 227° 24′ 23″. Thus, as previously stated, the tilt found by this more exact method checks the value found by the graphical method to 0.1 min, the swing checks within 5 min, and the azimuth of the principal plane within 15 min.

SUMMARY

It is believed that the main contribution of this paper lies in the use of the cosines of the base angles of the ground pyramid to obtain a rapid solution in finding the lengths of the edges of this pyramid, and in the application of "position circles" in fixing the position of the vertical point, first on the photograph and then on the map. Although the method of solving equations of the second degree such as those arising in finding the coordinates of the camera station by approximations, using Taylor's theorem as a basis, is well known, it is thought that the use of a constant *k* to represent the sum of the squares of the corrections is an innovation. The special application of the methods of spherical trigonometry to find the tilt, the swing, and the azimuth of the principal plane, it is believed, also has some merit.

APPENDIX. NOTATION

The following letter symbols are introduced in this paper. Discussers are requested to adapt their comments to these letters to avoid confusion of concept. In general, ground-point locations and line designations are not italicized, single letters in italics being used to designate quantities that can take a numerical equivalent. In designating points or lines small letters refer to locations on the map or photograph and capital letters are the corresponding points or lines on the ground

a = a mathematical coefficient (with b and c in Eqs. 20, 21, and 22);
b = a coefficient (see a);
c = a coefficient (see a);
e = spherical excess;
f = focal length of a camera;
h = a distance along a tangent to a circle, measured from the point of tangency;
i = tilt angle of a photograph;
k = a substitution constant (see Eq. 22);
L = length between traverse points measured on the ground;
l = length between traverse points measured on the map or photograph, corresponding to L;
M = angle to any ground object from a vertical through the lens center (see Fig. 5);
m = angle to any map or photograph object, measured from the plate perpendicular, corresponding to M (see Fig. 1);
q = a substitution constant (see Eq. 22);
r = radius of a circle;
s = swing, or angle made by the principal line, with the Y-axis;
u = a constant (see Eq. 12);
X = variable distances parallel to the X-axis, as observed on the ground; ΔX = correction to be applied to an X-value;
x = variable distances on a map or photograph corresponding to X;
Y = variable distances parallel to the Y-axis, as observed on the ground; ΔY = correction to be applied to a Y-value;
y = variable distances on a map or photograph corresponding to Y;
Z = variable value of height or ground elevation; ΔZ = correction to be applied to an elevation;
α = azimuth from the y-axis in the xy-plane, or from the Y-axis in the XY-plane, the direction line being denoted by appropriate subscript;
β = face angles of a triangular pyramid, the particular face being shown by appropriate subscript;
κ = an angle boa (Fig. 8) in a plane that is tangent to a sphere at point o;
λ = an angle aov (Fig. 8) in a plane that is tangent to a sphere at point o; and
ρ = radial distances from the lens center to ground points designated by appropriate subscripts.

DISCUSSION

Earl Church,[5] Assoc. M. ASCE.—Although this paper is entitled "Space Resection Problems in Photogrammetry," in reality it gives a solution of the space orientation problem (finding certain quantities or elements which serve to specify uniquely the orientation of the aerial photograph in space) as well as of the space resection problem (finding the survey space coordinates of the exposure station). These two problems are often treated together, as has been done by Professor Underwood.

In this paper the space resection problem has been approached by the often tried but seldom used pyramid method—determining the desired position of the exposure station by the lengths of the pyramid edges. The particular solution of this problem is very interesting, especially the use of Eq. 13 for finding the ratio u representing $d\rho_{A'}/d\rho_A$.

The method has two detractive features: (1) The solutions of the triangles forming the pyramid faces are tedious, inasmuch as they must always be made twice and often three times; and (2) the completion of this phase of the problem gives only the corrected lengths of the pyramid edges, still leaving the desired survey coördinates of the exposure station undetermined.

The part of the computation for finding these coordinates, simple as it is in principle, involving only the solution of the three simultaneous Eqs. 18, is, of course, troublesome in practice. Professor Underwood proposes a solution by approximations involving two preliminary computations, one consisting of the three-point problem in a plane and the other of the simple scale problem of photogrammetry. He then solves Eqs. 18, by the use of k as explained in the text following Eqs. 22 and cited in the "Summary." Possibly the straightforward algebraic solution of Eqs. 18 (of three spheres) with no preliminary computations, might even be more time saving than this method of approximations.

In either case, Professor Underwood's solution of the space resection problem is interesting; and, although perhaps somewhat longer than the usual space-analytics method, it is certainly not long enough to be impracticable.

Then space orientation is considered. The solution shows an interesting method of determining both the intersection of the photo plane with the vertical line through the exposure station and the survey position of the intersection of the camera axis with the ground plane. The former obviously determines at once the tilt and the swing, and the latter determines indirectly the azimuth of the principal plane of the photograph. The graphical determination of these two points, despite the large-scale drawings (Figs. 12 and 13), leaves something to be desired. What appear to be "triangles of error" in these graphs, as Professor Underwood states, must not be regarded as triangles of error, as far as errors in either control surveying or photographic measuring are concerned. They are triangles of error in either the analytical or the graphical phases of the computation itself.

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[5] Prof. of Photogrammetry, College of Applied Science, Syracuse Univ., Syracuse, N. Y.

On the whole, Professor Underwood's solution of the space orientation problem is of considerable interest because of the manner in which it attacks essential features of the problem. However, instructive as it is, the method compares unfavorably with several equally precise space-analytics methods in both simplicity and time required for the computations.

Professor Underwood's illustrative computation, although entirely correct, scarcely does justice to his method. The initial values for the desired quantities in his solution, obtained by methods of approximation, are much nearer to the correct values than would ordinarily be available in actual practice. This result might cause suspicion regarding convergence of the solution, should less accurate initial values be used. However, in the writer's experimental work with Professor Underwood's method, the solutions have been found to converge very satisfactorily under far less favorable conditions than those shown in the paper.

It must always be remembered that only by consideration of all the various solutions advanced can the most practical methods be evolved for handling these complex problems in practice. Professor Underwood's paper, therefore, deserves thorough reading and careful consideration by all photogrammetrists.

RALPH O. ANDERSON,[6] Assoc. M. ASCE.—The noteworthy fact about the work reported by Professor Underwood is the solution of the space equations without resorting to partial differential calculus (Eqs. 22). The implications of the foregoing statement are of technical importance in more ways than one. The advantages thus gained are as follows: High tilt in high relief may be computed with the same amount of work as that required for low tilt in low relief; and the method is considerably more simple in both derivation and application than the calculus method.[7] The first innovation consists of developing the ground pyramid for the purpose of determining the lengths of the three pyramid edges (LA, LB, and LC, Fig. 1). The suggested method will yield the correct lengths when the calculus method of adjustment is applied.

However, the graphical method will yield fairly close results. It was found that with careful drafting these pyramid edge lengths could be determined graphically within 10 ft. The errors were found to be 5 ft, − 4 ft, and 10 ft for the three edges. For example, lay off the sloping ground lengths on three transparent strips to some convenient scale, such as 1 in. equals 1,000 ft. Starting at a random point on leg A determine length A′ by fitting the sloping ground lengths between their respective pyramid edges. Repeat the operation using the starting length (leg A) equal to $\frac{1}{2}$ (LA + LA′), and repeat until LA equals LA′. This repetition is quite simple and can be done rapidly. Errors as great as 10 ft should not cause more than a 3-min tilt error. This is quite permissible in photogrammetry.

It was also found unnecessary to assume that k vanishes to zero in Eq. 22. Starting with a graphically determined exposure station expressed as a function of the principal point (letting k equal zero): $\Delta X = 252.22$, $\Delta Y = 262.20$, and $\Delta Z = -6.52$. The resulting exposure station coordinates are: $X_L = 34{,}463.22$, $Y_L = 11{,}245.20$, and $Z_L = 15{,}424.48$. Repeating the solution and letting

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[6] Mathematician II, TVA, Chattanooga, Tenn.

[7] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, 1944, Chapter XII, p. 536.

$k = 66{,}964$: $X_L = 34{,}463.20$, $Y_L = 11{,}245.25$, and $Z_L = 15{,}419.74$. These values agree precisely with the values computed by means of Professor Church's method.[7]

Professor Underwood's method of determining the tilt, when the exposure station coordinates are known, is excellent; but his method of adjustment is not clear to the writer. By referring to Figs. 5 and 12, it should be obvious that angles m_a and M_A do not lie on the same plane. Therefore, the incremental difference cannot be plotted on line ao extended (Fig. 12) without introducing an error. However, there is nothing else to do but plot the angles on their respective principal point radials. Consequently the three raised perpendiculars will not intersect at a common point. Therefore, it is necessary to apply an adjustment. Using the approximate v-position (Fig. 12) as a point within the triangle of error, compute a tilt and swing. Locate the nadir point v ($f \tan t$ from o) on the photograph (contact size). Drop perpendiculars from v to the three radials (oa, ob, and oc). Measure the lengths of these dropped perpendiculars (w) and also the lengths from the foot of these dropped perpendiculars to their respective control points (a, b, and c) and denote them as l. The angle M_A, when projected to oa, then becomes:

$$M'_A = M_A \left(\frac{l}{\frac{w^2}{2\,l} + l} \right) \qquad \text{(34)}$$

For radial ao: $M_A = 1{,}281.1$; $l = 3.2$ in.; and $w = 0.18$ in. Therefore,

$$M'_A = 1{,}281.1 \left(\frac{3.2}{3.20506} \right) = 1{,}279.08 \text{ min of arc}$$

$$m_a = \underline{1{,}243.22}$$

$$m_a - M'_A = -\ 35.86 \text{ min of arc.}$$

Using the revised value (M'_A, M'_B, and M'_C), the raised perpendiculars intersect to form a very small triangle of less than 0.01 in. in depth. All other data being correct, this would mean a tilt error of only 0° 00′ 15″, which is negligible. In large nadir throws or large tilts, possibly a second or third repetition would be needed to make the triangle of error vanish.

It is hoped Professor Underwood will treat this matter in his closing discussion.

Determination of Tilt.—For purposes of comparison a numerical example of computations to determine tilt by the "dropped perpendicular" method[8] is presented in Table 6.

Values under "First Determination" pertain to the initial computation using the principal point as the argument. Under "Second Determination," point v and the tilt i are used as arguments. Datum ratios (Table 6(a)) in the first determination are obtained by dividing the datum scales S_d by S_{do}. The value of $S_{do}(1{,}723 \pm)$ is determined by interpolating (visually) the datum scales (situated at their respective scale-point positions) into the principal point. The ratios of the second determination are determined as a function of the three computed tilt-axis scales (S_{di}), one for each check line. The computed tilt-axis scale for a given line is:

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[8] "Applied Photogrammetry," by Ralph O. Anderson, Edward Bros., Inc., Ann Arbor, Mich.

$$S_{di} = \frac{S_d}{1 \pm M_1 \tau} \quad \text{(35)}$$

in which M_1 is the normal distance from the tilt axis to its corresponding scale

TABLE 6.—DROPPED PERPENDICULAR METHOD OF COMPUTING TILT; NUMERICAL EXAMPLE

(a) DATUM RATIOS—R_d

Symbol[a]	Line ab		Line bc		Line ca	
	First determination	Second determination	First determination	Second determination	First determination	Second determination
P	7.6640		5.3695		6.2150	
D	13,137.22		9,137.46		10,609.77	
S_p	1,714.147		1,701.734		1,707.123	
h_e	129.393	128.821	173.962	174.663	97.606	94.672
h_e/f	15.684	15.615	21.086	21.171	11.831	11.475
δ_d	1,729.831	1,729.762	1,722.820	1,722.905	1,718.954	1,718.598
R_d	1.003965	1.003465	0.999896	0.999487	0.997652	0.996989
M_1	......	1.145	......	−0.200	......	−0.965
$1 + M_1 \tau$	......	1.00331	......	0.999422	......	0.997209
S_{di}	......	1,724.054	......	1,723.905	......	1,723.408
Average S_{di}	$S_{do} = 1723\pm$	1,723.789				

(b) TILT DETERMINATION

Symbol[a]	First Determination		Second Determination	
	$R_H - R_{int}$	$R_H - R_L$	$R_H - R_{int}$	$R_H - R_L$
	1.003965	1.003965	1.003465	1.003465
	0.999896	0.997652	0.999487	0.996989
Difference	0.004069	0.006313	0.003978	0.006476
l'	3.315		3.312	
C_1 (in.)	0.004069 (3.315)/0.006313 = 2.137		0.003978 (3.312)/0.006476 = 2.034	
l (in.)	2.183		2.140	
Sin t	0.006313 (8.25) /2.183 = 0.02386		0.006476 (8.25) /2.140 = 0.02497	
t	1° 22′		1° 26′	
s	225° 00′		225° 40′	

[a] In addition to the notation of the paper: P is the photographic length of the check line; D is the horizontal ground distance; $S_p = D/P$; t is the tilt angle; S_{di} is the tilt axis scale; R_H, R_{int}, and R_L represent the highest, intermediate, and lowest datum ratios, respectively; l' represents the length between the scale points conforming to R_L and R_H; and l represents l' projected normal to the line of constant ratios. The lengths l' and l are determined graphically when the datum scales are computed.

point and τ equals the sine of the tilt angle divided by f. The mean of the three S_{di}-values (1,723.789) is used as S_{di}. Accordingly, for a given check line,

$$R_d = \frac{S_d}{S_{di}} \text{ (average)} \quad \text{(36)}$$

The flying height above sea level is equal to H = (average) $S_{di}(f)$ + datum elevation = 15,421.3 ft. This is in error 1.6 ft.

The results of the first determination would generally be accepted in most photogrammetric work. The tilt and swing error is only 0° 05′ and 1° 00′, respectively. However, the second determination, which can be computed in a few minutes, will yield more accurate results. The resulting errors of tilt and swing (of the second determination) are 0° 01′ and 0° 40′, respectively. In addition, a more accurate determination is possible by applying the corrections, ΔM, ΔM_c, and h_c. These corrections are not needed in the current example because of the low tilt and relief.

When horizontal and vertical control is to be extended (without the aid of ground surveys except at flight ends) by means of computed tilts, elevations, and flying heights, all refinements must be used. A method of control extension (horizontal and vertical) is possible, using a triple parallax differential equation in conjunction with various expedients. This method is exact mathematically, but lends itself to a rapid graphical determination. The graphic errors introduced are well within the allowable tolerance. These tests were made on hypothetical examples. The photographic lengths were used to the nearest thousandth of an inch (computed) but all work from there on was performed graphically wherever possible. In a stereoscopic model of 6.5 sq miles, it was found that the maximum error in horizontal or vertical position was about 3 ft. When all values are computed to seven decimal places, the resultant position errors are less than a thousandth of a foot. The latter is really not an error but a difference due to fact that figures beyond the seventh decimal places were ignored. The method is then numerically proved.

The triple parallax differential equation consists of conjugate differences of three values which are readily determined graphically. Some simple arithmetical computations are also needed in the course of computing the elevation difference between two image points.

Professor Underwood's paper is most welcome, as diversified methods of spatial mathematics of the photogrammetric problem are practically nonexistent. The only precedent is the solution conceived by Professor Church, cited by the author.[7] Therefore, a comparison of Professor Underwood's method (A) against Professor Church's method (B) is unavoidable.

The early objective of method (A) is to determine the correct lengths of the three ground pyramid edges. These lengths can be determined graphically with a surprising degree of accuracy and also by means of a calculus adjustment. The graphically determined lengths will be close enough for all practical purposes. Knowing the correct (or nearly correct) pyramid edges, Eqs. 22 (method A) are solvable without excessive work. This solution is a near approach to a direct solution even though it consists of two parts—(1) letting k equal zero; and (2) inserting the value of k. When the value of k is inserted, the resolution is quite simple as only the constant terms are refactored.

The spatial equations in method (B) are mathematically correct but the complexity forbids a direct solution. Therefore, the partial differential calculus method of adjustment is employed. The approximate exposure station coordinates are obtained by means of the well-known, graphical three-point method. These coordinates conform approximately to the principal point ground values.

The exposure station coordinates, as initially used, are then in error. These errors, which are solved for as differentials, vary as the magnitude of the tilt. By definition, a differential is the smallest assignable value of the variable approaching zero as a limit. Therefore, differential equations express precise conditions only when the variable approaches zero as a limit. As these differentials grow in magnitude, the accuracy of the differential condition diminishes.

By this reasoning the number of repetitions required by method (B) varies as the magnitude of tilt. As before stated, the equations of method (A) are in error initially as k is dropped. However, when it is inserted the initial error vanishes at a rapid rate. Re-solution is quite simple as only the constant terms are affected and they alone are factored by the same initial factors. It may then be said that method (A) is more efficient than method (B) in solving for the exposure station. However, it must be remembered that method (A) utilizes the lengths of the three pyramid edges. Therefore, in order to compare the two methods it is necessary to contrast the efficiency (ease of computation) of determining the lengths of the pyramid edges (A) against the two different exposure station computations. Method (A) will probably be more efficient than method (B). The latter would be definitely so if the pyramid edges were determined graphically and if a tolerance of from 3 min to 5 min were allowed on the tilt. The graphical method of determining tilt, method (A), is decidedly more efficient than method (B). The tilt (using exact pyramid lengths) could be determined graphically within 0° 0′ 30″ by method (A).

The general trend in photogrammetric analysis leans toward the promotion of streamlined procedures in the general direction of extending horizontal and vertical control without the aid of ground surveys other than at the flight ends. Much has been done in this direction.

G. H. DELL,[9] Assoc. M. ASCE.—The graphical methods in this paper, supplemented in places by numerical calculations, provide a comparatively rapid determination of the required orientation data. The accuracy of the solution should prove adequate in most instances, particularly with the refinements afforded by Eqs. 29 and 28a. The author's adaptation of the theory of the line of position is noteworthy as a step in extending this concept to new fields.

The writer has found that the angles 1 to 6, Fig. 3, can be obtained fairly rapidly, to the nearest second, by the use of six-place log sines (see Eqs. 9). For the numerical example given by the author the equations are as follows:

$$\log \sin \angle(5) = \log \sin \angle(2) + 31{,}259 \qquad (37a)$$

$$\angle(6) = 139^\circ 07' 18'' - \angle(5) \qquad (38a)$$

$$\log \sin \angle(3) = \log \sin \angle(6) - 10{,}350 \qquad (37b)$$

$$\angle(4) = 146^\circ 36' 35'' - \angle(3) \qquad (38b)$$

$$\log \sin \angle(1) = \log \sin \angle(4) - 20{,}909 \qquad (37c)$$

$$\angle(2) = 131^\circ 03' 15'' - \angle(1) \qquad (38c)$$

TABLE 7.—DETERMINATION OF

Number	FIRST CYCLE		SECOND CYCLE		THIRD CYCLE		FOURTH CYCLE	
	Value	Log sin	Value	Log sin	Value	Log sin	Value	Log sin
2.	62°00′	9.94593	61°09′	9.94245	60°58′	9.94168	60°55′30″	9.941504
5.	71 35	(9.97719)	70 16	(9.97371)	69 59	(9.97294)	69 55 10	(9.972763)
6.	(67 32)	9.96572	(68 51)	9.96971	(69 08)	9.97054	(69 12 08)	9.970737
3.	64 28	(9.95537)	65 36	(9.95936)	65 50	(9.96019)	65 52 55	(9.960387)
4.	(82 09)	9.99591	(81 01)	9.99464	(80 47)	9.99436	(80 43 40)	9.994288
1.	70 45	(9.97500)	70 16	(9.97373)	70 10	(9.97345)	70 08 35	(9.973379)
2.	(60 18)	. . . .	(60 47)	. . . .	(60 53)	. . . .	(60 54 40)	. . . .

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[9] Asst. Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

To begin with, an approximate value of $\angle(2)$ is used. This quantity may be found by the graphical method described by Professor Underwood (see Fig. 2), or may be obtained from the following equation, in which it is assumed that the photograph is without tilt:

$$\angle \text{LBA} = \angle \text{Lba} - \sin^{-1} \frac{\sin i_{BA} \sin \text{Lba}}{\cos m_b} \quad \ldots\ldots\ldots\ldots (39)$$

in which i_{BA} is the inclination of line BA with respect to the horizontal (positive for upward inclination). The values obtained by the writer for $\angle$LBA ($= \angle 2$) were: By the graphical method, 62°00′; and, by Eq. 39, 60°51′44″. The solution shown in Table 7, which verifies that presented by the author, begins with the former value. Quantities obtained from Eqs. 37 and Eqs. 38 are shown in parentheses in Table 7.

TABLE 8.—COMPUTATION OF TILT, SWING, AND AZIMUTH OF PRINCIPAL LINE

Symbol	Equations	Numerical results
p_a......	$p_a = \overline{\text{La}} \cos M_A$	208.663
p_b......	$p_b = \overline{\text{Lb}} \cos M_b$	208.859
p_c......	$p_c = \overline{\text{Lc}} \cos M_c$	211.542
$\overline{\text{ce}}$......	$\overline{\text{ce}} = \frac{p_c - p_b}{p_c - p_a} \overline{\text{ac}}$	147.107
$\angle$cbe...	$\tan \frac{\angle \text{cbe} - \angle \text{ceb}}{2} = \frac{\overline{\text{ce}} - \overline{\text{cb}}}{\overline{\text{ce}} + \overline{\text{cb}}} \tan \frac{\angle \text{cbe} + \angle \text{ceb}}{2}$	51°13′32″
i.......	$\sin i = \frac{\sin i_{bc}}{\sin \angle \text{cbe}}$	−1°26′46″
s.......	$s = \alpha_{bc} - (90° - \angle \text{cbe})$	225°59′00″
$\angle$obf...	$\angle \text{obf} = \angle \text{cbe} - (\alpha_{bo} - \alpha_{bc})$	11°26′12″
$\angle$fbv...	$\tan \angle \text{fbv} = \tan \angle \text{obf} \cos i - \frac{\overline{\text{Lo}} \sin i}{\overline{\text{ob}} \cos \angle \text{obf}}$	13°47′38″
α_{VO}...	$\alpha_{VO} = \alpha_{VB} - (90° - \angle \text{fbv})$	227°23′49″

A method consisting primarily of plane trigonometry was used by the writer to check the calculation of the coordinates of L and V, and the results agreed within 0.1 ft, on the average, with those given by the author. Accordingly, in the calculation of i, α_{ov}, and α_{VO}, by a similar procedure, the author's data were used. Referring to Fig. 14, the vertical distances of points a, b, and c, with respect to point L—denoted by p_a, p_b, and p_c, respectively—were computed and the horizontal line be was located. Line fo is the trace of a vertical plane through point L perpendicular to line be. Point o is the principal point of the photograph, and point v is the vertical projection of point L on the horizontal plane through line be. Angle fbv is a horizontal

BASE ANGLES

FIFTH CYCLE		SIXTH CYCLE		SEVENTH CYCLE		Number
Value	Log sin	Value	Log sin	Value	Log sin	
60°55′05″	475	60°55′14″	485	60°55′15″	9.941487	2
69 54 32	(734)	69 54 45	(744)	69 54 48	(9.972746)	5
(69 12 46)	768	(69 12 33)	757	(69 12 30)	9.970755	6
65 54 28	(418)	65 54 16	(407)	65 54 14	(9.960405)	3
(80 42 07)	256	(80 42 19)	260	(80 42 21)	9.994261	4
70 07 53	(347)	70 07 59	(351)	70 08 00	(9.973352)	1
(60 55 22)	....	(60 55 16)	....	(60 55 15)	....	2

angle, whereas angle fbo lies in the plane abc, the inclination of which is i_{fo} = the tilt, i. The equations and numerical results are given in Table 8, and again it will be seen that the author's solution is substantially verified.

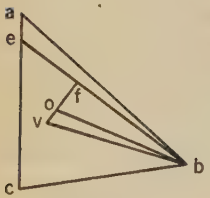

Fig. 14.

In the course of these calculations it was noted that a relatively small error in the β-angles had a pronounced effect on the calculated coordinates of points L and V. The writer would like to inquire whether Professor Underwood has made an estimate of the probable error of these coordinates, in the example in question, assuming that the solution is without error other than in the determination of the coordinates of the control points on the photograph.

Attention is called to the fact that the computation of the angle λ, shown beneath Table 5, is accurate to only four significant places. Using the numerals given in brackets, the writer obtained 0.451024 as the value of the tangent, corresponding to an angle of 24°16′36″. The value obtained by the use of six-place logarithms was 24°14′53″, and, by seven-place natural functions, 24°16′19″. The solution in Table 8 involves a similar weakness, although to a lesser extent.

This paper deals with a very interesting problem, and the author has presented a commendably clear and well-written treatment.

P. H. Underwood,[10] M. ASCE.—Since the paper was written with the idea that it might be of interest theoretically and with the hope that it might be of some practical value, it was expected that the methods outlined would be compared with other methods, as has been done in the interesting discussions. Professor Church's method has been used to a considerable extent in practice. As a result, routine procedures have been established to facilitate calculations. It is possible that the writer's method could be simplified by a more careful arrangement of the work and by certain approximations in the calculations.

As stated in the paper, the calculations and the final results were carried out to more decimal places than would be warranted by any practical application. With an exposure interval of 1/100 sec in taking a photograph and with a plane speed of 200 miles per hr, the camera would move some 3 ft during the exposure. Considering this and also the errors from other sources, such as lens errors and the precision required in coordinates for mapping purposes, there would seem to be little value in determining results to fractions of a foot. As Mr. Anderson has stated, a graphical solution to obtain the lengths of the edges of the ground pyramid yields values within a few feet. Following this, a single solution by calculation of the plate pyramids should generally yield results of sufficient accuracy for practical purposes, especially if corrections are made by the cosine multipliers.

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Mr. Anderson's solution using scale check lines to determine tilt, swing, and height of camera station is interesting. The writer has been much interested in this method and its modification by Jack Rihn[11] and has published a paper[12] on this subject, which leads to a fairly exact solution even under such adverse conditions as high relief and considerable tilt. It is believed that this method may be still further extended to give quite accurate solutions for space coordinates of exposure stations.

In his discussion Mr. Anderson raises questions relative to determining tilt when the exposure station coordinates are known, stating that the writer's method of adjustment is not clear to him. To attempt to clarify this point requires considerable explanation.

It should be considered that the position circles involved in the solution are on the surface of a sphere. Since the graphical solution must be made on a plane surface, it is necessary to apply some form of projection. In this case a "zenithal" projection was used, with the center of the projection at the principal point o and with points on this projection located at distances in proportion to their arc distances on the sphere and in directions corresponding to those on the sphere from the point of tangency o. This procedure necessarily introduces some distortion, which has not been investigated, but which is believed to be small in its effect on the use to be made of the projection.

On the sphere it is known that the distances from v (the point where the vertical from the exposure station L cuts the sphere) to a, b, and c are equal to M_A, M_B, and M_C, respectively. On the projection this condition would be true to a high degree of approximation if the tilt is small, since these distances would be measured nearly radially from o.

When the plotting of the projection is done to a large scale (as in Fig. 12), the radii of the "position circles" become so large that it would be impractical to draw them with centers and a beam compass. Thus, M_B is equal to 31° 04′ or to 1,864 min of arc. At the scale of the original drawing for Fig. 12 (1 mm = 1 min), the radius of the circle would be about 75 in.—that is, line no, if drawn to scale, would be more than 6 ft long. It is known that the center of this circle at b is at a distance from o equal to m_b with m_b less than M_B by 19.0 min. Thus, n, at the relatively short distance of 19.0 min to scale from o on the line ob, is the point where the position circle with b as center would cut that line. Consequently the perpendicular nn′ is the tangent to this circle at n. Similarly, perpendiculars kk′ and pp′ are tangents. Compared with the radii of the position circles, these tangents are short and approximate to their respective circles. They should not meet in a point but the "triangle of tangents" formed with their points of intersection as vertices should be small and should aid in closely approximating, as explained in the paper, the point where the circles would intersect, or point v. Computed offsets from the tangents give an even closer approximation to the point sought. Thus, the tangents, giving what was termed first a "triangle of error" and then "the triangle of the tangents," should not meet in a point theoretically, whereas the position circles should: (1) Meet if drawn on the sphere; and (2) fail to meet by only a small amount (which has not been evaluated but which for small tilts should certainly be small) if drawn on the projection. The position of each one of these three circles in the vicinity of the point sought can be found closely by use of the offsets from the tangents as shown in the paper, thus obtaining a close approximation to the vertical point.

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[11] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, 1944, p. 274.

[12] "The Determination of Tilt from Scale Check Lines," by P. H. Underwood, *Photogrammetric Engineering*, March, 1947, pp. 143-155.

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Paper No. 2317

DESIGN OF PLYWOOD I-BEAMS

By Howard J. Hansen,[1] Assoc. M. ASCE

With Discussion by Messrs. Dick W. Ebeling, I. Oesterblom, C. J. Hogue, Sidney Novick, and Howard J. Hansen.

Synopsis

The general formulas and methods presented herein are applicable to I-sections and box sections of various dimensions, but the M/f-values and W-values presented in Figs. 2(*a*) and 2(*b*) are based on a flange width of five times the web thickness and a flange depth of ten times the web thickness for I-beams. Eq. 12, which is also based on these ratios, is not applicable for other values; but, by substituting any other ratios in Eqs. 10*a*, 10*b*, and 11, a similar expression may be derived.

Since the greatest problem in designing any I-beam or box section is the determination of stiffener spacing, a method is suggested based on the application of a formula for isotropic materials.

No attempt has been made to determine the most efficient section with unequal flanges. From Eq. 2, as the depth of the compression flange increases the form factor increases. Accompanying this is an increase in I/C, up to a certain point, and these two factors will increase the strength in bending.

Introduction

The use of Douglas fir plywood webs and solid or laminated wood flanges for large I-beams is a comparatively recent development. The assembly consists of a plywood web of standard construction with the flanges and stiffeners fastened to the web by glue, bolts, or bolts and connectors. Plywood girders built up in this manner have several distinct advantages over the usual type of heavy timber construction; for example:

1. Larger cross sections and lengths can be manufactured than are available as single pieces;
2. The girder can be constructed so that highly stressed parts contain pieces with the least number of defects, and lower grades of lumber and plywood can be used elsewhere;

Note.—Published in June, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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3. The girder is composed of relatively small pieces, which are rapidly seasoned, and green lumber and shrinkage effects are at a minimum; and
4. The members are easily fabricated and erected.

The design of a plywood girder involves several factors that are not normally considered in the design of solid members. For instance, the shear between the flange and the web will often determine the depth of the flange. If the flange is glued to the web, the allowable unit shear at this point is one eighth of the allowable unit horizontal shear for the plywood web. The allowable horizontal shear stress is 240 lb per sq in. and the allowable web-flange shear is 30 lb per sq in. These values decrease as the grade of plywood is reduced below a 100% grade. Descriptions of the various grades of Douglas fir plywood, as well as other valuable data, are available elsewhere.[2] Since the unit horizontal shear is eight times the web-flange shear, the depth of a flange glued to the web should be eight times the thickness of the web when the full allowable shear is developed in the web. In preparing the design curves for this paper, the depth of the flange was taken as 10 b_w, b_w being the web thickness.

Because horizontal shear will govern the design of plywood girders except for very long spans, it seems unnecessary to make the flange width greater than 5 b_w. Based on this flange width, the fiber stress is approximately six times the horizontal shear stress and the allowable fiber stress in bending will not be reached until the length of the girder is about sixteen times the depth. Of course, by adding material to the flanges, the girder can be designed to carry a greater load. With these factors known, the design of a plywood girder becomes fairly easy. The difficult features are found in the design of splices and the placement of stiffeners.

NOTATION

The letter symbols in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically in the Appendix for convenience of reference.

FLEXURE

Before the allowable fiber stress can be used in the customary flexure formula—

$$f = \frac{M\,c}{I} \qquad \text{.........................(1)}$$

—it must be reduced by a factor whose value is dependent on the shape of the cross section. Because wood is much stronger in tension than in compression parallel to the grain, it might be expected that the allowable stress in compression parallel to the grain should be used in Eq. 1 when computing moments of resistance. Actually, however, the fibers in a wood beam do give way first on the compression side, but the adjacent fibers then receive a higher stress, which tends to move the neutral axis toward the tension side and increases the stress in tension. This procedure continues until tension failure occurs. In a wood beam the individual fibers help support each other; and, since the fibers at the neutral axis are less stressed, they act as supports for the extreme fibers, enabling them to take a higher load. This is not the case when a wood member is subjected to direct compression parallel to the grain. Under this condition all the fibers are stressed alike and will not give as much support to each other as in a beam.

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[2] "Technical Data on Plywood," Douglas Fir Plywood Assn., Tacoma, Wash., 1945.

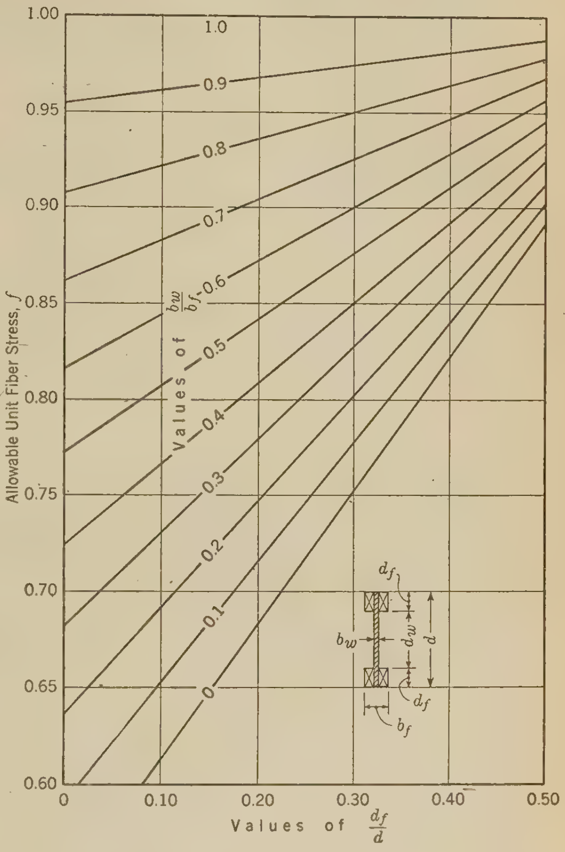

Fig. 1.—Form Factors

The original tests[3] used as a basis for developing structural grades were made on specimens 2 in. by 2 in. in cross section. Additional tests on larger sizes showed that, as the height of a beam increased, the modulus of rupture decreased. Since the effect of this decrease has been included in the working stresses for structural grades of lumber, the allowable fiber stress does not need to be reduced in designing rectangular sections. However, in an I-beam the complete supporting action of the wood fibers is only over a width equal to the width of the web. The supporting action of the fibers outside the web depends on the depth of the compression flange. For this reason an I-beam is weaker than a solid beam of the same height and the same section modulus. Therefore, it is necessary to multiply the allowable fiber stress by a form factor depending on the dimensions of the I-beam. The form factor may be calculated from the following formula[4] or taken directly from Fig. 1:

$$B = 0.58 + 0.42\left[u_d\left(1 - \frac{b_w}{b_f}\right) + \frac{b_w}{b_f}\right] \dots\dots\dots\dots\dots (2)$$

in which B is the form factor; u_d depends on the ratio of d_f/d, as given in Table 1; d_f is the depth of compression flange; d is the total depth; b_w is the thickness of web; and b_f is the width of flange.

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[3] "Strength and Related Properties of Woods Grown in the United States," by L. J. Markwardt and T. R. C. Wilson, *Technical Bulletin No. 479*, U. S. D. A., 1935.

[4] "Form Factors of Beams Subjected to Transverse Loading Only," *No. 1310* (mimeographed), Forest Products Laboratory, U. S. D. A., 1941.

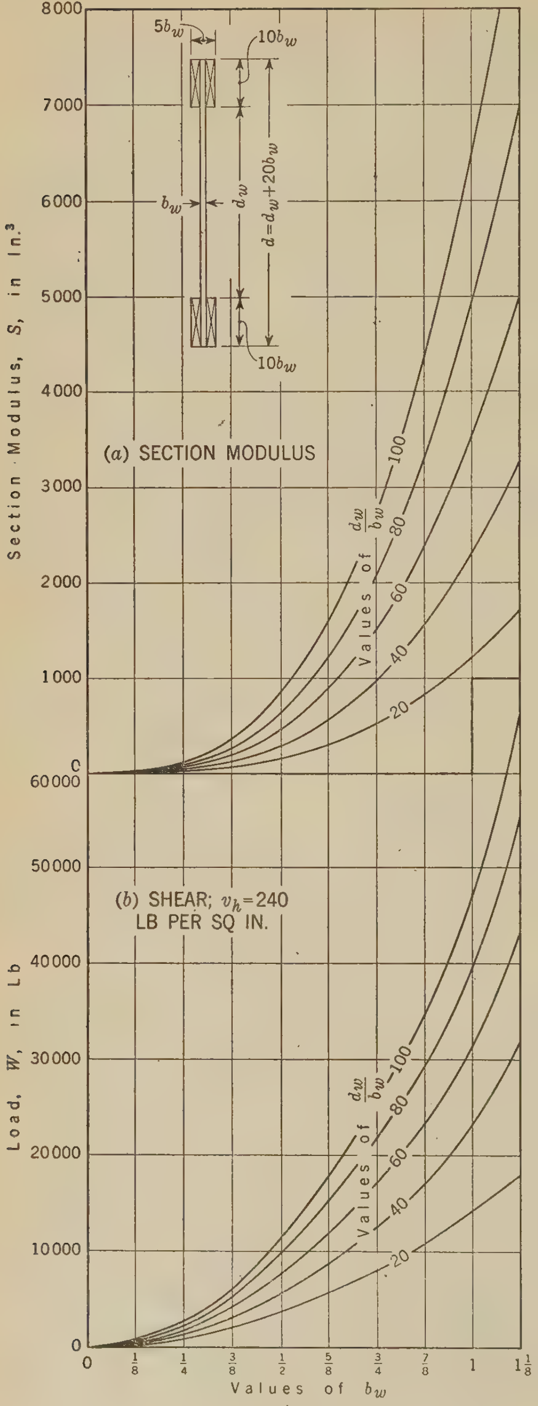

FIG. 2.—DESIGN CHARTS

Values of the section modulus $S = \frac{M}{f} = \frac{I}{c}$ for plywood I-beams having various web thicknesses are given in Fig. 2(*a*). Before calculating the moment from the S-value obtained from this chart, the fiber stress, f, must be multiplied by the appropriate form factor B.

TABLE 1.—COEFFICIENT u_d IN EQ. 1

d_f/d	u_d
0.10	0.085
0.15	0.155
0.20	0.230
0.25	0.315
0.30	0.400
0.35	0.490
0.40	0.575
0.45	0.660
0.50	0.740
0.55	0.810
0.60	0.875
0.65	0.920
0.70	0.950
0.75	0.970
0.80	0.985
0.85	0.995
0.90	0.998
0.95	1.000
1.00	1.000

SHEAR

In computing the unit horizontal shear in the plywood web the general formula—

$$v_h = \frac{V_v\,Q}{I\,b_w} \quad \ldots\ldots(3)$$

—may be used, in which v_h is the unit horizontal shear; V_v is the external shear; Q is the statical moment of area above or below neutral axis; and I is the moment of inertia.

The values in Fig. 2(*b*) are based on Eq. 3 and the total uniformly distributed load may be obtained for any values of b_w and $\frac{d_w}{b_w}$ when the unit horizontal shear v_h is 240 lb per sq in. For total loads at values less than $v_h = 240$, the value taken from Fig. 2(*b*) is simply multiplied by the ratio of the new shear value to 240. Fig. 2(*b*) is based on a flange width, b_f, of 5 b_w and a flange depth, d_f, of 10 b_w.

It will not be necessary to compute web-flange shear because the depth of the flange is sufficient to keep the shear less than the allowable when the flanges are glued to the web. When the flanges are bolted or joined by connectors and bolts to the web, the number and spacing of the fastenings must be determined.

LATERAL STABILITY

For I-beams unsupported laterally, the maximum ratio of the moment of inertia about the *x*-axis to the moment of inertia about the *y*-axis is usually 25. For the beam with a flange width b_f of 5 b_w and flange depth d_f of 10 b_w, the value of d_w could not exceed 9 b_w to satisfy this condition. However, most beams are supported laterally by purlins, joists, or heavy flooring.

WEB SPLICES

For long spans it will be necessary to splice the plywood webs and solid wood flanges. Probably the simplest method of splicing the plywood web is to glue a plywood panel on both sides of the web joint. The thickness of the plies in the splice plates with the direction of grain parallel to the span must be the same as that for those in the web. The width b_p of the splice plate will depend on the fiber stress at the point of the splice, the web thickness, and the allowable shear. The width may be found from the following formula:

$$b_p = \frac{f\,b_w}{v_p} \qquad (4)$$

in which b_p is the width of splice plate; f is the maximum fiber stress at point of splice; and v_p is the allowable shear between web and splice plate—60 lb per sq in.

Flange splices should be at the points of minimum bending moment, and can be made by overlapping an additional piece and fastening with glue, bolts, or connectors.

WEB STIFFENERS

At the supports and points of concentrated loads, vertical lumber or plywood stiffeners should be glued to the web between the flanges. These stiffeners are used as bearing members and should fit tightly against the flanges. Their size may be calculated by dividing the vertical shear by the allowable stress in compression parallel to the grain.

Intermediate stiffeners are required to prevent the web from buckling and their width must be sufficient to transmit the shear. The width may be calculated from the formula:

$$b_s = \frac{v_h\,b_w}{v_s} \qquad (5a)$$

in which b_s is the width of stiffener; v_h is the unit horizontal shear; and v_s is the allowable shear between web and stiffener—60 lb per sq in.

The spacing of stiffeners depends on the thickness of the web, the number and arrangement of the plies in the web, the clear distance between the flanges, and the horizontal shear stress developed. The critical shear stress v_c may be expressed in terms of convenient design factors discussed in detail elsewhere by the writer:[5]

$$v_c = \frac{4\, u_\theta \sqrt[4]{m_1\, m_2^3}}{b_w\, d_w^2} \qquad \text{(5b)}$$

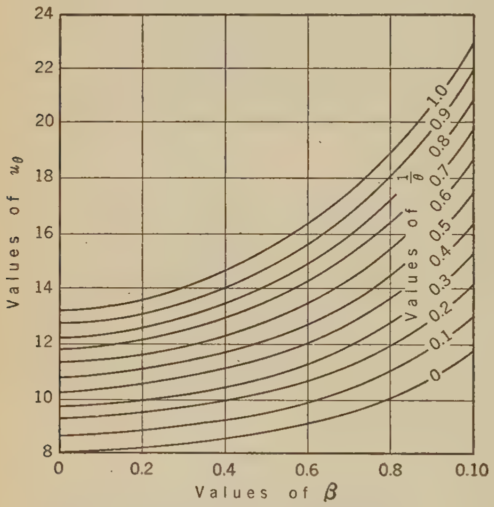

FIG. 3.—VALUES OF u_θ FOR SOLVING EQ. 5

in which u_θ is a factor introduced by Edgar Seydel[6] as c_a. Values of u_θ can be selected from Fig. 3 after complex functions of m have been converted to the convenient design constants[6] β and θ. Thus, the constants m are functions of the reciprocal of Poisson's ratio μ. The moduli of elasticity of the entire panel, taken along the face grain, E_s, and across the face grain, E_d, are, respectively:

$$E_s = E_d = \frac{1}{I} \sum_{i=1}^{i=n} (E_i\, I_i) \qquad \text{(6)}$$

in which i is the number of plies; n is a number in a mathematical expression; E_i is the modulus of elasticity of a single ply; and I_i is the moment of inertia of a single ply about the neutral axis of the cross section. As expressed by Eq. 6, E_s and E_d are summations of the product of the moment of inertia of each ply about the center of the panel and its modulus of elasticity, divided by the moment of inertia of the entire cross section.[7] In computing E_s and E_d, the modulus of elasticity along the grain for a single ply may be taken as 1,600,000 lb per sq in. and that across the grain as 0.045 times the modulus of elasticity along the grain or 72,000 lb per sq in. The modulus of rigidity in shear is equal to $G_d = \frac{1{,}600{,}000}{16} = 100{,}000$ lb per sq in. Poisson's ratio is expressed as:

$$\mu = 1 - \mu_s\, \mu_d \qquad \text{(7)}$$

In Eq. 7 the product of the Poisson's ratios along the grain (μ_s) and across the grain (μ_d) is equal to 0.01 and $\mu = 0.99$. With the foregoing definitions m_1, defining the flexural stiffness along the grain against flexure of the web across the grain, is

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[5] "Modern Timber Design," by Howard J. Hansen, John Wiley & Sons, Inc., New York, N. Y. 1943, p. 168.

[6] "The Critical Shear Load of Rectangular Plates," by Edgar Seydel, *Technical Memorandum No. 705*, National Advisory Committee for Aeronautics, Washington, D. C., 1933, p. 4, Fig. 3.

[7] "Modern Timber Design," by Howard J. Hansen, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 169.

$$m_1 = \frac{E_s\, b^3_w}{12\, \mu} \qquad \text{(8a)}$$

Similarly, m_2 defines the flexural stiffness across the grain against flexure of the web along the grain, or

$$m_2 = \frac{E_d\, b^3_w}{12\, \mu} \qquad \text{(8b)}$$

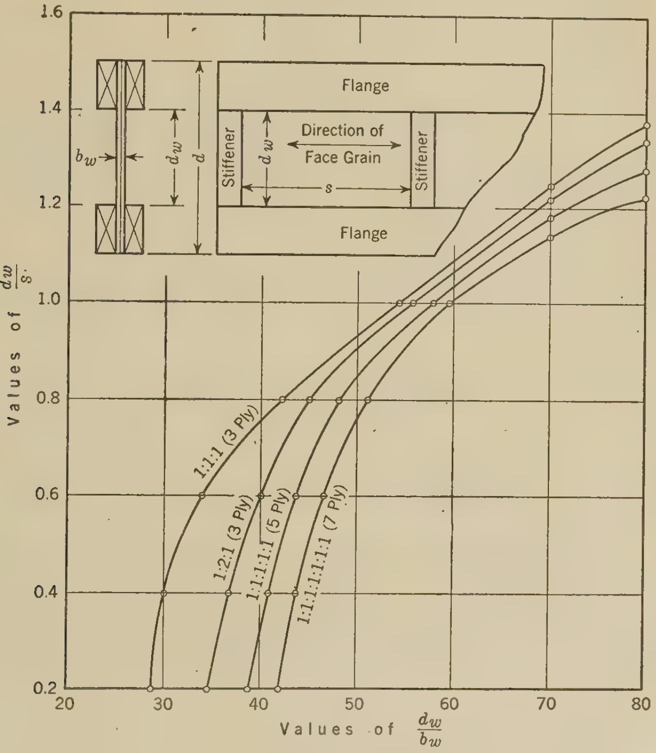

FIG. 4.—SPACING OF STIFFENERS

and m_3 defines the torsional stiffness, or

$$m_3 = \frac{E_s\, \mu_s\, b^3_w}{12\, \mu} + \frac{G_s\, b^3_w}{6} \qquad \text{(8c)}$$

Eqs. 8 apply to Douglas fir plywood, with the face grain parallel to the span. The design factors β and θ may now be written:

$$\beta = \frac{d_w}{s}\left(\frac{m_1}{m_2}\right)^{\frac{1}{4}} \qquad \text{(9a)}$$

and

$$\theta = \frac{(m_1\, m_2)^{\frac{1}{2}}}{m_3} \qquad \text{(9b)}$$

By calculating the values of E_s and E_d for the standard thicknesses of Douglas fir plywood, a solution for m_1, m_2, and m_3 is easily obtained. Then u_θ can be determined for any value of $\frac{d_w}{s}$. The curves in Fig. 4 were obtained by computing values of v_c for various combinations of $\frac{d_w}{b_w}$ and $\frac{d_w}{s}$. Since the ultimate shearing stress is approximately five times the allowable, the values of v_c have been divided by five. It is recommended that the distance, s, between stiffeners should not exceed 5 d_w.

DEFLECTION

The total deflection of an I-beam is the summation of the deflection computed by the usual formulas plus the deflection due to shear. Formulas for shear deformation may be found by equating the internal and external work. The general equations for the flange and web shear are as follows: For the flange—

$$v_h = \frac{V_v}{5\,b_w\,I} \int_y^{0.5d_w+10b_w} 5\,b_w\,y\,dy = \frac{V_v}{2\,I}\left[(0.5\,d_w + 10\,b_w)^2 - y^2\right] \;. \;. (10a)$$

and, for the web—

$$v_h = \frac{V_v}{I\,b_w}\left(\int_{0.5d_w}^{0.5d_w+10b_w} 5\,b_w\,y\,dy + \int_y^{0.5d_w} b_w\,y\,dy\right)$$

$$= \frac{V_v}{2\,I}\left[5\,(0.5\,d_w + 10\,b_w)^2 - 4\,(0.5\,d_w)^2 - y^2\right] \ldots\ldots\ldots (10b)$$

in which V_v is the total vertical shear; the width of the flange, b_f, is 5 b_w; the depth of the flange, d_f, is 10 b_w; and y is the distance from the neutral axis.

For a concentrated load, P, at the center the external work is equal to 0.5 $P\,\Delta_v$, in which Δ_v is the deflection due to shear. The general expression for internal work per unit volume becomes: $\frac{v_h}{2} \times \frac{v_h}{G} \times b_{w1}\,dy\,dx$, in which $b_{w1}\,dy$ represents the area and dx is equal to l, the span; and G is the modulus of elasticity in shear. The integral of the expression for internal work may be set equal to the external work so that:

$$\frac{P\,\Delta_v}{2} = 2\,\frac{l}{2\,G}\int_0^{0.5d_w+10b_w} v^2_h\,b_{w1}\,dy \ldots\ldots\ldots\ldots\ldots (11)$$

By substituting the values of flange and web shear in the expression for v_h and using the flange and web thickness for b_{w1}, the expression becomes

$$\frac{P\,\Delta_v}{2} = \frac{l}{G}\left\{\frac{5\,b_w\,V^2_v}{4\,I^2}\int_{0.5d_w}^{0.5d_w+10b_w}\left[(0.5\,d_w + 10\,b_w)^2 - y^2\right]^2 dy\right.$$

$$\left. + \frac{b_w\,V^2_v}{4\,I^2}\int_0^{0.5d_w}\left[5\,(0.5\,d_w + 10\,b_w)^2 - 4\,(0.5\,d_w)^2 - y^2\right]^2 dy\right\} \ldots (12)$$

Values for Δ_v based on several $\frac{d_w}{b_w}$-ratios $\left(\text{to be multiplied by } \frac{P\,l\,b^6_w}{I^2}\right)$ are as follows:

$\frac{d_w}{b_w}$	Deflection Δ_v for— Concentrated load, P	Uniformly distributed load, P
40	198	99
60	644	322
80	1,578	789
100	3,271	1,635
120	6,063	3,032

CONCLUSIONS

Although the section modulus values and the maximum loads based on the allowable shear given in Figs. 2(*a*) and 2(*b*) are only applicable when the flange width is five times the web thickness and the flange depth is ten times the web thickness, there are many instances when it will be necessary to increase the flange dimensions. For higher ratios than those used, the load-carrying capacity based on the fiber stress in bending will increase rapidly, but the total load based on the allowable shear will not. For example, if b_f is equal to 10 b_w, d_f is equal to 16 b_w, and d_w is equal to 40 b_w, the total load based on shear for 1-in. plywood becomes 25,800 lb whereas the value from Fig. 2(*b*) is 23,000 lb. It appears that Figs. 2(*a*) and 2(*b*) represent a method of selecting a section for the preliminary design.

The deflection due to shear in I-beams and box sections is of considerable magnitude and must be added to the usual deflection formulas or serious errors may result.

The theoretical method of determining the stiffener spacing has not been checked experimentally, but it is believed that test results would conform rather closely.

Additional information is needed on the lateral buckling of beams and experimental data are also lacking on the load capacity of beams with the flanges attached to the webs by all types of fastenings.

APPENDIX. NOTATION

The following letter symbols, used in the paper and in its discussions, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = area of cross section, $b\,d$;
B = a form factor defined by Eq. 2;
b = breadth, or thickness, of a section:

b_f = breadth or width of flange;
b_p = thickness of a splice plate;
b_s = width of stiffener;
b_w = thickness of the web;
d = depth, with subscripts corresponding to b;
E = modulus of elasticity:
E_d = entire panel across the face grain;
E_i = individual ply;
E_s = entire panel along the face grain;
f = fiber stress;
G = modulus of rigidity;
I = rectangular moment of inertia; I_i = individual ply;
i = number of plies;
l = clear distance between stiffeners;
M = moment;
m = a function of Poisson's number, the reciprocal of μ;
n = a number;
P = total load, either concentrated or distributed;
Q = statical moment of area above or below the neutral axis;
S = section modulus $= I/c = M/f$;
u = a factor:
u_d = a constant depending on the ratio d_d/d;
u_θ = a factor in Eq. 5, function of m and μ;
V = total shear, subscripts h or v denoting horizontal or vertical, respectively;
v = unit shear:
v_c = critical;
v_h = horizontal;
v_p = allowable stress between web and splice plates;
v_s = allowable stress between web and stiffener;
v_v = vertical;
y = distance from the neutral axis to the extreme fiber;
β = a factor (Eq. 9a) useful in design;
Δ_v = deflection due to shear; and
θ = a factor in Eq. 9b.

DISCUSSION

Dick W. Ebeling,[8] Jun. ASCE.—There are a few points in this very useful contribution which are not entirely clear: First, the author states (see "Introduction") that the unit horizontal shear is eight times the web-flange shear; but he fails to state that the unit horizontal shear in the plywood is twice the allowable horizontal shear for the wood from which the plywood is made. Second, from Fig. 1, it would seem that the flange depths must be equal, which is not true. The form factor depends only on the depth of the compression flange. The Forest Products Laboratory in Madison, Wis., has developed a formula[9] for the most efficient section with unbalanced flanges. In using this formula, the section is first designed with equal flanges; then part of the tension flange is transferred to the compression flange keeping the total area, height, and width constant:

$$x = \frac{A\, b_f\, d^2 - \sqrt{A^2\, b^2_f\, d^4 - 4\, A\, I_s\, b_f\, d(b_f - b_w)\, d_w}}{2\,(b_f - b_w)\, d_w\, b_f\, d} \qquad (13)$$

in which x is the thickness to be transferred from the tension to the compression side; and I_s is the moment of inertia of the symmetrical section.

Also, the spacing of stiffeners given in Fig. 4 is actually the minimum spacing. Just as the stirrup spacing in a concrete beam increases as the shear decreases, the stiffener spacing can be increased as the shear decreases. However, it is not possible to increase the spacing beyond 5 d_w because the stiffeners are needed as flange spacers in fabrication.

To increase the strength in shear and to decrease the deflection due to shear, the plywood web may be placed with the face grain at an angle of ± 45° to the span—thus doubling the allowable horizontal shear in the web. However, the strength in bending is reduced to one fourth of the value for plywood parallel to the span, although the allowable web-flange shear is not affected. The modulus of rigidity of the plywood is increased five times, which reduces the deflection.

If the plywood is placed with the grain at an angle to the span, the beam will probably have to be fabricated completely in the shop. In most cases, it is better to have the beams completely fabricated in the shop, where the trained men and proper facilities are available. Also, in shop fabrication, the splice plates can be eliminated by scarfed splice joints which develop the full strength of the plywood.

The author's theoretical method of spacing stiffeners agrees closely with that recommended by the Douglas Fir Plywood Association[2] as checked by tests.[10] Also a number of planes were designed from the Forest Products Laboratory recommendations[11] which are similar to the author's method; and, as far as the writer knows, there were no structural failures of the plywood

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[8] Asst. Engr., U. S. Army Engrs., Portland Dist., Powerhouse Development Branch, Portland, Ore.

[9] "ANC Handbook on Design of Wood Aircraft Structures." U. S. Govt. Printing Office, Washington, D. C., 1943, Supplement No. 2, p. 3.

[10] "Structural Application of Plywood," Douglas Fir Plywood Assn. Laboratory Repts., Tacoma, Wash., 1945.

[11] "ANC Handbook on the Design of Wood Aircraft Structures," U. S. Govt. Printing Office, Washington, D. C., 1943.

I-beams and box beams. Therefore, the author's method is adequate and conforms to conventional factors of safety for wooden structures.

I. OESTERBLOM,[12] M. ASCE.—To see papers on plywood construction is refreshing in a double sense. It reminds one of the not so far away past when rolled iron sections were "thrown" at an unsuspecting world; a keen search then followed to discover all their potential uses. It also affords a view of a future in which the United States will certainly go abroad begging for the iron ores she needs to maintain industrial supremacy, having tried valiantly—almost to fanaticism—to exhaust her own natural wealth as quickly as possible. This, Great Britain has done for a century or more—to her sorrow.

In this future, plywood is certain to come into its own on a tremendous scale. With moisture from the heavens, and energy from the sun, the forests will continue to grow long after domestic iron ore is a mere memory—unless man sets his mind on the task of exhausting the forests too—through economic mismanagement.

Visualizing this, it is easy to see a complex development in all directions—not only as to beams and girders but also as to every conceivable combination useful in home building and industry—and these are many. Some time ago a friend of the writer built a functional plywood home with double sheathing on 1-in. by 3-in. joists—all prefabricated and glued in his home basement during spare time, and then quickly erected. He was scorned for building a house of cards; but the proof of strength came quickly; a truck out of control ran into the house and pushed it sideways several inches without injury to the basic frame.

Experiences of similar order have multiplied recently; hence, papers on the theory and the use of plywood are highly welcome. Plywood will be of help to the builders of the future; and it will slow down the terrific rate at which iron ore is taken from the bowels of the earth.

Professor Hansen has supplied one of the chapters most needed to make plywood history. He has written about I-beams; and he has written well. He has verified his formulas by experiments. Now the engineer can design plywood I-beams with greater confidence.

There is a weakness in such beams—as he states—which one may possibly eliminate. It is not the plywood, seemingly, that determines permissible unit shear stress, but the glue between web and flange; thus the plywood itself could do a great deal more work if its own strength were governing. Why not change the section? At present there is a small glue area as compared with a large flange area. It should be the other way around to get the most out of the section. Therefore, it is proposed that the beams—at least the heavier girder types—be built up as are steel plate girders, with fillet blocks (of plywood if preferable) to take the place of the four angles connecting flange and web plates. This would greatly increase the shear area holding the web to the flanges so that even with the relatively low shear strength of the glue much greater flange stress could be transmitted to the web.

Does Professor Hansen agree? Are not such girders made? If they are not made, why not? If they are made, what new formulas are needed to use them properly in structural plywood designs?

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[12] Engr., Carbide & Carbon Chemicals Corp., South Charleston, W. Va.

C. J. Hogue,[13] M. ASCE.—An interesting contribution to the data on a useful and efficient combination of lumber and plywood, utilizing the tensile and compressive strength of sawed lumber and the shearing strength of plywood is presented by Professor Hansen. As the author notes, this development is comparatively recent; and further information and test checks are needed. The paper offers methods of determining size and spacing of intermediate stiffeners, and of calculating deflection due to shear, which differ from those in "Technical Data on Plywood."[14] Through such presentation of different methods, the best will be found.

As information on wood is developed and wood is used more technically, its anisotropic character leads to complicated analyses. Formerly, a designer in wood need only be equipped with a formula for bending and an empirical, straight-line column formula. With present advances, there is need for simplification of design data.

A complicated analytical formula may often be expressed in simplified form within close accuracy, and charts and tables can be prepared for convenient determination of complicated factors. This trend tends to more convenient use of wood and plywood, to quicker calculation, and to safer design where an involved formula might be misinterpreted and solved erroneously. The writer is glad to note that this is done both in the paper and in "Technical Data on Plywood."[14]

Sidney Novick,[15] Jun. ASCE.—The method for determining stiffener spacing (see heading, "Web Stiffeners") is based on the theoretical buckling behavior of plywood plates in shear. Fig. 4 appears to have been derived by finding simultaneous values of $\frac{d_w}{s}$ and $\frac{d_w}{b_w}$ for a fixed value of $v_c = 5 \times 240$, rather than "* * * by computing values of v_c for various combinations of $\frac{d_w}{b_w}$ and $\frac{d_w}{s}$" as stated by the author (paragraph following Eqs. 9). These curves, therefore, give spacings such that the theoretical factor of safety against shear buckling will be at least 5 when the horizontal shear is equal to one fifth of the maximum horizontal shear stress of the material, v_h (max)—that is, when $v_h = \frac{v_h\,(\max)}{5} = 240$ lb per sq in. The value of Fig. 4 is limited by its theoretical basis and by the failure to provide for increasing spacings when the design value v_h is less than $\frac{v_h\,(\max)}{5}$.

In 1944, the Army-Navy Civil Committee (ANC) on Aircraft Design Criteria, summarizing a more comprehensive study by the Forest Products Laboratory,[16] presented an "Experimental Buckling Curve"[17] which demonstrates that plywood beam webs cannot be assumed to buckle always at stresses close to the values predicted by theory. Actual shear stresses at buckling are equal to, or greater than, the predicted values only when the plate dimensions are in such relation as to cause a limiting value to be exceeded; below this limit, the actual stresses are less than predicted.

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[13] Cons. Timber Engr., Seattle, Wash. Mr. Hogue died in November, 1946.

[14] "Technical Data on Plywood," Section 9, Douglas Fir Plywood Assn., Tacoma, Wash.

[15] Structural Engr., General Panel Corp., New York, N. Y.

[16] "Design of Plywood Webs in Box Beams," *Mimeograph No. 1318* (and supplements), Forest Products Laboratory, Madison, Wis., 1943–1944.

[17] "Design of Wood Aircraft Structures," *Bulletin No. 18*, Army-Navy-Civil Committee on Aircraft Design Criteria, Washington, D. C., 1944, Fig. 2–41.

In the notation of the paper, the limit above which the theoretical buckling values are valid is that $\frac{d_w}{d_{w_o}}$ is equal to 2.2, in which d_{w_o} is the theoretical depth between flanges at which the critical buckling stress v_c would be equal to the shear strength of the material, v_h (max); that is,

$$d_{w_o} = \left(\frac{4\, u_\theta \sqrt{m_1\, m^3_2}}{b_w\, v_h\, (\text{max})} \right)^{\frac{1}{2}} \dots\dots\dots\dots\dots\dots (14)$$

Fig. 5 has been plotted from the ANC curves[17] to show the variation of the ratio $\frac{v_c\,(\text{actual})}{v_c\,(\text{theoretical})}$ (denoted by r_v) with $\frac{d_w}{d_{w_o}}$.

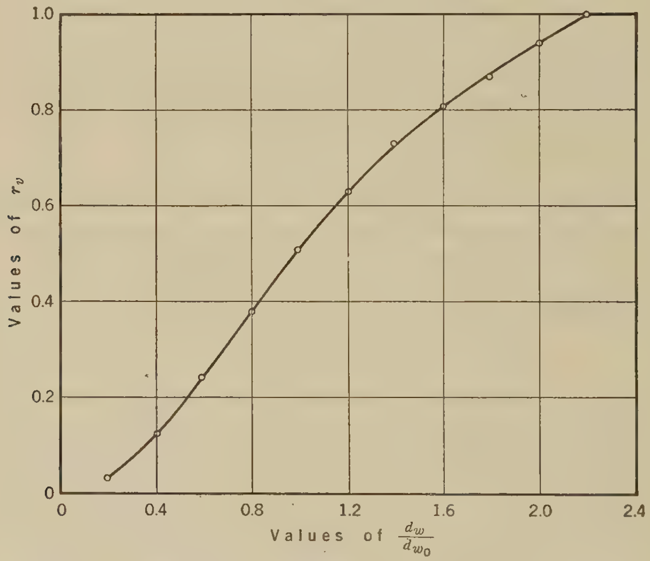

Fig. 5.—Ratio of Actual to Theoretical Buckling Values

From these empirical data it appears that shear buckling will always occur before the maximum horizontal shear stress, v_h (max) is reached, regardless of stiffener spacing: The absolute maximum value of $\frac{v_c\,(\text{actual})}{v_h\,(\text{max})}$ is only 0.83 $\left(\text{for a value of } \frac{d_w}{d_{w_o}} \text{ equal to } 0.06 \right)$. Therefore, if equal factors of safety against horizontal shear and shear buckling are desired, the value of the design horizontal shear, v_h, must be taken into account in determining stiffener spacing, and the maximum value of v_h must always be limited to less than $0.83 \times \frac{v_h\,(\text{max})}{5}$.

With these considerations in mind, Fig. 6 has been prepared to supplant Fig. 4. The same types of plywood and design constants as used by the author were employed; they will enable the designer to determine the spacing of stiffeners after v_h and the ratio $\frac{v_h}{v_h\,(\text{max})/5}$ $(= p_v)$ have been found. For commercial grades of plywood, $v_h\,(\text{max})/5$ should be taken as the maximum safe horizontal shear stress recommended for that particular grade.

The curves in Fig. 6 were obtained as follows: From the theoretical shear buckling equation, the buckling stress varies inversely as the square of the depth between flanges; therefore,

$$\frac{v_c\,(\text{theoretical})}{v_h\,(\text{max})} = \left(\frac{d_{wo}}{d_w}\right)^2 \ldots\ldots (15a)$$

If the value of v_h is such that $\frac{v_h}{v_c\,(\text{actual})/5} = 1$, then

$$p_v = \frac{v_h}{v_h\,(\text{max})/5} = \frac{v_c\,(\text{actual})/5}{v_h\,(\text{max})/5}$$

$$= \frac{v_c\,(\text{actual})}{v_h\,(\text{max})} \ldots\ldots\ldots (15b)$$

Since $\frac{v_c\,(\text{actual})}{v_c\,(\text{theoretical})} = r_v$,

$$p_v = r_v \times \frac{v_c\,(\text{theoretical})}{v_h\,(\text{max})} = \frac{r_v}{\left(\frac{d_w}{d_{wo}}\right)^2} \ldots (16)$$

For each type of plywood and for various ratios of $\frac{d_w}{s}$, values of $\frac{d_{wo}}{b_w}$ were computed by a method recommended by the Forest Products Laboratory[17,18] and

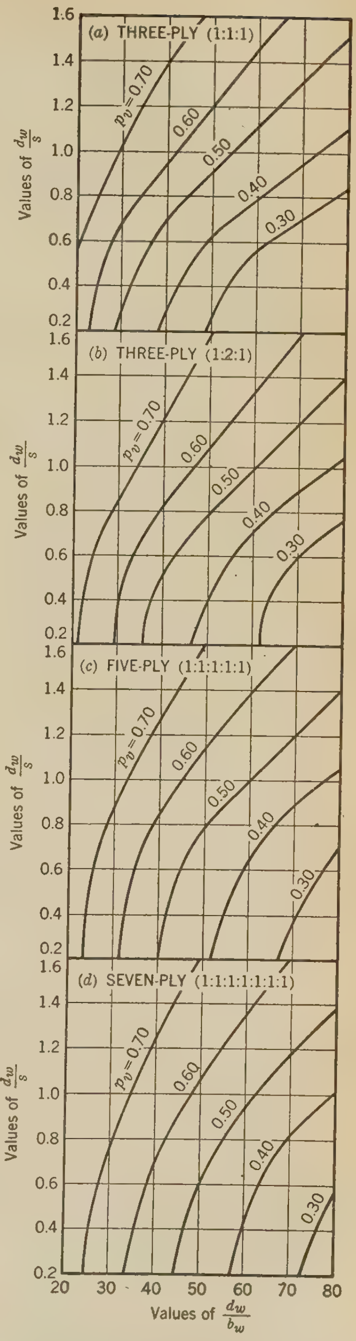

FIG. 6.—SPACING OF STIFFENERS IN PLYWOOD BEAMS

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[18] "Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear," *Mimeograph No. 1316* (and supplements), Forest Products Laboratory, Madison Wis., 1942–1945.

equivalent to that used by the author. These values checked the $\frac{d_w}{b_w}$-values in Fig. 4 by a small margin. Then, for various ratios of $\frac{d_w}{b_w}$, values of $\frac{d_w}{d_{w_o}}$, r_v, and p_v were found, successively. By interpolation between the values of p_v for each combination of $\frac{d_w}{b_w}$ and $\frac{d_w}{s}$, the curves for even-numbered values of p_v, as given in Fig. 6, were found.

Howard J. Hansen,[19] Assoc. M. ASCE.—The encouraging comments of those who discussed the paper seem to indicate that plywood I-beams have a definite place in the future of structural engineering. However, considerable test data and analyses are still needed. This is true not only of the I-beam but also of other types of cross sections.

As stated by the late Mr. Hogue, the anisotropic character of wood makes any analysis complicated; but, by expressing complicated equations in the form of simplified charts and tables, convenient design factors are established. With the advances made in timber and plywood analyses in the past few years, the need for the simplification of design data is apparent.

The writer did not plan his paper to cover all the possible conditions, and the limitations of the methods presented were noted in the "Synopsis." Mr. Ebeling pointed out several factors which are known by most engineers familiar with timber and plywood design. However, for those who are unacquainted with the Forest Products Laboratory (Madison, Wis.) formula for the most efficient section with unbalanced flanges and the effects of placing the face grain of the plywood web at an angle of ±45° to the span, his comments are timely.

Mr. Oesterblom's comments concerning the future of plywood are in accord with the writer's views. His suggestions as to possible girder cross sections seem feasible. It is the writer's observation that the box beam with two or more webs probably has more possibilities than any other cross section. The same design methods can be extended to this type of section with little variation.

The extension of the writer's method by Mr. Novick and the inclusion of the curves shown in Figs. 5 and 6 are appreciated. They have already saved considerable time and effort. The paper was originally written in 1944 and edited while the writer was overseas; consequently, access to the publications mentioned by Mr. Novick was impossible. The writer was pleased to note that the $\frac{d_w}{b_w}$-values checked by a close margin. This also was noted by Mr. Ebeling in his statement that the writer's theoretical method of spacing stiffeners agreed closely with that recommended by the Douglas Fir Plywood Association as checked by tests.

This paper should not be closed without mentioning a few of the many unsolved problems concerning wood and plywood when used in this or similar types of structures. Considerable information is needed on nailed joints.

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[19] Associate Prof., Civ. Eng. Dept., Univ. of Florida, Gainesville, Fla.

The paper considers flanges glued to the plywood web, but there is no reason why these cannot be nailed when sufficient test data are available on the spacing of nails. Since the flanges of an I-beam or box beam may be built up of several pieces nailed together, more information is needed on the size and quantity of nails to use. There is no question that glued laminated flanges fastened to the plywood web are more efficient than nailed flanges, but just how much more efficient is not known at present. Nailed girders have been used extensively in Europe, and, in place of plywood, diagonal 1-in. boards have been used for the web. This seems to present an interesting possibility, and tests should be made of girders of this type with American species. Complete frames or bents of this type of construction or with a plywood web should be analyzed and tested. Additional data are also needed on the use of plywood for gusset plates in timber trusses and arches.

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Paper No. 2318

RECHARGE AND DEPLETION OF GROUND-WATER SUPPLIES

By Charles L. McGuinness,[1] Esq.

With Discussion by Messrs. E. A. Vaubel, Raphael G. Kazmann, and Charles L. McGuinness.

Synopsis

It is widely believed that the water table is declining and that ground-water supplies are being depleted because of deforestation, drainage, improper methods of land cultivation, and pumping from wells. In addition, many fantastic explanations are offered to account for depletion of ground-water supplies. The paper describes the nature of ground-water reservoirs or aquifers, their recharge from precipitation and stream flow, and their discharge through seeps and springs and by evaporation and transpiration. It shows that changes in ground-water levels are caused by climatic changes or by human activities that affect the rates at which the ground-water reservoirs are recharged and discharged. Although additional investigation is needed, there is no reason to believe that changes in ground-water levels due to deforestation, drainage, and land cultivation have been progressive over long periods in most parts of the United States. The most pressing problems are those resulting from the withdrawal of water from wells. The paper divides the productive aquifers into two broad classes—those with high rates of recharge and large perennial supplies and those with low rates of recharge and small perennial supplies—and describes numerous examples of aquifers of both types. Artificial recharge as a method of conserving ground water is described briefly and is shown to be both practicable and necessary at certain places. The paper concludes that at some places the present rates of withdrawal from wells are excessive and will have to be curtailed sooner or later, but at many other places the rate of recharge is high and large supplies will be perennially available to wells.

Introduction

A persistent belief has arisen that the water table is declining progressively because of deforestation, drainage, improper methods of land cultivation, and

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Note.—Published in September, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Associate Geologist, U. S. Geological Survey, Washington, D. C.

pumping from wells. The growth of such a belief has been especially rapid since 1930, and is particularly strong in the Middle West. Ground-water levels may decline as the result of deficient or poorly distributed precipitation, pumping from wells, installation of drainage ditches, or other factors. The causes of a decline in ground-water levels at any particular place are generally so complex that attempts to define them without careful investigation may lead to confusion. Thus, it is to be expected that a large number of people might believe that ground-water supplies are diminishing to such an extent that sooner or later they will be largely depleted. It is obviously impossible to present this large and complicated subject in a single paper, but an attempt will be made to give some information on the sources of ground water, the extent of depletion, and the prospects for continuing supplies. General discussions of these phases of ground-water hydrology are found in several papers by O. E. Meinzer (1*a*)(2)(3)(4)(5)(6)(7).[2]

Recharge and Discharge of Ground-Water Reservoirs

A discussion of recharge and depletion of ground-water supplies involves the concept of ground-water reservoirs. The rock formations of the earth—which include both consolidated rocks such as sandstone and limestone and unconsolidated sand and gravel—constitute great natural ground-water reservoirs. These reservoirs, or aquifers, are replenished by water derived directly from rain and snow or from streams, and they are discharged under natural conditions through seeps and springs and by evaporation and transpiration of plants. The water levels in the subterranean reservoirs fluctuate seasonally and with cycles of drought and excessive precipitation. Both recharge and discharge are high in wet years, and the water table is high. In dry years recharge is diminished, water is withdrawn from storage, the water table declines, and the discharge diminishes. In a sense, the water then remaining in the reservoir is in dead storage, but some of it can always be made available by wells.

The storage capacity of the major subterranean reservoirs is very great, and therefore in times of need, such as periods of drought or other emergencies, quantities of water far in excess of the current recharge, or of the perennial safe yield, can be withdrawn from storage for a few years. However, all heavily pumped aquifers should be thoroughly investigated to determine the rates at which given installations of wells can be pumped indefinitely, so that estimates can be made of the extent to which emergency rates of pumping may have to be reduced.

Under natural conditions, the aquifers are in an approximate state of dynamic equilibrium, although the water table fluctuates seasonally and from year to year. The utilization of ground water by means of wells is an additional discharge imposed on the natural system of recharge and discharge, and upsets the equilibrium (8). Before equilibrium can again be reached, the new discharge must be balanced by an increase in the rate of recharge or by a decrease in the rate of natural discharge, or by a combination of the two. Until a new equilibrium is reached, water must be pumped from storage with a consequent decline in water levels. Ground-water reservoirs, or aquifers, differ widely in areal extent, opportunity for recharge, ability to transmit and store water, and conditions of natural discharge. The properties of the individual aquifers determine how much water can be withdrawn from them by wells and the relative extent to which withdrawal from storage, increase in recharge, and decrease in natural discharge are involved.

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

A decline in water levels caused by withdrawal of water from wells increases the hydraulic gradient from the areas of recharge and reduces the gradient toward the areas of natural discharge. A decline of the water table in the areas of recharge may result in an increase in the rate of recharge, but only if there is "rejected recharge" (8). Recharge is rejected if the potential rate of recharge exceeds the rate at which water can flow laterally through the aquifer, so that the water table stands at or near the surface and water is disposed of by evaporation and transpiration and by flow into streams. Lowering the water table in such an area reduces the rate at which water is lost, and thus increases the rate of recharge. If the rate of recharge is normally less than the rate at which the aquifer can carry the water away laterally, recharge is not rejected and the rate of recharge is governed (*a*) by the rate at which water is made available by precipitation or by the flow of streams, or (*b*) by the rate at which water can move vertically downward through the soil to the water table and thus escape evaporation. Lowering the water table by withdrawing water from wells does not cause an increase in the rate of recharge in such an area.

A decline of the water table in the areas of natural discharge caused by withdrawal of water from wells reduces the discharge by evaporation and transpiration and also by seepage into streams. Water that is derived not from a loss in storage but from increased recharge or decreased discharge has been termed "salvage" (9).

Over a long period under natural conditions, depletion of the water supply of an aquifer can occur only as the result of a climatic change or a geologic process that causes a reduction in recharge or an increase in discharge. For example, a reduction in recharge might be caused by a blanket of loess deposited by wind over the recharge area of a permeable formation, or by deposition of silt in the bottom of a stream that contributes water to an aquifer. An example of a process that might cause an increase in discharge would be the lowering, by downcutting, of the level of a stream into which ground water is discharging. Depletion of the supply can occur as the result of the activities of man, who not only withdraws water from wells but also performs other actions that may affect the rates of recharge and discharge.

A "Symposium on Fluctuations of Ground-Water Levels" that appeared in 1936 (1) contains papers giving long-time records of ground-water levels at several places in the United States and one place in England. Some of these records illustrate the principle that, under natural conditions, ground-water levels fluctuate with the amount and distribution of precipitation. The well whose hydrograph is shown in Fig. 1 is in the northeastern tip of Oneida County, New York (1*b*), and penetrates glacial sand and gravel. The observations were made by the Black River Regulating District. The graph shows no progressive decline in water level during the period of record. The graph of the water level in this well (Fig. 1(*a*)) in the latter part of 1944 was about the same as in the latter part of 1935, although before the end of 1944, because of drought, the water level declined a little lower than in 1935.

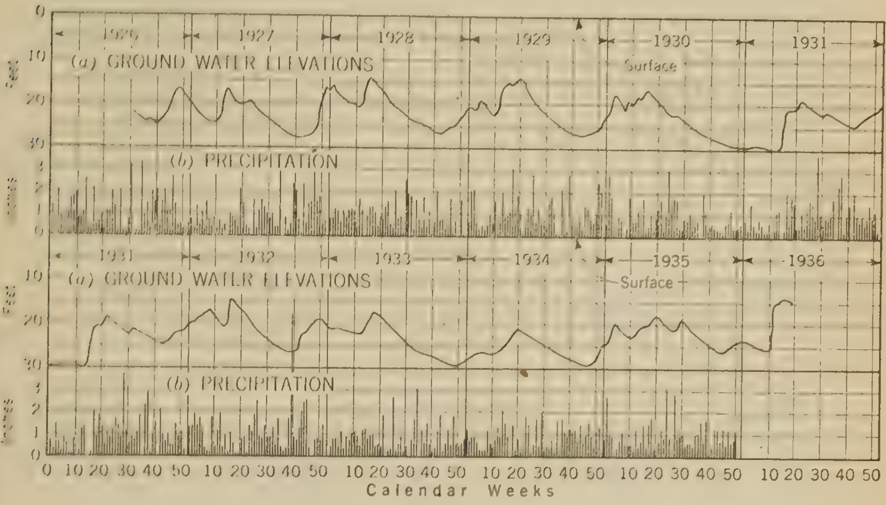

FIG. 1.—HYDROGRAPH OF WELL 6 AT WOODGATE, N. Y., 1926–1936

Fig. 2 illustrates a feature of ground-water recharge that is typical of some areas, particularly in the West, in which the pumpage from irrigation wells is in excess of the rate of recharge in years of normal or low precipitation, resulting in withdrawal of water from storage. However, the water is replenished during the infrequent periods of excessive precipitation. The Williams well (Fig. 2) is near the Santa Ana River, northwest of Redlands, Calif., and penetrates alluvium in the San Bernardino Valley (10). The observations were made by the Gage Canal Company.

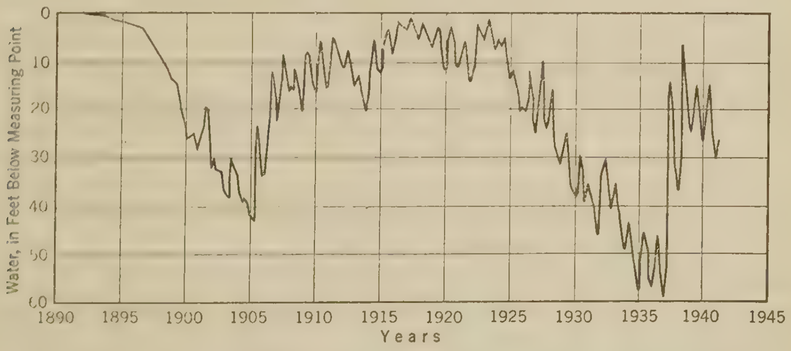

FIG. 2.—FLUCTUATIONS OF WATER LEVEL IN THE WILLIAMS WELL, SAN BERNARDINO AREA, SANTA ANA RIVER BASIN (CALIFORNIA) 1892–1940

Following a period of deficient precipitation in the late nineties, the water level in the Williams well—the fluctuations in which are reasonably typical of those in wells in the San Bernardino Valley—declined to a low stage in 1904, and then rose persistently until 1917. From 1917 to 1937 the trend was predominantly downward, although small rises occurred in 1922 and 1926. Large rises in water level occurred in 1937 and 1938 as the result of increased precipitation and recharge, after which the water level resumed a downward trend. In 1941 the water level rose again, reaching a stage only about 3 ft lower than that in 1917. The trend was again downward in 1942, but in 1943 the water level rose within 2 ft of the 1941 stage. On the average, the period beginning with 1937 has been a period of relatively high recharge, as was the period from 1904 to 1917.

In areas with a more humid climate the conditions are somewhat different. Recharge generally occurs each year, and only in exceptionally dry years does little or no recharge occur. Fig. 3 shows fluctuations of water level in a shallow observation well penetrating glacial outwash sand near Roscommon, in the north-central part of the lower peninsula of Michigan. In the vicinity of the well the ground-water supply is recharged by melting snow and rainfall, and is discharged by evaporation, by transpiration by young trees, and by seepage into a stream several hundred feet away. The water level is not affected by pumping. Substantial recharge has occurred during each year of record, although the amount varies from year to year. Fig. 3 shows no net decline in water level during the period of record. The relatively high stages in 1938, 1941, 1942, and 1943 were caused by excessive or favorably distributed precipitation. The relatively low stage in 1944 was the result of unfavorably distributed precipitation rather than lack of precipitation. The total precipitation in 1944 through October, at the precipitation station nearest the Roscommon well, was about average, but it was unfavorably distributed for ground-water recharge. Precipitation was light during the winter of 1943–1944—which accounts for the relatively small spring rise in water level. Rainfall was considerably above normal in June, 1944, but, because of evaporation and transpiration, only a small rise in water level resulted.

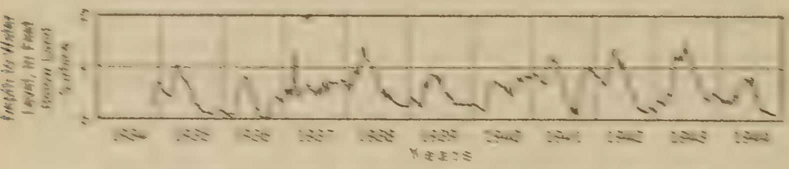

FIG. 3.—HYDROGRAPH OF THE ROSCOMMON WELL IN MICHIGAN, 1934–1944

The abrupt rise and decline in water level in the Roscommon well in February, 1937 (shown in detail in Fig. 4), was obviously exceptional. It was caused mostly, if not entirely, by local recharge from melting snow. Water from melting snow was observed to collect in a surface depression near the well. Although recharge undoubtedly occurred throughout the region as a result of the thaw, the recharge in the surrounding area was not as great as that at the well, and the water was able to drain away quickly into the adjacent areas.

In some places large and sudden rises, followed by rapid declines, in water-table wells have been attributed to pressure effects produced when the soil becomes saturated; but the data do not show whether such a mechanism may have accounted for part of the February, 1937, rise in the Roscommon well.

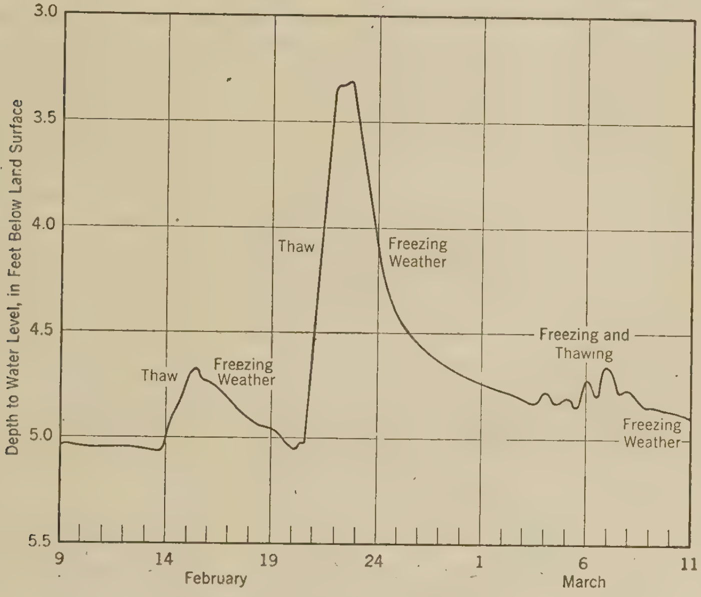

FIG. 4.—ENLARGEMENT OF HYDROGRAPH, FEBRUARY AND MARCH, 1937; THE ROSCOMMON WELL IN MICHIGAN

The relation of forests and the cultivation of land to ground-water supplies has received much discussion, but further rigorous investigation of the subject is needed. It seems inevitable that changes in the land such as deforestation and crop production should have produced changes in the average annual infiltration, changes in the average annual loss by evaporation and transpiration, and hence changes in the average stage of the water table and average annual amount of ground-water runoff. There is no reason to believe, however, that such effects have been progressive over long periods in most parts of the United States.

Drainage ditches, of course, accomplish their purpose of lowering the water table in the areas drained, with a resultant loss in ground-water storage. However, this effect may be expected to be complete within a relatively short time after the drains are installed (a year or a few years at the most) so that a progressive decline of the water table over a long period is not to be expected as the result of the installation of a given system of drains.

EFFECTS OF WITHDRAWAL OF WATER FROM WELLS

Although much investigation will be necessary to establish fully the effects on ground-water supplies of such activities as deforestation, land cultivation, and drainage, the most pressing problems are those created by withdrawal of large quantities of water through wells.

Water-bearing formations differ greatly in their capacity to transmit water and in the quantity of water that they yield from storage with a decline in head. Formations of low permeability that are capable of yielding only small supplies of water for domestic and stock use are found in most parts of the United States. Formations that are capable of yielding sufficient water for large public supplies, irrigation, extensive industrial use, or other purposes requiring large quantities of water are of more limited extent. In their capacity to transmit and store water, these productive aquifers differ widely. Some are of low permeability but have high artesian heads or are buried deeply so that large drawdowns in wells are practicable, and thus the aquifers can be made to yield substantial quantities of water to wells—at least for a limited period. Other aquifers are quite permeable but have low rates of recharge because of lack of precipitation or because of limited outcrop areas. Still others are highly permeable and have high rates of recharge. For the purpose of this paper, the aquifers from which large quantities of water are withdrawn can be divided into two very broad classes: (a) Those with high rates of recharge, transmission, and discharge, which are therefore capable of furnishing large perennial supplies; and (b) those with low rates of recharge and small perennial supplies, where heavy withdrawals are largely from storage.

The effects of withdrawal of water from wells are aggravated in some places by droughts, or by encroachment of salty water. The effect of droughts is most evident on aquifers with small storage capacities, which depend on sustained recharge to support heavy withdrawals. Aquifers with low rates of recharge and small perennial supplies, where heavy withdrawals are largely from storage, are relatively unaffected by droughts.

Encroachment of salty water affects aquifers of both classes but especially those of the second class. It occurs when the head of the fresh water is lowered by withdrawal of water through wells sufficiently to permit the salty water to move toward the wells. Some aquifers in the interior of the United States are subject to encroachment of water of poor quality; but most aquifers in danger of salt-water encroachment are near the coast, where water from the sea, or salty water from part of an aquifer that has not been flushed out by fresh water, tends to move toward the wells as the water levels are lowered by pumping or natural flow. In areas threatened by salt-water encroachment, the best method of detecting the approach of salty water is by means of so-called outpost wells, and the only method of guaranteeing a perennial supply of uncontaminated water is to permit the establishment of a ground-water divide between the salty water and the wells, by reduction or redistribution of pumpage or artificial recharge. In places where contamination has already occurred, the value of the ground-water supply can be salvaged to a considerable extent by using the water for purposes in which quality is not so important, such as cooling.

Examples of aquifers with high rates of recharge and discharge are the productive Tamiami limestone in the Miami (Fla.) area, the Ocala limestone of central and northern Florida and coastal Georgia, and some deposits of glacial outwash sand and gravel in the northern states—for example, (a) those in the valleys of the Ohio River and several of its northern tributaries; and (b) some highly productive glacial gravels in the Pacific Northwest. Aquifers with low rates of recharge (where heavy withdrawals are largely from storage) include the Ogallala formation in parts of the High Plains (as in the Panhandle of Texas), the Dakota sandstone of the Dakotas and adjacent states, and some of the desert basins or bolsons of the West—for example, the Mimbres Valley in New Mexico. Several representative aquifers of both classes will be described to show the great variety of ground-water conditions in the United States.

Examples of Aquifers with High Rates of Recharge and Large Perennial Supplies

The water-bearing limestone in the Miami area is one of the most permeable and productive aquifers in the United States (11). The rate of recharge from precipitation is high, and of course so is the rate of natural discharge through seeps and springs and by evaporation and transpiration. The coefficient of transmissibility (expressed as the number of gallons of water per day that will move through a section of the aquifer 1 mile wide under a hydraulic gradient of 1 ft per mile) has a value of 2,000,000 to 3,000,000 in the vicinity of Miami, whereas in many heavily pumped aquifers in the United States the value is less than 50,000.

In some places the water supply of the productive aquifer in the Miami area is jeopardized by salt water, which has access to certain areas of heavy pumping, especially by way of drainage canals. The well field of the Miami public water supply was actually affected by salt water that gained access through a canal, and it has been necessary to install a movable dam in the canal to prevent salt water from reaching the well field during periods of low flow in the canal. Also, a "tongue" of salty water is moving slowly toward the well field underground along the canal, and eventually will reach the well field unless a certain minimum fresh-water head, established by redistribution of pumpage or by some other method, is maintained continuously in the area between the salt-water tongue and the well field (11). However, the investigations of the U. S. Geological Survey have shown that, with proper management, the cities of Miami, Miami Beach, and Coral Gables (in Florida) have a very large perennial supply of fresh water.

Areas of productive glacial outwash deposits include those in the Spokane Valley-Rathdrum Prairie area, in Washington and Idaho. The glacial outwash deposits, at least 500 ft thick in many places and very permeable, serve as the underground outlet for water from an arm of Lake Pend Oreille, and the underflow is estimated to be not less than 1,000 cu ft per sec, or about two thirds of a billion gallons per day (unpublished reports by A. M. Piper and L. C. Huff in the files of the U. S. Geological Survey). An average of about 100 cu ft per sec, or about 65 mgd is withdrawn for use in the Spokane Valley.

Glacial outwash deposits in the valleys of the Ohio River and some of its tributaries yield large supplies of water to wells at many places. There is much recharge from rainfall and melting snow, but the river forms a major source of recharge at some places where wells have been constructed near the river and are pumped heavily enough to lower the water table adjacent to the stream. Examples of such supplies (unpublished reports in the files of the U. S. Geological Survey) include the public water supply at Parkersburg, W. Va., and the water supplies at large war plants at Point Pleasant, W. Va., and Charlestown, Ind. At one war plant as much as 50 mgd has been pumped from seven wells in a 2-mile stretch along the banks of the Ohio River. Similar conditions exist in many other valleys. Many large industrial and municipal water supplies have been developed from wells that are so situated as to induce recharge from the streams, and many others undoubtedly will be developed in the future.

In the Mill Creek Valley Water Supply Project, installed by the Federal Works Agency, water is pumped from wells penetrating permeable glacial outwash sands and gravels in the Miami River Valley south of Hamilton, Ohio, and is piped to a large plant in the Mill Creek Valley to the southeast. The designed capacity of the system is 15 mgd. The wells are located so as to take advantage of recharge from precipitation in the immediate vicinity and to intercept ground water flowing westward from a ground-water divide at the head of the Mill Creek Valley; but considerable water will be recharged from the Miami River when wells near the river are pumped heavily (12).

In the Louisville (Ky.) area, large quantities of water are pumped from wells, mostly from wells penetrating glacial outwash deposits in the Ohio Valley. The average pumpage increased from about 37 mgd in 1937 and 1938 to about 62 mgd in 1943 (13). A large part of the total recharge, which is estimated to be from 30 mgd to 40 mgd, is believed to come from the Ohio River.

The pumpage at Louisville has been in excess of recharge for some years, and water has been withdrawn from storage with resulting declines in water levels. In 1943 from 20 mgd to 30 mgd was withdrawn from storage. Inasmuch as the outwash deposits are only about 100 ft thick, the storage has been greatly depleted, and substantial reduction in the rate of pumping will be necessary unless the supply can be greatly increased by wells near the river or by artificial recharge.

In the Platte River Valley of Nebraska, which is cut into deposits of sand, gravel, silt, and clay of glacial and fluviatile origin, recharge of the ground-water supply is largely from precipitation. There is some recharge from the river, however, when increased flows occur following the dry season and also during floods, whenever they occur (14)(15). During most of the year, however, the water table is above stream level and water moves underground toward the river rather than away from it. The contours of the water table show that water recharged from precipitation moves toward the river and discharges into it. However, the movement has a downstream component because of the slope of the valley, and thus the individual particles of water have an oblique path with respect to the river. The downstream component can be considered underflow; but it is small in comparison with the total amount of water recharged and discharged and can scarcely be compared with true underflow such as that in the Spokane Valley.

An example of productive water-bearing formations in which droughts may aggravate the effects of heavy pumping is the Indianapolis (Ind.) area. The principal aquifer consists of glacial outwash deposits, but these deposits overlie and furnish water to limestones from which a large quantity of water is also pumped. The effects of two successive years of drought, 1940 and 1941, combined with the effects of record pumping in 1941 to produce critical ground-water conditions in some parts of the area (16). The average pumpage in 1941 was about 52 mgd. During 1942, however, the precipitation was about normal, the pumpage of ground water decreased to about 47 mgd, and the over-all ground-water conditions improved considerably (16).

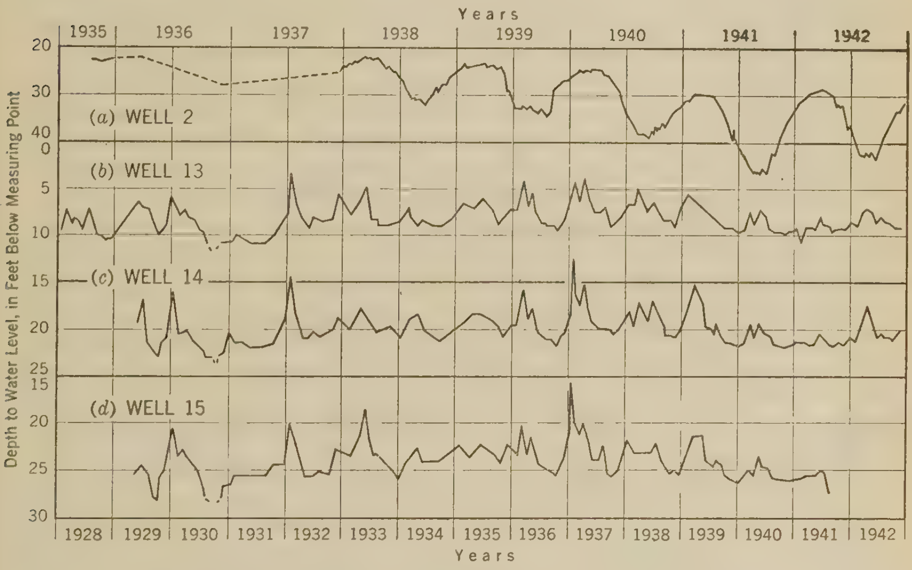

FIG. 5.—HYDROGRAPHS OF SELECTED WELLS, 1928–1942; INDIANAPOLIS, IND.

Wells 13, 14, and 15, Fig. 5, show fluctuations of the water table in an area where recharge is high and where pumping from wells is fairly heavy. The wells penetrate glacial outwash sand and gravel. A total of several million gallons per day is pumped from a gravel pit and from a number of drilled wells. The water levels in the wells were low in 1940 and 1941 as the result of drought and pumping. However, they were not as low as during the drought of 1930–1931. The water levels recovered considerably in 1942, but were much lower than in years of large recharge, such as 1932 and 1937. It is apparent that, because of a high average rate of recharge, the effects of pumping in this area are relatively slight except in years of drought.

Fluctuations in an area where the rate of recharge is relatively low are shown in Fig. 5(*a*). Well 2 penetrates glacial outwash sand and gravel in the downtown district of Indianapolis, where percolation of water from above is reduced or prevented by pavements and buildings and recharge largely results from lateral percolation from surrounding areas. As the result of heavy pumping, the water levels in wells showed net declines for several years and reached a low stage in 1941, when recharge was reduced by drought. In the summer of 1941 the pumping water levels in some of the wells (about 100 ft deep) penetrating sand and gravel in the downtown district were not far above the bottoms of the wells. As a result of increased recharge in 1942 the water levels rose somewhat but, because of continued heavy pumping, were considerably lower than in 1940 and previous years.

It is estimated that, since pumping began in the Indianapolis area, a total of 400,000,000,000 gal to 500,000,000,000 gal of water has been pumped from wells. Of this volume, about 97% is estimated to have come from recharge and only about 3% from storage. It is because of the dependence on a high rate of recharge to support the heavy pumping that the supply is affected so seriously by prolonged droughts.

These examples illustrate aquifers in which water-table conditions exist at most places. An example of an artesian aquifer with a high rate of recharge and discharge in the Ocala limestone of Florida and Georgia (9)(17)(18). This aquifer is recharged principally in its outcrop area in Georgia, Alabama, and South Carolina, and in two areas in northern and central Florida where recharge occurs through lakes and sinkholes. The water is discharged through submarine springs along the Atlantic and Gulf coasts and through large springs in north-central Florida. Silver Springs and Blue Springs and most of the other large springs in the Florida peninsula are derived from the Ocala limestone and associated limestones. Water supplies of many millions of gallons per day are derived from wells penetrating the Ocala limestone and associated limestones in Florida and Georgia, and are used for public water supply, irrigation, and industrial purposes.

The aquifer is not everywhere equally permeable. It is most permeable and productive in and near the recharge areas, and shows a general decrease in permeability away from the recharge areas. However, there are substantial differences in permeability in areas at approximately equal distances from the recharge areas. For example, at Brunswick, Ga., and Jacksonville, Fla., the Ocala limestone is relatively productive, but at Fernandina, Fla., between Jacksonville and Brunswick, the aquifer is somewhat less productive. In 1943, the draft at the three places ranged from 34 mgd to 40 mgd, as shown in Fig. 6(*a*), which also shows the pumpage at Savannah, Ga. In Fig. 6, the contours (interval, 10 ft) represent approximately the height to which water would rise above mean sea level in tightly cased wells penetrating the principal artesian aquifer in 1943 (Fig. 6(*a*)) and in the first wells ever drilled into the Ocala limestone (Fig. 6(*b*)). The piezometric surface was still above the land surface at Jacksonville and Brunswick, except in the immediate vicinity of pumped wells, whereas, at Fernandina, the piezometric surface was below the land surface in the heavily pumped area. The contrast between conditions at Fernandina and those at Brunswick and Jacksonville is shown strikingly in Fig. 6(*a*). The pumping at Brunswick has caused only a slight lowering of the

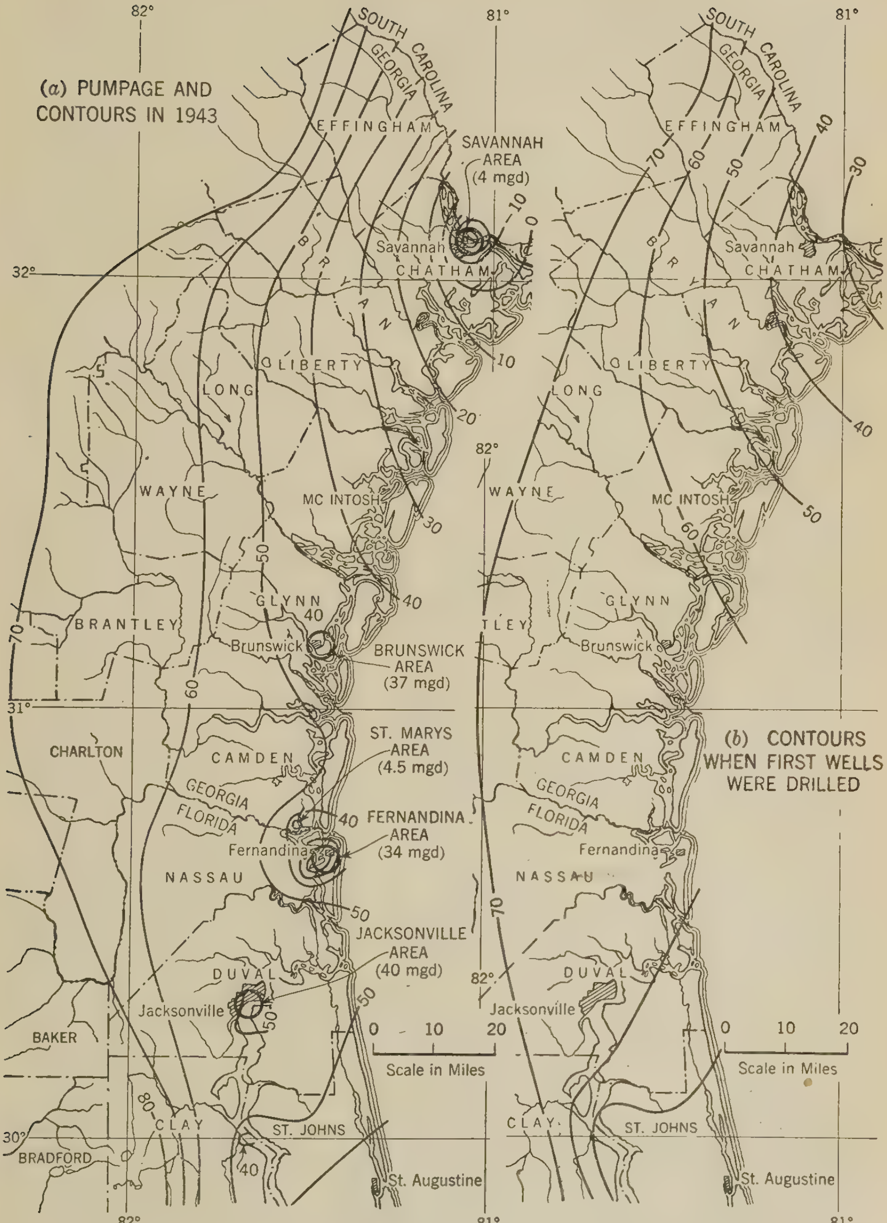
(a) PUMPAGE AND CONTOURS IN 1943
82°
81°
32°
31°
30°
SOUTH CAROLINA
GEORGIA
EFFINGHAM
SAVANNAH AREA (4 mgd)
Savannah
CHATHAM
BRYAN
LIBERTY
LONG
WAYNE
MC INTOSH
GLYNN
BRANTLEY
Brunswick
BRUNSWICK AREA (37 mgd)
CHARLTON
CAMDEN
ST. MARYS AREA (4.5 mgd)
GEORGIA
FLORIDA
FERNANDINA AREA (34 mgd)
Fernandina
NASSAU
JACKSONVILLE AREA (40 mgd)
DUVAL
Jacksonville
BAKER
CLAY
BRADFORD
ST. JOHNS
St. Augustine
0
10
20
30
40
50
60
70
80
0 10 20
Scale in Miles
(b) CONTOURS WHEN FIRST WELLS WERE DRILLED
SOUTH CAROLINA
GEORGIA
EFFINGHAM
Savannah
CHATHAM
BRYAN
LIBERTY
LONG
WAYNE
MC INTOSH
GLYNN
TLEY
Brunswick
CAMDEN
GEORGIA
FLORIDA
Fernandina
NASSAU
DUVAL
Jacksonville
CLAY
ST. JOHNS
St. Augustine
82°
81°
30
40
50
60
70
0 10 20
Scale in Miles

FIG. 6.—PIEZOMETRIC SURFACE OF THE OCALA LIMESTONE IN THE COASTAL AREA OF GEORGIA AND NORTHEASTERN FLORIDA

piezometric surface, whereas at Fernandina it has created a substantial cone of depression. The pumping at Jacksonville has created a cone of depression slightly deeper than that at Brunswick, but considerably shallower than that at Fernandina. A substantial cone of depression has formed at Savannah, where the productiveness of the aquifer is roughly comparable to that at Fernandina.

Where problems of depletion by pumping, not aggravated by drought, have developed in productive aquifers, the cause has generally been concentrated pumping from closely-spaced wells. In many such areas the remedy consists in wider spacing of wells and redistribution of pumpage. In the Baton Rouge (La.) area, as much as 50 mgd has been pumped from the so-called 400-ft and 600-ft sands in an area of about 6 sq miles—much of it in an area of not much more than 1 sq mile (unpublished reports in the files of the U. S. Geological Survey by J. C. Maher and G. C. Taylor, Jr.). Although the water-bearing sands are very productive, the concentrated pumpage in a small area has lowered the water levels in the wells to such an extent that it has been necessary to adopt emergency conservation measures at some of the important industrial plants to insure continued production. Among these measures is the development of supplementary water supplies from the Mississippi River. The ground-water problem in the Baton Rouge area can be solved by distributing the wells that penetrate the 400-ft and 600-ft sands over a larger area and by developing additional water from other sources, including deeper sands in the heavily pumped area and the virtually untouched deposits of alluvium adjacent to the Mississippi River.

Examples of Aquifers with Relatively Small Perennial Supplies

An example of a less productive aquifer which has a fairly high potential rate of recharge but which is strongly affected by drought is the shallow aquifer in the Elizabeth City (N.C.) area (19). The aquifer consists of surficial sands of Pleistocene age, about 30 ft thick. The climate is humid, the average rate of recharge is fairly high, and a moderately large quantity of water can be developed from a system consisting of numerous wells with small individual yields distributed over a large area. A supply of this kind has been developed for Elizabeth City. Because of the small saturated thickness of the aquifer, the successful development of a large supply depends on sustained recharge from precipitation. During prolonged droughts, the small amount of storage is quickly depleted, but with the return of wet weather the available storage space is refilled.

Among the aquifers with a low average rate of recharge, where heavy withdrawals are largely from storage, is the Ogallala formation, the principal aquifer underlying the High Plains. The formation consists largely of a sheet of alluvium deposited in Tertiary time, overlying older dissected rocks. It ranges in thickness from a featheredge at the border of the plains to more than 500 ft in some places. The formation, especially the lower part, consists largely of sand and gravel and contains an enormous amount of water in storage.

Investigations of the hydrology of the Ogallala formation by the U. S. Geological Survey and cooperating agencies are under way in most of the area underlain by the formation (20)(21)(22)(23)(24)(25)(26)(27)(28)(29). In general, they show that the formation is recharged by precipitation but that the rate of recharge is low. In different places the annal recharge from precipitation probably ranges from a fraction of an inch to a few inches of water.

A report by W. L. Broadhurst and W. H. Alexander, Jr. (20), in 1944, shows the situation that exists in the High Plains in Texas. From 1935 to the end of 1943 the number of irrigation wells in this region increased from about 300 to about 2,950, and the area irrigated increased from 35,000 acres to 400,000 acres, or about 13% of the total area within the boundaries of the irrigated districts. From 1937 to 1941, a period of subnormal precipitation, the increase in irrigation was accompanied by a persistent decline of the water table (Fig. 7). During 1941 the precipitation was heavy, the greatest on record in parts of the region. Consequently, the requirements for irrigation were small, recharge to the underground reservoirs was unusually large, and observation wells in each pumping district showed a pronounced rise in water level. During 1942 the precipitation was again above average, pumping was again light, and the water levels in most of the wells either remained nearly stationary or rose slightly. During 1943 the precipitation was light, the pumping was greater than ever before, and practically all the wells showed a decline in water level (Fig. 7).

It should be stated that the rainfall in 1941 was exceptional and the pumping for irrigation in both 1941 and 1942 was light. The recharge that occurred in 1941 was probably greater than in any year since 1915 and several times the recharge in most years (21). Nevertheless, net declines in water level occurred in the Plainview and Hereford districts; and, although net rises occurred in the Lubbock-Littlefield and Muleshoe districts, the water levels generally resumed their downward trend in 1943. It is evident that the rate of pumping will eventually have to be curtailed greatly in this region.

The Dakota sandstone (which underlies most of North Dakota and South Dakota and considerable parts of near-by states, extending to the south as far as New Mexico) is an aquifer in which water is confined under pressure by thick beds of younger rocks, mostly shale (30). The sandstone crops out at relatively high altitudes in the foothills of the Rockies and those of the Black Hills, and is recharged in these areas. The artesian pressure was 200 lb per sq in., oı even more, at some places when the first wells were drilled, and the wells had initial flows of several hundred gallons per minute (31)(32)(33)(34)(35). However, the average permeability of the sandstone is rather low, the specific capacity of the wells—that is, their yield per unit of drawdown—is small, and most of the water represented by the large initial yields was derived from storage by compaction of the beds as the head was lowered.

The high initial yields of the wells gave a false impression of the productiveness of the aquifer and encouraged the drilling of many wells and the withdrawal of excessive quantities of water throughout the Dakota artesian basin. However, as the artesian head was lowered, the yield of the wells decreased; and in some areas the yield is slowly approaching the perennial rate of recharge. For example, in one row of townships in the Ellendale-Jamestown area of North Dakota, the total yield of flowing wells penetrating the Dakota sandstone was about 520 gal per min in 1938. The estimated perennial recharge is about 500

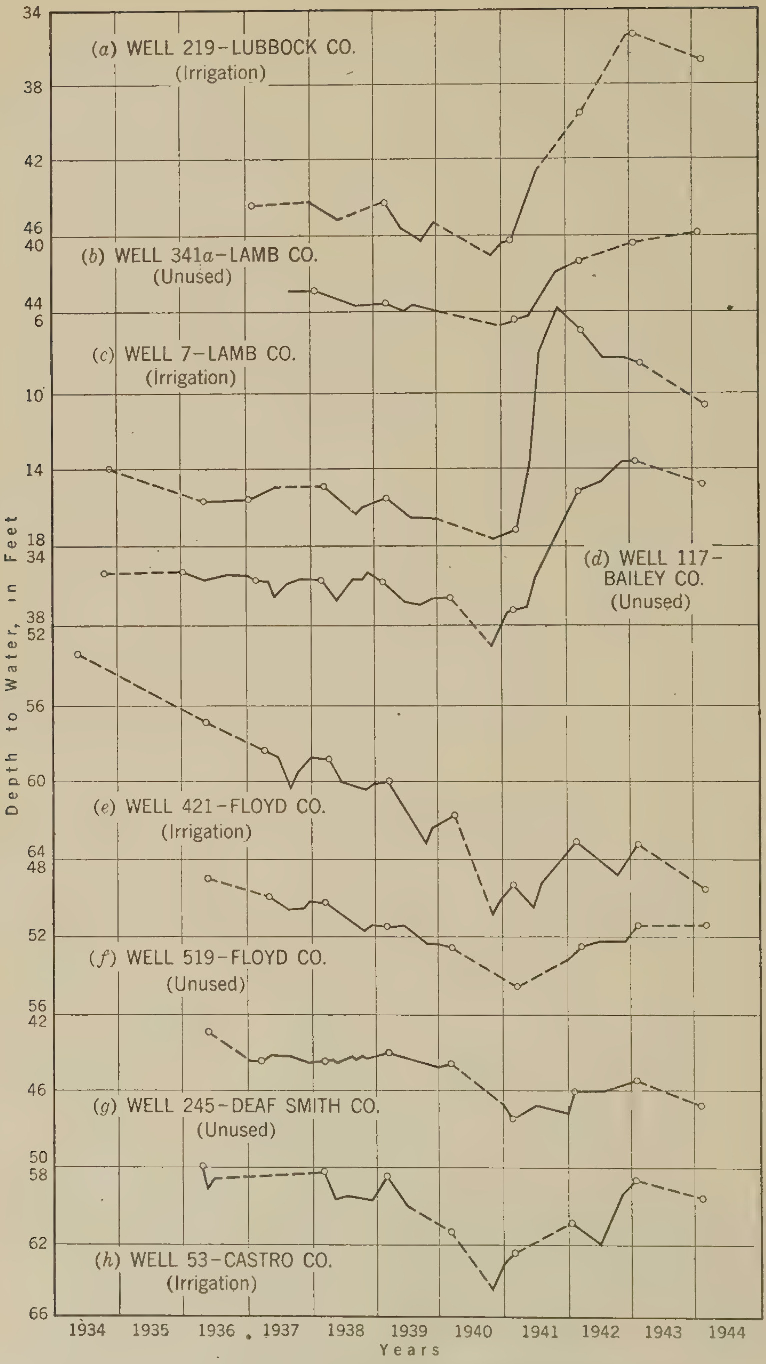

Fig. 7.—Hydrographs of Wells in the High Plains of Texas

gal per min (30). The yield of the wells in this row of townships in 1923 was about 1,000 gal per min, and the total yield when the wells were being drilled, largely between 1900 and 1920, was doubtless much greater.

Somewhat analogous conditions are found in the region in Illinois, Iowa, and Wisconsin underlain by Cambrian and Ordovician sandstones, although the artesian head was never as high as that in the Dakota Basin, partly because of a smaller difference in altitude between the outcrop areas and the areas of withdrawal. Comparatively few detailed investigations have been made of these aquifers, but great declines in water level have occurred in large parts of the region and much of the water withdrawn has been derived from storage (36)(37). The head of a well drilled at Chicago, Ill., in 1864 was 80 ft above the surface and about 692 ft above sea level (36). This well penetrated the Galena-Platteville limestone, which doubtless had a head lower than that of the deeper Cambrian sandstones that have since been developed so heavily. In 1943, according to data issued by the Illinois State Water Survey, the water levels in some wells at Chicago, penetrating the Cambrian sandstones, were at altitudes as low as 230 ft above sea level, and the decline undoubtedly continued during 1944 (Fig. 8).

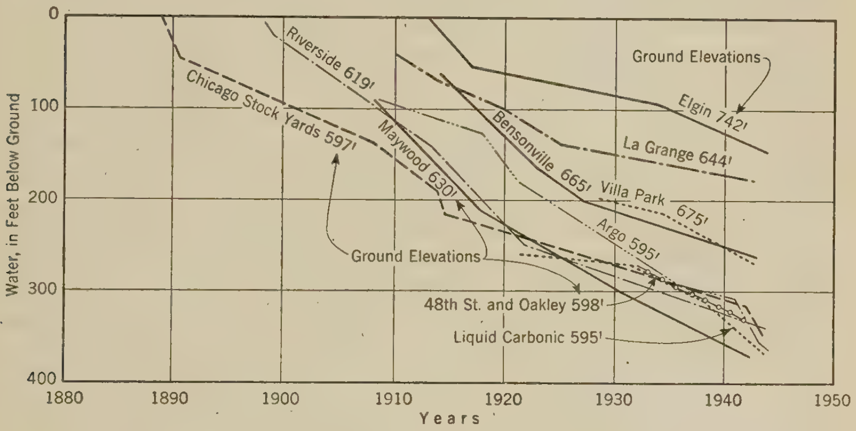

FIG. 8.—DECLINE OF WATER LEVELS IN WELLS IN AND NEAR CHICAGO, ILL., 1899–1943.

The Mimbres Valley in New Mexico has a very large quantity of water in subterranean storage and many productive wells. It was assumed to have a large perennial supply until critical investigation showed that the average annual recharge is actually quite small. The valley consists of a structural basin, floored with relatively impervious consolidated rocks and filled with alluvial deposits of gravel, sand, silt, and clay, derived from the adjacent mountains (38). The ground-water supply is recharged largely by the Mimbres River, which sinks into the alluvium after leaving the mountains. Rainfall on the floor of the valley and return flow from irrigation contribute relatively minor quantities of water. The average annual recharge was estimated to be about 10,000 acre-ft for the period from 1908 to 1928—an average of about 9 mgd. Records of water levels in wells, and of stream flow, show that recharge was slight during the period from 1929 to 1940, inclusive. Even in 1941, an exceptionally wet year, the recharge was not very great. Recharge from the Mimbres River is probably less than it was formerly because of the greater use or river water for irrigation in the headwater areas.

The total pumpage, largely for irrigation, increased from about 10,000 acre-ft per yr during the period from 1929 to 1933 to about 25,000 acre-ft in 1940 (an average of about 22 mgd), and decreased only to about 21,000 acre-ft in the exceptionally wet year of 1941. The records of water levels in wells show that most of the water pumped has been derived from storage in the subterranean reservoir. Even in 1941 there was a net loss in storage of several thousand acre-feet in the valley as a whole.

The Mimbres Valley is one of the areas in New Mexico in which legal control over the ground-water supply is exercised by the state. Although the pumpage is in excess of the average rate of recharge, the decline in ground-water levels at the present rate of pumping will be gradual, and the present policy is not to reduce the pumpage but simply to restrict additional developments.

Legal control of the use of ground water is a broad subject that cannot be treated adequately in this paper, but an example of a sound approach may be cited—namely, the New Mexico ground-water law just mentioned in connection with the Mimbres Valley. The law was an outgrowth of the overdevelopment of the artesian water supply of the Roswell basin in New Mexico. In this area the high initial yields of flowing wells penetrating cavernous Permian limestone encouraged the drilling of so many wells and the withdrawal of so much water that the artesian head declined sharply; wells ceased flowing along the margin of the flowing well area, which was becoming smaller and smaller; and large investments in land were threatened. The U. S. Geological Survey, in cooperation with the state engineer, made a thorough investigation of the region, determined the perennial safe yield of the artesian aquifer, and assisted state officials in drafting a law for the control of the ground-water supply. The law was enacted and has been enforced effectively by the state engineer with the cooperation of the ground-water users; thus the valuable artesian water supply is both efficiently developed and secure (39)(40). Conservation of water is effected by preventing wasteful use of water for irrigation and by reducing loss of water from the limestone into the overlying valley fill by locating and plugging leaky wells.

The New Mexico ground-water law is regarded as a model. Its development was made possible by the thorough investigation of the ground-water supply of the aquifer concerned and the determination of the rate at which water could be withdrawn perennially. Unless a good estimate of the perennial water supply of an aquifer has been made available through adequate investigation, a law enacted for control of the water supply of the aquifer is likely to be difficult or impossible to enforce.

Probably the most serious problems of ground-water depletion will arise in the areas underlain by aquifers that have low rates of recharge but from which wells can obtain initial yields of several hundred gallons per minute—or even 1,000 gal per min—giving the impression of perennial supplies more abundant than they actually are, and leading to overoptimistic and excessive development. The impression of abundance may be caused by high permeability, as in the Mimbres Valley, or to high artesian head, as in the Dakota artesian basin.

ARTIFICIAL RECHARGE

A method of conserving ground-water supplies that will be used more and more in the future is artificial recharge, in which water from streams or water pumped from wells is recharged to the ground-water reservoirs (41). Artificial recharge has been practiced on a large scale for some years in California, where recharge has been accomplished by passing water from streams or other bodies of surface water over the ground—in ditches, in furrows, in basins, or in a thin sheet, by a method commonly known as "water spreading." The infiltration galleries from which the public water supply of Des Moines, Iowa, is obtained are recharged artificially from shallow basins excavated over the galleries. Similar projects are in operation at a few other places in the eastern part of the United States, including Dayton, Ohio. Recharge from an artificial lake is accomplished in the Duhernal water-supply development near Old Bridge, N. J.

A method that has been used on a large scale in relatively few places is artificial recharge through wells. The most notable example is on western Long Island in New York, where water pumped for cooling from wells at more than a certain minimum rate must be returned to the ground through wells constructed for the purpose (42)(43). The New York State Water Power and Control Commission has the authority to require similar practices elsewhere on Long Island if necessary.

At Louisville where the present pumpage of ground water is in excess of the recharge and the ground-water levels are declining, artificial recharge of certain wells was undertaken at the suggestion of the U. S. Geological Survey and through the efforts of the War Production Board and the industries concerned. This recharge permitted continued production of industrial alcohol at certain plants whose production otherwise would have had to be curtailed because of lack of water (44). Wells are also being recharged on a fairly large scale at Indianapolis (16), at a large industrial plant in Virginia, and probably at several other places. It is to be expected that, at many places, wells will be recharged with cold filtered surface water in the winter, where the value of ground water for cooling purposes makes this practice economically feasible. This type of recharge is practiced at Louisville and at the industrial plant in Virginia.

METHODS OF INVESTIGATION

The hydrology of ground-water reservoirs is a somewhat complicated subject, and each reservoir has its own peculiar characteristics, due both to natural conditions and to the extent and distribution of the ground-water developments. Thus, each reservoir must be given individual study. The science of ground-water hydrology has advanced greatly in recent years, and methods are becoming available for the comprehensive study of the major ground-water reservoirs that will show the quantities of water that are perennially available and how to develop and utilize them efficiently.

The U. S. Geological Survey is engaged actively in the development of methods for making pumping tests and analyzing the results with respect to: (*a*) The quantities of ground water that are available in specific areas, (*b*) the extent of interference between different wells or groups of wells, and (*c*) the additional supplies that can be developed by proper distribution of the pumpage (45)(46)(47). Important developments have been made in recent years on the basis of the Theis nonequilibrium method, which permits the determination, from simple pumping tests, of the coefficients of transmissibility and storage of an aquifer—that is, the rate at which water is transmitted under a given gradient and the quantity of water released from storage with a given lowering in head. These two fundamental coefficients are then utilized to predict the effects of pumping given installations of wells penetrating the aquifer. Refined methods are coming into use for analyzing the results of pumping tests in aquifers that are limited by relatively impermeable boundaries within a short distance from the wells or in aquifers whose sources of recharge are near by. Under favorable conditions the results of pumping tests can even be used to determine the presence of geologic structures, such as a fault or other barrier that may not be apparent from surface observation or well records.

Conclusion

Deforestation, cultivation of land, drainage, and similar practices have undoubtedly affected ground-water supplies to some extent in the areas involved, but there is no reason to believe that the effects have been progressive over long periods of years in most parts of the United States. The effects of pumping or natural flow from wells are of great concern; and, as has been shown by the numerous examples of aquifers of different kinds that have been described, the conservation and development of ground-water supplies in the future will require thorough investigation of each major aquifer and effective legal control of the withdrawals from the more heavily developed aquifers. In some places heavy withdrawals from wells are depleting the ground-water supply and cannot be continued indefinitely; in many other places recharge is large and therefore large supplies will be perennially available to wells.

APPENDIX. BIBLIOGRAPHY

(1) **"Symposium on Fluctuations of Ground-Water Level,"** *Transactions*, Am. Geophysical Union, Vol. 17, 1936, Pt. II, p. 337. (*a*) "Review of the Work of W. J. McGee on Ground-Water Levels," by O. E. Meinzer, pp. 386–390. (*b*) "Fluctuations in Ground-Water at Woodgate, N. Y.," by E. S. Cullings, p. 359, Fig. 2.

(2) **"Our Water Supply,"** by O. E. Meinzer, *Journal*, Washington Academy of Sciences, Vol. 27, No. 3, March 15, 1937.

(3) **"Ground Water in the United States, a Summary,"** by O. E. Meinzer, *Water-Supply Paper No. 836-D*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1939.

(4) **"Ground Water, a Vital National Resource,"** by O. E. Meinzer, *Journal*, A. W. W. A., November, 1942, pp. 1595–1606.

(5) **"Protection of Ground-Water Supplies,"** by O. E. Meinzer, *Johnson National Drillers Journal*, September–October, 1944, pp. 10–11 and 14.

(6) **"Occurrence, Origin, and Discharge of Ground Water,"** by O. E. Meinzer, Physics of the Earth Series, Vol. 9, "Hydrology," McGraw-Hill Book Co., Inc., New York, N. Y., 1942, Chapter 10-A.

(7) **"Movement of Ground Water and Its Relation to Head, Permeability, and Storage,"** by O. E. Meinzer and L. K. Wenzel, *ibid.*, Chapter 10-B.

(8) **"The Source of Water Derived from Wells,"** by C. V. Theis, *Civil Engineering*, May, 1940. pp. 277–280.

(9) **"Ground-Water Investigations in Florida, with Special Reference to Duval and Nassau Counties,"** by H. H. Cooper, Jr., *Journal*, A. W. W. A February, 1944, pp. 169–185.

(10) **"Water Levels and Artesian Pressure in Observation Wells in the United States in 1940,"** by F. C. Ebert, *Water-Supply Paper No. 911*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1941, Pt. 6, Section on California.

(11) **"Interim Report on the Investigation of Water Resources in Southeastern Florida, with Special Reference to the Miami Area in Dade County,"** by G. G. Parker, G. E. Ferguson, and S. K. Love, Florida Geological Survey (in cooperation with U. S. G. S., Dade County, and the cities of Miami, Miami Beach, and Coral Gables), June, 1944 (mimeographed).

(12) **"A Quantitative Study of the Well Fields of the Mill Creek Valley Water Supply Project, Butler County, Ohio,"** by F. H. Klaer, Jr., and R. G. Kazmann, U. S. G. S. (in cooperation with Federal Works Agency), July, 1943 (hectographed).

(13) **"Progress Report on the Ground-Water Resources of the Louisville Area, Kentucky,"** by W. F. Guyton, W. T. Stuart, and G. B. Maxey, U. S. G. S. (in cooperation with Kentucky Dept. of Mines and Minerals and the City of Louisville), March, 1944 (mimeographed).

(14) **"Geology and Ground-Water Resources of South-Central Nebraska, with Special Reference to the Platte River Valley Between Chapman and Gothenburg,"** by A. L. Lugn and L. K. Wenzel, *Water-Supply Paper No. 779*, U. S. G. S. (in cooperation with Conservation and Survey Div., Univ. of Nebraska, Lincoln), Govt. Printing Office, Washington, D. C., 1938.

(15) **"Ground Water in Keith County, Nebraska,"** by L. K. Wenzel and H. A. Waite, *Water-Supply Paper No. 848*, U. S. G. S. (in cooperation with Conservation and Survey Div., Univ. of Nebraska, Lincoln), Govt. Printing Office, Washington, D. C., 1941.

(16) **"Ground-Water Resources of the Indianapolis Area, Marion County, Indiana,"** by C. L. McGuinness, Indiana Div. of Geology (in cooperation, with U. S. G. S.); January 1, 1943 (mimeographed).

(17) **"Artesian Water in the Florida Peninsula,"** by V. T. Stringfield, *Water-Supply Paper No. 773-C*, U. S. G. S. (in cooperation with Florida Geological Survey), Govt. Printing Office, Washington, D. C., 1936.

(18) **"Artesian Water in the Coastal Area of Georgia and Northeastern Florida,"** by V. T. Stringfield, M. A. Warren, and H. H. Cooper, Jr., *Economic Geology*, November, 1941, pp. 698–711.

(19) **"Geology and Ground-Water Resources of the Elizabeth City Area, North Carolina,"** by S. W. Lohman, *Water-Supply Paper No. 773-A*, U. S. G. S. (in cooperation with North Carolina Dept. of Conservation and Development), 1936.

(20) **"Progress Report on Ground Water in the High Plains in Texas,"** by W. L. Broadhurst and W. H. Alexander, Jr., Texas State Board of Water Engineers (in cooperation with U. S. G. S.), 1944 (mimeographed), pp. 11–12.

(21) **"Water Levels and Artesian Pressure in Observation Wells in the United States in 1942,"** by R. W. Sundstrom, *Water-Supply Paper No. 947*, Govt. Printing Office, Washington, D. C., 1944, Pt. 4, Section on Texas.

(22) **"Geology and Ground-Water Resources of Cimarron County, Oklahoma,"** by S. L. Schoff, *Bulletin No. 64*, Oklahoma Geological Survey (in cooperation with U. S. G. S.), 1943.

(23) **"Geology and Ground-Water Resources of Texas County, Oklahoma,"** by S. L. Schoff, *Bulletin No. 59*, Oklahoma Geological Survey (in cooperation with U. S. G. S.), 1939.

(24) **"Geology and Ground-Water Resources of Hamilton and Kearny Counties, Kansas,"** by T. G. McLaughlin, *Bulletin No. 49*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), December, 1943.

(25) **"Geology and Ground-Water Resources of Meade County, Kansas,"** by J. C. Frye, *Bulletin No. 45*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1942.

(26) **"Geology and Ground-Water Resources of Stanton County, Kansas,"** by B. F. Latta, *Bulletin No. 37*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1941.

(27) **"Geology and Ground-Water Resources of Morton County, Kansas,"** by T. G. McLaughlin, *Bulletin No. 40*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1941.

(28) **"Geology and Ground-Water Resources of Ford County, Kansas,"** by H. A. Waite, *Bulletin No. 43*, Kansas State Geological Survey (in cooperation with U. S. G. S. and Kansas State Boards of Health and Agriculture), 1942.

(29) **"Geology and Ground-Water Resources of Box Butte County, Nebraska,"** by R. C. Cady and O. J. Scherer, U. S. G. S. (in cooperation with Conservation and Survey Div., Univ. of Nebraska, Lincoln, in course of publication as a *Water-Supply Paper*).

(30) **"Water Supply of the Dakota Sandstone in the Ellendale-Jamestown Area, North Dakota, with Reference to Changes Between 1923 and**

1938," by L. K. Wenzel and H. H. Sand, *Water-Supply Paper No. 889-A*, U. S.G. S. (in cooperation with North Dakota Geological Survey), Govt. Printing Office, Washington, D. C., 1942.

(31) **"Artesian Waters in the Vicinity of the Black Hills,"** by N. H. Darton, *Water-Supply Paper No. 227*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1918.

(32) **"The Artesian-Water Supply of the Dakota Sandstone in North Dakota, with Special Reference to the Edgeley Quadrangle,"** by O. E. Meinzer and Herbert A. Hard, *Water-Supply Paper No. 520-E*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1925.

(33) **"Preliminary Report on the Geology and Underground Water Resources of the Central Great Plains,"** by N. H. Darton, *Professional Paper No. 32*, U. S. G. S., Govt. Printing Office, Washington, D. C., 1905.

(34) **Geology and Water Resources of the Northern Portion of the Black Hills and Adjoining Regions in South Dakota and Wyoming,"** by N. H. Darton, *Professional Paper No. 65*, U. S. G. S., Govt. Printing Office, Washington, D. C.

(35) **"Preliminary Description of the Geology and Water Resources of the Southern Half of the Black Hills and Adjoining Regions in South Dakota and Wyoming,"** by N. H. Darton, *Twenty-First Annual Report*, U. S. G. S., Govt. Printing Office, Washington, D. C., Pt. IV(b), 1901.

(36) **"The Artesian Waters of Northeastern Illinois,"** by C. B. Anderson, *Bulletin No. 34*, Illinois State Geological Survey, 1919.

(37) **"Ground Water Supplies of the Chicago-Joliet-Chicago Heights Area,"** *Bulletin No. 35*, Illinois State Water Survey, 1943.

(38) **"Progress Report on the Ground-Water Supply of Mimbres Valley, New Mexico, 1938-41,"** by C. S. Conover and P. D. Akin, State Engr. of New Nexico (in cooperation with U. S. G. S., in course of publication as part of 14th and 15th biennial repts. of the State Engr.).

(39) **"Geology and Ground-Water Resources of the Roswell Artesian Basin, New Mexico,"** by A. G. Fiedler and S. S. Nye, *Water-Supply Paper No. 639*, U. S. G. S. (in cooperation with State Engr. of New Mexico), Govt. Printing Office, Washington, D. C., 1933.

(40) **"Report of Artesian Well Supervisor,"** State Engr. of New Mexico, 12th and 13th biennial repts., 1934–1938.

(41) **"Artificial Recharge of Ground Water,"** by D. G. Thompson, U. S. G. S., February 10, 1942 (mimeographed circular).

(42) **"Ground-Water for Air Conditioning on Long Island, New York,"** by M. L. Brashears, Jr., and R. M. Leggette, *Transactions*, Am. Geophysical Union, Vol. 19, 1938, Pt. I, pp. 412–418.

(43) **"Ground-Water Temperature on Long Island, New York, as Affected by Recharge of Warm Water,"** by M. L. Brashears, Jr., *Economic Geology*, December, 1941, pp. 811–829.

(44) **"Conservation of Ground Water in the Louisville Area, Kentucky,"** by W. F. Guyton and Hazel E. Sublett, Kentucky Dept. of Mines and Minerals (in cooperation with U. S. G. S.), 1944 (mimeographed).

(45) **"Methods for Determining Permeability of Water-Bearing Materials, with Special Reference to Discharging-Well Methods,"** by L. K. Wenzel, *Water-Supply Paper No. 887*, Govt. Printing Office, Washington, D. C., 1942.

(46) **"Some Field Applications of Water Transmissibility and Storage Coefficients,"** by R. G. Kazmann, *Agricultural Engineering*, August, 1944, pp. 299–300 and 304.

(47) **"Drawdown Test to Determine Effective Radius of Artesian Well,"** by C. E. Jacob, *Transactions*, ASCE, Vol. 112, 1947, pp. 1047–1064.

(48) **"Induced Infiltration Supplies Most Productive Well Field,"** by RAPHAEL G. KAZMANN, *Civil Engineering*, December, 1946, p. 544.

DISCUSSION

E. A. VAUBEL,[3] Assoc. M. ASCE.—The thought that depletion of ground-water supplies does not occur suddenly but results from hydrological withdrawal from storage at rates greater than recharge is strikingly presented in this paper. Because many variables enter into a decline in water levels and differ somewhat even at two points in a restricted area of a few square miles, it is reasonable that many people might assume that declining levels would spell exhaustion of ground-water supplies within a few years.

As the author states, the properties of the individual aquifers determine how much water can be drawn from them by wells. When progressive declining levels occur due to a concentrated pumping requirement in a limited area, it is certainly proper to consider and to put into effect certain engineering concepts which promote effective utilization and conservation of ground-water resources. These concepts are:

1. Proper well spacing, projected by use of the Theis nonequilibrium formula applied to measured production and pumping levels of existing adjacent wells and pilot wells, can enable large municipal and large industrial users to secure their ground-water requirements without fear of depletion.
2. Sometimes in an existing well field, a redistribution of pumpage to equalize the effect of the water levels will help to forestall a rapid decline in a given sector of the field. This is attained by taking production from outlying wells in order to enlarge and equalize the withdrawal per given unit of area. The piezometric surface then tends to be devoid of the depression cones created by excessive pumpage in a small area in a well field. In terms of operation procedure, it may be stated that the quantities withdrawn from outlying wells should be equivalent to those withdrawn from strategically located wells in the center of the well field.
3. In many areas additional exploration will reveal the exact depth and thickness of aquifers and permit withdrawal of water for piping from a relatively virgin area to the point of use, without undue economic expense. A planned test drilling program is advisable when considering a long range ground-water supply requirement so as to be certain that the extent of the aquifer is definitely known.
4. Where known sands afford water of different mineral quality, it is expedient and within good conservation practice to utilize the better quality of water for boiler and domestic use. By this is meant water of low hardness and with chlorides generally within the United States Public Health Service standards. Where supplies of greater hardness and chloride content occur, these can generally be better utilized for certain industrial processes and cooling.
5. In serving the water supply needs of a large community the fact must not be overlooked that certain surface streams may supplement a ground-water supply program or vice versa. Sometimes there is a tendency to balance production, storage, and distribution costs of one against the other, although both

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[3] Engr., The Layne Texas Co., Ltd., Houston, Tex.

may be required to supply the present and future needs of a large and growing center of population adequately. Surface and ground water each have inherent advantages and disadvantages; the advantages of each are sometimes necessary to complete the water supply picture.

The points outlined, which are suggested procedure for attacking a ground-water supply depletion problem, were covered in part by the author, but item 5 supplements his presentation. Delineation by items serves to emphasize the points covered in detail and by example in the paper.

The Houston, Tex., area presents an example of an aquifer which has a high rate of recharge due to heavy precipitation, a large outcrop area with average permeability, and a series of sands totaling approximately 600 ft in thickness; and which, as a result, affords a large perennial supply. However, the discharge required from the aquifer is becoming increasingly greater due to industrial and municipal growth and irrigation usage. Therefore, the previously enumerated factors will have to be utilized to provide ample quantities for future water supply requirements.

Raphael G. Kazmann,[4] Assoc. M. ASCE.—An excellent summary of the present status of prevailing thought on the permanence and productivity of ground-water supplies is contained in this paper. Unfortunately the paper is too short to do more than summarize various situations. To focus the attention of the profession on the economic implications resulting from just one study of a specific ground-water situation, reference is made to the High Plains of Texas where, in 1943, 400,000 acres of land were irrigated from wells. Under the heading, "Examples of Aquifers with Relatively Small Perennial Supplies," the author states: "It is evident that the rate of pumping will eventually have to be curtailed greatly in this region."

Although the author does not state how long "eventually" may be, it is probable that within the next generation violent changes in the agricultural activity of the area will result from a shortage of ground water. A number of farmers will lose the major part of their capital investments in pumping plants, distribution systems, and farm buildings as a result of ground-water depletion; yet a sound, controlled program of ground-water development would enable a smaller amount of irrigation farming to continue indefinitely in the High Plains area.

In the other direction are the economic implications of the possibilities of increasing the usable ground-water supplies by improved methods of ground-water extraction. Mr. McGuinness states (under the heading, "Examples of Aquifers with High Rates of Recharge and Large Perennial Supplies"):

> "There is much recharge from rainfall and melting snow, but the [Ohio] river forms a major source of recharge at some places where wells have been constructed near the river and are pumped heavily enough to lower the water table adjacent to the stream. Examples of such supplies * * * include the public water supply at Parkersburg, W. Va., and the water supplies

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[4] Hydrologic Engr., Ranney Method Water Supplies, Inc., Columbus, Ohio.

> at large war plants at Point Pleasant, W. Va., and Charlestown, Ind. At one war plant as much as 50 mgd has been pumped from seven wells in a 2-mile stretch along the banks of the Ohio River. * * * Many large industrial and municipal water supplies have been developed from wells that are so situated as to induce recharge from the streams, and many others undoubtedly will be developed in the future."

Mr. McGuinness did not mention that the "wells," referred to at the war plants, were a relatively new development in ground-water engineering—that is, the radial, horizontal water collectors which are the equivalent of infiltration galleries laid on the bottom of an aquifer. The special virtue of these units is their capacity to operate with utmost efficiency in relatively thin aquifers where conventional wells are impracticable because of the smallness of the available drawdown. Horizontal collectors have their screens projected, horizontally, as close as possible to the bottom of the aquifer. Thus, they operate with equal efficiency as long as a little water remains in the aquifer. In the installations supplied by the infiltration of river water, the potential size of the differential head between the river and the collector or infiltration gallery is a maximum, thus making the supply as large as possible, and permanent as well.

Probably the outstanding installation of horizontal infiltration collectors was made in 1942 at the Wabash River Ordnance Works near Clinton, Ind., where an average production of 72 mgd and a maximum production of almost 89 mgd was obtained from six units (48).

The value of the additional, permanent ground-water supplies to municipalities and industries is incalculable. However, the horizontal infiltration collector, or any other type of infiltration gallery, although it will develop many new ground-water supplies, is certainly not the last technical advance which will affect the economic availability of ground water. The history of ground-water development shows that improved drilling and pumping methods have increased available ground-water resources. The horizontal water collector too has increased the potential ground-water supply by making thin infiltration aquifers economically available. Finally, a lessening of power costs through the use of new sources of energy would tend to agument available ground-water supplies.

Thus, from an engineering viewpoint, the principal addition needed in Mr. McGuinness' excellent paper is a short section dealing with the effect of technology in increasing the potentially useful quantity of ground water. Applied hydrology must consider engineering techniques.

Charles L. McGuinness,[5] Esq.—Mr. Vaubel makes several interesting and useful comments from the standpoint of the development and operation of water supply systems based on wells. He points out that proper well spacing, using the nonequilibrium formula as adapted by Charles V. Theis, is necessary. It might be emphasized that the use of the unmodified Theis formula is quite successful in many parts of the Coastal Plain in Texas and other coastal states, where extensive aquifers are typical. In many places where the aquifers are bounded—or where sources of recharge exist—at short distances from the wells, modification of the formula by intricate mathematical methods may be necessary.

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[5] Geologist, U. S. Geological Survey, Washington, D. C.

The statement that water of poor chemical quality should be used for purposes in which quality is relatively unimportant deserves great emphasis, especially in the many areas where water of the best quality is limited in quantity. Equally important is the statement concerning the need to consider both ground-water supplies and surface-water supplies in designing each installation.

Mr. Kazmann points out that it was not specified how long was meant by "eventually" in stating that curtailment of irrigation pumping in the High Plains of Texas will eventually be necessary. The length of time depends on the extent to which irrigation pumping, so far entirely uncontrolled, continues to increase. Obviously, adequate legal control by the states of ground-water withdrawals, based on quantitative estimates of the long-term yield obtained by thorough investigation, would go a long way in preventing undue hardship and excessive financial losses in such situations as that in the High Plains.

Mr. Kazmann also points out that the original paper was deficient in not containing a discussion of the effects of technological developments on the availability of ground-water supplies. Such developments in the past, especially the improvement of pumps, have greatly increased the amount of water economically available from wells. They have even led to overpumping in some areas where the aquifers are permeable but the rate of recharge is low. However, with the constant development of improved methods for determining the safe yields of aquifers and with the growth of effective and equitable legal control of ground-water withdrawals, technological progress in methods of extracting ground water may be expected to be entirely beneficial in the future.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 2319

MODEL STUDY OF BROWN CANYON DEBRIS BARRIER

By Karl J. Bermel,[1] Assoc. M. ASCE, and
Robert L. Sanks,[2] Jun. ASCE

With Discussion by Messrs. Clifford A. Betts, Roger E. Amidon, Ettore Scimemi, J. W. Johnson, and Karl J. Bermel and Robert L. Sanks.

Synopsis

The principal hydraulic tests of a 1:50 scale model of a debris barrier and the contiguous channel, with some corroborative prototype data, are included in this paper. Tests of the barrier include those for determining the spillway capacity, the performance of the overflow, the development of a stepped overflow crest, and the operation of the barrier with and without detrital material impounded upstream from the barrier. Tests in the channel cover the problems of scour downstream and deposition upstream from the barrier. Because of meager field data, especially regarding bed load and rates of transportation, the initial tests of deposition upstream from the barrier were based on bed load and rates of transportation as developed in the laboratory. These results were reasonably verified in subsequent tests, based on data obtained from the first major storm after completion of the prototype structure.

A discussion of these data was presented before the Waterways Division at the Los Angeles (Calif.) Meeting of the Society in July, 1943.

Introduction

As part of the effort to reduce flood damage in the heavily populated and highly developed metropolitan area of southern California, the U. S. Forest Service selected the Arroyo Seco watershed to inaugurate a comprehensive program of upstream flood control. The work in general is based on the findings and recommendations in a report by the U. S. Department of Agriculture dated January 14, 1939.[3]

Note.—Published in May, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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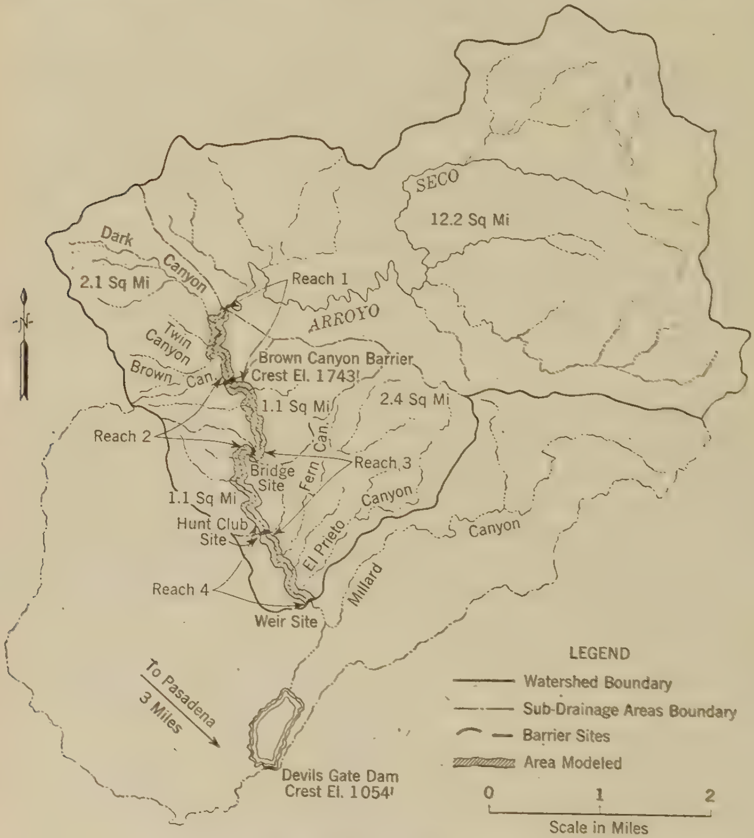
[2] Asst. Engr., Univ. of California, Berkeley, Calif.

[3] "Partial Plan for Runoff and Water-Flow Retardation and Soil Erosion Prevention for Flood Control Purposes, Arroyo Seco Flood Source Area," Survey Rept., Los Angeles River, U.S.D.A., January 14, 1939.

The watershed, north of Pasadena, Calif. (see Fig. 1), is approximately 30 sq miles in area, ranges from 1,100 ft to 6,000 ft in elevation, and is composed for the most part of a much fractured, deeply weathered, and disintegrating rock mass. The slope of the main channel in the lower part of the arroyo varies from 1.9% to 3.2% and the slopes of the tributaries vary from 10% to 67%. Records from twenty-two precipitation stations for a period from two to thirty-eight years indicate that seasonal rainfall varies from 6 in. to 34 in. near the mouth of the canyon, and from 11 in. to 60 in. at the higher elevations. The flow varies from 0 to a maximum recorded amount of 11,400 cu ft per sec (380 cu ft per sec per sq mile) near the mouth of the arroyo.


N
SECO
12.2 Sq Mi
Dark
Canyon
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Reach 1
ARROYO
Twin Canyon
Brown Can
Brown Canyon Barrier
Crest El. 1743'
1.1 Sq Mi
2.4 Sq Mi
Reach 2
Fern Can.
Bridge Site
Reach 3
Canyon
1.1 Sq Mi
Hunt Club Site
El Prieto
Canyon
Millard
Reach 4
Weir Site
To Pasadena 3 Miles
Devils Gate Dam
Crest El. 1054'
LEGEND
Watershed Boundary
Sub-Drainage Areas Boundary
Barrier Sites
Area Modeled
0
1
2
Scale in Miles

Fig. 1.—Arroyo Seco Watershed Showing Barrier Location, Subdrainage Areas, and the Parts of the Area to Be Modeled

These factors of geology, precipitous slopes, and rains of high intensity, in combination with the loss of much of the protective vegetal covering because of fires and road construction, make this watershed a serious flood problem area.

Since 1825, eighteen known floods have occurred in this locality. Devil's Gate Dam, downstream from the mouth of the Arroyo Seco, in operation since

1920, has lost approximately one third of its storage capacity, of 4,500 acre-ft, because of detrital material from this watershed.

THE PROBLEM

As part of the plan to reduce the silting rate of Devil's Gate Reservoir and flood destruction along the stream, the U. S. Forest Service has contemplated the construction of a series of four debris barriers to be spaced approximately a mile apart, starting at a point about one mile upstream from Devil's Gate Dam (see Fig. 1). Because of the large fluctuations in discharge and debris load in the Arroyo Seco, adequate information for designing these barriers and data on the influence of the barriers on the regime of the stream were not available. To provide such information, a series of model studies was undertaken by the Forest Service in cooperation with the University of California, at Berkeley.

In brief, the plan of model studies for each of the contemplated barriers is as follows:

1. To test the proposed spillway designed and to develop, if necessary, modifications in design to obtain adequate flow capacity and proper operation;
2. To determine the location and type of structure required to dissipate the energy of the maximum design flow over the spillway, and to prevent or minimize the resultant scour in the downstream channel bed which would impair the stability of the barrier;
3. To determine whether protection to the downstream canyon walls is necessary to prevent excessive erosion during major floods; and
4. To determine the slope of the debris surface and the approximate volume of debris that would be impounded by the barriers.

Since the Brown Canyon Barrier, which is the uppermost of the four barriers (see Fig. 1), is the only barrier constructed, this paper is concerned with the description of the model tests of this structure.

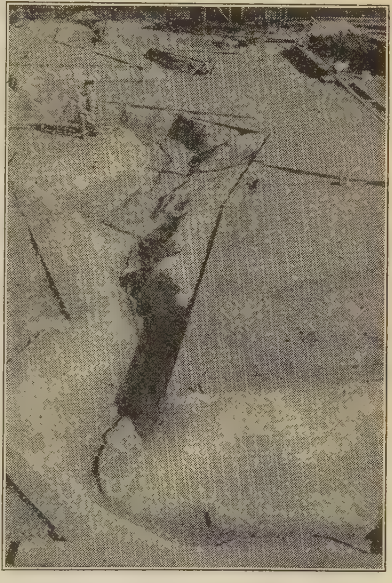


FIG. 2.—MODEL OF THE ARROYO SECO, REACH 1, FACING UPSTREAM

THE MODEL

The model of the reach of the Arroyo Seco containing the Brown Canyon Barrier was constructed to an undistorted scale of 1:50. The part modeled is indicated by the hatched area marked "reach 1" in Fig. 1. The channel was shaped in a sand-clay mixture, using male templets as a guide in forming the topography. The shaped channel was then covered with a thin protective coat of cement mortar, brush finished. Additional roughness for the canyon bottom was obtained by pressing model bed-load material into the fresh mortar. Fig. 2 is a view of the completed model of reach 1.

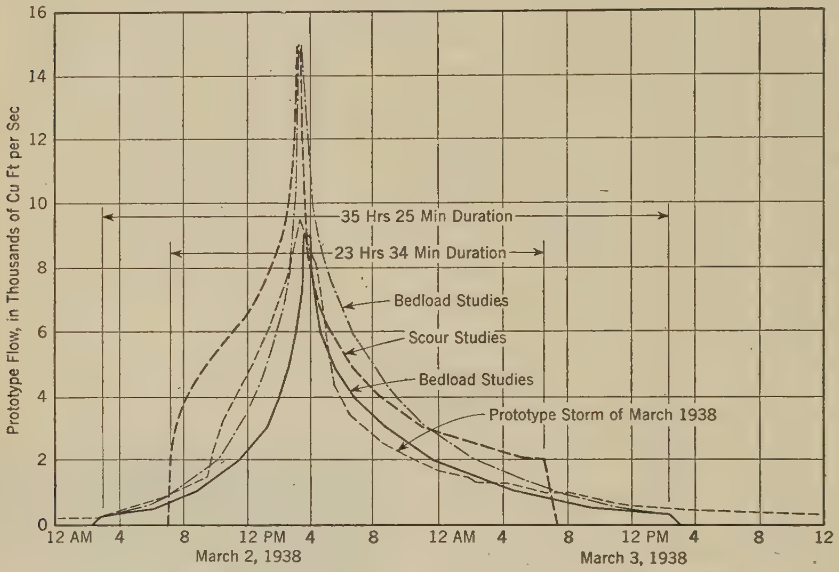


Fig. 3.—Hydrographs and Model Studies of the Arroyo Seco Storm

The barrier model was constructed of wood with a smooth varnish finish, reduced to one fiftieth of actual size without distortion. The crest and wing walls were built separately to allow for possible changes in design.

Discharge in the model was measured by a 45° V-notch weir. A continuous record of the head-discharge relationship was obtained by the use of a water-stage recorder, and was supplemented by a point gage for more accurate measuring and for checking the adjustment of the recorder.

The factor for converting discharge in the model to discharge in the prototype was based on Froude's law since it was considered that the most important measurements of flow Q would be made at the barrier, and the factor of gravity would be more important than that of friction.

The formula used was as follows:

$$Q = A\ V = L^2_r\, D^{1/2}_r = L^{5/2}_r \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

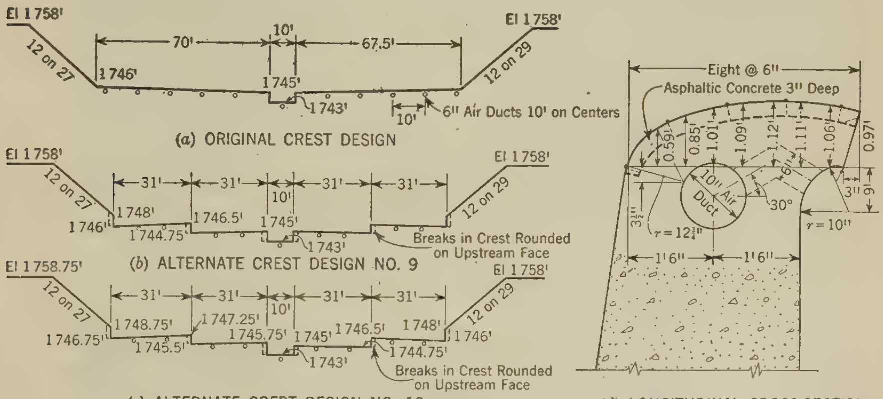
in which A is the cross-section area; V is the average velocity in the section; L_r is the ratio of model length L_m to prototype length L_p $\left(L_r = \frac{1}{50}\right)$; and D_r is the ratio of model diameter D_m to the prototype diameter D_p $\left(D_r = \frac{1}{50}\right)$.

The hydrographs used in the model studies, and the hydrograph of the March, 1938, storm on which they were based, are shown in Fig. 3. Flow in the prototype was measured over the Devil's Gate Dam (for the March, 1938, storm) and by the gaging station of the United States Geological Survey in the Arroyo Seco.

CREST STUDIES

The Brown Canyon Barrier is a constant-angle arch dam approximately 65 ft high and 220 ft long, with its spillway made roughly three fourths of this length to accommodate the maximum design flow. Since part of the flow is used for domestic purposes, the barrier was provided with a "grizzly" (an outlet 2 ft in diameter, adequately protected to prevent clogging) and numerous 6-in. weep holes to allow continuous flow passage.

The design crest (see Fig. 4) was provided with a 10-in. air duct, with 6-in. outlets spaced at 10-ft intervals under the lip of the crest, for aeration of the nappe. Tests indicated that for flows as great as 1,500 cu ft per sec, which were accommodated in the flat part of the spillway, complete aeration was not obtained (see Fig. 5(*a*)). The depth and velocity of flow at the outside edges were insufficient, causing the nappe to adhere to the downstream face, and preventing aeration other than that provided by the air ducts.


El 1758'
12 on 27
70'
10'
67.5'
El 1758'
12 on 29
1746'
1745'
1743'
10'
6" Air Ducts 10' on Centers
El 1758'
12 on 27
31'
31'
31'
31'
10'
El 1758'
12 on 29
1748'
1746"
1744.75'
1746.5'
1745'
1743'
Breaks in Crest Rounded on Upstream Face
El 1758.75'
12 on 27
31'
31'
10'
31'
31'
El 1758'
12 on 29
1748.75'
1746.75'
1745.5'
1747.25'
1745.75'
1745'
1746.5'
1748'
1746'
1743'
1744.75'
Breaks in Crest Rounded on Upstream Face
Eight @ 6"
Asphaltic Concrete 3" Deep
0.59'
0.85'
1.01'
1.09'
1.12'
1.11'
1.06'
0.97'
10" Air Duct
6"
30°
3"
9'
3½"
r = 12¾"
r = 10"
1'6"
1'6"

(*a*) ORIGINAL CREST DESIGN

(*b*) ALTERNATE CREST DESIGN NO. 9

(*c*) ALTERNATE CREST DESIGN NO. 10

(*d*) LONGITUDINAL CROSS-SECTION

FIG. 4.—CREST PROFILES AND SECTION, BROWN CANYON BARRIER, FACING UPSTREAM

For flows larger than 1,500 cu ft per sec, aeration was more complete, flow at the outside edges intermittently opening and then sealing off complete aeration, but not materially altering the profile of the nappe. At the maximum flow of 15,000 cu ft per sec, however, the area available for aeration on the right abutment was quite small (see Fig. 5(*b*)). Since measurements indicated ample freeboard, a minimum of 3 ft being stipulated, it was felt that reduction of the spillway length, by movement of the right wing wall, would be desirable to improve aeration and to decrease the flow over the right or east canyon wall. A total reduction in crest length of 12.5 ft improved conditions considerably. Further reduction was limited by the undesirability of increasing the stresses developed in the barrier by displacement of the right wing wall. Reduction of crest length produced no improvement in aeration for flows of less than 1,500

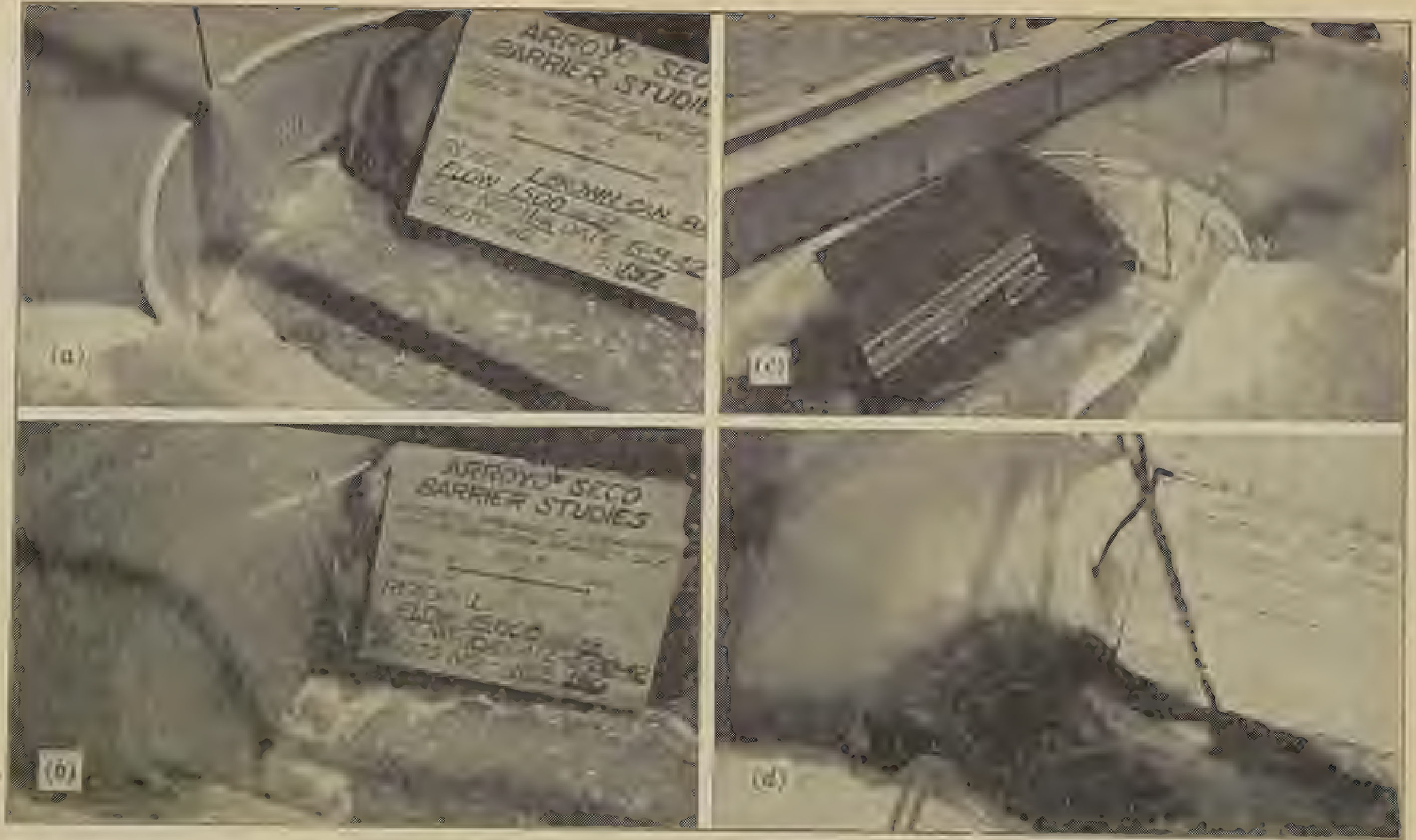

(a)
ARROYO SECO
BARRIER STUDIES
FLOW 1500
(c)
(b)
ARROYO SECO
BARRIER STUDIES
FLOW 15000
(d)

FIG. 5.—VIEWS OF THE MODEL IN OPERATION

(*a*) Original Designed Crest, Q = 1,500 Cu Ft per Sec
(*b*) Original Designed Crest, Q = 15,000 Cu Ft per Sec
(*c*) Final Crest, Q = 295 Cu Ft per Sec
(*d*) Final Crest, Q = 15,000 Cu Ft per Sec

cu ft per sec. Modifications of original design, such as increasing the flow section in the center or increasing the slope of the crest, were also unsatisfactory.

By resorting to a stepped crest, the low flows were confined to a narrower section, with improvement in aeration. With the steps level, considerable lag was encountered before flow over the side steps attained sufficient depth to spring free of the downstream face—the time of lag, of course, depending on the rate of rise or fall of flow.

Further improvements in crest design were directed toward reducing the time during which this undesirable condition was operative. Several variations in crest design were tested. The most effective was one in which the steps sloped up to the center of the barrier, thus making the flow on a step deeper toward the bank and shallower toward the center. As a result, a well-developed nappe falling free of the barrier was developed at the outside of a step by the time water was flowing over the highest part and joining with the water in the adjoining lower step. Therefore, the nappes under both steps were fully aerated at all times except at the instant when the flow over both was joined. At this instant a curtain of water was formed between the barrier and the nappe as before but was almost instantly broken, leaving the entire nappe well developed and falling free of the barrier.

Later tests, with the barrier filled with detritus, indicated that the flow had a marked tendency to pile up on the west side of the barrier. This condition was considered undesirable because of the excessive scour on the downstream canyon wall. To counteract this condition, the entire west half of the crest was raised 9 in. The modification was also beneficial for clear water flows (no detritus above barrier) by alternating the operation of the steps, thereby ensuring complete aeration of the nappe at all times (see Fig. 5(*c*) for operation of the final crest design at a flow of 295 cu ft per sec).

At the maximum flow of 15,000 cu ft per sec, steps in the barrier crest had no objectionable effect on the nappe. With this final design, considerable reduction in flow over the right abutment has been obtained (compare Fig. 5(*d*) with Fig. 5(*b*)). For comparison of head-discharge relationship of the various crests tested refer to Fig. 5. The extent of flow over the abutments was plotted on a grid system. In the prototype this area will be protected with gunite.

SCOUR STUDIES

For a barrier approximately 65 ft above bedrock, with flow about 7 ft deep over the crest at the maximum design discharge and with the bedrock fairly erosive, it was apparent that some form of protection to the downstream channel would be necessary. Observations as to the location and extent of scour that would occur without benefit of the various proposed types of protective structures were made before testing.

For this purpose five rates of clear steady flow were used, increasing from 1,500 cu ft per sec to 15,000 cu ft per sec (prototype values). With a sand-clay mixture for bedrock, overlain with loose bedrock material, each rate of flow was continued until scour conditions appeared stabilized. These tests indicated that the extent of scour, the maximum depth of scour, and the distance downstream to the point of maximum depth increased with increase in flow (see Fig. 6). Since the flow durations to achieve stability, especially for the two higher rates of discharge, are beyond any conceivable durations which might obtain in the prototype, the magnitude of depth of scour was considered indicative only of that which ultimately might develop.

Tests using an assumed hydrograph of the design storm, having a peak of 15,000 cu ft per sec, eliminating flows of less than 2,000 cu ft per sec and enduring for 171 hours (prototype time), produced a maximum depth of scour of 4.5 ft between 25 ft and 35 ft downstream. As a result, the use of the sand-clay mixture as bedrock was considered satisfactory, yielding results comparable to those which might occur in the prototype under similar flow conditions. With fixed bedrock, overlain with loose bedrock material, the results indicated that, where no scour hole could develop in the bedrock, more of the bedrock was exposed and, for the maximum flow of 15,000 cu ft per sec, the exposed bedrock extended downstream 290 ft.

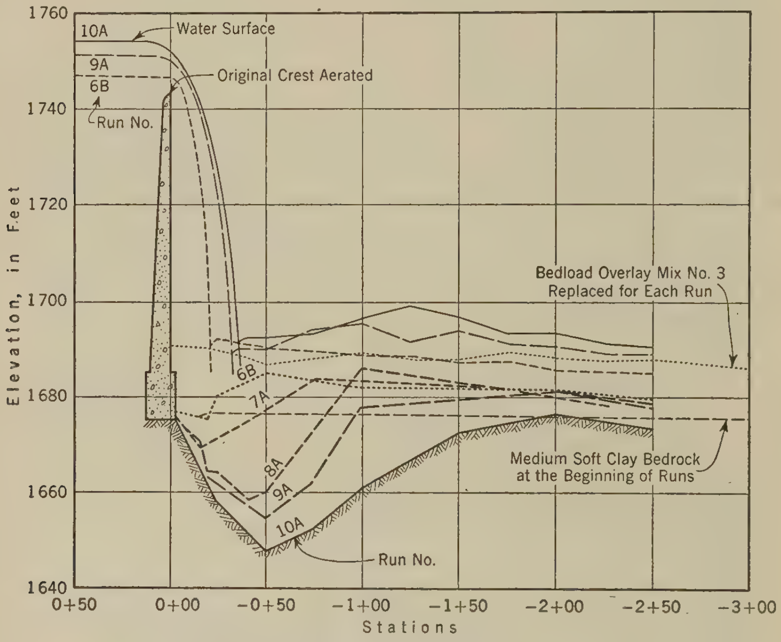


FIG. 6.—PROGRESSIVE SCOUR BELOW BROWN CANYON BARRIER

In an attempt to determine the extent and duration to which the bedrock might be exposed, tests were made using hydrograph flow, of the design storm, capable of transporting bed load over the barrier. At the peak of the flow the extent of bedrock exposed was practically the same as that exposed for the maximum steady flow without bed load. However, with the recession of flow, bed load was redeposited over the bedrock. Bedrock between 100 ft and 285 ft downstream from the barrier was exposed for a little more than a third of the total flow duration.

Preliminary tests of an ogee sill (crest elevation 1,690), approximately 14 ft above bedrock, tested at 80 ft and 170 ft downstream from the barrier indicated that, to be effective, the height and the distance downstream would have to be increased considerably. Tests of two trapezoidal sills 380 ft downstream from the barrier (crest elevations 1,693 ft and 1,697 ft) reduced the depth of scour by approximately the amount that they increased the backwater elevation. The main objection to these structures was their large size and the scour problem they developed.

Tests of an apron 70 ft long showed good protection against development of a scour hole; but the effect of this apron was to expose the downstream bedrock as in the previous fixed bedrock studies. Thus, as far as dissipating energy was concerned, the development of a scour hole was desirable provided that the barrier itself was not endangered. An apron 13 ft long, designed primarily as a cutoff wall (which would act as an apron for flows less than 500 cu ft per sec), allowed a scour hole to develop, reduced the undercutting effect, and was considered the most desirable of any of the structures tested (see Fig. 7).

Deposition Studies

Field examination of bed load impounded by the Devil's Gate Dam, as well as of that remaining in the arroyo (excluding any material that was considered residual), gave a fair approximation of the size and grading of material transported. In the absence of any actual measurements of rate of bed-load transportation, it was believed that past measurements of the loss of storage capacity

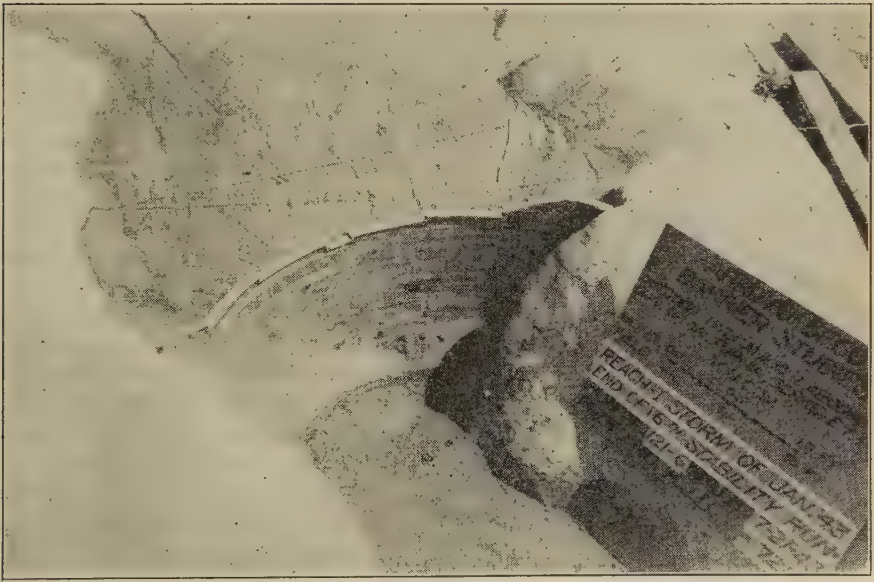


Fig. 7.—Combination Apron and Cutoff Wall That Was Considered the Most Desirable of Those Tested

of the Devil's Gate Dam could be correlated with flow measurements to yield approximate loading rates. Unfortunately, however, upon investigation there were many confusing factors. Measurements of loss of storage capacity included material contributed by a tributary, and did not include a substantial amount of material deposited above the crest elevation of the dam and the mouth of the arroyo. This was further complicated by the lack of complete flow data.

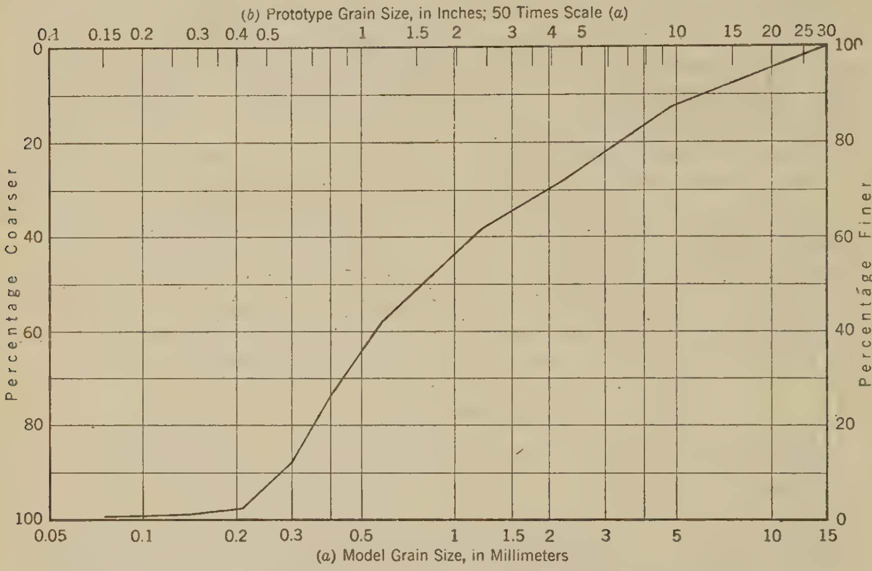


FIG. 8.—GRADING CURVE FOR MODEL BED-LOAD MATERIAL, SERIES A, B, C, E, AND F

As a result, it was felt that more satisfactory information could be obtained from the model. A series of steady flow runs were made in reach 1 of the model without the Brown Canyon Barrier in place. For each flow, bed load was introduced at the upper end of the reach at a rate that would not create excessive deposition or scour at the point of introduction. Observation of deposition throughout the reach and measurements and analysis of the bed load introduced and carried away provided a method for adjusting the size, grading, and rate to obtain a bed load that would attain general movement and have approximately equal rates of inflow and outflow. The bed load finally used in the model was composed of commercial sands and gravels, varying in prototype size from $\frac{1}{8}$ in. to 28 in. in diameter (see Fig. 8 for grading curve). By determining steady rates of flow, loading curves were established for hydrograph flows.

For the model studies of bed-load deposition, two hydrographs were used—one, a hydrograph similar to that of the maximum recorded flow in the arroyo (9,000 cu ft per sec), and the other, a hydrograph of the design storm with a peak of 15,000 cu ft per sec.

With the barrier in place, the first series of runs to fill the barrier were made with the design storm, and bed load was introduced at the established rate—

177,100 cu yd per run (prototype value). At the conclusion of the first run, the detrital front extended downstream from the upper end of the reach to within approximately 900 ft of the barrier. With succeeding runs, the front continued to move downstream until, during the tenth run, the initial filling of the barrier was sufficient to cause about 100 cu yd of bed load to be carried over the barrier. With additional runs the detritus slopes, upstream from the barrier, continued to increase, but at a decreasing rate. The volume of stored material likewise increased. With each succeeding run the volume of bed load transported over the barrier increased until, at the end of the twenty-second run, when little change was observed in the channel slope, stability was assumed to exist. The average slope of the detrital material, as based on the canyon center-line length, was 1.75% with a storage of 1,014,000 cu yd (see series A, Fig. 9, and Table 1). This compares favorably with computed estimates of 1,013,000 cu yd and a slope of 1.88%. The estimate was based on measurements of debris slopes behind existing small barriers where the measured slope averaged approximately seven tenths of the original channel slope.

It was felt that size of flow, rates of loading, and size of bed-load material would vary in the prototype and would have some

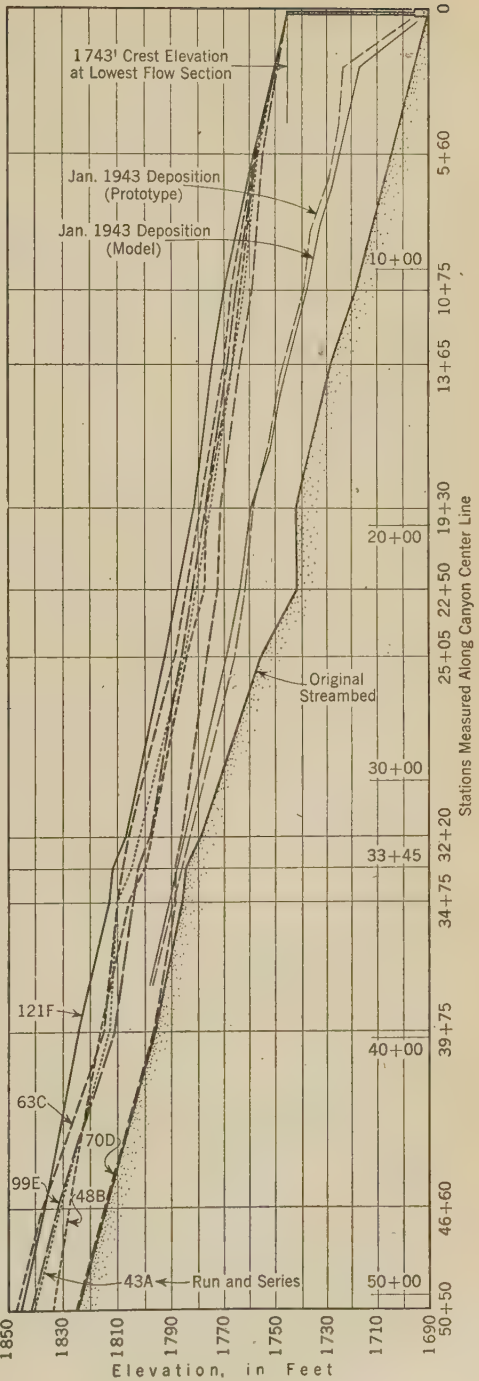


Fig. 9.—Profiles of Stream Center Line Upstream from Brown Canyon Barrier, Showing Final Profile of the Barrier Filling that Results from Using the January, 1943, Storm (See Legend in Table 1)

influence on the detrital slopes and volume. As a result three other series of runs were made to observe the variations that might occur.

Reduction of the bed load introduced, to one third of the volume in series A, caused noticeable scour at the upper end of the channel and reduced the slope and volume to 1.71% and 928,000 cu yd, respectively (see series B, Fig. 9, and Table 1).

TABLE 1.—STABILITY RUNS, BROWN CANYON BARRIER; LEGEND AND DATA APPLYING TO FIG. 9

Symbol (see Fig. 9)	Run No.	Series		Hydrograph[a]	Bed Load		Cubic yards per run	Storage[b] (cu yd)	Slope[c] (%)	
		Letter	Run Nos.		No.	Median diameter (in.)			Canyon	Stream
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
············	43	A	22–43	15,000	4	1.5	77,100	1,014,000	1.75	1.73
- - - - - -	48	B	44–48	15,000	4	1.5	24,600	928,400	1.71	1.54
– – – – –	63	C	49–63	9,000	4 6	1.5	33,500	1,085,300	1.88	1.87
— — —	70	D	66–70	9,000	7	0.98	33,500	681,000	1.29	1.27
—— ——	99	E	71–99	9,000	6A 6B 6C	0.65 0.98 5.2	33,500	1,088,200	1.72	1.58
▨	....	Original stream bed			....	....	....	....	....	2.69
— - —	....	Canyon center line			....	....	....	....	2.68	....
———	121	F	116–121	4,400	13	0.45	243,000	1,140,000	2.01	2.12

[a] Maximum flow in cubic feet per second. [b] Storage volume between Brown Canyon Barrier and ordinate line N 4,202,300 (Fig. 9). [c] Average stream-bed slope based on the canyon center line and the stream center line, respectively.

Reducing the flow to the hydrograph for 9,000 cu ft per sec, using the original loading rates, increased the average slope to 1.88% and the volume to 1,085,300 cu yd (see series C, Fig. 9, and Table 1).

With the same flow and loading rates, as in the preceding series of runs, but with the size of bed load reduced, the slope was decreased to 1.29% and the storage volume to 681,000 cu yd (see series D, Fig. 9, and Table 1).

In all the foregoing runs, no attempt was made to vary the size of the material introduced with respect to flow. To check whether, at the upper end of the channel, the accumulation of the larger sized particles which were not transported by the low flows had much influence on the slope of the bed, a series of runs similar to series C was made. However, the bed load was separated into three size groups and was introduced in flow ranges that could develop general movement of all particles in a given group. The resultant storage volume was practically the same as in series C (see series E, Fig. 9, and Table 1).

Stream-bed profiles and center lines were plotted for each run. Those for the final runs, of each series, indicate significant changes in slope and those for the channel meander indicate the locations of those parts of the side canyon walls that would be subjected to scour. For the range of conditions tested, the detrital slopes varied from a minimum of 0.5 to a maximum of 0.7 of the original stream-bed slope of 2.69%.

VERIFICATION STUDIES—BED-LOAD DEPOSITION

The occurrence of the January, 1943, storm (peak flow 4,400 cu ft per sec, see Fig. 3) shortly after the completion of the Brown Canyon Barrier afforded an excellent opportunity to verify the model studies. With a negligible amount of detritus being carried beyond the barrier, surveys before and after the storm indicated that 243,000 cu yd were impounded by the barrier. Mechanical analysis of samples of some 10,000 lb of bed load from twelve pits throughout the reach above the barrier showed that the bed load varied from fines to occasional boulders larger than 12 in. in diameter. The average was weighted on the basis of representative volume (see Fig. 10). If this range in material size was reduced to model scale, a large part of the material would be so fine that it would be transported in suspension. To rectify this difficulty, the size range was reduced to a grain size varying from $\frac{1}{10}$ in. to 14 in. (prototype values).

Loading rates were determined as before, using steady flows. Manipulation of the time scale in connection with the hydrograph of the storm allowed the required bed load to be introduced. Three series of runs of the January, 1943, storm were made before results were considered satisfactory. Better conformity was achieved mainly by manipulation of the time scale and loading rates and also by the closer conformity of backwater elevations maintained by the barrier. The final average slope in the model was 2.26% as compared to 2.03% in the prototype.

Using the same loading rates and time scale (1 to 4.42) as in the last test with the January, 1943, storm, an analysis of five storms was made to achieve stabilized slopes above the barrier. Even though the rates of bed load transported over the barrier did not attain the rates of bed load introduced at the upper end of the model, a small change in slope was concluded to represent, approximately, the maximum slope that would be developed by storms similar to the January, 1943, storm. The storage volume (1,140,000 cu yd) and the average slope (2.01%) of this final stream bed (see series F, Fig. 9, and Table 1) are both 0.13% greater than the maximum of the previous tests. If the original discrepancy in model verification of the January, 1943, storm is considered, it can be concluded that previous results, based on model determination of loading rates and bed-load size, were applicable to the prototype.

As a result of these bed-load studies, since the probability of having identical storms as to flow, bed-load size, and amount available for transportation is rather remote, true stability of slopes or volume will never be achieved, but will vary depending on these and other factors. Also, the range of results obtained in the model studies appears to predict the reasonable variations that can be expected in the prototype.

PROTOTYPE CREST PERFORMANCE

Measurement of depth of flow over the barrier, for the peak of 4,400 cu ft per sec, during the January, 1943, storm, accounting for flow through the weep holes and the "grizzly," compared favorably with head-discharge relationships as determined in the model.

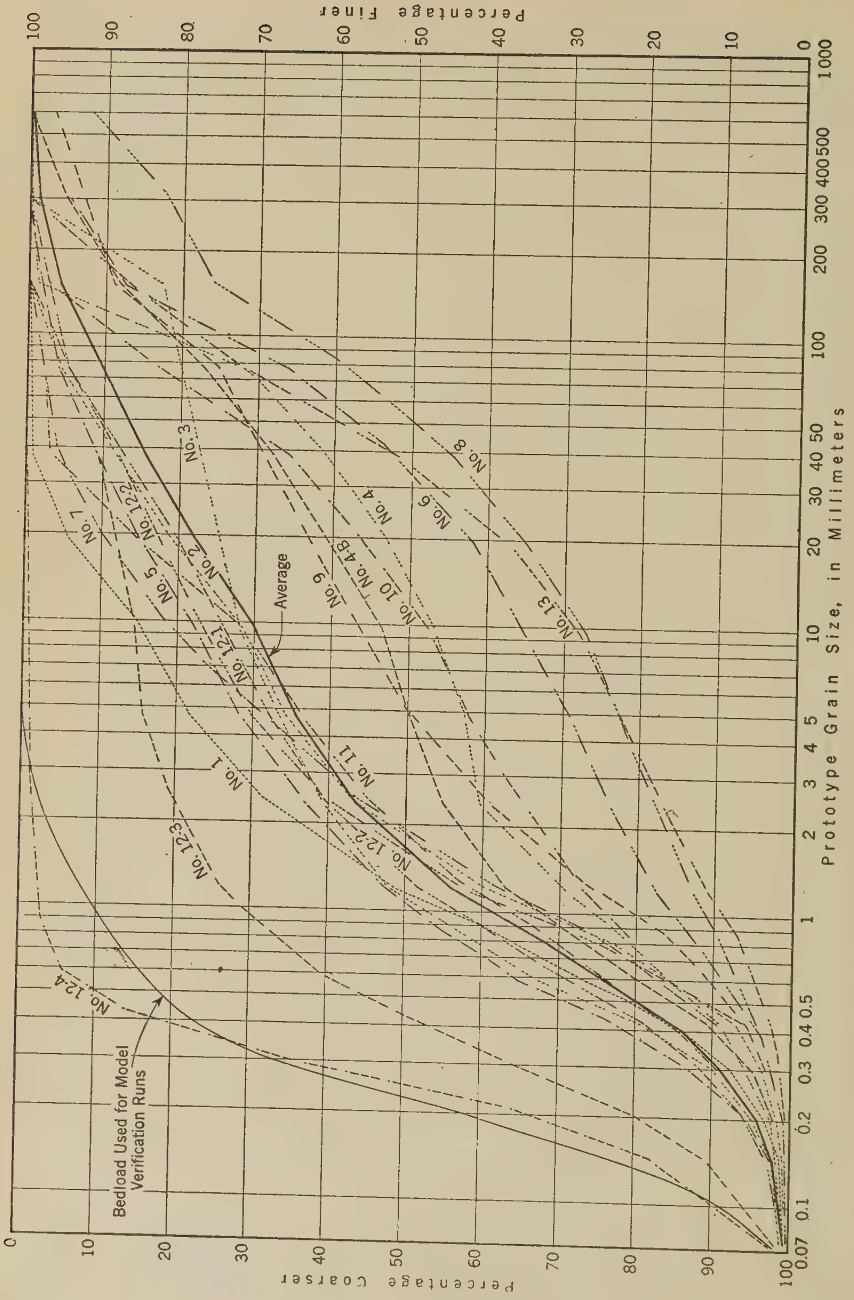


FIG. 10.—GRADING CURVES, FIELD SAMPLES OF DEBRIS DEPOSITS, STORM OF JANUARY 21 TO 23, 1943

With a flow of approximately 300 cu ft per sec, part of the spillway is in operation, as shown in Fig. 11. The most noticeable difference between model and prototype performance occurs with shallow depths of flow over the crest. In the prototype there is less tendency of the nappe to adhere to the lip of the crest.

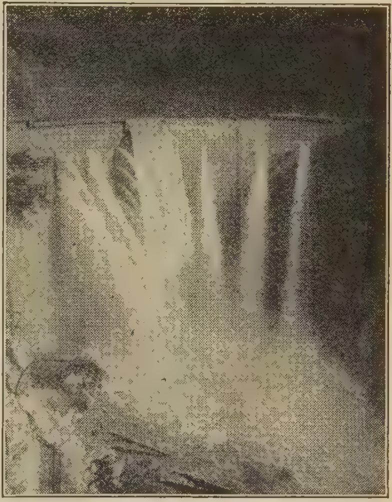


Fig. 11.—Operation of the Brown Canyon Barrier at a Flow of Approximately 300 Cu Ft Per Sec Over the Crest

Prototype Scour Downstream from the Barrier

The storm of January, 1943, occurred before the combination apron and cutoff wall were constructed. As a result no comparison can be made with the installation as recommended by the model studies. The scour that did occur, however, without the benefit of any structure, was similar to that obtained in the model where the bedrock was fixed and overlain with loose bedload material. In the prototype, the channel downstream from the barrier was mostly scoured to bedrock and extended downstream for several hundred feet.

With the removal of the overlying loose bed-load material, a stilling pool developed immediately below the barrier. This pool extended approximately 40 ft downstream, with the greatest depth of 10 ft at approximately 25 ft downstream from the barrier. Field inspection of the development of this depression in the bedrock profile indicated that it was not caused by wear of the bedrock but rather by the removal of the loose overlying material.

Concluding Remarks

Observation of discharge over the crest of the barrier, while covering less than a third of the maximum design flow, provides reasonable reliance in the results as obtained from the model.

Scour downstream from the barrier (the combination apron and cutoff wall had not been installed at the time of the storm) produced conditions similiar to those that occurred in the model—that is, the overlying detrital material was scoured and bedrock was exposed for a considerable distance downstream.

Although a comparison of the detrital slopes that eventually will exist upstream from the barrier cannot be made at this time, the slopes and disposition of the initial filling are comparable to those attained in the model.

As a result of the studies involving the transportation and deposition of bed-load material, it becomes increasingly evident that, before much confidence can be placed in the use of models to reproduce or to predict changes in existing conditions, much more effort will have to be made to determine what takes place in nature.

ACKNOWLEDGMENTS

These experiments were undertaken during 1942 by the Department of Agriculture, U. S. Forest Service, Region 5, in cooperation with the Department of Mechanical Engineering of the University of California. General supervision, frequent consultation, and the use of hydraulic laboratory facilities were furnished by the University, under the direction of Morrough P. O'Brien, M. ASCE, assisted by J. W. Johnson, Assoc. M. ASCE, and by E. H. Taylor, Jun. ASCE. Close collaboration was maintained throughout the experiments with E. R. Huber, M. ASCE, and W. L. Minaker and John F. Serex.

DISCUSSION

CLIFFORD A. BETTS,[4] M. ASCE.—The debris bed-load investigations constitute an interesting phase of the model studies of the Brown Canyon Barrier. They furnish information that may be helpful in the design of other structures or series of barriers for channel control. Particularly significant is the 1.73% to 2% slope of the debris upstream from the barrier. It indicates an ultimate valley storage of three quarters of a million cubic yards of debris upstream from the structure and above spillway level, or nearly double the crest level storage capacity. This occurs in a foothills arroyo having an average stream bed slope of 2.7%. The fact that the flood (4,400 cu ft per sec) of January, 1943, subsequent to construction, corroborated the model study predictions, adds to their value.

The use of channel barrier dams in narrow canyons, to stabilize mountain sides by holding the channel regimen and thereby preventing channel undercutting, is not new. However, there is need for more authoritative data on the complicated problem of predicting the behavior of bed loads of various materials under different volumes of flow.

In this case the bed load consisted of Lowe grandiorite, San Gabriel formations, and Wilson diorite, graded from fines to boulders, 3 ft or 4 ft in diameter, with sizes less than 6 in. predominating. As was to be expected, deposition in one bar might be localized fines or all coarse gravel; but the over-all distribution from the upper to the lower end of the canyon was relatively uniform, with the proportion of large-size material decreasing downstream. Differences between the behavior of the model and the prototype resulted from the artificial fixed channel, lack of tree-root obstructions, and the fact that no attempt was made to introduce material smaller than would be retained on a 200-mesh sieve ($\frac{1}{8}$ in. and smaller in the prototype) because of suspension. Nevertheless, there has been sufficient evidence in the subsequent behavior of the prototype channel to confirm the findings that the slope of the stream bed becomes greater (1) when debris-carrying flows are reduced, (2) when debris quantities are increased, and (3) when the size of bed-load debris is increased.

The introduction of the bed load to simulate actual watershed conditions seems to be an important factor. It requires meticulous study and calibration in order that the quantities applied to the various flows approximate the quantities of material actually transported.

Since these quantities increased with the rate of flow, it is obvious that extensive tests covering wide ranges of hydrographs would have added greatly to the information in this field had the project requirements justified the extra cost. As usual, however, this load will have to fall on some larger project.

ROGER E. AMIDON,[5] ASSOC. M. ASCE.—The interesting model study of the Brown Canyon Debris Barrier is well presented in the paper under discussion. Valuable information pertaining to the spillway and downstream scour action was obtained. However, it is regrettable that more definite data did not result relating to the movement of debris and to the probable ultimate debris slope to be stabilized by the structure.

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[5] Civ. Engr., U. S. Forest Service, California Forest and Range Experiment Station, Berkeley, Calif.

The original estimate that debris would assume an ultimate slope above the structure approximately equal to 70% of the original channel slope was based on measurements made above existing structures. These measurements were made during the period 1939 through 1941 at sites of approximately one hundred debris dams and check dams with small drainage areas above, seldom exceeding 5 sq miles. The age of the structures varied from 1 to 15 years. In most cases the channel gradient was affected from the site of the dam to the toe of the next upstream barrier. In some cases deposition at the toe of the next upstream structure was several feet deep. The dams included in the study vary from 2 ft to 35 ft in height, and are spaced from 20 ft to 2,500 ft apart. The original channel slopes varied from 4% to 35% and the debris slopes varied from 0% to 21%. The debris slopes varied from 0% to 100% of the original channel slopes, and averaged 52% for all barriers surveyed. The average debris slope above sixty-seven dams, where the original channel slope was 14% or less, was 66% of the original channel slope. On this basis it seemed reasonable that over a much longer period of time, on the lower gradient channels, the debris slope would at least approach 70% of the original.

The great number of variables that affect the slope which debris will assume above barriers forestalls the possibility of making precise estimates based on generalized curves for any specific structure. The size and quantity of material available for transportation, slope of the channel, volume of flow, obstructions in the channel, and period of years are some of the influencing factors.

In planning debris barriers it is extremely important to estimate correctly the volume of slope storage that will result in order to determine the economic feasibility of the structure. Detailed studies of field conditions and accurate measurements of progressive barrier storage, over a period of time, will provide data from which reasonably accurate estimates of debris slope can be made.

The increasing importance of the control of erosion and debris movement above reservoirs designed for flood control, water conservation, and hydroelectric power generation justifies serious consideration of this problem.

Ettore Scimemi,[6] Esq.—Among the problems treated in this most interesting paper is the depth of the scour at the foot of a spillway dam such as Brown Canyon.

Experiments were performed on a model and since, in the prototype, the material encountered was a fairly erosive bedrock, the authors were obliged to substitute for this material, in the model, some other material with similitude characteristics reduced in the scale 1:50. Such a transformation presents considerable difficulties, not only in the selection of a suitable material, but in obtaining or developing one that will have the proper size and size distribution. In a reduction of 1:50 it is inevitable that the effect of some of the finer material in the prototype will be lost entirely in the model. In addition to these difficulties (which could make the results questionable), there are some general considerations in the design of a spillway dam.

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[6] Prof., Hydr. Inst., Univ. of Padua, Padua, Italy.

A nappe that falls vertically from the top of a dam, and which can scour the bottom of the river, cuts the channel to a depth that depends on the balance of the forces engaged; that is, equilibrium between energy of the nappe and the energy destroyed in the whirlpool. Therefore, the volume and the depth of the scour are not more rigorous functions of the size of the material on the bottom, but only a function of the difference in the elevation of the overflowing nappe and that of the stilling pool below. Alessandro Veronese[7] examined this problem, and gave the following formula for the depth (in metric units) of the scour y_s:

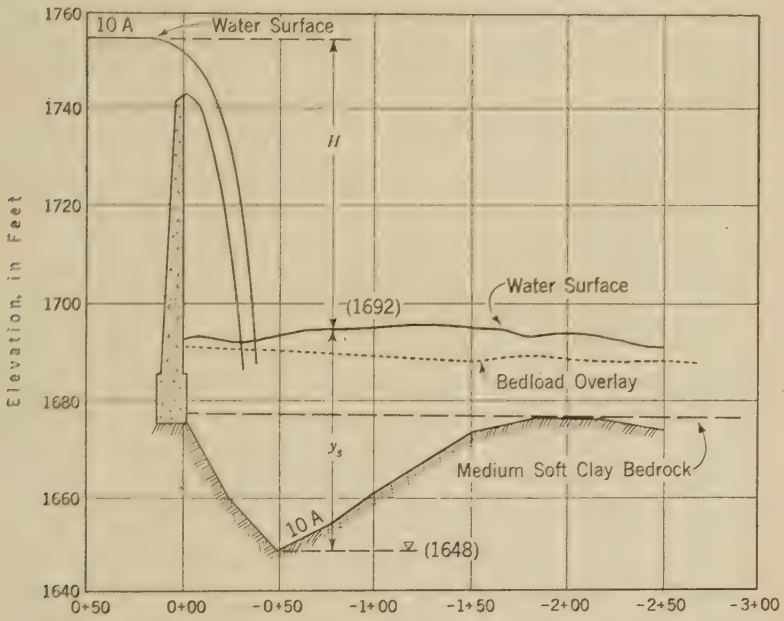

FIG. 12

$$y_s = 1.90\,H_f^{0.225}\,q^{0.54} \qquad \text{(2a)}$$

in which y_s is the depth of the scour, under the downstream water surface; H_f is the effective drop (see Fig. 12); and q is the discharge per linear unit of the dam. This formula is translated into English units as

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[7] "Erosioni di fondo a valle di uno scarico," by A. Veronese, *Annali dei Lavori Pubblici*, Roma, Settembre, 1937, p. 717.

$$y_s = 1.32\, H_f^{0.225}\, q^{0.54} \ldots\ldots\ldots\ldots\ldots\ldots (2b)$$

in which y_s and H_f are in feet; and q is in cubic feet per second per foot of dam.

In Fig. 12, which is analogous to Fig. 6: $H_f = 1{,}755 - 1{,}692 = 63$ ft; $q = 15{,}000/134 = 112$ cu ft per ft; and, therefore, $y_s = 1.32 \times 63^{0.225} \times 112^{0.54} = 43$ ft. According to Fig. 6, $y_s = 1{,}692 - 1{,}648 = 44$ ft. In other words, Eqs. 2 conform perfectly to the experiences observed in the Brown Canyon model.

When formulas such as Eqs. 2 cannot be used for the design of hydraulic structures, at locations where the tailrace is subject to scour, and where it is not possible to reduce the model material to the relative scale of the prototype material, observations to determine the maximum scour, or scouring limit, can be made by another experimental method. This consists in making successive observations on the model with smaller and smaller grain sizes without regard to the grading of the prototype material. Trials must be begun with rather large gravel and then repeated with successively smaller gravel. The scour will be deeper and deeper with each reduction in grain size until a limiting depth is soon reached.

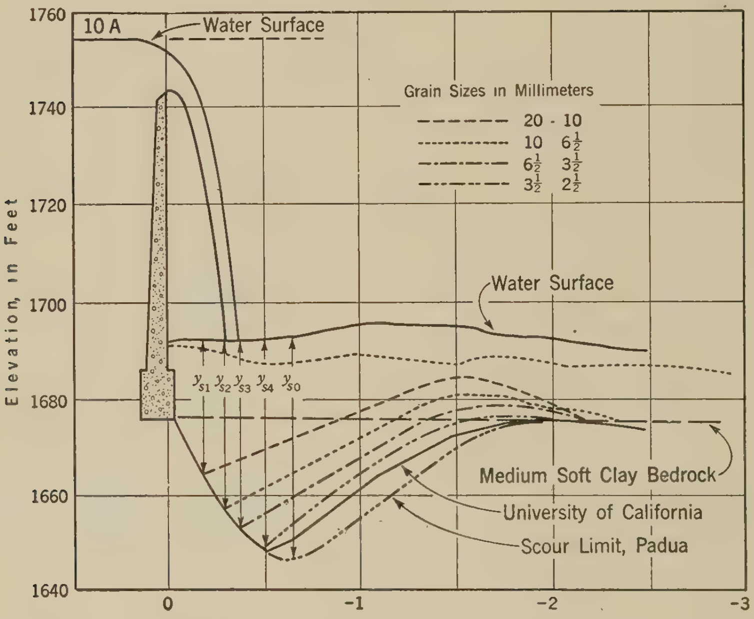

Fig. 13

This method has been used by the writer on a model of Brown Canyon Dam (scale 1:50) similar to that used by the authors, constructed in the hydraulic laboratory of the University of Padua in Padua, Italy. The results of these trials are plotted in Figs. 13 and 14. Fig. 14 gives the relation between depth of scour and grain size. At grain size zero the scour limit, $y_{s0} = 47$ ft, is obtained. This value is near enough to that found by the authors (44 ft) so that the difference may be said to be due to errors in reproducing various accessories and parts of the model to an accurate scale ratio.

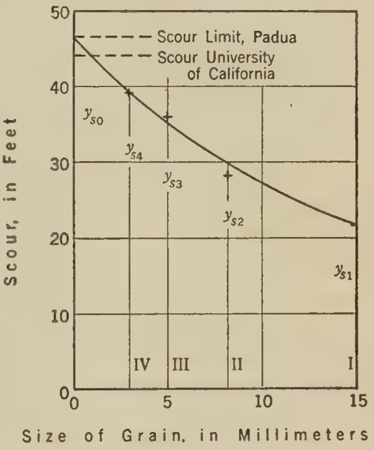

Fig. 14

This method should lead to good results for any type of hydraulic structure, without necessarily conforming to any reduction, in scale, of the prototype material, and of its grading curve (if available). Results of this kind, with some limitations, were described in a previous paper regarding scouring experiences at Conowingo Dam, at Sennar Dam, and at other structures.[8]

J. W. Johnson,[9] Assoc. M. ASCE.—A search through published reports on hydraulic model studies shows that investigations have been made on a large number of hydraulic problems. However, the study by the authors of the Arroyo Seco structures appears to be the first comprehensive model investigation on the problem of the movement and deposition of sediment in the vicinity of debris barriers. The advantages of model studies for debris barrier investigations are many; a large variety of factors such as barrier spacings, order of constructing a series of barriers, and sediment loading can be studied in detail. Observations on field installations would require years to yield information of comparable value. The experiments reported by the authors and those made at a later date on the other barriers of the Arroyo Seco system[10,11,12] have permitted the formulation of certain general principles of the action of debris barriers.[13]

The section of the paper concerning the deposition studies is of particular interest to the writer. It was fortunate that the studies on sediment deposition were in the early stages of the model work when the January, 1943, storm occurred on the Arroyo Seco watershed. During, and following, this storm, prototype data were obtained on: (*a*) The total volume and slope of the debris deposited behind the Brown Canyon Barrier, (*b*) a hydrograph of stream flow at a gaging station a short distance downstream, and (*c*) a record of water levels in the basin during the flood. Thus, with the total volume of debris transported by the stream during the flood (except for the volume of very fine suspended matter which passed over the spillway) and the slope of the final deposit being known, the model of the Brown Canyon Barrier was operated by varying the base length of the hydrograph until the volume and slope of the debris deposits in the model corresponded with those observed in the prototype. In all subsequent investigations of the Brown Canyon Barrier and the proposed barriers located farther downstream, the hydrograph and rate of sand feed developed from the data of the January, 1943, storm gave an excellent means of comparing the effectiveness of the various barrier systems, bank protection works, channel correction measures, and other proposed works.

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[8] "Sulla relazione che intercede fra gli scavi osservati nelle opere idrauliche originali e nei modelli," by E. Scimemi, *L'Energia Elettrica*, Milan, November, 1939.

[9] Associate Prof. of Mech. Eng., Univ. of Calif., Berkeley, Calif.

[10] "Brown Canyon and Weir Debris Basins and Barriers," by W. K. Bermel, Fluid Mechanics Laboratory, Vol. II, Univ. of Calif., Berkeley, August, 1945 (unpublished).

[11] "Report on Low and Intermediate Barriers in the Weir Barrier Reach," by G. R. Loucks, Fluid Mechanics Laboratory, Vol. III, Univ. of Calif., Berkeley, August, 1945 (unpublished).

[12] "Report on Bank Protection," by W. U. Garstka, Fluid Mechanics Laboratory, Vol. IV, Univ. of Calif., Berkeley, May, 1945 (unpublished).

[13] "Movement and Deposition of Sediment in the Vicinity of Debris-Barriers," by J. W. Johnson and W. L. Minaker, *Transactions*, Am. Geophysical Union, 1944, p. 901.

The most important factor in estimating the efficiency of a reservoir as a debris trap appears to be the ratio of original capacity for debris storage to the area of the watershed.[14] From the model studies of the various barrier systems the volume of debris that could be stored in each system was estimated and the capacity-watershed ratios were computed and are summarized in Table 2 (the various systems include the Brown Canyon Barrier, a high barrier at the Weir Site (Fig. 1), and two possible systems of low barriers in reach 4 instead of the high barrier at the Weir Site).

TABLE 2.—RATIOS OF CAPACITY VERSUS WATERSHED AREA, ARROYO SECO

(Units Are Acre-Feet per Square Mile)

System	Barriers		Capacity (cu yd)	Capacity-area ratio
	No.	Height (ft)		
Brown Canyon...	1	65	1,140,000	49.4
Weir Site........	5	7	120,000	5.3
	3	15	340,000	11.2
	1	60	1,528,000	50.0

The ratios of capacity versus watershed area (capacity-area ratio) for the various barrier systems are of interest in connection with the relationship between annual storage loss and capacity-area ratio as computed for existing water-supply reservoirs in Southern California, as shown in Fig. 15; the plotting data for this figure being obtained from published data on reservoir silting.[15] Carl B. Brown, Assoc. M. ASCE,[14] states that in reservoirs used for water storage an annual storage loss of one half of 1% is considered serious enough to require measures to control silting. A comparison of the data in Table 2 with Fig. 15 indicates that the Brown Canyon Barrier and the proposed high barrier at the Weir Site might be expected to have an annual loss of storage capacity near this critical rate; however, the 7-ft and 15-ft barriers in reach 4 (see Table 2) definitely would be expected to have excessive silting rates. The model studies, in fact, indicated that the basins in these two low-barrier systems would become almost completely filled during a single flood of the magnitude of that of January, 1943. It is admitted, of course, that the data used in plotting Fig. 15 are too meager to give a definite relationship between annual storage loss and capacity-area ratio; however, the general magnitude of the critical value of the ratio appears reasonable. Future sedimentation surveys of water and debris storage basins in Southern California will provide needed data on reservoir silting rates in that area.

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[14] "The Control of Reservoir Silting," by C. B. Brown, *Miscellaneous Publication No. 521*, U.S.D.A., 1943.

[15] "Silting of Reservoir," by H. M. Eakin and Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., revised August, 1939.

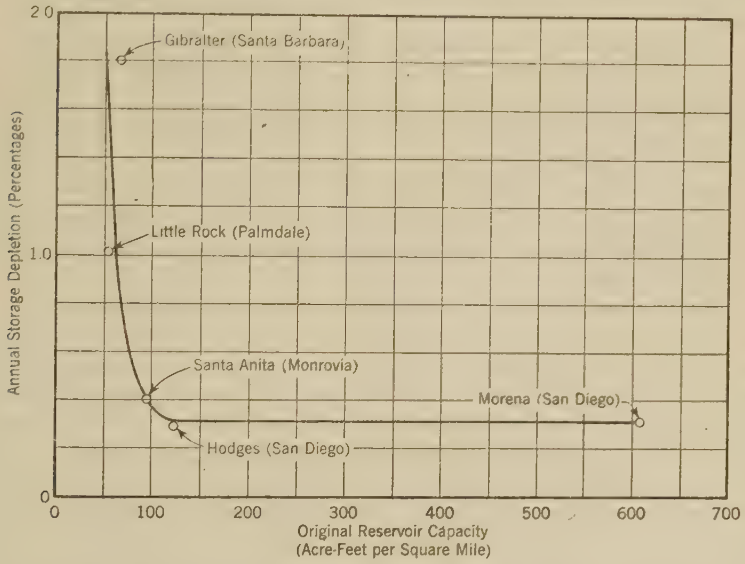

FIG. 15.—RELATIONSHIP BETWEEN ANNUAL DEPLETION OF RESERVOIR CAPACITY AND CAPACITY-WATERSHED-AREA RATIO FOR VARIOUS SOUTHERN CALIFORNIA RESERVOIRS

KARL J. BERMEL,[16] ASSOC. M. ASCE, AND ROBERT L. SANKS,[17] JUN. ASCE.—The writers wish to express their appreciation of the interest taken by those who submitted discussions; for the valuable contributions, substantiating data, and constructive criticisms of this paper.

Mr. Betts, discussing that phase of the studies dealing with detrital slopes upstream of the barrier, states, "* * * extensive tests covering wide ranges of hydrographs would have added greatly to the information in this field had the project requirements justified the extra cost." Although cost is certainly a consideration, justification of more extensive testing was limited primarily by the lack of sufficient data or knowledge as to the laws governing flood flows and the transport of bed-load material. To obtain this information, aside from the attendant difficulties, is both time consuming and costly and is certainly beyond the scope of these studies; however, until such time as measurements of this type are made and laws established governing both model and prototype behavior, models at best only can be relied upon to indicate general tendencies. This also applies to the first part of the comment by Mr. Amidon, that "* * * it is regrettable that more definite data did not result relating to the movement of debris and to the probable ultimate debris slope to be stabilized by the structure."

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[16] Senior Engr., Div. of Mech. Eng., Univ. of Calif., Berkeley, Calif.

[17] Asst. Engr., Univ. of California, Berkeley, Calif.

As for the latter part of Mr. Amidon's comment, it was previously mentioned in the paper that true stability (ultimate debris slopes) would never be achieved since variations in flow, bed-load size, and amount available for transportation, as well as changes in flow sections, changes of vegetative growth in the channel bottom, and many other factors would influence the debris slopes that would exist after each successive flood. If these factors and the variation of these factors were known, and could be predicted for the future, then model studies could be made that would provide information as to the maximum, minimum, or average storage that would attain for the period of time considered.

It is unfortunate that Mr. Amidon did not include more detailed data regarding the measurements of debris slopes above the approximately one hundred existing structures. These measurements cover a sufficient range of original channel slopes so that it might be possible to determine if the debris slopes as measured vary as a function of the original channel slopes. In the paper, the final detrital slopes, as achieved in the model, were compared with the original channel slopes; however, the findings of the model studies indicate that the detrital slopes vary as a function of the flow, the size and gradation of the bed-load material, and the amount of bed load transported or available for transport.

For the experiments performed to determine the extent of scour, the maximum depth of scour, and the distance downstream to the point of maximum depth of scour, the authors used the experimental method as suggested by Mr. Scimemi, with the exception that a graded bed-load material was used instead of material successively smaller in grain size. This material was the same as that used for the deposition studies upstream of the barrier; that is, it varied from 0.1 mm to 15 mm in size. The discrepancy of 3 ft less depth of scour, as compared to the experiments performed at the University of Padua, Italy, is no doubt due to the retention of the larger sized particles. Whereas the latter experiments are more exact as to the limiting depth of scour, the use of the bed-load material was considered more indicative of the conditions that would exist after bed-load material was transported over the barrier, which would be the conditions of operation for the greatest part of the life of the structure.

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Paper No. 2320

TRUCK SPEED AND TIME LOSS ON GRADES

By J. W. Stevens,[1] Assoc. M. ASCE

With Discussion by Messrs. Jonathan Jones, Elliott J. Dent, F. B. Ogle, and J. W. Stevens.

Synopsis

In the design of grades it is frequently necessary to ascertain the delay to traffic resulting from the decrease in speed of heavily loaded trucks on adverse grades. On routes involving heavy traffic it is particularly important to determine the effect of speed reduction and time loss incurred by loaded trucks on hill and overpass grades upon the operation of traffic at a constant design speed. Where the time loss is large it may be necessary to use preventive or remedial measures, such as an alternate route in rough terrain, a reduction of the rise in grade at an overpass, or in an extreme case the construction of an additional traffic lane, in order to avoid congestion for the anticipated traffic density.

A procedure for calculating the velocity curve and determining the time lost by a given vehicle traveling over an adverse grade is described in this paper. It is possible to determine the relative advantages of alternate grade designs with regard to the operation of truck traffic. The method outlined was developed in the design of a freeway project, and the examples used in the paper are concerned with overpass grades on that project. However, the application of the method is wider than that covered by the examples. The entire range of problems involving time loss on grades can be handled by the outlined methods, modified as suggested in the paper.

In the freeway design problem, certain conclusions were reached concerning the effect of variations in grade rise and grade rate at overpasses upon time loss. These conclusions and some general comments regarding the effect of truck speed reduction upon highway traffic capacity are included in the paper.

1. Basic Data and Principles

Basic data for computing truck velocities on grades were published in 1942 by Carl C. Saal,[2] including tests made by the U. S. Public Roads Administration

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Note.—Published in September, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Bldg. Insp. Engr., City of Amarillo, Amarillo, Tex.

[2] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 33.

(USPRA) to determine the velocities that could be maintained on various grades by trucks carrying various loads, and to determine the decrease in performance of trucks due to age. Although the direct conclusions to be drawn from Mr. Saal's findings pertain to the maximum sustained speed of loaded trucks on long grades, the tests also included the determination of various data which are required for the computation of truck speeds on short grades and on combinations of up-and-down grades under different conditions of loading and power.

In the calculation of velocity and time curves for trucks, operating over a given profile, it is possible to utilize principles which have been used for some time in studies of railroad train operation. Since the operating characteristics of motor vehicles are considerably more flexible than those of railway trains, it is probable that the results of calculations for trucks are subject to errors which would not be encountered in railroad work. Also, since experimental data on the tractive resistance of trucks are scarce, it will often be necessary to use assumed values for this factor. Furthermore, the difference in driving habits of vehicle operators and the total lack of knowledge regarding what may be expected as an average procedure for the application of the available tractive effort of the vehicle in question necessitate the use of rather broad assumptions relative to the manner of applying the motor power, especially on descending grades. It is desirable that additional test data on tractive resistance and driver habits be obtained, in order that the limitations which these features impose on velocity-curve determination will be removed.

2. Fundamental Equations of Motor Vehicle Dynamics

The forces that affect the speed of a motor vehicle may be grouped under three main classifications: (1) The tractive effort exerted by the motor through the driving wheels; (2) the total tractive resistance of the vehicle, including rolling and air resistance; and (3) the accelerating or decelerating force created by the grade on which the vehicle is operating. The first two classes of forces are subject to determination by more or less elaborate tests; the third is easily computed from the gross weight of the vehicle and the rate of grade.

The familiar equation, $F = m\,a$, with modifications for the problem under consideration, is used for calculating velocity changes with varying forces. Mr. Saal uses the equation:[3]

$$P_e = a\,(m + k) + W_g\,(f) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which P_e is the net tractive effort, in pounds, at drive wheels; a is the acceleration of vehicle, in feet per second per second; m is the mass of vehicle $= W_g/32.2$, in pounds mass; k is the mass equivalent constant for rotating parts of vehicle, in pounds mass; W_g is the gross vehicle weight, in pounds; and f is the coefficient of tractive resistance, in pounds per pound.

Eq. 1 is applicable for level grade, and it may be modified to the following form, which is applicable, with only small error, for inclined grades:

$$P_e = a\,(m + k) + W_g\,(f) + W_g\,(S) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which S is the rate of grade, in feet of rise per foot of horizontal distance.

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[3] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 49, Eq. 15.

Eq. 2 may be rearranged in the following form:

$$a = \frac{P_e - [W_g(f) + W_g(S)]}{m + k} \quad \text{.....................(3)}$$

and the acceleration a thus evaluated may be used in calculating the change in velocity of the vehicle in a given distance or time interval, when the initial velocity is known.

3. USE OF NUMERICAL INTEGRATION FOR CALCULATING VELOCITY CURVES

The method described herein for computing velocity curves for motor vehicles involves the use of a numerical integration process, which is based on the following principles:

Fig. 1 shows velocity, acceleration, and time curves, referred to distance plotted along the abscissa. If the velocity v_1 is known, an approximation to v_2 may be obtained by means of the relation,

$$v_2 = v_1 + a_m(t_2 - t_1) \quad \text{....(4)}$$

in which a_m is the mean value of the acceleration between s_1 and s_2. It is evident that the time interval, $(t_2 - t_1)$, is equal to the distance interval, $(s_2 - s_1)$, divided by the mean velocity between s_1 and s_2. If sufficiently small intervals of distance are used, any desired degree of accuracy may be obtained in the approximation; the arithmetic mean values, $(v_1 + v_2)/2$ and $(a_1 + a_2)/2$, may be used, instead of the actual mean values, for velocity and acceleration, respectively, obtained by dividing the areas under the corresponding curves in the interval (s_1 to s_2) by the length of the interval.

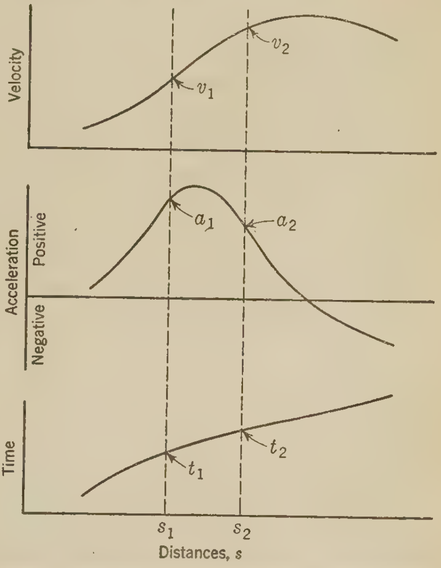

FIG. 1.—CHARACTERISTIC VELOCITY, ACCELERATION AND TIME CURVES

4. OUTLINE OF METHOD FOR COMPUTING VELOCITY CURVES

The method for computing the velocities of a given vehicle at successive points along a given profile may be stated briefly as follows:

Uniform distance intervals, sufficiently short to afford the desired degree of accuracy in velocity and time values, are marked off on the profile throughout the section under consideration. The uniform intervals may be subdivided if necessary at points where one or more of the forces acting on the vehicle changes abruptly. The change in velocity in each distance interval is calculated by the procedure outlined in the next paragraph; and the velocity at the end of each interval, the mean velocity in the interval, and the time required to traverse the interval are computed. From the series of values obtained for the successive intervals through the section of profile being studied, the velocity curve may be plotted, and the time lost by the vehicle, compared to the time required for traveling over the section at a given uniform velocity, may be determined.

The forces acting on the vehicle at a given point, such as the beginning or the end of a distance interval, are determined by the power developed by the motor, the rate of grade at the given point, and the velocity of the vehicle. These forces are the tractive effort P_e in Eq. 3 which may be computed from the motor power by a method explained in Section 7, the grade resistance $W_g\ (S)$ in Eq. 3, and the tractive resistance $W_g\ (f)$ in Eq. 3 (which varies with the vehicle velocity). The acceleration or deceleration of the vehicle at the beginning of the distance interval under consideration is computed by means of Eq. 3; that is, by dividing the resultant force (which is the algebraic sum of the three forces, P_e, $W_g\ (S)$, and $W_g\ (f)$) by the sum of the mass (m) of the vehicle and a mass equivalent constant (k) for the rotating parts of the vehicle. In a similar manner the forces acting on the vehicle and the acceleration or deceleration at the end of the distance interval are computed. Since the velocity at the end of the interval is unknown at this stage, an assumed value of f must be used to obtain the tractive resistance. The use of the value of f corresponding to the velocity at the beginning of the interval in question will afford satisfactory accuracy; although, if desired, the value of f corresponding to the approximate anticipated velocity at the end of the interval may be used for closer results. The average value for acceleration or deceleration in the distance interval is computed by taking the arithmetic mean of the values at the beginning and the end of the interval. Then the change in velocity through the distance interval is calculated by multiplying the average acceleration or deceleration by an assumed period of time for the vehicle to traverse the distance interval under consideration. This change in velocity is added algebraically to the initial velocity for the interval to obtain the velocity at the end of the interval, and the arithmetic mean velocity for the interval is computed. The time required for traversing the interval at this average velocity is computed by dividing distance by velocity. If the time thus obtained does not check that previously assumed, the computation of velocity change and the succeeding calculations are repeated until corresponding values for time and mean velocity are obtained. Fair accuracy usually may be secured on the first trial, since the time can be estimated closely from the initial velocity, and from a knowledge of whether the acceleration is positive or negative. When satisfactory agreement between time and mean velocity is obtained, the corresponding values for these items and for the velocity at the end of the interval are taken as final for the interval under consideration. The foregoing series of operations are repeated for the successive distance intervals.

5. Vehicle Data Required for the Computations

The vehicle data required for the computations consist of the tractive effort available at the driving wheels, the tractive resistance of the vehicle for the entire range of velocities involved in the problem, the mass of the vehicle, and the mass equivalent constant for the inertia of the wheels and other rotating parts of the vehicle. Fairly complete data for the trucks and truck-trailer combinations tested by the USPRA are given by Mr. Saal.[2] The tractive resistance of a vehicle varies considerably for different types of pavement or road surface. The values which he gives are applicable for concrete pavement, and should be used only for concrete and closely similar types of surfacing. The tractive-resistance data presented by Mr. Saal are applicable to the low-type bodies used in the tests, and higher values should be used for large van-type bodies.

6. Application of Available Tractive Force

There is great uncertainty as to the manner in which the available tractive force of a vehicle will be applied by an individual driver; and there are little or no observed data to indicate an average habit pattern for the application of the tractive force. Therefore, it is necessary to use a driving procedure, for velocity-curve computations, based almost entirely upon assumption. For the 15-ton truck and the type of profile considered in this study, involving relatively low rises in grade in level terrain, it was decided to consider operation in high gear only. For a heavily loaded truck on ascending grade, it appears entirely logical to assume that the tractive force applied will be at or near the maximum available value, as long as the speed of the vehicle is decreasing. Such an assumption was used in the various cases covered in this study. It was also assumed that the tractive effort is increased uniformly from the value required for constant initial speed on level grade to the maximum available value as the vehicle traverses the sag vertical curve at the beginning of the grade rise. Thus it is assumed that other traffic prevents the operator of a heavily loaded truck from increasing speed prior to reaching the rising grade to compensate for the expected loss in speed, and that full power is applied by the time the maximum rate of grade is encountered.

It is more difficult to state what procedure can logically be assumed for application of the available motor power on a descending grade, or even after the vehicle has begun to accelerate in passing over a summit at full throttle. The most serious limitation upon the accuracy of computed velocity curves results from this feature. On a descending grade the vehicle may be operated in a great variety of ways—with the throttle fully open, partly open, or closed, in gear, or in neutral, and with or without application of the brakes. Any combination of these operating conditions in succession may be encountered. It is evident that very complicated tests would be required in order to determine, under such a variety of possibilities, a typical or average driving pattern of procedure. Lacking such test data, one can only attempt a fairly logical assumption for driving procedure on descending grades.

In the cases for which velocity curves were computed in this study, it is assumed that the truck is moving on a level grade at a speed of $V = 40$ miles per hr prior to entering the grade up to an overpass; and that the tractive effort is uniformly increased from the value required for operation on level grade to the maximum available value as the vehicle moves through the vertical curve from level to ascending tangent grade. The maximum tractive effort is maintained until the vehicle has begun to accelerate near the summit of the grade rise. Then it is gradually reduced to zero and maintained there until the truck is near the end of the descending grade, when it is gradually increased to the value for operation on level grade at the initial speed of 40 miles per hr. It is the intention to apply the power or tractive effort in accordance with the foregoing procedure in such a way that the vehicle speed will be restored to exactly 40 miles per hr at the end of the descending grade. This procedure does not result in the minimum possible time loss on the grades, for it does not take account of a possible increase in speed to a value greater than 40 miles per hr on the descending grade. Operation on the descending grade at a speed greater than 40 miles per hr might require the use of the brakes to avoid running too close to the vehicle ahead of the truck. In such a case, a time gain rather than a time loss is involved. Operation over the profile with minimum time loss could be effected only by the most skilful drivers, and certainly not by the average driver. The assumptions used in the problems are chosen to represent average conditions.

The objective of restoring the speed to 40 miles per hr at the end of the descending grade can be attained only by a trial procedure; but it is possible to use the acceleration curve as a basis for assumptions regarding the use of the tractive effort, so that this curve encloses approximately equal areas above and below the zero axis, within the limits of the rising and descending grades. In other words, the speed at the end of the overpass grade will be restored to the original value if the product of the deceleration and the distance through which it operates is made approximately equal to the product of the acceleration and the distance through which it operates. This technique is subject to a minor error resulting from the fact that the acceleration curve is plotted to a distance abscissa, rather than to a time abscissa.

For the grade problems discussed herein, it is assumed that the vehicle remains in high gear at all times when power is applied. This assumption is legitimate for the range of speeds involved, since about 30 miles per hr is the minimum speed encountered in normal overpass grade problems. On a given grade, the characteristics of the truck and the habits of the driver will determine the speed at which gear changes will be made. It is considered very unlikely that a change from high gear to second would be made under ordinary circumstances at a speed above 30 miles per hr. For the grade conditions considered herein, it appears probable that operation in high gear will result in a minimum time loss, since the time and speed lost in shifting gears twice would most likely counteract any possible saving in time realized by the utilization of the higher tractive effort available at a higher gear ratio.

In problems concerning long steep grades, of course, it will be necessary to consider changes in gear ratio in computing the velocity curves. This feature affects the tractive-force diagram and the mass equivalent constant for rotating parts of the vehicle, but it does not introduce serious complications in the calculations.

7. Formulas for Maximum Tractive Effort

The maximum tractive effort available at the drive wheels for a given vehicle may be calculated for various engine speeds and gear ratios, from data relative to the engine horsepower or torque and the over-all mechanical efficiency of the transmission and the rear end drive mechanism. For conversion of truck speed to engine speed, or vice versa, the following formula[4] is applicable:

$$\omega = \frac{168\, G\, V}{r} \qquad (5)$$

in which ω is the engine speed, in revolutions per minute; G is the ratio of total gear reduction; V is the road speed, in miles per hour; and r is the effective radius of drive wheels, in inches.

When the net engine horsepower for a given engine speed for full throttle operation is known, the net engine torque may be computed by means of the following relation:

$$T = \frac{33{,}000\, E}{2\, \pi\, \omega} \qquad (6)$$

in which T is the net engine torque, in pound-feet; and E is the net engine horsepower. If the net engine torque curve for a given truck is available, the tractive-effort curve for a given gear ratio may be derived from it by using the following formula (given in slightly different form[5] by Mr. Saal):

$$P_e = \frac{12\, T\, G\, \eta}{r} \qquad (7)$$

in which P_e is the net tractive effort, in pounds, available at drive wheels, corresponding to a given engine speed, at full throttle; and η is the ratio of over-all efficiency of drive mechanism, for given speed and gear ratio.

Substitution of the expression for torque, Eq. 6, in Eq. 7 gives the following formula for tractive effort:

$$P_e = \frac{12\, E \times 33{,}000\, G\, \eta}{2\, \pi\, r\, \omega} = \frac{198{,}000\, E\, G\, \eta}{\pi\, r\, \omega} \qquad (8)$$

8. Performance Data for Truck-Trailer Combination

The truck-trailer combination selected for example purposes in the computed velocity curves included in this study is a medium truck and a high van-type semi-trailer, having a gross weight (W_g) of 30,000 lb. Average performance data, rather than actual data for any one make and model of truck, are used. The maximum net horsepower output of the motor (E) is taken at 90 at a speed (ω) of 2,800 revolutions per min, and the over-all efficiency of the drive mechanism (η), exclusive of the motor, is assumed to be 90% throughout the range of speeds involved. Actually, the efficiency factor for a given gear ratio varies slightly with speed; however, sufficiently accurate results for the purpose of this study will be obtained by the use of a constant efficiency factor. Also, the tractive effort available at full throttle at a given gear ratio varies with the speed, but it was considered satisfactory to use a constant value for this factor, for the range of speeds involved. Using Eqs. 5 and 8, with a total gear reduction (G) of 7.15 and an effective drive wheel radius (r) of 18.5 in., the tractive effort (P_e) available at the wheels for full throttle operation was found to be 750 lb for a truck speed (V) of 40 miles per hr and 780 lb for a speed of 30 miles per hr. These values correspond to horsepower values (E) of 89.0 and 69.3, respectively, taken from a typical power curve for a 90-hp engine. A maximum tractive-effort value of 760 lb is used in the computations.

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[4] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 45, Eq. 10.

[5] *Ibid.*, p. 49, Eq. 16.

Values for the mass equivalent constant (k) used to represent the energy stored in rotating parts of the truck and trailer combinations were determined experimentally and are shown, in charts, by Mr. Saal.[6] For the truck and trailer type considered in this study, operating in high gear at a gear ratio of 7.15, Mr. Saal indicates that a mass equivalent constant of about 70 is applicable. The total mass of the vehicle effective in resisting acceleration or deceleration is computed as follows: Total mass $= m + k = \frac{W_g}{32.2} + 70 = \frac{30{,}000}{32.2} + 70 = 1{,}002$. To simplify the computations in this case, it is sufficiently accurate to use a value of 68 for k, which makes the total mass equal to 1,000 lb.

Two tractive-resistance curves are shown in Fig. 2. These curves represent data for the average tractive resistance of medium tractor trucks with low-bed semi-trailers, operating on concrete pavement in very good condition.[7] These curves represent the data for a combination with gross loads of 30,000 lb and 20,000 lb, as shown. In the computations of Section 11 the tractive resistances used are based on the 20,000-lb curve, on the assumption that it is approximately correct for the van type of bed with a load of 30,000 lb, since the air resistance will be higher than that for a low bed.

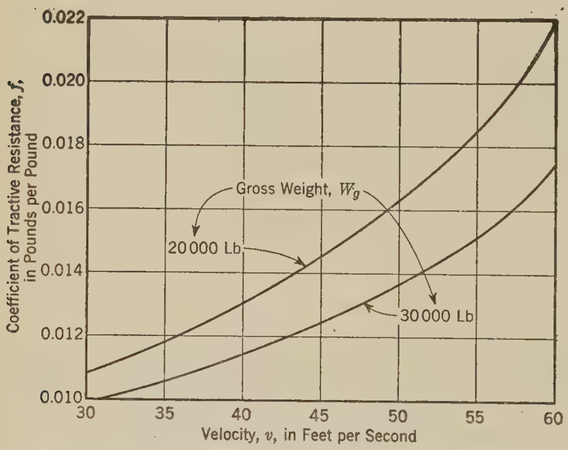

FIG. 2.—TRACTIVE-RESISTANCE CURVES, LOW-BED TRAILER ON CONCRETE PAVEMENT

The foregoing data constitute all the information relative to the vehicle required for computation of the velocity curve. Information regarding the profile is necessary to complete the data for the calculations.

9. DATA RELATIVE TO PROFILE

In the present study, made in connection with a divided highway in an urban area, where the average speed will be about 40 miles per hr, it is considered that a vertical sight distance of 500 ft should be provided, based on a line of sight 4.5 ft above the pavement surface at each end. This design requirement governs the length of summit vertical curves for various rates of grade on each side of the summit. The grade lines used in the speed-loss computations for rises of various heights were made symmetrical about their high point, the midpoint of the summit vertical curve, except in the case involving a rise without an equal grade drop. The lengths of the short vertical curves connecting to level grade at each end of the grades over the overpass were chosen in most cases so that a grade change of 5% per station results. For a given rise in grade above a level line there is a certain maximum rate of grade that can be used under the aforementioned conditions. This maximum grade will be that for which the ends of the summit vertical curve coincide with the ends of the sag vertical curves connecting to the level grade. Generally, this maximum rate of grade will provide the most economical solution from the standpoint of construction cost. There is a possibility, however, that it may involve excessive delay to traffic, on account of steep grades, if a heavy volume of truck traffic is involved. It might even be excessive for any type of traffic.

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[6] "Hill-Climbing Ability of Motor Trucks," by Carl C. Saal, *Public Roads*, May, 1942, p. 47, Figs. 17 and 18.

[7] *Ibid.*, p. 43, Table 15.

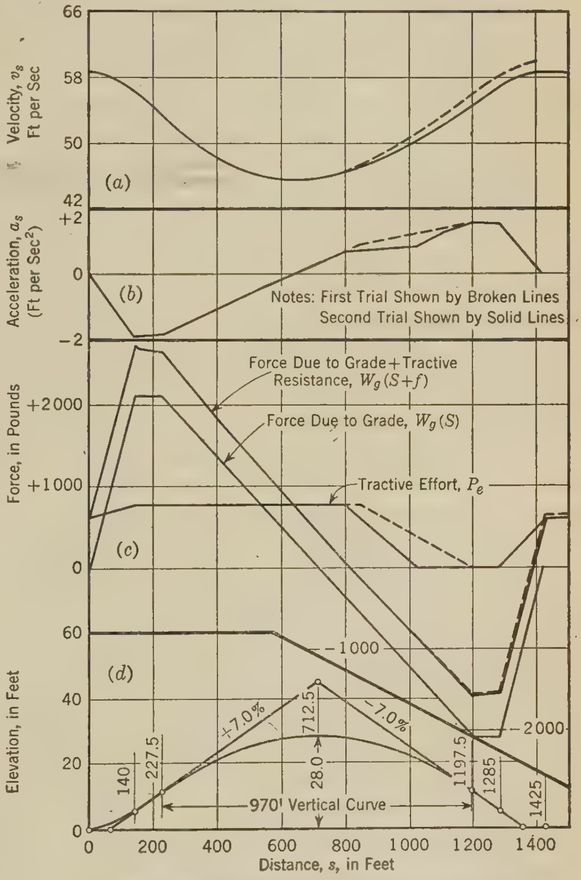

FIG. 3.—FORCE, ACCELERATION, AND VELOCITY CURVES (GROSS WEIGHT, 30,000 LB; 90-HP MOTOR; HIGH GEAR)

The grade line for the illustrative problem, for which detailed computations are outlined, provides a rise of 28 ft above the level approach grades, which is approximately the height required for a railroad overpass. A rate of grade of 7.0% was chosen as being about the maximum that is satisfactory for the type of highway under consideration. The length of vertical curve required to provide a sight distance of 500 ft between the +7.0% and −7.0% grades is 970 ft. The length of sag vertical curves required to connect the 7.0% grade and the level grade is 7.0/5.0 = 1.4 stations or 140 ft. The essential features of the profile are now known, and the grade line is plotted in Fig. 3, for convenience in visualizing the problem in the computations of Section 11.

10. Force Diagrams and Acceleration Curve

The next step is to plot the force diagram for the given profile: The force created by the grade, acting to change the velocity of the vehicle, is the product of the gross vehicle weight and the rise or fall of the grade in feet per foot of distance measured along the roadway surface. It is sufficiently accurate and more convenient to use the grade ratio of feet of rise or fall per foot of horizontal distance in computing the force due to grade. The error involved for a 7.0% grade is slightly less than 0.25%, which is negligible.

Where the grade is tangent, the force due to grade is constant. On vertical curves the rate of grade, and as a consequence, the force due to grade, vary uniformly. Since the force due to grade is determined by the profile, the complete diagram for this force may be drawn at once; but the diagrams for the tractive effort exerted by the truck and the tractive resistance cannot be computed at this stage of the work. The diagram for tractive resistance can only be plotted as the velocity computations are made, because the resistance varies with the speed of the vehicle. The curve for tractive effort is projected by trial, in an attempt to provide for the restoration of the vehicle speed to the initial value of 40 miles per hr at the end of the descending grade. In Fig. 3(*c*) the tractive-effort diagram is plotted upward from the zero force axis. The diagram for force due to grade is plotted upward for plus grades and downward for minus grades; and the tractive resistance is plotted upward from the grade-force diagram, as the successive tractive-resistance values are determined. This method of plotting provides a curve representing the resultant of the external forces acting on the vehicle—that is, all the forces except the tractive effort. The difference between the ordinate to this resultant curve and the ordinate to the tractive-effort curve, at a given point on the profile, is the net force acting to accelerate or decelerate the vehicle. When the resultant curve is above the tractive-effort curve, the computed net force is negative, decelerating the vehicle; when the resultant curve is below the tractive-effort curve, the net force is positive, and the vehicle is accelerating. Furthermore, the ordinates to the acceleration curve are proportional to the corresponding differences between the ordinates to the resultant curve and the tractive-effort curve. With this arrangement, it is easy to visualize the effect of changes in the applied tractive effort. By studying the acceleration curve from time to time after the vehicle is on the descending grade, the tractive-effort curve may be projected in such a manner that fewer trials will be required to attain the desired vehicle speed at the end of the grade.

11. Description of Detailed Computations

With the data in Sections 8 and 9, the numerical computations can be arranged in the form shown in Table 1, or in any shorter form suitable to the computer. The normal interval of distance is 50 ft, but, wherever the force acting on the vehicle changes sharply inside the normal 50-ft interval, the normal interval is subdivided. For instance, the normal interval is divided at the end of the sag vertical curve, distance $s = 140$, where the force due to grade reaches a maximum. The use of short intervals in this manner improves the accuracy of the approximation.

For the profile under consideration, the grade is level at distance $s = 0$ and the grade resistance, therefore, is zero. The velocity of the truck is 40 miles per hr or 58.667 ft per sec at distance $s = 0$. The coefficient of tractive resistance (f), taken from the curve in Fig. 2, is 0.0206 lb per lb of truck weight. The tractive resistance (R_t) corresponding to the initial velocity, and used also for

TABLE 1.—COMPUTATIONS FOR THE VELOCITY CURVE

FEET	POUNDS				FEET PER SEC²		FEET PER SEC		LB PER LB	FT PER SEC	SECONDS	
s	P_e	R_t	R_g	P_n	a_e	a_a	Δv_e	v_e	f	V	Δt	t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(*a*) DECELERATION ON RISING GRADE; MAXIMUM P_e APPLIED												
0	+618	−618	0	0	0	....	....	58.667	0.0206	....	....	0
50	+669	−618	−750	−699	−0.699	−0.350	−0.299	58.368	0.0205	58.518	0.854	0.854
100	+719	−615	−1,500	−1,396	−1.396	−1.048	−0.904	57.464	0.0198	57.916	0.863	1.717
140	+760	−594	−2,100	−1,934	−1.934	−1.665	−1.171	56.293	0.0193	56.879	0.703	2.420
150	+760	−579	−2,100	−1,919	−1.919	−1.926	−0.343	55.950	0.0190	56.122	0.178	2.598
200	+760	−570	−2,100	−1,910	−1.910	−1.914	−1.738	54.212	0.0181	55.081	0.908	3.506
227.5	+760	−543	−2,100	−1,883	−1.883	−1.896	−0.971	53.241	0.0176	53.727	0.512	4.018
250	+760	−528	−2,003	−1,771	−1.771	−1.827	−0.778	52.463	0.0173	52.852	0.426	4.444
300	+760	−519	−1,787	−1,546	−1.546	−1.658	−1.605	50.858	0.0167	51.661	0.968	5.412
350	+760	−501	−1,570	−1,311	−1.311	−1.428	−1.424	49.434	0.0161	50.146	0.997	6.409
400	+760	−483	−1,353	−1,076	−1.076	−1.194	−1.223	48.211	0.0156	48.823	1.024	7.433
450	+760	−468	−1,136	−844	−0.844	−0.960	−1.006	47.205	0.0153	47.708	1.048	8.481
500	+760	−459	−920	−619	−0.619	−0.732	−0.782	46.423	0.0150	46.814	1.068	9.549
550	+760	−450	−704	−394	−0.394	−0.507	−0.549	45.874	0.0148	46.148	1.083	10.632
600	+760	−444	−487	−171	−0.171	−0.282	−0.309	45.565	0.0147	45.720	1.094	11.726
650	+760	−441	−271	+48	+0.048	−0.062	−0.068	45.497	0.0147	45.531	1.098	12.824
700	+760	−441	−54	+265	+0.265	+0.156	+0.171	45.668	0.0148	45.583	1.097	13.921
750	+760	−444	+162	+478	+0.478	+0.372	+0.405	46.073	0.0149	45.871	1.090	15.011
800	+760	−447	+379	+692	+0.692	+0.585	+0.631	46.704	0.0151	46.388	1.078	16.089
(*b*) FIRST TRIAL FOR ACCELERATION												
850	+760	−453	+595	+902	+0.902	+0.797	+0.846	47.550	0.0154	47.127	1.061	17.150
900	+651	−452	+812	+1,001	+1.001	+0.952	+0.991	48.541	0.0158	48.046	1.041	18.191
950	+543	−474	+1,028	+1,097	+1.097	+1.049	+1.069	49.610	0.0162	49.075	1.019	19.210
1,000	+434	−486	+1,245	+1,193	+1.193	+1.145	+1.140	50.750	0.0166	50.180	0.996	20.206
1,050	+326	−498	+1,461	+1,289	+1.289	+1.241	+1.209	51.959	0.0171	51.354	0.974	21.180
1,100	+217	−513	+1,678	+1,382	+1.382	+1.336	+1.271	53.230	0.0177	52.594	0.951	22.131
1,150	+109	−531	+1,894	+1,472	+1.472	+1.427	+1.324	54.554	0.0183	53.892	0.928	23.059
1,200	0	−549	+2,100	+1,551	+1.551	+1.512	+1.368	55.922	0.0190	55.238	0.905	23.964
1,250	0	−570	+2,100	+1,530	+1.530	+1.540	+1.360	57.282	0.0198	56.602	0.883	24.847
1,285	0	−594	+2,100	+1,506	+1.506	+1.518	+0.920	58.202	0.0204	57.742	0.606	25.453
1,300	+66	−612	+1,875	+1,329	+1.329	+1.418	+0.364	58.566	0.0206	58.384	0.257	25.710
1,350	+287	−618	+1,125	+794	+0.794	+1.062	+0.900	59.466	0.0213	59.016	0.847	26.557
1,400	+508	−639	+375	+244	+0.244	+0.519	+0.435	59.901	0.0217	59.683	0.838	27.395
1,425	+618	−651	0	−33	−0.033	+0.106	+0.044	59.945	0.0217	59.923	0.417	27.812
1,450	+618	−651	0	−33	−0.033	−0.033	−0.014	59.931	0.0217	59.938	0.417	28.229
1,500	+618	−651	0	−33	−0.033	−0.033	−0.028	59.903	.....	59.917	0.835	29.064

TABLE 1.—(*Continued*)

FEET	POUNDS				FEET PER SEC²		FEET PER SEC		LB PER LB	FT PER SEC	SECONDS	
s (1)	P_e (2)	R_t (3)	R_g (4)	P_n (5)	a_s (6)	a_a (7)	Δv_s (8)	v_s (9)	f (10)	V (11)	Δt (12)	t (13)
(*c*) SECOND TRIAL FOR ACCELERATION												
800	+760	−447	+379	+692	+0.692	....	....	46.704	0.0151	....	....	16.089
850	+591	−453	+595	+733	+0.733	+0.712	+0.756	47.460	0.0154	47.082	1.062	17.151
900	+422	−462	+812	+772	+0.772	+0.752	+0.786	48.246	0.0157	47.853	1.045	18.196
950	+254	−471	+1,028	+811	+0.811	+0.792	+0.814	49.060	0.0160	48.653	1.028	19.224
1,000	+84	−480	+1,245	+849	+0.849	+0.830	+0.839	49.899	0.0163	49.480	1.011	20.235
1,025	0	−489	+1,353	+864	+0.864	+0.856	+0.427	50.326	0.0164	50.113	0.499	20.734
1,050	0	−492	+1,461	+969	+0.969	+0.916	+0.453	50.779	0.0166	50.553	0.495	21.229
1,100	0	−498	+1,678	+1,180	+1.180	+1.074	+1.047	51.826	0.0171	51.303	0.975	22.204
1,150	0	−513	+1,894	+1,381	+1.381	+1.280	+1.221	53.047	0.0176	52.437	0.954	23.158
1,200	0	−528	+2,100	+1,572	+1.572	+1.476	+1.374	54.421	0.0182	53.734	0.931	24.089
1,250	0	−546	+2,100	+1,554	+1.554	+1.563	+1.418	55.839	0.0190	55.130	0.907	24.996
1,285	0	−570	+2,100	+1,530	+1.530	+1.542	+0.958	56.797	0.0195	56.318	0.621	25.617
1,300	+66	−585	+1,875	+1,356	+1.356	+1.443	+0.380	57.177	0.0197	56.987	0.263	25.880
1,350	+287	−591	+1,125	+821	+0.821	+1.088	+0.943	58.120	0.0203	57.649	0.867	26.747
1.400	+508	−609	+375	+274	+0.274	+0.548	+0.470	58.590	0.0206	58.355	0.857	27.604
1,425	+618	−618	0	0	0	+0.137	+0.058	58.648	0.0206	58.619	0.426	28.030
1,450	+618	−618	0	0	0	0	0	58.648	0.0206	58.648	0.426	28.456
1,500	+618	−618	0	0	0	0	0	58.648	....	58.648	0.853	29.309

the interval from distance $s = 0$ to distance $s = 50$, is the gross vehicle weight (W_g) multiplied by the coefficient, or $30{,}000 \times 0.0206 = -618$ lb. Since the truck speed (V) is constant on the level approach grade, the tractive effort (P_e) at distance $s = 0$ is just sufficient to balance this resistance; that is, +618 lb. In the numerical work, the forces tending to decelerate the vehicle are marked negative and those tending to accelerate the vehicle are marked positive. The foregoing data are then recorded at $s = 0$ ft in Table 1.

The tractive resistance (R_t) used at distance $s = 50$ is −618 lb, as stated in the preceding paragraph. The engine tractive effort (P_e) at distance $s = 50$ is equal to $618 + (760 - 618) \times 50/140 = +669$ lb. The grade resistance (R_g) is equal to $30{,}000 \times 0.07 \times 50/140 = -750$ lb (see $s = 50$ ft under Cols. 1 to 4, Table 1). Taking the algebraic sum of the engine tractive effort, the tractive resistance, and the grade resistance, the net force P_n is found to be −699 lb; the acceleration (a_s) at the end of the interval between $s = 0$ and $s = 50$ is computed by dividing the net force by the total mass of the truck, $-699/1{,}000 = -0.699$ ft per sec per sec; and the average acceleration (a_a) during the passage of the truck from distance $s = 0$ to distance $s = 50$ is the arithmetic mean of the accelerations at the beginning and the end of the interval, $(0.000 - 0.699)/2 = -0.350$ ft per sec per sec (see $s = 50$ under Cols. 5, 6, and 7, Table 1).

The velocity change (Δv_s) in the interval from $s = 0$ to $s = 50$ is computed by multiplying the average acceleration by the time (Δt) taken for the truck to pass through the distance interval at an average velocity (V) equal to the arithmetic mean of the velocities at the beginning and the end of the interval. It is necessary to assume an average velocity (V); then find the time for this velocity; compute the velocity change and the velocity at the end of the interval; and observe whether the mean velocity checks that assumed. Using an assumed average velocity of 58.500 ft per sec, the time is 50/58.5 = 0.855 sec, and the velocity change is $-0.350 \times 0.855 = -0.299$, giving a velocity of $58.667 - 0.299 = 58.368$ ft per sec at distance $s = 50$, and an average velocity in the interval of $(58.667 + 58.368)/2 = 58.518$ ft per sec. This velocity is used for the second trial, since it does not check the value previously assumed. The time for the second trial is $50/58.518 = 0.854$ sec, and the velocity change is $-0.350 \times 0.854 = -0.299$ ft per sec. The velocity at the end of interval is $58.667 - 0.299 = 58.368$; and the mean velocity is the same as computed in the first trial (see $s = 50$, under Cols. 8, 9, 11, and 12, Table 1).

The time for the distance interval, 0.854 sec, is added to the previous value for total time. The coefficient of tractive resistance for the velocity at the end of the interval is read from the curve in Fig. 2, and found to be 0.0205 lb per lb (see $s = 50$, under Cols. 10 and 13, Table 1). The complete data for the second line of the table have thus been computed.

These several steps are repeated for the computation of the data for distance $s = 100$, and for the subsequent distance intervals. The computations for time and velocity may be performed very easily and quickly with a slide rule, with sufficient accuracy for the purpose. On certain types of rules, only one setting of the slide is required for each trial; the second trial is usually sufficiently close to the correct value, so that, in reality, the computations are much simpler than the description indicates.

In Fig. 3 the curves for the complete computations shown in Table 1 are plotted. The tractive-effort curve indicated by a broken line in Fig. 3(c) represents the first trial to increase the speed of the vehicle to the initial value of 40 miles per hr at the end of the descending grade. The corresponding velocity curve (Fig. 3(a)) is also indicated by a broken line. The tractive-effort curve shown by the solid line in Fig. 3(c) was projected as the second trial, and the corresponding velocity curve is also shown by a solid line in Fig. 3(a). The numerical values for the second trial are indicated in Table 1(c).

12. Time and Distance Loss

The time lost by the assumed vehicle for a given distance is the difference between the computed time for traversing this distance and the time required to cover the same distance at a constant speed equal to the initial speed, which is 40 miles per hr in the illustrative problem. Fig. 4 shows the time and distance lost for any point on the profile of the example. The computed time curve represents the total time, shown in Col. 13, Table 1, plotted against the distance for each 50-ft interval point on the profile, the plotted curve being that of the values for the second trial computation for points beyond distance $s = 800$ (see Table 1(c)). The time curve for uniform velocity is a straight line, with a slope of one second on the time scale to 58.667 ft on the distance scale.

The time loss at a given point on the profile, as compared with a uniform speed of 40 miles per hr, may be scaled from Fig. 4, as the difference between the ordinates to the computed time curve and the time curve for uniform velocity, for the distance in question. For distance $s = 1{,}425$ the time loss scales 3.7 sec. Since this point is the end of the descending grade of the profile used in the example, the time loss at this point may be called the total for the profile.

The total time loss is the answer sought in most overpass grade problems, since it may be used as a basis for comparison with other grade lines between control points, and intermediate values for time loss are not important. For such cases, the total time loss can be calculated directly without plotting the time curves. In the example, the total computed time for the distance of 1,425 ft, in the second trial, is 28.03 sec. The time required to cover this distance at a constant speed of 40 miles per hr is $1{,}425/58.667 = 24.29$ sec. The total time loss is $28.03 - 24.29 = 3.74$ sec.

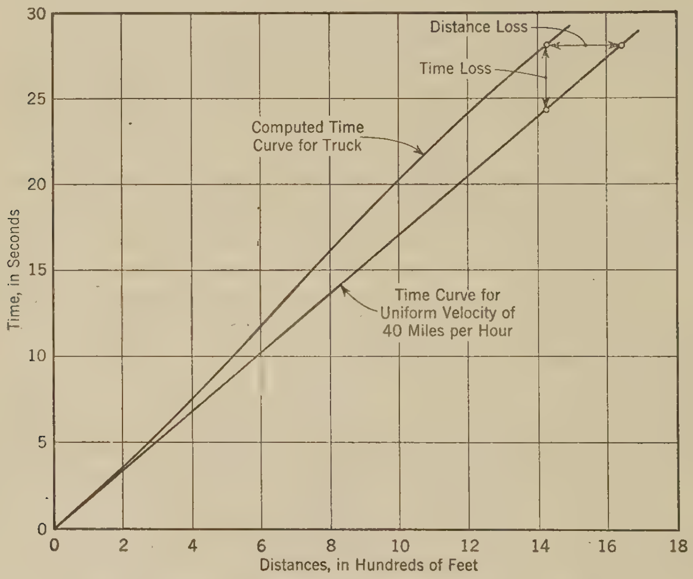

Fig. 4.—Time-Distance Curves Corresponding to Fig. 3

The distance lost by the truck at a given point on the profile is defined as the product of the time loss in seconds at the given point and the governing initial velocity, using the unit of feet per second to obtain the distance loss in feet. In the example, the total distance loss for the profile, at $s = 1{,}425$, is $3.74 \times 58.667 = 219$ ft. In other words, when the truck in the example reaches $s = 1{,}425$, it is 219 ft back of the position it would have reached in the same time at a constant speed of 40 miles per hr. The distance loss may be scaled from Fig. 4, as the distance between the two time curves—that is, by measuring horizontally from a point on the computed time curve, corresponding to the actual distance or time in question, to the time curve for uniform velocity.

For determining time or distance, lost or gained, by scaling from a time-distance curve, the measurement should be made in all cases from the point in question on the computed curve to the curve for uniform velocity.

13. USE OF TIME INTERVALS, RATHER THAN DISTANCE INTERVALS, FOR VELOCITY-CURVE COMPUTATIONS

It is practicable to use a computation procedure involving time intervals, instead of distance intervals, as used in the illustrative problem. This procedure would simplify the detailed computations for velocity slightly, since it would eliminate the necessity for trial determination of the time interval during which a certain average acceleration operates; but the use of this procedure would complicate the determination of the forces acting upon the vehicle at the end or the beginning of the constant time interval, and would make the plotting of the acceleration and velocity curves more difficult.

14. GRAPHIC METHODS FOR DETERMINING VELOCITY CURVES

Graphic methods for the construction of velocity curves, providing satisfactory accuracy for the type of problem under consideration, have been used in connection with similar studies in railway work, and there are apparently no circumstances that would prevent the adaptation of these graphic methods for studies of truck velocities. The description of such a method, including a bibliography, was presented in 1941 by A. I. Lipetz.[8] It is considered unlikely that the graphic method would prove advantageous, from the standpoint of saving time and labor, as compared with the numerical method described herein, for the short profiles under consideration in this study. In the comparison of different grade lines, such as might be encountered on different routes between control points in mountainous terrain, the use of a graphic method might be distinctly advantageous.

15. EFFECT OF HEIGHT OF RISE AND RATE OF GRADE ON TIME LOSS

In an effort to determine the factors that would provide the best designs for grade lines over grade separation structures on a freeway, computations of the velocity curves and the time loss for the truck described in Fig. 3 were made for a variety of grade lines. Values between 5 ft and 28 ft were used for the rise in grade. The velocities were not restored to exactly 40 miles per hr at the end of the descending grade in some cases, since it appeared that variations up to about 0.3 mile per hr would permit sufficiently accurate computation of time loss. An endeavor was made to use procedures for applying the tractive effort on the descending grade for the different profiles which followed the same general principles in all cases.

In Fig. 5 the time losses for the various profiles are plotted against the rise in grade. The curve drawn through points A to I is a quadratic parabola. From the relation between the plotted points and the parabola it is evident that the time loss is approximately proportional to the square of the rise in grade, and the four profiles studied for the 28-ft value of rise indicate that the time loss is not greatly affected by the rate of grade. These conclusions are based on the assumed truck-trailer combination described in Section 8, and on a definite general procedure for the application of the tractive effort of the truck;

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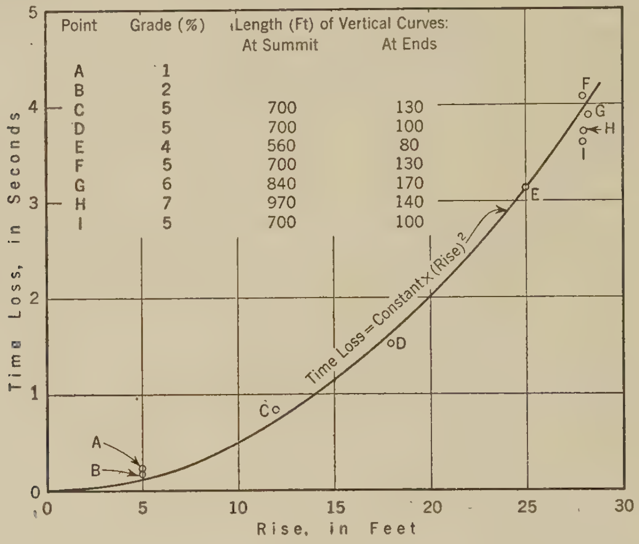
[8] "Graphical Methods for Plotting Time-Speed-Distance Curves for Railway Trains," by A. I. Lipetz, *Transactions*, A.S.M.E., October, 1941, p. 603.

and they should not be applied directly for other truck designs or loads, or for grades that require a shifting of gears. However, it appears that the assumptions that time loss is approximately proportional to the square of the rise and is independent of the rate of grade may be used as a general basis for designing overpass grades, provided the same standard for vertical sight distance governs for the different profiles considered.



Point	Grade (%)	Length (Ft) of Vertical Curves: At Summit	At Ends
A	1		
B	2		
C	5	700	130
D	5	700	100
E	4	560	80
F	5	700	130
G	6	840	170
H	7	970	140
I	5	700	100

Fig. 5.—Time Loss for a Truck on Overpass Grades (90-Hp Motor, High Gear; W_g = 30,000 Lb; Vertical Sight Distance, 500 Ft)

16. Effect of Overpass Grades on Highway Capacity

The time lost by a single truck on an up-and-down grade such as that discussed herein is a fairly definite quantity, subject to the aforementioned limitations on accuracy. The effect of a rise in grade to overpass a railroad or street on the carrying capacity of a single traffic lane or on the capacity of a complete highway facility is somewhat more difficult to evaluate. It is possible, however, to give some general comments relative to this question, which may be useful in the study of specific problems.

In 1942, O. K. Normann[9] presented a paper containing data and comments pertaining to the maximum theoretical capacity and the practical working capacity of various types of highways. In the remarks that follow, use is made of ideas expressed by Mr. Normann, with only a brief explanation of the fundamentals involved. Reference to the original source will provide the reader with a broader background for review of the remarks contained herein.

There is an average minimum spacing, which depends upon velocity, for vehicles moving in a single lane of a given highway, and this spacing may be determined by the proper test methods when traffic conditions on the highway are suitable. On the basis of the average minimum spacing between vehicles for various speeds, it is possible to calculate the maximum theoretical traffic capacity of the traffic lane under consideration. Under actual operating conditions, the traffic volume will rarely reach the maximum theoretical capacity, since this capacity is based on continuously maintaining the minimum spacing. There is a definite practical working capacity for a given traffic lane of a given highway, however, and a working capacity for a given traffic facility, taken as a whole. The practical working capacity of a given section of highway is a relative value depending on various local conditions.

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[9] "Results of Highway-Capacity Studies," by O. K. Normann, *Public Roads*, June, 1942, p. 57.

If a truck travels over a certain section of a highway which includes an overpass grade, and there is no other traffic on the section of highway at the time, the reduction in speed of the truck in moving over the grade rise affects the truck alone. That is, no other vehicle is delayed as a result of the reduction in speed of the truck. On the other hand, if the same section of highway is carrying its maximum theoretical traffic volume, for the assumed speed of 40 miles per hr (which condition may exist for a short time) and one of the vehicles moving over the section is a truck such as that of Section 8, it is evident that each vehicle following the truck over the grade rise will lose time in an amount equal to the time lost by the truck, provided the average minimum spacing is maintained through the section, since under the condition of maximum theoretical capacity there will be no passing. If a stream of traffic feeds into a section of highway containing an overpass grade at a speed of 40 miles per hr, at the average minimum spacing for this speed, and a considerable percentage of trucks is involved, the time lost by a truck at the grade rise will result in delay for each vehicle following the truck. The loss of time incurred by each additional truck will cause additional delay for the vehicles behind it, and congestion will eventually result at some point on the highway back of the grade rise. The maximum theoretical traffic capacity of the section of highway for a speed of 40 miles per hr will be reduced as a direct result of the presence of the grade rise, provided, of course, the trucks and cars do not regain the time lost at the overpass grade by operating at a speed in excess of 40 miles per hr at any place in the section.

The conditions outlined in the preceding paragraph are extreme. The practical working capacity of a lane of a divided multilane highway will probably be less than two thirds of its theoretical maximum capacity; and, although two, three, or more vehicles will be encountered in a group, spaced at or near the minimum distance, such groups will be separated by spacings greater than the minimum. Under these conditions, the reduction in speed of a given truck passing over a grade rise would result in delay for the vehicles following the truck and not separated from it or the preceding vehicle by a spacing sufficiently long to obviate the effects of speed reduction by the vehicles ahead.

This last point may be clarified by an example, in which it is assumed that, on one lane of the section of road approaching the grade rise, vehicles are traveling in groups of five, at 100-ft average minimum spacing for the initial speed of 40 miles per hr; and these groups of five vehicles are spaced 1,000 ft apart, net. Then, if a truck included in one of the groups incurs a time loss and corresponding distance loss on the given profile, only the vehicles behind the truck in its group will be delayed as a result of the truck time loss, unless the distance loss for the last vehicle in this group amounts to more than 900 ft; that is, the original group spacing of 1,000 ft, less the average minimum spacing between vehicles for the speed involved. If the latter condition occurs, vehicles in the group following that containing the truck will incur delay; if the delay to this second group is sufficiently great, a third group also will be delayed, and so on. A succession of grade rises in the profile under consideration would involve cumulative time losses. From these considerations, it appears that in a practical problem the number of vehicles that would be delayed by a reduction in speed of a truck on a given profile can be determined only on the basis of mathematical probability, taking proper account of the average minimum spacing between vehicles, the average number of vehicles in a group, the average spacing between groups, and the proportion of vehicles that are unable to maintain speed on the given profile.

Furthermore, under working conditions, vehicles following the truck would not necessarily be prohibited from passing the truck while its velocity is less than 40 miles per hr, or from increasing speed to a value above 40 miles per hr and passing the truck after traveling over the grade rise; so it is possible that, on a sufficiently long section of highway involving one overpass grade, some of the vehicles in a group following a truck at the grade rise might pass the truck and thus permit the remainder of the vehicles to make up their lost time, with the result that none of the vehicles except the truck would incur a time loss for the section of highway under consideration. Finally, it is possible that the truck might operate at a speed greater than 40 miles per hr after passing the overpass grade and thus overcome the loss of time on the grade.

The foregoing remarks indicate the wide range of conditions that may be encountered in a study of the effect of grade rises on the capacity of a highway. Ordinary conditions of the profile and ordinary proportions of truck traffic apparently are not likely to justify the construction of an extra traffic lane on overpass grades to permit passing of heavily loaded vehicles. However, such extra lanes are often justified and have been built at many locations on rural highways where fairly long and steep grades are involved, to eliminate excessive delays to traffic.

17. Time Loss on Rising Grade Without Compensating Falling Grade

As may be seen from a study of Fig. 3, the force due to grade is large in comparison with the tractive effort available from the engine for heavily loaded vehicles on ordinary overpass grades. In cases where the grade rise is not accompanied by a corresponding drop in grade beyond the overpass, a considerable increase in time loss prior to a restoration of the velocity to its initial value is to be expected, as compared to time loss in the case where the rise is compensated by an equal fall. Fig. 6 shows complete force, acceleration, velocity, and time curves for two profiles involving a rise of 12 ft. The broken lines represent values for a profile with a 12-ft drop in grade beyond the overpass, and the solid lines represent values for a profile with a level grade beyond the overpass. The total time loss for the first case is 0.83 sec and the distance loss

49 ft. For the second case the time and distance losses are 1.50 sec and 88 ft, or about 80% greater than for the first case.

The rise of 12 ft involved in these cases is an ordinary value for urban grade separations between streets, and the value of 1.5 sec for time loss is small. For a rise of 25 ft or more, not compensated by a drop in grade, it appears that excessive time loss might result. To permit the use of a convenient scale, the time curves between the beginning of the profile and distance $s = 500$ are not shown in Fig. 6.

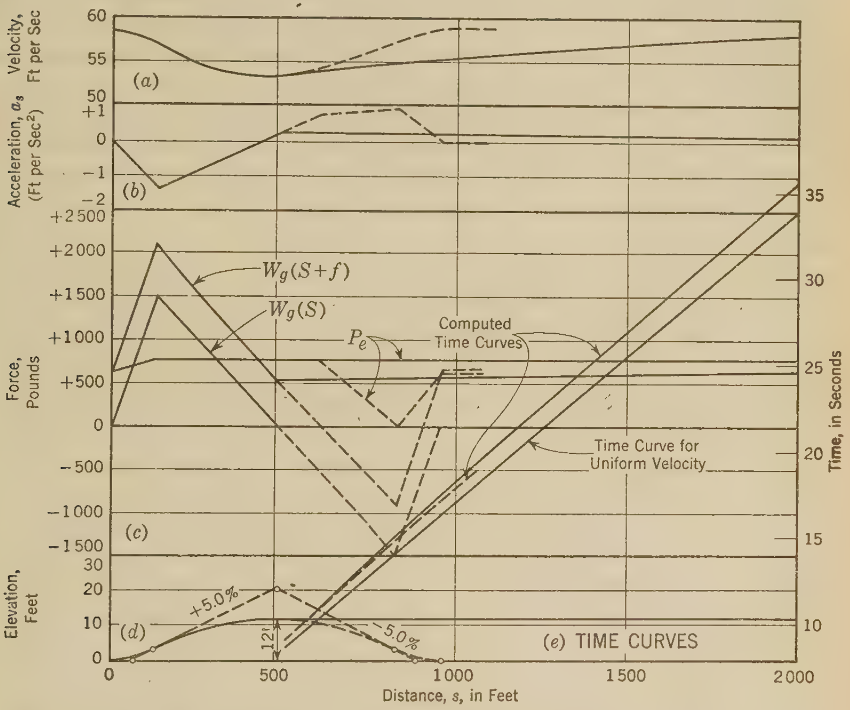


FIG. 6.—TRUCK OPERATION CURVES FOR A GRADE RISE WITH OR WITHOUT A COMPENSATING FALL

18. CONCLUSION

This paper is presented with the primary purpose of calling attention to the method of obtaining the velocity curve for a given vehicle traveling over a given profile. Although the original studies on which the paper is based concern overpass grades, and the examples are taken from those studies, the applicability of the outlined methods to a wide variety of grade problems is evident.

It is felt that the comments regarding the effect of low rises in grade, on truck speed and highway traffic capacity, may be useful in a field wider than that of the overpass grades for which they were originally formulated, and that they are of sufficient interest to warrant their inclusion with the exposition of the method for velocity-curve computation.

It is hoped that the method and corollary ideas will provide the engineer with improved means for selecting the best grade line, from the standpoint of operation of trucks and mixed highway traffic, not only for short adverse grades, but for longer profiles as well.

19. ACKNOWLEDGMENT

The methods outlined were developed under the direction of H. G. Bossy, of the Texas Highway Department. For his numerous suggestions during the work, and for his aid in preparing the paper, Mr. Bossy deserves special credit. The work was done while the writer was employed as designing engineer by the Texas Highway Department of which D. C. Greer, M. ASCE, is state highway engineer, and J. L. Dickson was engineer of road design.

The writer has also received assistance, in the final preparation of the manuscript, from the Civil Engineering Department of the University of Texas at Austin.

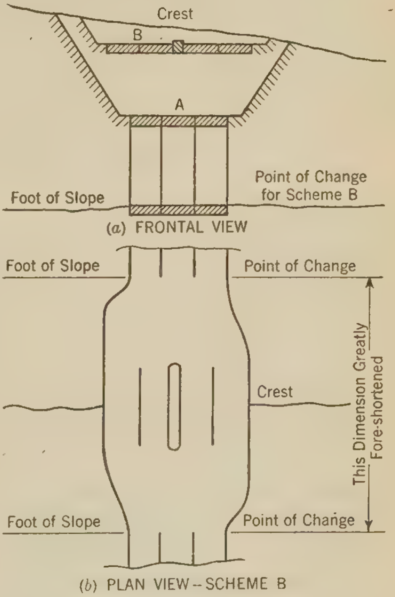
DISCUSSION

Jonathan Jones, M. ASCE.—The impression a layman receives from reading this paper is that the adverse effect upon general traffic of the slowing down of trucks on ascending grades is not a very serious matter; this impression is summarized in the two concluding sentences of Section 16. The author may be justified in reaching this conclusion for his locality, and for overpasses required in otherwise flat terrain. He considers, however, that his study is more widely applicable than that. In the rapidly succeeding hills and valleys of the state of Pennsylvania, the backing-up effect of trucks upon general traffic on ascending grades is seriously exasperating and, human nature being what it is, a source of great danger.

All may be well on four-lane divided highways, assuming that all slow-moving trucks will occupy the right lane in each direction; but on two-lane and three-lane highways, ascending, faster vehicles cannot pass a truck without extreme danger; nor, after passing the crest, can they often pass because at that stage the trucks will accelerate quickly to the legal limit of speed. Consequently there is no greater boon desired by the general driver in a hilly state, than the elimination of the backing-up effect of slow-moving trucks on steep ascending grades.

The author's statement "Ordinary conditions of the profile and ordinary proportions of truck traffic apparently are not likely to justify the construction of an extra traffic lane * * *" should be taken as applying to overpass grades and not to the means of crossing a natural rise. There is in such situations an alternative means which has taken hold, to what the writer considers an excessive degree; and the intent of these remarks is to express a belief that here the construction of the extra lane, where relief is needed, will usually be superior to the alternative practice of making a deep cut to reduce the grade.

Fig. 7 will serve to illustrate this point in terms of a highway practice of which the writer is well aware through observing the construction of improved highways in his vicinity. This diagram shows roughly the frontal view and the plan view of an incline from a valley to a crest, on which it may be assumed that the natural grade would be severe, and that this may be reduced to half or less by making the deep cut indicated as scheme A. So doing does not, however, enable all trucks to climb the grade without reduction of speed, but only



(a) FRONTAL VIEW

(b) PLAN VIEW—SCHEME B

Fig. 7

some of them. The others reduce their speed somewhat, although not so much as on the original grade. The delay to following traffic is therefore somewhat mitigated; the danger is not removed. Passing on the center lane is dangerous and in many instances forbidden by law. The cost of the cut is extreme; in one observed instance it added at least three months to the construction time, during which traffic was detoured, because of the necessity for blasting solid rock.

If, however, this road were constructed from valley to crest as shown as scheme B, and in the plan view, Fig. 7(*b*), providing an additional lane on the right side of the ascending pair, and separating the ascending and descending pairs with a raised element, then all passenger and light truck traffic could negotiate the steep grade in high gear, without any danger from oncoming traffic, and therefore without the need for special sight distance. The cost (because the ground would be only barely scratched instead of deeply cut) should be little more, and frequently would be less, then alternative A. The required right of way would be reduced. The pleasure of the average passenger would be enhanced, because he would be viewing a landscape instead of the side of a cut. Trucks on the right-hand lane would naturally be traveling at considerably slower speed by scheme B than by scheme A; but their loss in this respect would be the gain of all others.

As a user of the highway both for business and pleasure, the writer believes that the construction of an extra traffic lane over natural rises is very frequently justified, and that the fullest consideration should be given to providing this lane in accordance with scheme B rather than seeking to avoid it by the adoption of scheme A.

ELLIOTT J. DENT,[10] M. ASCE.—In discussing this interesting subject there are certain practical considerations that the writer believes should receive more emphasis.

On the upgrades encountered on main highways the modern passenger car can, ordinarily, maintain normal speed; in fact, there is often a considerable reserve of power for pickup. Trucks, on the other hand, are slowed down to a marked degree. On the downgrades trucks can, and do, maintain the legal or safe speed limits.

On an upgrade a car can readily slow down or stop; on a downgrade the control of the vehicle is much less efficient. Other things being equal, the safe speed on an upgrade is greater than that on a downgrade.

If the foregoing were the only considerations, the easy and safe places for passenger cars to pass trucks would be on the upgrades. It is unfortunate that in such locations the summits limit the sight distances and make it unsafe to pass, whereas on the downgrades the visibility is much better.

In rolling country, on heavily traveled two-lane highways, it is inconvenient and most exasperating that, when the safe and available speeds are favorable for passing, the visibility so often inhibits such action. On the other hand, when the visibility is satisfactory, the available relative speeds are unfavorable.

In the majority of cases the logical solution is the provision of an additional traffic lane on the upgrade side of the crest.

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[10] Cons. Engr., Washington, D. C.

F. B. Ogle,[11] Assoc. M. ASCE.—The proper maximum grades for highways, required to meet a specific set of conditions, can be determined by the long-needed method presented in this paper. Certain variable factors, particularly driver habits and truck operating characteristics, will be minimized by driver training and by the standardization of truck building and maintenance, which will be forced by competition. This development is foretold by the experience in railroad operation of the past. The method proposed by Mr. Stevens is sound, simple, and easily applied.

The first designers of the modern highway were primarily concerned with providing an all-weather surface for light, self-propelled vehicles of relatively low speeds. The alinement usually followed existing roads; and the grade line followed the existing grade closely. Based on present-day standards, this alinement was poor, and the adverse grades were excessive in many cases.

As the traffic load increased, improvements consisted primarily of widening pavements, inserting curves at angle turns, flattening existing curves, and reducing excessive grades to an assumed maximum rate of rise. The assumed maximum grade rates were low enough to avoid any serious reduction in speed of the then predominately automobile traffic. Highway traffic capacity did not appear as a problem.

The appearance of the freight hauling truck in substantial numbers in the past few years has changed the picture materially; and highway capacity has become an important problem in highway design. The trend clearly indicates that in the near future, certainly within the normal life-span of the highways now being designed, truck traffic will predominate. This trend will be accelerated by the adoption of uniform traffic regulations by all the states. Such regulations will come in time, and their adoption will increase interstate movement greatly. For economic reasons, commercial trucks in interstate haul must operate with heavier loads and at higher speeds.

Grades that can be negotiated by the passenger automobile at 50 miles per hr or more will force the trucks into lower gears, requiring them to crawl up at 10 miles per hr, or less. On long grades where traffic is heavy, this produces a serious condition, both from the standpoint of safety and loss of highway capacity. This is apparent to all drivers. One method proposed by Mr. Stevens for relieving this congestion is to provide passing lanes which will release the faster moving vehicle; but it will give no relief to the trucks, which can be relieved only by proper grade design with the truck as the controlling factor.

Grade design based on truck operation will increase the construction or first cost; but it will reduce the unit cost of truck operation and increase the highway capacity. The net result will be to reduce the actual cost based on number of vehicles accommodated; and these are, in fact, the true costs.

It appears that the expenditure of public funds for the convenience of trucks operated for private profit is not justified, as has been contended by the railroad companies. This is unquestionably true in so far as the benefits accrue to the truck operator; but the movement of freight on the highways has become vital to the economic life of the United States, and any reduction in the cost of freight movement will accrue to the public.

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[11] Senior Resident Engr., Texas Highway Dept., Lubbock, Tex.

J. W. Stevens,[12] Assoc. M. ASCE.—The discussions indicate an opinion which is evidently held by many private drivers: That trucks on hills delay traffic to an annoying extent. The writer heartily agrees with the comments pointing out the need for passing lanes on many existing highway grades. It is easy to determine whether there is serious need for a passing lane on an existing highway grade merely by observation of traffic in the field and without analysis of the profile and possible truck speed.

Passing lanes should be provided in the original design, instead of waiting until the pavement is built before determination of their need. Therefore, the application of the speed and time loss studies is most valuable in the general design of a project on new location. In many cases the need for passing lanes will be evident without detailed study, and the necessity for such study will be reduced to the doubtful or borderline cases.

Special emphasis should be given to the limitation of the conclusion at the end of Section 16 (which applies strictly to overpass grades in otherwise flat terrain) as pointed out by Mr. Jones. It is probable that passing lanes might be amply justified on fairly steep grades involving rises of the order of 50 ft on highways where a rise of 25 ft would not require an extra lane.

Mr. Ogle's discussion indicates that the question of permitting automobiles to pass slow-moving trucks is only part of the designer's problem. The need for grade design to provide for efficient operation of truck traffic is increasing, and will continue to increase as the proportion of truck traffic grows.

The writer feels that the published discussions are valuable for their additions to the ideas covered in the paper, and wishes to express appreciation for these discussions.

Attention should be called again to the possibility of combining arithmetical operations to shorten the computations, as mentioned in the first sentence in Section 11. Any user of the method will find additional short cuts particularly suited to his procedure. It is interesting to note that both the shape and height of the velocity curve in Fig. 3 show considerable resemblance to the profile curve. This resemblance, of course, depends on the scales of the two curves and on the applied power of the vehicle, and possibly is purely coincidental. However, remarkable similarity of the velocity and profile curves was noted in all cases covered by Fig. 5. It is conceivable that the similarity of these curves might be established for short sections of profile in many cases, so that the velocity curve could be drawn with satisfactory accuracy by inspection of the profile, thus eliminating computations.

It is hoped that the remarks in Section 14 will discourage no one from making full use of graphic methods in speed and time loss problems. The writer's attention has been called to the possibility of graphic solutions shorter than those of which he was aware when the paper was prepared. The graphic methods for time loss determination might well form the subject of a valuable paper for the benefit of engineers concerned with this phase of highway design.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2321

DRAWDOWN TEST TO DETERMINE EFFECTIVE RADIUS OF ARTESIAN WELL

By C. E. Jacob,[1] Assoc. M. ASCE

With Discussion by Messrs. N. S. Boulton, Carl Rohwer, R. M. Leggette, M. R. Lewis, and C. E. Jacob.

Synopsis

The drawdown in an artesian well that is pumped has two components: The first, arising from the "resistance" of the formation, is proportional to the discharge; and the second, termed "well loss" and representing the loss of head that accompanies the flow through the screen and upward inside the casing to the pump intake, is proportional approximately to the square of the discharge. The resistance of an extensive artesian bed increases with time as the ever-widening area of influence of the well expands. Consequently, the specific capacity of the well, which is discharge per unit drawdown, decreases both with time and with discharge.

The multiple-step drawdown test outlined in this paper permits the determination of the well loss and of the "effective radius" of the well. The trend of drawdown is observed in the pumped well and in one or more near-by observation wells as the discharge is increased in stepwise fashion. A simple graphical procedure gives the permeability and the compressibility of the bed. From these several factors it is possible to predict the pumping level at any time for any given discharge.

Notation

The letter symbols in this paper are defined where they first appear and are assembled alphabetically, for convenience of reference, in the Appendix.

Introduction

It has long been known that the discharge of an artesian well is almost, but not quite, proportional to the drawdown. In a well that is pumped the drawdown is the difference between the static water level and the pumping

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Note.—Published in May, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C.

water level, customarily measured after several hours of continuous operation. Usually the major part of this loss of head, or drawdown, occurs in the formation, where the energy expended in overcoming the frictional resistance of the sand against the slowly moving water is directly proportional to that rate of motion. A smaller although no less important part of the loss of head occurs as the water moves at relatively high velocities through the screen and upward inside the casing to the intake of the pump. This head loss is approximately proportional to some higher power of the velocity approaching the square of the velocity. Adding these two components of drawdown:

$$s_w = B\,Q + C\,Q^2 \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

—approximately. Considering the drawdown, s_w, analogous to electric potential drop and the discharge, Q, analogous to electric current, the factor B can be defined as the "resistance" of the formation. This factor represents the total hydraulic resistance of the formation, from the face of the well to some distance where the head drop is virtually zero and where the radial motion of water toward the discharging well has not yet begun. The ratio of discharge to drawdown, called "specific capacity," is seen from Eq. 1 to be

$$Q/s_w = 1/(B + C\,Q) \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Clearly, then, the specific capacity must vary, however slightly, with the discharge. Also, it must vary with time because, as will be shown, the resistance B increases with time as the ever-widening area of influence of the well expands.

It is the purpose of this paper to demonstrate that the factors B and C can be determined by a procedure that is little more elaborate than the usual "drawdown test" made to determine the specific capacity and to check the performance of the pump and motor. This is accomplished simply by controlling, more closely, the stepwise variation of the discharge and by observing, more frequently and more accurately, the trend of the pumping level as it is lowered.

Distribution of Drawdown in and Near an Artesian Well

Fig. 1 shows three typical examples of artesian wells that completely penetrate extensive formations of assumedly homogeneous structure and uniform thickness. Fig. 1(*a*) is the ideal case of an uncased hole drilled through a water-bearing sandstone confined above and below by impervious shales. Virtually all the head loss occurs in the formation, since no well screen or slotted casing is present to impede the flow of water into the hole. There must inevitably be some additional friction as the water moves up the hole, and consequently the pumping water level in the hole does not coincide exactly with the head at the face of the hole just inside the formation, but rather stands somewhat lower, perhaps as indicated by the dashed line in Fig. 1(*a*).

Fig. 1(*b*) shows a common type of construction in an unconsolidated artesian sand, where a screen is necessary. For comparison, this sand is shown as having the same thickness and permeability as the sandstone of Fig. 1(*a*).

In the second case, with the same discharge there is the same drop in head within the formation; but in addition there is a "well loss" which includes the head lost through friction as the water flows upward inside the screen and casing to the pump intake as well as to the head drop across the screen. The radial distribution of head within the formation is about the same in both cases, which may indicate that the sand in Fig. 1(*b*) is not greatly affected by drilling or developing operations; the screen effectively retains all the sand including fines.

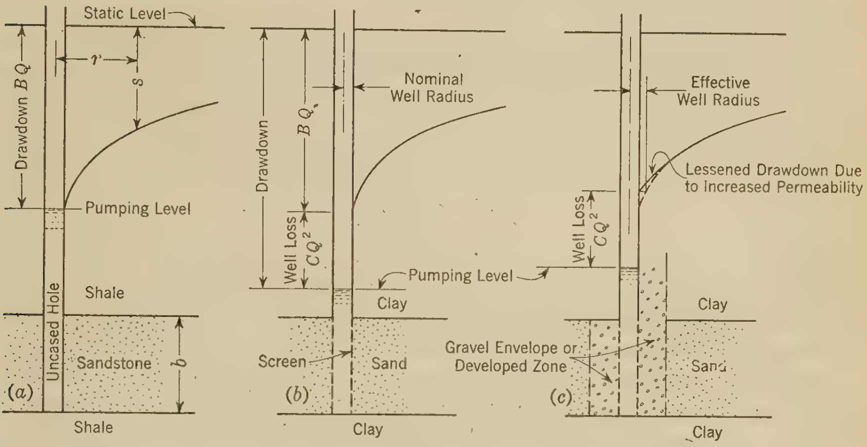
Static Level
Drawdown BQ
r
s
Pumping Level
Shale
Uncased Hole
Sandstone
b
(*a*)
Shale
Drawdown
BQ
Well Loss CQ^2
Nominal Well Radius
Pumping Level
Clay
Screen
Sand
(*b*)
Clay
Effective Well Radius
Lessened Drawdown Due to Increased Permeability
Well Loss CQ^2
Clay
Gravel Envelope or Developed Zone
Sand
(*c*)
Clay

FIG. 1.—TYPICAL EXAMPLES OF ARTESIAN WELLS, SHOWING DISTRIBUTION OF DRAWDOWN

A common, although not always essential, item of artesian well construction in unconsolidated formations—the gravel envelope—is shown in Fig. 1(*c*). The gravel envelope is particularly effective when the water-bearing sand is fine and of uniform grading. When properly constructed, it is also useful in other situations—to prevent the fines from being drawn into the well. If the size of gravel is properly chosen, the head loss in the immediate vicinity of the screen is reduced to less than it would be if the natural undisturbed water-bearing formation which the gravel replaced were there. Developing a well to remove the fines from the material surrounding the screen has a similar effect. In some sands, developing operations alone are adequate and gravel-wall construction is not needed. In either case the increased permeability of the material surrounding the well lessens the drawdown and increases the effective radius of the well. "Effective radius" is defined as that distance, measured radially from the axis of the well, at which the theoretical drawdown based on the logarithmic head distribution (defined subsequently by Eq. 4) equals the actual drawdown just outside the screen (see Fig. 1(*c*)).

The dashed curved line in Fig. 1(*c*) represents the head distribution that would exist if the water-producing bed were left in place undisturbed, with uniform permeability. That curve duplicates the drawdown curves in Figs. 1(*a*) and 1(*b*). It is so shaped because, to maintain a steady flow of water (at the rate Q) toward the well, the hydraulic gradient must be inversely proportional to the radial distance; or, actually:

$$\frac{ds}{dr} = -\frac{Q}{2\,\pi\,k\,b\,r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which b is the thickness of the bed and k is the "permeability" or transmission constant of the sand, defined as:

> "* * * the quantity [volume] of water that would be transmitted in unit time through a cylinder of the soil of unit length and unit cross-section under unit difference in head at the ends."[2]

Integrating Eq. 3 between the fixed limit r_w and the variable limit r:

$$s_w - s = \frac{Q}{2\,\pi\,T}\log_e\frac{r}{r_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

In Eq. 4, the "transmissibility," T, is the product of k and b.[3] Eq. 4 gives a logarithmic distribution of drawdown that holds in the immediate vicinity of a well pumping from an artesian bed which it penetrates completely. Assuming that the drawdown, s_w, is known at the effective well radius, r_w, the drawdown at some greater distance may be determined easily. Actually, as a matter of common knowledge, the drawdown in an artesian well increases continuously with time (rapidly at first, of course, and then more slowly) as long as the discharge continues at a steady rate and also provided that the well is not too near the margin of the aquifer where the head may be maintained essentially constant despite the withdrawal of water. To determine the drawdown at the well and its distribution throughout an extensive aquifer at any time, it is necessary to study the flow of the confined water in response to varying heads more closely, taking into consideration the compressibility of the water and also the compressibility of the sand bed.

Theory of Nonsteady Radial Flow in an Extensive Artesian Aquifer

Consider a cylindrical shell of height b, inner radius r, and outer radius $(r + \delta r)$ concentric with the axis of the well. By the principle of continuity, the net outward flow of water from this shell must equal the time rate of decrease of the volume of water within the shell, referred to a constant (atmospheric) pressure. The total volume of water in the shell is

$$V_w = 2\,\pi\,r\,\delta r\,b\,n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which n is the porosity of the sand. The time rate of decrease of this volume is $2\,\pi\,r\,\delta r\,b\,n\,\beta\,\frac{\partial(\gamma\,s)}{\partial t}$, in which γ is the specific weight of the water and β is its compressibility. To allow for the compressibility of the water-producing bed, which is assumed to be compacted elastically as the pressure is reduced and as the water is allowed to expand, an apparent compressibility, β', is substituted for β. Experience[4,5] indicates β' to be several times the actual water compressibility, β. Combining several factors into a nondimensional coefficient,

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[2] "Theoretical Investigation of the Motion of Ground Waters," by Charles S. Slichter, *Nineteenth Annual Report*, U. S. Geological Survey, 1899, Pt. II, p. 323.

[3] "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," by Charles V. Theis, *Transactions*, Am. Geophysical Union, Pt. II, 1935, p. 520.

$$S = \gamma\,\beta'\,b\,n \qquad (6a)$$

and $2\,\pi\,r\,\delta r\,S\,\frac{\partial s}{\partial t}$ is the time rate of decrease of the volume of water. In Eq. 6a S is the "coefficient of storage,"[6,7] which defines the volume of water that a unit decline in head releases from storage in a vertical prism of the aquifer of unit cross-sectional area.

The apparent fluid compressibility, β', is related to the respective compressibilities of the water and the sand as follows:

$$\beta' = \beta + \frac{\alpha}{n} \qquad (6b)$$

Again β is the compressibility of the water, and n is the porosity of the sand. The symbol α represents the "compressibility" of the sand bed—that is, the relative decrease in thickness of the bed per unit increase of the vertical component of compressive stress in the sand.

The foregoing relation in somewhat different notation was derived in an earlier paper[8] by the writer in which the theory of nonsteady flow in elastic artesian aquifers was developed.

The outward flow of water from the shell through its inner cylindrical surface is equal to $-\,2\,\pi\,r\,T\left(\frac{\partial s}{\partial r}\right)$. Similarly, the outward flow through the outer cylindrical surface is $2\,\pi\,T\,(r + \delta r)\left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\,\delta r\right)$. Assuming the upper and lower bounding planes of the aquifer to be impermeable, the sum of these two terms may be equated to the time rate of decrease of the enclosed volume of water. Expanding, eliminating differentials of higher order, simplifying, and dividing through by $(2\,\pi\,r\,T\,\delta r)$:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad (7)$$

The solution of this fundamental differential equation that is sought here must satisfy the following conditions:

$$s = 0 \qquad \text{for} \qquad t \leqq 0 \qquad (8a)$$

$$\lim_{r \to \infty} s = 0 \qquad \text{for} \qquad t > 0 \qquad (8b)$$

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[4] "Notes on the Elasticity of the Lloyd Sand on Long Island, New York," by C. E. Jacob, *Transactions*, Am. Geophysical Union, 1941, Pt. III, pp. 783–787.

[5] "Application of Coefficients of Transmissibility and Storage to Regional Problems in the Houston District, Texas," by W. F. Guyton, *ibid.*, pp. 756–770.

[6] "The Significance and Nature of the Cone of Depression in Ground-Water Bodies," by Charles V. Theis, *Economic Geology*, 1938, p. 894.

[7] "The Source of Water Derived from Wells," by Charles V. Theis, *Civil Engineering*, May, 1940, p. 277.

[8] "On the Flow of Water in an Elastic Artesian Aquifer," by C. E. Jacob, *Transactions*, Am. Geophysical Union, 1940, Pt. II, pp. 574–586.

and

$$\lim_{r \to 0} \left(r \frac{\partial s}{\partial r} \right) = - \frac{Q}{2 \pi T} \quad \text{for} \quad t > 0 \ldots\ldots\ldots\ldots (8c)$$

The answer is given in terms of an infinite series[8,9,10] as follows:

$$s = \frac{Q}{4 \pi T} \left(- 0.5772 - \log_e u + u - \frac{u^2}{2 \times 2!} - \frac{u^3}{3 \times 3!} + \cdots \right) \ldots (9)$$

in which

$$u = \frac{t^*}{t} = \frac{r^2 S}{4 T t} \ldots\ldots\ldots\ldots (10)$$

In still simpler notation,

$$s = \frac{Q}{4 \pi T} W(u) \ldots\ldots\ldots\ldots (11)$$

Fig. 2 gives a nondimensional plotting of Eq. 9 or of Eq. 11. The well starts pumping at a steady rate Q at time zero ($t/t^* = 0$). The drawdown at a given distance from the well increases very slowly at first and reaches a maximum rate of increase at $t/t^* = 1$. As this is the point of inflection on the time-drawdown curve, t^* may be called the "inflectional time." Thereafter the rate of increase of drawdown diminishes continually but never vanishes. Theoretically, the drawdown becomes infinite at infinite time.

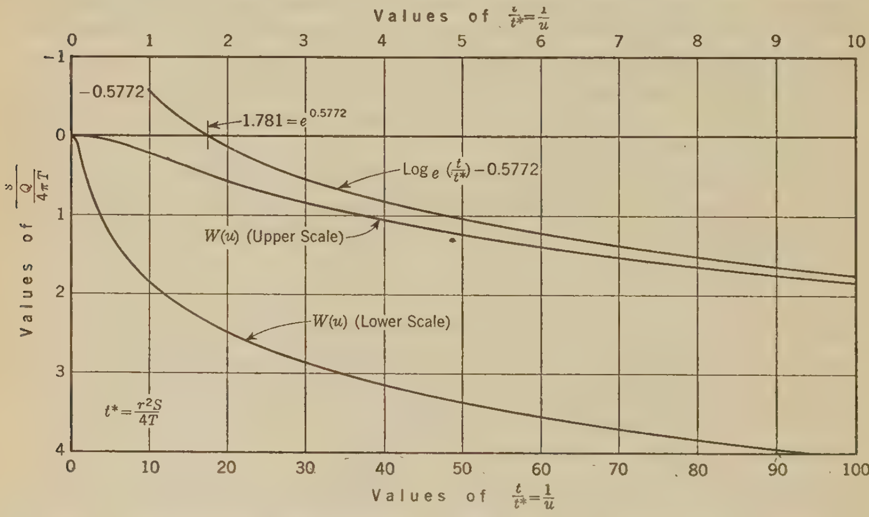

FIG. 2.—NONDIMENSIONAL TIME-DRAWDOWN CURVES, EXACT AND APPROXIMATE, FOR SINGLE WELL DISCHARGE AT A STEADY RATE, FROM AN EXTENSIVE ARTESIAN AQUIFER

In Fig. 3 the same equation is plotted on semilogarithmic paper, again in nondimensional form. For sufficiently large values of t, the W-function may be approximated by a simple logarithmic expression that plots as a straight

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[9] "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," by Charles V. Theis, *Transactions*, Am. Geophysical Union, 1935, Pt. II, pp. 519–524.

[10] "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, p. 667.

line on that graph. Thus, the drawdown after sufficient time has elapsed is given approximately by

$$s = \frac{Q}{4\,\pi\,T}\left(\log_e \frac{t}{t^*} - 0.5772\right) \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

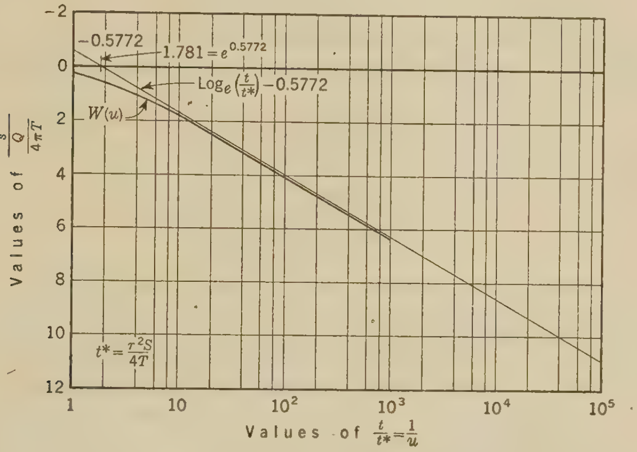

FIG. 3.—SEMILOGARITHMIC PLOTTING OF THEORETICAL TIME-DRAWDOWN CURVE AND STRAIGHT-LINE APPROXIMATION

The drawdown in a well with a negligible well loss (Fig. 1(*a*)) is then:

$$s_w = \frac{Q}{4\,\pi\,T}\left(\log_e \frac{t}{t^*_w} - 0.5772\right) \ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

in which $t^*_w = r^2{}_w \frac{S}{4\,T}$, r_w being the effective radius of the well. When the well loss is appreciable (Figs. 1(*b*) or 1(*c*)), the drawdown in the well is

$$s_w = \frac{Q}{4\,\pi\,T}\left(\log_e \frac{t}{t^*_w} - 0.5772\right) + C\,Q^2 \ldots\ldots\ldots\ldots (13b)$$

Comparing Eq. 13*b* with Eq. 1, the resistance of the aquifer is

$$B = \frac{\log_e \frac{t}{t^*_w} - 0.5772}{4\,\pi\,T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

According to Eq. 12 or Eq. 13*a*, S and T may be determined from a series of drawdown observations by plotting values of s against values of the logarithm of t. For sufficiently large values of t, relative to t^* or t^*_w, the points should fall on a straight line. Taking two points on that line to determine the slope,

$$T = \frac{2.30\,Q\,\log_{10}\frac{t_2}{t_1}}{4\,\pi\,(s_2 - s_1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

Eq. 15*a* can be simplified further by choosing, arbitrarily, the two points one log cycle apart. Then, $\log_{10} \frac{t_2}{t_1} = 1$, and

$$T = \frac{2.30\,Q}{4\,\pi\,(s_2 - s_1)} \qquad (15b)$$

Knowing T, theoretically the value of S may now be determined from the intercept of the straight line with the zero-drawdown line because at this point $(t, 0)$—

$$\log_e \frac{4\,T\,t}{r^2\,S} = 0.5772 \qquad (16)$$

from which

$$S = \frac{4\,T\,t}{r^2\,e^{0.5772}} = \frac{2.25\,T\,t}{r^2} \qquad (17)$$

Application of Theory to a Simple Drawdown Test

Fig. 4 is a semilogarithmic graph of data from a simple drawdown test of a pumping well and a near-by observation well in glacial outwash near Meadville, Pa. The pumping well is of the gravel-wall type and has 15 ft of 18-in. screen between depths of 49 ft and 64 ft. During the test it was pumped at $Q = 1{,}350$ gal per min, or about 3.0 cu ft per sec. Observations of drawdown were made periodically by an air line in the pumping well. An automatic gage gave a continuous record of the drawdown and subsequent recovery in an observation well 1,200 ft away from the pumping well. The data for the drawdown period are plotted as open circles. The data for the recovery period are plotted as solid circles.

The transmissibility may be determined from the slope of either straight line. In both cases the change in drawdown over one log cycle is 2.27 ft. According to Eq. 15*b*, with Q equal to 3.0 cu ft per sec, the value of T is $\frac{2.30 \times 3.0}{4\,\pi \times 2.27} = 0.24$ sq ft per sec. The storage coefficient may be determined from the intercept of the upper straight line with the zero-drawdown line by substituting $t = 693$ sec in Eq. 17. The result of this calculation is $S = \frac{2.25 \times 0.24 \times 693}{1.44 \times 10^6} = 0.00026$.

Assuming the porosity of the sand to be 35% and its effective thickness about 100 ft, the apparent water compressibility is computed as follows: By Eq. 6*a*,

$$\beta' = \frac{S}{\gamma\,n\,b} = \frac{0.00026}{62.4\ (\text{lb per cu ft}) \times 0.35 \times 100\ (\text{ft})}$$

$$= \frac{1}{8{,}400{,}000\ (\text{lb per cu ft})} = \frac{1}{58{,}000\ (\text{lb per cu in.})}$$

The bulk modulus of gas-free water at ordinary temperatures is about 300,000 lb per sq in. The apparent water compressibility (which is the reciprocal of bulk modulus) in this case is then about five times the actual compressibility of water.

Assuming that this value of S holds within the immediate vicinity of the well, the well loss and the factor C may be determined under the further provisional assumption that the effective well radius is equal to the nominal well radius (see Fig. 1(b)).

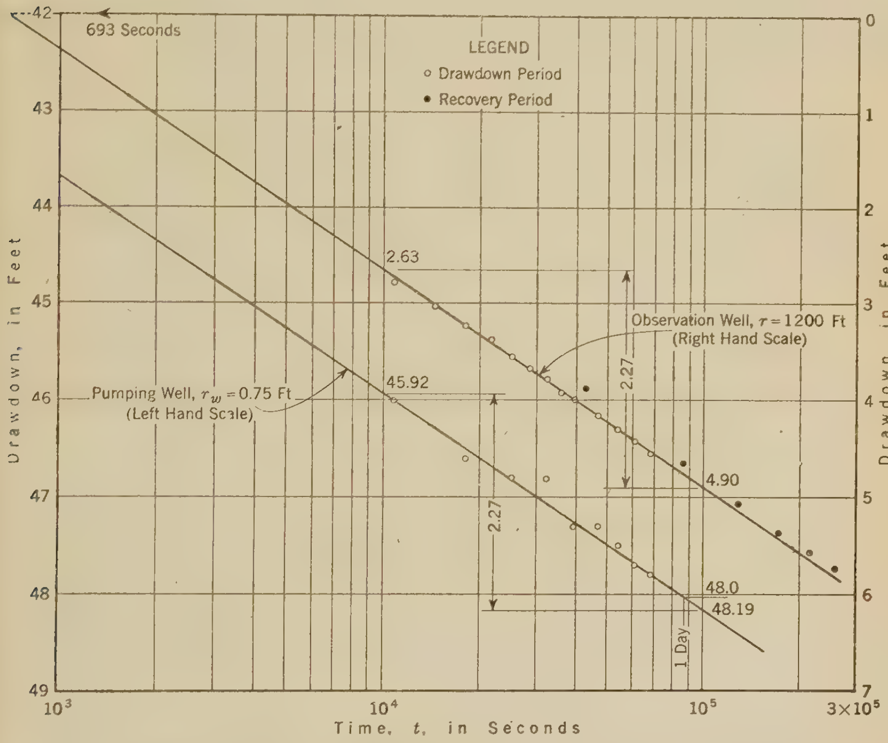

FIG. 4.—SEMILOGARITHMIC PLOTTING OF DATA FROM A SIMPLE DRAWDOWN TEST OF A PUMPING WELL AND A NEAR-BY OBSERVATION WELL TO DETERMINE TRANSMISSIBILITY, STORAGE COEFFICIENT, AND WELL LOSS

For example, to determine the drawdown for one day, with the use of Fig. 4, solve the formula:

$$s_w = \frac{Q}{4\,\pi\,T}\left(2.303\log_{10}\frac{4\,T\,t}{r^2_w\,S} - 0.5772\right) + C\,Q^2 \ldots\ldots\ldots (18)$$

The fraction $\frac{4\,T\,t}{r^2_w\,S} = \frac{4 \times 0.24 \times 86{,}400}{0.5625 \times 2.6 \times 10^{-4}} = 5.67 \times 10^8$; $\log_{10}\frac{4\,T\,t}{r^2_w\,S} = 8.753$; the value in parentheses in Eq. 18 equals $(8.753 \times 2.303) - 0.58 = 19.5$; and $s_w = \frac{3.0 \times 19.58}{4\,\pi \times 0.24} + C\,Q^2 = 19.5 + C\,Q^2 = 48.0$ ft. Finally, $C\,Q^2 = 9.0\,C = 48.0 - 19.5 = 28.5$ ft; from which $C = 3.2\left(\frac{\text{sec}^2}{\text{ft}^5}\right)$. The foregoing calculations indicate that $B\,Q$ in this case was about 19.5 ft after 24 hours of continuous pumping. The observed drawdown in the pumping well at that time was 48.0 ft, leaving 28.5 ft for the well loss. Inasmuch as Q was about 3.0 cu ft per sec, the numerical value of C was approximately 3.2.

Fig. 5 shows how the specific capacity of the well under discussion would vary with the discharge or with time. Because of the relatively high well loss, the one-day specific capacity for $Q = 3$ cu ft per sec is only about 60% of that for $Q = 1$ cu ft per sec. The one-year specific capacity for 3 cu ft per sec is about 65% of that for 1 cu ft per sec. Since at lower discharge rates a greater proportion of the total drawdown is attributable to head loss occurring within the formation (which increases with time while the other component remains constant), the specific capacity shows a greater percentage decline at the lower discharge rates. This fact is shown clearly in Fig. 5. With $Q = 1$ cu ft per sec, the specific capacity declines from about 0.104 cu ft per sec per ft at one day to about 0.086 cu ft per sec per ft at one year—a drop of about 17%. On the other hand, with $Q = 3$ cu ft per sec, the specific capacity declines from about 0.063 cu ft per sec per ft at one day to about 0.056 cu ft per sec per ft at one year—a drop of about 11%.

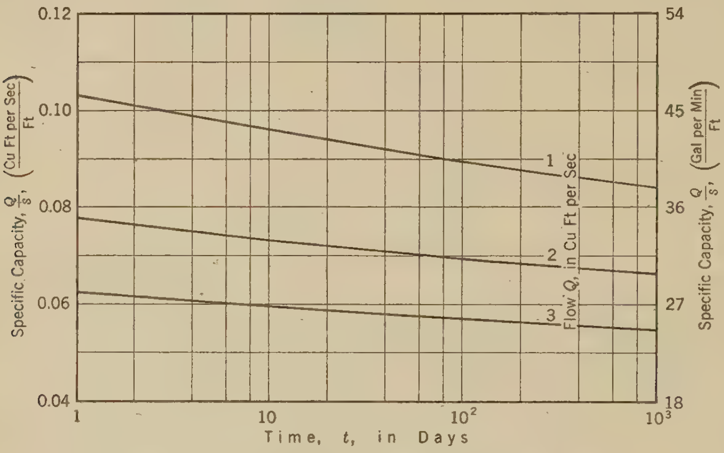

FIG. 5.—VARIATION OF THE SPECIFIC CAPACITY OF PUMPING WELL OF FIG. 4 WITH DISCHARGE AND WITH TIME

Fig. 5 illustrates the importance of stating the length of the pumping period during which the discharge remains constant and at the end of which the reported specific capacity is to be determined. It is also important to state the discharge. Too often in the past the specific capacity has been regarded as invariable, only passing attention being given to its variation with discharge and little or nothing being noted about its variation with time. This neglect may perhaps be attributed to the fact that, most commonly, measurements of drawdown are made by an air line, with a pressure gage reading to the nearest pound per square inch or an altitude gage reading to the nearest foot—no effort being made to interpolate closer than half a scale division. Very often the discharge is allowed to vary unmethodically to obtain several points quickly on the discharge-head curve for checking the characteristics of the pump and motor.

In the example given in this section it was assumed that the gravel envelope was not particularly effective for reducing the drawdown in the vicinity of the well and that therefore the nominal radius of the well screen might be used for the effective radius of the well. Actually, a more practical line of attack is to assume that conditions are as indicated in Fig. 1(*c*) and then to devise a method of determining the well loss that is independent of the effective well radius, which is to be determined last. The theory of such a method is outlined in the following section.

THEORY OF MULTIPLE-STEP DRAWDOWN TEST TO DETERMINE WELL LOSS AND EFFECTIVE WELL RADIUS

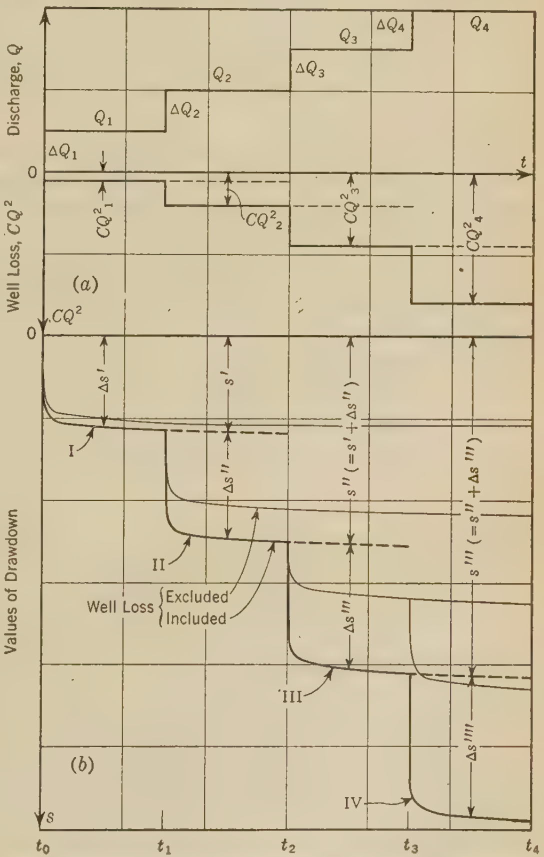

FIG. 6.—VARIATION OF DISCHARGE, DRAWDOWN, AND WELL LOSS IN MULTIPLE-STEP DRAWDOWN TEST

Eq. 13*b* gives the drawdown in an artesian well with an appreciable well loss. It applies to a single drawdown period preceded by a period during which the well is idle. By modifying the second term of the right-hand member, Eq. 13*b* may be made to apply to increments of drawdown occurring in successive periods, at the beginning of each of which the discharge is increased abruptly.

Fig. 6 depicts the progressive lowering of head in a multiple-step drawdown test. Two sets of drawdown curves are shown: The light lines in Fig. 6(*b*) show the draw-down that would occur during the successive periods of the test if there were no well loss; and the heavy lines, to which the notations refer, include the well losses, $C\,Q^2$, values of which are indicated in Fig. 6(*a*).

At time $t = t_0$ if the well (which theretofore has been idle) is started pumping at a rate $Q_1 = \Delta Q_1$, the drawdown at any time t thereafter is given by

$$\Delta s' = \frac{\Delta Q_1}{4\,\pi\,T}\left(\log_e \frac{t'}{t^*_w} - 0.5772\right) + C\,Q^2_1 \ldots\ldots\ldots\ldots (19a)$$

in which $t' = t - t_0$. If at some later time t_1 the discharge is increased by an amount ΔQ_2 to a new rate Q_2, the second increment of drawdown obeys the relation,

$$\Delta s'' = \frac{\Delta Q_2}{4\pi T}\left(\log_e \frac{t''}{t^*_w} - 0.5772\right) + C\,(Q^2_2 - Q^2_1) \quad \text{(19b)}$$

with $t'' = t - t_1$.

Similarly, the third increment of drawdown beginning at time t_2 obeys the relation,

$$\Delta s''' = \frac{\Delta Q_3}{4\pi T}\left(\log_e \frac{t'''}{t^*_w} - 0.5772\right) + C\,(Q^2_3 - Q^2_2) \quad \text{(19c)}$$

in which $t''' = t - t_2$. The generalized equation of this progression is

$$\Delta s^{(i)} = \frac{\Delta Q_i}{4\pi T}\left(\log_e \frac{t^{(i)}}{t^*_w} - 0.5772\right) + C\,(Q^2_i - Q^2_{i-1}) \quad \text{(20)}$$

in which $t^{(i)} = t - t_{i-1}$.

The value of T might be determined from any one of these equations by plotting $\Delta s^{(i)}$ against $\log_e t^{(i)}$, as before. However, to determine $r^2_w S$, from which to solve for r_w, the constant C must first be found, as this factor shifts the intercept according to the magnitude of Q_{i-1} and Q_i.

Dividing Eqs. 19 and 20 by the respective increments of discharge, and fixing $t' = t'' = t''' \cdots = t^{(i)}$, after simplifying the differences in squares of values of discharge:

$$\left.\begin{aligned}
\frac{\Delta s'}{\Delta Q_1} &= B + C\,\Delta Q_1 \\
\frac{\Delta s''}{\Delta Q_2} &= B + C\,(2\,Q_1 + \Delta Q_2) \\
\frac{\Delta s'''}{\Delta Q_3} &= B + C\,(2\,Q_2 + \Delta Q_3) \\
&\cdots\cdots\cdots \\
\frac{\Delta s^{(i)}}{\Delta Q_i} &= B + C\,(2\,Q_{i-1} + \Delta Q_i)
\end{aligned}\right\} \quad \text{(21)}$$

Taking the differences between successive pairs of equations,

$$\left.\begin{aligned}
\frac{\Delta s''}{\Delta Q_2} - \frac{\Delta s'}{\Delta Q_1} &= C\,(\Delta Q_1 + \Delta Q_2) \\
\frac{\Delta s'''}{\Delta Q_3} - \frac{\Delta s''}{\Delta Q_2} &= C\,(\Delta Q_2 + \Delta Q_3) \\
&\cdots\cdots\cdots \\
\frac{\Delta s^{(i)}}{\Delta Q_i} - \frac{\Delta s^{(i-1)}}{\Delta Q_{i-1}} &= C\,(\Delta Q_{i-1} + \Delta Q_i)
\end{aligned}\right\} \quad \text{(22)}$$

Considering log t to be variable, Eqs. 21 are the equations of a series of parallel straight lines whose spacings are given by Eqs. 22. Just as the ratio of discharge to drawdown is termed "specific capacity," the ratio of drawdown to discharge may be termed "specific drawdown." Similarly, the ratio of an increment of drawdown to the increment of discharge producing it may be termed "specific incremental drawdown." Eqs. 21, then, give the specific incremental drawdown as a function of time, the factor B increasing with time while the factor C remains constant.

The factor C may be determined from any one of Eqs. 22. Then, knowing C, there may be determined the difference:

$$\frac{\Delta s'}{\Delta Q_1} - \frac{\Delta s^\circ}{\Delta Q_0} = C\,(\Delta Q_0 + \Delta Q_1) \qquad (23)$$

In Eq. 23, $\frac{\Delta s^\circ}{\Delta Q_0}$ is an abbreviation for the limiting value that the specific incremental drawdown approaches as the discharge increment approaches zero and as the well loss consequently becomes negligible. Subtracting Eq. 23 from the first of Eqs. 21, recalling Eq. 14, and keeping in mind that $\Delta Q_0 \rightarrow 0$:

$$\frac{\Delta s^\circ}{\Delta Q_0} = B = \frac{1}{4\,\pi\,T}\left(\log_e \frac{t}{t^*_w} - 0.5772\right) \qquad (24)$$

If the values of specific incremental drawdown given by Eqs. 21 are plotted against log t, as suggested previously a series of straight lines is obtained. Eq. 24 can be plotted on the same graph as a straight line parallel to the others and spaced with reference to the first of the other straight lines in accordance with Eq. 23. Inasmuch as the C-term is lacking in Eq. 24, that equation simply expresses the component of drawdown that occurs in the formation. From its intercept with the zero-drawdown line, then, the effective radius of the pumping well may be determined by the following modification of Eq. 17:

$$r^2_w = 2.25\,\frac{T\,t}{S} \qquad (25)$$

In Eq. 25, S can be determined from observations in a near-by observation well at a known distance r, as shown in Fig. 4.

DATA FROM MULTIPLE-STEP DRAWDOWN TEST

Figs. 7 and 8 give data from a multiple-step drawdown test run in August, 1943, at Bethpage, Long Island, N. Y. The well that was tested was gravel packed and had 50 ft of 8-in. screen with a No. 60 slot, and with bottom at 350 ft. It was equipped with a 1,200 gal per min deep-well turbine pump. The flow was metered with a propeller-type meter in the discharge line. Water levels inside the casing were measured by an air line with a pressure-gage reading in pounds per square inch. During the first period of the test, measurements of depth to water were made with a weighted steel tape. Thereafter it was not possible to lower the tape to the water surface.

This test had four periods of approximately one hour's duration each. By timing the dial on the nonrecording flow meter with a stop watch, it was possible to secure virtually instantaneous readings of the discharge. Actually, the pumping rate declined slightly during each period of the test as the constant-speed pump adjusted itself to the lowering water level in accordance with the head-discharge characteristic. As these variations were slight, only the average discharge for each period is shown in Fig. 7. Readings of the air-line pressure gage (converted) are indicated by ×'s. The circles plotted one hour after the beginning of each period are interpolated points that are carried over to the discharge-drawdown diagram (Fig. 7(*c*)). Curve A, drawn through these points, is the type of curve ordinarily obtained, drawdown readings being taken only at the end of each period of the test. Customarily, the specific capacity is determined from an average secant of curve A passing through the origin.

Values of drawdown increments taken from Fig. 7(*b*), divided by corresponding increments of discharge, are plotted in Fig. 8 against appropriate values of time on a logarithmic scale. Using the tape readings during the first period as a guide, the slope of the several lines (which theoretically are parallel) is determined. The spacing of the lines in units of feet per cubic feet per second is given in the first column of the tabulation in Fig. 8. Also tabulated are the numbers of the periods, the discharge during each period, the increment of discharge at the beginning of each period, and the sums of neighboring increments of discharge. In accordance with Eqs. 22, data in the first and last columns of this tabulation are plotted in Fig. 7(*d*) to determine C. The three experimental points, for i = 2, 3, and 4, show considerable scattering, partly

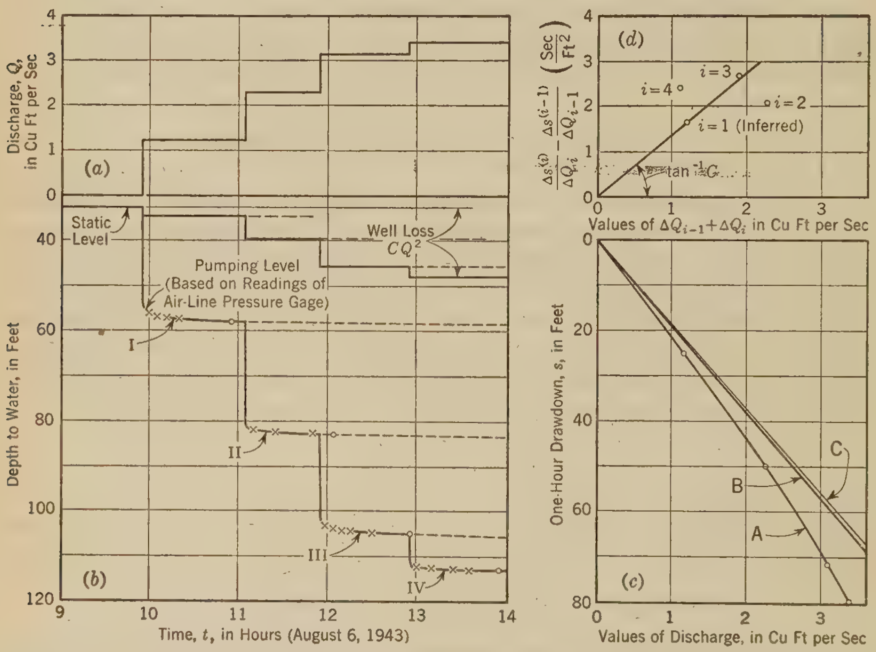

Fig. 7.—Determination of Well Loss When Effective Well Radius Is Not Known

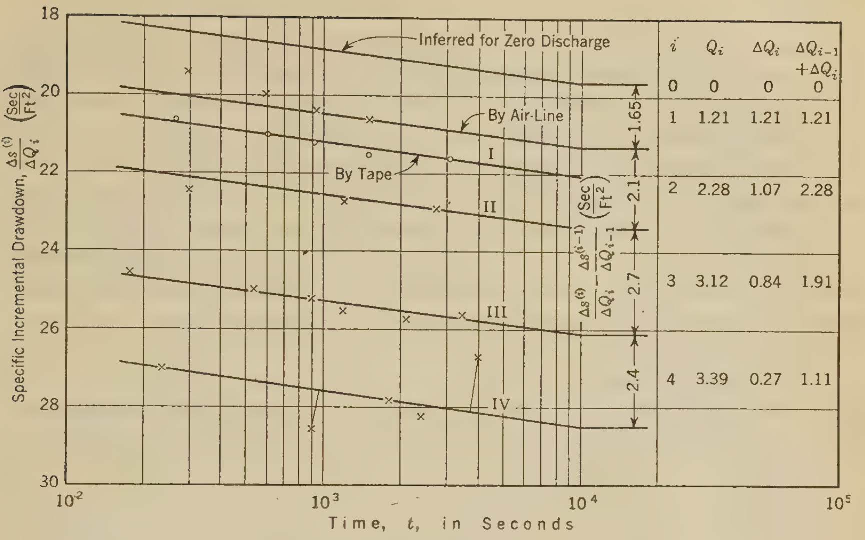

FIG. 8.—SEMILOGARITHMIC GRAPH OF DATA FROM FIG. 7(*b*)

perhaps because of the insufficiency of the theory but more likely because of inaccuracies of the air-line pressure-gage readings. From the slope of the straight line drawn through the center of mass of the three points, C is found to be $1.35 \left(\frac{\sec^2}{\text{ft}^5}\right)$.

Knowing C, the amount of well loss during each period of the test may now be determined. For the fourth period of the test, when the discharge was 3.39 cu ft per sec (1,520 gal per min), the well loss is computed as 15.5 ft. During the earlier periods of the test, when the discharge was smaller, the well loss was correspondingly smaller, as indicated in Fig. 7(*a*). Subtracting the appropriate value of well loss from each point plotted in Fig. 7(*c*), curve B is obtained. This curve then represents the discharge-drawdown relation for zero well loss. The straight line C (Fig. 7(*c*)) represents the relation between discharge and drawdown that would be obtained with zero well loss if separate one-hour tests were run at each rate, starting from rest with long intermediate periods of shutdown.

Theoretically, it should be possible to determine the transmissibility of the bed from the slope of the straight lines in Fig. 8; but the situation is complicated somewhat by the lenticular structure of the material penetrated by the well. Although the sand within a few hundred feet of the well is definitely confined, at greater distances it is effectively although somewhat circuitously interconnected with overlying beds of sand. If the setup were more nearly ideal and if there were a near-by observation well from which to determine the value of S, the effective radius of the pumping well might be determined.

SUMMARY

Because of limitations of the data from the two examples given in the paper, a composite of both examples is needed to illustrate the theory of the multiple-step drawdown test completely. With this in mind, the practical application of the theory can be summarized somewhat as follows:

(1) The test should be run following a period in which the well has been inactive, beginning at a fraction of the capacity of the pump and increasing the discharge in steps, each of which is a fair fraction of the pump capacity (Fig. 7). During each period of the test the metered discharge should remain essentially constant. (Small variations in discharge arising from the automatic adjustment of the pump to the declining water level when pumping against a constant discharge pressure are permissible.)

(2) During each period of the drawdown test, frequent readings of the drawdown should be made by air line or, if possible, preferably by a steel tape or by an electric-contact device. If an air line is used, care should be taken to use a reliable pressure gage that has been calibrated and to read it to the nearest fifth or tenth of a scale division.

(3) Frequent drawdown readings should also be taken in one or more observation wells tapping the same sand. If the screen of the pumping well does not completely penetrate the sand, the nearest observation well for this purpose should not be closer than about twice the sand thickness from the pumping well.

(4) Plot the data obtained under item (3) on a semilogarithmic graph (Fig. 4), using increments of drawdown against logarithm of time. Determine the transmissibility from the slope of the straight lines and the storage coefficient from their average intercept.

(5) Plot the data obtained under items (1) and (2) on rectangular coordinates (Figs. 7(*a*) and 7(*b*)). Extrapolate the drawdown curve for each period through the following period to determine the increments of drawdown.

(6) Plot values of specific incremental drawdown against the logarithm of time on semilogarithmic paper (Fig. 8). Draw parallel straight lines through the plotted points. (Extensions of these straight lines should check the extrapolations on the other graph. Secondary adjustments may be made to improve the extrapolations.)

(7) Plot differences of specific incremental drawdown given by neighboring lines against the sum of neighboring discharge increments (Fig. 7(*d*)). Determine C from the slope of the straight line through the origin and through the center of mass of the plotted data.

(8) Compute the well loss for each period of the test from $C\,Q^2$.

(9) Infer the limiting straight-line relation for zero discharge (Fig. 8) and from its intercept with the zero-drawdown line, using the value of storage coefficient determined under item (4), compute the effective radius of the well.

If the storage coefficient and transmissibility of the bed and the effective radius of the well are determined, it is possible to compute the resistance, B, at any time. Knowing the factor C, it is possible to compute the well loss. Combining the two, the total drawdown in the pumping well may be determined for any time and for any pumping rate.

In extensive artesian aquifers such predictions of drawdown are often trustworthy for periods of several months or a few years, but longer-term predictions must be based upon further consideration and closer evaluation of the outer "boundary conditions" of the aquifer. In local artesian beds this type of analysis may be required even for short-term predictions. In either case the concepts and procedures advanced in this paper should constitute useful implements, although not displacing in any way that knowledge of the geology and hydrology of an aquifer that is so necessary for a complete understanding of its performance.

The procedure itemized in this "Summary" should make possible, at any time during the life of a well, the accurate determination of both components of its specific drawdown, thus, for example, facilitating the recognition of the effects of encrustation of the screen or sand packing of the gravel wall, which too often have been ascribed to "depletion of the sand." Furthermore, it should enable the evaluation of the effectiveness of gravel packing and of the various development operations practiced in well construction. Through the accumulation of data, as wells are developed and placed in operation, this procedure should greatly aid in the selection of the proper gravel size and the appropriate screen opening so that the efficiency of wells will be increased and much needless waste of pumping energy will be prevented. Similarly, where the gravel wall is not called for, unnecessary expenditures for this type of construction may be avoided by referring to cases that have been tested under similar circumstances. Through predictions of the trend of pumping levels with time, proper selection of the pump and motor may be made that will give optimum performance throughout the life of the well, thus avoiding the wasteful practice of operating a pump with its discharge throttled to keep within the limits set by the diminishing capacity of the well.

The decline in production of oil wells is perhaps even more troublesome than that in the production of water wells. Often, it is difficult to determine whether the decline is due merely to depletion of the reservoir or whether it is due to the plugging of the perforations in the "liner," to the transportation of the fines, to the deposition of asphaltic substances, or to other causes. With some modifications the procedure outlined in this paper can be applied to oil wells as an aid in answering such questions. However, more accurate determinations of fluid level would be required than are now generally feasible while pumping steadily at different rates of production.

ACKNOWLEDGMENT

The writer wishes to express his appreciation of their helpfulness to O. E. Meinzer, chief of the Ground-Water Division, Water Resources Branch, United States Geological Survey, and to M. L. Brashears, Jr., district geologist, in charge of ground-water investigations in New York and New England, under whose direction he worked during the development of this paper.

APPENDIX. NOTATION

The following letter symbols conform essentially with American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942) and with ASCE *Manual of Engineering Practice No. 22* on "Soil Mechanics Nomenclature":

B = "hydraulic resistance" of formation, head loss per unit discharge;

b = thickness of confined sand bed;

C = coefficient in term $C\,Q^2$ expressing "well loss," a component of drawdown, the other term of which is $B\,Q$;

k = transmission constant, or "coefficient of permeability";

n = porosity of sand;

Q = discharge of well;

ΔQ_i = increment of discharge; $i = 1, 2, 3, \cdots$;

r = radial distance from axis of well;

r_w = "effective radius" of well;

S = "coefficient of storage" $= (b/V)(dV_w/ds)$;

s = drawdown at distance r, the difference between initial head and head at time t at that distance;

s_w = drawdown at r_w, according to theoretical logarithmic distribution;

$\Delta s^{(i)}$ = increment of drawdown produced by ΔQ_i;

$\Delta s^{(i)}/\Delta Q_i$ = "specific incremental drawdown" during i^{th} period of test;

$\Delta s^{\circ}/\Delta Q_0$ = limiting value of specific incremental drawdown for discharge approaching zero $(= B)$;

T = "transmissibility" of sand bed $= k\,b$;

t = time;

t^* = "inflectional time" $= r^2\,S/4\,T$;

u = $r^2\,S/4\,T\,t = t^*/t$, a nondimensional variable;

V = volume;

V_w = volume of water;

$W(u)$ = "well function" of u, or the negative "exponential integral" of $-u$, for which tables are available;

α = "compressibility" of solid skeleton of sand bed, relative decrease in thickness per unit increase of vertical component of compressive stress in sand bed;

β = compressibility of water in sand bed;

β' = apparent compressibility of water $= \beta + \alpha/n$; and

γ = specific weight of water.

DISCUSSION

N. S. Boulton,[11] Esq.—The importance of carefully recording both the small variations in pumping level, which may occur during pumping tests at constant discharge, and the duration of the test, are appropriately stressed in this paper. From such information it is possible to predict, as the author has shown, the probable steady decline in specific capacity "for periods of several months or a few years" when the well is pumped at constant discharge. It is important to remember, however, that the accuracy of this prediction depends essentially on the assumption that the compressibility of the aquifer (which enters into the coefficient of storage) has the same value for the very small pressure releases which occur at large distances from the pumped well as for the comparatively large pressure releases near to the well. It would be appreciated if the author could present evidence in support of this assumption, based on long-period observations of declining well levels. In addition, it would be interesting to know whether the author has been able to check the values for "well loss" by direct estimates of the pipe friction loss as the water flows inside the well casing and also of the loss of head due to the screen.

For the fourth period of the test at Bethpage, Long Island, N. Y., the depth of water in the well was apparently about 238 ft. Allowing for the water entering the well uniformly along the bottom 50 ft, a reasonable estimate (from a usual formula) for the head lost in pipe friction in the 8-in.-diameter tube is about 10.5 ft, including 1.5 ft for the velocity head. The computed well loss (see heading, "Data from Multiple-Step Drawdown Test") is stated to be 15.5 ft, which leaves 5 ft for the loss due to the screen. It is easy to calculate the latter loss on the assumption of flow through a uniform permeable medium outside the screen to which Darcy's law may be applied. Thus, for long vertical slots spaced equally around the circumference of the well, it can be shown from the potential solution for the flow net that the head loss due to the restricted inlet area provided by a slotted tube is closely given by:

$$h = \frac{Q}{2\,\pi\,N\,b\,k} \log_e \left(\frac{2}{1 - \cos \nu\,\pi} \right) \quad \text{.............. (26)}$$

in which N is the number of vertical slots around the circumference of the tube; and ν is the slot-width ratio or width of slot divided by the distance between the centers of two adjacent slots.

According to Eq. 26, the head loss is proportional to the discharge and, for a given slot-width ratio, inversely proportional to the number of slots.

If $Q = 3.39$ cu ft per sec and $b = 50$ ft, as in the fourth period of the Bethpage test, and if $k = 0.004$ ft per sec (as deduced from Fig. 8), assuming $\nu = \frac{1}{4}$ (since the dimensions of the slotted tubing are not given in the paper), it is found on substitution in Eq. 26 that $h = 5.2/N$ ft. For one hundred slots, each 0.063 in. wide, uniformly spaced around the circumference, $h = 0.052$ ft which is negligible. On the other hand, if the slots are arranged in batteries numbering, say, ten in the circumference, the batteries being 0.5 in. wide with 2 in. between them, $h = 0.52$ ft which is still small.

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It should be emphasized that this calculation makes no allowance for any clogging of the slots. Such clogging may account for the discrepancy between the small calculated screen loss and the value of 5 ft deduced from the test result.

Carl Rohwer,[12] M. ASCE.—The serious depletion of ground-water supplies in many areas during World War II has focused attention on the problems of ground-water hydrology. In this connection the investigations of the engineers of the Water Resources Branch of the U. S. Geological Survey are adding important information regarding the characteristics of wells and the capacity of ground-water formations. The analysis of drawdown tests of artesian wells by the author is a valuable contribution to this subject.

The writer is in agreement with the objectives of the author's investigations but he is of the opinion that the analysis of the problem would have been simplified if some of the factors that have only a slight effect on the results had been ignored. Under most conditions met with in the field of engineering the compressibility of water can be ignored. The coefficient is approximately 4×10^{-6} per pound pressure at ordinary temperatures and pressures. A reduction in pressure of 10 lb per sq in. would increase the volume of 1 cu ft of water by only 0.00004 cu ft, a difference of 1 in 25,000. In view of the large unavoidable errors involved in other measurements it seems that this factor could well be neglected. The same may be stated of the compression of the aquifer. As indicated by the author (see heading, "Application of Theory to a Simple Drawdown Test"), the combined effect of compressibility of the water-bearing formation is only five times the actual compressibility of water. Consequently, the combination of these two factors would produce a change of only 1 in 5,000 for a drop in pressure of 10 lb (approximately 23 ft). If the change in pressure were increased to 100 ft the effect produced by the compressibility of the water and aquifer would not be significant.

In reference to the tests on a shallow well at Meadville, Pa., the author states in the sentences following Eq. 18, that:

> "The foregoing calculations indicate that $B\,Q$ in this case was about 19.5 ft after 24 hours of continuous pumping. The observed drawdown in the pumping well at that time was 48.0 ft, leaving 28.5 ft for the well loss."

Such a large well loss seems unusual for an inflow of 1,350 gal per min through 15 ft of 18-in. screen unless the screen were badly encrusted or improperly perforated. Immediate steps should be taken to improve the performance of the screen in this well.

In the solution of problems involving many variables of which only a few can be determined by direct measurement, the use of multiple equations provides a method of determining the unknowns. However, there are difficulties inherent in this method which may lead to contradictory or inconsistent results.

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As shown by the author in reference to the determination of C (see heading, "Data from Multiple-Step Drawdown Test"), there is considerable scattering in the values obtained for the "specific incremental drawdown," Fig. 7(*d*). No doubt, this is due in part to the inaccuracies in the drawdown readings. If this method were used on problems in which all readings could be made accurately, consistent results should be expected. Since this is not generally true, the multiple-equation method results in solutions in which the final answers may have errors greatly in excess of the observed data. The writer is not aware of the mathematical basis for this assumption, but he observed the same effect when attempting to use a similar method to determine the values of the factors involved in the seepage from canals. The conclusion was reached that, in the elimination of variables from the series of equations by subtraction, the variables eliminated were forced to conform exactly to the law; and, as a result, all the discrepancies accumulated and finally appeared in the solution of the unknown. A solution based on another pair of equations may, therefore, yield a result widely different from the first one.

Since the author has had the opportunity to observe how the solutions vary when he uses different equations it would be of interest to study the mathematical principles causing the variations. No doubt rules could be formulated which would make it possible to obtain more consistent results from the observed data. Such an analysis would be useful in the solution of problems in other fields of engineering.

R. M. LEGGETTE,[13] AFFILIATE, ASCE—Although it covers a highly technical subject, this paper clearly demonstrates the practical importance of a number of factors of well design. It seems desirable to emphasize these practical considerations because they are often given too little attention. Frequently water works men and well-drilling contractors greatly belittle or fail to recognize the magnitude of what Mr. Jacob calls "well loss."

It is obvious, of course, that the water level in a pumping well must be lower than the water level immediately outside the well. In many wells, much of this difference in head is screen friction loss which results from the use of a poorly designed screen. This difference in head is sometimes presumed to be only a few inches, or a fraction of a foot; however, actual observations have shown that in some wells the well loss is a considerable part of the total drawdown. Thus, from the point of view of economy of operation, well loss may be an important factor.

The paper also indicates the desirability of increasing the effective radius of a "sand and gravel" well by development to remove the fine material surrounding the screen, or by artificial gravel packing. It should be noted that the advantages of development or gravel packing may be largely overcome if an inefficient well screen is used.

The process of development by surging, swabbing, and brushing is being used more and more in uncased wells (rock wells), the walls of which apparently become "mudded up" during the drilling process. This clogging of the uncased wall of the well has the same effect as an inefficient well screen in a "sand

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[13] Cons. Ground-Water Geologist, New York, N. Y.

and gravel" well. The well loss in many wells of this type has been greatly reduced by development.

From the point of view of economy of pumping well water, this paper indicates the following: A well should be of large diameter; it should be extensively developed; and, if a well screen is used, it should be designed so as to produce a minimum of screen friction loss.

M. R. Lewis,[14] M. ASCE.—A valuable method has been presented, in this paper, for analyzing the capacity of artesian wells in spite of the necessary assumptions that there is uniformity in the aquifer and that the total supply to the well is drawn from the aquifer by the release of elastic forces. Such theoretical or mathematical treatments of the flow of fluids assist greatly in the understanding of practical problems even though the latter seldom are based on ideal conditions.

The multiple-step test proposed by the author should give very useful information on the points mentioned in the "Summary" wherever the assumptions are approximately fulfilled. It is hoped that the author will explore the possibility of a similar analysis under other conditions. Two important types of wells that might be studied are those of a simple water-table situation and those in which the bed overlying an aquifer is relatively impermeable but permits recharge from the soil surface surrounding the well.

It appears to the writer that two other factors besides the compaction of the aquifer are important elements in making the "apparent compressibility, β'," greater than the compressibility of water, β. These are (*a*) the increase in volume of the solid material of the aquifer because of the reduced hydrostatic pressure and (*b*) the reduction in the pore space because of the deformation of the solid particles by reason of the increased pressure on the mineral skeleton. In his earlier paper,[8] the author mentioned these factors but, apparently, considered them to be of negligible importance. Whether they are, or are not, important makes no difference in the author's analysis.

C. E. Jacob,[15] Assoc. M. ASCE.—Although few in number and brief, the discussions have added much to the paper and, moreover, have suggested the direction that further work might profitably take. The writer wishes to thank those who have contributed.

The question raised by Mr. Boulton is a pertinent one—regarding the need for evidence to support the assumption that the compressibility of the aquifer has the same value near to, and far from, the pumped well despite the wide range of pressure release. The writer knows of no close observations of long-period decline under constant and continuous pumping that might clarify this problem. Of course, it is to be expected that an unconsolidated sand would have a variable compressibility, depending on the rate and magnitude of the loading occasioned by the release of pressure. Moreover, sight should not be lost of the fact that the flexural rigidity of the overlying beds complicates the problem, especially in the immediate vicinity of pumped wells tapping deep aquifers. Fortunately, however, as long as the compressibility (thus modified) may be assumed reasonably constant in time at a given distance the uncertainties arising from ignorance of its absolute value are reflected only in the degree of approximation of r_w. Actually, in predictions of future drawdown based on the theory of an elastic aquifer, the product $S\,r^2_w$ is used. This product is determined empirically and it is not necessary to break it down into S and r_w, except to compare different wells. If S varies with t, for one reason or another, then some other theory must be used or the elastic theory must be modified.

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[15] Senior Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C.

Mr. Boulton's estimate of 10.5 ft for the friction and velocity head losses accompanying the upward flow inside the well casing of the Bethpage well narrows down the well loss to that part that may be termed "screen loss." His calculation of the "convergence loss" under laminar flow into an assumed system of vertical slots shows that such loss would be quite small. Actually the openings in the screen are in the form of a helix, but the "convergence loss" there should be of the same or even of a smaller order of magnitude. Clogging of the slots is a distinct possibility, as pointed out by Mr. Boulton. Furthermore, departure from laminar flow may begin as the water passes through the screen openings, especially if they are clogged.

By calculation Mr. Rohwer shows that the combined relative compression of the aquifer near Meadville, Pa., and its contained water is only about 1 in 5,000 for a 23-ft drop in head. He states:

> "In view of the large unavoidable errors involved in other measurements it seems that this factor [compressibility of water] could well be neglected. The same may be stated of the compression of the aquifer."

Perhaps Mr. Rohwer has in mind estimating the ultimate or steady-state discharge of wells in a limited aquifer, in which case the volume of water derived from artesian storage might soon become small by comparison with the volume drawn through the aquifer from an outer boundary. However, until that steady state of flow is established, water is withdrawn from storage through its elastic expansion and through the concomitant compression or compaction of the aquifer. Indeed, in extensive and deep-lying aquifers such as the Dakota sandstone, the supply may be furnished entirely from storage for decades. Because of the tremendous volume of water in such an aquifer and because of the sizable lowering of head that may be produced, the total volume of water withdrawn from storage through wells may itself reach an astounding magnitude. In any event, however insignificant this factor may appear, the study of the transient behavior of an elastic aquifer, on which the paper is based, requires an appraisal of its magnitude.

With reference to the large well loss of 28.5 ft at 1,350 gal per min in the well near Meadville, Pa., the suggestion is offered by Mr. Rohwer that the screen may be encrusted or improperly perforated. As the well was newly constructed when tested, it would seem that sand packing of the gravel envelope and of the screen might be the explanation. The formation is a uniform fine sand—difficult to handle in drilling and developing a well.

Mr. Rohwer adds a valuable point in remarking on the unavoidable magnification of errors through elimination of variables by subtraction from the series of equations. He justifiably emphasizes the need for care in treating such data as those presented in the paper. Whereas from a theoretical standpoint, with ideal data, the procedure outlined in the paper is sound; in practice it needs modification. Higher precision of measurement may warrant the assumption that the drawdown obeys the law $s_w = B\,Q + C\,Q^n$, introducing a third unknown, the exponent n (<2), to be determined by trial-and-error computation, or by graphical procedure together with the coefficients B and C.

An important point is raised by Mr. Leggette—that the advantages of gravel packing or developing a well may be offset by poor design or improper choice of screen. The writer feels that amassing empirical values of C and of r_w, together with pertinent data on the details of design and construction of wells, may eventually make possible the accurate appraisal of these various factors. The selection of screen type, slot opening, and gravel size—and even the determination of whether or not a gravel envelope is required—may be lifted from the realm of guesswork to a rational plane through the future study of existing and newly constructed wells and through the measurement of their characteristics of performance, due consideration being given to the transient behavior of the aquifer.

In emphasizing the magnitude of well losses, condemnation of the well driller is not intended, for much of the friction loss in and near a well is unavoidable and will never be entirely eliminated. Nevertheless, it behooves the engineer and the well-drilling contractor alike to strive for as efficient design and construction as possible to meet the stringencies of economic demands. The points summarized by Mr. Leggette thus are objectives toward which progress should be made.

Mr. Lewis suggests that similar analyses be made for unconfined flow under simple water-table conditions, and for confined flow in which recharge from the soil surface occurs through a relatively impermeable confining bed. To the writer's knowledge a satisfactory analysis of nonsteady unconfined flow has not been given. Even in the case where the storage coefficient (S, ultimately approaching "specific yield") is constant, there are insuperable difficulties. Only by analogy to confined flow, and then in cases where the maximum drawdown is but a small fraction of the initial depth of flow, has an approximate solution been obtained. It may be stated, however, that even in the absence of well losses the specific capacity of a water-table well would vary with the discharge, the curve of drawdown versus discharge at constant time being a parabola under certain approximative assumptions. Further work should be done on this problem, both in the laboratory and in the field.

A solution has been given for the nonsteady radial flow toward a steadily discharging well in a leaky confined aquifer.[16] The leakage is assumed proportional to the drawdown. Whether this is exactly the condition Mr. Lewis has in mind is not known, but in the early phase of a transient state such a system acts like an ideal elastic aquifer without leakage. Accordingly, a short multiple-step drawdown test could be analyzed under those conditions on the basis of the elastic theory, although long-term predictions would consider the leakage.

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[16] "Radial Flow in a Leaky Artesian Aquifer," by C. E. Jacob, *Transactions*, Am. Geophysical Union, Vol. 27, 1946, Pt. II, pp. 198–205.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2322

RIGID-FRAME STRUCTURES SUBJECT TO NONUNIFORM THERMAL ACTION

By Carl C. H. Tommerup,[1] Assoc. M. ASCE

With Discussion by Messrs. I. Oesterblom, Frank R. Higley, A. L. Miller, Charles O. Boynton, William A. Conwell, and Carl C. H. Tommerup.

Synopsis

The purpose of this paper is to clarify the problem of determining stresses and strains at any point in rigid frames whose members are subject to a nonuniform temperature differential; that is, structural members in which the temperature on one face is different from that on the other face. Many engineers are unaware of the exceedingly high stresses created by this quite common loading condition. The paper develops a rational setup which is of general scope since it is applicable to any type of framework.

This paper is divided into five parts, of which Sections 2, 3, and 4 contain the complete thermal analyses of three basic types of frames.

1. Introduction

Thermal stress analysis is of considerable importance in many branches of the metal industry in connection with castings, structural frames, etc., as well as in the field of such reinforced-concrete structures as flues, flue portals in plinths of stack foundations, hot liquid flumes, foundations for boilers, oil refinery furnaces, and blast furnaces.

Although it is generally recognized that reinforced concrete is far from being an ideal material in structures that are subject to high nonuniform temperature changes, it is continually used under these circumstances with reasonable success. The main reason for this is undoubtedly the fact that no other material is so readily applied to a variety of structures. It is equally well known, however, that a reinforced-concrete structure built in the shape of a rigid frame (a closed ring, for example), and then heated to a fairly high degree from within, is certain to develop a number of noticeable fractures along all outside edges. These fractures, which are caused entirely by thermal action,

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Note.—Published in June, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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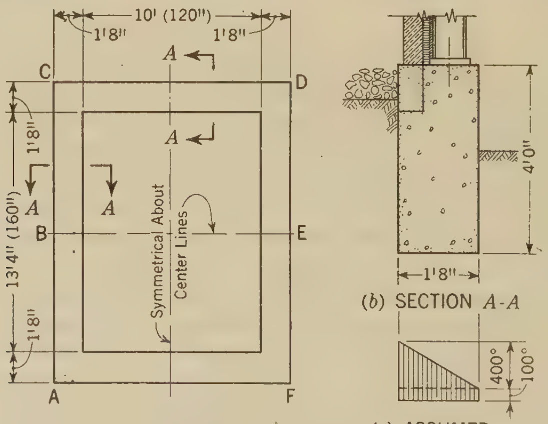
vary in width with the depth of the frame member; that is, a deep frame member produces a wider fracture than a shallow member produces. In some instances such fractures do not impair the efficiency of the frame and may safely be neglected; yet, in many cases, the formation of fractures is an indication that the structure in question should have been designed along entirely different lines.

As stated in the "Synopsis," this paper analyzes thermal stresses in three distinct types of frames. The frames chosen are reinforced-concrete foundations for boilers and oil refinery furnaces. The analyses, of course, are equally applicable to frames of any other shape and built of any material or combination of materials. For simplicity, the frames are symmetrical and their members are of constant cross section from joint to joint. If the cross sections of the members vary, the amount of computation is naturally increased but the basic principles remain the same. No loading other than that produced by nonuniform thermal action is treated. Because of the greater ease, member lengths, as a rule, are taken along their inside surfaces. In order to gain knowledge as to just what thermal action does to frames, the shape of the elastic curves is determined.



(*a*) PLAN

(*b*) SECTION *A*-*A*

(*c*) ASSUMED TEMPERATURE VARIATION

Fig. 1.—Hypothetical Rectangular Frame

Notation.—The letter symbols in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically in the Appendix, for convenience of reference.

2. Rectangular Frame

A plan view of the simple rectangular frame to be analyzed first is shown in Fig. 1. Fig. 1(*b*) is an enlarged typical cross section through any frame member, showing the lower part as a reinforced-concrete foundation wall, 4 ft high by 1.67 ft thick. Superimposed on this wall is the furnace wall proper with its columns, buckstays, steel casing, insulation, and refractory lining.

The furnace floor, which is formed by broken firebrick laid directly on the elevated soil, is at the left. The upper part of the foundation wall is protected from the intense heat, to some extent, by a cast-in-place refractory facing of high insulating value. Heat is transmitted from the furnace floor through the soil to the inside face of the foundation wall. The right-hand side of Fig. 1(*b*) is exposed to the atmosphere and indicates the outside grade level as being approximately at the midheight of the foundation wall. The assumed temperature variation is illustrated in Fig. 1(*c*). The inside wall face is at 500°, and the outside face at 100°, leaving a temperature differential of 400°.

Assume now that the frame is cut at each of its corners and that the temperature differential is subsequently applied. This will cause each of the four separate beams to bow inward since their inside faces have received an increase in length, whereas the length of their outside faces remains unchanged. The amount of bow, curvature, etc., is obtained from Fig. 2 which shows the right-hand half of wall FD. With the temperature differential of 400° and with a coefficient of expansion equal to 0.0000079 in. per in. per degree, the wall length ED receives an increase in length of $\Delta_L = 0.0000079 \times 400° \times 80$ in. $= 0.2528$ in. and point D moves to point D′, Fig. 2.

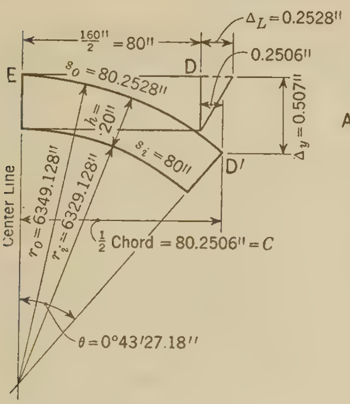


FIG. 2

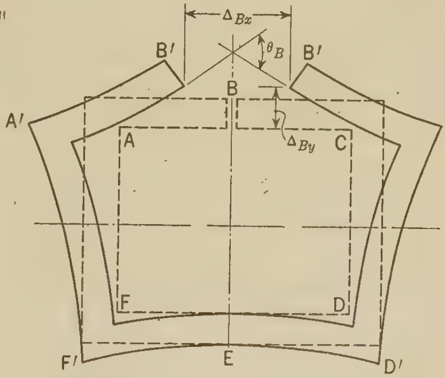


FIG. 3

The expressions for the lengths of arc s_o and s_i can be written:

$$s_o = \frac{\pi r_o \theta}{180} \qquad (1a)$$

and

$$s_i = \frac{\pi r_i \theta}{180} \qquad (1b)$$

The measure of the difference between the inner and outer arc lengths is Δ_L, Fig. 2, or

$$\Delta_L = s_o - s_i = \frac{\pi \theta}{180} (r_o - r_i) \qquad (2)$$

Since $r_o - r_i = h$ (Fig. 2), $\Delta_L = \frac{\pi \theta h}{180}$, or, in degrees:

$$\theta = \frac{180\,\Delta_L}{\pi h} \qquad \text{(3)}$$

Substituting numerical values in Eq. 3, $\theta = 0.724219° = 0° 43' 27.18''$. By Eq. 1*a*, $r_o = \frac{180\,s_o}{\pi \theta} = 6{,}349.128$ in. The vertical deflection Δ_y equals:

$$\Delta_y = 2\,r_o \sin^2 \tfrac{1}{2}\,\theta \qquad \text{(4)}$$

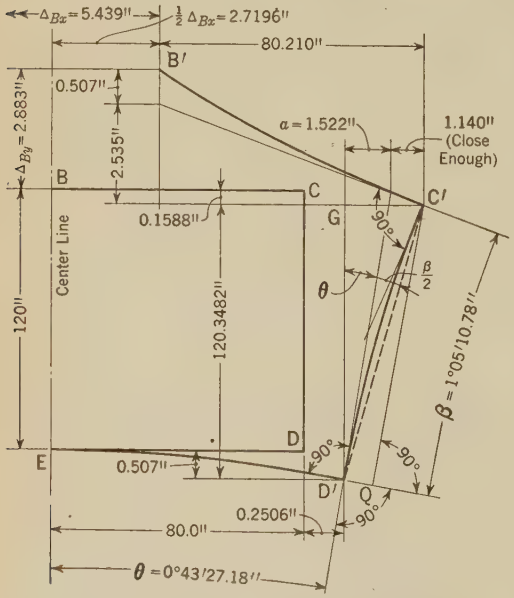
from which, by proper substitution, $\Delta_y = 0.507$ in. Similarly, half the chord length C is equal to

$$C = r_o \sin \theta \qquad \text{(5)}$$

or $C = 80.2506$ in. (see Fig. 2).

The various values found, by Eqs. 1 to 5, for the length ED also apply to lengths FE, AB, and BC, Fig. 1(*a*), by symmetry. The corresponding values for members AF and CD are found in an identical manner.

For purpose of analysis, the frame in Fig. 1 is assumed cut at a convenient point B. Applying the temperature differential, the frame will take the shape as shown in Fig. 3. Points A, B, C, D, and F move to A′, B′, C′, D′, and F′, respectively. A gap of Δ_{Bx} forms at B, which moves vertically a distance Δ_{By}, and the tangents to the elastic curve at point B assume an angle ϕ_B. Point E remains stationary. (In the case of unsymmetrical frames, assume some convenient point as being fixed in location and let the structure displace itself thermally in all directions from that one point. Should the temperature differential vary from joint to joint, displacement must be modified accordingly. If a member or a part of a member has no temperature differential, its elastic line in a displacement diagram remains unaltered.) The numerical values of all displacements are given in Fig. 4. The rotational displacement at point B is $\phi_B = 2(2\,\theta + \beta) = 5° 04' 10.28'' = 0.0884$ radian. The computations for the vertical and horizontal displacements are, respectively (see Fig. 4): $\Delta_{By} = (0.507 + 2.535 + 120.3482) - (0.507 + 120.0000) = 2.8832$; and $\Delta_{Bx} = 2\,(80.0000 + 0.2506 + 1.5220$

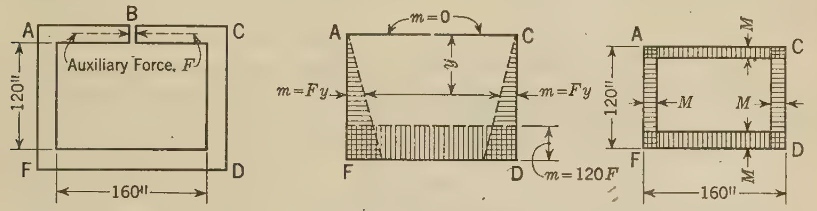


Δ_{Bx}=5.439″
$\frac{1}{2}\,\Delta_{Bx}$=2.7196″
80.210″
B′
0.507″
Δ_{By}=2.883″
2.535″
a=1.522″
1.140″ (Close Enough)
B
C
C′
0.1588″
G
90°
$\frac{\beta}{2}$
θ
Center Line
120″
120.3482″
β=1°05′10.78″
D
90°
90°
E
0.507″
D′
Q
90°
0.2506″
80.0″
θ=0°43′27.18″

Fig. 4.—Numerical Value of Displacements

$+ 1.1400 - 80.2100) = 5.4052$. Various slight approximations have been made in computing these displacements.

The unknown thermal moments are solved for by the elastic energy theory[2] because of its universal application and the ease with which it lends itself to this type of problem. The effect of axial forces in frame members are neglected in the following computations unless otherwise noted. All four corner moments must be equal because both the frame and its loading are symmetrical about the same center lines. Since no external forces are acting between joints, the moment curve becomes a straight line from joint to joint. It follows, therefore, that neither shears nor axial forces are present. Any point of the frame is affected only by a moment that is of uniform magnitude all around. Each wall is reinforced by eight 1-in. round rods in the outside face, and four ½-in. rounds in the inside face. The moment of inertia of the wall, I, transformed into terms of concrete, is 14,230 in.^{4}; the corresponding area, A, is 1,060 sq in.; the modulus of elasticity E is assumed equal to 2,000,000 lb per sq in. (E and I are constant throughout); and $n = 15$. Figs. 5(a) and 5(b) show the auxiliary loading, F, and moments M, respectively. Fig. 5(c) sets forth the actual, or M, moments.



(a) AUXILIARY LOADING (b) AUXILIARY MOMENTS (c) ACTUAL MOMENTS

Fig. 5.—Moment Curves, Linear Displacements

Computing the unknown thermal moment M in terms of longitudinal displacement Δ_{Bx}:

$$F\,\Delta_{Bx} = \int_B^B \frac{m\,M\,ds}{E\,I} \quad\ldots\ldots (6a)$$

or

$$E\,I\,\Delta_{Bx} = \int_B^B \frac{m\,M\,ds}{F} \quad\ldots\ldots (6b)$$

Furthermore,
$$E\,I\,\Delta_{Bx} = \int_A^F \frac{F\,y\,M\,dy}{F} + \int_C^D \frac{F\,y\,M\,dy}{F} + \int_A^C \frac{0\,M\,dx}{F} + \int_D^F \frac{120\,F\,M\,dx}{F} = \int_A^F M\,dy \times \bar{y} + \int_C^D M\,dy \times \bar{y} + 120 \int_D^F M\,dx;$$ or

$$E\,I\,\Delta_{Bx} = (\text{area}) \times \bar{y}\,\Big]_A^F + (\text{area}) \times \bar{y}\,\Big]_C^D + 120 \times (\text{area})\,\Big]_D^F \quad\ldots (7)$$

Substituting the numerical values: $2{,}000{,}000 \times 14{,}230 \times 5.439 = 2 \times 120\,M \times 60 + 120 \times 160\,M$; and $M = 4{,}600{,}000$ in.-lb (tension outside).

---

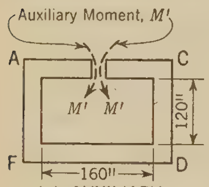
[2] "Elastic Energy Theory," by J. A. Van den Broek, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1942.

That the moment produces tension in the outside face of the members is evident by reasoning as follows: If members were cut loose at corners, A, C, D, and F, the beams would, upon being heated along their inner faces, bow inwardly as shown in Figs. 2, 3, and 4, and their ends would form acute angles at the joints. However, the ends are compelled to remain at angles of 90° apart because of the rigidity of the system. This can be accomplished only by "negative" joint moments; that is, resisting moments that cause tension in the concave or outer faces.

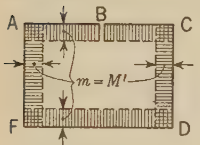
The thermal moment of 4,600,000 in.-lb produces the following stresses:

Material	Computation	Stress (Lb Per Sq In.)
Tensile steel	$\frac{4{,}600{,}000 \times 10.6 \text{ in.}}{14{,}230} \times 15$	51,300
Compressive steel	$\frac{4{,}600{,}000 \times 3.4 \text{ in.}}{14{,}230} \times 15$	16,500
Concrete	$\frac{4{,}600{,}000 \times 6.4 \text{ in.}}{14{,}230}$	2,070

These stresses, of course, are far from being safe. Moreover, they are beyond the limit within which the equations of this paper apply. Nevertheless, they serve to emphasize the danger incurred in the design of unequally heated structures when due precaution is not paid to thermal stresses that are likely to be introduced. Calling attention to this fact is the foremost purpose of the paper.



(a) AUXILIARY LOADING



(b) AUXILIARY MOMENTS

FIG. 6.—MOMENT CURVES, ANGULAR ROTATION

It is interesting to note that even if the temperature differential is as little as 200° F instead of 400° F the resulting moment (one half *M*) would still be of enormous magnitude considering the fact that stresses produced by it are in reality secondary stresses.

In case one is not interested in obtaining a complete picture of the linear displacements, the moment *M* may be more promptly arrived at through application of the angular rotation θ_B which is readily computed by Eq. 3. For this setup, Fig. 6 shows the auxiliary loading and moment diagrams, respectively. The actual or *M*-diagram is the same as previously indicated in Fig. 5(*c*). With $\theta_B = 0.0884$ radian:

$$M' \theta_B = \int_B^B \frac{m\, M\, ds}{E\, I} \qquad (8a)$$

or

$$\theta_B = \frac{1}{E\, I} \int_B^B \frac{M'\, M\, ds}{M'} \qquad (8b)$$

and

$$E\, I\, \theta_B = \int_B^B M\, ds = (\text{area}) \Big]_B^B \qquad (9)$$

Substituting the numerical values in Eq. 9: 2,000,000 × 14,230 × 0.0884 = M (2 × 160 + 2 × 120); which yields, M = 4,500,000 in.-lb. The small discrepancy between this value and that of 4,600,000 in.-lb previously found is caused by the slight approximations made in computing displacements.

If the problem at hand concerns merely that of a simple frame, and assuming that the designer is desirous of ascertaining neither linear nor rotational displacements, the thermal moment may be determined quickly on the basis of the arch theory,[3] by the formula,

$$\text{Moment } M_t = \frac{\alpha . \Delta t \, E \sum \frac{ds}{h}}{\sum \frac{ds}{I}} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which: α = coefficient of thermal expansion; Δt = temperature differential; and ds = element of length along the arc of a member, measured along the gravity axis. Since the frame is symmetrical about both center lines, the elastic center coincides with the geometrical center of the structure and their results: $\sum \frac{ds}{h} = \frac{160}{20} \times 2 + \frac{120}{20} \times 2 = 28$; and $\sum \frac{ds}{I} = \frac{160}{14{,}230} \times 2 + \frac{120}{14{,}230} \times 2 = 0.0394$. The moment is: $M_t = \frac{0.0000079 \times 400 \times 2{,}000{,}000 \times 28}{0.0394} = 4{,}500{,}000$ in.-lb.

Having obtained the stresses created in the rectangular frame by a 400° temperature differential between inside and outside faces of frame members, the shape of the elastic curve, as it exists when frame comes to rest after thermal loading is applied, can be established.

Considering the four frame members as simple beams (cut at corners) and applying the temperature differential, they would deform as indicated in Fig. 7(*a*). The deflection Δ_{ml} at midpoint of long beams is $\frac{M l^2}{8 E I} = \frac{4{,}500{,}000 \times 160^2}{8 \times 2{,}000{,}000 \times 14{,}230} = 0.507$ in. (see Fig. 2). The deflection Δ_{ms} at the midpoint of short beams is $\frac{4{,}500{,}000 \times 120^2}{8 \times 2{,}000{,}000 \times 14{,}230} = 0.285$. The deflections at the center of beams caused solely by the uniform restraining moments are shown in Fig. 7(*b*).

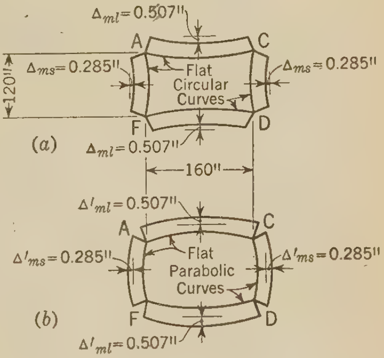


FIG. 7

The foregoing values of Δ_m reveal the interesting fact that when the members of a simple closed frame, of any shape, are subjected to a nonuniform temperature change through their "depth" (h) they undergo no deformation. As long as temperature change, depth, and moment of inertia remain constant for all members, the elastic curve remains a straight line throughout (in cases where frame members were originally straight) despite the fact that bending moments of enormous magnitude are created.

Various other facts of interest may be observed from Figs. 5(*c*) and 6. For example:

---

[3] "Reinforced Concrete and Masonry Structures," by George A. Hool and W. S. Kinne, McGraw-Hill Book Co., Inc., New York, N.Y., 1924, p. 477, Eq. 115.

(1) If frame members were doubled in length, the linear expansions Δ_L and Δ_s and angles θ, β, and ϕ_B would double in magnitude. Hence, the left-hand term of Eq. 9 would double; but so would the right-hand term since the moment area now has twice its former value. The result is that the moment of 4,500,000 in.-lb remains constant regardless of lengths of members so long as the ratio of lengths, temperatures, and depth of members are not changed. The foregoing facts might also have been deduced by considering Fig. 5. With the member lengths doubled, linear displacements Δ_{Bx} and Δ_{By} would become four times as great as shown. However, the right-hand side of Eq. 7 would also be increased to four times its former value, since both moment areas and their arms are doubled in magnitude.

(2) Assuming that instead of doubling member lengths, the thickness h is doubled. Eq. 3 shows that in this case angles θ, β, and ϕ_B would be halved. However, the moments of inertia of the members are now increased to eight times their original value (assuming that the proper percentage of steel reinforcement is added to the new section). Therefore, the net result is that the left-hand term of Eq. 9 becomes four ($= \frac{1}{2} \times 8$) times as great as before. Since moment areas have not changed in size, the right-hand side of Eq. 9 remains unchanged. Consequently, doubling the wall thickness quadruples the moment.

If, as in the foregoing example, the walls are of equal cross section, the "ratio of lengths" mentioned in Section 1 is of no significance.

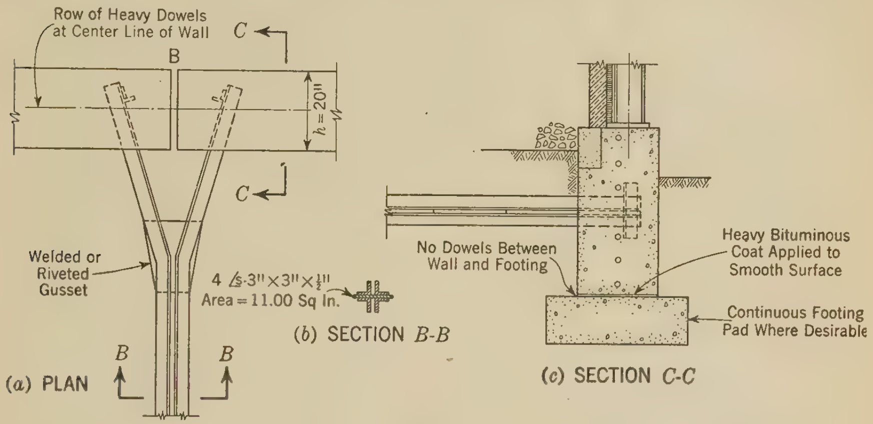
3. Rectangular Frame with Tie Rod and Hinges

The analysis in Section 2 serves to indicate that high temperature differentials in rigid frames are certain to create troublesome problems. It is a logical step, therefore, to attempt to make the frames less rigid in an effort to minimize thermal stresses.

Again, consider the frame discussed in Section 2 and place a hinge at each of the points B and E. Upon application of the temperature differential, the frame would then be able to move without any restraint other than that caused by friction between the bottom of the walls and the soil upon which the frame rests, so long as the frame is symmetrical about both center lines. Disregarding this friction, no temperature stresses whatever would be created in the frame. In many instances such hinges would serve well as an inexpensive expedient. In other cases the considerable increase of distance between points B and E, as well as the comparatively abrupt change in curvature at these points, would have a most unfavorable effect on the superimposed equipment such as structural framing and refractory walls. For this reason it is generally advantageous to introduce a fairly heavy, yet slender, tie member between points B and E, Fig. 1, as illustrated for point B in Fig. 8.

The setup for the frame shown in Fig. 1(*a*), with the addition of links at points B and E and a tie rod between these points, then becomes as shown in Fig. 9. It is assumed that the tie rod BE receives a temperature increase of 300° F through the soil. This causes the tie rod to elongate an amount $120 \text{ in.} \times 300° \times 0.0000067 = 0.242 \text{ in.}$; and $80.2293 \text{ in.} \times \tan \frac{\beta}{2} = 80.2293 \times \tan (0° \, 32' \, 35.39'') = 0.761 \text{ in.}$

Fig. 9 shows the deformation of one half of the frame without the restraining action of a tie rod. The change of span length between points B and E, that causes stress in the members, then is 122.914 − 120.242 = 2.672 in.; but the tie rod also elongates, due to its stress, an amount equal to $\frac{T_{BE}\,L}{A\,E}$, in which: T = the force sustained by the rod; and L = length of member. Hence, the displacement becomes $2.672 - \frac{T_{BE} \times 120}{5.50 \times 30{,}000{,}000}$.


Row of Heavy Dowels at Center Line of Wall
C
B
h = 20"
C
Welded or Riveted Gusset
B
B
4 ∠s-3"×3"×½"
Area = 11.00 Sq In.
No Dowels Between Wall and Footing
Heavy Bituminous Coat Applied to Smooth Surface
Continuous Footing Pad Where Desirable

(*a*) PLAN (*b*) SECTION *B-B* (*c*) SECTION *C-C*

FIG. 8

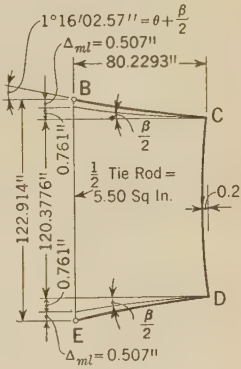

1°16′02.57″ = θ + β/2
Δml = 0.507"
80.2293"
B
C
β/2
0.761"
½ Tie Rod = 5.50 Sq In.
0.285"
122.914"
120.3776"
0.761"
D
E
β/2
Δml = 0.507"

FIG. 9

For a slender tie rod, stored-up elastic energy of both axial forces and moments must be included in the elastic energy equation, which for one half of the complete frame reads:

$$\sum \frac{F\,T\,L}{A\,E} + \int \frac{m\,M\,ds}{E\,I} = 2.672 - \frac{T_{BE} \times 120}{5.50 \times 30{,}000{,}000} \quad \ldots\ldots(11)$$

The force in the tie rod is the only redundant. Once this force is known, the frame is statically determinate. Since one equation is all that is necessary for its solution, the solution of tie-rod force T_{BE} can be based upon the displacement, $2.672 - \frac{T_{BE} \times 120}{5.50 \times 30{,}000{,}000}$, of point E relative to point B.

The auxiliary and actual stress diagrams, respectively, for axial forces in the frame members are included in Table 1 which is used to evaluate the term,

$$\sum \frac{F\,T\,L}{A\,E} = +\,0.000000784 \times T' \times T_{BE} \ldots\ldots\ldots\ldots\ldots(12)$$

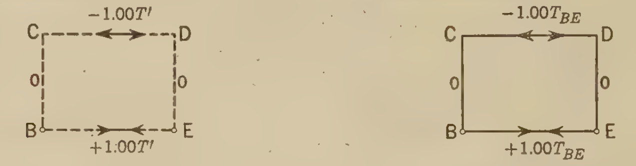
in which T' is an auxiliary tie-rod force. In order to evaluate the term

$\int \frac{m\,M\,ds}{E\,I}$, in Eq. 11, refer to the auxiliary and actual moment diagrams in Figs. 10(*a*) and 10(*b*), respectively:

$$\int \frac{m\,M\,ds}{E\,I} = \int_B^C \frac{T'\,x\,M\,dx}{E\,I} + \int_C^D \frac{80\,T'\,M\,dy}{E\,I} + \int_E^D \frac{T'\,x\,M\,dx}{E\,I}$$

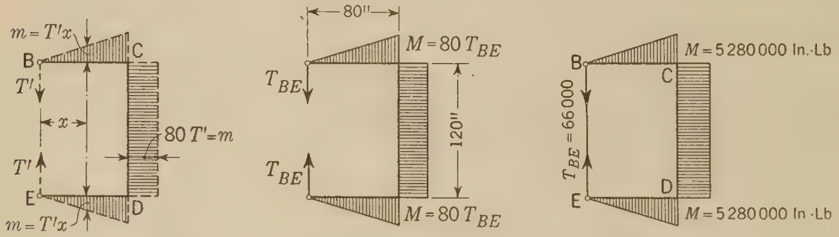
$$= \frac{T'}{E\,I} \int_B^C M\,dx\,\bar{x} + \frac{80\,T'}{E\,I} \int_C^D M\,dy + \frac{T'}{E\,I} \int_E^D M\,dx\,\bar{x},$$

TABLE 1.—EVALUATION OF EQ. 12 (TENSION IS POSITIVE)


−1.00T′
C
D
0
0
B
E
+1.00T′
−1.00T_{BE}
C
D
0
0
B
E
+1.00T_{BE}

AUXILIARY STRESS DIAGRAM ACTUAL STRESS DIAGRAM

Member	Force F (lb)	Force T (lb)	L (in.)	Area A (in.²)	E (lb per sq in.)	$\frac{F\,T\,L}{A\,E}$
BC.......	0	0	80	1,060	2,000,000	0
CD.......	−1.00 T'	−1.00 T_{BE}	120	1,060	2,000,000	+0.0000000566 $T'\,T_{BE}$
DE.......	0	0	80	1,060	2,000,000	0
BE.......	+1.00 T'	+1.00 T_{BE}	120	5.50	30,000,000	+0.0000007272 $T'\,T_{BE}$
Σ.........	....	....	....	....	....	+0.000000784 $T'\,T_{BE}$


m = T′x
B
C
T′
x
80 T′=m
T′
E
D
m = T′x
80″
M = 80 T_{BE}
T_{BE}
120″
T_{BE}
M = 80 T_{BE}
B
C
M = 5 280 000 In.-Lb
T_{BE} = 66 000
D
E
M = 5 280 000 In.-Lb

(*a*) AUXILIARY MOMENTS (*b*) ACTUAL MOMENTS (*c*) FINAL MOMENTS

FIG. 10.—MOMENT CURVES AS AFFECTED BY A TIE ROD

that is,

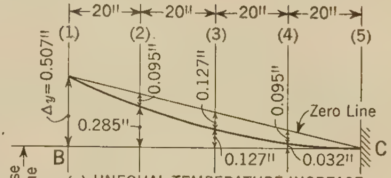
$$\int \frac{m\,M\,ds}{E\,I} = \frac{T'}{E\,I}\Big[(\text{area}) \times \bar{x}\Big]_B^C + \frac{80\,T'}{E\,I}\Big[(\text{area})\Big]_C^D + \frac{T'}{E\,I}\Big[(\text{area}) \times \bar{x}\Big]_E^D \quad \text{............(13)}$$

Substituting the known values in Eq. 13, $\int \frac{m\,M\,ds}{E\,I} = \frac{T'}{2{,}000{,}000 \times 14{,}230}$ $(80\,T_{BE} \times 40 \times 53.3 + 80 \times 80\,T_{BE} \times 120 + 80\,T_{BE} \times 40 \times 53.3) = 0.000039\,T'\,T_{BE}$.

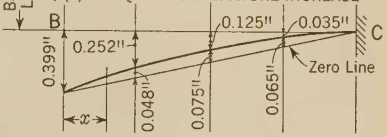
With the two terms evaluated, Eq. 11 yields:

$$0.000000784\,T'\,T_{BE} + 0.000039\,T'\,T_{BE} = 2.672 - 0.000000727\,T_{BE} \quad \text{..(14)}$$

The auxiliary force T' might have been assumed as unity at the outset since it cancels; hence, $0.000040511 \times T_{BE} = 2.672$; and $T_{BE} = \frac{2.672}{0.000040511} = 66{,}000$ lb. The maximum moment now is $66{,}000 \times 80 = 5{,}280{,}000$ in-lb, or greater than for the hinge-and-tieless frame. The final moment diagram is shown in Fig. 10(*c*).


20"
20"
20"
20"
(1)
(2)
(3)
(4)
(5)
Δy = 0.507"
0.095"
0.127"
0.095"
Zero Line
0.285"
B
0.127"
0.032"
C
Base Line

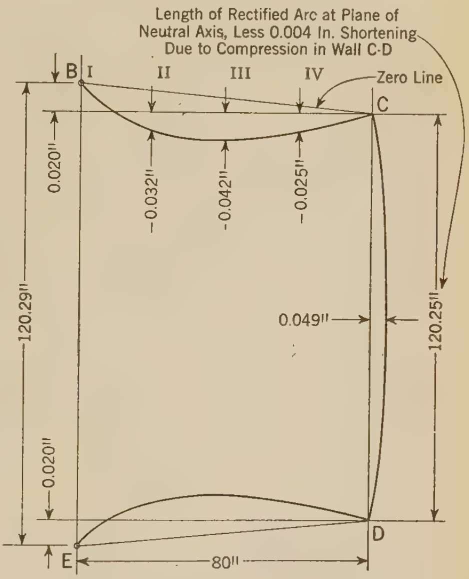
(*a*) UNEQUAL TEMPERATURE INCREASE


B
0.125"
0.035"
C
0.252"
0.399"
Zero Line
0.048"
0.075"
0.065"
x

(*b*) TIE-ROD FORCE OF 66 000 LB

0.095	0.127	0.095
−0.048	−0.075	−0.065
0.047	0.052	0.030

(*c*) DISTANCES BELOW ZERO LINE, IN INCHES


Length of Rectified Arc at Plane of Neutral Axis, Less 0.004 In. Shortening Due to Compression in Wall C-D
B
I
II
III
IV
Zero Line
C
0.020"
−0.032"
−0.042"
−0.025"
120.29"
0.049"
120.25"
0.020"
D
E
80"

(*d*) FINAL DEFLECTION DIAGRAM

FIG. 11.—CONSTRUCTION OF DEFLECTION DIAGRAM IN RELATION TO THE ZERO LINE

With respect to the end wall CD, its moment has been increased by 15%. Nevertheless, the moments are eased over the middle part of the side walls by the introduction of hinges and a tie rod.

The extremely strange shape of the elastic curve is illustrated in Fig. 11(*d*) as compiled from Fig. 11(*c*). For end wall CD the tie-rod pull of 66,000 lb (Fig. 11(*b*)) is sufficient to overcome the inward bow caused by the temperature differential (Fig. 11(*a*)), since tie-rod pull forces the end wall to bow outward. Regarding wall pieces BC and ED, Fig. 11(*d*), the inward temperature bow remains predominant in spite of tie-rod pull. The outside faces of these two wall pieces are in tension, even though the elastic curve apparently indicates that the opposite is true.

The exact deflections at various points of the frame are determined as follows: The length that wall CD shortens as the result of compression, $\frac{T_{BE}\,L}{A\,E}$, is equal to $\frac{66{,}000 \times 120}{1{,}060 \times 2{,}000{,}000}$ or 0.004 in. In other words, the length of the rectified arc at the plane of the neutral axis (see Fig. 11(*d*)), less a shortening of 0.004 in. as the result of compression in wall CD, is 120.25 in. The tie-rod elongation, in inches, is—

For stress, $\frac{66{,}000 \times 120}{5.50 \times 30{,}000{,}000}$	0.048
For a 300° temperature increase, Δt, through the soil	0.242
Total increased length	0.290

Deflections as given in Fig. 11(*d*) are established in part by means of the setup in Fig. 11(*c*) as being the difference between deflection caused by thermal bow (see Fig. 2) and deflection produced in opposite direction by tie-rod pull, from the formula,

$$\Delta_{BE} = \frac{T_{BE}}{6EI}(2 \times 80^3 - 3 \times 80^2 x + x^3) \qquad (15)$$

The ultimate displacements, in inches, are obtained as follows:

Section (see Fig. 11)	Computation	Δ	Fig. No.
1	....	−0.020	11(*d*)
2	$0.020 \times \frac{60}{80} - 0.047$	−0.032	11(*c*)
3	$0.020 \times \frac{40}{80} - 0.052$	−0.042	11(*c*)
4	$0.020 \times \frac{20}{80} - 0.030$	−0.025	11(*c*)

The outward bow of wall CD equals $\frac{M l^2}{8EI} - 0.285$ (see Fig. 7(*a*))

$$= \frac{5{,}280{,}000 \times 120^2}{8 \times 2{,}000{,}000 \times 14{,}230} - 0.285 = 0.049 \text{ in.}$$

The foregoing computations serve to indicate that the introduction of hinges and tie rods at properly selected points of the frame may bring a relieving effect to some parts of the structure, but an aggravating effect to other parts. It is natural, therefore, that the next step should be to investigate what can be gained by allowing the half frame BCDE, Fig. 11(*d*), to acquire a somewhat greater deformation than a fixed-length the tie rod permits. For this purpose assume that the tie rod used in the preceding example is placed with an initial slack of 1.20 in. The distance between hinges, then, must be increased 1.20 in. (entirely by a nonuniform heating of the walls) before the tie rod comes into action as a load-carrying member of the system. If the new tie rod is of the same sectional area as the four angles previously used, the expression for the tie-rod force T_{BE} becomes $\frac{2.672 - 1.20}{0.000040511} = 36{,}300$ lb. The corner moment in the frame equals 36,300 × 80 = 2,900,000 in.-lb. The corrected tie-rod elongation, in inches, is now determined as before—

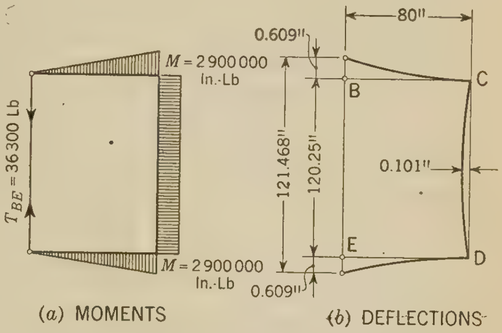
For stress, $\frac{36{,}300 \times 120}{5.50 \times 30{,}000{,}000}$ 0.026
For a 300° temperature increase Δt through the soil 0.242
Plus the initial slack in the tie rod 1.200
Total increased length 1.468

The center deflection of wall CD, in inches, will be—

Thermal curvature inward (see Fig. 7(*a*)) .... +0.285
Restraining corner moments (outward) $\frac{M l^2}{8EI}$
$= \frac{2{,}900{,}000 \times 120^2}{8 \times 2{,}000{,}000 \times 14{,}230}$ .... −0 184
Ultimate deflection (inward) .... +0.101

Fig. 12 shows the final moment and deformation diagrams, respectively. It is interesting to note by comparing Figs. 10(*c*) and 12(*a*) that the initial tie-rod slack has reduced the frame moment by almost one half. The increase in distance between points B and E is only half that which would have been produced in the frame with hinges at points B and E—but without any tie rod. This also holds true for the abruptness of the change in curvature at points B and E.

Comparison between Figs. 11(*d*) and 12(*b*) reveals that the elastic curve of end wall CD in one case is outside, and in the other case inside, its original position.


80"
0.609"
$M = 2\,900\,000$ In.-Lb
C
B
$T_{BE} = 36\,300$ Lb
121.468"
120.25"
0.101"
E
D
$M = 2\,900\,000$ In.-Lb
0.609"

(*a*) MOMENTS (*b*) DEFLECTIONS

FIG. 12.—MOMENTS AND DEFLECTIONS, RECTANGULAR FRAME WITH TIE ROD AND HINGES

The analysis demonstrates that the introduction of a tie rod with an initial slack, in many instances, should constitute an economical means by which to bring thermal stresses within a reasonable range. The greater the permissible slack, the less will be resulting thermal stresses. The proper amount of slack is governed by the magnitude of abrupt displacement which the equipment (refractory wall, steel structure, etc.) will safely withstand, and should be subject to close scrutiny by the designer in each individual case.

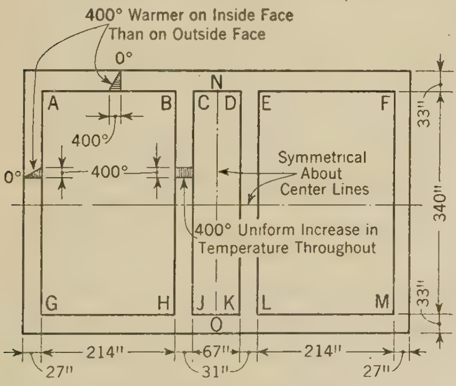

400° Warmer on Inside Face Than on Outside Face
0°
N
A
B
C
D
E
F
400°
0°
400°
Symmetrical About Center Lines
33"
340"
400° Uniform Increase in Temperature Throughout
G
H
J
K
L
M
33"
O
214"
67"
214"
27"
31"
27"

FIG. 13.—RECTANGULAR FRAME WITH TWO CROSS WALLS

4. RECTANGULAR FRAME WITH TWO CROSS WALLS

The last type of frame to be analyzed is a foundation consisting of six monolithic walls. A simplified plan of the structure is essentially as shown in Fig. 13. It differs from the frames described in Section 3 in that it is much larger and contains members of different cross section. Side walls AF and GM, Fig. 13, are identical. Cross walls BH and EL are also identical but of a somewhat lighter section. End walls AG and FM are of a lighter section than any of the others. A cross section through the exterior walls is substantially as shown in Fig. 1(*b*).

The exterior walls are subject to the same temperature differential (400° F) as the frames previously analyzed. The two cross walls, however, receive a uniform temperature increase of 400° F throughout since they are exposed to similar heat from both sides.

Axial forces are neglected in the following computations. Moments of inertia, transformed into terms of concrete of the three different walls, are computed for a "slice" of wall, 1 ft thick. The properties of the sections are as listed in Table 2.

TABLE 2.—PROPERTIES OF SECTIONS, RECTANGULAR FRAME WITH TWO CROSS WALLS

Wall (Fig. 15)	Area of Steel, A_s (in.^{2})		I (in.^{4})
	Outside	Inside	
Side, AF......	3.00	0.50	22,020
End, FM......	2.30	0.50	10,940
Cross, EL......	1.30	1.30	10,260

To obtain a picture of thermal displacements that occur when the structure is heated, cut the frame at points A, N, and F. This makes the frame statically determinate and causes it to deviate from its cold shape, as shown in dotted lines, to that shown in full lines in Fig. 14. Point O is the only spot that remains stationary. All displacements are symmetrical about line ON. Since both frame and loading are symmetrical about both center lines of the frame, the moment diagram will likewise be symmetrical about the center lines.

Fig. 15 is the complete geometrical displacement diagram for the right-hand side of the structure. The linear displacements (both horizontally and vertically), as well as the rotational displacements at points N and F, are: $\Delta_{Nx} = 2 \times 1.601 = 3.202$ in.; $\Delta_{Ny} = 0.955 + 0.231 + 0.115 = 1.301$ in.; $\phi_N = 2 \times 2\gamma = 4\gamma = 1° 4' 30.72'' = 0.01876$ radian; $\Delta_{Fx} = 278.500 + 0.847 + 9.087 + 6.786 - (230.211) + 49.150 + 1.601) = 14.258$ in.; $\Delta_{Fy} = 3.050 + 0.955 + 1.450 = 5.455$ in.; and $\phi_F = \theta + \beta + \theta - 2\gamma = 2\theta + \beta - 2\gamma = 4° 47' 53.16'' = 0.08369$ radian.

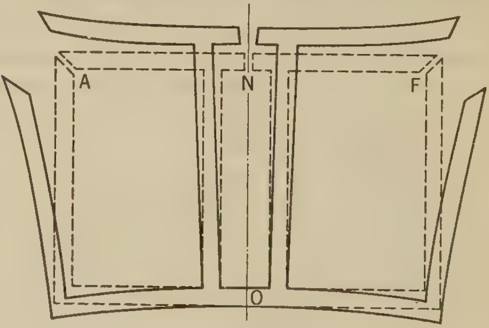


FIG. 14

It follows from Fig. 13 that there are only four unknowns—namely, moments M_D, M_E, M_F, and the moment in the cross wall M_{DK}. As the latter is the difference between M_D and M_E, it may be expressed as such, thereby reducing the number of unknowns to three.

Proceeding along the same lines as in the example Section 3, it is possible to write three elastic energy equations based on the vertical, horizontal, and rotational displacements at each of the points N and F. Examination reveals, however, that due to symmetry the equation (Eq. 4) based on vertical deflection Δ_{Ny} is unusable, still leaving an abundant number of equations. Choosing to utilize the three elastic energy equations that are based on Δ_{Nx}, Δ_{Fy}, and ϕ_F, the auxiliary loading diagrams and auxiliary moment diagrams shown in Figs. 16(*a*), 16(*b*), and 16(*c*), respectively, are constructed. The moment areas as sketched in these illustrations are drawn on that side of the members where auxiliary force produces compression.

Fig. 17(*a*) is the assumed actual, or *M*-moment diagram. The moment areas are shown on that side of the member where compression will presumably occur. It will also be noted that, for simplicity, the suffixes for moments in the upper right quadrant are repeated in the three other quadrants. The modulus of elasticity *E* is constant throughout and, as in the other examples of this paper, is assumed to be 2,000,000 lb per sq in. As before, $n = 15$.

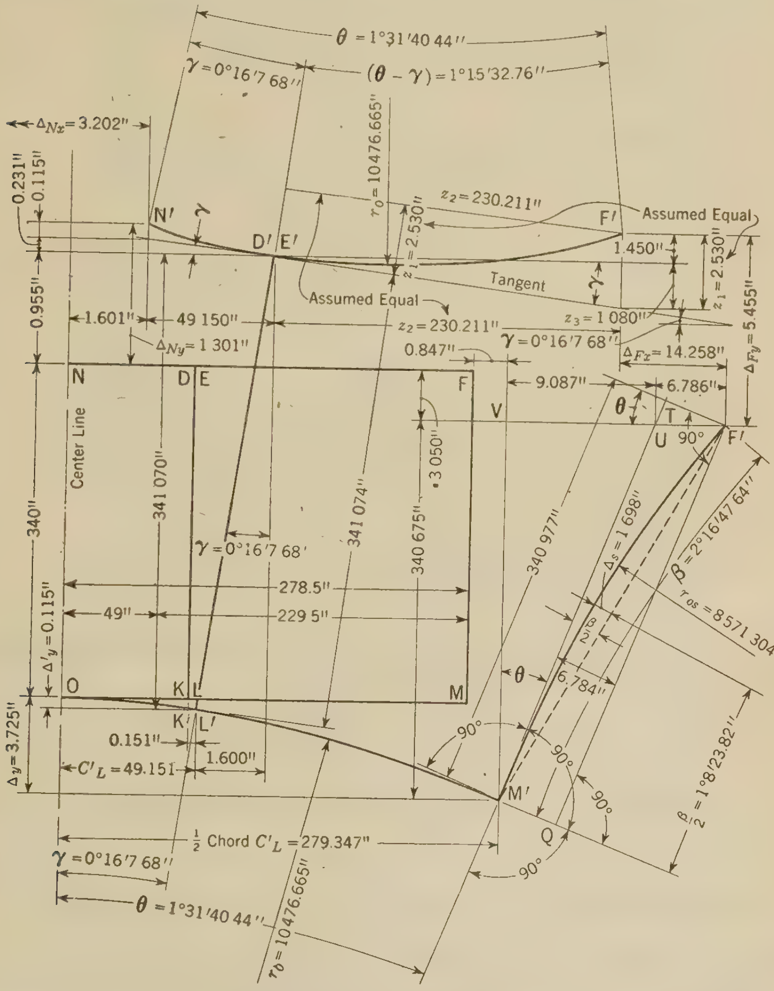

θ = 1°31′40 44″
γ = 0°16′7 68″
(θ − γ) = 1°15′32.76″
Δ_Nx = 3.202″
0.231″
0.115″
0.955″
r₀ = 10476.665″
z₂ = 230.211″
z₁ = 2.530″
N′
γ
D′ E′
F′
Assumed Equal
1.450″
z₁ = 2.530″
Δ_Fy = 5.455″
Tangent
Assumed Equal
z₃ = 1 080″
1.601″
49 150″
z₂ = 230.211″
Δ_Ny = 1 301″
0.847″
γ = 0°16′7 68″
Δ_Fx = 14.258″
N
D E
F
V
9.087″
6.786″
θ
T
U 90°
F′
Center Line
340″
341 070″
3 050″
341 074″
340 675″
340 977″
Δs = 1 698″
β = 2°16′47 64″
r_os = 8571 304″
γ = 0°16′7 68′
278.5″
229 5″
49″
Δ′y = 0.115″
β/2
6.784″
θ
O
K L
M
K′ L′
Δy = 3.725″
0.151″
1.600″
C′_L = 49.151
90°
90°
90°
M′
Q
β/2 = 1°8′23.82″
½ Chord C′_L = 279.347″
γ = 0°16′7 68″
θ = 1°31′40 44″
90°
r₀ = 10476.665″

Fig. 15.—Geometrical Displacement Diagram

The elastic energy equation based on horizontal displacement Δ_{Nx} at point N (compare Eqs. 6 and 7) is written as: $E\,\Delta_{Nx} = \int_B^H \frac{F_1\, y\, M\, dy}{F_1\, I_{EL}} + \int_H^L \frac{340\, F_1\, M\, dx}{F_1\, I_{EF}}$

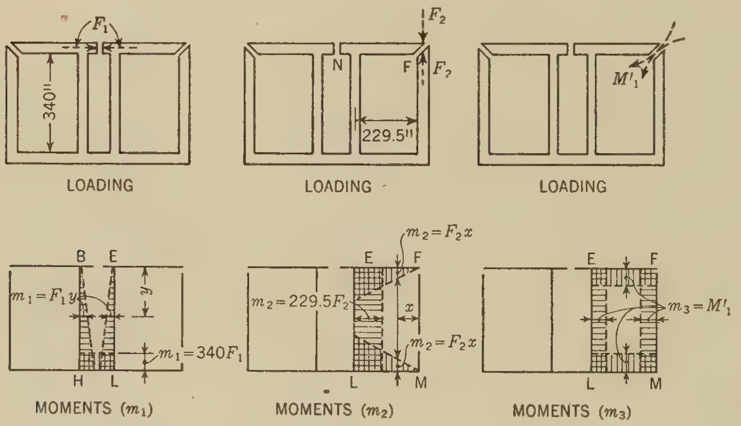
$$+ \int_L^E \frac{F_1\, y\, M\, dy}{F_1\, I_{EL}} + \int_E^B \frac{0\, M\, dx}{F_1\, I_{EF}} = \int_B^H \frac{M\, dy}{I_{EL}} \times y + 340 \int_H^L \frac{M\, dx}{I_{EF}} + \int_L^E \frac{M\, dy}{I_{EL}}$$

$\times \bar{y} + 0$. Similar to Eq. 7:

$$E\,\Delta_{Nx} = \frac{\text{(area)}}{I_{EL}} \times \bar{y}\,\Big]_B^H + 340 \times \frac{\text{(area)}}{I_{EF}}\,\Big]_H^L + \frac{\text{(area)}}{I_{EL}} \times \bar{y}\,\Big]_L^E \quad \text{....(16)}$$

Substituting the known values—$2{,}000{,}000 \times 3.202 = \dfrac{(M_D - M_E) \times 340}{10{,}260}$ $\times\, 170 + 340 \times \dfrac{98\,M_D}{22{,}020} + \dfrac{(M_D - M_E) \times 340}{10{,}260} \times 170$—and reducing:

$$12.79\,M_D - 11.28\,M_E = 6{,}400{,}000 \quad \text{...............(17)}$$



(a) FOR Δ_{Nx} (b) FOR Δ_{Fy} (c) FOR ϕ_F

Note: Moment Areas Are Shown on That Side of the Member Where Compression is Produced

FIG. 16.—AUXILIARY LOADING DIAGRAMS AND CORRESPONDING MOMENTS

Precisely similar to Eq. 16, the elastic energy equation expressing stored-up energy due to vertical displacement at point F is:

$$E\,\Delta_{Fy} = \frac{\text{(area)}}{I_{EF}} \times \bar{x}\,\Big]_F^E + 229.5\,\frac{\text{(area)}}{I_{EL}}\,\Big]_E^L + \frac{\text{(area)}}{I_{EF}} \times \bar{x}\,\Big]_L^M \quad \text{.....(18)}$$

and substituting the known values as for horizontal displacements, reducing as for Eq. 17:

$$-\,7.6\,M_D + 9.2\,M_E - 0.8\,M_F = 10{,}900{,}000 \quad \text{...........(19)}$$

Also, the elastic energy equation based on rotational displacement at joint F is:

$$E\,\phi_F = \frac{\text{(area)}}{I_{EF}}\,\Big]_F^E + \frac{\text{(area)}}{I_{EL}}\,\Big]_E^L + \frac{\text{(area)}}{I_{EF}}\,\Big]_L^M + \frac{\text{(area)}}{I_{FM}}\,\Big]_M^F \quad \text{......(20)}$$

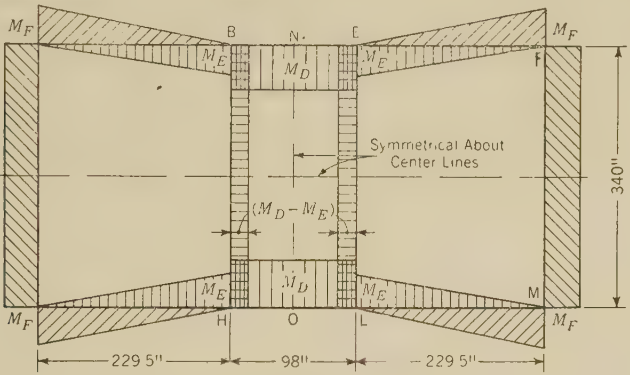
From which, by substitution and reduction:

$$-\,0.0332\,M_D + 0.0436\,M_E - 0.0415\,M_F = 167{,}300 \quad \text{........(21)}$$

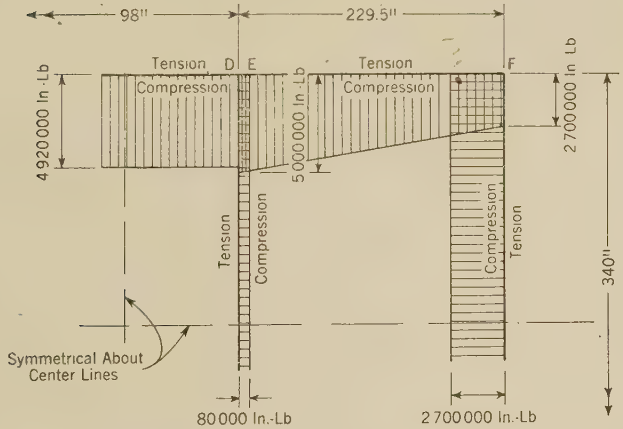
Solution of the three simultaneous Eqs. 17, 19, and 21 yields, in inch-pounds: $M_D = +\,4{,}920{,}000$; $M_E = +\,5{,}000{,}000$; $M_F = -\,2{,}700{,}000$; and $M_D - M_E = 4{,}920{,}000 - 5{,}000{,}000 = -\,80{,}000$.

In accordance with the sign convention used, the moments that proved to be positive were assumed correctly; those that became negative are opposite to what was assumed at the beginning (see Fig. 17(*a*)). The final moment diagram is as indicated in Fig. 17(*b*).

Side walls AF and GM are not subject to axial load. Cross walls and end walls, however, do carry an axial force (compression in cross walls and tension in end walls), the numerical value of which is $\dfrac{M_E - M_F}{229.5} = \dfrac{5{,}000{,}000 - 2{,}700{,}000}{229.5} = 10{,}000$ lb. The greatest stresses in the side wall occur at point E, the magnitude of which (in pounds per square inch) is:


M_F
B
N
E
M_F
M_E
M_D
M_E
F
Symmetrical About
Center Lines
340"
$(M_D - M_E)$
M_E
M_D
M_E
M
M_F
H
O
L
M_F
229 5"
98"
229 5"

(*a*) ASSUMED ACTUAL


98"
229.5"
Tension
D
E
Tension
F
Compression
Compression
4 920 000 In.-Lb
5 000 000 In.-Lb
2 700 000 In.-Lb
Tension
Compression
Compression
Tension
340"
Symmetrical About
Center Lines
80 000 In.-Lb
2 700 000 In.-Lb

(*b*) FINAL

Fig. 17.—Moments, *M*, in a Rectangular Frame with Two Cross Walls

Material	Computation	Stress
concrete, f_c	$\dfrac{5{,}000{,}000 \times 11.4}{22{,}020}$	2,590
tensile steel, f_s	$\dfrac{5{,}000{,}000 \times (30 - 11.4)}{22{,}020} \times 15$	63,300

These stresses also are far beyond the limits within which the equations of Section 4 hold.

In view of the aforementioned stresses, the foundation would be unusable in its present rigid form, except possibly in cases where its superimposed load is quite uniformly distributed. Even in such cases, the superimposed equipment must be of a nature that will withstand the effects of a somewhat cracked-up foundation. However, when the designer undertakes to decide whether or not the equipment is capable of absorbing effects of a more or less failing foundation, the difficulty immediately arises of determining where the foundation is likely to yield. The probability is that the foundation in Fig. 13 would fracture between points B and E, or between points H and L. It might crack anywhere in these two regions, as well as at various other points, since the moments and the corresponding resistance values of the frame members are fairly proportionate.

In connection with a frame as analyzed in Section 2, it is impossible, theoretically, to predict where the frame will rupture since both moment and wall section are constant throughout. Practice shows that fractures appear at intervals along the outside face of all four frame members, although they appear mostly near the corners.

Two factors contribute to the comparatively much higher moments in the frame of Fig. 13 over those in the frame of Fig. 1(*a*). The two factors are (1) the "thicker" cross sections and (2) the detrimental effect introduced when the cross walls push the side walls outward.

5. Summary

The foregoing computations furnish an explanation of the numerous fractures that inevitably occur in rigid-frame members when subject to a nonuniform temperature variation. They show also the certainty with which it is possible to analyze such structures, with the view of minimizing thermal stresses, when the calculations are set up in a rational manner.

The actual occurrence of 500° F is by no means exceptional for the temperature of structural concrete. In connection with the type of problem discussed in this paper, the entire danger lies in the fact that only one side of each of the frame members attains the higher temperature increase. The analyses of Sections 2 and 4 disclose that neither of the two frames are capable of withstanding safely, the effects of a temperature differential materially in excess of 100° F, since the frames are already stressed to some extent by vertical loading.

For all frames analyzed in this paper, it has been assumed that the concrete maintained its elasticity even at the very high stresses encountered. This is not the case, as the stresses were computed to be way into the plastic range of concrete. For this reason it is logical to assume that once the computed stresses reach the plasticity range of the concrete they increase only slightly with further increase in displacements. This fact is most likely one of the reasons that concrete stands up as well as it does even under quite unfavorable nonuniform thermal conditions.

The degree of accuracy with which this type of analysis should be executed depends not only on the structure in question, but also on various individual circumstances. Displacements must be computed by logarithms, whereas the slide rule offers abundant accuracy for all other numerical values.

Since the average designer has little interest in ascertaining linear displacements and deflections, considerable time may be saved by computing the frame moment entirely from rotational displacements as shown in Section 2.

One factor of uncertainty entering into thermal problems is the determination of the exact temperature on each side of the frame members. This is evidently a field in which much research should, and could, be done in order to enable the engineer to begin with reasonably correct temperature values instead of "makeshift" values that are largely a matter of guesswork. A question which (to the best of the writer's knowledge) is shrouded in total obscurity is that of how to determine intelligently the "average gradient" in a wall of limited extent whose temperature gradient varies from one edge to the other. A typical example of this is shown in Fig. 1(*b*). The thermal condition at the top of the foundation wall is quite different from that at the lower part of the wall; the upper part of wall is exposed to the cooling influence of the atmosphere; and the lower part of wall is affected by the insulating action of the surrounding soil.

In general, the investigations reported herein apparently sound a strong warning against continuity in structures whose members are subject to a fairly high nonuniform thermal action, unless it is ascertained beforehand, by an analysis, that stresses thus produced are within the allowable range.

In some instances, thermal fractures in a rigid frame are of little consequence. A good example of this is the foundation in Fig. 1(*a*). In this type of structure the superimposed load is distributed fairly uniformly so that little vertical beam action is required of the frame members.

When the frame in question forms a foundation, it is at times advisable to eliminate its continuity by cutting its members at well-chosen points, thus leaving the various parts free to assume their natural thermal deformations without restraint. However, this uncontrolled expedient usually results in a complicated and expensive foundation structure in order to maintain linear and rotational displacements within the permissible limits.

Again considering the frame of Fig. 1(*a*) assume that it has been decided to eliminate continuity altogether. Assuming also that no relative vertical settlement between the various foundation elements is permissible, the logical step would be to sever each of the two side walls AC and FD completely by a 1-in. thick rock-wool block expansion joint at a point about 2.5 ft from the inside face of each end wall AF and CD. Each end wall would be severed by the same type of joint at its center. This would create a foundation "ring" composed of six loose parts—two straight pieces of wall and four unequal angle-shaped parts. To avoid relative vertical settlement between any of these pieces, it would become necessary to place the wall parts on a footing pad with a smooth top and bituminous joint as shown in section CC of Fig. 8. The footing pad should be made discontinuous by a 1-in. thick rock-wool joint at corners A, C, F, and D. To prevent a lateral shifting of wall parts relative to footing pads, each wall part, at its extreme fourth or fifth points, must be doweled, to footing pads at two places. The dowels should be large-diameter, round, smooth bars heavily coated in mastic. Any unequal vertical bending, which before was supported by the walls, now must be supported entirely by the footing pads.

Some degree of controlled continuity may easily be established along the lines suggested in Section 3 (see Fig. 8). Whether or not an initially slacked tie rod is used at a strategic location, this foundation might be further improved (at added cost) by thinning the wall between buckstays. To avoid exceeding the allowable soil-bearing pressure, it probably would be necessary to place the walls on a continuous footing pad as shown in section CC which, in turn, would require a smooth bearing surface and a bituminous joint, as indicated, in order to facilitate sliding. Resistance against sliding should be checked; and if of considerable magnitude it should be taken into account as a more or less uniform load acting toward the inside of the frame. It increases corner moments but decreases the moments between joints.

Offhand, it would seem that the two cross walls BH and EL, Fig. 13, should be designed as beams (to carry their vertical load) and should be supported on movable end bearings at points B, E, H, and L. In practice, however, it will generally be found that the large end reactions create frictional forces at these points considerably in excess of the thrust of 10,000 lb existing in the cross wall.

In many instances it is possible to improve, materially, the design of a nonuniformly hot foundation by substituting comparatively slender steel members for some of the bulky concrete members. Such structural steel should be encased in a suitable type of concrete of a mix that yields high insulating value but low compressive strength. In order to ascertain that this protective concrete remains in place, even though badly fractured, it should be reinforced by continuous wire mesh or welded fabric placed fairly close to the surface.

In the case of a traditional, rectangular, concrete flue, an improved design is usually achieved by eliminating dowels between base slab and walls—at the same time providing a smooth steel-troweled surface on top of the base slab. This procedure enables flue walls (as they are heated from within) to bow and displace themselves outward without any restraint other than that caused by a slight friction between walls and base slab. If the flue is underground, the thermal displacement will become less due to the lowered temperature differential. In cases where the floor of the flue is below water table, the procedure just mentioned should be reversed in order to prevent water from entering the flue; that is, flue walls should be "fixed" at floor by through dowels, and flue roof then must rest loosely on walls without any continuity.

In connection with a time-honored type of foundation—namely, that of an octagonal mat with a centrally located plinth such as is used for stacks and blast furnaces—it is the writer's belief that ring stresses of enormous magnitude exist in the outer part of the mat. The central area (under the plinth) becomes very hot; whereas the ring part undoubtedly remains at a considerably lower temperature. It appears that the most satisfactory solution is to provide eight, or at least four, 1-in. thick rock-wool joints radially from the edge of the mat to within a couple of feet of the face of the plinth. These joints, of course, must extend through the full thickness of the mat. When placing eight radial slots in the mat it is obvious that the less economical four-way type of reinforcing must be adopted. If only four slots are used, it is still possible to utilize two-way reinforcement.

The analyses in Sections 2, 3, and 4 would seem to argue against the use of hard-grade reinforcing bars in unequally heated concrete structures. Intermediate or structural grade bars should be used because of their greater ductility. Bars rolled from the milder grades of steel offer some degree of security against the possibility that an overstressed structure will partly collapse, suddenly, due to their capacity for plastic deformation without fracture. Herein lies, undoubtedly, one of the reasons why a number of thermally overstressed concrete structures are today giving very satisfactory service despite development of numerous large fractures.

Much could be accomplished toward the exclusion of thermal stresses in concrete foundations by designing the equipment in such a manner as to prevent the heat from being transmitted to the load-carrying members. The hot equipment should be kept from coming in contact with the concrete, either by a sufficient amount of effective insulating material, or, by circulating air currents that are induced by stack action.

When at all feasible, the use of an overhead steel breeching is much more preferable than the old style of concrete flue.

Numerous troublesome problems could probably be eliminated, or at least simplified, by suspending a large proportion of the hot superstructure from carefully chosen key points. The transfer of heat from the superstructure to the foundation would then be restricted to the isolated key points where it may be dealt with more conveniently.

APPENDIX. NOTATION

The following letter symbols conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) and for Heat and Thermodynamics (ASA—Z10.4—1943), prepared by a Committee of the American Standards Association, with Society representation.

A = area of cross section, subscripts s and c denoting "steel" and "concrete," respectively;
C = one half the length of a chord;
E = modulus of elasticity;
F = force;
f = unit force or unit stress;
h = thickness of a wall or depth of a frame member;
I = rectangular moment of inertia;
L = axial length of a member;

M = moment:

M' = auxiliary moment;

M_t = moment produced by differential temperature, Δt;

m = moment of a unit force;

n = ratio of elastic moduli, E_s/E_c;

r = radius; subscripts o and i denoting outside and inside of a curved member, respectively; r_{os} = outer radius of the side member;

s = length of arc measured along the neutral axis of a curved member; ds = elemental part of length s;

T = tie-rod force;

t = temperature (thermometric); Δt = difference in temperature between two faces of a wall:

x = horizontal ordinate distance;

y = vertical ordinate distance;

z = distances, with appropriate numerical subscripts, in Fig. 15;

α = coefficient of thermal expansion;

γ = angle of curvature of a beam (Fig. 15) see also θ and ϕ;

Δ = change in length, or the displacement of a point as the result of a differential temperature Δt:

Δ_{Bx} = horizontal component of the displacement of;

Δ_L = increase in total span length;

Δ_m = displacement of the midpoint of a member, with subscripts s and l to denote "short beam" and "long beam," respectively;

Δ_v = vertical displacement at the end of a beam (Fig. 2);

θ = angle of curvature of a beam; a construction angle (see Fig. 15); and

ϕ = angle of curvature of a beam.

DISCUSSION

I. Oesterblom,[4] M. ASCE.—It is a sign of wakening to important but neglected facts that thermal action in structures has been more frequently discussed recently. There is again a sample of this trend in the publication of the paper by Mr. Tommerup. The author has succeeded in presenting a method that is quite applicable to simple frames. This commentator agrees wholeheartedly and thankfully with the logic of the method, but believes that the details must be modified to make the method truly useful.

There seem to be two basic points that require more thought and possibly correction—and some minor points, also. One of the basic points—if the author should agree—would materially lower the stresses; the effect of the other is the reverse. Presumably, the author has neglected the second point with the very good intention of simplifying his method; but, in doing it, he may lead novices astray unless he issues at least a warning. One may well question if the omission can be made fairly, when it is connected so intimately with the designer's problem that a separate superimposed solution for the omission is not permissible.

The two items may be presented, as follows:

(1) When heat is added on one side there is not only elongation and compression on that side, but also tension on the opposite side; and

(2) The direct longitudinal forces cannot be ignored, for they are very large and also set up bending moments by acting over a curved and deflected element.

Contrary to the conclusion of Mr. Tommerup, a correction for tension on the opposite side (item (1)) yields a deflection. Item (1) is predicated on the necessity of having a tensile reaction to the compressive stresses on any small section *dx* exactly in that element, and that condition sets up a neutral axis in the center line. Otherwise, where could the reactions be? They must be included somewhere, and analysis of the element *dx* would indicate that they must be there, regardless of the fact that the accumulation appears as a moment at the corner. This also sets up a longitudinal shear, which the author—quite correctly from his point of view—has omitted.

Figs. 2, 3, and 4 are correspondingly subject to criticism, and need correction. Fig. 2 should show center lines of the elements; the deflection should apply to these lines. The gap and the deflection in Fig. 3 should be similarly adjusted, and so should the displacements in Fig. 4. It follows then that the stresses would be different—roughly, about one half of the values shown.

There is also doubt as to the value of E assumed for the rationalized concrete that is made today. With the aggregates properly selected, graded, and proportioned, and with the quantity of water under control, E is more likely to be nearer to 3.5×10^6 than the 2×10^6 as assumed[5]—thus a reduction of the concrete stress approximately to $\left(\frac{1}{2} \times \frac{2}{3.5} =\right)$ 29% of what is shown. The revised value of 600 lb does not seem so devastating as the 2,070 lb computed by Mr. Tommerup; yet, the revised value must be increased considerably due to the direct normal strains and stresses.

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[4] Engr., Carbide & Carbon Chemicals Corp., South Charleston, W. Va.

[5] "Johnson's Materials of Construction," by J. B. Johnson, rewritten and revised by M. O. Withey and J. Ashton, John Wiley & Sons ,Inc. ,New York ,N. Y .,8th Ed., 1939, p. 479.

To compute these corrections the designer must know the initial construction temperature in addition to the 100° assumed at the time of the thermal addition of 400°. If the frame were built in cold weather with the temperature at 40° a rise of $(60° + 200° =)$ 260° would be required to reach the average of the frame at that time. With the corners fixed against translation the stress is $\delta = E\,\Delta t\,\alpha = 3.5 \times 260 \times 7.9 = 7{,}200$ lb per sq in., which is to be added to the previous concrete figures, algebraically. As a result there is compression on the heated side equal to $(7{,}200 + 600 =)$ 7,800 lb and compression, also, on the cool side equal to $(7{,}200 - 600 =)$ 6,600 lb. In addition, there are the bending stresses caused by the thrust being applied to an independently created eccentricity.

To compute this bending is a very laborious task. Almost the only way to combine the two problems is by the aid of Castigliano's second law, which presents such difficulties as to make the simple method proposed by Mr. Tommerup very attractive. For this reason the writer sincerely hopes that his argument concerning the deflection can be proved erroneous.

There is a saving feature which bears directly and heavily on the high stresses—in actual construction it is difficult to establish complete fixation against translation. Thus, if the elements in question could be completely freed from external restraint there would be no direct thermal stress; and, depending on how much restraint there is, the stresses would vary between 0 lb and 7,200 lb.

This brings to the fore a point that Mr. Tommerup has demonstrated so well and so fairly: Investigators concerned with thermal forces must know how they act, and then must design the details so that these forces are nil, if possible, or at least very small. It is only by proper details that a structure can be prevented from cracking or crumbling or even complete disaster. This is an extremely important point to which very few designers give proper emphasis.

Gratitude is due Mr. Tommerup; structural engineers are fortunate that he has presented this phase of a complex subject so well. He has advanced it in the right direction; but he has still left much work to be done to amplify his method. As presented, it does not seem to be complete enough for safety. When it is finished no one should be able to charge a thermal failure in a simple frame to "the act of God" or "force majeure," as is so often done today by the formula fraternity inside the profession.

FRANK R. HIGLEY,[6] ESQ.—A sheet metal unit developed in 1944 for a mechanical cycling operation responsive to nonuniform thermal action serves to demonstrate the possible danger from thermal stresses in unequally heated structures. The unit (see Fig. 18) comprises a single piece of steel, 0.040 in. thick and 5 in. long which, secured at one end, deflects its opposite end laterally 0.20 in. in 60 sec against a 1-lb load responsive to a minimum size gas pilot flame, substantially independent of ambient temperatures up to 1,000° (these data being approximate).

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[6] Cleveland, Ohio.

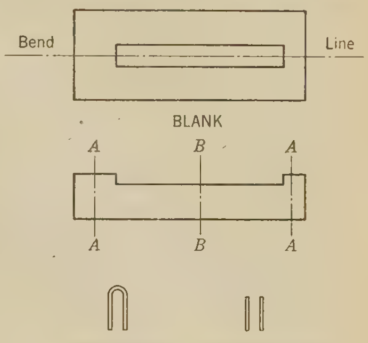

Fig. 18.—Thermally Responsive Unit.

With one objective in mind, it is often enlightening to study efforts leading toward the opposite objective. Of course, the purpose of designing such a thermal unit was opposite to that of designing the structures so ably analyzed in the paper. The sheet metal thermal unit is provided with a single slot and is simply bent along the line of the slot to provide a pair of members disposed in a parallel, slightly spaced, relation, with an integral interconnection adjacent to their ends along one edge of the resultant unit.

The principal features of the unit are: (*a*) Heat transfer between the members is inhibited to the greatest extent possible so that the thermal differential may be as great as possible; (*b*) the members have positive rigid interconnection at their ends to make best use of the relative expansive forces set up in the heated leg; and (*c*) while remaining rigid in the direction of generated force, the members are arranged to permit flexure in the resultant force direction desired.

Application of the same considerations to the type of structure described by the author indicates that (1) the wall should first be made laminate in so far as possible, to provide stratification between high-temperature and low-temperature faces; and (2) adjacent extremities of the laminas should be free from each other in the direction of their relative expansion. The author has at least implied the former of these two considerations under the heading, "2.—Rectangular Frame"; and has, in effect, quite thoroughly analyzed the second.

Inherently, from both the paper and the foregoing discussion, it appears that a plane of lamination, which in section determines what may be considered as a neutral thermal axis, should as nearly as possible be located to coincide with the common neutral axis of gyration between the same members at that section. Thus, as far as possible the members may have free relative expansion under the superimposed temperature differential, with relative expansion only on one side and relative contraction only on the other side of the axis.

A. L. Miller,[7] M. ASCE.—A distinct service has been rendered by the author in directing attention to problems arising from differential temperature change, and by presenting a feasible interpretation and theoretical evaluation. Necessarily, theory is abstract. Limited by hypothesis and assumption, it produces results which must be interpreted with due regard for these limitations. In addition to the generally recognized stipulations of the theory of flexure and continuity, the materials are presumed to possess their usual properties and behavior above the ordinary range of temperature. Also, it will be observed that the dimensional changes of the convex face of a member are assigned to its centroidal axis. Both premises are justifiable and undoubtedly give results that are more severe than the real effects. In other words, the theoretical results predict the upper limit of effects within which to exercise engineering judgment.

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[7] Prof. of Mechanics and Structures, Dept. of Civ. Eng., Univ. of Washington, Seattle, Wash.

Subscribing to the proposals of the paper, this discussion aims to validate the results of the illustrative examples and to indicate procedures for the application of "elastic line" methods of calculation for those who prefer them to the "elastic energy" method used by the author. "Moment Distribution"[8] and "The Theorem of Joint Translation"[9] (an extended form of the theorem of three moments) are convenient tools for this purpose. The examples of the paper will be solved by moment distribution, a most useful elastic line method. Solution by the theorem of joint translation will not be illustrated but the essential load term will be derived for use by those who are familiar with the theorem.

An initially straight member, when subjected to differential thermal action throughout its length, tends to assume a circular arc unless resistance is afforded by its attachments or supports. Since the curvature is relatively small, the theory applicable to straight members applies with negligible error. The extent to which the member is prevented from assuming the span-end slopes of the circular arc determines the span-end moments induced by the action and the consequent stresses and deflections throughout a rigid frame.

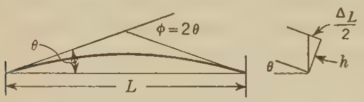

FIG. 19

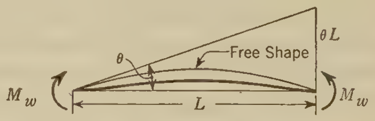

FIG. 20

Following the notation of the paper, the geometry of an initially straight member of uniform section which assumes a circular arc without stress when subjected to thermal differential is expressed (see Fig. 19):-

$$\Delta_L = L\,\alpha\,\Delta t; \qquad \theta = \frac{\Delta_L}{2\,h} = \frac{L\,\alpha\,\Delta t}{2\,h}; \qquad \text{and } \phi = 2\,\theta = \frac{L\,\alpha\,\Delta t}{h} \ldots (22)$$

in which θ and ϕ are expressed as slopes, consistent with the basic procedures of the theory of flexure. Accordingly, the fixed-beam moment $M_{F,N}$ caused by the thermal differential, for use in the moment-distribution method, is as follows (Fig. 20): By the moment-area principle,

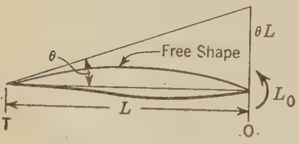

FIG. 21

$$\theta\,L = \frac{M_w}{E\,I}\,\frac{L^2}{2}; \qquad \text{and } M_w = \frac{2\,E\,I\,\theta}{L} \ldots . (23a)$$

Therefore—

$$M_w = \frac{E\,I\,\alpha\,\Delta t}{h} \ldots\ldots\ldots (23b)$$

—with rotational signs determined by inspection. Eq. 23*b* demonstrates that the fixed-beam moment M_w caused by the thermal differential is no function of the span length.

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[8] "Moment Distribution," *Bulletin No. 64*, Eng. Experiment Station, Univ. of Washington, Seattle.

[9] "The Theorem of Joint Translation," *Bulletin No. 89*, Eng. Experiment Station, Univ. of Washington Seattle.

The load term, L_o, for use with the theorems of two moments, three moments, and joint translation is derived as follows (Fig. 21): By the moment-area principle,

$$\theta L = \frac{L_o}{E I}\left(\frac{L}{2}\right)\frac{L}{3}; \qquad \text{and } L_o = \frac{6 E I \theta}{L} \ldots\ldots\ldots\ldots (24a)$$

Therefore—

$$L_o = \frac{3 E I \alpha \Delta t}{h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

The differential temperature Δt that causes upward deflection is positive. The rectangular frame of Fig. 1 is solved by moment distribution as follows:

The joint coefficients of the span ends are computed in the conventional manner, being 0.429 and 0.571 for the 160-in. and 120-in. members, respectively.

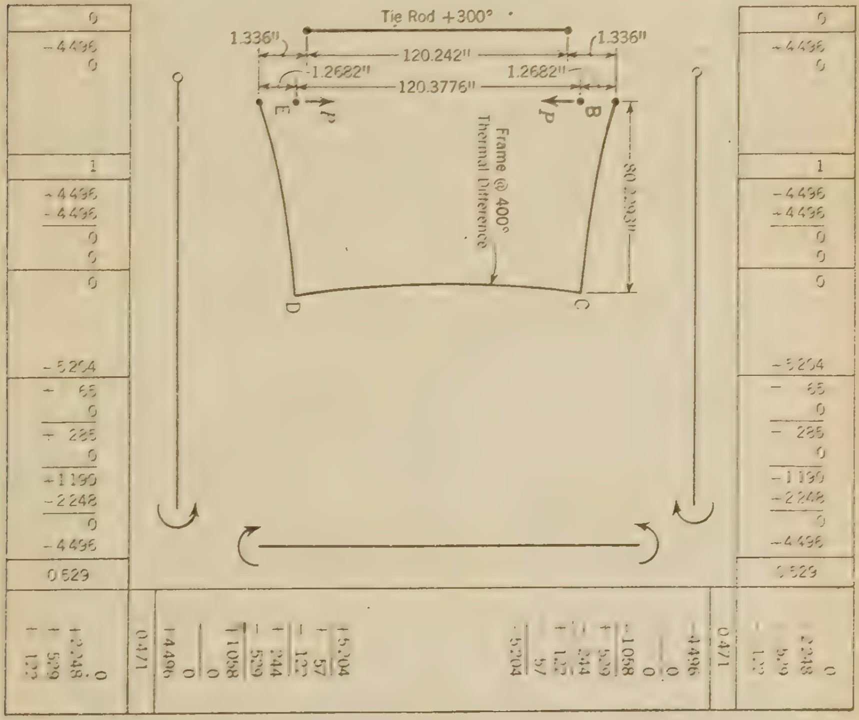
Tie Rod +300°
1.336"
120.242"
1.336"
1.2682"
120.3776"
1.2682"
E
P
P
B
Frame @ 400° Thermal Difference
80.293"
D
C
0
−4 496
0
1
−4 496
−4 496
0
0
0
−5 204
− 65
0
− 285
0
−1 190
−2 248
0
−4 496
0.529
0.471
+4 496
+1 058
+5 204
−5 204
−1 058
−4 496
0.471
+2 248
0

FIG. 22

The carry-over factors are 0.5 throughout. The fixed-beam moments are found by the equation,

$$M_w = \frac{E I \Delta t}{h} \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Thus, $M_w = \dfrac{2{,}000 \times 14{,}230 \times 0.0000079 \times 400}{20} = 4{,}496.68$ kip-in. for all members since each is subjected to the same thermal differential and possesses the same values of E, I, and h. Since no balancing moment appears in the first operation, subsequent distributions and carry-overs are zero, giving 4,496.68 kip-in. as the precise result at each span end. When arithmetical and geometrical approximations are eliminated, the solutions presented in the paper produce this result.

The rectangular frame with tie rod and hinges (Fig. 9) is analyzed by moment distribution as follows:

The joint coefficients of each span end are computed in the usual manner with due regard for the pins at points B and E. Carry-over factors are all 0.5 except those to the pins, B and E. The form of tabular procedure shown in Fig. 22 is convenient for any rigid frame and is self-explanatory to one familiar with the method of moment distribution. The boxes representing the joints provide a space in which to record the balancing moments; the joint coefficients appear on the span ends adjacent to the joints; and the steps in the successive cycles proceed outward from the joints in the sequence: Fixed-beam moment—distribution—carry-over—distribution—carry-over—distribution—etc. The span-end moment is the sum of the items at that span end. The number of cycles is the number of balancing moments appearing in the joint. Each step must proceed concurrently in all members throughout the structure.

The fixed-beam moments are calculated by Eq. 25, which yields 4,496 kip-in. for members BC and ED and 5,204 kip-in. in member CD. Rotational signs are determined by inspection. The computation is completed as follows:

Item	Description	
1	The load P (in kips), necessary to cause a displacement Δ of 1.2682 in., is $\frac{5{,}204}{80.2293}$, or	65.00
	Displacement Indexes, in Inches per Kip—	
2	Bending throughout the frame is $\frac{1.2682}{65.0}$, or	0.0195
3	Compression in member CB is $\frac{P\,L}{2\,A\,E} = \frac{1 \times 60.189}{1{,}060 \times 2{,}000}$, or	0.000028
4	Bending and compression combined	0.019528
5	Elongation of tie rod is $\frac{P\,L}{2\,A\,E} = \frac{1 \times 60.121}{5.5 \times 30{,}000}$, or	0.000364
6	Total index (items 4 and 5)	0.019892
7	The displacement Δ (in inches) of the frame plus the tie rod is $T_{BE} \times 0.019892$, or	1.336
8	The tie-rod force T_{BE} (in kips) is $\frac{1.336}{0.019892}$, or	67.0
9	The bending moment M, in kip-inches, is	5,360
10	If the tie rod is lengthened 1.20 in., item 5 becomes $\frac{1 \times 60.721}{5.5 \times 30{,}000}$, or	0.000368
11	The tie-rod force (item 8) becomes $\frac{1.336 - 0.60}{0.019896}$, or	37.0
12	The corresponding bending moment is	2,960

The rectangular frame with two cross walls in Fig. 13 must be analyzed in two separate solutions. The span-end moments due to curvature, computed in Fig. 23(*a*), are superimposed on the moments due to the differential expansion between the cross walls and end walls in Fig. 23(*b*) to produce the final results in Fig. 24. The magnitude of differential displacement between the ends of the 229.5-in. members is 0.015 in., based on the dimensions shown in Fig. 15. Because the frame is biaxially symmetrical, only one quarter, as shown in Fig. 17(*b*), will be needed for the solutions since a carry-over can be regarded as reflected back from an axis of symmetry with reversal of sign.

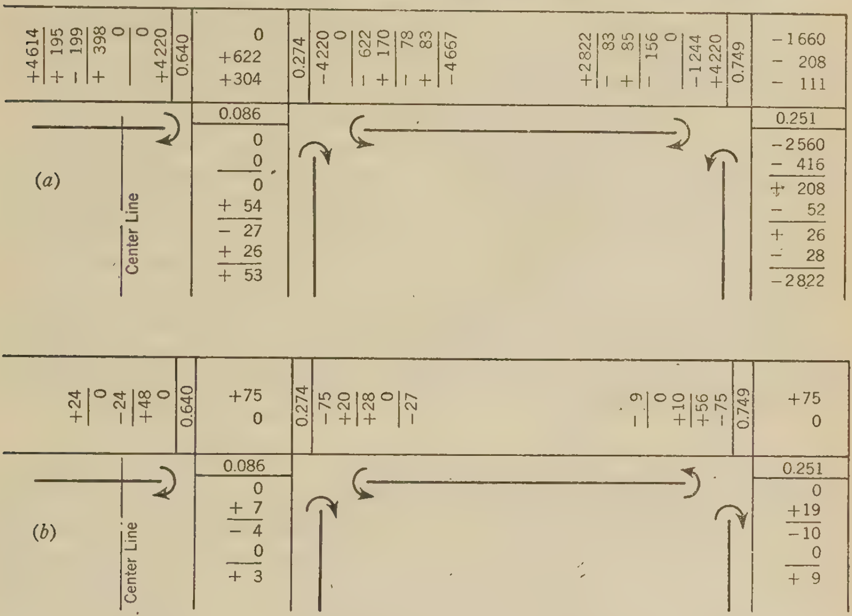
(a)
+4614
+ 195
− 199
+ 398
0
0
+4220
0.640
0
+622
+304
0.274
−4220
0
− 622
+ 170
− 78
+ 83
−4667
+2822
− 83
+ 85
− 156
0
−1244
+4220
0.749
−1660
− 208
− 111
0.086
0
0
0
+ 54
− 27
+ 26
+ 53
Center Line
0.251
−2560
− 416
+ 208
− 52
+ 26
− 28
−2822
(b)
+24
0
−24
+48
0
0.640
+75
0
0.274
−75
+20
+28
0
−27
− 9
0
+10
+56
−75
0.749
+75
0
0.086
0
+ 7
− 4
0
+ 3
Center Line
0.251
0
+19
−10
0
+ 9

Fig. 23.—Rectangular Frame with Two Cross Walls; Check Solution

The joint coefficients are found in the usual manner and all carry-over factors are 0.5. Fixed-beam moments due to curvature are found by Eq. 25 for each member, and three cycles of operation follow.

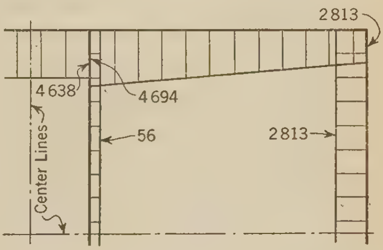
2813
4638
4694
56
2813
Center Lines

Fig. 24.—Final Moments in Kip-Inches

The coefficients of the frame having been found, the fixed-beam moments due to end translation are computed by:

$$M_d = \frac{6\,E\,I\,\Delta}{L^2} \qquad (26)$$

and found to be 75 kip-in. for each of the 229.5-in. members when = 0.015 in. Δ

The term is zero for all other members. Two cycles produce precise results.

Combining the results of Figs. 23(*a*) and 23(*b*) and expressing them in accordance with the general sign convention for comparison with Fig. 17(*b*) produces the moments shown in Fig. 24.

The results appearing in this discussion differ slightly from those of the paper. However, it will be observed that the problems are very sensitive numerically, slight initial variations producing noticeable final differences. Absolutely precise numerical results were obtained by means of the theorem of joint translation (not illustrated herein). The precise value of the moment for the rectangular frame with the tie rod in a rectangular position is 5,188.48 kip-in. which produces a value of T_{BE} = 66.8 kips and M = 5,344 kip-in.

Precise results for the span-end moments with two cross walls are: M_{BA} = 4,687.12; M_{AB} = 2,810.64; M_{BH} = 57.61; and M_{BE} = 4,636.6—all in kip-inches.

Elastic line methods have the distinct advantage of eliminating inaccuracies inherent in the complicated geometrical calculations required by elastic energy methods. Reliability of theoretical results is enhanced and the required time and effort greatly reduced thereby.

Charles O. Boynton,[10] Assoc. M. ASCE.—The bending stresses in a closed continuous rectangular reinforced concrete frame, caused by heat applied to the inner face, have been analyzed in Part 2, to which this discussion appertains principally. Mr. Tommerup has ably demonstrated the excessive bending stresses inherent in such a frame, recommending that it be separated into sections by expansion joints.

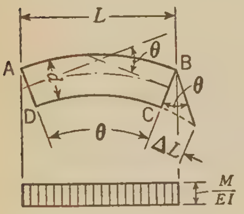

Fig. 25

It should be emphasized that the full magnitude of the uniform bending moment obtains in any part of the beam, however short, when restrained from bending. This condition may be illustrated by reference to Fig. 25, in which ABCD represents any very short part of the beam of length L and depth d, free from restraint. Side AB has been heated $t°$ F hotter than side CD, with relative elongation ΔL. If the temperature coefficient of linear expansion is α, then $\Delta L = \alpha\, t\, L$ and the central angle formed by the tangents to the ends of the elastic curve, $\theta = \frac{\Delta L}{d} = \frac{\alpha\, t\, L}{d}$ (θ being very small).

Let the conjugate beam be loaded with the $\frac{M}{E\,I}$-diagram, M being constant since the curvature is uniform and just sufficient to "straighten" the curved beam. The area of this diagram, $\frac{M\,L}{E\,I}$ by the moment area method equals the angle θ, or $\frac{M\,L}{E\,I} = \frac{\alpha\, t\, L}{d}$ and $M = \frac{\alpha\, t\, E\, I}{d}$. Substituting the author's values, $M = \frac{0.0000079 \times 400 \times 2{,}000{,}000 \times 14{,}230}{20} = 4{,}500{,}000$ in.-lb. This bending moment agrees with the author's determination and is about twice the safe resisting moment of the beam. However, had the more common grade of concrete been employed, with E equal to about 3,000,000 lb per sq in., the resulting stresses would be 50% greater.

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[10] Engr., Structural Design, Blanchard & Maher–Frederic R. Harris, Inc.–Keller & Gannon, San Francisco, Calif.

In view of the extremely large uniform bending stress in this design, it would be logical to destroy the bond of the tensile steel by applying a coating, thus allowing the concrete to crack at very frequent intervals. The tensile bars would help to prevent disintegration of the concrete.

If the ends of the frame are restrained, there will be, in addition to the bending stress, an axial thrust due to an average temperature rise of 200° F, the unit compressive stress being $E \alpha t = 2{,}000{,}000 \times 0.0000079 \times 200 = 3{,}160$ lb per sq in.

The stresses developed in this example are so far beyond safe values that some basic changes in the design would be indicated, such as protecting the hot face of the concrete with firebrick and ventilation, in addition to providing expansion joints.

This paper affords good information on a type of design which is frequently not given careful consideration.

William A. Conwell,[11] M. ASCE.—In analyzing rigid frames subject to thermal action, the author has fully demonstrated the severity of the stresses involved. It is unfortunate, however, that the demonstration is largely a qualitative one rather than a quantitative one. With few exceptions, as stated in the paragraphs following Eq. 7, the stresses are "beyond the limit within which the equations of this paper apply." The stresses encountered are beyond the elastic range, but the methods of analysis are based on elastic behavior and real solution lies only in drastic revision of the type of structure.

The author's method of computing deformations due to thermal action, as demonstrated in Fig. 2 and the subsequent detailed applications to frames, is unique in that it sets aside the usual approximations made in small-angle geometry and treats the problem as a precise surveying layout. This action has forced the author out of the usually accepted method of slide-rule computation and has led him to the statement (in Section 5) that "Displacements must be computed by logarithms, whereas the slide rule offers abundant accuracy for all other numerical values." With a view to determining whether the comparatively large deflections involved in this study required such means of computation, the writer computed the deflections by 10-in. slide rule in the usual manner, as shown in Table 3. The comparison with the author's results (shown in Col. 8, Table 3) would seem to indicate that properly read slide-rule computations can be expected to give reasonably accurate results.

Although it is often desirable and sometimes necessary to know the deflections of a frame under various hypothetical conditions of cutting, support, etc., it is believed that the importance of such deflections has been emphasized unduly in this paper. The designer is first interested in bending moments, shear and axial forces, and then stresses. If these can be obtained without consideration of hypothetical deflection conditions he is usually happy to obtain them in that manner. Having obtained these elements of design he may later elect, or be required, to compute the actual deflections, and these furnish real information as to the ultimate action of the structure.

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[11] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

TABLE 3.—COMPARISON OF COMPUTATIONS FOR DEFLECTIONS, SLIDE RULE VERSUS LOGARITHMS, IN THE FRAME OF FIG. 4

Deflections	Point D in Relation to Point E		Point C from Point D		Point B from Point C		Total, Point B from Point E	
	Computations (1)	Inches or radians (2)	Computations (3)	Inches or radians (4)	Computations (5)	Inches or radians (6)	Slide rule (7)	Logarithms (8)
Δx	0.00316×80	0.2528	$0.000158 \times 120^2 \times \frac{1}{2}$ 0.01264×120	1.137 1.518 2.655	$0.00316 \times (-80)$	−0.2528	2.655	2.720
Δy	$-0.000158 \times 80^2 \times \frac{1}{2}$	−0.506	0.00316×120	0.379	$0.000158 \times 80^2 \times \frac{1}{2}$ 0.0316×80	0.506 2.528 3.034	2.907	2.883
θ	0.000158×80	0.01264	0.000158×120	0.01896	0.000158×80	0.01264	0.04424	0.0442

Section 2 furnishes an example of the emphasis on deflections. In the sixth paragraph of Section 2, prior to the application of the elastic energy equations, the author logically concludes, without reference to deflections, that "Any point of the frame is affected only by a moment that is of uniform magnitude all around." At this point the entire problem may be solved by considering the 1-in. element of the frame shown in Fig. 26. Under thermal action this element tends to assume a wedge shape with its inside fiber $0.0000079 \times 400 \times 1 = 0.00316$ in. longer than the unheated outside fiber. The bending moment induced by this tendency is sufficient to maintain the faces of the element parallel, and is equal to that necessary to turn the face of the element through an angle $d\phi = 0.00316 \times 1/20 = 0.000158$. Such bending moment is quickly determined by substituting known quantities in the expression $d\phi = \frac{M\,ds}{E\,I}$, whence $M = 0.000158 \times 2{,}000{,}000 \times 14{,}230 = 4{,}500{,}000$ in.-lb.

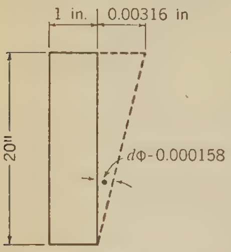

FIG. 26.—DEFORMATION OF THE FRAME ELEMENT

If there is justification for an elaborate computation of thermal deformations, such nicety should be extended to other phases of the calculations. Experience shows that the best results in the solution of statically indeterminate structures are obtained when the same basis is used for each part of the computations. For instance, the use of precise methods in certain steps of a calculation, and not in others, does not necessarily improve the ultimate result over a balanced calculation, on a lower level of accuracy, in which the same factors have been disregarded throughout. The author uses precise methods of calculation for thermal deformations; but he uses them in connection with the less accurate equations of elastic energy, the derivation of which disregards all quantities resulting from the multiplication of two or more differentials. It is this lack of balance which contributes to the discrepancy between the bending moments of 4,500,000 in.-lb and 4,600,000 in.-lb in Section 2 rather than "the slight approximations made in computing displacements," the reason assigned by the author. In support of this contention is the writer's calculation of the preceding paragraph which checks that made by the author on the basis of angular rotation. In further corroboration of this point of view is the fact that, if the writer's value for Δx (taken from Col. 7, Table 3, as $2 \times 2.655 = 5.310$ in.) is substituted with other known quantities in Eq. 7, a value of 4,500,000 in.-lb, consistent with other balanced calculations, is obtained for M. It is not claimed that these checks, which are the product of consistency, indicate a more accurate value of M; but they certainly enhance the confidence of the designer in his calculations. A more accurate value may be obtained only by conducting a completely balanced calculation on a higher plane, not by improving only one step in the process.

In Section 1 the author states: "Because of the greater ease, member lengths, as a rule, are taken along their inside surfaces." No difficulty is attached to the use of inside surfaces as a basis for calculating thermal deformations except the fact that the comparatively small effect of the corners is thus neglected. Some allowance for corners could have been made by taking member lengths along a plane midway between their outside and inside surfaces, although this in itself is a matter of small moment. Since the bending moment in the frame of Section 2 is dependent only on angular deformations of the frame and not on change in length of the members, the place of measurement of member lengths is immaterial. The same, however, may not be stated of the structures studied in Section 3 and Section 4.

An examination of Fig. 12 reveals that not only have the lengths of the members been taken along their inside faces, but the neutral planes have likewise been taken in that position. The neutral plane of member CD is actually some 90 in. from the line of action of T_{BE} if it is assumed to be about midway between the inner and outer surfaces of the member. This is not a bad assumption. The bending moment actually developed in member CD because $T_{BE} = 36{,}300$ lb would be $36{,}300 \times 90$ or 3,267,000 in.-lb. Although specifications would permit the use of a face-of-support bending moment of 2,900,000 in.-lb in the design of member BC, they would normally require the higher value for member CD; but, more deeply, T_{BE} has been determined on the basis of a bending moment of 2,900,000 in.-lb in member CD and its value would change were the moment other than this value. There is thus some question as to the conformance of the frame analyzed by the author and that which it was intended to investigate. The properties of, and the conditions affecting, the frame analyzed by the author are, upon study and inspection, as follows:

1. Tie member BE is treated substantially as required by the actual design data;
2. Members BC and DE have lengths of 80 in. each and have their neutral planes initially at a distance of 120 in. from each other;

3. Member CD has a length of 120 in. and has its neutral plane 80 in. from the line of action of member BE;
4. Members BC, CD, and DE are subject to the angular deformation resulting from a temperature differential of 400° between their outside and inside faces; and
5. Members BC, CD, and DE are subject to linear expansion resulting from a uniform temperature increase of a full 400° throughout their length.

There is some question as to whether this frame sufficiently conforms to the one being investigated. In the latter, members BC and DE have effective lengths of about 90 in. and member CD, a length of about 140 in. The linear expansion to which each of these members is subject is that resulting from an average increase in temperature of only 200° throughout its length.

A similar situation obtains as regards the frame of Section 4. A study reveals the following with respect to the frame analyzed by the author:

(1) Members AF and GM have lengths of 557 in. each and have their neutral planes initially at a distance of 340 in. from each other;
(2) Members AG and FM have lengths of 340 in. each and have their neutral planes initially at a distance of 557 in. from each other;
(3) Members BH and EL have lengths of 340 in. each and have their neutral planes initially at a distance of 98 in. from each other;
(4) Members AF, GM, AG, and FM are subject to the angular deformations resulting from a temperature differential of 400° between their inside and outside faces; and
(5) All members, not only BH and EL, are subject to linear expansion resulting from a uniform temperature increase of a full 400° throughout their length.

Again there is question as to whether this frame meets the design conditions outlined in Fig. 13; particularly with regard to the fact that spans BH and EL are the only members of the given frame that undergo the full 400° rise in temperature, whereas all other members are subjected to an average increase of only 200°. That this condition is a fact is further borne out by Fig. 15 in which member FM is shown as elongating an amount about equal to that of EL.

With a view to checking the statement of the foregoing paragraph (and, incidentally, demonstrating how well the method of moment distribution lends itself to these problems) a solution was developed for a frame having the foregoing characteristics and represented as the frame actually solved by the author in Section 4. This solution is shown in Fig. 27. The fixed-end moments for each member were determined by the method outlined by the writer in Fig. 26 and the fourth paragraph of this discussion. Moments were distributed in the usual manner. The values are seen to differ somewhat from those of the author but this difference is again the result of the unbalance of elaborate deformation calculations used in conjunction with the ordinary equations of elastic energy. As a means of further checking this point, the writer, in the manner previously demonstrated in Table 3, calculated Δx at point N, Δy at point F, and θ at point F, obtaining 3.19 in., 5.04 in., and 0.08375 radians, respectively. Substitution of these values in Eqs. 16, 18, and 20, and simultaneous solution, yielded the following data which check those obtained by moment distribution: M_D, 4,595,000 in.-lb; M_E, 4,645,000 in.-lb; and M_F, 2,836,000 in.-lb. By methods similar to those used in the treatment of sidesway, any difference in linear expansion between exterior and interior members of this frame may also be easily handled by moment distribution.

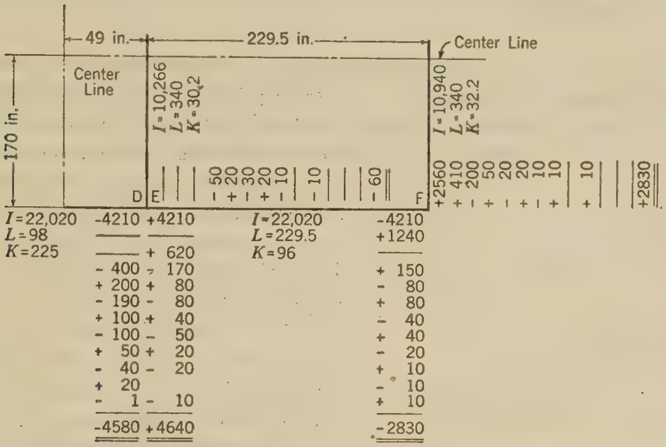
49 in.
229.5 in.
Center Line
Center Line
170 in.
I = 10,266
L = 340
K = 30.2
I = 10,940
L = 340
K = 32.2
D
E
F
- 50
+ 20
- 30
+ 20
- 10
- 10
- 60
+2560
+ 410
- 200
+ 50
- 20
+ 20
- 10
+ 10
+ 10
+2830
I = 22,020
L = 98
K = 225
-4210
- 400
+ 200
- 190
+ 100
- 100
+ 50
- 40
+ 20
- 1
-4580
+4210
+ 620
- 170
+ 80
- 80
+ 40
- 50
+ 20
- 20
- 10
+4640
I = 22,020
L = 229.5
K = 96
-4210
+1240
+ 150
- 80
+ 80
- 40
+ 40
- 20
+ 10
- 10
+ 10
-2830

FIG. 27.—SOLUTION, BY MOMENT DISTRIBUTION, OF THE FRAME ANALYZED IN SECTION 4

In the solution of reinforced-concrete structures subject to elastic deformation only, it is not important that the absolute moment of inertia of the section of each member be determined. It is usually sufficient to establish relative values with respect to some base, the most important item being that the same procedure be used in determining the moment of inertia of each member. An example of such procedure is the often-used assumption that each member is adequately reinforced, and, accordingly, that the moments of inertia are proportional to the cubes of the depths of the members.

When other than elastic deformations are involved, as in the present study, determination of the absolute moment of inertia for the purpose of computing deflections is essential. This paper uses the same moment of inertia for calculating deformations as for determining stresses, a procedure which is open to question.[12] After delineating the differences in action between reinforced concrete and homogeneous beams but arriving at the conclusion that it would be desirable to employ the same methods for calculating deflections, F. E. Turneaure, Hon. M. ASCE, and E. R. Maurer specify the following procedure to bring calculated deflections within the range of observed test deflections:[12]

> "For the reasons stated above, the deflection formulas for homogeneous beams will be used for reinforced-concrete beams, but modified in accordance with the following assumptions:
>
> "1. That the representative or mean section has a depth equal to the distance from the top of the beam to the center of the steel;

---

[12] "Principles of Reinforced Concrete Construction," by F. E. Turneaure and E. R. Maurer, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1919, Chapter VI.

> "2. That it sustains tension as well as compression, both following the linear law;
>
> "3. That the proper mean modulus of elasticity of the concrete equals the average or secant modulus up to the working compressive stress; and
>
> "4. That the allowance for steel in computing the moment of inertia of the mean section should be based on the amount of steel in the mid-sections."

Application of the foregoing assumptions to the frames of this paper would influence the results considerably. For example, the moment of inertia of the section of the frame of Section 2 would be based on the section shown in Fig. 28. Its value would be 26,200 in.^{4} and the corresponding bending moment would be 8,250,000 in.-lb as compared with 4,500,000 in.-lb previously computed. In physical terms, it will take much more than 4,500,000 in.-lb to restore the deformed frame of Fig. 4 and reduce Δx at point B to zero because this frame is much stiffer than the net moment of inertia of its section would indicate.

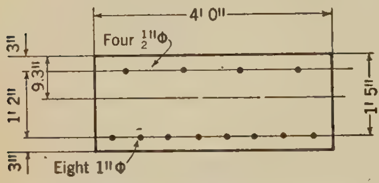

Fig. 28.—Effective Frame Section for Deflection Computations

Use of the gross area of the section for computation of the axial elastic deformations as employed in this paper is scarcely consistent with the use of net moment of inertia but would fit well into the use of the total section outlined by Messrs. Turneaure and Maurer. (The upper 17 in. of concrete in Fig. 28 is considered effective in both tension and compression, in the computation of deflections.)

As to notation, x and y are defined as horizontal and vertical ordinate distances, while actually all values of x and y lie in a horizontal plane. The symbol m is defined as the moment of a unit force, whereas throughout the paper it is used as the bending moment caused by the various auxiliary forces and moments. Letter symbols $\bar{x}$ and $\bar{y}$ are not defined, although investigation of their use usually discovers their meaning. It is suggested that some definite and compact form of notation be adopted for the undefined quantity "area" which has not the same value for each member of a frame although it seems to have, as it appears in the various equations and formulas, devoid of subscripts or other means of identification.

In paragraph 15 of Section 2 the statement that "when the members of a simple closed frame, of any shape, are subjected to a nonuniform temperature change through their 'depth' (h) they undergo no deformation" should be modified by the words "except axial elongation."

The author states Eq. 14 as four terms, only two of which contain the quantity T'. He then follows with the statement that "The auxiliary force T' might have been assumed as unity at the outset since it cancels * * *." It is not clear how T' can be canceled from an equation when it appears in only two of the four terms. Since the value of the auxiliary force is at the choice of the computer there is no reason why it should not have originally been taken as unity. In any event it should have been assumed as unity or disposed of by cancellation before it had an opportunity to appear in Eq. 14, which is satisfied only when $T' = 1$.

Another observation which may be made of Eq. 14 is the smallness of the terms expressing axial deformations as compared with that concerned with angular deformations. Although the author normally disregards axial deformations, he has made a particular point of including them in this problem by stating that "For a slender tie rod, stored-up elastic energy of both axial forces and moments must be included in the elastic energy equation * * *." With a view to ascertaining this necessity the writer solved the frame by moment distribution as shown in Fig. 29, disregarding all axial deformation including that of the tie rod. The results are not far from the author's results.

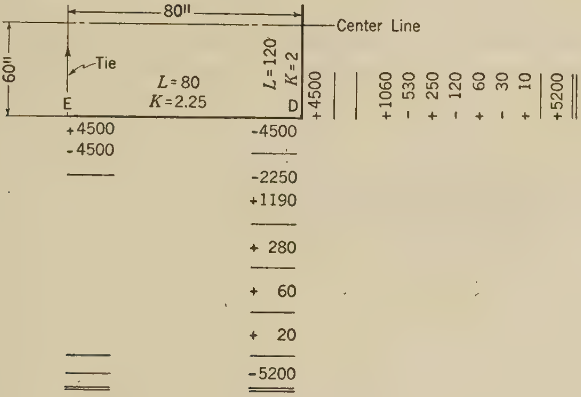

FIG. 29.—SOLUTION BY MOMENT DISTRIBUTION, NEGLECTING ALL AXIAL DEFORMATION; FRAME ANALYZED IN SECTION 3 ($I = 14{,}230$ THROUGHOUT; MOMENTS ARE IN INCH-KIPS)

A study of this paper would have been made more facile by inclusion of such basic data as the location of reinforcing rods, the positions of which had to be ascertained in each instance by working back from dimensions which the author substituted in formulas; by including a statement that the values of A_s and I given in Table 2 are per-foot height of wall (or that the walls are only 12 in. high); and by avoidance of generalities in the statement of problems such as that in the first paragraph of Section 4: "A cross section through the exterior walls is substantially as shown in Fig. 1(*b*)."

The computations of Section 4 would have been much simplified by making use of both axes of symmetry, taking the midpoint of member EL as a base, cutting the frame at point N and at the midpoint of member FM, and so confining all treatment to the upper right-hand quadrant.

Much of the "Summary" is devoted to a discussion of construction details which were not previously treated and which, accordingly, could well have been placed in a section of their own.

Carl C. H. Tommerup,[13] M. ASCE.—Many new ideas, twists, and short cuts have been advanced in the interesting discussions, for which the writer wishes to express his gratitude.

Some of the discussers criticize the fact that the computations in the paper are not sufficiently accurate and that various approximations are made. Be that as it may. A far greater number of engineers have criticized the paper on the ground that the computations were needlessly exact.

Enlightening as the discussions are, the writer is greatly disappointed that the discussers submit so little actual field data on similar problems. By actual data are meant observations, tests, and unusual design features. It seems reasonable to assume that many engineers have in the past been confronted with the same type of problem in one form or another.

The paper was submitted in the hope of extracting replies from engineers who not only had dealt with similar problems, but who had also collected experimental data on uneven heat transfer through structural members. A large amount of experimental work in this field is in order, and it is sincerely hoped that engineers who have already compiled such data will make them available to the profession. It is equally desirable that properly equipped institutions conduct tests along these lines in order to eliminate much of the present guesswork in structural thermal computations.

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MATRIX ANALYSIS OF CONTINUOUS BEAMS

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With Discussion by Messrs. I. Oesterblom, Harris Solman, Rufus Oldenburger, Horace A. Johnson, Lloyd T. Cheney, Paul W. Norton, A. Floris, Sven Olof Asplund, and Stanley U. Benscoter.

Synopsis

An introduction to the use of matrix algebra in the development and explanation of various methods of structural analysis is presented in this paper. The possibilities of making rapid design calculations by matrix methods are also illustrated. The paper is divided into four parts: In the first part an explanation is given of the elementary operations of addition, subtraction, multiplication, and division of matrices; in the second part a general formula is derived for the solution of any continuous beam (on unyielding supports) in matrix notation; in the third part the moment distribution process is developed by matrix algebra; and in the fourth part two successive correction methods of computation which may be regarded as variations of the method of moment distribution are illustrated and discussed.

PART I. ELEMENTARY OPERATIONS

Introduction

Matrix algebra may be regarded as a "shorthand" technique for representing a system of linear equations by a single equation and then solving that single equation. The rules of matrix algebra provide a computational procedure which is often more rapid than the numerical processes in common usage. Since all indeterminate structures are governed by systems of linear equations, the possibility of useful application of matrix methods by the structural engineer is suggested. The ability of the reader to understand, appreciate, and evaluate the matrix methods presented in this paper will be dependent, in a large measure, on his willingness to familiarize himself with the elementary operations. These mechanical procedures can be learned only by practice on

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Note.—Published in October, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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numerical examples, just as the mechanical procedure of moment distribution must be learned.

EVALUATION OF DETERMINANTS

It is essential that the reader recall the process of evaluating a determinant. The determinant of A is a square array of numbers which is written in the following form:

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{(1)}$$

In order that the reader may check his ability to evaluate determinants, the following examples are given of second-order and third-order determinants:

$$\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = 12 - 2 = 10 \quad \text{(2a)}$$

and

$$\begin{vmatrix} 5 & 1 & 2 \\ 0 & 3 & 1 \\ 1 & 2 & 4 \end{vmatrix} = 5 \times 10 - 1 \times (-1) + 2 \times (-3) = 45 \quad \text{(2b)}$$

ADDITION AND SUBTRACTION OF MATRICES

A matrix is also an array of numbers, but it may be either square or rectangular. A matrix will be indicated by a bracket and written in the following form:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{(3)}$$

The subscripts of the elements of the matrix or determinant are chosen so that the first subscript corresponds to the row containing the element and the second subscript corresponds to the column. To make use of matrices it is necessary to know the rules for addition, subtraction, multiplication, and division of matrices. These rules will be stated as simply as possible and should be learned by application to numerical cases.

Consider the matrix $[A]$ in Eq. 3 and the matrix $[B]$ given in the following form:

$$[B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad \text{(4)}$$

If the matrices $[A]$ and $[B]$ are added, a matrix $[C]$ is obtained and $[C]$ is also a 3×3 array of numbers:

$$[A] + [B] = [B] + [A] = [C] \quad \text{(5)}$$

in which

$$[C] = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \quad \text{(6)}$$

Each element of $[C]$ is obtained by adding the corresponding elements of $[A]$

and $[B]$. Thus, the elements of $[C]$ are computed as follows:

$$c_{11} = a_{11} + b_{11} \tag{7a}$$

$$c_{12} = a_{12} + b_{12} \tag{7b}$$

$$c_{13} = a_{13} + b_{13} \tag{7c}$$

etc. Matrices are subtracted in the same manner.

The following examples illustrate the processes of addition and subtraction; let

$$[M] = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}; \text{ and } [N] = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \tag{8}$$

then

$$[S] = [M] + [N] = \begin{bmatrix} 3 & 1 \\ 3 & 7 \end{bmatrix} \tag{9}$$

and

$$[T] = [M] - [N] = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \tag{10}$$

MULTIPLICATION OF MATRICES

If the matrices $[A]$ and $[B]$ are multiplied, a matrix $[D]$ is obtained which is also a 3 × 3 array of numbers:

$$[A][B] = [D] \tag{11}$$

in which

$$[D] = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \tag{12}$$

The elements of $[D]$ are computed from the elements of $[A]$ and $[B]$ by using the "row by column" rule. The rows of $[A]$ are multiplied by the columns of $[B]$. The row of $[A]$ to be used corresponds to the row of the element of $[D]$ being computed and the column of $[B]$ to be used corresponds to the column of the element of $[D]$. Thus, d_{11} is computed by multiplying the first row of $[A]$ by the first column of $[B]$. This multiplication may be indicated by

$$d_{11} = [a_{11}\ a_{12}\ a_{13}] \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} \tag{13}$$

These quantities are multiplied by adding the products of corresponding elements:

$$d_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} \tag{14}$$

A curious but important property of matrices is that they are noncommutative in multiplication. Thus, in general, the product $[A][B]$ does not equal the product $[B][A]$. This property of matrices deserves emphasis since the structural engineer is not accustomed to dealing with noncommutative numbers.

The following example illustrates the process of multiplication; let

$$[A] = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}; \text{ and } [B] = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \qquad (15)$$

then

$$[A][B] = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 13 & 6 \end{bmatrix} \qquad (16a)$$

and

$$[B][A] = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 7 & 4 \end{bmatrix} \qquad (16b)$$

Eqs. 16 show how the reversal of the order of multiplication gives a different answer. Two more examples will illustrate the multiplication of rectangular matrices:

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 1 \end{bmatrix}\begin{Bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 3 \end{Bmatrix} = \begin{bmatrix} 7 & 20 \\ 3 & 7 \end{bmatrix} \qquad (17)$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \end{bmatrix}\begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix} \qquad (18)$$

In Eq. 17 a 3 × 2 matrix is multiplied by a 2 × 3 matrix and the result is a square 2 × 2 matrix. In Eq. 18, a 3 × 2 matrix is multiplied by a column matrix, commonly called a column vector. The result is a column vector of two elements. The elements of a column vector are frequently called the components of the vector. The reader should attempt to multiply the matrices in Eqs. 17 and 18 in reverse order. In the first case the multiplication is possible whereas in the second case it is impossible. The first matrix in a multiplication must have the same number of columns as there are rows in the second matrix.

If a matrix is to be multiplied by a constant, each element of the matrix must be multiplied by the constant. Thus,

$$2\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 0 & 6 \end{bmatrix} \qquad (19)$$

DIVISION OF MATRICES

To explain the process of dividing matrices, it is necessary to introduce the following definitions which are applicable only to square matrices:

1. The "principal diagonal" of a matrix is the set of elements extending from the upper left-hand corner to the lower right-hand corner.
2. The "transpose" of $[A]$ is obtained by rotating $[A]$ about its principal diagonal. The rows and colums thus become interchanged. A transpose is indicated by a prime. Thus,

$$[A] = \begin{bmatrix} 2 & 7 \\ 3 & 1 \end{bmatrix}; \text{ and } [A]' = \begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix} \qquad (20)$$

3. A matrix is "symmetrical" if the matrix is equal to its transpose. Thus,

$$[A] = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}; \text{ and } [A] = [A]' \quad \text{(21)}$$

4. Corresponding to each element of a matrix (or determinant) there is a "minor" which is obtained by striking out the row and column of the matrix (or determinant) containing the element. The minor corresponding to an element of a matrix (or determinant) is a determinant that can be evaluated to obtain an ordinary number. Considering the determinant of Eq. 1 or the matrix of Eq. 3, the following minors illustrate the definition:

$$\text{Minor corresponding to } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \quad \text{(22a)}$$

and

$$\text{Minor corresponding to } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \quad \text{(22b)}$$

5. The "cofactor" of an element of a matrix (or determinant) is the signed minor corresponding to that element. A "signed minor" is a minor with the appropriate sign, either plus or minus, prefixed. The proper sign for the cofactor of the first element a_{11} is positive. The signs for the cofactors of the other elements alternate according to their position relative to a_{11}. Thus, the cofactor of a_{12} is negative, that of a_{13} is positive, that of a_{21} is negative, etc. It is convenient to use a corresponding capital letter to indicate a cofactor. Thus,

$$\text{Cofactor of } a_{11} = A_{11} = + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \quad \text{(23a)}$$

and

$$\text{Cofactor of } a_{12} = A_{12} = - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \quad \text{(23b)}$$

6. The "adjoint" of a matrix $[A]$ is the matrix obtained by replacing each element of $[A]$ by its cofactor and then transposing the matrix. Thus,

$$\text{Adjoint of } [A] = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \quad \text{(24)}$$

The following numerical example is an illustration:

$$[A] = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 1 \\ 2 & 4 & 3 \end{bmatrix} \quad \text{(25)}$$

$$\text{Adjoint of } [A] = \begin{bmatrix} 2 & -9 & 3 \\ -1 & 6 & -2 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{(26)}$$

7. The "unit matrix" is a matrix in which the elements on the principal diagonal are 1's whereas the remaining elements are 0's. It is indicated by

$[I]$; thus,

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (27)$$

This matrix is also called the identity matrix. It plays a part in the algebraic manipulation of matrices which is similar to the part played by unity in the algebra of ordinary numbers. With ordinary numbers:

$$1 \times a = a \times 1 = a \qquad (28)$$

With matrices:

$$[I][A] = [A][I] = [A] \qquad (29)$$

The verity of Eq. 29 can be easily demonstrated by applying the "row by column" rule to a numerical example.

8. The "reciprocal" (or "inverse") of $[A]$ is a matrix which, when multiplied by $[A]$, produces the unit matrix. Thus,

$$[A][A]^{-1} = [A]^{-1}[A] = [I] \qquad (30)$$

The following example can be checked by the "row by column" rule for multiplication:

$$[A] = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}; \text{ and } [A]^{-1} = \begin{bmatrix} 4/3 & -1/3 \\ -5/3 & 2/3 \end{bmatrix} \qquad (31)$$

Multiplying $[A]$ by $[A]^{-1}$ gives:

$$[A][A]^{-1} = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 4/3 & -1/3 \\ -5/3 & 2/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (32)$$

The foregoing definition of a reciprocal gives no clue as to how it can be obtained. The formula for computing the reciprocal of a matrix is as follows:

$$[A]^{-1} = \frac{\text{Adjoint of } [A]}{|A|} \qquad (33)$$

The reciprocal is equal to the adjoint divided by the determinant. Considering the example of Eq. 31,

$$\text{Adjoint of } [A] = \begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix} \qquad (34)$$

and

$$|A| = 2 \times 4 - 1 \times 5 = 3 \qquad (35)$$

Substituting into Eq. 33, the reciprocal is obtained as given in Eq. 31.

The process of division can now be explained. If $[B]$ is to be divided by $[A]$, the result may be obtained by multiplying $[B]$ by $[A]^{-1}$. This multiplication can be performed in two ways: $[B]$ can be premultiplied by $[A]^{-1}$ as indicated by $[A]^{-1}[B]$, or it can be postmultiplied by $[A]^{-1}$ to obtain $[B][A]^{-1}$. In general, these two methods of multiplication will give two different results. Consequently, it should be apparent that a representation of division by writing a fraction, such as $\frac{[B]}{[A]}$, would be ambiguous and hence not permissible. Whenever a matrix equation is to be divided by $[A]$, either every term in the equation must be premultiplied by $[A]^{-1}$, or every term in the equation must be postmultiplied by $[A]^{-1}$.

SOLUTION OF SIMULTANEOUS EQUATIONS

The usefulness of matrix algebra in solving simultaneous equations can now be explained. Consider the following system of three equations in three unknowns:

$$3x_1 + x_2 - 2x_3 = 2 \qquad (36a)$$

$$x_1 + 2x_2 + 3x_3 = 1 \qquad (36b)$$

$$2x_1 - x_2 + 4x_3 = 3 \qquad (36c)$$

Eqs. 36 can be expressed, by using matrices, in the following manner:

$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \qquad (37)$$

If the matrices are multiplied, and corresponding elements on each side of the matrix equation are then equated, the original Eqs. 36 will be reproduced.

Introduce the following definition:

$$[A] = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix} \qquad (38a)$$

with

$$[x] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \text{ and } [k] = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \qquad (38b)$$

Eq. 37 becomes

$$[A][x] = [k] \qquad (39)$$

The original system of linear algebraic equations has now been reduced to a single linear matrix equation. Eq. 39 is solved by multiplying through by $[A]^{-1}$. Thus,

$$[A]^{-1}[A][x] = [A]^{-1}[k] \qquad (40a)$$

or

$$[I][x] = [A]^{-1}[k] \qquad (40b)$$

or

$$[x] = [A]^{-1}[k] \qquad (40c)$$

Eq. 40*c* gives the values of the components of the column vector $[x]$ by a simple multiplication of matrices, provided $[A]^{-1}$ can be determined. Following the rules and definitions previously given, $[A]^{-1}$ can be computed as

$$[A]^{-1} = \frac{1}{45}\begin{bmatrix} 11 & -2 & 7 \\ 2 & 16 & -11 \\ -5 & 5 & 5 \end{bmatrix} \qquad (41)$$

Eq. 40c becomes

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{45} \begin{bmatrix} 11 & -2 & 7 \\ 2 & 16 & -11 \\ -5 & 5 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \qquad (42)$$

By performing the multiplication indicated on the right, the following values of x_1, x_2, and x_3 can be obtained:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41/45 \\ -13/45 \\ -2/9 \end{bmatrix} \qquad (43a)$$

or

$$x_1 = \frac{41}{45}\,;\ x_2 = \frac{-13}{45}\,;\ \text{and } x_3 = \frac{2}{9} \qquad (43b)$$

Since continuous structures are governed by linear algebraic equations, it should be apparent that matrix algebra may provide a useful technique for dealing with certain types of structures.

PART II. GENERAL MATRIX FORMULAS FOR CONTINUOUS BEAMS

INTRODUCTION

An analysis of a continuous beam will be developed in matrix notation on the assumption that the supports do not deflect. If the reactions of a continuous beam can be determined, the values of bending moments and shears at various sections can be calculated by statics; or, if the bending moment over each support can be determined, the moments and shears at various sections can likewise be determined. Many methods of analysis of continuous beams have been developed. In each method certain fundamental quantities appear which must be determined for each span individually. These fundamental quantities are dependent on the physical properties of the beam and on the loads in the spans. Consequently, it is necessary first to consider an individual span. The algebraic relationships between the fundamental quantities that appear in various methods of continuous beam analysis have been shown by Fang-Yin Tsai.[2]

COLUMN ANALOGY

A development of the properties of a single span may begin by considering either a simply-supported beam or a fixed-end beam. The latter case will be used. Any complete group of characteristics of a fixed-end beam can be expressed in terms of five fundamental properties. Three of these properties are functions of the physical shape of the beam and two of them are dependent on both the beam shape and the loading. The column analogy method,[3] which was introduced by Hardy Cross, M. ASCE, offers a very convenient notation for stating these beam properties in a general case. Fig. 1 shows a nonprismatic beam with its load, the corresponding statical moment diagram, the analogous column section, and the indeterminate moment diagram. The diagrams of m_s and m_i have thicknesses normal to the paper equal to the thickness of the analogous column at a corresponding point.

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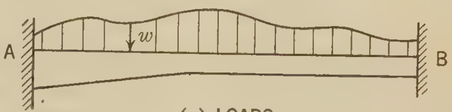
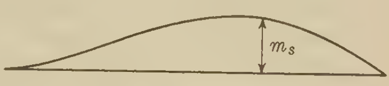
[2] *Transactions*, ASCE, Vol. 102, 1937, p. 44.

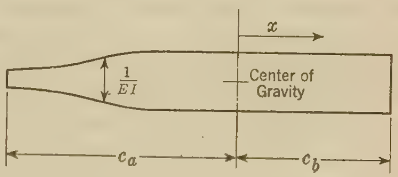
[3] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1932.



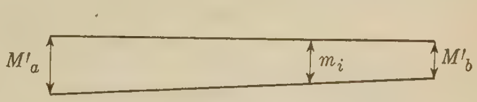
(a) LOADS



(b) STATICAL MOMENTS



(c) ANALOGOUS COLUMN



(d) INDETERMINATE MOMENTS

FIG. 1.—COLUMN ANALOGY FOR A SINGLE-SPAN BEAM

The center of gravity of the analogous column section is indicated in Fig. 1(*c*). This point is considered as the origin of coordinates with x measured positive to the right. The distances to the extreme fibers are shown as c_a and c_b, the former being a negative number. The five fundamental properties are: (*a*) The load P_o on the analogous column, (*b*) its first moment about some reference axis, (*c*) the area of the analogous column A_o, (*d*) its first moment, and (*e*) its second moment.

Corresponding to the fiber distances c_a and c_b, two section moduli S_a and S_b can be defined by

$$S_a = \frac{I_o}{c_a}\text{, and } S_b = \frac{I_o}{c_b}. \quad (44)$$

in which I_o is the moment of inertia of the column section. The modulus S_a is a negative number since I_o is always positive.

The end moments may be expressed in matrix notation as follows:

$$\begin{bmatrix} M'_a \\ M'_b \end{bmatrix} = \begin{bmatrix} A^{-1}_o & S^{-1}_a \\ A^{-1}_o & S^{-1}_b \end{bmatrix} \begin{bmatrix} P_o \\ M_o \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

in which M_o is the moment acting on the analogous column, or the moment of P_o about the origin. The column analogy method uses a bending-moment sign convention which defines positive moment as that producing compressive stress in the top fiber. The prime on a moment (M') indicates the fixed-end value of the moment.

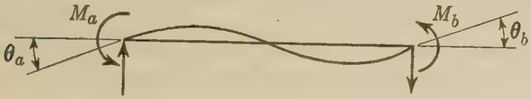


FIG. 2.—ELASTIC CURVE OF AN UNLOADED SPAN

END ROTATION EFFECT

Next it is necessary to consider the relationship between end moments and end slopes for a single span. Fig. 2 shows an unloaded span with applied end moments. The end moments and end slopes are linearly related. Each end moment may be expressed as a linear function of the two end slopes, or each end slope may be expressed as a linear function of the two end moments. The former relationship may be written in matrix notation as

$$\begin{bmatrix} M_a \\ M_b \end{bmatrix} = \begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix} \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} \qquad (46)$$

in which K_a and K_b are the standard stiffness values for the member and the quantity R is a secondary stiffness value which gives the change in moment at one end of a member caused by a unit rotation at the opposite end. It is equal to the product of the carry-over factor multiplied by the stiffness, these values being taken at either end. Thus,

$$R = r_a K_a = r_b K_b \qquad (47)$$

in which r_a and r_b are the carry-over factors. A statical moment sign convention is used in expressing the moment-slope relationship. (The stiffness values and carry-over factors are all positive numbers.) Although a bending moment sign convention may be used in continuous beam analysis, the statical moment convention becomes more convenient when the method of analysis is extended to frame works or trusses. Moment and slope are both chosen to be positive in the counterclockwise direction at either end of the member.

To express the slopes θ_a and θ_b as linear functions of the end moments, multiply Eq. 46 by the reciprocal of the coefficient matrix; thus,

$$\begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix}^{-1} \begin{bmatrix} M_a \\ M_b \end{bmatrix} = \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} \qquad (48)$$

The reciprocal matrix may be evaluated to give

$$\begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix}^{-1} = \frac{1}{K_a K_b - R^2} \begin{bmatrix} K_b & -R \\ -R & K_a \end{bmatrix} \qquad (49)$$

That is,

$$\begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix}^{-1} = \frac{1}{D K_a K_b} \begin{bmatrix} K_b & -R \\ -R & K_a \end{bmatrix} \qquad (50)$$

in which D is the end rotation constant [4] (commonly indicated by $1/C$) for the beam corresponding to simply-supported end conditions and is given by the formula:

$$D = 1 - r_a r_b \qquad (51)$$

The product of the end rotation constant D and the standard stiffness value of a beam gives a modified stiffness value which is the moment required to produce a unit rotation at one end when the other end of the beam is simply supported. When the scalar coefficient of the matrix of Eq. 50 is taken inside the bracket and multiplied by each term, it becomes convenient to introduce new letters to represent the reciprocals of modified stiffness values; thus, let

$$\phi_a = \frac{1}{D K_a}\,;\ \phi_b = \frac{1}{D K_b}\,;\ \text{and } \gamma = \frac{R}{D K_a K_b} \qquad (52)$$

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[4] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 119.

Eq. 48 now becomes

$$\begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} = \begin{bmatrix} \phi_a & -\gamma \\ -\gamma & \phi_b \end{bmatrix} \begin{bmatrix} M_a \\ M_b \end{bmatrix} \quad \text{(53)}$$

By Eq. 46, each end moment is expressed as a linear function of the two end slopes. By Eq. 53, each end slope is expressed as a linear function of the two end moments.

The numerical values of the stiffness quantities K_a, K_b, and R may be computed by using the column analogy concepts. The value of K_a is the indeterminant moment that would be computed at point A if the analogous column is loaded with a unit concentrated load at point A. The value of R is the indeterminant moment computed at point B with the unit load at point A, and K_b is determined by placing the unit load at point B. Thus, the stiffness values are given by

$$\begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix} = \begin{bmatrix} A^{-1}{}_o & S^{-1}{}_a \\ A^{-1}{}_o & S^{-1}{}_b \end{bmatrix} \begin{bmatrix} 1 & 1 \\ c_a & c_b \end{bmatrix} \quad \text{(54)}$$

Since the quantities on the right side of Eq. 54 are in accord with a bending moment sign convention, the quantity R as computed will have a negative sign. This sign should be ignored in a statical moment sign convention.

Slope-Deflection Equations for Single Span

The next step in the analysis requires consideration of a loaded span as shown in Fig. 3. The end moments may be expressed as linear functions of the end slopes as before, but a constant term must be added:

$$\begin{bmatrix} M_a \\ M_b \end{bmatrix} = \begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix} \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} + \begin{bmatrix} M'_a \\ M'_b \end{bmatrix} \quad \text{(55)}$$

The elements M'_a and M'_b of the column vector that has been added to Eq. 46 to obtain Eq. 55 represent the values of M_a and M_b when both θ_a and θ_b are zero. Thus, M'_a and M'_b are the fixed-end moments for the given loading and are calculated as shown by Eq. 45. (The values of M'_a and M'_b in Eq. 45 have their signs in accord with a bending moment sign convention and must be changed to a statical moment convention.) Eq. 55 is a special case of the general slope-deflection equations which include an additional term for the effect of relative end, or joint, translation.

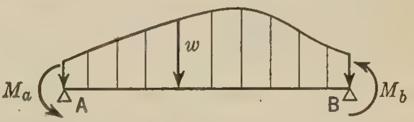

Fig. 3.—Loaded Span

It is also possible to use Eq. 53 rather than Eq. 46 to obtain an expression for end slopes in terms of moments for a loaded beam. As before, a constant term is added to obtain

$$\begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} = \begin{bmatrix} \phi_a & -\gamma \\ -\gamma & \phi_b \end{bmatrix} \begin{bmatrix} M_a \\ M_b \end{bmatrix} + \begin{bmatrix} \theta'_a \\ \theta'_b \end{bmatrix} \quad \text{(56)}$$

The elements θ'_a and θ'_b of the column vector that has been added to Eq. 53 to obtain Eq. 56 represent the values of θ_a and θ_b when both M_a and M_b are zero. Thus, θ'_a and θ'_b are the simple beam end slopes for the given loading. To evaluate θ'_a and θ'_b, the fixed-end beam is first considered with end moments M'_a and M'_b. If it is now assumed that a moment $-M'_a$ is applied at end A and a moment $-M'_b$ is applied at end B, then the fixed-end beam becomes a simply-supported beam with zero end moments. The angles through which each end rotates, such as $-M'_a$ and $-M'_b$ are applied, will be the angles θ'_a and θ'_b. Therefore, the angles θ'_a and θ'_b are linear functions of $-M'_a$ and $-M'_b$. This relationship is expressed by Eq. 53 as follows:

$$\begin{bmatrix} \theta'_a \\ \theta'_b \end{bmatrix} = - \begin{bmatrix} \phi_a & -\gamma \\ -\gamma & \phi_b \end{bmatrix} \begin{bmatrix} M'_a \\ M'_b \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

This linear relationship between simple beam end slopes and fixed-end moments, for a given loading, has some usefulness in practical design work. It provides a convenient procedure for computing end slopes that are not well known to the designer from stiffness values and fixed-end moments which are well known, either by formula for prismatic beams or by graphical representations for nonprismatic beams.

Slope-Deflection Equations for Entire Beam

A four-span continuous beam is now to be considered as shown in Fig. 4. Beneath each span is shown a 2 × 2 matrix of stiffness values for the span corresponding to the coefficient matrix of Eq. 46. Since the ends of the beam are fixed, the rotation of joints 4 and 5 is known to be zero. Thus, there are only three effective joints. Only one of the two slope-deflection equations for the end spans and one of the stiffness values for the end spans are needed in determining the bending moments at the effective joints. As a secondary part of the solution, the bending moments at joints 4 and 5 can be computed by proper carry-over calculations according to the process of moment distribution. If the ends of the beam were simply supported rather than fixed, the standard stiffness values K_{15} and K_{34} would be replaced by modified stiffness values using the appropriate end rotation constant. The joints have been numbered by beginning with the first effective joint and continuing consecutively.

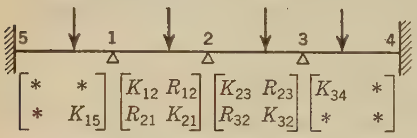
5
1
2
3
4
$\begin{bmatrix} * & * \\ * & K_{15} \end{bmatrix}$
$\begin{bmatrix} K_{12} & R_{12} \\ R_{21} & K_{21} \end{bmatrix}$
$\begin{bmatrix} K_{23} & R_{23} \\ R_{32} & K_{32} \end{bmatrix}$
$\begin{bmatrix} K_{34} & * \\ * & * \end{bmatrix}$

Fig 4.—Four-Span Beam, with Stiffness Values

It should be noted that, in dealing with the continuous beam, the necessity arises for using double subscripts on the stiffness values. The first subscript corresponds to the joint, or support, and the second subscript indicates the member. The 2 × 2 matrices shown in Fig. 4 are symmetrical; and, hence,

$$R_{23} = R_{32}; \text{ and } R_{34} = R_{43} \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Bending moments will also require double subscripts whereas joint rotations will require only a single subscript.

It is now necessary to define several column vectors. Consider first that the fixed-end moments at the left end of each member form a column vector

$[M'_a]$ and the fixed-end moments at the right end of each member form a column vector $[M'_b]$. Thus,

$$[M'_a] = \begin{bmatrix} M'_{12} \\ M'_{23} \\ M'_{34} \end{bmatrix}; \text{ and } [M'_b] = \begin{bmatrix} M'_{15} \\ M'_{21} \\ M'_{32} \end{bmatrix} \quad \text{(59)}$$

Define similar vectors of the actual moments by

$$[M_a] = \begin{bmatrix} M_{12} \\ M_{23} \\ M_{34} \end{bmatrix}; \text{ and } [M_b] = \begin{bmatrix} M_{15} \\ M_{21} \\ M_{32} \end{bmatrix} \quad \text{(60)}$$

Also define a column vector of joint rotations by

$$[\theta] = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad \text{(61)}$$

Only the effective joints are included. The vectors $[M_a]$ and $[M_b]$ can be shown to be linear matrix functions of the vector $[\theta]$. Hence,

$$[M_a] = [K_a][\theta] + [M'_a] \quad \text{(62}a\text{)}$$

and

$$[M_b] = [K_b][\theta] + [M'_b] \quad \text{(62}b\text{)}$$

The similarity in form between Eqs. 62 and Eq. 55 for a single span should be noted. The vector $[M'_a]$ is the value of the vector $[M_a]$ when all elements, or components, of the vector $[\theta]$ are zero. The coefficient matrices $[K_a]$ and $[K_b]$ have been introduced and must be defined. Each is a 3 × 3 matrix composed of stiffness values properly chosen from Fig. 4. The matrix $[K_a]$ is formed in the following manner: Take the first row of elements of each 2 × 2 matrix in Fig. 4 and write them one below another—each being staggered one place to the right. Fill the remaining places of the matrix thus formed with zeros. The result is

$$[K_a] = \begin{bmatrix} K_{12} & R_{12} & 0 \\ 0 & K_{23} & R_{23} \\ 0 & 0 & K_{34} \end{bmatrix} \quad \text{(63}a\text{)}$$

Using the second row of elements of each 2 × 2 matrix of Fig. 4, the vector $[K_b]$ is formed as

$$[K_b] = \begin{bmatrix} K_{15} & 0 & 0 \\ R_{21} & K_{21} & 0 \\ 0 & R_{32} & K_{32} \end{bmatrix} \quad \text{(63}b\text{)}$$

To demonstrate that Eqs. 62 are true they will be written out in full; thus,

$$\begin{bmatrix} M_{12} \\ M_{23} \\ M_{34} \end{bmatrix} = \begin{bmatrix} K_{12} & R_{12} & 0 \\ 0 & K_{23} & R_{23} \\ 0 & 0 & K_{34} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} M'_{12} \\ M'_{23} \\ M'_{34} \end{bmatrix} \quad \text{(64}a\text{)}$$

and

$$\begin{bmatrix} M_{15} \\ M_{21} \\ M_{32} \end{bmatrix} = \begin{bmatrix} K_{15} & 0 & 0 \\ R_{21} & K_{21} & 0 \\ 0 & R_{32} & K_{32} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} M'_{15} \\ M'_{21} \\ M'_{32} \end{bmatrix} \quad \text{(64}b\text{)}$$

Each of these two matrix equations corresponds to three algebraic equations. If the multiplication indicated on the right side of the equations is done, a column vector of three components will exist on each side of each equation. Equating corresponding elements of these column vectors will give the two families of three algebraic equations each. Consider the first algebraic equation of the first family as follows:

$$M_{12} = K_{12}\,\theta_1 + R_{12}\,\theta_2 + M'_{12} \quad \text{(65)}$$

Eq. 65 is one of the slope-deflection equations for the second span and corresponds exactly to the first of the two algebraic equations represented by Eq. 55. Similarly all the six algebraic equations corresponding to Eqs. 64 can be shown to be slope-deflection equations for the various spans. Hence, Eqs. 62 are valid.

Eqs. 62 represent a complete statement in matrix notation of the slope-deflection equations for any continuous beam with supports that are fixed against translation. If there are ten effective joints, the column vectors will have ten components and the coefficient matrices will be 10 × 10 square matrices.

Any numerical values for the components of $[\theta]$—that is, the joint rotations—may be assumed and substituted into Eqs. 62. The resulting moments will correspond to a continuous elastic curve for the beam. The moment on the left side of each joint is not equal in magnitude to the moment on the right side of the joint. From a physical standpoint this state of deflections and moments could exist only if artificial moments of sufficient magnitude to maintain the joint rotations at the assumed values were introduced at the joints. These artificial moments might be introduced by considering that torques are applied to axles which are fixed to the beam at the joints and are normal to the plane of the page.

Actually, the foregoing solution is not correct for moments resulting from the applied loads. There is one, and only one, correct set of values of the components of $[\theta]$. With this correct set of values the moment on the left side of each support is equal to the moment on the right side of the same support, but of opposite sign. In other words, each joint must be in equilibrium so that no artificial torques are required. Thus, the correct solution must satisfy both the continuity of the elastic curve and the equilibrium of the joints. Eqs. 62 guarantee continuity but do not guarantee equilibrium. Hence, the equilibrium conditions must also be imposed on the solution.

If each joint is in equilibrium, the elements of the vector $[M_a]$ must be equal and opposite in sign to the elements of $[M_b]$. Thus, the equilibrium condition is expressed by

$$[M_a] + [M_b] = 0 \quad \text{(66)}$$

Eqs. 62 and 66 define the solution completely. The vector $[\theta]$ can be eliminated from these three matrix equations to give convenient formulas for design practice.

THEOREM OF THREE SLOPES

If Eqs. 62 are substituted into Eq. 66,

$$[K][\theta] + [u] = 0 \quad \text{(67)}$$

in which

$$[K] = [K_a] + [K_b] = \begin{bmatrix} \Sigma_1 K & R_{12} & 0 \\ R_{21} & \Sigma_2 K & R_{23} \\ 0 & R_{32} & \Sigma_3 K \end{bmatrix} \quad \text{(68)}$$

and

$$[u] = [M'_a] + [M'_b] = \begin{bmatrix} M'_{12} + M'_{15} \\ M'_{23} + M'_{21} \\ M'_{34} + M'_{32} \end{bmatrix} \quad \text{(69)}$$

The elements of the stiffness matrix $[K]$ should be considered. The first element of the principal diagonal is the sum of stiffness values at joint 1. The second element of the principal diagonal is the sum of stiffness values at joint 2, etc. The matrix contains one diagonal row of elements above and one below the principal diagonal, and is symmetrical. These properties are true for the stiffness matrix of all continuous beams regardless of the number of effective joints. The diagonal row above the principal diagonal is called the "superdiagonal" and the diagonal row below the principal diagonal is called the "subdiagonal." A matrix that has a principal diagonal, subdiagonal, and superdiagonal, with all other elements equal to zero, is called a "continuant matrix." [5] (The mathematical reason for this name has been stated by A. C. Aitken.) The stiffness matrix $[K]$ is a continuant matrix. This will only be true if the effective joints are numbered in consecutive order. The subdiagonal and superdiagonal are composed of secondary stiffness values, or R-values. In any given row the diagonal element corresponds to a particular joint. The R-values in this row correspond to the members adjacent to the joint.

The vector $[u]$ is composed of components that may be recognized as the "unbalanced" moments of the moment distribution process of analysis. The unbalanced moment at a joint is the sum of the fixed-end moments at the joint. The vector $[u]$ may be written

$$[u] = \begin{bmatrix} \Sigma_1 M' \\ \Sigma_2 M' \\ \Sigma_3 M' \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \text{(70)}$$

For example, from Eq. 69, $u_2 = M'_{23} + M'_{21}$.

Each of the algebraic equations represented by the matrix Eq. 67 contains three slopes, or joint rotations, irrespective of the number of spans of the continuous beam. Hence, Eq. 67 may be called the theorem of three slopes (or "angles," which is a term used by W. L. Schwalbe[6]). This equation can be solved for $[\theta]$ by multiplying the equation by $[K]^{-1}$:

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[5] "Determinants and Matrices," by A. C. Aitken, Interscience Publishers, Inc., New York, N. Y., 1939, p. 126.

[6] "Simultaneous Equations in Mechanics Solved by Iteration," by W. L. Schwalbe, *Transactions* ASCE, Vol. 102, 1937, p. 941.

$$[\theta] = -[K]^{-1}[u] \qquad (71)$$

Eq. 71 states that the joint rotations created by a given loading are linear functions of the unbalanced moments. Substituting Eq. 71 in Eqs. 62 gives

$$[M_a] = [M'_a] - [K_a][K]^{-1}[u] \qquad (72a)$$

and

$$[M_b] = [M'_b] - [K_b][K]^{-1}[u] \qquad (72b)$$

The second term on the right-hand side of Eqs. 72 may be considered a correction or modification to be added, with its negative sign, to the fixed-end moment. The final moment at a joint may be considered to be composed of a preliminary estimate, or fundamental value—the fixed-end moment plus a correction that is a linear function of the unbalanced moments of all the joints.

It is convenient to introduce the following matrices:

$$[C_a] = -[K_a][K]^{-1} \qquad (73a)$$

and

$$[C_b] = -[K_b][K]^{-1} \qquad (73b)$$

Eqs. 72 become

$$[M_a] = [M'_a] + [C_a][u] \qquad (74a)$$

and

$$[M_b] = [M'_b] + [C_b][u] \qquad (74b)$$

The numerical values of joint moments may be computed by using either Eq. 74*a* or Eq. 74*b*. However, it is advisable to use both and to compare the values as this procedure checks the computation of the elements of $[C_a]$ and $[C_b]$. Eqs. 74 give a complete statement in matrix notation of the solution of any continuous beam of any number of spans. The column vectors $[M'_a]$, $[M'_b]$, and $[u]$ are dependent on the loading and physical characteristics of the individual spans. The coefficient matrices $[C_a]$ and $[C_b]$ are dependent only on the physical properties of the structure. They can be computed for a given structure before the loading is known and need to be computed only once. The information necessary for this computation is shown in Fig. 4.

The matrix $[C_a]$ is computed in two steps: First, $[K]^{-1}$ is computed; and then this value is premultiplied by $[K_a]$. A matrix multiplication can be performed in a few minutes, particularly on modern calculators which retain and add products of numbers. Large matrices can be multiplied faster by using a calculating machine than by using a slide rule since the result of each individual multiplication does not have to be written down but is retained in the machine. The principal labor involved is in the computation of $[K]^{-1}$. The time required for computating the reciprocal of a matrix increases very rapidly with the size of the matrix. For example, the calculation of the reciprocal of a 6 × 6 matrix would require the evaluation of thirty-six 5 × 5 determinants—a very lengthy process.

It is convenient, for discussion, to introduce the correction vectors $[a]$ and $[b]$ by the definitions:

$$[a] = [C_a][u]; \text{ and } [b] = [C_b][u] \qquad (75)$$

Eqs. 74 become

$$[M_a] = [M'_a] + [a] \qquad (76a)$$

and

$$[M_b] = [M'_b] + [b] \qquad (76b)$$

The design computations for a given loading can be written in a manner similar to moment distribution after the elements of the coefficient matrices $[C_a]$ and $[C_b]$ have been computed. The fixed-end moments may be written, as usual, and directly below them the components of the correction vectors $[a]$ and $[b]$ are written. Direct addition at each joint gives the final answer. Because of the length of time required to compute the reciprocal of the stiffness matrix $[K]$, and thus to obtain $[C_a]$ and $[C_b]$, a given beam can be analyzed faster for one loading condition by moment distribution than by using matrices. The advantage of using matrices would become apparent if a beam were to be analyzed for many loading conditions, since the reciprocal of $[K]$ would need to be computed only once—suggesting that matrix methods should prove to be useful in developing influence lines.

THEOREM OF THREE MOMENTS

The theorem of three moments is better known to structural engineers than the theorem of three slopes. In developing the theorem of three moments, it is necessary to begin with the moment-slope relationship of Eq. 56. The end slopes are considered linear functions of the end moments. Proceeding in a manner similar to the development of the theorem of three slopes, a complete set of moment-slope relationships for the entire continuous beam can be written. This set of equations will guarantee equilibrium rather than continuity. The condition of continuity at each joint must be introduced and the theorem of three moments is then obtained. It expresses the joint moments as functions of the simple beam end slopes and can be solved directly for the moments. Eq. 57 has been given for obtaining simple beam end slopes from fixed-end moments.

To develop the theorem of three moments, it is necessary to shift to a bending moment sign convention. It is also necessary to differentiate between the slopes (rather than the moments) on the right-hand side and left-hand side of a support. Thus, slopes must have double subscripts whereas joint moments need have only single subscripts. Since these changes are very confusing, the development has been omitted, the outline merely being given to indicate a possibility for those who may wish to pursue the study further.

NUMERICAL EXAMPLE

In Fig. 5, a continuous beam is shown with parabolic haunches and simple supports. A row of standard stiffness values for each span is shown as determined from graphs published by the Portland Cement Association.[7] Since these graphs give the carry-over factors, the secondary stiffness values must be computed. Just below the stiffness values are the stiffness matrices for each span, showing a modified stiffness for the end spans caused by the simple supports at the ends. The left-end and right-end stiffness matrices $[K_a]$ and $[K_b]$ are

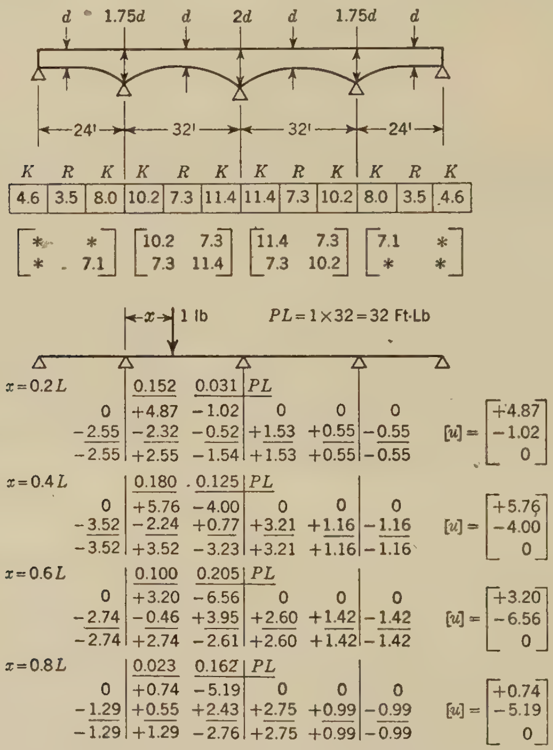
d 1.75d d 2d d 1.75d d
24' 32' 32' 24'
K R K K R K K R K K R K
4.6 3.5 8.0 10.2 7.3 11.4 11.4 7.3 10.2 8.0 3.5 4.6
* * / * 7.1
10.2 7.3 / 7.3 11.4
11.4 7.3 / 7.3 10.2
7.1 * / * *
x 1 lb
PL = 1×32 = 32 Ft-Lb
x = 0.2L
0.152 0.031 PL
0 +4.87 −1.02 0 0 0
−2.55 −2.32 −0.52 +1.53 +0.55 −0.55
−2.55 +2.55 −1.54 +1.53 +0.55 −0.55
[u] = +4.87 −1.02 0
x = 0.4L
0.180 0.125 PL
0 +5.76 −4.00 0 0 0
−3.52 −2.24 +0.77 +3.21 +1.16 −1.16
−3.52 +3.52 −3.23 +3.21 +1.16 −1.16
[u] = +5.76 −4.00 0
x = 0.6L
0.100 0.205 PL
0 +3.20 −6.56 0 0 0
−2.74 −0.46 +3.95 +2.60 +1.42 −1.42
−2.74 +2.74 −2.61 +2.60 +1.42 −1.42
[u] = +3.20 −6.56 0
x = 0.8L
0.023 0.162 PL
0 +0.74 −5.19 0 0 0
−1.29 +0.55 +2.43 +2.75 +0.99 −0.99
−1.29 +1.29 −2.76 +2.75 +0.99 −0.99
[u] = +0.74 −5.19 0

FIG. 5.—ANALYSIS OF A FOUR-SPAN CONTINUOUS BEAM

$$[K_a] = \begin{bmatrix} 10.2 & 7.3 & 0 \\ 0 & 11.4 & 7.3 \\ 0 & 0 & 7.1 \end{bmatrix} \qquad (77a)$$

and

$$[K_b] = \begin{bmatrix} 7.1 & 0 & 0 \\ 7.3 & 11.4 & 0 \\ 0 & 7.3 & 10.2 \end{bmatrix} \qquad (77b)$$

By adding Eqs. 77*a* and 77*b*, the stiffness matrix $[K]$ is obtained

$$[K] = \begin{bmatrix} 17.3 & 7.3 & 0 \\ 7.3 & 22.8 & 7.3 \\ 0 & 7.3 & 17.3 \end{bmatrix} \qquad (78)$$

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[7] "Continuous Concrete Bridges," Portland Cement Assn., Chicago, Ill., 1939.

The reciprocal is given by

$$[K]^{-1} = \frac{1}{4{,}980}\begin{bmatrix} 341 & -126 & 53.3 \\ -126 & 299 & -126 \\ 53.3 & -126 & 341 \end{bmatrix} = \begin{bmatrix} 0.0685 & -0.0253 & 0.0107 \\ -0.0253 & 0.0600 & -0.0253 \\ 0.0107 & -0.0253 & 0.0685 \end{bmatrix} \quad ..(79)$$

The matrices $[K]$ and $[K]^{-1}$ are symmetrical about the principal diagonal and also about the diagonal running from the upper right-hand corner downward (the secondary diagonal). This second symmetry occurs because the beam is itself symmetrical about its center support.

The matrices $[C_a]$ and $[C_b]$ are computed to be

$$[C_a] = -[K_a][K]^{-1} = \begin{bmatrix} -0.514 & -0.180 & +0.076 \\ +0.210 & -0.499 & -0.212 \\ -0.076 & +0.180 & -0.486 \end{bmatrix} \quad ......(80a)$$

and

$$[C_b] = -[K_b][K]^{-1} = \begin{bmatrix} -0.486 & +0.180 & -0.076 \\ -0.212 & -0.499 & +0.210 \\ +0.076 & -0.180 & -0.514 \end{bmatrix} \quad ......(80b)$$

It should be noted that $[C_b]$ can be obtained from $[C_a]$ by two rotations of the matrix—one about the principal diagonal and one about the secondary diagonal. This condition is caused by symmetry of the beam about its center support.

The correction vectors $[a]$ and $[b]$ can now be written as

$$[a] = \begin{bmatrix} -0.514 & -0.180 & +0.076 \\ +0.210 & -0.499 & -0.212 \\ -0.076 & +0.180 & -0.486 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad ....(81a)$$

and

$$[b] = \begin{bmatrix} -0.486 & +0.180 & -0.076 \\ -0.212 & -0.499 & +0.210 \\ +0.076 & -0.180 & -0.514 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad ....(81b)$$

For any given loading condition, fixed-end moments can be computed from graphical representations of moment coefficients. The unbalanced moments $[u]$ can then be determined, and the corrections $[a]$ and $[b]$ can be computed by a matrix multiplication as shown in Eqs. 81. The convenience of using this method for calculating influence lines should be apparent.

Four solutions for four positions of a unit load in the second span are shown in Fig. 5. The components of the correction vectors have been computed in each case by a matrix multiplication using Eqs. 81. The fixed-end moment coefficients have been taken from a booklet issued by the Portland Cement Association.[7] The column vector of unbalanced moments is shown at the right for each case.

PART III. MATRIX DEVELOPMENT OF MOMENT DISTRIBUTION

INTRODUCTION

This development is also limited to the consideration of continuous beams on supports which are fixed against translation. The process of moment distribution consists of a sequence of cycles of numerical computations. Each cycle has two steps. In the first step all joints are balanced and in the second step the "carry-over calculation" is performed in each span. Although other sequences of performing the operations are possible, the foregoing order has generally been adopted because of the ease in checking the computations by an individual other than the original computer. A statical moment sign convention will be used through this part of the paper. (After submission of the manuscript for publication a discussion by Leon Beskin, Assoc. M. ASCE, appeared in which several matrix equations were presented corresponding closely to some of those contained in this part of the paper.[8])

BEAM WITH ONE EFFECTIVE JOINT

It is possible to give a simple illustration of the process of moment distribution by considering a beam for which the final moments can be obtained by performing only one cycle of computations. Such a beam is one having only one effective joint. Three examples are shown in Fig. 6. These beams do not need to be prismatic. The beam in Fig. 6(*a*) has modified stiffness values, computed by using the appropriate end rotation constants. It is only necessary to perform the first step of the first cycle of moment distribution to obtain the final solution. A similar type of case is shown in Fig. 6(*c*) in which a single distribution of moments at joint 1 gives the final solution. The opportunity for using an end rotation constant in the center span arises from the symmetry of the structure and its loading. A similar opportunity would exist if the loading were antisymmetrical. In Fig. 6(*b*) is shown a beam that requires both steps of the cycle for its complete solution. The stiffness matrix for each span is shown, from which the distribution and carry-over factors are computed. The distribution factors are shown in parentheses above the moment distribution. The distribution step of the cycle yields the correction values which,

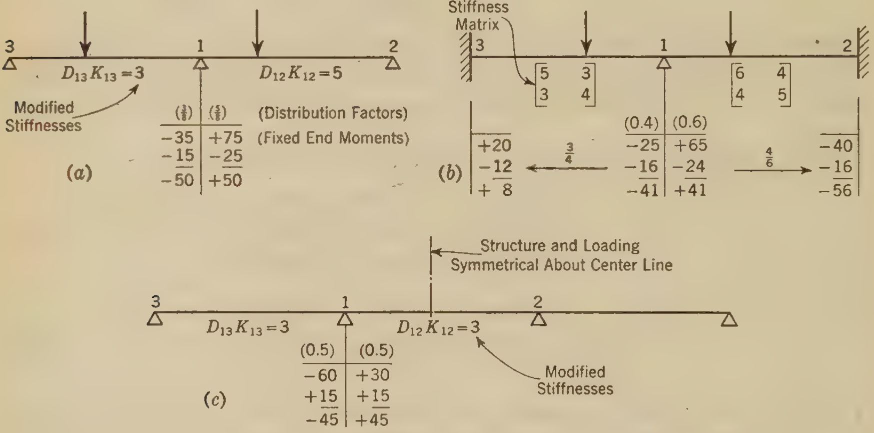
3
1
2
$D_{13}K_{13}=3$
$D_{12}K_{12}=5$
Modified Stiffnesses
($\frac{3}{8}$)
($\frac{5}{8}$)
(Distribution Factors)
−35
+75
(Fixed End Moments)
−15
−25
−50
+50
(a)
Stiffness Matrix
3
1
2
5 3
3 4
6 4
4 5
(0.4)
(0.6)
+20
$\frac{3}{4}$
−25
+65
$\frac{4}{6}$
−40
(b)
−12
−16
−24
−16
+ 8
−41
+41
−56
Structure and Loading Symmetrical About Center Line
3
1
2
$D_{13}K_{13}=3$
$D_{12}K_{12}=3$
(0.5)
(0.5)
−60
+30
Modified Stiffnesses
(c)
+15
+15
−45
+45

FIG. 6.—BEAMS WITH ONE EFFECTIVE JOINT

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[8] *Proceedings*, ASCE, September, 1945, pp. 1111–1120.

when added to the fixed-end moments, give the final moments at joint 1. The carry-over step determines the outer end moments.

In each of the examples of Fig. 6, the computations at the effective joint conform to the following algebraic equations:

$$M_{12} = M'_{12} + a_1 \quad \text{(82a)}$$

and

$$M_{13} = M'_{13} + b_1 \quad \text{(82b)}$$

in which

$$a_1 = -\frac{K_{12}}{\Sigma_1 K} u_1 = -d_{12} u_1 \quad \text{(83a)}$$

and

$$b_1 = -\frac{K_{13}}{\Sigma_1 K} u_1 = -d_{13} u_1 \quad \text{(83b)}$$

The final moments are obtained by adding to the fixed-end moments the corrections a and b. This procedure corresponds exactly to the method of solution described in Part II and illustrated by Eqs. 76. In the present case the column vectors have only one element and, hence, are ordinary numbers. The distribution factors d_{12} and d_{13} have been introduced and defined in agreement with the usual moment distribution procedure.

MATRIX RECIPROCAL EXPRESSED BY POWER SERIES

As was stated in Part I, most of the work involved in solving a continuous beam of many spans lies in the evaluation of the reciprocal of the stiffness matrix $[K]$. Instead of calculating this reciprocal by using determinants as explained in Part I, it is possible to obtain this reciprocal by a converging approximation method. The method can be developed by expressing $[K]^{-1}$ as a power series of matrices. This process leads directly to a complete mathematical explanation of moment distribution.

The matrix $[K]$, for a beam with three effective joints (see Fig. 4), is given by

$$[K] = \begin{bmatrix} \Sigma_1 K & R_{12} & 0 \\ R_{21} & \Sigma_2 K & R_{23} \\ 0 & R_{32} & \Sigma_3 K \end{bmatrix} \quad \text{(84)}$$

This matrix can be separated into two parts defined as follows:

$$[D] = \begin{bmatrix} \Sigma_1 K & 0 & 0 \\ 0 & \Sigma_2 K & 0 \\ 0 & 0 & \Sigma_3 K \end{bmatrix} \quad \text{(85)}$$

and

$$[R] = \begin{bmatrix} 0 & -R_{12} & 0 \\ -R_{21} & 0 & -R_{23} \\ 0 & -R_{32} & 0 \end{bmatrix} \quad \text{(86)}$$

The matrix $[D]$ is a diagonal matrix containing the diagonal elements of $[K]$. The matrix $[R]$ contains the nondiagonal elements of $[K]$ with negative signs. The reciprocal of $[D]$ will appear in the development and a convenient rule for computing the reciprocal of a diagonal matrix can be established. Evaluating $[D]^{-1}$ by determinants gives

$$[D]^{-1} = \frac{1}{\Sigma_1K\,\Sigma_2K\,\Sigma_3K}\begin{bmatrix} \Sigma_2K\,\Sigma_3K & 0 & 0 \\ 0 & \Sigma_1K\,\Sigma_3K & 0 \\ 0 & 0 & \Sigma_2K\,\Sigma_3K \end{bmatrix}$$

$$= \begin{bmatrix} (\Sigma_1K)^{-1} & 0 & 0 \\ 0 & (\Sigma_2K)^{-1} & 0 \\ 0 & 0 & (\Sigma_3K)^{-1} \end{bmatrix} \quad \text{(87)}$$

Inspection shows that $[D]^{-1}$ is also a diagonal matrix formed by replacing each of the diagonal elements of $[D]$ by its reciprocal.

From Eqs. 84 to 86:

$$[K] = [D] - [R] \quad \text{(88)}$$

Eq. 88 may be written as

$$[K] = \{[I] - [R][D]^{-1}\}[D] \quad \text{(89)}$$

It becomes convenient to define the matrix $[Q]'$ by

$$[Q]' = [R][D]^{-1} \quad \text{(90)}$$

The physical significance of the elements of $[Q]'$ and the reason for using the transpose $[Q]'$ instead of $[Q]$ will be explained subsequently. Eq. 89 becomes

$$[K] = \{[I] - [Q]'\}[D] \quad \text{(91)}$$

To write the reciprocal of $[K]$, it is necessary to establish another rule of matrix algebra regarding the reciprocal of a product of two matrices. For example, assume that

$$[C] = [A][B] \quad \text{(92)}$$

Premultiplying both sides of Eq. 92 by $[A]^{-1}$,

$$[A]^{-1}[C] = [I][B] = [B] \quad \text{(93)}$$

Premultiplying both sides of Eq. 93 by $[B]^{-1}$,

$$[B]^{-1}[A]^{-1}[C] = [I] \quad \text{(94)}$$

Postmultiplying both sides of Eq. 94 by $[C]^{-1}$,

$$[B]^{-1}[A]^{-1} = [C]^{-1} \quad \text{(95)}$$

Comparing Eq. 95 with Eq. 92, it is apparent that the reciprocal of a product of two matrices equals the product of their reciprocals—in reverse order. From this rule, and from Eq. 91, $[K]^{-1}$ can be written

$$[K]^{-1} = [D]^{-1}\{[I] - [Q]'\}^{-1} = [D]^{-1}\left\{\frac{[I]}{[I] - [Q]'}\right\} \quad \text{(96)}$$

In general, it is not permissible to write a matrix division as a fraction because of the ambiguity that arises as explained in Part I. However, the fraction shown in Eq. 96 has the identity matrix for a numerator; and, hence, no ambiguity arises. The reciprocal has been written in the form of a fraction in order that it can be compared with the following well-known algebraic formula:

$$\frac{1}{1-\epsilon} = 1 + \epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4 + \cdots \qquad (97)$$

This infinite power series can be derived by applying the binomial theorem or by direct long division. The number unity of ordinary numbers is comparable to the identity matrix in matrix algebra. Hence, by comparison:

$$\{[I] - [Q]'\}^{-1} = [I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots \qquad (98)$$

The validity of Eq. 98 can be demonstrated by multiplying both sides by the binomial $[I] - [Q]'$. A rigorous mathematical proof of the correctness of this series requires that the power matrix $\{[Q]'\}^n$ shall approach the zero matrix as n approaches infinity. The reciprocal $[K]^{-1}$ can now be expressed by

$$[K]^{-1} = [D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \qquad (99)$$

Moment Distribution

As was shown in Part II, the correction vectors $[a]$ and $[b]$ are given by the following formulas:

$$[a] = -[K_a][K]^{-1}[u] \qquad (100a)$$

and

$$[b] = -[K_b][K]^{-1}[u] \qquad (100b)$$

Substituting Eq. 99 into Eqs. 100,

$$[a] = -[K_a][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \{[Q]'\}^4 + \cdots\}[u] \qquad (101a)$$

and

$$[b] = -[K_b][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \{[Q]'\}^4 + \cdots\}[u] \qquad (101b)$$

The physical significance of these equations can be noted by writing the matrices in expanded form and considering the operations term by term. First, consider the two matrices in front of the heavy brace in each equation. Assume a beam having three effective joints:

$$[K_a][D]^{-1} = \begin{bmatrix} K_{12} & R_{12} & 0 \\ 0 & K_{23} & R_{23} \\ 0 & 0 & K_{34} \end{bmatrix} \begin{bmatrix} (\Sigma_1 K)^{-1} & 0 & 0 \\ 0 & (\Sigma_2 K)^{-1} & 0 \\ 0 & 0 & (\Sigma_3 K)^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} d_{12} & r_{21}\,d_{21} & 0 \\ 0 & d_{23} & r_{32}\,d_{32} \\ 0 & 0 & d_{34} \end{bmatrix} \qquad (102)$$

In this matrix the diagonal elements are the distribution factors applicable at the left end of the members (or the right side of the joints) as used in moment distribution. The nondiagonal elements are products of a distribution factor and a corresponding carry-over factor. It is convenient to represent these nondiagonal elements by a single letter q with appropriate subscripts. Thus, for example,

$$q_{21} = r_{21}\,d_{21};\ \text{and}\ q_{32} = r_{32}\,d_{32} \qquad (103)$$

It is appropriate to discuss the relationship of the quantity q to other physical quantities which are well known. If the distribution factor d_{23}, as used in moment distribution, is referred to as a primary distribution factor, then the quantity q_{23} may be called a secondary distribution factor. This is analogous to calling K_{23} the primary stiffness value and R_{23} the secondary stiffness value as introduced in Part II. In both cases the secondary value is obtained by multiplying the primary value by the carry-over factor r_{23}. Thus,

$$R_{23} = r_{23}\,K_{23};\ \text{and}\ q_{23} = r_{23}\,d_{23} \qquad (104)$$

It is also possible to compute the secondary distribution factor in a manner which is exactly the same as that used for the primary factors. Thus,

$$d_{23} = \frac{K_{23}}{\Sigma_2 K};\ \text{and}\ q_{23} = \frac{R_{23}}{\Sigma_2 K} \qquad (105)$$

Since the distribution factors can be calculated directly from the corresponding stiffness values, an analysis of a continuous beam by matrix methods does not require computation of the carry-over factor. Introducing secondary distribution factors, Eq. 102 becomes

$$[K_a][D]^{-1} = \begin{bmatrix} d_{12} & q_{21} & 0 \\ 0 & d_{23} & q_{32} \\ 0 & 0 & d_{34} \end{bmatrix} \qquad (106a)$$

and, similarly,

$$[K_b][D]^{-1} = \begin{bmatrix} d_{15} & 0 & 0 \\ q_{12} & d_{21} & 0 \\ 0 & q_{23} & d_{32} \end{bmatrix} \qquad (106b)$$

Now consider the terms of the power series of Eqs. 101, term by term. First, assume that the correction vectors are to be computed using only the first term of the series:

$$[a] = -[K_a][D]^{-1}[I][u] = -[K_a][D]^{-1}[u] \qquad (107a)$$

and

$$[b] = -[K_b][D]^{-1}[I][u] = -[K_b][D]^{-1}[u] \qquad (107b)$$

Substituting Eqs. 106,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = -\begin{bmatrix} d_{12} & q_{21} & 0 \\ 0 & d_{23} & q_{32} \\ 0 & 0 & d_{34} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (108a)$$

and

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = -\begin{bmatrix} d_{15} & 0 & 0 \\ q_{12} & d_{21} & 0 \\ 0 & q_{23} & d_{32} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (108b)$$

When the first element of the first row of each coefficient matrix is multiplied by the first element of the vector $[u]$, the two quantities obtained are $-d_{12}\,u_1$ and $-d_{15}\,u_1$—the products of the unbalanced moment at joint 1 and the distribution factors. The contribution $-d_{12}\,u_1$ to the value of a_1 is thus exactly the value of the contribution to the final joint moment which is made by the first step of a moment distribution cycle applied to the unbalanced moment at joint 1. Similarly, the products of all the diagonal elements of the coefficient matrices of Eqs. 108 and the vector $[u]$ provide the same contributions to the solution as are provided by the first step of the first cycle of moment distribution in which all the joints are balanced.

Consider next the effect of the nondiagonal elements, or secondary distribution factors, of the coefficient matrices. Consider only the elements of the second row of the matrix of Eq. 108*a*. The nondiagonal element q_{32} contributes $-q_{32}\,u_3$ to the value of a_2. From the definition of q, the contribution is $-r_{32}\,d_{32}\,u_3$. This quantity is the contribution to the joint moment on the right side of joint 2 brought about by the carry-over calculation in span 23. Thus, the nondiagonal elements of the coefficient matrices correspond to the second step of a moment distribution cycle.

Consequently, the coefficient matrices of Eqs. 108 represent, in matrix notation, one complete cycle of moment distribution. These matrices—$-[K_a][D]^{-1}$ and $-[K_b][D]^{-1}$—when applied to a column vector $[u]$ of unbalanced moments make the same contribution to the final solution as is made by one complete cycle of moment distribution.

Consider now that the correction vectors $[a]$ and $[b]$ are to be computed by using the first two terms of the series of Eqs. 101:

$$[a] = -[K_a][D]^{-1}[u] - [K_a][D]^{-1}[Q]'[u] \qquad (109a)$$

and

$$[b] = -[K_b][D]^{-1}[u] - [K_b][D]^{-1}[Q]'[u] \qquad (109b)$$

The second terms could be expressed in a manner similar to Eqs. 108, given for the first term, if the vector $[u]$ were replaced by the product $[Q]'[u]$. In expanded form, this product (see Eq. 90) becomes

$$[Q]'[u] = [R][D]^{-1}[u]$$

$$= \begin{bmatrix} 0 & -R_{12} & 0 \\ -R_{21} & 0 & -R_{23} \\ 0 & -R_{32} & 0 \end{bmatrix} \begin{bmatrix} (\Sigma_1 K)^{-1} & 0 & 0 \\ 0 & (\Sigma_2 K)^{-1} & 0 \\ 0 & 0 & (\Sigma_3 K)^{-1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (110)$$

Let

$$[u'] = [Q]'[u] \qquad (111)$$

Then

$$[u'] = -\begin{bmatrix} 0 & r_{21}\,d_{21} & 0 \\ r_{12}\,d_{12} & 0 & r_{32}\,d_{32} \\ 0 & r_{23}\,d_{23} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = -\begin{bmatrix} 0 & q_{21} & 0 \\ q_{12} & 0 & q_{32} \\ 0 & q_{23} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (112)$$

Consideration of the coefficient matrix $[Q]'$ of Eq. 112 shows why the transpose $[Q]'$ was introduced rather than $[Q]$. The subscripts of the elements show that the matrix is in transposed form. Calculating the components of $[u']$ individually gives

$$u'_1 = -r_{21}\,d_{21}\,u_2 \qquad (113a)$$

$$u'_2 = -r_{12}\,d_{12}\,u_1 - r_{32}\,d_{32}\,u_3 \qquad (113b)$$

and

$$u'_3 = -r_{23}\,d_{23}\,u_2 \qquad (113c)$$

From consideration of these formulas, the components of $[u']$ are shown to be the unbalanced moments remaining at the joints after one cycle of moment distribution has been completed. Since it was previously shown that the coefficients $-[K_a][D]^{-1}$ and $-[K_b][D]^{-1}$ correspond to one cycle of moment distribution, it should now be apparent that the first term of Eqs. 109 corresponds to the first cycle of moment distribution and the second term corresponds to the second cycle.

Similarly, it can be shown that the third term of the power series of Eqs. 101 corresponds to the third cycle of moment distribution, etc. Thus, it is established that a complete mathematical statement of the method of moment distribution can be given in matrix notation as follows:

$$[M_a] = [M'_a] - [K_a][D]^{-1} \{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\}[u] \qquad (114a)$$

and

$$[M_b] = [M'_b] - [K_b][D]^{-1} \{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\}[u] \qquad (114b)$$

Although, by making certain limiting assumptions, R. Oldenburger proved the convergence of moment distribution, this theory has not been given very extensive treatment by mathematicians.[9] Eqs. 114 show that this convergence is dependent upon the convergence of the series within the braces. In any given practical case, a rigorous proof of convergence can be developed; but it will not be attempted here.

PART IV. VARIATIONS FROM MOMENT DISTRIBUTION

INTRODUCTION

In Part II, it was shown that a complete solution can be obtained by matrix computation methods in which the reciprocal of the stiffness matrix is computed by using determinants. With Eqs. 114, it is possible to devise a number of different converging approximation methods of solution using matrix methods of computation. The method of moment distribution, with its cycles of numerical computations, is in exact correspondence with Eqs. 114; yet it makes no use of matrix methods. It is possible to devise combined procedures which use both matrix methods and moment distribution. A development of two methods, based upon Eqs. 114, will be given.

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[9] "Convergence of Hardy Cross's Balancing Process," by R. Oldenburger, *Journal of Applied Mechanics*, December, 1940, pp. A-166 to A-170.

A Combined Method

If a given beam is to be analyzed for only one loading condition, the designer who is familiar with moment distribution will undoubtedly find it the fastest method of solution available. However, if it is desired to use matrix methods, an interesting combined procedure can be developed by writing the correction vectors $[a]$ and $[b]$ as follows:

$$[a] = -[K_a][D]^{-1}[\bar{u}] \qquad \text{(115}a\text{)}$$

and

$$[b] = -[K_b][D]^{-1}[\bar{u}] \qquad \text{(115}b\text{)}$$

in which $[\bar{u}]$ is a vector of transformed unbalanced moments defined by

$$[\bar{u}] = [u] + [u'] + [u''] + [u'''] + \cdots \qquad \text{(116)}$$

The vectors $[u']$, $[u'']$, etc., are the unbalanced moments which remain after each cycle of moment distribution and may be computed by the formulas:

$$[u'] = [Q]'[u] \qquad \text{(117}a\text{)}$$

$$[u''] = \{[Q]'\}^2\,[u] = [Q]'[u'] \qquad \text{(117}b\text{)}$$

$$[u'''] = \{[Q]'\}^3\,[u] = [Q]'[u''] \qquad \text{(117}c\text{)}$$

Each of these vectors is computed by multiplying the previous one by $[Q]'$. It is interesting, and rather surprising, to discover that the unbalanced moments which will exist after any given number of cycles of moment distribution can be computed directly without actually writing out the moment distribution computations. The elements of the vectors $[u']$, $[u'']$, etc., decrease and rapidly approach zero. To obtain the accuracy, to three significant places, which is customary in structural design practice, in general, the four vectors of the type given by Eqs. 117 will be sufficient. Each additional vector usually gives one additional significant figure accurately. The accuracy of the final solution, however, cannot be greater than the accuracy of the fixed-end moments, which are seldom obtainable to an accuracy greater than three significant places. For several practical reasons, significant figures in excess of three are generally found of little use in design practice.

After the transformed vector $[\bar{u}]$ has been computed, the correction vectors $[a]$ and $[b]$ can be determined from Eqs. 115. However, it has been shown previously that the contributions to a solution made by premultiplying a vector of unbalanced moments by the factors $-[K_a][D]^{-1}$ and $-[K_b][D]^{-1}$ correspond to operating on the vector with one cycle of moment distribution. Thus, it becomes apparent that a solution can be obtained by first computing $[\bar{u}]$ and then performing one cycle of moment distribution with this set of unbalanced moments rather than with the actual values of $[u]$. If any joint is slightly out of balance at the end of the cycle, such a joint should be balanced. The foregoing procedure employs both matrix methods and moment distribution. It is recommended only for beams that are to be analyzed for one or two loading conditions or, more specifically, when the number of loading conditions is less than the number of effective joints.

A Matrix Method

If a continuous beam is to be analyzed for a number of loading conditions, equal to or greater than the number of effective joints, a different process of analysis will be found to require less time than the one previously described. This second method employs only matrix methods and becomes especially useful when the number of loading conditions greatly exceeds the number of effective joints. One such case is evidently the computation of influence lines by placing a unit load at various points along the beam.

Referring to Eqs. 101, the complete coefficients of $[u]$ may be evaluated to obtain the coefficient matrices $[C_a]$ and $[C_b]$ as introduced in Part II (see Eq. 75). The vectors $[a]$ and $[b]$ become

$$[a] = [C_a][u]; \text{ and } [b] = [C_b][u] \quad \ldots\ldots\ldots\ldots (118)$$

From Eqs. 101, the formulas for $[C_a]$ and $[C_b]$ become

$$[C_a] = -[K_a][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \ldots (119a)$$

and

$$[C_b] = -[K_b][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \ldots (119b)$$

Eqs. 118 and 119 provide a method of successive corrections for calculating $[C_a]$ and $[C_b]$ that may be used instead of determinants as explained in Part II.

The method of successive corrections will be found to require less time than the method of determinants when the beam has four or more effective joints—that is, when $[Q]$ is of the fourth or higher order. Substituting Eqs. 106 into Eqs. 119,

$$[C_a] = -\begin{bmatrix} d_{12} & q_{21} & 0 \\ 0 & d_{23} & q_{32} \\ 0 & 0 & d_{34} \end{bmatrix}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \ldots (120a)$$

and

$$[C_b] = -\begin{bmatrix} d_{15} & 0 & 0 \\ q_{12} & d_{21} & 0 \\ 0 & q_{23} & d_{32} \end{bmatrix}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \ldots (120b)$$

The first step in the numerical computations is the evaluation of the matrix obtained by summing the power series within the large braces. Usually the fourth-degree term in the series will be the highest power of $[Q]'$ which is needed. Each term of the series is computed by multiplying the previous term by $[Q]'$. In order to perform this calculation, it is necessary for the designer to have a convenient rule for setting up the matrix $[Q]'$ in expanded form. Referring to Eq. 112, for a beam with three effective joints, $[Q]'$ has the form:

$$[Q]' = -\begin{bmatrix} 0 & q_{21} & 0 \\ q_{12} & 0 & q_{32} \\ 0 & q_{23} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots (121)$$

This matrix consists solely of secondary distribution factors. Considering the matrix $[Q]$,

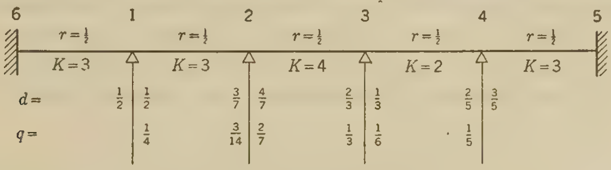
$$[Q] = -\begin{bmatrix} 0 & q_{12} & 0 \\ q_{21} & 0 & q_{23} \\ 0 & q_{32} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots (122)$$

In this matrix the elements have subscripts that agree with their position in the matrix. The first subscript of an element indicates its row and the second subscript indicates its column. The q-values occupy the subdiagonals and the superdiagonals whereas all other elements of the matrix are zero. The subscripts also correspond to the joint and member, the first subscript indicating the joint. It is convenient to set up $[Q]$ first and then transpose to obtain $[Q]'$.

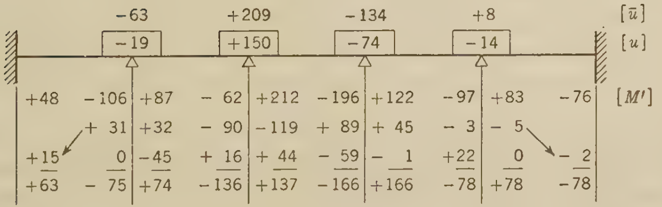
If the coefficient matrices of Eqs. 120 are considered, the nondiagonal elements are the same as the elements of $[Q]'$. The superdiagonal enters the formula for $[C_a]$; and the subdiagonal, the formula for $[C_b]$. The diagonal elements of these coefficients are the distribution factors for the left and right ends of the members, respectively. Consequently, the only quantities that enter the evaluation of $[C_a]$ and $[C_b]$ by Eqs. 120 are primary and secondary distribution factors.

Numerical Examples

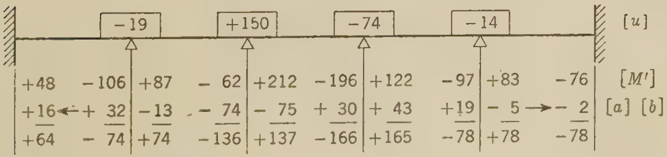
An example will be solved using each of the two variations from moment distribution which have been described. A five-span beam, having four effective joints, is shown in Fig. 7(a). Each span is prismatic, thus having the same stiffness value at either end and a carry-over factor of one half. The


6
1
2
3
4
5
r=½
r=½
r=½
r=½
r=½
K=3
K=3
K=4
K=2
K=3
d=
½
½
3/7
4/7
2/3
1/3
2/5
3/5
q=
¼
3/14
2/7
1/3
1/6
1/5

(a) FIVE SPAN PRISMATIC BEAM


−63
+209
−134
+8
[ū]
−19
+150
−74
−14
[u]
+48 −106 +87 −62 +212 −196 +122 −97 +83 −76
[M′]
+31 +32 −90 −119 +89 +45 −3 −5
+15 0 −45 +16 +44 −59 −1 +22 0 −2
+63 −75 +74 −136 +137 −166 +166 −78 +78 −78

(b) COMBINED METHOD OF ANALYSIS


−19
+150
−74
−14
[u]
+48 −106 +87 −62 +212 −196 +122 −97 +83 −76
[M′]
+16 +32 −13 −74 −75 +30 +43 +19 −5 −2
[a] [b]
+64 −74 +74 −136 +137 −166 +165 −78 +78 −78

(c) MATRIX METHOD OF ANALYSIS

Fig. 7.—Comparison of Methods

stiffness values are in boxes below the center of each span. The primary distribution factors are shown for each joint and below them the secondary factors. The matrix $[Q]$ is given by

$$[Q] = -\begin{bmatrix} 0 & q_{12} & 0 & 0 \\ q_{21} & 0 & q_{23} & 0 \\ 0 & q_{32} & 0 & q_{34} \\ 0 & 0 & q_{43} & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1/4 & 0 & 0 \\ 3/14 & 0 & 2/7 & 0 \\ 0 & 1/3 & 0 & 1/6 \\ 0 & 0 & 1/5 & 0 \end{bmatrix} \ldots(123)$$

Transposing and converting to decimals,

$$[Q]' = -\begin{bmatrix} 0 & 0.214 & 0 & 0 \\ 0.250 & 0 & 0.333 & 0 \\ 0 & 0.286 & 0 & 0.200 \\ 0 & 0 & 0.167 & 0 \end{bmatrix} \ldots\ldots\ldots\ldots(124)$$

This matrix must be used in either of the two methods of analysis.

A set of fixed-end moments are assumed as shown in Fig. 7(*b*). The unbalanced joint moments which form the components of the vector $[u]$ are noted in the box over each joint. Using Eqs. 116 and 117, the components of the transformed vector $[\bar{u}]$ may be conveniently computed as shown in Table 1. Each column is obtained by multiplying the previous column by $[Q]'$.

TABLE 1.—CALCULATION OF $[\bar{u}]$

$[Q]'$				$[u]$	$[u']$	$[u'']$	$[u''']$	$[u'''']$	$[\bar{u}]$
0	−0.214	0	0	−19	−32	−6	−5	−1	−63
−0.25	0	−0.333	0	150	29	21	5	4	209
0	−0.286	0	−0.2	−74	−40	−11	−7	−2	−134
0	0	−0.167	0	−14	12	7	2	1	8

The last column is obtained by adding all the previous columns. These transformed unbalanced moments are shown in Fig. 7(*b*) just above the actual values. The application of one cycle of moment distribution to the components of $[\bar{u}]$ yields the solution as shown.

The second method of solution requires the computation of the terms of the power series in $[Q]'$. These terms are computed as follows:

$$[I] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots(125a)$$

$$[Q]' = -\begin{bmatrix} 0 & 0.214 & 0 & 0 \\ 0.25 & 0 & 0.333 & 0 \\ 0 & 0.286 & 0 & 0.2 \\ 0 & 0 & 0.167 & 0 \end{bmatrix} \ldots\ldots\ldots\ldots(125b)$$

$$\{[Q]'\}^2 = \begin{bmatrix} 0.054 & 0 & 0.071 & 0 \\ 0 & 0.149 & 0 & 0.067 \\ 0.072 & 0 & 0.129 & 0 \\ 0 & 0.048 & 0 & 0.033 \end{bmatrix} \tag{125c}$$

$$\{[Q]'\}^3 = -\begin{bmatrix} 0 & 0.032 & 0 & 0.014 \\ 0.038 & 0 & 0.061 & 0 \\ 0 & 0.052 & 0 & 0.026 \\ 0.012 & 0 & 0.022 & 0 \end{bmatrix} \tag{125d}$$

$$\{[Q]'\}^4 = \begin{bmatrix} 0.008 & 0 & 0.013 & 0 \\ 0 & 0.025 & 0 & 0.012 \\ 0.013 & 0 & 0.022 & 0 \\ 0 & 0.009 & 0 & 0.004 \end{bmatrix} \tag{125e}$$

Each matrix is obtained by multiplying the preceding matrix by $[Q]'$. Adding these matrices,

$$[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \{[Q]'\}^4$$
$$= \begin{bmatrix} 1.062 & -0.246 & 0.084 & -0.014 \\ -0.288 & 1.174 & -0.394 & 0.079 \\ 0.085 & -0.338 & 1.151 & -0.226 \\ -0.012 & 0.057 & -0.189 & 1.037 \end{bmatrix} \tag{126}$$

The products $[K_a][D]^{-1}$ and $[K_b][D]^{-1}$ are given by

$$[K_a][D]^{-1} = \begin{bmatrix} 0.5 & 0.214 & 0 & 0 \\ 0 & 0.571 & 0.333 & 0 \\ 0 & 0 & 0.333 & 0.2 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \tag{127a}$$

and

$$[K_b][D]^{-1} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.25 & 0.429 & 0 & 0 \\ 0 & 0.286 & 0.667 & 0 \\ 0 & 0 & 0.167 & 0.4 \end{bmatrix} \tag{127b}$$

The diagonal elements of these matrices are the left-end and the right-end primary distribution factors. The nondiagonal elements are taken directly from the matrix $[Q]'$ in the preceding computation (Eq. 125*b*). The coefficient matrices $[C_a]$ and $[C_b]$ are then computed (see Eq. 119):

$$[C_a] = -\begin{bmatrix} 0.5 & 0.214 & 0 & 0 \\ 0 & 0.571 & 0.333 & 0 \\ 0 & 0 & 0.333 & 0.2 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \begin{bmatrix} 1.062 & -0.246 & 0.084 & -0.014 \\ -0.288 & 1.174 & -0.394 & 0.079 \\ 0.085 & -0.338 & 1.151 & -0.226 \\ -0.012 & 0.057 & -0.189 & 1.037 \end{bmatrix}$$

$$= -\begin{bmatrix} 0.469 & 0.128 & -0.042 & 0.010 \\ -0.136 & 0.558 & 0.158 & -0.030 \\ 0.026 & -0.101 & 0.346 & 0.132 \\ -0.007 & 0.034 & -0.113 & 0.622 \end{bmatrix} \tag{128a}$$

and

$$[C_b] = -\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.25 & 0.429 & 0 & 0 \\ 0 & 0.286 & 0.667 & 0 \\ 0 & 0 & 0.167 & 0.4 \end{bmatrix}\begin{bmatrix} 1.062 & -0.246 & 0.084 & -0.014 \\ -0.288 & 1.174 & -0.394 & 0.079 \\ 0.085 & -0.338 & 1.151 & -0.226 \\ -0.012 & 0.057 & -0.189 & 1.007 \end{bmatrix}$$

$$= -\begin{bmatrix} 0.531 & -0.123 & 0.042 & -0.007 \\ 0.142 & 0.442 & -0.148 & 0.030 \\ -0.026 & 0.110 & 0.655 & -0.128 \\ 0.009 & -0.034 & 0.117 & 0.377 \end{bmatrix} \cdots\cdots\cdots (128b)$$

The correction vectors can now be written as

$$[a] = \begin{bmatrix} -0.469 & -0.128 & 0.042 & -0.010 \\ 0.136 & -0.558 & -0.158 & 0.030 \\ -0.026 & 0.101 & -0.346 & -0.132 \\ 0.007 & -0.034 & 0.113 & -0.622 \end{bmatrix}[u] \cdots\cdots (129a)$$

and

$$[b] = \begin{bmatrix} -0.531 & 0.123 & -0.042 & 0.007 \\ -0.142 & -0.442 & 0.148 & -0.030 \\ 0.026 & -0.110 & -0.655 & 0.128 \\ -0.009 & 0.034 & -0.117 & -0.377 \end{bmatrix}[u] \cdots\cdots (129b)$$

The components of the correction vectors are thus expressed as linear functions of the unbalanced moments. The elements of the coefficient matrices are dependent only on the physical characteristics of the beam. In Fig. 7(*c*) the same fixed-end moments and unbalanced moments as were assumed in the previous solution are shown. The components of the correction vectors, as computed from Eqs. 129, are written directly below the fixed-end moments. Direct addition gives the final solution. If solutions for additional loading conditions are desired, it should be apparent that the solutions could be computed very quickly from Eqs. 129.

CONCLUSION

The linear equations that govern continuous beams have been expressed and solved in the notation of matrix algebra. A complete algebraic statement of the method of moment distribution has been given. Matrix computation methods have been illustrated. The problem treated in this paper is the most elementary type of indeterminate structure. The joints have only one degree of freedom. This limitation in the physical scope of the paper was considered desirable in order that the emphasis could be placed on the methods and concepts of matrix algebra.

DISCUSSION

I. Oesterblom,[10] M. ASCE.—The use of matrices in structural engineering as presented in this paper constitutes a well-arranged review of the basic ideas and also a few sensible applications. Some matrix applications are already well known to engineers who have analyzed their continuity problems by the aid of energy, deflection, or slope methods, because all these, ultimately, lead to a system of equations for which a solution is possible only by the aid of determinants. (Among the many writers in this field, the following are noteworthy for comprehensive coverage and accessibility in American engineering libraries: Max Foerster,[11] David Molitor,[12] and J. I. Parcel and G. A. Maney.[13])

Too many engineers, however, have kept aloof from these methods, perhaps because they have been frightened by the unknown simplicity of matrix operations. Since the publication of this paper, however, they can no longer offer this as an excuse. On the other hand, others—the physics-minded type—have been led in the opposite direction. Albert Einstein's "generalized relativity" opened the entire field of physics to tensor operations, and this has led even to problems of elasticity where today, especially the French are offering most interesting simplifications[14] to some of the old and most discouraging problems; and tensors are nothing but the supreme of matrices, highly rationalized.

Perhaps there was also something beyond the matrices that frightened the early designers of indeterminate structures. It might not have been their presence but their extent. The nodal points in most commercial structures were many, as were, therefore, the elements in the matrix. It required a staff of good mathematicians under very keen leadership to set up the equations, and then to solve them, for five stories and three spans only—not to mention the many larger frames. A maximum of fifteen redundants was as much as one man could handle.

The utmost in this respect was discovered in the Zeppelin frames—with one hundred or more nodal points.[15] When these airborne visitors, on destruction bent, arrived over the British Isles during World War I fear came to every man's heart: "What can we do to master these wicked monsters?" R. V. Southwell in England and J. C. Hunsaker in the United States worked on the problem, intensely and fearfully, to build better airships. Mountain high matrices as wide as an ocean—or so it must have seemed to those who did the work—were required.

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[10] Engr., Carbide & Carbon Chemicals Corp., South Charleston, W. Va.

[11] "Taschenbuch für Bauingenieure," edited by Max Foerster, Vol. I, Julius Springer, Berlin, 1928, pp. 309–316.

[12] "Kinetic Theory of Engineering Structures," by David Molitor, McGraw-Hill Book Co., Inc., New York, N. Y., 1911.

[13] "Statically Indeterminate Structures," by J. I. Parcel and G. A. Maney, John Wiley & Sons, Inc., New York, N. Y., 1936.

[14] "Les Tenseurs en Mécanique et en Élasticité," by Leon Brillouin, Masson et Cie, Paris, 1938; American edition by Dover Publications, New York, N. Y., 1946.

[15] *Popular Aviation*, January, 1934, p. 29.

Mr. Southwell's magnificent work now reposes in the archives of the Royal Society in London—a bewildering array of mass matrices of group elements; and a child was born out of this labor, too—the magnificent but ill-fated dirigible R101 which went to its destruction in the mountains of eastern France on its maiden trip to India. This was the only airship that was ever truly and rationally designed—so the world found when peace came. As a measure necessary to prove that the war was truly over, the foremost "matricians" of the world were ordered to surrender all their designs and calculations for the Zeppelins. It was a sorry collection they presented. The master mathematicians had been defeated by too many nodal points; they had nothing to show except reams of empiricism heavily loaded with guesswork.

This leads one to ask: "What good today are the matrices in dealing with frameworks?" If the latter are complicated, matrices are inadequate, and if they are simple, the modern relaxation methods are as good for the simple tasks as they are for the most complicated—and much simpler in application.

The engineer must think further than that, however, even though Mr. Benscoter's applications have been somewhat limited, for there are other problems—as in the field of dynamics and elasticity—many of them not yet born. When these rise out of the twilight of the unknown, designers will be thankful to Mr. Benscoter for providing such a fine light to lead them through the darkness.

Quite incidentally, the paper reveals that the stiffness of continuity may be expressed as a power series. Is this for a single sequence only, or does it apply also for a frame in two or three dimensions? For such a frame the discovery would have an epochal significance, because it would lighten the burden considerably when one has to deal with changes of sections or loads in a frame. As things are now, much guessing must be done because the actual work of rational redesign would be prohibitive for most such problems, because of the time required.

HARRIS SOLMAN,[16] M. ASCE.—By using the theory of matrices in the analysis of continuous beams, the author has made a valuable contribution to the field of structural analysis. The presentation of relationships in the form of a matrix or a system of matrices not only has the advantage of a convenient, systematic, and easily visualized arrangement but also possesses a mathematical elegance, which—it is hoped—engineers will not be too "practical" to appreciate.

Aside from the theoretical interest of the author's method, however, there is also a practical feature involved which has not been sufficiently emphasized in the paper. The method presented can be utilized conveniently in setting up general values of end moments for a given system of continuous beams in terms of loads, so that they can be useful in computing influence coefficients.

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[16] Highway Bridge Engr., U. S. Public Roads Administration, Division 1, Albany, N. Y.

A number of years ago the writer became interested in setting up general formulas to express the end moments of continuous beams on supports fixed against translation in terms of the fixed-end moments. Since a load placed at any one point on the structure produces only two fixed-end moments (one at each end of the loaded span), only two terms of each such formula would need to be evaluated for any one position of the load. Furthermore, the coefficient for each term would be a function of the beam constants only and would be an invariant for all positions of the load. Such formulas were deduced and successfully used by the writer.

These formulas, transcribed in the same general notations as those used by the author, are given subsequently. The following minor changes in notation are to be observed: The first end support (noneffective) was designated as zero, the symbol n being reserved to indicate the last effective support (the degree of the system). Additional symbols p and Δ were introduced for convenience and are defined by the following relationships:

Description	1		2		n	
Fixed-end moments..................	M'_{10}	M'_{12}	M'_{21}	M'_{23}	$M'_{n(n-1)}$	$M'_{n(n+1)}$
Distribution factors..................	d_{10}	d_{12}	d_{21}	d_{23}	$d_{n(n-1)}$	$d_{n(n+1)}$
Carry-over factors....................	0	r_{12}	r_{21}	r_{23}	$r_{n(n-1)}$	0
q-factors ($d \times r$).........................	0	q_{12}	q_{21}	q_{23}	$q_{n(n-1)}$	0

FIG. 8.—BEAM CONSTANTS FOR AN n-DEGREE SYSTEM

$$p_{uv} \text{ (for } 0 < u \leqq n \text{ and } 0 > v \geqq n) = q_{uv}\, q_{vu} \qquad \text{(130a)}$$

and, for an n-degree system:

$$\Delta_n = \text{determinant} \begin{vmatrix} 1 & q_{12} & 0 & \cdots & 0 \\ q_{21} & 1 & q_{23} & \cdots & 0 \\ 0 & q_{32} & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \end{vmatrix} \qquad \text{(130b)}$$

The beam convention for the algebraic sign is used throughout, in order that the formulas may be readily applied to the beams with their proper signs without the necessity of considering the rotation effects on the joints.

In general, for an n-degree system (see Fig. 8):

$$\left.\begin{aligned}
M_{10} &= M'_{10} + b^{(n)}_{11}(M'_{10} - M'_{12}) + b^{(n)}_{12}(M'_{21} - M'_{23}) + \cdots \\
&\qquad + b^{(n)}_{1n}[M'_{n(n-1)} - M'_{n(n+1)}] \\
M_{12} &= M'_{12} + a^{(n)}_{11}(M'_{10} - M'_{12}) + a^{(n)}_{12}(M'_{21} - M'_{23}) + \cdots \\
&\qquad + a^{(n)}_{1n}[M'_{n(n-1)} - M'_{n(n+1)}] \\
M_{21} &= M'_{21} + b^{(n)}_{21}(M'_{10} - M'_{12}) + b^{(n)}_{22}(M'_{21} - M'_{23}) + \cdots \\
&\qquad + b^{(n)}_{2n}[M'_{n(n-1)} - M'_{n(n+1)}] \\
M_{23} &= M'_{23} + a^{(n)}_{21}(M'_{10} - M'_{12}) + a^{(n)}_{22}(M'_{21} - M'_{23}) + \cdots \\
&\qquad + a^{(n)}_{2n}[M'_{n(n-1)} - M'_{n(n+1)}] \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
M_{n(n-1)} &= M'_{n(n-1)} + b^{(n)}_{n1}(M'_{10} - M'_{12}) + b^{(n)}_{n2}(M'_{21} - M'_{23}) + \cdots \\
&\qquad + b^{(n)}_{nn}[M'_{n(n-1)} - M'_{n(n+1)}] \\
M_{n(n+1)} &= M'_{n(n+1)} + a^{(n)}_{n1}(M'_{10} - M'_{12}) + a^{(n)}_{n2}(M'_{21} - M'_{23}) + \cdots \\
&\qquad + a^{(n)}_{nn}[M'_{n(n-1)} - M'_{n(n+1)}]
\end{aligned}\right\}\ldots(131)$$

For $n = 1$, coefficient $b^{(1)}_{11} = -d_{10}$ and coefficient $a^{(1)}_{11} = d_{12}$ so that Eqs. 131 reduce to:

$$\left.\begin{aligned}
M_{10} &= M'_{10} - d_{10}(M'_{10} - M'_{12}) \\
M_{12} &= M'_{12} + d_{12}(M'_{10} - M'_{12})
\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(132)$$

For $n = 2$, Eqs. 131 reduce to

$$\left.\begin{aligned}
M_{10} &= M'_{10} - \frac{d_{10}}{\Delta_2}(M'_{10} - M'_{12}) + \frac{q_{21}\,d_{10}}{\Delta_2}(M'_{21} - M'_{23}) \\
M_{12} &= M'_{12} + \frac{d_{12} - p_{12}}{\Delta_2}(M'_{10} - M'_{12}) + \frac{q_{21}(1 - d_{12})}{\Delta_2}(M'_{21} - M'_{23}) \\
M_{21} &= M'_{21} - \frac{q_{12}(1 - d_{21})}{\Delta_2}(M'_{10} - M_{12}) - \frac{d_{21} - p_{12}}{\Delta_2}(M'_{21} - M'_{23}) \\
M_{23} &= M'_{23} - \frac{q_{12}\,d_{23}}{\Delta_2}(M'_{10} - M'_{12}) + \frac{d_{23}}{\Delta_2}(M'_{21} - M'_{23})
\end{aligned}\right\}\ldots(133)$$

For the general n-degree system:

$$\left.\begin{aligned}
&b^{(n)}_{uv} \text{ (for } 0 < u < n \text{ and for } 0 < v < n) = b^{(n-1)}_{uv} + b^{(n-1)}_{u(n-1)} \\
&\qquad\qquad \times a^{(n-1)}_{(n-1)v}\, r_{(n-1)n}\, q_{n(n-1)} \frac{\Delta_{(n-1)}}{\Delta_n} \\
&b^{(n)}_{un} \text{ (for } 0 < u < n) = -\, b^{(n-1)}_{u(n-1)}\, q_{n(n-1)} \frac{\Delta_{(n-1)}}{\Delta_n} \\
&b^{(n)}_{nv} \text{ (for } 0 < v < n) = -\, a^{(n-1)}_{(n-1)v}\, r_{(n-1)n} [1 - d_{n(n-1)}] \frac{\Delta_{(n-1)}}{\Delta_n} \\
&b^{(n)}_{nn} = \frac{p_{(n-1)n}\, \Delta_{(n-2)} - d_{n(n-1)}\, \Delta_{(n-1)}}{\Delta_n} = \frac{[1 - d_{n(n-1)}]\, \Delta_{(n-1)}}{\Delta_n} - 1 \\
&a^{(n)}_{uv} \text{ (for } 0 < u < (n-1) \text{ and } 0 < v < n) = a^{(n-1)}_{uv} \\
&\qquad\qquad + a^{(n-1)}_{u(n-1)}\, a^{(n-1)}_{(n-1)v}\, r_{(n-1)n}\, q_{n(n-1)} \frac{\Delta_{(n-1)}}{\Delta_n} \\
&a^{(n)}_{un} \text{ (for } 0 < u < (n-1)) = -\, a^{(n-1)}_{u(n-1)}\, q_{n(n-1)} \frac{\Delta_{(n-1)}}{\Delta_n} \\
&a^{(n)}_{(n-1)v} \text{ (for } 0 < v < n) = a^{(n-1)}_{(n-1)v} + [a^{(n-1)}_{(n-1)(n-1)} - 1] \\
&\qquad\qquad \times a^{(n-1)}_{(n-1)v}\, r_{(n-1)n}.\, q_{n(n-1)} \frac{\Delta_{(n-1)}}{\Delta_n} \\
&a^{(n)}_{nv} \text{ (for } 0 < v < n) = -\, a^{(n-1)}_{(n-1)v}\, r_{(n-1)n}\, d_{n(n+1)} \frac{\Delta_{(n-1)}}{\Delta_n} \\
&a^{(n)}_{(n-1)n} = [1 - a^{(n-1)}_{(n-1)(n-1)}]\, q_{n(n-1)} \frac{\Delta_{(n-1)}}{\Delta_n} \\
&a^{(n)}_{nn} = d_{n(n+1)} \frac{\Delta_{(n-1)}}{\Delta_n}
\end{aligned}\right\} \quad ..(134)$$

Eqs. 134 operate by providing a procedure for building up the coefficients of a higher degree system from those of the lower degree system. Eqs. 132 and 133 for $n = 1$ and $n = 2$ may serve as the initial step in the progressive building up of the coefficients. These formulas are equivalent to the following matrix equations suggested in the author's presentation:

$$\begin{pmatrix} a^{(n)}_{11}\, a^{(n)}_{12} \cdots a^{(n)}_{1n} \\ a^{(n)}_{21}\, a^{(n)}_{22} \cdots a^{(n)}_{2n} \\ \\ \cdots\cdots\cdots\cdots\cdots \\ a^{(n)}_{n1}\, a^{(n)}_{n2} \cdots a^{(n)}_{nn} \end{pmatrix} = \begin{pmatrix} d_{12} & q_{21} & 0 & \cdots 0 \\ 0 & d_{23} & q_{32} & \cdots 0 \\ 0 & 0 & d_{34} & \cdots 0 \\ \cdots & \cdots & \cdots & \cdots\cdots \\ 0 & 0 & 0 & \cdots d_{n(n+1)} \end{pmatrix} \begin{pmatrix} 1 & q_{21} & 0 & \cdots 0 \\ q_{12} & 1 & q_{32} & \cdots 0 \\ 0 & q_{23} & 1 & \cdots 0 \\ \cdots & \cdots & \cdots & \cdots\cdots \\ 0 & 0 & 0 & \cdots 1 \end{pmatrix}^{-1} \quad ..(135a)$$

and

$$\begin{Bmatrix} b^{(n)}{}_{11} & b^{(n)}{}_{12} \cdots b^{(n)}{}_{1n} \\ b^{(n)}{}_{21} & b^{(n)}{}_{22} \cdots b^{(n)}{}_{2n} \\ \cdots & \cdots \\ b^{(n)}{}_{n1} & b^{(n)}{}_{n2} \cdots b^{(n)}{}_{nn} \end{Bmatrix} = - \begin{Bmatrix} d_{10} & 0 & 0 & \cdots 0 \\ q_{12} & d_{21} & 0 & \cdots 0 \\ 0 & q_{23} & d_{32} & \cdots 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots d_{n(n-1)} \end{Bmatrix} \begin{Bmatrix} 1 & q_{21} & 0 & \cdots 0 \\ q_{12} & 1 & q_{32} & \cdots 0 \\ 0 & q_{23} & 1 & \cdots 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots 1 \end{Bmatrix}^{-1} \quad \ldots(135b)$$

or, in abbreviated symbol form,

$$[a] = [K_a][D]^{-1}\{[1] - [Q']\}^{-1} \ldots\ldots\ldots\ldots (136a)$$

and

$$[b] = -[K_b][D]^{-1}\{[1] - [Q']\}^{-1} \ldots\ldots\ldots\ldots (136b)$$

The author proceeds to break up the matrix $\{[1] - [Q']\}^{-1}$ into a summation of successive approximations—$[1] + [Q'] + [Q']^2 + [Q']^3 + [Q']^4 + \cdots$. Although this development is interesting from the standpoint of a theoretical discussion, the writer questions the advantage of such a further transformation, since it changes an absolute value for each coefficient to an approximation, for which the degree of accuracy in each case must be predetermined. Furthermore, the breaking up of the reciprocal matrix into a summation is based on the application of Eq. 97; but that relationship has a condition of convergence imposed upon it and is known not to apply to some mathematical units, as for instance to negative numbers and to complex numbers. Its application in this instance, therefore, would have to be substantiated by a rigorous proof.

Fortunately, it is not difficult to set up a matrix equal to the original reciprocal matrix without resolving it into a summation:

$$\{[1] - [Q']\}^{-1} = \begin{Bmatrix} 1 & q_{21} & 0 & 0 & \cdots 0 & 0 \\ q_{12} & 1 & q_{32} & 0 & \cdots 0 & 0 \\ 0 & q_{23} & 1 & q_{43} & \cdots 0 & 0 \\ 0 & 0 & q_{34} & 1 & \cdots 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots 1 & q_{n(n-1)} \\ 0 & 0 & 0 & 0 & \cdots q_{(n-1)n} & 1 \end{Bmatrix}^{-1}$$

$$= \frac{\begin{Bmatrix} e^{(n)}{}_{11} & e^{(n)}{}_{12} \cdots e^{(n)}{}_{1n} \\ e^{(n)}{}_{21} & e^{(n)}{}_{22} \cdots e^{(n)}{}_{2n} \\ \cdots & \cdots \\ e^{(n)}{}_{n1} & e^{(n)}{}_{n2} \cdots e^{(n)}{}_{nn} \end{Bmatrix}}{\Delta_n} \quad \ldots\ldots\ldots\ldots (137)$$

For $n = 1$, the e-matrix of Eq. 137 is 1; for $n = 2$, the e-matrix is $\begin{bmatrix} 1 & -q_{21} \\ -q_{12} & 1 \end{bmatrix}$; and, for $n = 3$, the e-matrix is $\begin{Bmatrix} 1 - p_{23} & -q_{21} & q_{21}q_{32} \\ -q_{12} & 1 & -q_{32} \\ q_{12}q_{23} & -q_{23} & 1 - p_{12} \end{Bmatrix}$. In general, in the

e-matrix of degree n:

$$\begin{aligned}
e^{(n)}{}_{uv} \text{ (for } 0 < u < \overline{n-1} \text{ and } 0 < v < \overline{n-1}) &= e^{(n-1)}{}_{uv} - p_{(n-1)n}\, e^{(n-2)}{}_{uv} \\
e^{(n)}{}_{u(n-1)} \text{ (for } 0 < u < \overline{n-1}) &= e^{(n-1)}{}_{u(n-1)} \\
e^{(n)}{}_{(n-1)v} \text{ (for } 0 < v < \overline{n-1}) &= e^{(n-1)}{}_{(n-1)v} \\
e^{(n)}{}_{un} \text{ (for } 0 < u < n) &= -\, e^{(n)}{}_{u(n-1)}\, q_{n(n-1)} \\
e^{(n)}{}_{nv} \text{ (for } 0 < v < n) &= -\, e^{(n)}{}_{(n-1)v}\, q_{(n-1)n} \\
e^{(n)}{}_{nn} &= \Delta_{(n-1)}
\end{aligned}$$

Thus, in general, the elements of the e-matrix of any degree can be built up from the elements of the e-matrices of the lower degrees in a way analogous to the formation of the coefficients in the writer's formulas.

That Eqs. 134 yield the same results as those obtained from the matrix equations (Eqs. 135) is evident, except for the values of $a^{(n)}{}_{uv}$ and $b^{(n)}{}_{uv}$, from a study of the general properties of matrices and their adjoints as explained in the author's presentation. The values of $a^{(n)}{}_{uv}$ and $b^{(n)}{}_{uv}$ are not evident from the general matrix properties but can be derived from a special characteristic of the e-matrix, namely:

$$e^{(n)}{}_{un}\, e^{(n)}{}_{nv} = e^{(n)}{}_{uv}\, \Delta_{(n-1)} - e^{(n-1)}{}_{uv}\, \Delta_n \ldots\ldots\ldots\ldots\ldots (138)$$

which relationship can be proved by mathematical induction.

The writer's formulas were developed some time ago independently of matrix analysis and do not depend on matrix analysis for their derivation. In deducing these formulas the writer made use of the relationship quoted by the author as Eq. 97 but in the reverse order from that used by the author—that is, the fractional form was obtained from the infinite series.

For a two-degree system, apply a unit fixed-end moment at each of the beam ends, successively, and in each case distribute in accordance with the principle of moment distribution. Fig. 9 shows such a distribution for a fixed-end moment at 12. (Note that $1 - p_{12} = \Delta_2$). Other coefficients are obtained similarly. For each final end moment an infinite power series is obtained in p_{12}. The latter can be proved greater than zero and less than one, which is a necessary and sufficient condition for the convergence of the power series and the validity of relationship expressed by Eq. 97. The end moment coefficients are therefore summed up in accordance with this relationship.

	1		2
d_{10}	1 $-\,d_{12}$		
		q_{12} $-\,d_{21}\,q_{12}$	$d_{23}\,q_{12}$
$d_{10}\,p_{12}$	p_{12} $-\,d_{12}\,p_{12}$	$q_{12}\,p_{12}$ $-\,d_{21}\,q_{12}\,p_{12}$	$d_{23}\,q_{12}\,p_{12}$
	$p^2{}_{12}$		
....	....	....	....
$\dfrac{d_{10}}{1-p_{12}} = -\,b^{(2)}{}_{11}$	$1 - \dfrac{d_{12}-p_{12}}{1-p_{12}} = 1 - a^{(2)}{}_{11}$	$\dfrac{q_{12}\,(1-d_{21})}{1-p_{12}} = -\,b^{(2)}{}_{21}$	$\dfrac{d_{23}\,q_{12}}{1-p_{12}} = -\,a^{(2)}{}_{21}$

FIG. 9.—COEFFICIENTS OF $(M'_{10} - M'_{12})$ FOR A TWO-DEGREE SYSTEM

The same procedure can be used for higher degree systems, but the power series becomes complicated as the degree of the system increases and the convergence of the series becomes less evident. The relationships expressed by Eqs. 134 then become useful. To derive these relationships, the writer divided the n-degree system into two parts, the first part containing supports 1 to $(n - 1)$, inclusive, and the second part containing support n.

Fig. 10 shows the coefficients of fixed-end moments applied at points $v(v - 1)$

	1		2	
Fixed-end moment at point $v(v - 1)$	$b^{(n-1)}{}_{1v}$	$a^{(n-1)}{}_{1v}$	$b^{(n-1)}{}_{2v}$	$a^{(n-1)}{}_{2v}$
Fixed-end moment at point $(n - 1)n$	$-b^{(n-1)}{}_{1(n-1)}$	$-a^{(n-1)}{}_{1(n-1)}$	$-b^{(n-1)}{}_{2(n-1)}$	$-a^{(n-1)}{}_{2(n-1)}$

	$n - 1$		Carry-over to second part
Fixed-end moment at point $v(v - 1)$	$b^{(n-1)}{}_{(n-1)v}$	$a^{(n-1)}{}_{(n-1)v}$	$-r_{(n-1)n}\, a^{(n-1)}{}_{(n-1)v}$
Fixed-end moment at point $(n - 1)n$	$-b^{(n-1)}{}_{(n-1)(n-1)}$	$-a^{(n-1)}{}_{(n-1)(n-1)}$	$r_{(n-1)n}\, a^{(n-1)}{}_{(n-1)(n-1)}$

FIG. 10.—COEFFICIENTS FOR THE FIRST PART OF THE n-DEGREE SYSTEM

and $(n - 1)n$ for the end moments at the various supports of the first part of the system and for carry-overs to the second part. The carry-overs act as fixed-end moments in the second part and bring back corresponding carry-overs to the first part to act as new fixed-end moments at $(n - 1)n$. The sum total of all the carry-over moments induced in the first part is an infinite power series which can be summed up by Eq. 97. The coefficients of the second part were treated similarly. The effects of both the original fixed-end moments and the summation of the infinite series of carry-overs combine to yield Eqs. 134.

The summation of the carry-overs and the final form of Eqs. 134 are conditioned on the following relationships:

$$0 < q_{n(n-1)}\, r_{(n-1)n}\, a^{(n-1)}{}_{(n-1)(n-1)} < 1 \quad \ldots\ldots\ldots\ldots (139)$$

$$a^{(n)}{}_{nn} = \frac{d_{n(n+1)}\, \Delta_{(n-1)}}{\Delta_n} \quad \ldots\ldots\ldots\ldots (140)$$

and

$$q_{n(n-1)}\, r_{(n-1)n}\, a^{(n-1)}{}_{(n-1)(n-1)} = \frac{p_{(n-1)n}\, \Delta_{(n-2)}}{\Delta_{(n-1)}} \quad \ldots\ldots\ldots\ldots (141)$$

These relationships are interdependent and can be proved together by mathematical induction, remembering that

$$\Delta_n = \Delta_{(n-1)} - p_{(n-1)n}\, \Delta_{(n-2)} \quad \ldots\ldots\ldots\ldots (142)$$

which follows from the peculiar characteristic of the determinant Δ.

Although the higher degree coefficients are thus obtained from those of the lower degrees, the writer has found it convenient to develop Eqs. 131 for a system of a sufficiently high degree and to apply them also to lower degree systems by equating to zero the constants of those spans which are not a part of the system under consideration.

The moments for a five-degree system (which is probably of a high enough degree for ordinary work with continuous beams) are as follows:

$$\left.\begin{aligned}
M_{10} &= M'_{10} - \frac{d_{10}(1 - p_{23} - p_{34} - p_{45} + p_{23}\,p_{45})}{\Delta_5}(M'_{10} - M'_{12} \\
&\quad + \frac{d_{10}\,q_{21}(1 - p_{34} - p_{45})}{\Delta_5}(M'_{21} - M'_{23}) - \frac{d_{10}\,q_{21}\,q_{32}(1 - p_{45})}{\Delta_5} \\
&\quad \times (M'_{32} - M'_{34}) + \frac{d_{10}\,q_{21}\,q_{32}\,q_{43}}{\Delta_5}(M'_{43} - M'_{45}) \\
&\qquad\qquad - \frac{d_{10}\,q_{21}\,q_{32}\,q_{43}\,q_{54}}{\Delta_5}(M'_{54} - M'_{56}) \\
M_{12} &= M'_{12} + \frac{d_{12}(1 - p_{23} - p_{34} - p_{45} + p_{23}\,p_{45}) - p_{12}(1 - p_{34} - p_{45})}{\Delta_5} \\
&\quad \times (M'_{10} + M'_{12}) + \frac{(1 - d_{12})\,q_{21}(1 - p_{34} - p_{45})}{\Delta_5}(M'_{21} - M'_{23}) \\
&\quad - \frac{(1 - d_{12})q_{21}\,q_{32}(1 - p_{45})}{\Delta_5}(M'_{32} - M'_{34}) + \frac{(1 - d_{12})\,q_{21}\,q_{32}\,q_{43}}{\Delta_5} \\
&\qquad \times (M'_{43} - M'_{45}) - \frac{(1 - d_{12})\,q_{21}\,q_{32}\,q_{43}\,q_{54}}{\Delta_5}(M'_{54} - M'_{56}) \\
M_{21} &= M'_{21} + \frac{q_{12}\,p_{23}(1 - p_{45}) - q_{12}(1 - d_{21})\,(1 - p_{34} - p_{45})}{\Delta_5} \\
&\quad \times (M'_{10} - M'_{12}) - \frac{(d_{21} - p_{12})\,(1 - p_{34} - p_{45})}{\Delta_5}(M'_{21} - M'_{23}) \\
&\quad + \frac{q_{32}(d_{21} - p_{12})\,(1 - p_{45})}{\Delta_5}(M'_{32} - M'_{34}) - \frac{q_{32}\,q_{43}(d_{21} - p_{12})}{\Delta_5} \\
&\qquad \times (M'_{43} - M'_{45}) + \frac{q_{32}\,q_{43}\,q_{54}(d_{21} - p_{12})}{\Delta_5}(M'_{54} - M'_{56}) \\
M_{23} &= M'_{23} + \frac{q_{12}\,p_{23}(1 - p_{45}) - q_{12}\,d_{23}(1 - p_{34} - p_{45})}{\Delta_5} \\
&\quad \times (M'_{10} - M'_{12}) - \frac{p_{23}(1 - p_{45}) - d_{23}(1 - p_{34} - p_{45})}{\Delta_5} \\
&\quad \times (M'_{21} - M'_{23}) + \frac{q_{32}(1 - d_{23} - p_{12})\,(1 - p_{45})}{\Delta_5}(M'_{32} - M'_{34}) \\
&\quad - \frac{q_{32}\,q_{43}(1 - d_{23} - p_{12})}{\Delta_5}(M'_{43} - M'_{45}) \\
&\qquad\qquad + \frac{q_{32}\,q_{43}\,q_{54}(1 - d_{23} - p_{12})}{\Delta_5}(M'_{54} - M'_{56}) \\
M_{32} &= M'_{32} + \frac{q_{12}\,q_{23}(1 - d_{32} - p_{34} - p_{45} + d_{32}\,p_{45})}{\Delta_5}(M'_{10} - M'_{12}) \\
&\quad - \frac{q_{23}(1 - p_{34} - p_{45} - d_{32} + d_{32}\,p_{45})}{\Delta_5}(M'_{21} - M'_{23}) \\
&\quad + \frac{(p_{23} - d_{32} + d_{32}\,p_{12})\,(1 - p_{45})}{\Delta_5}(M'_{32} - M'_{34}) \\
&\quad - \frac{q_{43}(p_{23} - d_{32} + d_{32}\,p_{12})}{\Delta_5}(M'_{43} - M'_{45}) \\
&\qquad\qquad + \frac{q_{43}\,q_{54}(p_{23} - d_{32} + d_{32}\,p_{12})}{\Delta_5}(M'_{54} - M'_{56})
\end{aligned}\right\}\ldots(143)$$

and

$$\Delta_5 = 1 - p_{12} - p_{23} - (1 - p_{12})p_{34} - (1 - p_{12} - p_{23})p_{45} \ldots\ldots (144)$$

The remainder of the moments can be obtained from the foregoing through the symmetry of the system. In matrix form,

$$\begin{Bmatrix} M_{12} - M'_{12} \\ M_{23} - M'_{23} \\ M_{34} - M'_{34} \\ M_{45} - M'_{45} \\ M_{56} - M'_{56} \end{Bmatrix} = \begin{bmatrix} d_{12} & q_{21} & 0 & 0 & 0 \\ 0 & d_{23} & q_{32} & 0 & 0 \\ 0 & 0 & d_{34} & q_{43} & 0 \\ 0 & 0 & 0 & d_{45} & q_{54} \\ 0 & 0 & 0 & 0 & d_{56} \end{bmatrix} \frac{[e]^{(5)}}{\Delta_5} \begin{Bmatrix} (M'_{10} - M'_{12}) \\ (M'_{21} - M'_{23}) \\ (M'_{32} - M'_{34}) \\ (M'_{43} - M'_{45}) \\ (M'_{54} - M'_{56}) \end{Bmatrix} \ldots (145a)$$

and

$$\begin{Bmatrix} M_{10} - M'_{10} \\ M_{21} - M'_{21} \\ M_{32} - M'_{32} \\ M_{43} - M'_{43} \\ M_{54} - M'_{54} \end{Bmatrix} = \begin{bmatrix} -d_{10} & 0 & 0 & 0 & 0 \\ -q_{12} & -d_{21} & 0 & 0 & 0 \\ 0 & -q_{23} & -d_{32} & 0 & 0 \\ 0 & 0 & -q_{34} & -d_{43} & 0 \\ 0 & 0 & 0 & -q_{45} & -d_{54} \end{bmatrix} \frac{[e]^{(5)}}{\Delta_5}$$

$$\times \begin{Bmatrix} (M'_{10} - M'_{12}) \\ (M'_{21} - M'_{23}) \\ (M'_{32} - M'_{34}) \\ (M'_{43} - M'_{45}) \\ (M'_{54} - M'_{56}) \end{Bmatrix} \ldots (145b)$$

in which

$$[e]^{(5)} = \begin{bmatrix} 1-p_{23}-p_{34}-p_{45}(1-p_{23}) & -q_{21}(1-p_{34}-p_{45}) & q_{21}q_{32}(1-p_{45}) & -q_{21}q_{32}q_{43} & q_{21}q_{32}q_{43}q_{54} \\ -q_{12}(1-p_{34}-p_{45}) & 1-p_{34}-p_{45} & -q_{32}(1-p_{45}) & q_{32}q_{43} & -q_{32}q_{43}q_{54} \\ q_{12}q_{23}(1-p_{45}) & -q_{23}(1-p_{45}) & (1-p_{12})(1-p_{45}) & -q_{43}(1-p_{12}) & q_{43}q_{54}(1-p_{12}) \\ -q_{12}q_{23}q_{34} & q_{23}q_{34} & -q_{34}(1-p_{12}) & 1-p_{12}-p_{23} & -q_{54}(1-p_{12}-p_{23}) \\ q_{12}q_{23}q_{34}q_{45} & -q_{23}q_{34}q_{45} & q_{34}q_{45}(1-p_{12}) & -q_{45}(1-p_{12}-p_{23}) & 1-p_{12}-p_{23}-p_{34}(1-p_{12}) \end{bmatrix} \ldots (146)$$

In conclusion, the writer is grateful for the opportunity of having the results of his own studies verified by a new approach and hopes that this discussion will contribute to the general study of the subject.

Rufus Oldenburger,[17] **Esq.**—In a paper[18] by the writer, matrices were introduced to clarify the mathematics of the Cross method of structural analysis[19] because the operations involved in this analysis are linear, and the convergence questions can be expressed simply in terms of matrices. The Cross method is an infinite process whereby fixed joints are relaxed so that the moment distribution for a given structure is obtained from the moment distribution when all joints are fixed.

Mr. Benscoter shows that by matrices one can proceed directly from the fixed joint moments to the final moment distribution. This method, however, involves computation of the inverse of a matrix. Because generally it is exceedingly difficult to obtain the inverse of a matrix directly, the series (Eq. 98) is used, in which $[I]$ is the identity matrix and $[Q]$ is a matrix of the same order. This expansion is suggested by the similar expansion (Eq. 97) for $1/(1 - \epsilon)$ in the case of a number ϵ.

That one can work directly from the initial to the final moment distribution by matrices follows from the fact that the final moments can be obtained from the initial moments by solving linear equations. By "directly" is meant "in a finite number of steps." The importance of Mr. Benscoter's use of matrices lies in the fact that this point of view leads to an infinite process through the use of Eq. 98. It is sometimes simple to obtain successive powers of a matrix $[Q]$, so that if the series converges rapidly, a good approximation to the inverse of $[I] - [Q]$ is obtained. Computation of complicated determinants is thus avoided; but to use Eq. 98 is to employ an approximation method of computation. In fact, it has been shown that each term in that equation corresponds to a standard computation cycle and that each cycle is composed of two steps. "In the first step all joints are balanced and in the second step the carry-over calculation is performed in each span."

The great value of Eq. 98 thus lies in the fact that the terms in the series correspond to the standard accepted procedure for "balancing" moments. It is, however, only one method out of infinitely many for applying the Hardy Cross type of method. The approach in the writer's paper[18] on the subject is general and therefore covers the balancing process corrresponding to Eq. 98. In particular, the convergence theory contained in that paper can be applied to Eq. 98. For reasons of exposition both Mr. Benscoter and the writer have restricted themselves to the case of the continuous beam. If the numbers 1, 2, 3, · · ·, n be assigned to the supports along a given beam, the convergence proof of the writer for balancing alternately odd-numbered and even-numbered supports goes over without change to the standard process corresponding to Eq. 98.

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[17] Prof. of Math., Illinois Inst. of Tech., Chicago, Ill.

[18] "Convergence of Hardy Cross's Balancing Process," by Rufus Oldenburger, *Journal of Applied Mechanics*, December, 1940, pp. A-166 to A-170.

[19] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE Vol., 96, 1932, pp. 1–10.

In the writer's matrix treatment of continuous beams the vector $[M]$ of the desired moments is computed from the relation:

$$[M] = \lim_{n \to \infty} [A]^n [M'] \quad \text{.................... (147)}$$

for the vector $[M']$ of given moments, provided that the balancing process is a cycle repeated forever. For the elements of the vectors occurring in Eq. 147 the total moments at the supports have been used. This was done for mathematical simplicity. Mr. Benscoter's technique of working with left end moments and right end moments, instead of lumping the moments at each support, has considerable value for understanding the theory of the balancing process, and is needed to determine the final left end moments and right end moments. However, the writer's treatment readily extends to the case where the elements in the vectors of Eq. 147 are the individual unlumped moments. Of course, $[A]$ must be changed accordingly.

Matrices are of unquestionable value in understanding methods for obtaining moment distributions in structures. When it comes to actual computations the situation is different. The writer has still to see a computational process in which the explicit use of matrices gives an economy of effort, although matrices are often of great value in prescribing what computational process should be employed. In any case, it is easier to use Eq. 147 for left end moments and right end moments, obtaining the result by computing powers of a matrix, than to use Eq. 98, where one must perform the additional operation of adding powers after they have been computed.

Mr. Benscoter's paper is an excellent exposition of the matrix point of view in structural analysis, and sheds much light on what moment balancing really is. There is still a need, however, for more specific information on the convergence of the Hardy Cross process, in particular for the case where the moments at supports are not lumped. Above all the problem of the rapidity of convergence should be studied. The designer should know how many steps must be taken to obtain a given accuracy. Because of the importance of the subject a complete physical and mathematical treatment should appear in a book on general structures, beginning from basic fundamentals. In such a treatment matrices would play a dominant role.

Horace A. Johnson,[20] Assoc. M. ASCE.—A service has been done for the engineering profession by the presentation of the subject of matrices and determinants. It has always been a source of wonder to the writer that so few engineers are acquainted with these powerful mathematical tools, or even with the most elementary use of determinants.

However, the author has possibly been overenthusiastic in the use of matrices. For example, Eqs. 36 are expressed as $[A][x] = [k]$ in Eq. 39. Although Eq. 39 is a more succinct expression of Eqs. 36, it conveys less meaning to a person not using matrices frequently, as is the case with most engineers, and it in no way reduces the amount of work required for the numerical solution of Eqs. 36.

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[20] Engr. (Civ.), Corps of Engrs., War Dept., Sacramento, Calif.

Eqs. 36 may be solved easily by use of determinants as follows:

$$x_1 = \frac{\begin{vmatrix} 2 & 1 & -2 \\ 1 & 2 & 3 \\ 3 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \\ 2 & 1 & 4 \end{vmatrix}} = \frac{41}{45}; \quad x_2 = \frac{\begin{vmatrix} 3 & 2 & -2 \\ 1 & 1 & 3 \\ 2 & 3 & 4 \end{vmatrix}}{45} = -\frac{13}{45};$$

$$x_3 = \frac{\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix}}{45} = \frac{10}{45} = \frac{2}{9}$$

In each of these cases the solution is obtained by substituting the constant terms for the coefficients of the desired unknown in the numerator determinants, the remaining terms being the coefficients of the other unknowns. The denominator determinant in each case is composed of the coefficients of the unknowns. This procedure seems simpler than the solution given by the author.

Solution of simultaneous equations by the use of determinants is a systematic approach to the problem. It does not, however, avoid a large amount of numerical work as the expansion of an nth-order determinant gives $n!$-terms; thus, the expansion of a fifth-order determinant gives 120 terms.

Lloyd T. Cheney,[21] Jun. ASCE.—The possible advantages of matrix algebra in various methods of structural analysis are called to the attention of engineers by the author of this commendable paper. The painstaking explanation of elementary operations gives the engineer a condensation of these processes, which would otherwise be found only after considerable search in one or more mathematics texts. This explanation should serve to demonstrate the relative ease of manipulation of matrix algebra and to reassure the engineer that with the necessary practice it can become a most powerful instrument.

Structural engineers are generally introduced to series of simultaneous equations in applying the classical theorem of three moments or the method of slope deflection.[22] A frequently suggested method of solution is that of arranging the coefficients and constant terms in tabular form.[23] By successive manipulations the series is reduced to a single equation containing a single unknown variable. With the variable thus determined, successive substitutions are made until all variables have been found. For example, the system illustrated in Eqs. 36*a*, 36*b*, and 36*c*, Table 2, has been arranged with the steps numbered and the operations noted so that the table is self-explanatory.

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[21] Asst. Prof. of Civ. Eng., Cornell Univ., Ithaca, N. Y.

[22] *Engineering Studies No. 1*, Univ. of Minnesota, Minneapolis, Minn.

[23] "Structural Theory," by Hale Sutherland and Harry Lake Bowman, John Wiley & Sons, Inc., New York, N. Y., 1935, p. 212.

TABLE 2.—FORM FOR SOLUTION OF SIMULTANEOUS EQUATIONS

Step number	Operation	COEFFICIENTS			Right side of equation, k
		x_1	x_2	x_3	
(1)	(2)	(3)	(4)	(5)	(6)
1	Eq. 36*a*	3	1	−2	2
2	Eq. 36*b*	1	2	3	1
3	Eq. 36*c*	2	−1	4	3
4	Step 2 ×step 1	6	2	−4	4
5	....	1	2	3	1
6	Step 2 ×step 3	4	−2	8	6
7	Step 4 +step 6	10	0	4	10
8	Step 5 +step 6	5	0	11	7
9	Step 2 ×step 8	10	0	22	14
10	Step 9 −step 7	0	0	18	4

Step 10 of Table 2 gives the value for x_3. Study of the table discloses that within it are the matrices shown by Eq. 37, but that no advantage has been taken of matrix algebra in the solution. This suggests that engineers have been working with matrices but have hesitated to use them as such.

Attention has been called to the fact that other methods yield quicker results for a single condition of load because of the possibility of lengthy computations in evaluating the reciprocal of the stiffness matrix $[K]$. It might be well to emphasize that methods for evaluating more cumbersome determinants are not so frightening as the size of the determinant itself would suggest.

The algebraic sum of the products of the elements of a row and their corresponding minors will give a quick means of evaluating larger determinants. Employing this method with the 4-by-4 determinant:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix}$$

$$+ a_{13} \begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} - a_{14} \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix} \ldots\ldots\ldots\ldots\ldots (148)$$

Still larger determinants may be evaluated by expressing them in terms of the algebraic sum of the products of smaller determinants. Consider the 6-by-6 determinant:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{vmatrix} = |\,A\,| \ldots\ldots\ldots\ldots\ldots (149)$$

Making the substitutions—

$$\left.\begin{array}{ll} |B| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} & |C| = \begin{vmatrix} a_{14} & a_{15} & a_{16} \\ a_{24} & a_{25} & a_{26} \\ a_{34} & a_{35} & a_{36} \end{vmatrix} \\ |D| = \begin{vmatrix} a_{41} & a_{42} & a_{43} \\ a_{51} & a_{52} & a_{53} \\ a_{61} & a_{62} & a_{63} \end{vmatrix} & |E| = \begin{vmatrix} a_{44} & a_{45} & a_{46} \\ a_{54} & a_{55} & a_{56} \\ a_{64} & a_{65} & a_{66} \end{vmatrix} \end{array}\right\} \ldots\ldots(150)$$

—Eq. 149 becomes:

$$|A| = |B||E| - |C||D| \ldots\ldots\ldots\ldots\ldots(151)$$

By using a combination of these methods, plus manipulations to reduce some of the elements to zero, the evaluations may be accomplished. However, although these computations will be routine, there is no avoiding the fact that they will be time consuming.

The new application of matrix algebra should stimulate interest in this mathematical instrument and result in its wider application to common engineering problems.

Paul W. Norton,[24] M. ASCE.—Attention has been directed by Mr. Benscoter to interesting possibilities in the use of matrix algebra for the solution of structural problems involving simultaneous equations. By taking advantage of the implement afforded by the matrix notation, the author achieves a noteworthy abbreviation of the process of solution by means of determinants in two important respects: (1) For the several determinant ratios expressing the several unknown quantities, he substitutes a single matrix expression, comprehending in itself the solution for all the unknowns; and (2) the reduction of this expression to the final numerical results involves, not the evaluation of as many determinants as there are unknown quantities, but the evaluation of only one, plus a relatively simple operation on the matrices.

The application of these methods to the determination of moments in rectangular frames, such as those commonly encountered in building construction, is of particular interest to the writer. Some earlier studies of a similar nature, followed by careful reading of this paper, have confirmed him in the conclusions that, as applied to such problems, the formation of the fundamental matrix equation follows even more immediately and obviously from an inspection of the stiffness properties of the frame, and of readily derived functions of its loading, and the indicated numerical operations can be done by processes even less laborious than revealed by the author.

To present clearly a discussion which seems to be justified by this view, and by the hope that its demonstration may contribute something to the usefulness of the author's paper, it will be expedient to show the application of the methods under consideration to the solution of a specific example. The derivation of the determinant and matrix expressions, and the routine of computation will be illustrated, with such explanatory comment as seems useful to the purpose. The adaptability of the method to frames in which occur members having varying moments of inertia, and to those subject to translation of joints, or sidesway, will be indicated without details of analysis or numerical illustration.

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[24] Cons. Engr., Boston, Mass.

Notation.—In addition to the notation of the paper the following symbols are introduced:

M_{pq} = clockwise moment acting at point P on the member (or part of member) PQ.

M_f = Fixed-end moment

$M_{f,pq}$ = a moment M_f at point P; that is, the value of M_{pq} if member PQ were fixed at both ends.

$M'_{f,pq}$ = a modified value of M_f; that is, the value of M_{pq} if member PQ were fixed at point P and freely supported at point Q; for a prismatic member, $M'_{f,pq} = M_{f,pq} - \frac{1}{2} M_{f,qp}$.

$\sum M_{f,p}$ = summation of all values of M_f or $(M_f)'$ for all members intersecting at joint P.

K_{pq} = stiffness coefficient of member PQ at end P; the actual stiffness divided by the constant 4 E. For a prismatic member K_{pq} is the $\frac{I}{L}$-value of member PQ. This is the stiffness coefficient as defined in the familiar form of the slope-deflection equation. Its value is one fourth of that defined by the author, and is preferable because it avoids fractions and simplifies numerical work.

K'_{pq} = a value of K_{pq} modified to account for freedom at end Q. For a prismatic member, $K'_{pq} = \frac{3}{4} \times K_{pq}$.

$\sum K_p$ = the summation of all values of K, or K', for all members intersecting at joint P.

Φ = the angular clockwise rotation of joint P, multiplied by 2 E.

z = a parameter moment; z_{pq} and z_{qp} are parameter moments pertaining to member PQ, the derivation of which (by methods to be described) leads to a determination of moment conditions throughout span PQ.

Determinant of the Frame.—The frame in Fig. 11 is that used by Hardy Cross, M. ASCE, to illustrate the procedure described in his paper, "Analysis of Continuous Frames by Distributing Fixed-End Moments."[25] The data are here transcribed as given by Professor Cross except: (*a*) The signs of the fixed-end moments are altered to conform to the convention here established; and (*b*) for the member hinged at point F the "modified" stiffness coefficient and fixed-end moment at point C are used. These are:

$$K'_{cf} = \tfrac{3}{4} \times K_{cf} = \tfrac{3}{4} \times 2 = 1.5 \ldots\ldots\ldots\ldots\ldots (152a)$$

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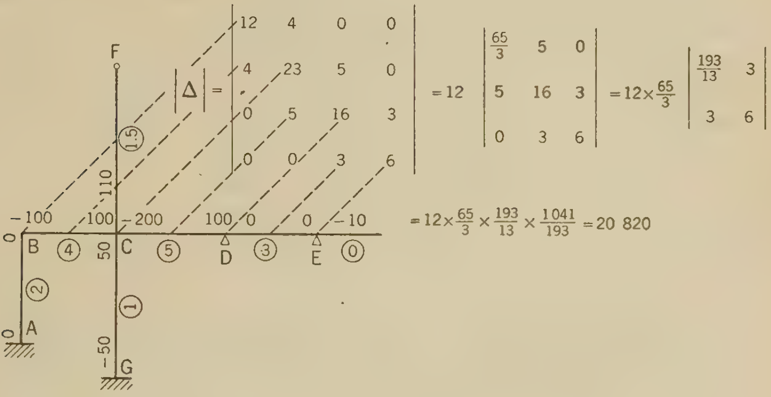
[25] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 8, Fig. 2.

and

$$M'_{f,ef} = M_{f,ef} - \tfrac{1}{2} M_{f,fe} = 80 - \tfrac{1}{2}(-60) = 110 \ldots\ldots (152b)$$

In Fig. 11, stiffness coefficients are indicated by numerals in circles, and fixed-end moments by unenclosed numerals at ends of members.



$$|\Delta| = \begin{vmatrix} 12 & 4 & 0 & 0 \\ 4 & 23 & 5 & 0 \\ 0 & 5 & 16 & 3 \\ 0 & 0 & 3 & 6 \end{vmatrix} = 12 \begin{vmatrix} \frac{65}{3} & 5 & 0 \\ 5 & 16 & 3 \\ 0 & 3 & 6 \end{vmatrix} = 12 \times \frac{65}{3} \begin{vmatrix} \frac{193}{13} & 3 \\ 3 & 6 \end{vmatrix}$$

$$= 12 \times \frac{65}{3} \times \frac{193}{13} \times \frac{1\,041}{193} = 20\,820$$

Fig. 11

If the slope-deflection equation were to be used to express the moment at each interior joint in terms of the rotations at that and at adjacent joints, four equations would result relating the four rotations at points B, C, D, and E. One method of solving this set of equations simultaneously involves forming a determinant whose elements are the coefficients of the unknown rotations. This determinant, often called the "determinant of the set of equations," will be referred to herein as $|\Delta|$, the determinant of the frame. It is important in the analysis of the frame, either by determinants or by the author's matrix expressions; it is independent of the loading; and it can be derived by inspection. For its elements, it has functions of the K-values of the members, related to the array of values recorded on the diagram of the frame as illustrated by Fig. 11. The order of $|\Delta|$ is equal to the number of interior joints; the principal diagonal consists of the double sum of the K-values of all members meeting at the corresponding joints; the adjacent diagonals (superdiagonal and subdiagonal) consist of the K-values of the members, in order; and all other elements are zero. Thus, the general expression for the determinant of such a frame, in which interior joints are denoted by B, C, D, E, is:

$$|\Delta| = \begin{vmatrix} 2\sum K_b & K_{cb} & 0 & 0 \\ K_{bc} & 2\sum K_c & K_{dc} & 0 \\ 0 & K_{cd} & 2\sum K_d & K_{ed} \\ 0 & 0 & K_{de} & 2\sum K_e \end{vmatrix} \ldots\ldots\ldots\ldots (153)$$

(it being understood that modified values K' and $(M_f)'$ are to be used in place of K and M_f for members hinged at the remote end).

Rotations of the Joints.—To continue with the analysis of the frame by means of determinants it would be necessary to form another determinant corresponding to each joint, and involving the load terms, or fixed-end moments. These also can be derived by inspection, and the numerical computations can be comparatively brief.

However, there is a marked advantage in being able to express the solution for all parts of the frame in a single matrix equation, as developed by the author. Mr. Benscoter shows that the equivalent of the four expressions for the rotations at the joints B, C, D, and E of the frame, which would result from the determinant analysis, is (letting Δ represent the array of elements in the frame determinant):

$$\begin{pmatrix}\Phi_b\\ \Phi_c\\ \Phi_d\\ \Phi_e\end{pmatrix} = [\Delta]^{-1}\times\begin{pmatrix}-M_{f,b}\\ -M_{f,c}\\ -M_{f,d}\\ -M_{f,e}\end{pmatrix} = \begin{pmatrix}12 & 4 & 0 & 0\\ 4 & 23 & 5 & 0\\ 0 & 5 & 16 & 3\\ 0 & 0 & 3 & 6\end{pmatrix}^{-1}\times\begin{pmatrix}100\\ -60\\ -100\\ 10\end{pmatrix}$$

$$= \frac{1}{20{,}820}\begin{pmatrix}23\times\frac{343}{23}\times\frac{1{,}851}{343} & -4\times16\times\frac{87}{16} & 4\times5\times6 & -4\times5\times3\\ -4\times16\times\frac{87}{16} & 12\times16\times\frac{87}{16} & 12\times5\times6 & -12\times5\times3\\ 4\times5\times6 & -12\times5\times6 & 12\times\frac{65}{3}\times6 & -12\times\frac{65}{3}\times3\\ -4\times5\times3 & 12\times5\times3 & -12\times\frac{65}{3}\times3 & 12\times\frac{65}{3}\times\frac{193}{13}\end{pmatrix}$$

$$\times\begin{pmatrix}100\\ -60\\ -100\\ 10\end{pmatrix} = \frac{1}{20{,}820}\begin{pmatrix}1{,}851 & -348 & 120 & -60\\ -348 & 1{,}044 & -360 & 180\\ 120 & -360 & 1{,}560 & -780\\ -60 & 180 & -780 & 3{,}860\end{pmatrix}\times\begin{pmatrix}100\\ -60\\ -100\\ 10\end{pmatrix}$$

$$= \frac{1}{20{,}820}\begin{pmatrix}185{,}100 & +20{,}880 & -12{,}000 & -600\\ -34{,}800 & -62{,}640 & +36{,}000 & +1{,}800\\ 12{,}000 & +21{,}600 & -156{,}000 & -7{,}800\\ -6{,}000 & -10{,}800 & +78{,}000 & +38{,}600\end{pmatrix}$$

$$= \frac{1}{20{,}820}\begin{pmatrix}193{,}380\\ -59{,}640\\ -130{,}200\\ 99{,}800\end{pmatrix} = \begin{pmatrix}9.288\\ -2.864\\ -6.253\\ 4.793\end{pmatrix} \tag{154}$$

—that is, $\Phi_b = 9.288$; $\Phi_c = -2.864$; $\Phi_d = -6.253$; and $\Phi_e = 4.793$.

Before proceeding, attention may be directed to the process of reducing a determinant to a numerical value, such as the 20,820 in Fig. 11, and to the routine of computation in determining the elements of the matrix reciprocal, as illustrated by Eq. 154.

Evaluation of a Determinant.—Numerical evaluation of a determinant by repeated summation of minors, as described in most elementary texts are illustrated by the author in Eqs. 2*a* and 2*b*, is a process that becomes increasingly laborious for those of orders higher than the third. (A determinant of the tenth order, with no zeros among the elements, would require writing and evaluating 2,606,500 minors of all orders.) A much briefer method, illustrated by the numerical work of Fig. 11, and of general application, is the following: In any determinant,

$$|D| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix} \dots\dots\dots\dots\dots\dots\dots (155)$$

let a "modified minor" of the element a_{11} be formed, by substituting for each element a_{jk} of the minor a value b_{jk} derived by the formula:

$$b_{jk} = a_{jk} - (a_{1k} \times a_{j1}/a_{11}) \dots\dots\dots\dots\dots\dots (156)$$

This modification of the minor does not change its value, by virtue of the theorem:

> "If the elements of any column (or row) of a determinant be altered by adding to each the corresponding element of another column (or row), multiplied by a constant, the value of the determinant is thereby unchanged."

Then $|D|$ has the form:

$$|D| = \begin{vmatrix} a_{11} & 0 & 0 & \cdots \\ a_{21} & b_{22} & b_{23} & \cdots \\ a_{31} & b_{32} & b_{33} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix} = a_{11} \begin{vmatrix} b_{22} & b_{23} & \cdots \\ b_{32} & b_{33} & \cdots \\ \cdots & \cdots & \cdots \end{vmatrix} \dots\dots\dots\dots (157)$$

and the evaluation will have proceeded by reduction of the determinant of the nth order to one of the $(n - 1)$th order. The numerical value of any element b_{jk} is usually ascertainable mentally with small effort. Thus, in $|\Delta|$ of the example considered, the element a_{22} is 23, and the "modified" element b_{22} is $23 - 4 \times 4/12 = 65/3$. The presence of zeros among the elements also greatly facilitates the computation; thus, the element a_{32} in $|\Delta|$ is 5, and $b_{32} = 5 - 0 \times 4/12 = 5$, no modification of the element resulting. Furthermore, the successive reductions in order accomplished by this process result in a continued product of numbers, which, expressed as fractions, invariably lead readily to the final result by cancellations of like figures in numerators and denominators.

Formation of the Reciprocal Matrix.—The author has shown (see Eq. 33) that the reciprocal of $[\Delta]$,

$$[\Delta]^{-1} = \frac{\text{adjoint of } [\Delta]}{|\Delta|} \dots\dots\dots\dots\dots\dots (158)$$

The value of $|\Delta|$ having already been found, it remains to discover the most direct and rapid way to write the adjoint. The author remarks that in this lies the chief difficulty of the computation; it seems to the writer that the device whereby the matrix reciprocal is expressed in the form of a power series (see heading, "Matrix Reciprocal Expressed by Power Series") is of academic interest, and makes little progress toward facilitating the computation.

In obtaining the results in the problem used as an example it has not been necessary to write down any figures other than those appearing in Eq. 154. The fact that all elements of $[\Delta]$ are zero, except those in the principal and two adjacent diagonals, makes easy the formation of its adjoint.

Let a matrix of this form, and of the nth order, be

$$[M] = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix} \dots\dots\dots\dots\dots\dots (159)$$

Proceeding to form the adjoint by substituting for each element its cofactor, and to evaluate each cofactor (which is a determinant of the $(n - 1)$th order) by the process outlined herein under the heading, "Evaluation of a Determinant," and illustrated in the equation in Fig. 11, a matrix (still of the nth order) is obtained such that each element is the product of $(n - 1)$ factors, as follows:

Adjoint of $[M]$

$$= \begin{bmatrix} a_{22}\,a'_{33}\,a''_{44} & -a_{21}\,a_{33}\,a'_{44} & a_{21}\,a_{32}\,a_{44} & -a_{21}\,a_{32}\,a_{43} & \cdots \\ -a_{12}\,a_{33}\,a'_{44} & a_{11}\,a_{33}\,a'_{44} & -a_{11}\,a_{32}\,a_{44} & a_{11}\,a_{32}\,a_{43} & \cdots \\ a_{12}\,a_{23}\,a_{44} & -a_{11}\,a_{23}\,a_{44} & a_{11}\,a'_{22}\,a_{44} & -a_{11}\,a'_{22}\,a_{43} & \cdots \\ -a_{12}\,a_{23}\,a_{34} & a_{11}\,a_{23}\,a_{34} & -a_{11}\,a'_{22}\,a_{34} & a_{11}\,a'_{22}\,a''_{33} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \dots (160)$$

Observation of easily stated rules to which the elements of the adjoint conform enables one to write it at once for a continuant matrix of any order:

(*a*) The subscripts of the $(n - 1)$ factors of each element of the adjoint are, in order, those of the elements in the principal diagonal of the cofactor of the corresponding element of the matrix.

(*b*) The sign of the element is the sign of the cofactor.

(*c*) Primes occur only on factors whose two subscripts are equal, and greater by unity than the two equal subscripts of the factor next preceding. Such a factor immediately following a primed factor is double primed.

(*d*) The numerical value of a primed factor (as a'_{77}) is a modified value of the corresponding element a_{77} of $[M]$, derived by the formulas:

$$a'_{77} = a_{77} - a_{67} \times a_{76}/a_{66} \dots\dots\dots\dots\dots\dots (161a)$$

and

$$a''_{77} = a_{77} - a_{67} \times a_{76}/a'_{66} \dots\dots\dots\dots\dots\dots (161b)$$

The application of these principles leads to the matrix shown in Eq. 154 as the adjoint of $[\Delta]$. For example, the cofactor of the element $b_{22} = 23$ of $[\Delta]$ is:

$$\begin{vmatrix} 12 & 0 & 0 \\ 0 & 16 & 3 \\ 0 & 3 & 6 \end{vmatrix} = 12 \times 16 \left(6 - \tfrac{9}{16}\right) = 1{,}044 \dots\dots\dots\dots (162)$$

—and similarly for the evaluation of the other cofactors. The factors of each element of the adjoint can be written quickly, either by using Eq. 160 as a formula, or directly from an inspection of the matrix. (It helps in concentrating the attention on the diagonal of the cofactor if one obscures the row and column of the matrix which contain the element whose cofactor is being observed.)

Bending Moments in Members of the Frame.—The solution for the joint rotations as shown by Eq. 154 leads easily to graphical determination of end moments in the members of the frame. Two parameters, z_{pq} and z_{qp}, are ntroduced for each member PQ, as defined in the notation. For example, for member CD, Fig. 11:

$$z_{cd} = K_{cd}\,\Phi_c = 5\,(-\,2.8645) = -\,14.323 \qquad (163a)$$

and

$$z_{dc} = K_{dc}\,\Phi_d = 5\,(-\,6.2536) = -\,31.268 \qquad (163b)$$

These quantities have the character and dimensions of bending moments.

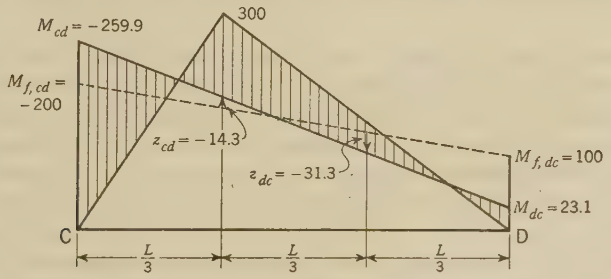


FIG. 12

From points C and D, Fig. 12, ordinates $M_{f,cd}$ and $M_{f,dc}$ (representing the fixed-end moments) are erected, and from the line joining them, at the third points of the span, the values of z_{cd} and z_{dc} are laid off, in the direction indicating negative rotation about points C and D, respectively. The extremities of these lie on the moment line M_{cd}-M_{dc}, whose ordinates at the ends of the span represent the end moments in span CD, under the given conditions of loading and restraint.

The analytical equivalent of the foregoing construction is the expression for the end moments given by the slope-deflection equations:

$$M_{cd} = M_{f,cd} + 2\,z_{cd} + z_{dc} = -\,200 - 28.645 - 31.268 = -\,259.913 \qquad (164a)$$

and

$$M_{dc} = M_{f,dc} + 2\,z_{dc} + z_{cd} = 100 - 62.536 - 14.323 = 23.141 \qquad (164b)$$

The intercepts of the ordinates between the line M_{cd}-M_{dc} and the moment diagram for member CD simply supported (supplied in Fig. 12 from data not given, but consistent with the fixed-end moments produced) show graphically the bending moment conditions throughout the span CD.

Sidesway.—The occurrence of joint translation, or sidesway, introduces into the problem additional unknown quantities relating to each of the members subject to rotational movement. The order of the matrices and determinants involved in the solution is increased correspondingly. For the ordinary rectangular building frame subject to horizontal displacement of joints, the number of different unknowns so involved is equal to the number of stories; thus for a one-story frame one additional row and column of elements occur in the matrix.

The matrix expression for the solution of this problem is derived without difficulty; however, the fact that the matrix contains elements other than zero in all diagonals, increases the labor of computation considerably, because the derivation of the reciprocal by inspection as illustrated in Eq. 154 is not possible.

Nonprismatic Members.—The method of analysis by matrix computations is applicable also to frames having haunched members. It is necessary, however, to derive modified values of the stiffness factors and load terms relating to the haunched members, and to generalize the fundamental expressions. The author has illustrated the procedure for the case of a continuous beam (see heading, "Numerical Example," and Fig. 5), using values derived from charts published by the Portland Cement Association,[7] and the method of moment distribution.

It is perhaps inappropriate to the purpose of a discussion, the aim of which is to demonstrate the practical usefulness of the matrix analysis in ordinary building design, to pursue the subject into the intricacies of precise results where nonprismatic members are involved. Usually the requirements of design can be satisfied with safety and reasonable economy, if based on a qualitative knowledge of the effects of haunching. However, the writer has found it helpful to his understanding of the principles involved, to derive the generalized values entering into the matrix and slope-deflection equations, in terms of constants made available in tables of the properties of haunched members by Walter Ruppel, M. ASCE.[26] The additional notation is as defined by Mr. Ruppel,[27] except that to avoid conflict I_o is used to represent the minimum moment of inertia of a member; and K_o, to represent the ratio I_o/L.

The stiffness coefficients and fixed-end moments are given by the following formulas:

$$K_{ab} = K_o \times \frac{1 - v}{4\,p\,(1 - u - v)} \qquad K_{ba} = K_o \times \frac{1 - u}{4\,q\,(1 - u - v)} \ldots (165a)$$

$$K'_{ab} = K_o \times \frac{1}{4\,p\,(1 - v)} \qquad K'_{ba} = K_o \times \frac{1}{4\,q\,(1 - u)} \ldots\ldots (165b)$$

$$M_{f,ab} = W\,L \times \frac{t\,u - s\,(1 - v)}{1 - u - v} \qquad M_{f,ba} = W\,L \times \frac{t\,(1 - u) - s\,v}{1 - u - v} \ldots (165c)$$

$$(M_{f,ab})' = -\,W\,L \times \frac{t}{1 - v} \qquad M_{f,ba} = W\,L \times \frac{s}{1 - u} \ldots\ldots\ldots (165d)$$

$$z_{ab} = \frac{K_o}{2\,p} \times \Phi_a \qquad z_{ba} = \frac{K_o}{2\,q} \times \Phi_b \ldots\ldots\ldots\ldots\ldots (165e)$$

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[26] *Transactions*, ASCE Vol. 90, 1927, p. 152.

[27] *Ibid.*, p. 158.

Eqs. 165 are general expressions, yielding the values for prismatic beams when the dimensions of the haunch disappear. The matrix expression (corresponding to Eq. 154 for prismatic beams) is:

$$\begin{bmatrix} \Phi_a \\ \Phi_b \\ \Phi_c \\ \Phi_d \\ \cdots \end{bmatrix} = \begin{bmatrix} 2\sum K_a & \frac{2u}{1-u}K_{ba} & 0 & 0 & \cdots \\ \frac{2v}{1-v}K_{ab} & 2\sum K_b & \frac{2u}{1-u}K_{cb} & 0 & \cdots \\ 0 & \frac{2v}{1-v}K_{bc} & 2\sum K_c & \frac{2u}{1-u}K_{dc} & \cdots \\ 0 & 0 & \frac{2v}{1-v}K_{cd} & 2\sum K_d & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}^{-1} \times \begin{bmatrix} -\sum M_{f,a} \\ -\sum M_{f,b} \\ -\sum M_{f,c} \\ -\sum M_{f,d} \\ \cdots \end{bmatrix} \quad ..(166)$$

When the joint rotations, Φ, have been ascertained, and the parameters, z, computed by Eq. 165*e*, the graphical construction for moments may proceed exactly as in Fig. 12, the ordinates z being erected at the u and v points of the span; or the moments may be computed as

$$M_{ab} = M_{f,ab} + \frac{1-v}{1-u-v} z_{ab} + \frac{u}{1-u-v} z_{ba} \ldots\ldots\ldots (167a)$$

and

$$M_{ba} = M_{f,ba} + \frac{1-u}{1-u-v} z_{ba} + \frac{v}{1-u-v} z_{ab} \ldots\ldots\ldots (167b)$$

these being the general forms of Eqs. 164.

Summary.—For a rectangular frame composed of columns and beams, the matrix expression for the joint rotations is derived directly from an inspection of the frame, without the writing of equations. The numerical evaluation of the expression follows an established series of arithmetical operations, which is accomplished easily, with little likelihood of error, and after a little practice, relatively rapidly. The result of the evaluation is translated at once, algebraically, into expressions for end moments in all members, or graphically, into a bending moment diagram for the entire frame.

The matrix solution is a routine of computation rather than a method of analysis. It has the merit of precision, being neither an approximate method nor a method of successive approximations, but an exact method just in so far as the physical data on which the solution is based can be said to be exact. From one point of view (which seems to the writer well grounded) the method shares a defect common to all formula methods. No visualization of the manner of action of the structure attends the solution. The process of computation is mechanical—one feeds the data into the receiving end and, all operations being faithfully performed, extracts the correct result. There is no possibility of applying common sense or judgment to detection of errors at intermediate stages. In this respect the process of approaching results by such means as moment distribution will continue to appeal to most designers as having advantages quite outweighing those of an analysis by equations or formulas.

Nevertheless, Mr. Benscoter's contribution is a valuable one, not only in its application to the analysis of indeterminate structures in the limited field covered by this discussion, but more importantly in promoting acquaintance with an algebraic notation, that is somewhat unfamiliar to many engineers, but which is capable of serving as a powerful tool in the solution of problems involving a number of linearly related unknown quantities.

A. Floris,[28] Esq.—From an academic viewpoint the paper is highly interesting and instructive. To a reader with a theoretical turn of mind, the study of this paper will undoubtedly be a source of pleasure and inspiration. Practicing engineers, however, have at their disposal other more elementary methods of stress analysis, which do not require the mathematical training and equipment necessary for the intelligent use of the method presented.

A brief and unusually clear introduction to the matrix algebra is given by the author for the benefit of those not familiar with the subject. Nevertheless, without a systematic study of matrices it is doubtful whether the author's methods can be used profitably.

The main practical difficulty arising in the stress analysis of hyperstatic structures is the solution of linear nonhomogeneous algebraic equations obtained from considerations of structural deformation. To a certain extent, the best methods are those which reduce the number of these elastic equations to a minimum, or eliminate them entirely. The relaxation methods of stress analysis meet this requirement. However, by their very nature the matrix methods do not.

The solution of linear nonhomogeneous algebraic equations by means of matrices is identical to G. Cramer's rule given in the theory of determinants. Although this rule leaves nothing to be desired theoretically, in practice it is almost useless. By the same token, matrix methods for formal, shorthand solutions have no equals. In numerical problems, however, their usefulness is rather restricted. The greatest success has been achieved in the application of matrix methods to the solution of linear ordinary differential equations with constant coefficients. In this case, use is made of the matric-exponential function.[29,30]

The statement is made that the stiffness factors are always positive. This is, of course, only true if the deformation due to shears, which are produced by the moments at the joints, is neglected. If this shearing deformation is greater than the bending deformation, the stiffness factors become negative.[31,32]

Inasmuch as the division of two matrices cannot be performed directly, the great importance of the inverse matrix, which transforms the division into multiplication, becomes apparent. Therefore, a mnemonic check on the correctness of the computed inverse matrix can be of some value. Using the matrix (Eq. 38a) and its inverse (Eq. 41) the self-explanatory checking scheme is arranged as in Fig. 13.

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[28] Asst. to Structural Engr., Pacific Electric Railway Co., Los Angeles, Calif.

[29] "Matrix and Tensor Calculus," by Aristotle D. Michal, John Wiley & Sons, Inc., New York, N. Y., 1947, p. 20.

[30] "Elementary Matrices," by R. A. Frazer, W. J. Duncan, and A. R. Collar, Cambridge Univ. Press, 1938, p. 156.

[31] "Shearing Deformation in Continuous Beams and Rigid Frames," by A. Floris, *Journal*, ACI, November–December, 1937.

[32] "Verallgemeinertes Ausgleichverfahren," by A. Floris, *Beton u. Eisen*, May 20, 1939.

Division of the bold-face numbers in this scheme, by the determinant of the given matrix (in this case 45), gives the unit matrix or L. Kronecker's delta. Thus, Eq. 30 is satisfied.

(3)(11) = 33	(3)(2) = 6	(3)(−5) = −15
(1)(−2) = − 2	(1)(16) = 16	(1)(5) = 5
(2)(7) = 14	(2)(−11) = −22	(2)(5) = 10
—	—	—
45	0	0
(1)(11) = 11	(1)(2) = 2	(1)(−5) = − 5
(2)(−2) = − 4	(2)(16) = 32	(2)(5) = 10
(−1)(7) = − 7	(−1)(−11) = 11	(−1)(5) = − 5
—	—	—
0	**45**	0
(−2)(11) = −22	(−2)(2) = − 4	(−2)(−5) = 10
(3)(−2) = − 6	(3)(16) = 48	(3)(5) = 15
(4)(7) = 28	(4)(−11) = −44	(4)(5) = 20
—	—	—
0	0	**45**

FIG. 13.—MNEMONIC CHECK OF INVERSE MATRIX

The inverse of a two-row square matrix is obtained in a simple manner by changing the signs of the elements in the secondary diagonal and by interchanging those in the principal diagonal. The result is then divided by the determinant of the original matrix, which for a unit matrix is equal to one.

Practically, the application of matrix methods to the analysis of continuous beams by means of the slope-deflection equations and the three-moment equations is not simple, although formally the process is excellent. It requires more numerical work than the corresponding older methods. In addition, the slope-deflection equations are solvable by iteration, and if the shearing deformation predominates, the three-moment equations can also be solved by iteration.[33]

The most valuable section of the paper is the treatment of the moment-distribution method from an advanced mathematical point of view. Here, as in the previous parts of the paper, the reader is refreshed by the exact formulation and generality of the statements as contrasted to the oversimplified and, to some extent, dull treatment accorded them in the ordinary textbooks.

The continuous beam is a comparatively simple element in the analysis of hyperstatic structures. It is to be regretted that the author did not include the more complicated continuous and multiple-story frames, inasmuch as the acid test of any new method lies in the analysis of multiple-story frames, for which simple and speedy methods are much needed.

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[33] "Shear Deformation Included in Three-Moment Equation," by A. Floris, *Civil Engineering*, October, 1937, p. 711.

In conclusion, the writer wishes to congratulate the author upon his efforts to bring to the attention of structural engineers the fruitful and elegant methods of matrix algebra through this pioneer work of considerable scientific value.

Sven Olof Asplund,[34] M. ASCE.—How matrices may be applied in the analysis of continuous beams is shown very constructively by the author, who uses matrix methods to demonstrate that the result of the iterative moment distribution method developed by Hardy Cross, M. ASCE, approaches that of the exact classical solution. Obviously in recognition of the generality of matrix methods, Mr. Benscoter suggests the possibility of useful application of matrices to the analysis of other statically indeterminate structures. The writer has actually employed matrices in the analysis of pile groups. In such a case the matrices originate from the vectors used in determining the acting forces and their positions.

In European practice[35] pile groups have for many years[36] been calculated according to the elastic theory. For plane pile groups the calculations are easily accomplished,[37] but for spatial pile groups the computation will become almost prohibitive if matrices are not employed.

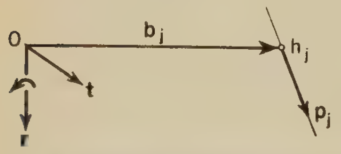

Fig. 14.—Vectors Representing Pile Forces and Movements of Pile Group

The external forces acting upon the rigid pile pier cause a small translation, defined by a vector $\mathbf{t}$ (Fig. 14), and a small rotation, r, about an axis of rotation, $\mathbf{r}$, through the origin O. The pile heads, h_j, in the pier are located by the vectors $O\,h_j = \mathbf{b}_j$, and the total movement of the pile head is given by $\mathbf{t} + [\mathbf{r}\,\mathbf{b}_j]$ (brackets indicate vector products). The projection of this movement upon the unit vector $\mathbf{p}_j$ along the pile axis is the scalar product $\mathbf{p}_j\,(\mathbf{t} + [\mathbf{r}\,\mathbf{b}_j]) = \mathbf{p}_j\,\mathbf{t} + [\mathbf{b}_j\,\mathbf{p}_j]\,\mathbf{r}$. The pile shortens by this length, causing an elastic pile reaction:

$$\lambda_j\,\mathbf{p}_j\,(\mathbf{p}_j\,\mathbf{t} + [\mathbf{h}_j\,\mathbf{p}_j]\,\mathbf{r})\ldots\ldots\ldots\ldots\ldots\ldots(168)$$

Aside from the pile reactions, all other forces that act upon the pier may be reduced to a force, $\mathbf{f}$, and a moment, $\mathbf{m}$, about O. The equilibrium of the pier demands that

$$\mathbf{f} = \Sigma\lambda\,\mathbf{p}\,(\mathbf{p}\,\mathbf{t} + [\mathbf{b}\,\mathbf{p}]\,\mathbf{r}) = \mathbf{T}\,\mathbf{t} + \mathbf{U}\,\mathbf{r}\ldots\ldots\ldots\ldots(169a)$$

$$\mathbf{m} = \Sigma\lambda\,[\mathbf{b}\,\mathbf{p}] \times \mathbf{p}\,\mathbf{t} + \Sigma\lambda\,[\mathbf{b}\,\mathbf{p}] \times [\mathbf{b}\,\mathbf{p}]\,\mathbf{r} = \mathbf{U}'\,\mathbf{t} + \mathbf{V}\,\mathbf{r}\ldots(169b)$$

or, in Mr. Benscoter's matrix notation,

$$[f] = [T]\,[t] + [U]\,[r]\ldots\ldots\ldots\ldots\ldots\ldots(170a)$$

$$[m] = [U]'\,[t] + [V]\,[r]\ldots\ldots\ldots\ldots\ldots\ldots(170b)$$

This matrix notation appears to the writer to be unnecessarily laborious.

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[34] Docent, Bridge Eng. Dept., Royal Inst. of Tech., Stockholm, Sweden.

[35] "Der Kampf des Ingenieurs gegen Erde und Wasser im Grundbau," by A. Agatz, Springer, Berlin, 1936.

[36] "Teori för Grundpålningar," by Per Gullander, Norstedt, Stockholm, 1914.

[37] "Beregning av Paelevaerker," by Chr. Nøkkentved, Gad, Copenhagen, 1923.

Henry Margenau and G. M. Murphy[38] and Harold Jeffreys[39] use bold face letters to denote matrices, whereas Richard Courant and David Hilbert[40] use ordinary letters:

$$\left.\begin{aligned} f &= T\,t + U\,r \\ m &= U'\,t + V\,r \end{aligned}\right\} \quad \text{(171)}$$

The symbols T, U, and V may be recognized as third-order square matrices with elements $T_{k1} = \Sigma\lambda\, p_k\, p_1$, $U_{k1} = \Sigma\lambda\, p_k\, [\mathbf{b}\ \mathbf{p}]_1$, etc. The symbol U' is the transpose of U. The solution of Eq. 171 is easily found to be

$$\left.\begin{aligned} t &= T^{-1}{}_0\, f - T^{-1}{}_0\, U\, V^{-1}\, m \\ r &= -\,V^{-1}{}_0\, U'\, T^{-1} f + V^{-1}{}_0\, m \end{aligned}\right\} \quad \text{(172)}$$

in which $T_0 = T - U\,V^{-1}\,U'$ and $V_0 = V - U'\,T^{-1}\,U$. Substitution of Eqs. 172 in Eq. 168 yields the force in each pile. The numerical computations may be systematically and effectively carried through according to Eqs. 172.

It is often possible to add to an actual pile group one or more fictive piles in such a manner that the resulting pile group may be more expediently analyzed —for example, by resulting symmetry. The forces that the original external forces **f**, **m** induce in the fictive piles are evaluated and added to **f**, **m** to neutralize the reactions from the fictive piles upon the pier. The added forces cause further reactions from the fictive piles which are again added, etc. This iteration often converges, and it is possible by summation of geometric matrix series (Neumann series) as explained by the author to show that the final result is identical to a direct solution of the actual original pile group in accordance with Eq. 172.

STANLEY U. BENSCOTER,[41] ASSOC. M. ASCE.—The discussions contain several interesting additions to the paper. Several writers question the usefulness of matrix methods in more complicated problems such as the analysis of frameworks with joint translations. In such problems matrix algebra provides a convenient process of formulating and defining the various calculating procedures that may be used. The best method of calculation will depend on such factors as the type of computing device to be used, the size of the matrix of coefficients, and the number of times that the equations are to be solved (number of loading conditions). One should not expect to analyze an enormous framework on a slide rule any more than one would expect to dig an enormous excavation with a hand spade. Just as large electrical or mechanical shovels are used for large excavation jobs, so must one use large electrical or mechanical computing machines for analyzing large frameworks.

For solving a general system of equations, evaluating a determinant, or computing a reciprocal matrix, on a hand-operated calculating machine, the method of P. D. Crout[42] is the most efficient that is known to the writer. This method is being adopted quite generally throughout the United States in design and research work. More powerful calculating machines have been developed in recent years such as the punch card machine and the sequence-controlled electronic or electrical relay machines. These machines operate automatically with iteration procedures and perform successive matrix multiplications conveniently. They solve as many as one hundred simultaneous equations and provide the only real answer to the civil engineers' need for a method of analyzing large frameworks. Matrix algebra and the modern calculating machine are complementary tools for use in the solution of physical problems for which solutions in terms of analytical functions cannot be obtained. Both of those tools must be developed simultaneously to obtain the maximum benefit from either.

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[38] "The Mathematics of Physics and Chemistry," by Henry Margenau and G. M. Murphy, D. Van Nostrand Co., Inc., New York, N. Y., 1943.

[39] "Methods of Mathematical Physics," by Harold Jeffreys, Cambridge Univ. Press, Cambridge, England, 1946.

[40] "Methoden der Mathematischen Physik," by Richard Courant and David Hilbert, Springer, Berlin, 1931.

[41] Structural Engr., Structures Div., N.A.C.A., Langley Memorial Aeronautical Laboratory, Langley Field, Va.

The historical note by Mr. Oesterblom is interesting. His question concerning the possibility of representing the reciprocal of the stiffness matrix for a framework by a power series can be answered by stating that the reciprocal of any square matrix can be represented by a power series if the determinant of the matrix does not vanish. Re-analysis of a structure for loading changes involves the use of the same stiffness matrix and, hence, is not a serious problem in design. Re-analysis for framework dimension changes is an entirely different matter because this procedure involves changes in the stiffness matrix, which require a new computation of the reciprocal matrix. However, if the changes in stiffness values are small relative to the originally assumed stiffnesses, a convenient method of obtaining the new reciprocal matrix can be developed.

Assume that the original stiffnesses give the square matrix $[K]$ as the coefficient matrix in the equation of three slopes (Eq. 67). From a stress analysis it is found that changes in the dimensions of the members are required. The new stiffness matrix becomes $[K + \Delta K]$. The square matrix $[\Delta K]$ contains small numbers representing the changes in stiffness of the members. Define the square matrix $[\delta]$ by

$$[\delta] = [K]^{-1} [\Delta K] \quad \ldots\ldots\ldots\ldots (173)$$

Eq. 173 can be computed immediately since the reciprocal matrix $[K]^{-1}$ is known. The new stiffness matrix may be written as

$$[K + \Delta K] = [K][I + \delta] \quad \ldots\ldots\ldots\ldots (174)$$

The reciprocal may be written as

$$[K + \Delta K]^{-1} \doteq \{[I] - [\delta] + [\delta]^2 - \cdots\}[K]^{-1} \quad \ldots\ldots\ldots\ldots (175)$$

Since the elements of $[\delta]$ are small relative to unity, the second and higher degree terms in the series may be neglected:

$$[K + \Delta K]^{-1} = [I - \delta][K]^{-1} \quad \ldots\ldots\ldots\ldots (176)$$

Thus, the original reciprocal matrix is easily converted into the new reciprocal matrix. The development of practical design procedures on the basis of this theory is a worthwhile objective for future research study.

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[42] "A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients," by P. D. Crout, *Transactions*, A.I.E.E., Vol. 60, 1941, pp. 1235–1240.

The formulas for moments given by Mr. Solman form an important contribution to the analysis of continuous beams. Since there are a number of other problems in both civil and aeronautical design which lead to a system of equations having a continuant matrix, the possibility of developing such formulas is of considerable interest. The formulas become especially useful with symmetrical structures. The simple formulas of Eqs. 132 and 133 may be used to analyze symmetrical beams having as many as six spans. This classification includes a large part of all highway bridges.

It is clearly shown by the formulas of Mr. Solman that the solution for moments is dependent only on the fixed-end moments and the primary and secondary distribution factors. The convenience of introducing product factors $p_{ij} = q_{ij}\, q_{ji}$ is also evident. The significant feature of these scalar formulas, from a general mathematical viewpoint, lies in the fact that they arise from formulas for the elements of the reciprocal of the matrix $\{[I] - [Q]'\}$. The development of the formulas, as given by Mr. Solman, is so brief as to be difficult for a reader to follow. The development employs certain special properties of a continuant matrix and its adjoint that are not known to the majority of readers.

The special case of two effective joints has been noted by Hardy Cross,[43] M. ASCE. In this case the power series of matrices degenerates into a scalar geometric series and direct formulas for the solution are readily developed.

The writer is in agreement with the comments of Professor Oldenburger. It was the classical paper of Professor Oldenburger[9] that inspired the writer to take up the study of matrix algebra. Studies of the convergence of the power series, which defines moment distribution, lead to new methods of computing the critical, or characteristic, values of buckling or vibratory loads. The problem of framework buckling is in serious need of research attention so that more rational methods of design may be developed.

Mr. Johnson has illustrated the use of the Cramer rule in solving a system of equations. The formula for computing a reciprocal matrix as given in Eq. 33, without proof, would be developed by using this rule. The Cramer rule is very useful for solving either two or three simultaneous equations when the equations are to be solved only once (one loading condition). For more than three equations other methods should be used even when solving the equations only once.

Professor Cheney has illustrated the standard engineering textbook method of successive elimination of unknowns. The evaluation of a 4 × 4 determinant, as given in Eq. 148, is correct. However, the evaluation of a 6 × 6 determinant, as shown in Eqs. 150 and 151, is erroneous. Unfortunately, the task cannot be simplified in this manner.

Mr. Norton has illustrated in Fig. 11 the association that exists between the stiffness matrix for a continuous beam and the structure itself. He has also explained in detail the Gauss method of pivotal condensation[44] for evaluating determinants. By using the Gaussian method it is found that the elements of the adjoint of a continuant matrix may be expressed as given by Eq.

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[43] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 125.

[44] "Determinants and Matrices," by A. C. Aitken, Interscience Publishers, Inc., New York, N. Y., 1939, p. 45.

160. This development is of considerable importance for a number of different structural problems. The pattern of formation of the adjoint is clear and convenient to use. The calculations are easily performed when the factors of an element are expressed as continued fractions. Consider the element of the first row and first column of the matrix of Eq. 160. Apply the formula to the matrix $\{[I] - [Q]'\}$ of the fifth order; thus:

$$A_{11} = a_{22}\, a'_{33}\, a''_{44}\, a'''_{55}$$
$$= 1 \times (1 - p_{23}) \left(1 - \frac{p_{34}}{1 - p_{23}}\right) \left[1 - \frac{p_{45}}{1 - \frac{p_{34}}{1 - p_{23}}}\right] \ldots\ldots(177)$$

in which $p_{ij} = q_{ij}\, q_{ji}$. If the multiplication indicated in Eq. 177 is performed, the resulting formula will be found to correspond to the first element of the matrix of Eq. 146, as given by Mr. Solman. The determinant of the matrix may also be expressed as a product of continued fractions.

Mr. Floris mentions the possibility of negative stiffness values when shearing deformations are considered. There is some doubt as to whether rotational stiffnesses are affected by shearing deformation—whether they are or are not is dependent on the definition of stiffness. There are at least two possible methods. The problem is in need of still further attention by research workers. It is possible that only the translational stiffnesses (which are not mentioned in the paper) are affected. The complicating effect of considering shear deformation is that it brings about a discontinuity in the slope of the elastic curve at points of application of concentrated loads or reactions. In the case of an applied load the magnitude of the discontinuity is known, whereas in the case of a reaction the magnitude is unknown.

The simple rule for inverting a 2 × 2 matrix, as noted by Mr. Floris, is very useful. The writer has used it many times in design and research work. The adjoint is formed by interchanging the diagonal elements of the matrix and by merely changing the signs of the nondiagonal elements.

Mr. Asplund has illustrated the use of matrices in the solution of a very interesting problem. The analysis of pile groups has received little attention in American literature. The plane problem was treated by C. P. Vetter,[45] M. ASCE. The space group with arbitrary pile batter, pile length, and head position is a more complex problem.

Since the development of the formulas for the elements of the matrices has been given by Mr. Asplund in the language of vector algebra, it may be interesting to show the same development in the language of matrix albebra. This statement is especially true since vector products arise in the analysis. Vectors in vector notation will be equated to vectors (column vectors or row vectors) in matrix notation.. The equality sign must be understood to mean equality of the components of the vectors.

The pier translation is indicated by the vector:

$$\mathbf{t} = [t]' = [t_1\, t_2\, t_3] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(178)$$

The pier rotation is indicated by

$$\mathbf{r} = [r]' = [r_1\, r_2\, r_3] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(179)$$

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[45] "Design of Pile Foundations," by C. P. Vetter, *Transactions*, ASCE, Vol. 104, 1939, pp. 758–811.

In developing relationships for a single pile, the subscript, j, will be omitted for convenience. The vector from the origin to the pile head is given by

$$\mathbf{b} = [b]' = [b_1\, b_2\, b_3] \qquad (180)$$

To define vector products in matrix notation, it is necessary to introduce two skew-symmetric matrices that may be constructed from the components of the vectors $[r]$ and $[b]$:

$$[R] = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} \qquad (181)$$

$$[B] = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix} \qquad (182)$$

A skew-symmetric matrix is one that is equal to the negative of its transpose:

$$[R] = -[R]'; \quad [B] = -[B]' \qquad (183)$$

A vector product may now be illustrated; thus:

$$\mathbf{r} \times \mathbf{b} = [R][b] = -[B][r] \qquad (184)$$

The resultant translation of the pile head may then be expressed by

$$\mathbf{t} + \mathbf{r} \times \mathbf{b} = [t] + [R][b] = [t] - [B][r] \qquad (185)$$

A unit vector along the pile axis is indicated by

$$\mathbf{p} = [p]' = [p_1\, p_2\, p_3] \qquad (186)$$

The components of this vector are the direction cosines of the pile axis. The shortening of the pile is given by

$$\mathbf{p} \times (\mathbf{t} + \mathbf{r} \times \mathbf{b}) = [p]'\{[t] - [B][r]\} \qquad (187)$$

The quantity on either side of Eq. 187 is a scalar number. When multiplied by an axial stiffness constant, λ, the magnitude of the pile reaction force is obtained. If this value is then multiplied by the vector, $\mathbf{p}$, the reaction will be expressed as a vector:

$$\lambda\, \mathbf{p}\, [\mathbf{p} \cdot \mathbf{t} + \mathbf{p} \cdot (\mathbf{r} \times \mathbf{b})] = \lambda\, [p]\{[p]'[t] - [p]'[B][r]\} \qquad (188)$$

or

$$\lambda\, \mathbf{p}\, [\mathbf{p} \cdot \mathbf{t} + \mathbf{p} \cdot (\mathbf{r} \times \mathbf{b})] = \lambda\, \{[P][t] - [P][B][r]\} \qquad (189)$$

in which

$$[P] = [p][p]' = \begin{bmatrix} p^2_1 & p_1 p_2 & p_1 p_3 \\ p_2 p_1 & p^2_2 & p_2 p_3 \\ p_3 p_1 & p_3 p_2 & p^2_3 \end{bmatrix} \qquad (190)$$

In the analysis given by Mr. Asplund five of the six stiffnesses that may be defined for the pile have been regarded as negligible relative to the axial stiffnesses. The neglected stiffnesses are the bending translational stiffnesses, the bending rotational stiffnesses, and the twisting rotational stiffness. The assumption is most directly applicable to piles driven through soft ground to bedrock. In the case of friction piles the foundation itself participates to a considerable extent in the action to complicate the problem. The assumption of a constant of proportionality between reaction force and displacement function should be replaced by a more general functional relationship, as determined by an appropriate influence function. To the equilibrium solution given by Mr. Asplund should be added a linear combination of a finite set of self-equilibrating characteristic distributions to allow for the foundation effect. The mathematical difficulties of obtaining such a solution are, at present, insurmountable. The theory of Mr. Asplund is probably as good, in most cases, as the commonly used assumption of planar distribution of reaction beneath piers, walls, dams, etc., without piles.

The summation of all reaction forces and their moments must equal the applied force, $[f]$, and the applied moment, $[m]$. The subscript, j, will now be introduced where necessary for the summation:

$$\Sigma\lambda_j \{[p_j][t] - [P_j][B_j][r]\} = [f] \qquad (191)$$

$$\Sigma\lambda_j [B_j]\{[P_j][t] - [P_j][B_j][r]\} = [m] \qquad (192)$$

Substituting Eq. 183 in Eq. 192 gives

$$\Sigma\lambda_j [B_j]' \{-[P_j][t] + [P_j][B_j][r]\} = [m] \qquad (193)$$

The following matrices may be introduced:

$$[T] = \Sigma\lambda_j [P_j] \qquad (194)$$

$$[U] = -\Sigma\lambda_j [P_j][B_j] \qquad (195)$$

$$[V] = \Sigma\lambda_j [B_j]' [P_j][B_j] \qquad (196)$$

These matrices were introduced by Mr. Asplund to give the two matrix formulas (Eq. 170*a* and Eq. 170*b*). The two equations may be written as a single equation, thus:

$$\begin{bmatrix} T & U \\ U' & V \end{bmatrix} \begin{bmatrix} t \\ r \end{bmatrix} = \begin{bmatrix} f \\ m \end{bmatrix} \qquad (197)$$

The matrix equation (Eq. 197) represents six linear algebraic equations that must be solved simultaneously. In the case of general 6 × 6 matrices the method of Professor Crout[37] will usually be found to be the most rapid on a hand-operated calculating machine. Partitioning of the matrices, as illustrated by Mr. Asplund, becomes advantageous when the submatrices $[T]$, $[U]$, and $[V]$ have special simplified forms. A study of these simplified forms for various typical pile arrangements would be an interesting research problem. It should be noted that the representation of matrix division by fractions, as shown in Eq. 172, is ambiguous. It does not indicate the correct order of the matrix multiplication.

The discussions have revealed a number of additional suggestions for the use of determinants and matrices in the solution of linear equations of structures. The paper has introduced only the simplest concepts of matrix algebra. It is to be hoped that there will be continued advancement in the application of matrix theory to structural analysis problems.

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Paper No. 2324

CORRECTION OF TAILWATER EROSION AT PRAIRIE DU SAC DAM

By C. N. Ward,[1] M. ASCE, and Henry J. Hunt,[2] Esq.

With Discussion by Messrs. George P. Steinmetz, Louis M. Laushey, Abraham Streiff, R. H. Keays, and C. N. Ward and Henry J. Hunt.

Synopsis

The hydroelectric power plant at Prairie du Sac, Wis., on the Wisconsin River was placed in operation in 1914.[3,4,5] A general lowering of the tailwater and of the river bottom downstream from the plant became obvious a few years later. Because of the presence of a large island immediately below the east half of the spillway section of the dam when constructed, the recession of the tailwater level was not appreciable until 1920 when the island had been completely removed. From then until 1932 the recession of tailwater was more rapid. The average rate of recession of minimum monthly tailwater levels over a 17-year period, 1915–1931, inclusive, was 0.43 ft per yr. Such river controls as an old bridge foundation, two highway bridges, and a railroad bridge and gravel shoals over certain river stretches caused the rate of recession to decrease to 0.10 ft per yr from 1932 to 1936, inclusive, and to practically a stable condition from 1937 to 1944, inclusive. During 1929 hydraulic model tests were made to determine a method of reducing excessive erosion of the river bed immediately downstream from the dam.

Expensive and extensive methods of improvement were not considered in the first study because a new power dam was then under consideration which would, when constructed, restore tailwater levels at Prairie du Sac. Model tests were confined to changes which could be built on the prototype without the need of a cofferdam. A partial or temporary remedy was all that was expected. A wooden baffle was designed on the basis of model tests and was constructed on the prototype in 1930.

Note.—Published in October, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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[3] "Construction of the Prairie du Sac Power Plant," *Engineering Record*, May 31, 1913, p. 603.

[4] "Method of Constructing a Hydro-Electric Power House and Dam on Sand Foundation," *Engineering and Contracting*, May 7, 1913, p. 533.

[5] "Building Power House and Dam on Sand Foundation," *Engineering News*, June 15, 1916, p. 1113.

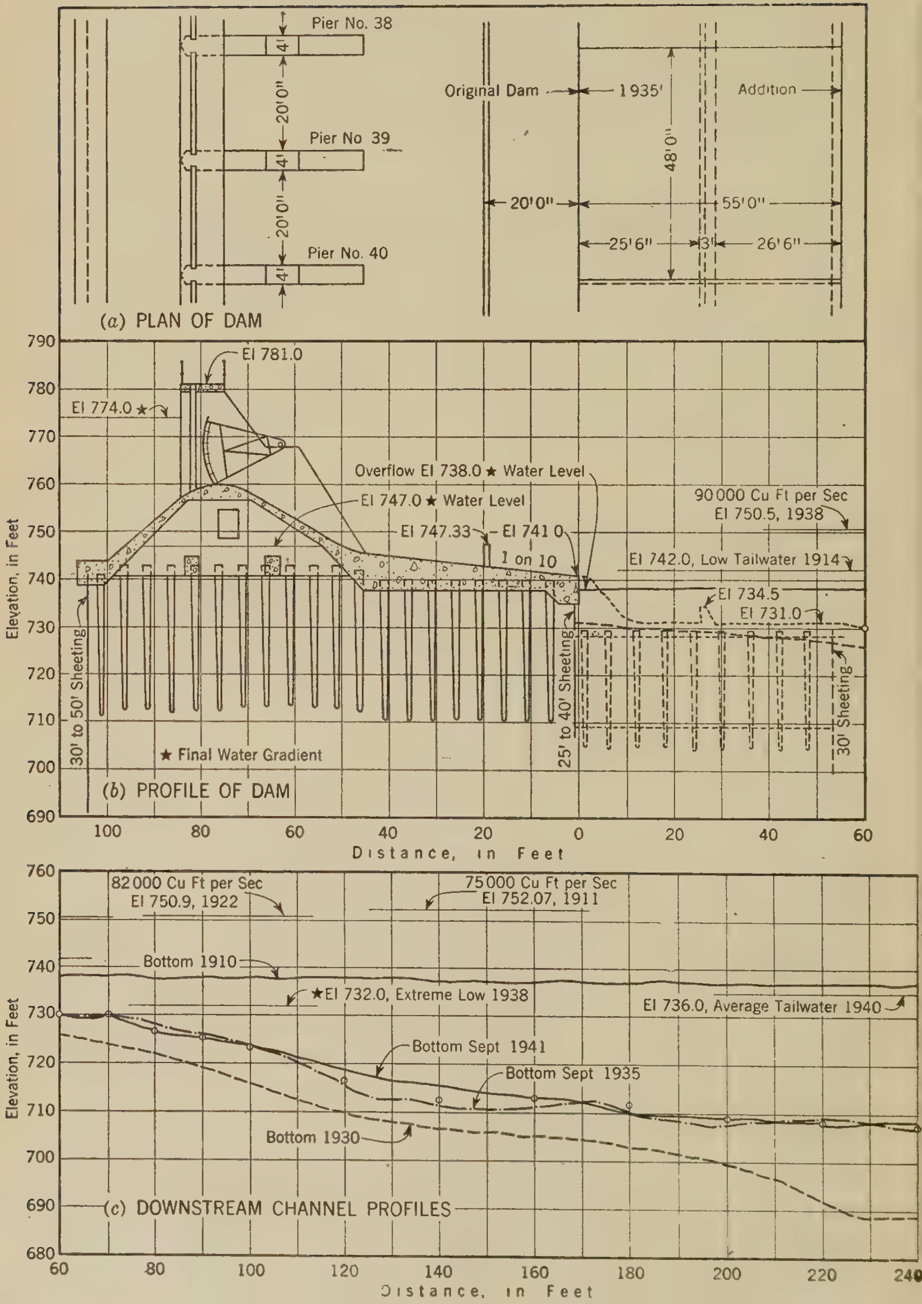

Pier No. 38
4'
20'0"
Pier No 39
4'
20'0"
Pier No. 40
4'
Original Dam
1935'
Addition
48'0"
20'0"
55'0"
25'6"
3'
26'6"
(a) PLAN OF DAM
El 781.0
El 774.0 ★
Overflow El 738.0 ★ Water Level
El 747.0 ★ Water Level
El 747.33
El 741.0
1 on 10
90 000 Cu Ft per Sec
El 750.5, 1938
El 742.0, Low Tailwater 1914
El 734.5
El 731.0
30' to 50' Sheeting
25' to 40' Sheeting
30' Sheeting
★ Final Water Gradient
(b) PROFILE OF DAM
Elevation, in Feet
790
780
770
760
750
740
730
720
710
700
690
100
80
60
40
20
0
20
40
60
Distance, in Feet
82 000 Cu Ft per Sec
El 750.9, 1922
75 000 Cu Ft per Sec
El 752.07, 1911
Bottom 1910
★El 732.0, Extreme Low 1938
El 736.0, Average Tailwater 1940
Bottom Sept 1941
Bottom Sept 1935
Bottom 1930
(c) DOWNSTREAM CHANNEL PROFILES
Elevation, in Feet
760
750
740
730
720
710
700
690
680
60
80
100
120
140
160
180
200
220
Distance, in Feet

Fig. 1.—Plan and Profiles of th

In 1933 it was decided that a new dam would not be constructed in the near future downstream from Prairie du Sac and that works would be designed and constructed at the site to reduce erosion further and to improve other unfavorable conditions arising from lowered tailwater. A new series of tests was then run to determine a basis for design of an improvement which would be complete in itself.

This paper briefly describes the model tests and types of improvements adopted and presents a general comparison of performance of models with that of the prototype.

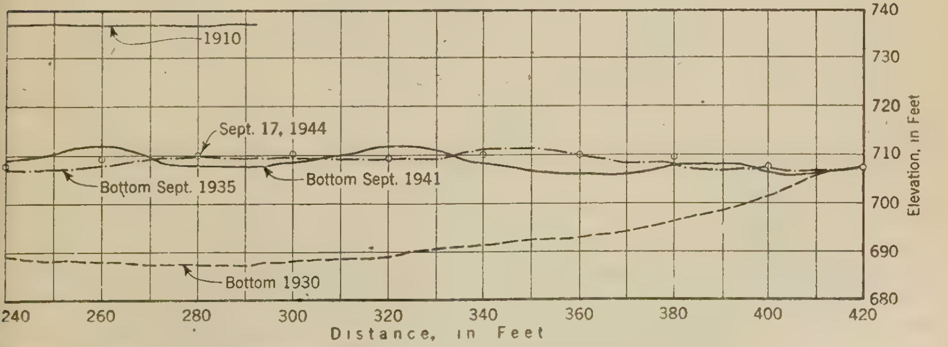
GENERAL DESCRIPTION OF PLANT

There is an earth embankment about 1,700 ft long on the east side of the river. In the main channel there are forty-one Tainter gates (20 ft wide by 14 ft high) with 4-ft piers, a lock 35 ft wide, a closed dam and sluices 38 ft long, and a powerhouse 329 ft long. Profiles of the river bottom downstream from the dam in a region of maximum bed cutting are shown in Fig. 1. Fig. 2, which was used as a form sheet in recording the direction of water currents and other data in some of the 1933 model tests, is a diagrammatic plan, showing the main structures of the plant and the contours of the river bottom as of 1932. The most serious erosion of the bottom of the river occurred about 220 ft downstream from the end of the original apron below gates Nos. 36 to 41 and near the east side of the lock. The deepest erosion extended 54 ft below the top of the original apron.

TABLE 1.—TAILWATER DISCHARGE AT PRAIRIE DU SAC, WIS.

Tailwater elevation (ft)	Discharge (Cu Ft per Sec)		
	1912	1929	1931
734	....	....	1,200
735	....	1,500	2,700
736	....	3,250	4,250
737	....	5,500	6,000
738	....	8,100	8,300
739	....	11,000	11,100
740	....	14,400	14,400
741	3,500	18,000	18,000
742	5,750	21,800	21,800
743	8,250	26,000	26,000
744	11,000	30,500	30,500
745	14,250	35,250	35,250
746	17,700	40,500	40,500
747	22,000	46,300	46,300
748	27,750	54,200	54,200
749	34,750	65,000	65,000
750	43,000	....	....

"Tailwater discharge" curves are shown for the period from 1909 to 1931 in Table 1. During this time the tailwater receded about 5.8 ft for a flow of 5,000



PRAIRIE DU SAC DAM IN WISCONSIN

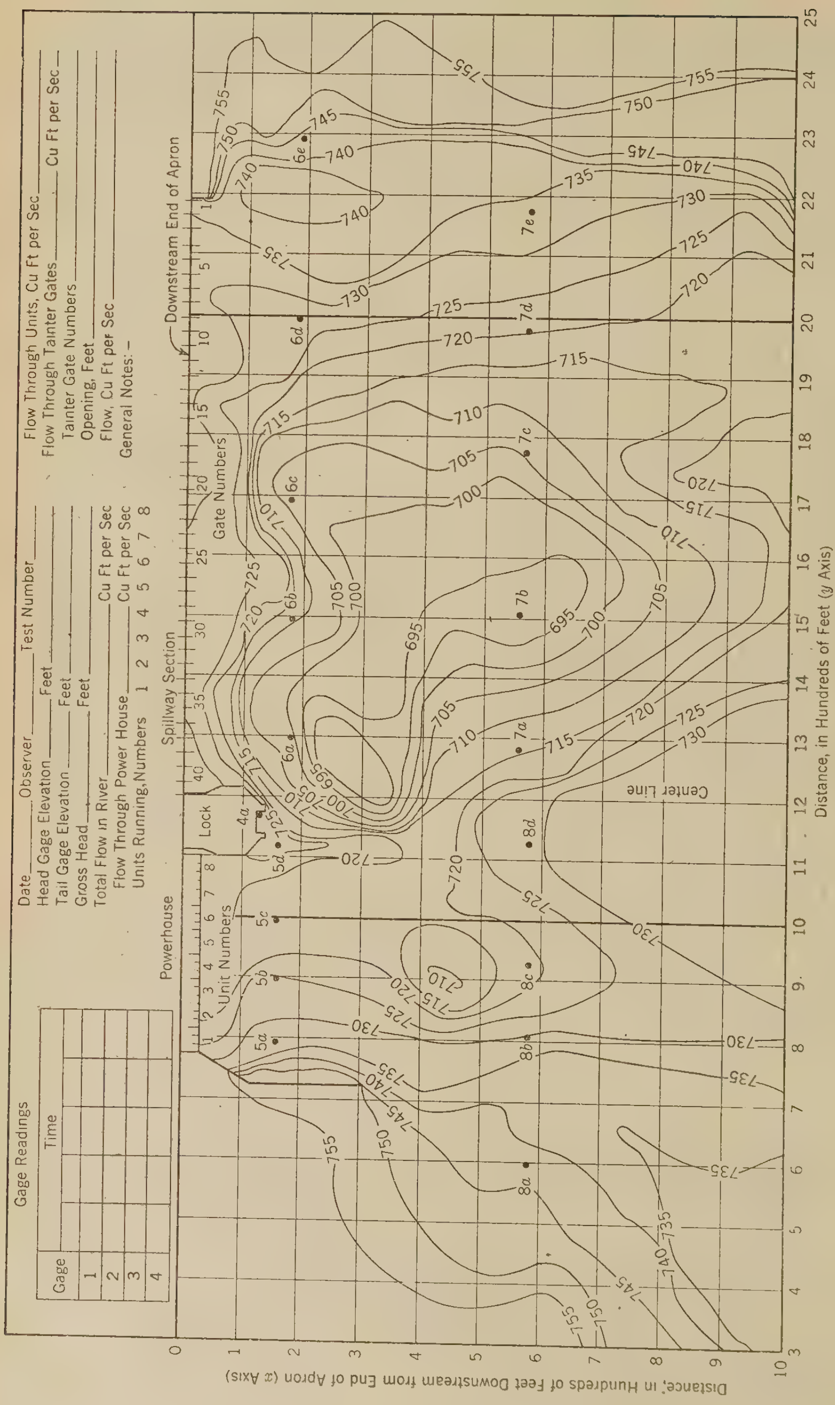

Date
Observer
Test Number
Head Gage Elevation
Feet
Tail Gage Elevation
Feet
Gross Head
Feet
Total Flow in River
Cu Ft per Sec
Flow Through Power House
Cu Ft per Sec
Units Running, Numbers 1 2 3 4 5 6 7 8
Flow Through Units, Cu Ft per Sec
Flow Through Tainter Gates
Cu Ft per Sec
Tainter Gate Numbers
Opening, Feet
Flow, Cu Ft per Sec
General Notes: –
Gage Readings
Gage
Time
1
2
3
4
Powerhouse
Unit Numbers
Lock
Spillway Section
Gate Numbers
Downstream End of Apron
Center Line
5a
5b
5c
5d
4a
6a
6b
6c
6d
6e
7a
7b
7c
7d
7e
8a
8b
8c
8d
695
700
705
710
715
720
725
730
735
740
745
750
755
Distance, in Hundreds of Feet (y Axis)
Distance, in Hundreds of Feet Downstream from End of Apron (x Axis)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25

FIG. 2.—FORM SHEET AND GENERAL CONTOUR PLAN OF RIVER BOTTOM AS OF 1932

cu ft per sec and 3.3 ft for a flow of 50,000 cu ft per sec. The actual period of recession began in 1914 so that the rate for 5,000 cu ft per sec was 0.34 ft per yr; and, for 50,000 cu ft per sec, the rate was 0.20 ft per yr.

1929 Tests

A model of three gates of the dam to a scale of 1 to 20 was used in the 1929 tests at the Hydraulic Laboratory of the University of Wisconsin at Madison. The dam, constructed of wood, was tested in a wooden box 30 ft long, which had glass sides permitting visual inspection of flow conditions on the apron of the dam for a distance of 70 ft (on prototype) downstream from the end of the apron. All measurements in the tests were recorded and are referred to in terms of units applicable to the prototype. Two venturi meters (4 in. and 8 in.) were used to measure the flow of water. Water was admitted to the space underneath the dam and apron which acted as a stilling basin. Baffles caused the water to flow to the gates smoothly. A gate used at the downstream end of the box to control tailwater levels was designed to allow water to discharge so that normal vertical velocity curves would occur a short distance upstream from the gate.

The control gate was of the register type consisting of two wooden members, each having rectangular orifices, all the same height and arranged in horizontal rows with different spacing at various depths. One member was fixed; and the other was placed in contact with it and was vertically adjustable. The orifices were designed to discharge water at different levels, thus giving a normal vertical velocity distribution in the model for the average conditions to be tested. The gate was found to be very effective in controlling tailwater heights, and velocity measurements made with a pitot tube showed that normal and desired velocity distribution actually occurred a short distance upstream from this gate.

TABLE 2.—Sieve Analysis of Sands from Prototype and of Sand Used in Model

Sieve No.	Percentage Retained (Weight)[a]						
	Sand No. 1	Sand No. 2	Sand No. 3	Sand No. 4	Sand No. 5	Sand No. 6	Sand No. 7
8	0	0	0	0	6.5	0	0
14	2.2	1.0	1.5	2.2	20.5	0	0.5
28	19.0	2.5	8.5	9.3	35.0	0	1.1
48	94.0	40.0	52.5	91.5	76.0	22.5	6.0
100	100	94.0	98.3	100	99.0	82.0	18.5
150	....	99.5	99.5	....	100	95.0	26.5
200	....	100	100	....	....	97.5	30.5
Pan	....	....	....	....	....	100	100

[a] The samples of the sands were taken at the following places: No. 1, center of river below locks; No. 2, near west shore; No. 3, near east shore; No. 4, center of tailrace near island; No. 5, Janesville, Wis.; and Nos. 6 and 7, model sands, at the Wisconsin foundry.

The bottom of the box was filled with sand and the surface of the sand was made to conform with various profiles of the river bottom. In some tests the bottom was shaped to conform to the river bottom downstream from gate No. 9 and in others to an average profile of the river bottom.

Since the exact laws of similitude relating to movement of soil grains by water were not known, it was decided to use the finest sand readily attainable for the bottom of the channel of the model. Sieve analyses of sand from the prototype are shown in Table 2, sands Nos. 1, 2, 3, and 4, and the sand used in the model is designated as sand No. 6. Sand No. 7 was rejected because it contained a large amount of fine material that was carried in suspension with a very slight movement of water. It was decided that a study of bed movement in the model would be relatively indicative of results attainable in the prototype even though the exact quantitative relationship was not known.

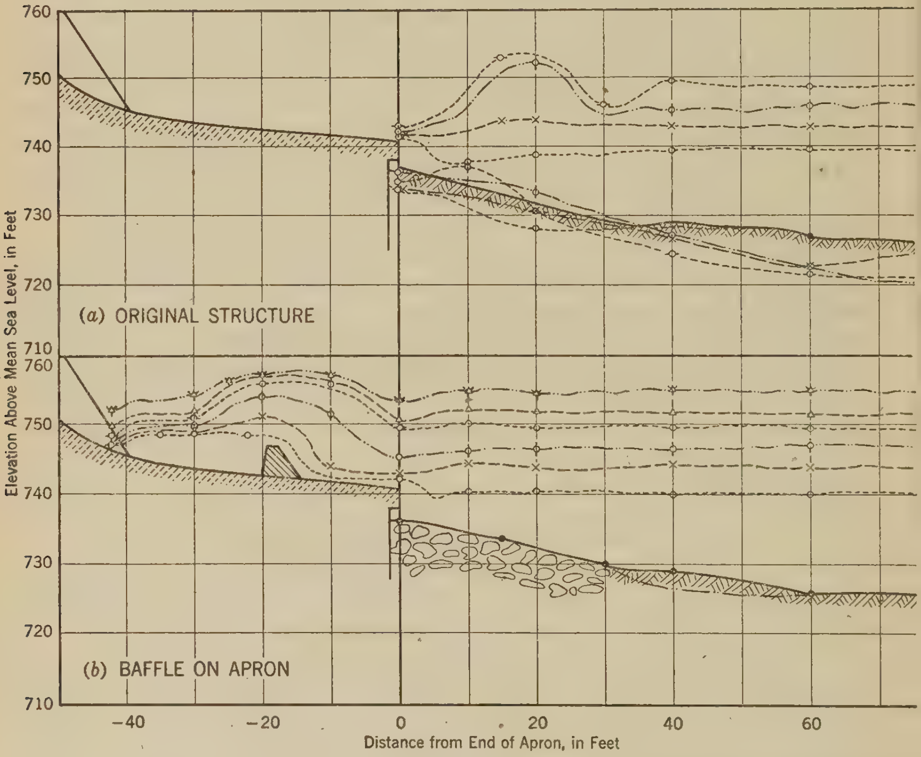


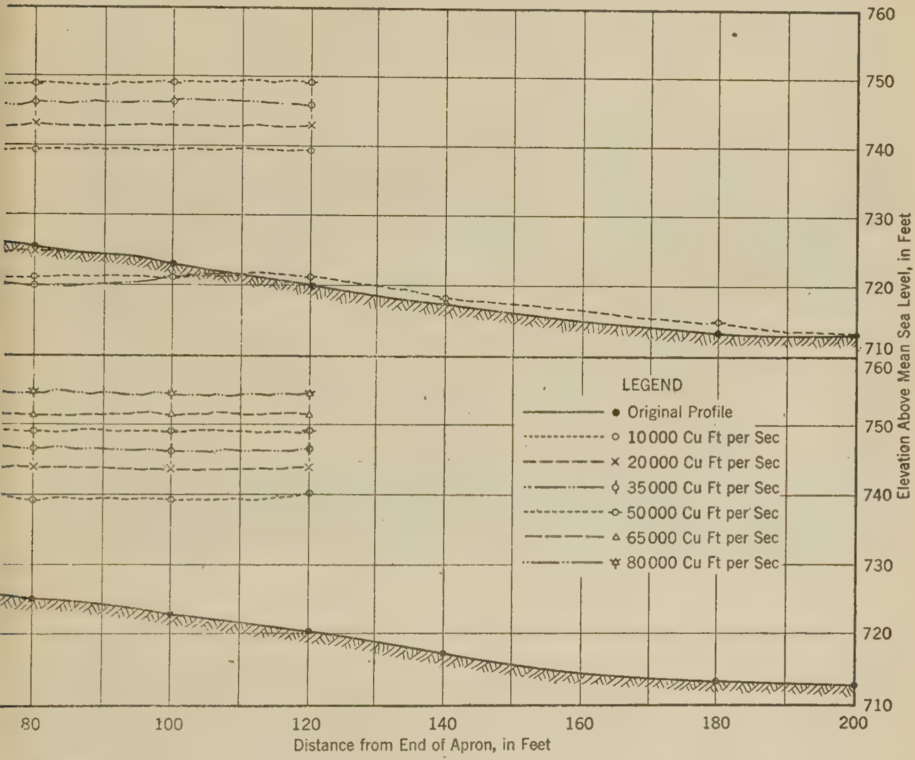
FIG. 3.—WATER SURFACE AND RIVER BOTTOM PROFILES

Attempts to measure water velocities in the model with a small current meter designed for this purpose and a pitot tube were not satisfactory in the 1929 tests. Yarn tied on small sticks was used to determine the presence and direction of water currents. Direct observations through the glass windows of the model indicated the general magnitude of bottom velocities since the movement of soil particles could be seen.

Early tests showed that the sand in the model reached a stable condition in about 1 hour after a run was started. Each of thirty-four different arrangements of the model were tested under from four to six rates of flow varying from

10,000 cu ft per sec to 80,000 cu ft per sec. Tailwater rating curves used in tests usually followed the 1924 rating curve.

Water profiles and sand bottom profiles were taken during and after each run. Fig. 3(*a*) shows water profiles and bottom profiles in one test with the original dam; and Fig. 3(*b*), a similar test with the exception of a 52-in. baffle wall placed on the apron of the dam. The sand lost from the bottom in the first 200 ft downstream from the dam for flows up to 50,000 cu ft per sec was 270 cu yd per gate (in terms of the prototype), for the dam as originally constructed, whereas no measurable loss resulted with the same range of flows when



ROM Tests of a 1-to-20 Scale Model (1929 Tests)

the 52-in. baffle was used. A baffle 52 in. high, 20 ft from the downstream end of the apron, appeared to improve conditions materially. Such a baffle was then constructed on the prototype below two Tainter gates. Because of the river flow conditions, the tailwater could not be built up to flood stages. However, the two gates were opened to discharge at rates equivalent to a flood flow of 30,000 cu ft per sec in the river. Flow conditions over the baffle appeared generally similar to those observed on the model. The pool in front of the baffle was maintained on the prototype as in the model. No movement of river bed material downstream from the gates occurred.

A baffle 52 in. high was built across the entire length of the prototype dam in 1930. It was constructed with 8-in. timbers used as stop logs supported by 12-in. H-beams placed in the apron of the dam. From 1930 to 1933 this baffle improved conditions materially.

1933 Tests

It was definitely decided in the early part of 1933 that a new dam downstream from Prairie du Sac would not be constructed in the near future. Because of recession of the tailwater, the Prairie du Sac Dam would have been subjected to a head considerably greater than that for which it was originally designed if normal headwater elevation were maintained. For a number of years the plant was operated with the headwater lowered to reduce the head on the dam. Deep holes, eroded in the river bottom prior to 1930, had not been filled. Thus, a new series of tests was run to develop plans for the permanent improvement of the structures at Prairie du Sac that would remedy the unfavorable conditions resulting from receding tailwater elevations.

After a brief study, it was decided to design new works which would be constructed adjacent to and downstream from the end of the Prairie du Sac apron—itself able to withstand a head of 13 ft and to maintain a downstream head on the old dam up to about El. 742. The model tests made in 1930 indicated quite clearly the procedure required to prevent erosion of the sand bottom. It was felt desirable, however, to make further studies using a model which incorporated the entire dam lock and powerhouse. Observations of the prototype indicated that serious crosscurrents and eddying were caused when currents of different velocities joined downstream from the dam and the powerhouse. The discharge from certain combinations of Tainter gates resulted in obvious disturbances. Eddying also appeared downstream from the lock, where water discharged through the gates and water discharged through the powerhouse joined. A survey was made of the river channel downstream from the Prairie du Sac Dam for a distance of about a mile.

A model of all structures and of the river bed to a scale of 1 to 100 was made outdoors near the powerhouse at Prairie du Sac. Accurate sheet metal templets were cut to the shape of the river channel at each change of section of the river. These templets were supported on short wooden piles and were held firmly by bolts placed to permit accurate adjustment of the templets. Concrete was then poured between templets and the surface was struck with a straight edge. Great care was taken to reproduce, accurately, the channel of the prototype. All details of the dam and the power plant which concerned river flow were carefully reproduced in the model. A gate at the downstream end of the model which controlled the tailwater consisted of a plank hinged at the bottom with its top cut to conform generally to the shape of the section of the river. In whatever position it was placed, the gate allowed water to be released from the model so that the distribution of flow across the section was very nearly the same as that in the prototype. The model of the river bottom was poured with concrete composed of white cement and light-colored sand. The concrete bottom was divided into 1-ft squares by coordinates painted with asphalt. One coordinate was parallel to the end of the apron of the dam.

Ten different arrangements (a total of one hundred and six runs) of the model were tested under various flood flows with different distributions of flow through the powerhouse and over the dam and with various combinations of gates to discharge flood flows. Fig. 4 shows one view of the 1-to-100 scale model. Average vertical velocities in the model were studied with small vertical floats loaded so that they would float with one end slightly exposed. Floats were started near the end of the original apron of the dam. One observer plotted the path of each float on the form sheet shown in Fig. 2 by observing the movement of the float relative to the reference lines painted on the surface of the model. Another observer with a stop watch noted the time at which each float crossed the reference lines. These time observations were recorded on the sheet. Floats were selected so that they would just clear the bottom of the channel. Whenever a float encountered shallower water and dragged on the bottom, a shorter float was started a short distance upstream from the obstruction and observations of velocities were continued. Bottom velocities were observed with wooden spheres about $\frac{1}{4}$ in. in diameter which were loaded so that they would sink very slowly in still water. The course followed by each ball and the time were recorded as for the vertical staff floats.

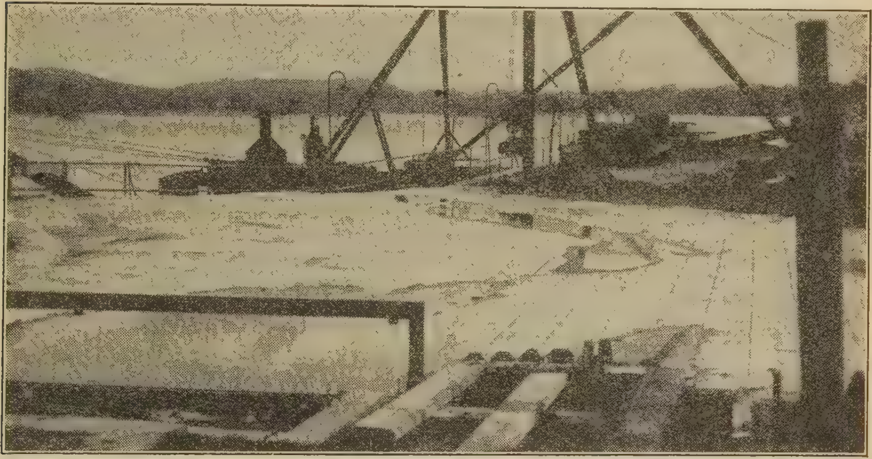


FIG. 4.—MODEL OF THE PRAIRIE DU SAC PLANT; SCALE, 1 to 100

It was decided that a new apron 55 ft wide would be constructed on the prototype at the downstream end of the old apron with the top elevation at El. 731 or 10 ft below the top of the end of the original apron. The 1929 tests indicated that the baffle placed on the old apron would be effective with the new apron. Tests were then run on the 1-to-20 scale model (which had been used in the 1929 tests) to determine the effect of a baffle wall on the new apron and to verify conclusions reached in the studies with the 1-to-100 scale model. Nine different conditions were studied on the 1-to-20 scale model; and it was found that there was no erosion in the bed of the model with: (*a*) The new apron constructed at El. 731; (*b*) the baffle on the old apron as built in 1930; or (*c*) an

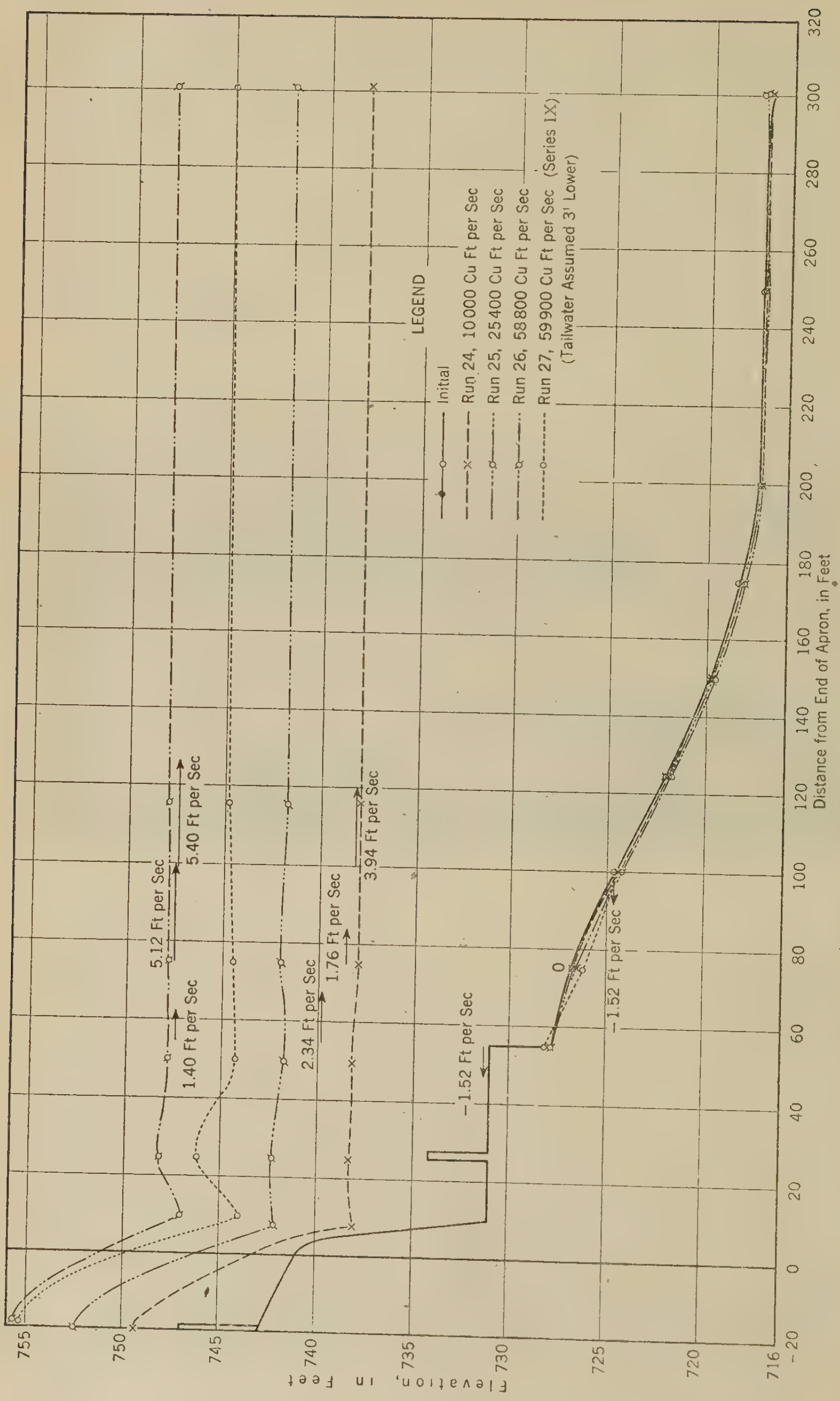


Fig. 5.—Tests in 1933; 1-to-20 Scale Model with New Apron and Baffles

additional baffle 3.5 ft high on the new apron about 30 ft from the downstream end of the new apron.

Tests with the 1-to-100 scale model with the new apron at El. 731 were made with the depressions in the river bottom all filled to El. 710, and bottom velocities were found satisfactory under this condition. Fig. 5 shows the results of the test of the 1-to-20 scale model for the arrangement which was finally adopted with various flows and with tailwater elevations in accordance with the 1929 rating curve. In this test, erosion of the river bed was very small.

The velocities of flow were measured in the model with a pitot tube at the end of the new apron, 20 ft downstream and 45 ft downstream, for surface, midsection, and bottom. In Fig. 5 one set of these velocities for run No. 26, series VIII, 58,800 cu ft per sec, shows the relative positions of observations and the magnitude of each. A tabulation of the velocities for the other runs shown in Fig. 5 is given in Table 3.

TABLE 3.—VELOCITY MEASUREMENTS (FEET PER SECOND) IN A 1-TO-20 SCALE MODEL

(Reduced to Prototype Values)

Location	DOWNSTREAM DISTANCE, IN FEET, FROM END OF APRON		
	0	20	45
(*a*) RUN NO. 24; SERIES VIII; 10,000 CU FT PER SEC			
Surface.......	+2.33	+3.64	+2.16
Middle.......	+1.53	+1.98	+1.53
Bottom.......	+1.76	0	0
(*b*) RUN NO. 25; SERIES VIII; 25,400 CU FT PER SEC			
Surface.......	+3.80	+3.64	+1.97
Middle.......	+3.42	0	+2.33
Bottom.......	+2.93	+1.98	−2.97
(*c*) RUN NO. 26; SERIES VIII; 58,800 CU FT PER SEC			
Surface.......	+1.40	+5.12	+5.40
Middle.......	+2.34	+1.76	+3.94
Bottom.......	−1.52	0	−1.52
(*d*) RUN NO. 27; SERIES IX; 59,900 CU FT PER SEC; TAILWATER ASSUMED 3 FT BELOW NORMAL FOR SAME FLOW			
Surface.......	+9.30	+5.23	+4.68
Middle.......	+4.85	+3.05	+2.55
Bottom.......	+2.34	−1.77	+1.77

A new apron was constructed on the prototype in 1935 at El. 731 with a baffle at El. 734.5 and with a smooth transition section from El. 741 to the top of the new apron.

Figs. 6(*a*) and 6(*b*) are photographs of the model with the new apron and baffles which were finally adopted. Fig. 6(*a*) shows the baffles and apron of the model; and Fig. 6(*b*), a flood flow of 58,800 cu ft per sec on the model. A flood of 90,000 cu ft per sec occurred on September 15, 1938, and was the highest recorded flood at Prairie du Sac. The second largest flood—82,000 cu ft per sec—occurred on April 14, 1922.

The new apron was designed with a water seal between it and the old apron so that either structure could move somewhat without causing a reaction on the other. Sheet piling was placed at the downstream end of the new apron. The apron was supported on wood bearing piles placed to withstand uplift—so that the new structure would safely withstand a head of 13 ft or any head up to El. 743. A drain was constructed of porous concrete under the new apron which discharged normally at El. 738.0 into manholes at the ends of the dam. Stop logs were provided in the manholes which permitted adjustment of the water level under the new structure. The hydraulic gradient on a section across the spillway structure would be: Headwater elevation, 774.0 ft; elevation within the hollow spillway section, 747.0 ft; elevation for drainage overflow at new apron, 738.0 ft; and minimum tailwater elevation, 732.0 ft.

The line labeled "September, 1935," in Fig. 1, represents the profile of the river bottom as it was just after construction of the apron on the prototype. The solid line shows the position of the river bottom in 1941; and the circles, the bottom in 1944. There has been no loss of material in the first 200 ft downstream from the end of the old apron. In fact, the river bottom has been raised somewhat by deposition of material brought upstream from points 400 ft or more from the downstream end of the apron. The condition at this section is quite typical of that at all sections. At gate No. 7 a slight amount of material was removed between points 220 ft and 400 ft downstream from the end of the old apron—the average removal being about $2\frac{1}{2}$ ft and the maximum 5 ft. In this region where there was a slight loss of material, the bottom had been left somewhat above the general bottom elevation of adjacent sections at the time of construction. In general, there has been no loss of material except in very small isolated areas which were not leveled off to conform with general elevations of surrounding areas. The new construction has been subjected to a greater range of flow conditions than had occurred within the period of record

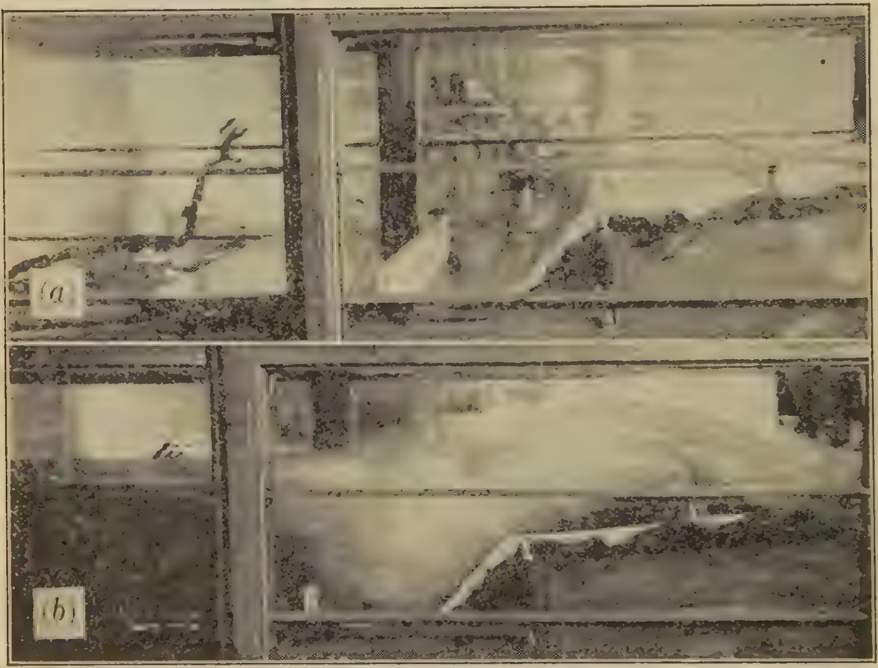


FIG. 6.—MODEL OF FINAL DESIGN
(a) View of Baffles and Apron
(b) Flow, 58,800 Cu Ft per Sec

prior to the construction of the new apron. The greatest flow ever recorded passed the river at Prairie du Sac at 8 a.m. on September 15, 1938. Soundings have been taken every year since the apron was completed; and there has never been any material change in the river bottom since 1935—except tendencies toward general raising of the river bottom in the first 200 ft downstream from the old apron and toward leveling off high spots which were left after construction.

Both the 1-to-100 scale model tests and the 1-to-20 scale model tests indicated that removal of bed material below the dam was caused by upstream currents which existed from a distance at least 180 ft downstream from the end of the old apron to the apron. In no case did the jet from the old apron plunge under causing high bottom velocities in a downstream direction. Excessive bottom cutting in the model was always caused by upstream currents which occupied about two thirds of the depth of the channel.

CONCLUSIONS

At various times during the tests of the 1-to-100 scale model at Prairie du Sac, the model was adjusted to simulate conditions then existing on the prototype. Both the model and the prototype could be observed at one time. Under one condition, there was a pronounced surface current in the prototype which began about 1,200 ft downstream from the powerhouse and extended across the channel upstream toward the center of the Tainter gate section of the dam. With comparable flow distribution and tailwater level, such a condition was found to exist in the model. In a similar manner various combinations of powerhouse units that caused eddies in the tailrace were duplicated on the model. The distribution and direction of currents in the two sizes of models were nearly alike. High-velocity upstream currents on the river bottom occurred over essentially the same areas in the two models.

In the model tests the new apron and baffles reduced bottom velocities near the dam under all conditions—the highest bottom velocities were small and were generally in an upstream direction. The baffle on the old dam used with the new apron gave good results; but a material improvement was experienced when the baffle was placed on the new apron.

The design of improvements, based on model tests, has proved entirely satisfactory. The profiles of river bottoms in Fig. 1 show that bottom velocities have been reduced so that there is no objectionable cutting of the river bed near the dam.

High bottom velocities and considerable river bed movement occurred in the models under conditions which caused serious erosion in the prototype. Ten years of experience with the new structures indicate that the new apron and baffles were as effective in eliminating erosive velocities in the prototype as they were in the model.

DISCUSSION

George P. Steinmetz,[6] M. ASCE.—The correction of tailwater erosion at the Prairie du Sac Dam (Wisconsin) by the construction of an additional apron and baffles was well planned, thoroughly investigated, and has proved to be highly successful. This project again emphasizes the economic advisability of careful model testing to determine the most advantageous corrective methods, as well as the nature of operation to be followed in making maximum use of corrective facilities after they have been installed. The erosion, necessitating corrective measures, further emphasizes the economy and desirability of model testing before initial construction of new projects when the behavior of foundation material cannot be anticipated with certainty. It is appreciated, however, that at the time this dam was constructed the science of model testing of this nature was not well developed.

The increase in bottom velocities in run No. 27 over run No. 26, with tailwater below normal in the latter run, as shown in Table 3, indicates the advisability of a gradual building up of tailwater elevation to normal before the gates are opened to the full extent necessary to pass a given flood so that excessive bottom velocities may be avoided.

The effects of flow distribution, as it might be controlled by particular combinations of gate openings, are indicated in the first paragraph of the "Conclusions." The determination and adoption of the most advantageous combinations of gate opening for various rates of flood and power generation discharges may be particularly important.

The entire problem of controlling tailwater erosion at the Prairie du Sac Dam resulted from the characteristics of the shifting sand bed of this stretch of the Wisconsin River. Some understanding of these conditions can be obtained by reading the 1913 and 1916 articles concerning the construction of the dam which were cited in the paper.[3,4,5] In this section of the river, where the sand bed is not controlled by a dam or other barrier, the bed is in a continual process of moving downstream in the form of moving sand bars. This movement is particularly evident at the stream gaging station at Muscoda, Wis., on the Wisconsin River, a number of miles downstream from the dam. At this gaging station, which was established at about the same time as the Prairie du Sac Dam was constructed, it is a common occurrence to observe a sand bar upstream from the gaging station during the early part of the open water season. As the season progresses, the bar moves closer to the station, finally reaching it; and at that time a normal depth of water of from 8 ft to 9 ft at the gaging station is replaced by 6 in. to 1 ft of water over the surface of the bar.

Soundings at the Muscoda Bridge for regular discharge measurements during the period 1914-1929 give some indication of the amplitude of moving sand bars. At eleven representative stations across the channel the extreme changes in bottom position were from 4 ft to 22 ft in 15 years, with an average range of nearly 8 ft for the eleven stations. Airplane photographs of the river show the sand bars quite clearly. No reliable information exists, however, on the speed with which these bars pass downstream, or the bed-load volume involved. Historically, this movement of sand bars has been in progress since early log rafting days on the river, but it appears that the downstream movement had not been seriously considered. When a large pond is created by a dam on the river, the drifting bars above the dam are arrested in the pond, but downstream from the dam the movement continues, thereby lowering the stream bed and the tailwater at the dam. Immediately downstream from the dam the movement of material is greatly accelerated by the new and rapid currents introduced by the dam.

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[6] Chf. Engr., Public Service Comm., Madison, Wis.

It appears that until such time as it may become economically feasible to develop slack water throughout the sandy reach of this river, about 200 miles in length, any dam constructed on the sand bed will experience a lowering of the stream bed similar to that experienced at Prairie du Sac, and, therefore, measures to retard or to take advantage of such erosion will be advisable. It is doubtful if such retardation can be effected without the creation of a deep pool or stilling basin, such as was rapidly created by nature at the Prairie du Sac Dam through the erosive action of flood discharges.

Perhaps the initial construction of an apron and baffle system on any future projects, such as has been developed at the Prairie du Sac Dam, over a period of time, would result in the creation of a relatively stable stilling basin in the sand bed by the natural action of floodwater discharge.

In the event that a project was designed, as indicated herein, to create a stilling basin or pool through designed erosion, it would be advisable to determine by model tests the extent of erosion in different periods of time and so to design draft tube outlets, locks (if any), and other parts of the structure for the ultimate tailwater elevation and head that would be created.

The piling foundation of the apron addition, which is designed to withstand an uplift of about 13 ft of head, is unique in Wisconsin. It is particularly important in this project, in that it allowed the effective head on the original structure to be decreased and, at the same time, allowed the total head on the project to be increased. The result was that the output was increased, particularly during periods of low stream flow when tailwater is lowest and firm power most essential.

Louis M. Laushey,[7] Jun. ASCE.—Different methods have frequently been used to dissipate the kinetic energy possessed by the high-velocity jet on the apron, the best of which has probably been the use of a hydraulic jump. The problem at Prairie du Sac was a gradual lowering of the tailwater to such a depth that apparently a jump would not form on the apron for all discharges.

If the discharge and initial depth are known, the downstream depth required for the jump to form can be computed from the momentum equation. When the tailwater depth is insufficient, a jump will not form on the apron. The authors have used the common method of interposing a baffle or sill in the path of the jet to create an additional upstream force to aid in the formation of the jump at the desired position. The obstructions also have the additional value of deflecting the high-velocity filaments toward the surface, the result being to keep the velocities on the bottom at a minimum. Frequently, a roller can be produced that actually causes the bottom velocities downstream from the end sill to travel toward the dam and to build up any scoured material behind the sill.

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[7] Instructor in Civ. Eng., Carnegie Inst. of Tech., Pittsburgh, Pa.

Tests are being made at Carnegie Institute of Technology in Pittsburgh, Pa., to determine the effectiveness of a rectangular end sill on the production of a hydraulic jump when the tailwater depths are below normal. Some experiments have been made in a rectangular channel 12 in. wide and 30 ft long. A thin, high-velocity jet was admitted to the upstream end by confining the discharge between the channel bottom and a low, rigid plate which extended across the channel and was parallel to the bottom. The inlet plate was adjusted to different depths in such a manner that the water surface upon exit into the channel was parallel to the bottom. The jump was formed at the lower end of the channel after traveling about 20 ft. It was assumed that a velocity distribution would be produced at the beginning of the jump closely resembling that of the discharge after flowing down the spillway of a dam. The initial and final depths, and the discharge, were measured when a jump was formed on the apron without a sill, and with sills of various heights. Tailwater depths were measured with piezometers at 2-ft intervals for a distance of 6 ft downstream from the sills. An adjustable gate at the lower end of the channel made it possible to adjust the tailwater to obtain the minimum depth required to form a jump. Minimum depth was defined as the lowest depth for which the piezometers downstream from the sills recorded the same height. This setup insured that the depth recorded in the model would be comparable to the actual depth in the prototype rating curve. Furthermore, any depth below this minimum caused a wavy surface that was highest over the sill, and the possibility of high velocities being deflected toward the bottom was considered more likely than when the depth downstream was constant. The use of a second baffle, as at Prairie du Sac, would decrease the piling up over a single baffle with a resulting drop in the water surface to the actual tailwater elevation.

Analysis of Fig. 3(*b*), given for the 1929 tests by the authors, shows that at low discharges a jump does not form to increase the depth below the sill. Actually the baffle for the smaller flows, when the tailwater is very low, acts as an overflow weir. As a result the water drops below critical depth on the apron downstream from the baffle to nearly its original depth. This condition holds true to a lesser extent for the higher flows. Any increase of tailwater depth to the height over the sill would not cause the jump to move upstream. The fact that a small amount of scouring resulted during operation of the spillway is probably a result of the small velocities at low flows, and of the higher depths and the increased effectiveness of the baffle to form a jump at the larger discharges. An added protection against scouring was obtained by placing the baffle 20 ft from the end of the apron. The "drop-off" from the end of the apron to the unpaved stream bed, obtained in the 1929 tests, would aid in giving good results by reducing the velocities because of the added depth of flow.

Perhaps of most importance in the correction of the erosion was the excellent velocity distribution obtained by the authors as shown in Fig. 5 where the bottom velocities either were zero or were directed upstream when the new apron was installed. This is a necessary function of any sill or baffle— the formation of a low bed velocity. The effect that such a device has on dissipating energy and forming a jump at lower tailwaters than otherwise required is of little importance if high velocities reach the bottom.

From experimental curves and work in process at Carnegie Institute of Technology, an attempt has been made to analyze the height of baffle of 52 in. used in the 1929 tests for forming a jump with low tailwater depths. The initial jet depth for 50,000 cu ft per sec (50.8 cu ft per sec per ft of apron), from Fig. 3(*b*), scales between 1.5 ft and 2 ft. For the former initial depth, it would be possible to produce a jump, with no drop in elevation anywhere downstream, with an actual stream depth only 80% of that required without a baffle. Using the other limit of the range would require a depth of 87% of normal. The ordinary hydraulic jump formula would require a tailwater depth at the baffle of between 8 ft and 9.5 ft, depending on the assumed initial depth. Thus, the required elevation of tailwater for 50,000 cu ft per sec with no baffle is between 751 ft and 752.5 ft. Fig. 3(*b*) shows an actual depth of 749 ft or a depth, without a baffle, at 20 ft, of 6 ft of water. This yields a percentage ratio, at the distance −20 ft, of depth available to depth required for a jump on the apron of between 63% to 75%. Therefore, it seems that the tailwater depth is about 15% too low for a jump with no loss in depth to form at this discharge. That this estimate is perhaps reasonable can be seen by inspection of the profile in Fig. 3(*b*) where the height over the baffle is about 6 ft above the stream elevation. Experiments indicate that at this flow a baffle less than two thirds of the height used would have produced a jump at 50,000 cu ft per sec. Of course, the baffle would have to be designed for 65,000 cu ft per sec, and this flow would require a higher baffle. The maximum discharge seems to be drowned out on the bucket, and the depth at distance −42 ft from the end of the apron seems to be high for the jet depth. Therefore, it is unfortunate that a similar analysis could not be made for this discharge. At any rate, the amount of scouring is of most importance, and undoubtedly the 56-in. baffle was used on the basis of best protection from scour, and not just on the formation of a hydraulic jump.

Abraham Streiff,[8] M. ASCE.—An important contribution toward the solution of several questions involved in the design of dams is contained in this paper. In particular, the publication of the 33-year experience record of the Prairie du Sac Dam is a valuable addition to the literature on scour below dams. This dam, on a sand foundation, designed by Daniel W. Mead, Hon. M. and Past-President, ASCE, is a remarkable structure in many respects. The authors describe mainly the conditions and corrections of tailwater erosion, which will be discussed first. Several other features are also mentioned which warrant special discussion.

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[8] Fargo Eng. Co., Jackson, Mich.

The main and important conclusion to be drawn from the paper is the following: To be considered successful, a dam structure should be permanent and secure. To this end the paper demonstrates that it is unnecessary to design against the occurrence of all erosion below the apron and that the aim of the designer should be to prevent such erosion. If the design is such that the erosion itself is stable, and does not endanger the structure, the design may be considered completely successful. Experience has taught how to attain this end.

There are many dams with erosion below the apron, not only on friable soil foundations, but also on the hardest rocks, such as gneiss.[9] In many of these cases corrective measures have been taken; but in others, with equally successful results, no such measures were necessary. These structures, built since the late 1880's, all illustrate the truth of the foregoing statement. An important additional proof has been given by the paper.

This principle should be emphasized because recent discussions frequently center around the building of aprons so that: (1) Erosion downstream is prevented; (2) the hydraulic jump is created; and (3) erosion occurs on the apron.

(1) *Prevention of Downstream Erosion.*[10]—This requirement is frequently advanced, but the necessity of preventive measures is not sustained by actual experience, nor is it followed in actual design. The Prairie du Sac Dam in Wisconsin is only one of many examples. It is important because the maximum erosion, of one and one-half times the normal head, occurred in a sand foundation. The dam is apparently entirely secure.

The width of the structure—104 ft from upstream sheet piling to downstream end of apron—is very short considering the head of 32 ft and the sand foundation. Comparison with the structures, listed in Table 4(*a*) built during the same period and on similar foundation material, reveals that they have aprons much longer than the apron of the original Prairie du Sac Dam. Hence, none of these dams shows any erosion below the apron. Although this condition is satisfactory, the question arose as to whether a considerable saving in cost might not be attainable by making shorter aprons and by allowing a certain amount of stable erosion, especially since a number of existing and older structures show that this result is quite feasible as has been proved again by the paper. The original apron of the Prairie du Sac Dam was lengthened 50 ft. The baffle directs the flow upward, and thus corrects the original downward inclination of the apron. The same effect is also obtainable by a toe weir with inclined upstream face. A number of dams built later than the Prairie du Sac Dam, on soil foundations, have shorter aprons and have permanent but harmless erosions below the structure (see Table 4(*b*)). These dams have experienced maximum flood discharges of 115,000 cu ft per sec several times, and have permanent but harmless downstream erosions.

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[9] "Technisch wirtschaftliche Fortschritte auf dem Gebiete der Wasserkraftanlagen in der Schweiz," by E. Meyer-Peter, World Power Conference, Vol. IX, 1930, p. 77.

[10] "Development and Hydraulic Design, Saint Anthony Falls Stilling Basin," by Fred W. Blaisdell, *Proceedings*, ASCE, February, 1947, p. 123.

There are much older structures on soil foundations which have permanent and harmless deep erosions downstream. An example is the Trowbridge Dam on the Kalamazoo River, in Michigan, which was built in the 1890's on a silt and sand foundation. The head is 22 ft and a permanent erosion 40 ft deep exists downstream from the pole apron.

TABLE 4.—OVER-ALL UPSTREAM AND DOWNSTREAM LENGTH OF EXISTING DAMS ON SOIL FOUNDATIONS

Dam	Location	Head (ft)	Year built	Length[a] (ft)
(a) LONG APRONS				
Croton	Muskegon River, Michigan	40	1907	289.75
Mio	Ausable River, Michigan	32	1915	245.21
Alcona	Ausable River, Michigan	40	1922	253
Junction	Manistee River, Michigan	55	1917	292.75
(b) SHORTER APRONS				
Prairie du Sac	Prairie du Sac, Wisconsin	32	1914	104
Dunlap	Guadalupe River, Texas	35.2	...[b]	122
Abbott	Guadalupe River, Texas	31.0	...[b]	110
Nolte	Guadalupe River, Texas	24.7	...[b]	93
Texas Power Company: IV	Guadalupe River, Texas	30.5	...[b]	112
Texas Hydro-electric Company, H4	Guadalupe River, Texas	28	...[b]	118
Texas Hydro-electric Company, H5	Guadalupe River, Texas	29	...[b]	118

[a] Length from upstream sheet piling to end of apron. [b] Various years between 1927 and 1931.

Many dams with permanent downstream erosions also exist on rock foundations.[11,12] Other examples, in Europe and in the United States, are numerous.[13,14,15] The only requirements are that the erosions be stable and that they not tend to undermine the structure. Considerable experience is recorded to assist in determining how such results can be attained. The paper clearly states this important conclusion.

(2) *Creation of a Hydraulic Jump.*—At low heads and at discharges of great depth, the hydraulic jump is an imperfect method of destroying energy, as may be noted from available tables of hydraulic jump data.[16] Therefore, there is no merit in attempting (as so often is done) to create a sufficient depth on the apron to cause the formation of a hydraulic jump at low heads. The authors illustrate this fact clearly in Fig. 6(*b*) where, although the upper baffle is too high to create a true jump, it, nevertheless, proved effective.

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[11] "Concentrated Flow Erodes Rock Below Wilson Dam," by Hugh P. Oram, *Engineering News-Record*, February 3, 1927, p. 190.

[12] "The O'Shaughnessy Dam and Reservoir," by John H. Gregory, C. B. Hoover, and C. B. Cornell *Transactions*, ASCE, Vol. 93, 1929, pp. 1501–1502.

[13] "Kolk abwehr," by A. Schoklitsch, Julius Springer, Vienna, 1935.

[14] "Mécanisme de l'Eau," by René Koechlin, Béranger, Paris, 1926.

[15] "Reservoirs," by James Dix Schuyler, John Wiley & Sons, Inc., New York, N. Y., 1908.

[16] "Hydraulics of Open Channels," by Boris Bakhmeteff, McGraw-Hill Book Co., Inc., New York N. Y., 1932, p. 247.

(3) *Locating the Jump on the Apron.*—In low-head design, this often-specified requirement has been proved unnecessary. It is more important to direct the nappe toward the surface, which the authors achieve by baffles, as stated under "Conclusions." This procedure creates as much room as possible for the upstream bottom velocities of the "ground whirl." The erosion caused by the extension of the turbulence below the apron, as the authors show, is permanent and harmless, even though the erosion depth is 20 ft below the new apron and the foundation consists of sand. Experience with many other structures, some of which are listed in Table 4, illustrates this fact.

(4) *Lowering of Tailwater Elevation.*—The lowering of tailwater is a familiar phenomenon on many rivers in alluvial beds. The dam intercepts the movement of bed load, and below the dam the existing grade creates new bed load and degrades the tailwater. The total for the Prairie du Sac Dam was fortunately not great; apparently it was not more than 6 ft after which it seems to have been stabilized. Greater increases have occurred in some Michigan dams. At the Junction Dam on the Manistee River the tailwater was lowered 10 ft and was only stabilized by the construction of a weir downstream, which is only temporary. The degradation continues. In 1917, when the dam was designed, previous experience with the Croton Dam on the Muskegon River caused the writer to place the Junction draft tubes 10 ft lower than necessary. Even this amount proved insufficient, for the draft tubes have been corrected since, and later a counter weir was built to stabilize the head. The apron, which was previously submerged, is "high and dry" at low water. This lowering of tailwater is much in evidence in the southwestern United States—for instance, at Boulder Dam.

(5) *Other Features of Prairie du Sac Dam.*—Several other interesting and successful features of the Prairie du Sac Dam are shown in the paper. Since there is no base slab, the upstream sheet piling forms the principal cutoff. There is no horizontal percolation distance; the reduction in head by the sheet piling is 27 ft or 75% of the present head. At the Mio Dam, which has a base slab, the upstream piling reduces the head by 50%. The advantage of the design shown in Fig. 1 is obvious and apparently has been quite successful.

The dam has forty-one Tainter gates 20 ft wide by 14 ft high. The dam is on a large river in an area that has a severe winter climate. Apparently a multiplicity of rather small gates has not been a handicap under severe ice conditions. It would be of great interest to learn from the authors how the spillway is guarded against ice thrust and is kept operative during the winter and during the breakup of the ice.

R. H. KEAYS,[17] M. ASCE.—The project described by the authors is very interesting to the writer, as several years ago he had a similar problem in connection with the flood control works on the Struma River in Macedonia, Greece.

The term "tailwater erosion" is not strictly accurate; the authors would do better to state that the erosion was caused by the turbulent flow from the dam. It is understood that the tailwater elevation is measured after the turbulent area is passed and the river has assumed a normal flow.

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[17] Jackson Heights, N. Y.

The "Synopsis" states that in the course of time there was a general lowering in the tailwater level and the river bottom downstream from the dam. It had been expected that long since, another dam would have been constructed downstream, which, presumably, would have backed up the water enough to maintain the tailwater elevation and prevent erosion of the river bed. As this was not done, however, erosion continued and a sharp drop developed at the Prairie du Sac Dam, in Wisconsin. It would be interesting, therefore, to know how much erosion has occurred at a distance from the dam sufficient to be free from the influence of the turbulent flow.

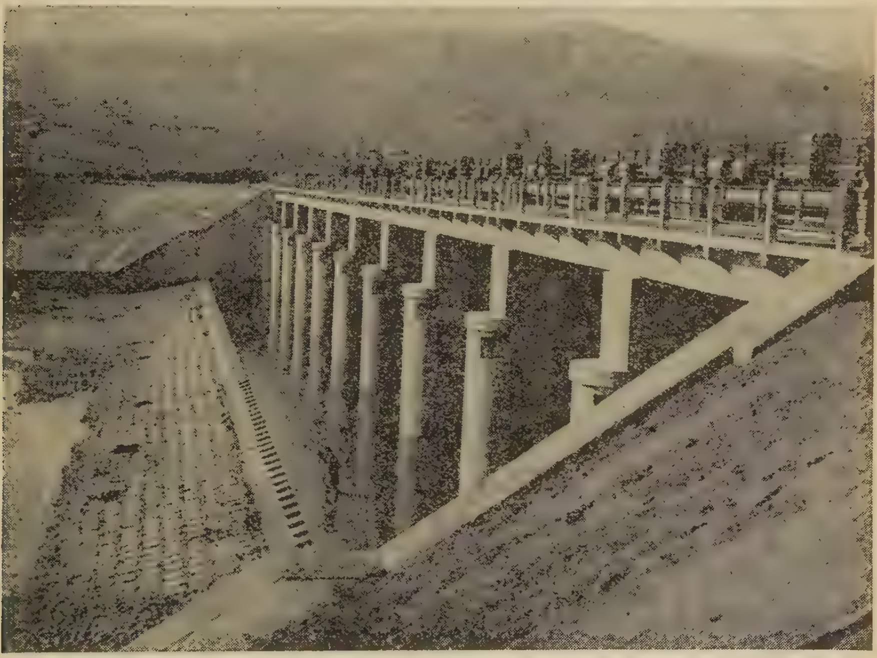

Fig. 7.—Downstream View of Weir, Showing Articulated Block Paving and Dentated Sill

The Kerkini weir on the Struma River near the north end of the Struma Plain is a structure similar to the Prairie du Sac Dam (see Fig. 7). It also is built on a floating foundation in sand with a line of steel sheet piling at the upstream and downstream ends. In all respects it is similar to the barrages on the Nile River in Egypt, and was designed by an engineer with long experience in the Nile Irrigation Service.

The weir creates a large detention basin Kerkini Lake to regulate a flood of 4,000 cu m (141,000 cu ft) per sec, and this basin catches all the sediments brought down through the Rupel Gorge from the watershed in the mountains of Bulgaria to discharge into the Struma Plain.

The design of the weir was referred to Karl Rehbock at Karlsruhe in Germany, and his modifications of the design were based on hydraulic model experiments in his laboratory. Professor Rehbock's design included what he called a dentated sill. The dentations were a row of concrete "teeth" projecting upward at the downstream edge of the apron. They corresponded to the baffle wall described by the author. Professor Rehbock laid great stress on the importance of these dentations in breaking up the velocity of the current.

With this design it was calculated that a so-called "roll" would develop in the flow downstream from the apron. A roll can be described as a vertical eddy with the return flow at the bottom. The experiments showed that the erosion downstream from the apron would be in the form of a shallow depression extending for only a hundred feet or so.

Apparently, Professor Rehbock gave no consideration to the fact that the Struma River was a heavy sediment carrier. The result was that the relatively clear water discharge from the weir soon began to erode the bottom downstream from the weir for a long distance. This erosion was neutralized to some extent by material brought down by tributaries below the weir, but it was increased because of a cutoff having been made in the river some distance below the weir to shorten its course. This was a permanent lowering of the bed as no new sediment was coming down as a replacement.

This lowering of the bed, if carried far enough, would endanger the stability of the weir. As nearly as the writer can remember, the weir was designed for a flood flow of 1,200 cu m (42,400 cu ft) per sec. There was no reason to anticipate, however, that the weir would ever have to operate at more than 500 cu m (17,700 cu ft) per sec. It is reported that, at one time in 1935, it was probably operated by revolutionists at a rate considerably in excess of 500 m per sec.

The weir consists of a series of ten steel vertical lift gates supported by intermediate piers and wing walls at the ends, all supported on a concrete foundation on wood piles in a sand formation. To prevent seepage, lines of steel sheet piling are driven at the upstream and downstream ends.

The weir was well along to completion when the writer became chief engineer on the project in 1932. One of the first studies undertaken thereafter was to examine the question of the prospective erosion downstream. Even with a flow of 500 cu m per sec, or less, it was calculated that it would not be long before the stability of the weir would be in danger because of the difference in head allowing water to find its way under the steel sheet piling.

The weir was placed in operation in February, 1933. There was a small flow through the weir for some time—just enough to wash away some small sand deposits above the weir and deposit them on the flatter slope below the weir. As soon as these deposits were exhausted, however, erosion began downstream all along the course of the lower river.

There is no reliable information available as to the present condition of the Struma improvements. It is reported that they were largely destroyed during World War II. The upper works of the Kerkini weir certainly were destroyed as evidenced by photographs in possession of the writer.

C. N. WARD,[18] M. ASCE, AND HENRY J. HUNT, ESQ.[19]—A number of pertinent and interesting factors associated with erosion below dams were emphasized by the various discussions of the paper. The writers appreciate the favorable comments as to the final plans of improvement which were adopted and as to the use of models in such studies.

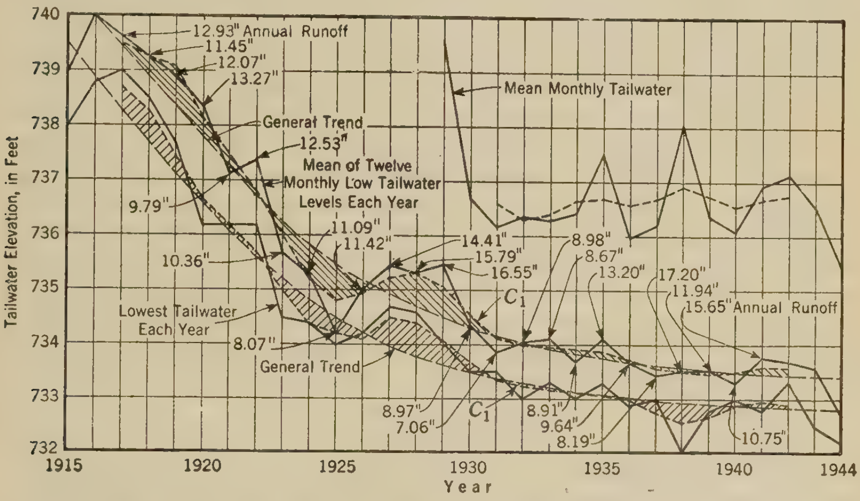

FIG. 8.—TAILWATER DROP AT PRAIRIE DU SAC PLANT ON WISCONSIN RIVER

At the time the Prairie du Sac Dam, Wisconsin, was designed and constructed, 1910-1914, little was known about drop in tailwater due to the movement of stream bed material. Inasmuch as this dam design was unique in many ways, and a departure from the standard practice, a careful check has been maintained on the structure from year to year. From the hourly plant records many interesting curves have been prepared, of which Fig. 8 is highly significant. For example, to understand the curves marked C_1, let a, b, c, d, and e be the annual tailwater elevations for successive years. Then C_1, the progressive mean for the middle year (in which the actual value is C), is expressed as:

$$C_1 = \frac{a + 4\,b + 6\,c + 4\,d + e}{16} \qquad (1)$$

The general trend of the drop in the lowest tailwater experienced each year since 1915 is shown on the lower curve. A similar curve for the mean of the twelve monthly low tailwater levels for each year is also shown for the same period, and the mean monthly tailwater levels for the period 1929 to 1947.

The general trend, as shown by Fig. 8, is toward a stabilized tailwater level. Several conditions may be mentioned that probably have a bearing on this stabilization. Originally, the slope of the river for some distance downstream was about 2.5 ft per mile, whereas today the slope in the first 7 miles is about 1.5 ft per mile. For corresponding flows, the channel width is greater and the area larger now (1947) than in 1915 because of the reduced slope and velocity. The amount of material moved out of the first 7 miles of river is estimated at 11,000,000 cu yd. A large part of this material was from the first 2 miles below the dam, in which reach one large island and several other smaller areas were carried downstream. From a visual inspection of the river bottom downstream from the dam, during the early morning hours when the flow is especially low, it is obvious that several controls play their part in the stabilization of the tailwater level. For instance, there are two highway bridges and one railroad bridge, a few old government wing dams and coarse gravel banks, which act as controls.

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Mr. Steinmetz' description of the shifting Wisconsin River bottom outlines the conditions very well. Some years ago A. Schoklitsch found, by laboratory experimentation, that the amount of bed load in a stream is proportional to the square of the hydraulic slope and the difference of the actual discharge Q and the discharge Q_o at which the material begins to move. With the change in gradient from 2.5 ft to 1.5 ft per mile below the Prairie du Sac Dam, the carrying capacity in the first seven miles downstream is probably less than one third of that experienced in the early history of the dam.

Before the final design for improving conditions below the Prairie du Sac Dam had been selected, many alternate plans were considered—for example, auxiliary dams for tailwater control in the immediate vicinity downstream, or a combined power plant and dam some distance downstream which would control the minimum tailwater at the Prairie du Sac plant. With the apron added in 1935 and the control weir with crest at El. 734 below the powerhouse, it was found that a maximum amount of benefit could be obtained with a minimum of expenditure. Prior to the improvement, the maximum head permitted was 32 ft, so that as the tailwater dropped the headwater was maintained accordingly. With the improvement make, the stability of the structure was assured, and the average annual output increase was estimated at 15,000,000 kw-hr per yr. For a low water year, this increase was estimated at 22,000,000 kw-hr per yr.; and for a high water year only 6,000,000 kw-hr per yr. On the basis of the average yearly increase in output at only 5 mills per kw-hr for the energy, the increase in income for four average years was sufficient to pay the cost of the improvement. In addition to the financial benefits, the stability of the structure is excellent. The step down of the hydraulic gradient through the structure is a valuable improvement.

Observations on the 100-to-1 scale model of the entire dam, lock, and powerhouse were quite convincing in their testimony that the island below the east half of the spillway and the method of gate operation were factors in the development of the 57-ft hole some distance below the lock and dam. It was soon found that all Tainter gates should be opened uniformly across the dam for best results. Much time was spent on the location of the baffle on the new apron, with and without the old baffle, the width of the new apron, and the depth of setting until all of these were adjusted so that the velocity at the end of the new apron was as nearly zero as possible; or, if any velocity were present, it was small and in an upstream direction, with the highest velocity on the surface of the water.

Mr. Steinmetz emphasized the economic advisability of model testing. The paper described only a few of the model tests. Many of them were run using types of improvements developed in other places and as suggested by engineers with many years of experience in the design of protective works on dams. Most of these corrective measures proved to be of little or no value. It is much more feasible to arrive at the solution of such a problem by a trial-and-error method in the laboratory than in the field. The adequacy of the hydraulic jump in destroying energy of flowing water was appreciated at the time the model testing started. The early testing consisted of using different methods of causing turbulence on the apron of the dam to destroy energy so that a depth of flow near the bucket of the dam could be attained which would permit the formation of a jump on or near the apron. Blocks attached to the apron to increase roughness and staggered piers were found to be somewhat effective under certain limited conditions of flow. A so-called "cutter pier," with an edge pointed upstream and with curved sides extending downstream and outward shaped like the moldboard of a plow, was placed with its center line downstream from each gate. It was of no value because it accentuated erosion at the end of the apron under certain conditions of flow. Water rose over this pier nearly to headwater elevation and dropped to the river bottom, eroding deep holes at the end of the apron. A continuous wall or weir of proper height, and placed a certain distance from the end of the apron, was found to be the most effective protection for the original dam and for the new apron.

Mr. Laushey describes the interesting work done at Carnegie Institute of Technology, Pittsburgh, Pa., including the conditions under which the baffle on the old apron of the Prairie du Sac Dam was effective in producing a hydraulic jump. It is true, as he states, that the production of a jump is of little importance if high velocities reach the bottom. In the model tests, particular attention was given to bottom velocities rather than to the formation and the exact location of the jump.

Mr. Streiff is quite right in stating that erosion below dams is not confined to friable soil foundations, but may occur with any type of foundation. The rapidity of the drop in tailwater depends upon the original natural slope in the river, the kind of material, and the location of the structure relative to the tailwater levels for various flows. A dam on the Niobrara River near Spencer, Nebr., was inspected in 1928, and again in 1934. This structure is on shale foundation, and the original slope of the river was about 7 ft per mile. In this 6-year period the tailwater had dropped 7 ft. The dam on the Wisconsin River at Wisconsin Dells, known as the Kilbourn Dam, is on sandstone rock, but the river bottom downstream is similar to that at the Prairie du Sac Dam. The tailwater at this plant dropped from El. 813 in 1909 to El. 806 in 1935 when modifications were made in this structure.

Mr. Streiff cites two dams in which tailwater erosion occurred on a rock bottom. Model testing is often advantageous even though the site of a dam is on rock and no future lowering of the tailwater elevations is anticipated. Subsequent to the Prairie du Sac Dam tests another model of an existing dam was tested because considerable erosion had occurred in a sandstone bottom of the prototype. This erosion resulted from the plunging of the jet from the apron to the river bottom through a considerable depth of tailwater. This unfavorable condition would have existed had there been no recession of general tailwater elevations. An interesting condition was noticed in the last mentioned test. It was found, under certain conditions of flow, that the jet might either plunge to the bottom, causing high erosive bottom velocities, or it might remain on top. Over a certain range of flow in the model this jet could be caused to plunge or to remain on top by directing it with a shovel or a board. On a rising flood the jet might continue to plunge to the bottom over a "flow-tailwater head" range in which it would remain near the surface on a receding flood. During the period of model testing, which was run at the dam site, a high flood occurred in the river. It was noted that the conditions of the jet as observed on the prototype appeared to be precisely the same as disclosed by the model. The model tests showed that a comparatively narrow apron with a continuous baffle wall would eliminate bottom erosion. There was found to be only one height and one location of baffle which would be effective under all conditions. At this dam, bottom velocities were influenced more by changes in the height and position of the baffle than on any other dam studied by the writers.

Generally, a dam should be designed, at the very beginning, to discharge flood flows without serious tailwater erosion. It is usually much more costly to add protective works at a later date. After serious erosion occurs, the old structure is usually less safe and there is often some hazard involved during the progress of construction of new protective works. Some provision should be made in the original design to take care of conditions should there be some future lowering of the tailwater due to the presence of the dam itself or to other changes such as removal of bridges or straightening of the river channel downstream from the dam.

On the other hand, it may be more economical to construct the original dam on a stream in a safe manner, but not design it for the ultimate expected drop in tailwater. Then, as necessity requires, model tests may be made and a design made to meet the conditions. In all probability the general plan of procedure used on the Prairie du Sac Dam was the economical way to do this work. The power company could not have spent the additional amount for the extra work in 1914, and to carry the extra investment over a long period of years would have been a burden.

The forty-one Tainter gates, 14 ft by 20 ft (mentioned by Mr. Streiff in item (5)), are capable of passing 160,000 cu ft per sec. The maximum flood since 1873 occurred in 1938 and amounted to 90,000 cu ft per sec. The pond area is about 9,000 acres at a normal water level of 774.0 ft. No difficulties from ice have been experienced. The ice usually deteriorates and softens before passing the gates. There has been no excessive ice pressure, and the pond is permitted to freeze with no provision for relieving the pressure. The turbine units are capable of using about 15,000 cu ft per sec, which is 1.65 cu ft per sec per sq mile on the 9,125 sq miles of drainage area. Since the power load on the company's system is large enough to absorb the output, there is little need to open the gates until a substantial flood reaches this plant. This gives sufficient time for the ice to break up.

It was stated in the paper that small current meters and pitot tubes did not give satisfactory results in the 1929 tests. In subsequent studies of various models of dams the use of pitot tubes has been found very feasible. In fact, the results of the velocity measurements would be relied upon in future tests. A type of pitot tube and its use was developed which avoided most of the troubles encountered in 1929. The pitot tubes were made from 1/8-in. glass tubes using a blow torch to form the bent ends. For approximate and preliminary studies, a rubber bulb attached to both legs of the tube permitted raising the liquid in the glass legs above water level in the channel where the differential head could be observed. Air carried in the flowing water and entering the pitot tube which gave so much trouble in other pitot tubes could readily be detected when it entered the glass tubes. For more accurate determinations a differential fluid gage, using gasoline, was connected to the pitot tube. A method of flushing was developed which permitted successful removal of air from the tubes. Because of great turbulence in most of the model tests and the considerable amount of entrained air, the pitot tubes and differential gage as originally used were not practicable. Clear water under pressure was brought to the gage below the lowest gage fluid level. A valve in each branch hose connecting to the legs could be adjusted so that when the flushing water was turned on the flow in each leg was such as not to cause the gage fluid to be blown out. The procedure was to place the pitot tube in proper position, open the flushing valve, close flushing valve, and read the gage. This was repeated until two or three consecutive readings gave consistent results. The pitot tubes were calibrated by observing the differential head as recorded by the differential gage when the tube openings were held near the surface of flowing water in a channel. The actual velocity of flow was determined by timing the motion of floats.

Mr. Keays took some exception to the writers' use of the term "tailwater erosion." Although tailwater elevations were measured downstream from the turbulent area, the disturbed or turbulent reach was considered to be "tailwater." The general lowering of the bed of the Struma River in Macedonia appears quite similar to that of the lower Wisconsin River. It is possible that flood flows of 500 cu m (17,700 cu ft) per sec or greater may not have created any greater erosion at the dam on the Struma River than existed at lower flows. Maximum floods at Prairie du Sac did not cause maximum erosion. Floods of about one-half maximum resulted in deeper cutting of the river bottom near the end of the apron than greater floods.

The idea submitted by Mr. Keays to develop a roll in the flow downstream from the apron does not seem desirable because of the varying size of floods that may be encountered. However, the design might work perfectly under certain conditions of flood and tailwater level. The Spencer Dam on the Niobrara River was designed so that the water formed a roll which ultimately cut back under the dam as the tailwater receded, and the dam finally failed.

Mr. Steinmetz compared bottom velocities in runs No. 27 and No. 26 and emphasized the need for building up the tailwater to normal for a given flood. Operation on this basis is vital where erosion of appreciable magnitude has occurred. All gates should be opened progressively, allowing flood flows to discharge as uniformly over the entire length of the dam as is practicable at all times. If only one gate at Prairie du Sac were opened to discharge 2,000 cu ft per sec it would be discharging into a tailwater at an elevation of about 735 ft. Since the rate for one gate corresponds to a flood flow of about 80,000 cu ft per sec for the entire dam, the single gate would be discharging into a tailwater which is about 15 ft too low. Erosion increases quickly with tailwater lowered below normal. When using models with sand bottoms, care must be taken to prevent the lowering of tailwater below the desired limit to be tested. Tailwaters must be adjusted downward in the tests.

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TRANSACTIONS

Paper No. 2325

EFFECT OF STRESS DISTRIBUTION ON YIELD POINTS

BY F. G. ERIC PETERSON,[1] ASSOC. M. ASCE

WITH DISCUSSION BY MESSRS. F. P. SHEARWOOD, EDWIN H. GAYLORD, L. J. MENSCH, GEORGE WINTER, O. SIDEBOTTOM AND T. J. DOLAN, J. E. BAKER AND J. W. RODERICK, R. K. BERNHARD, AND F. G. ERIC PETERSON.

SYNOPSIS

In practice, the yield point of mild and structural steel is obtained by a standard axial tension test. The stress-strain diagram for compression has been found to agree closely with that for tension. After the yield point has been obtained by this standard tension test it is commonly assumed that when any fiber in a steel structure reaches such stress intensity, yielding will occur. Such is not the case; yielding does not occur until higher stress intensities are reached when the distribution of stress over the critical section is not uniform. The stress at which yielding occurs is dependent on type of stress distribution and the shape of the cross section under stress.

The application of loads on a beam to cause bending, and eccentric tension tests, are two of the simplest ways in which to produce a nonuniform stress distribution. Both of these methods have been used in obtaining the data for this paper.

PART I. THEORIES OF PLASTIC FAILURE IN BENDING

Two theories of ultimate moment resistance, M_u, of a given cross section have been developed (1).[2] These will be referred to as the "old theory" and the "new theory." Both theories, no doubt, idealize the actual conditions that exist.

The "Old Theory."—This theory is based on an idealized stress-strain diagram (work line) as shown in Fig. 1(*a*) by the broken line OAB which is substituted for the curved line and is a very close approximation to the curved line. Fig. 1(*b*) shows the stress distribution over a given cross section in pure bending with the extreme fiber stress σ_y below the yield point. Fig. 1(*c*)

NOTE.—Published in April, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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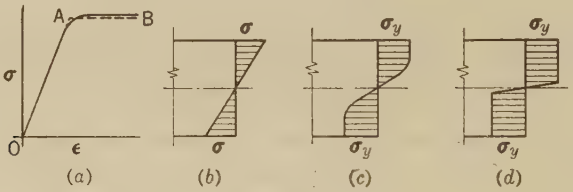
[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

shows the stress distribution after some of the outer fibers have exceeded the yield point and, on the basis of the idealized stress-strain relation, would be as shown in Fig. 1(*d*) where all moment resistance M_u is practically exhausted.

This theory assumes that the ultimate moment resistance is reached when all fibers (both tension and compression) have reached the lower yield point as determined by the usual tension test. It can be seen that to have fibers very near to the neutral axis reaching the yield point the necessary strain requires considerable curvature of the member at the cross section and theoretically cannot actually hold right up to the neutral axis. Nevertheless, this conception of yielding gives values that are good approximations to the actual since fibers very near to the neutral axis contribute very little to the moment resistance in any event. When a section has reached the ultimate value of M_u it is said to act as a yield hinge. The action of such a hinge is not reversible since the behavior on reversing becomes elastic. In the case of I-beams with the web vertical, failure is likely to occur before the "yield-hinge" condition is reached because the compression flange will usually buckle soon after this entire flange has reached the yield point.



(*a*) (*b*) (*c*) (*d*)

FIG. 1

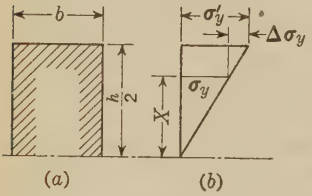
The "New Theory."—This theory is based on the idea that the yield point as obtained by the usual tension test does not necessarily hold for cases of nonuniform stress distribution. In the simple tension test of structural steel the yield point is determined by the occurrence of one or more phenomena. These are: (*a*) The drop of the scale beam of the testing machine, (*b*) the scaling off of the surface material in hot-rolled steel, and (*c*) the appearance of Lueders lines or yield lines in the case of a polished specimen. The appearance of Lueders lines in the polished specimen is without doubt the same phenomenon that causes the scaling of the surface in the hot-rolled specimen. In polished specimens which are subjected to nonuniform stress conditions the forming of Lueders lines occurs at a higher stress than in the simple tension test. From tests by A. Thum and F. Wunderlich (2) of pure bending of various forms of cross sections, the stress at which Lueders lines appear is dependent on the shape of the cross section. More particularly it seems to be dependent on whether the greater part of the material is distributed close to the neutral axis (as in the case of a round bar) or most of the material is distant from the neutral axis (as in the case of an I-beam or wide flange section). In the first case the yield point in bending appears to be raised a considerable amount (about 65% in the case of a round bar (3)) whereas in the second case the increase is not so marked. In the case of a rectangular cross section the increase in the yield point is somewhat more than 40%. Tests by the writer (detailed report of which immediately follows this discussion) seem to bear out these contentions. In the case of pure bending of two polished rectangular specimens the writer found that the stress at which Lueders lines formed was, respectively, 40.5% and 44.0% higher than in the simple tension test, for an average value of about 42.2%. In these tests readings of deflection of the beam at the midpoint were also taken. It is interesting to note (shown subsequently in Fig. 5) that the load-deflection line appears to continue to be a straight line practically up to the appearance of the Lueders lines. This tends to confirm the idea that the yield point is actually raised since otherwise the load-deflection line should tend toward the horizontal at points where the extreme fiber stress is higher than the yield point obtained from the simple tension test.

Further tests by the writer, as reported in Part II, were made on four specimens in eccentric tension to determine whether similar results would be obtained, and also to determine whether the ratio of thickness to width had any bearing on the results. These specimens were also of rectangular cross section. In all four cases the apparent raising of the yield point was more than 40% and the ratio of thickness to width seemed to have little effect. In the thinner specimens the Lueders lines or yield lines formed at right angles to the load, indicating yield across the thickness of the specimen governed. In the thickest specimen, lines were formed perpendicular to the load and also at 45° to the load indicating yield across the thickness and the width of the specimen simultaneously. It is well known that these lines form along the directions of maximum shear stress.

In the tests by Messrs. Thum and Wunderlich (2) a tension test on part of a bar of steel showed it to have a yield point of 38,400 lb per sq in. Next a rectangular specimen was cut from the same bar, polished, and then subjected to bending. A load which, by the usual bending formula, would cause an extreme fiber stress of 49,000 lb per sq in., or 27.5% greater than the tension yield point, was applied 200 times consecutively without the appearance of yield lines or any other sign of failure.

This "new theory" contends that in the case of nonuniform stress the yield point is higher than in the uniform state as indicated by the usual tension test. It also states that the increase $\Delta\sigma_y$ over the tension yield point σ_y is dependent on the shape of the cross section and that when this increased yield point is reached yield occurs suddenly and that the stress then subsides to the tension yield point. This phenomenon has been likened to the case of the delayed boiling of a liquid (4). The explanation of the phenomenon possibly lies in the field of metallography. A. M. Freudenthal, Assoc. M. ASCE (4), states that it is confined to metals that have a "stereocentric" type of lattice structure and is not evident in the "planocentric" or more simple types of lattice structure. The phenomenon of the raising of the yield point, no doubt, is confined to metals that have a very well-defined yield point and therefore includes mild steel and structural steel.

With the knowledge that is available at present it is probably impossible to give a rigid mathematical treatment of the phenomenon. A basic difficulty presents itself in the fact that all the investigator can measure is strain—that is, he cannot measure stress directly. The difficulty from the mathematical viewpoint is that although much is known about elastic behavior, and also about completely plastic behavior, little knowledge is available concerning the range between these states where plastic regions are surrounded by elastic material, and elastic portions are surrounded by plastic regions.



(a) (b)

FIG. 2

Nevertheless, W. Kuntze (3) has developed a method of determining the apparent increase in the yield point which agrees well with the results of numerous tests performed. This method can be stated as follows: Assuming that the triangular distribution of stress in bending holds, even if the tension yield point is exceeded, divide the total stress on one side of the neutral axis into two equal parts; then set the stress at the point which divides these two parts equal to the tension yield-point stress.

For illustration, assume that a rectangular bar is subjected to bending, neglecting the shearing stresses. Fig. 2 shows the upper half of a rectangular cross section in bending, in which X is the distance from the neutral axis to the point that divides the total stress into two equal parts. Then

$$X = \tfrac{1}{2}\, h \frac{\sigma_y}{\sigma'_y} \qquad (1a)$$

and

$$\frac{\sigma'_y}{2} \times \frac{b\,h}{2} = b\,X \frac{2\,\sigma_y}{2} \qquad (1b)$$

Eliminating X from Eqs. 1: $\sigma'_y = \sqrt{2}\,\sigma_y$; or $\Delta\sigma_y = 0.414\,\sigma_y$, which represents a 41.4% increase. It is seen that this value agrees well with the results obtained by experiment. In a series of eight beam tests by Messrs. Thum and Wunderlich (2) an average value of 40.5% increase was obtained. The tests by the writer in both bending and eccentric tension resulted in an average increase of 42.4%, test I in eccentric tension (see Part II) was omitted from this average. Fig. 3, a plat of data by Mr. Kuntze (3), affords a graphic comparison of the values of X obtained by computation and experiment for various types of cross sections. The correlation is very good.

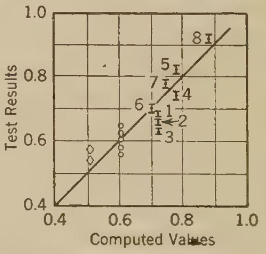


FIG. 3.—COMPARISON OF COMPUTED AND EXPERIMENTAL VALUES OF $X = \dfrac{\text{TENSION YIELD POINT}}{\text{BENDING YIELD POINT}}$

Calculation of the ultimate bending-moment resistance, M_u, on the basis of the "new theory" gives values slightly less than those given by the "old theory"; and they agree better with the results of tests as will be seen in a study made by Mr. Kuntze (3). Whether or not the "new theory" is actually correct, calculation on this basis is simplified since, after σ_y is found, the usual section modulus can be used in calculating M_u, that is

$$M_u = \sigma_y\, Z \qquad (2)$$

An account of the results obtained by the writer in some tests on bending and eccentric tension is presented in Part II.

PART II. TESTS RESULTS ON THE APPARENT RAISING OF THE YIELD POINT OF STEEL UNDER NONUNIFORM CONDITIONS OF STRESS

1. PURE BENDING

These tests were made to determine inconsistencies in results by the usual tension test and those obtained under nonuniform stress. One specimen was tested in pure tension and two specimens in pure bending. Fig. 4 shows the dimensions and the arrangement of the supports and points of application of the loads for the bending tests. The specimens were of cold-rolled flat bar steel $1\frac{1}{2}$ in. by $\frac{3}{4}$ in., machined to 0.700 in. by 1.430 in. at the narrow section. Consequently, $A = 1.002$ sq in. and $Z = 0.239$ in.^{3} A similar specimen was prepared for the standard tension test. All three specimens were annealed at 1,700° F and allowed to cool slowly in the furnace, after which the narrow section was polished so as to facilitate the observation of Lueders lines. Since 1,700° is well above the normalizing temperature the internal stresses due to cold rolling were eliminated. All three specimens were cut from the same bar.

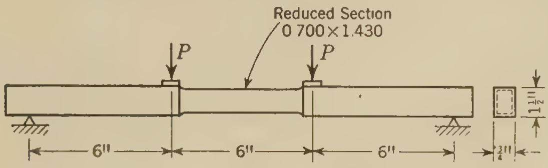


FIG. 4

The tension-test specimen showed an upper yield-point stress of 36,080 lb per sq in. and a lower yield point of 35,370 lb per sq in. The Lueders lines appeared immediately after the upper yield point was reached and spread quite rapidly over the entire specimen.

The specimens tested in bending were subjected to equal loads P at the third points so that the middle section was in pure bending. If the yield point in pure tension had governed the appearance of yielding in bending as evidenced by the formation of Lueders lines, these lines should have appeared at a load of 2,820 lb. Such was not the case, however. Table 1 shows the results obtained. According to the method described by Mr. Kuntze for pure bending of a rectangular cross section, the increase in the yield point should be $\Delta\sigma_y = (\sqrt{2} - 1)\sigma_y$ or about 41.4%. Considering experimental error the results of these tests agree very well with this value.

TABLE 1.—PERCENTAGE INCREASE IN YIELD POINT, BY BENDING TEST

Col. 1, specimens 1, 2, and their average.
Col. 2, Lueders line load.
Col. 3, unit stress σ in kips per square inch.
Col. 4, percentage increase beyond the tensile yield point.

Bar (1)	Load (2)	σ (3)	$\Delta\sigma_y$ (%) (4)
1	3,960	49.7	40.5
2	4,045	50.9	44.0
Average	(4,002)	(50.3)	(42.2)

As was to be expected, the Lueders lines appeared first on the upper and lower surfaces of the cross section in pure bending. They appeared almost simultaneously on these surfaces and spread quite rapidly down the sides of the specimen from the top surface, and up the sides of the specimen from the lower surface, with very little increase in the load. They reached a point about $\frac{1}{4}$ in. from the neutral axis each way. This seems to indicate that the process of yield in bending is a fairly rapid phenomenon rather than a slowly expanding region of yield.

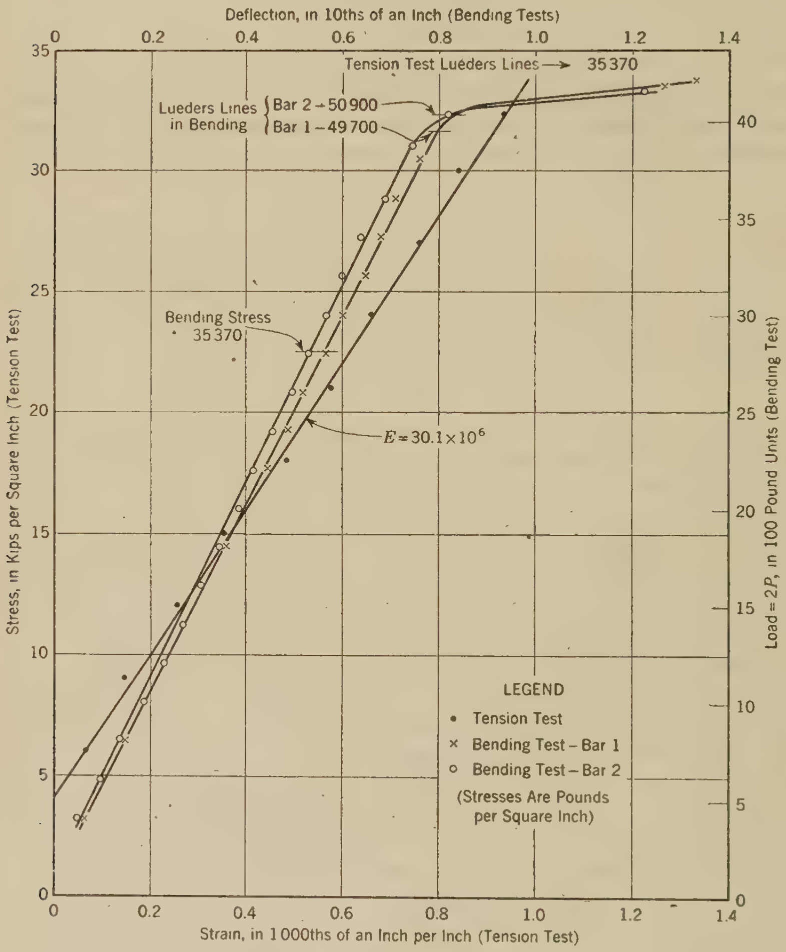


Fig. 5

The results of these tests are shown graphically in Fig. 5. Note that the load-deflection line appears to continue as a straight line for bending stresses far above the tension yield-point stress of 35,370 lb per sq in. Note also that after Lueders lines have appeared the load-deflection lines tend to become horizontal, and the bending resistance is practically used up.

2. Eccentric Tension Tests

These tests were made in order to determine the effect of the nonuniform stress condition in eccentric tension on the apparent raising of the yield point as compared to the yield point determined by axial tension. Four specimens of different thicknesses were used to discover whether the ratio of thickness to width had any effect on the results. Control tests in axial tension were made for each specimen. The steel used in these control tests was obtained from the same bars of steel, respectively, from which the eccentric specimens were obtained.

TABLE 2.—Dimensions of Specimens, Eccentric Tension Tests

Specimen	Thickness b, in Inches		Width h, in Inches		Area A, in Square Inches		Section modulus Z (in.^{3})
	Axial	Eccentric	Axial	Eccentric	Axial	Eccentric	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
I	0.048	0.048	0.627	1.800	0.0301	0.0855	0.0257
II	0.1115	0.111	0.627	1.800	0.0698	0.200	0.060
III	0.234	0.233	0.628	1.800	0.147	0.429	0.126
IV	0.351	0.350	0.626	1.800	0.220	0.630	0.189

The metal for these tests was selected at random from a commercial stock. Therefore, there was no reason to expect the yield point of the individual specimens to agree very closely. The steel was hot rolled. The dimensions of each specimen are presented in Table 2. The form and dimensions of the eccentric tension specimen are illustrated in Fig. 6. The eccentricity was one sixth of the width so that the stress varied from zero at one edge to twice the average at the other edge. Significant phenomena observed at various loads (in pounds) are as follows:

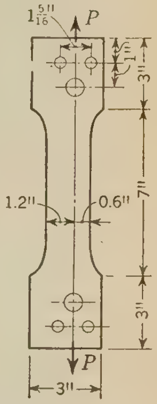


Fig. 6

Load (lb)	Remarks
Specimen I, Axial.—	
1,300	Lueders lines formed near the end curves but did not spread.
1,500	Lines appeared near the center of the specimen and spread over the entire narrow section.
1,600	This was the maximum load applied.
Specimen I, Eccentric.—	
3,300	Lines first appeared at the stressed edge and spread over the length of the specimen. As the load increased, the lines gradually extended laterally toward the zero stress edge.
4,000	This was the maximum load applied.

Load (lb)	Remarks
Specimen II, Axial.—	
2,610	Upper yield point. Lueders lines began to form.
2,590	Lower yield point.
3,000	This was the maximum load applied.
Specimen II, Eccentric.—	
4,700	Lueders lines began to form near the curves at the ends of the reduced section.
5,400	Lueders lines began to form near the middle of the reduced section, at the stressed edge.
6,540	This was the maximum load applied. The formation of Lueders lines near the curved ends, under the 4,700-lb load, must have been caused by stress concentration because, after they formed, no tendency to spread was noticed until lines began to appear near the middle section of the specimen (the 5,400-lb load). Even at these points the increase was 26.5%. At the first appearance of Lueders lines they extend ½ in. or more laterally.
Specimen III, Axial.—	
6,340	Upper yield point. Lueders lines began to form.
6,240	Lower yield point.
6,550	This was the maximum load applied.
Specimen III, Eccentric.—	
12,850	Lueders lines began to form near the middle of the reduced section.
15,000	This was the maximum load applied.
Specimen IV, Axial.—	
10,750	Upper yield point. Lueders lines began to form.
9,900	Lower yield point.
12,000	This was the maximum load applied.
Specimen IV, Eccentric.—	
18,250	Lueders lines began to form at the ends of the reduced section—that is, where the end curves begin.
20,000	Lines began to form near the middle of the narrow section at the stressed edge and spread over the entire specimen.
22,500	This was the maximum load applied.

From the foregoing data it was possible to compute the percentage increase $\Delta\sigma'_y$ as shown in Table 3.

The results of these tests indicate an appreciable increase in the yield point (assuming that Lueders lines indicate the yield point) in the nonuniform state of stress as compared with the yield point resulting from axial tension; they agree quite well with the results obtained in pure bending. Each of the specimens showed an increase of more than 40% and the thinner specimens seemed to show the greatest increase. This was contrary to what was expected as it was thought that, when the axial tension yield point was reached across the thickness of the section at one edge, the Lueders lines would appear. Fig. 7

TABLE 3.—INCREASE IN YIELD-POINT STRESS BY BENDING

Specimen No.	CRITICAL LOAD (POUNDS)[a]		STRESS (LB PER SQ IN.)		Increase, $\Delta\sigma'_y$, Col. 5 over Col. 4 (%)
	Axial	Eccentric	Axial[b] σ_y	Eccentric[c] σ'_y	
(1)	(2)	(3)	(4)	(5)	(6)
I	1,500	3,300	49,900	77,200	55
II	2,590	5,400	37,100	54,000	45.5
III	6,240	12,850	42,450	60,000	41.3
IV	9,900	20,000	45,000	63,500	41.0

[a] See tabulation in the text, and accompanying remarks. [b] $\dfrac{\text{Col. 2}}{\text{Col. 6, Table 2}}$. [c] $\dfrac{2 \times \text{Col. 3}}{\text{Col. 7, Table 2}}$.

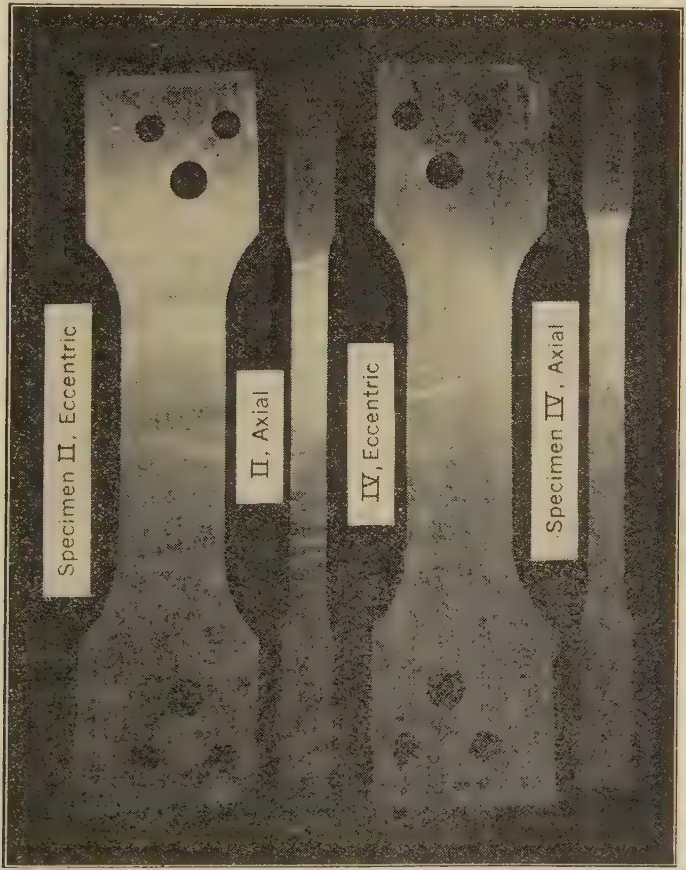
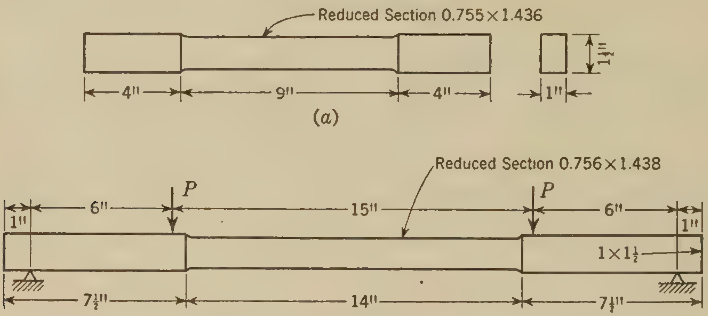

Specimen II, Eccentric
II, Axial
IV, Eccentric
Specimen IV, Axial

FIG. 7

is a photograph showing the Lueders lines on specimens II and IV, for axial and for eccentric tests. On specimens II, which are quite thin, note that the lines are predominantly horizontal whereas on specimens IV, which are relatively thick, there is a marked tendency of the lines to spread at 45° as well.



(a)

(b)

Fig. 8

This is apparently the only effect of thickness. If it is assumed that there is a straight line stress distribution up to the appearance of Lueders lines the apparent raising of the yield point is just as marked for the thin as for the thick specimens. On the three thinnest specimens the Lueders lines formed at right angles to the direction of the load although after reaching a considerable depth there was a tendency to form at 45°. On the thickest specimen the lines formed both at 90° and at 45° to the direction of the load. This seemed to be the only effect of the thickness-to-width ratio. As was shown by Messrs. Thum and Wunderlich (2) and by Mr. Kuntze (3) the shape of the cross section has a marked effect on the raising of the yield point.

The appearance of Lueders lines seems to be the first indication of physical breakdown of the internal structure of the metal. It may be open to question whether or not yield occurs before Lueders lines begin to form. Does the metal reach the plastic state in advance of the appearance of Lueders lines? Referring to Fig. 5, the load-deflection lines of bars 1 and 2 appear to continue in the same straight line far beyond the point at which the stress in the extreme fiber has reached the tension yield point. If this is true, the beam continues to react elastically even after the tension yield point has been reached.

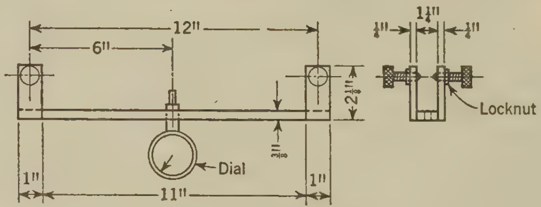


Fig. 9.—Dial Gage Holder

Check Tests.—As a check on the reliability of the foregoing tests, in support of the conclusion that the yield point is actually raised, the writer decided to make further tests in which Lueders lines would not be observed. The writer does not contend that Lueders lines are an exact indication of yield. Since the tension stress-strain diagram shows some curvature at a stress somewhat lower than the yield point (as usually defined), some slight yielding occurs below the yield point and before Lueders lines appear.

Two rectangular specimens were tested in tension and two in bending. The dimensions of these specimens are shown in Fig. 8. All specimens were cut from the same bar of commercial mild steel. Fig. 8(*a*) gives the dimensions of the two tension specimens. Fig. 8(*b*) gives the dimensions of the bending specimens and also the arrangement of loading. This loading arrangement causes the middle 15-in. section to be subjected to pure bending. By means of the frame shown in Fig. 9, which was attached to the reduced section of the bending specimen, it was possible to read, accurately, the deflection at the mid-point of a 12-in. section (subjected to pure bending moment) relative to the ends of that section. A dial, reading directly to 0.0001 in., was used.

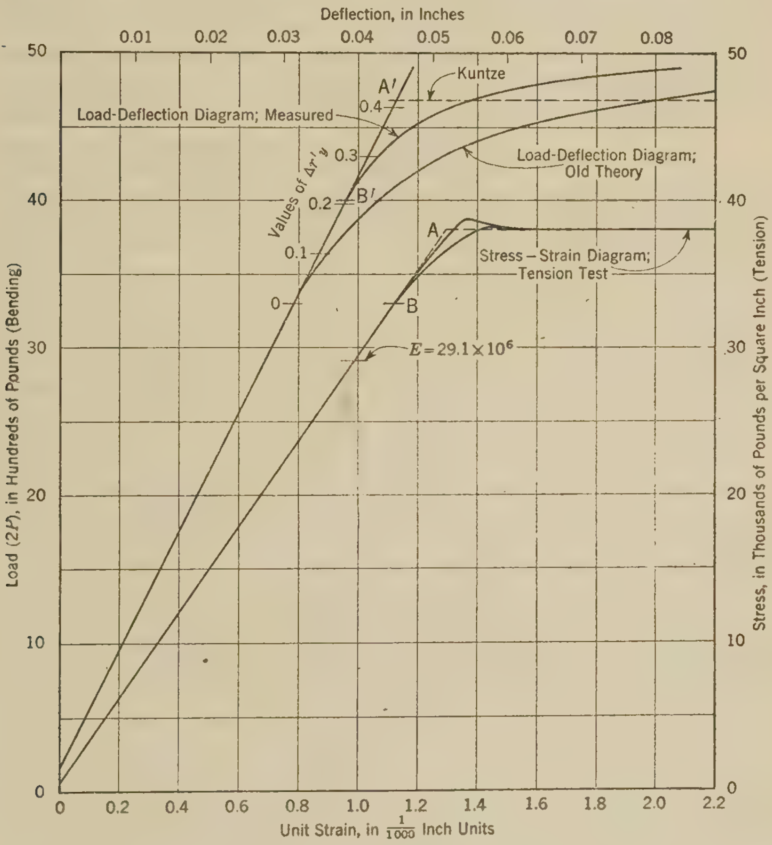


FIG. 10

The results of these tests are shown graphically in Fig. 10. The stress-strain diagrams for the tension specimens were identical except near the yield point where two distinct curved lines appear. The yield points from direct calculation of load over area were, respectively, 37,960 lb per sq in. and 38,050 lb per sq in. The difference between these two values is not sufficient to show on the graph. The dotted lines intersecting at point A represent the idealized stress-strain diagram later assumed in calculations. In Fig. 10 is also shown the measured load-deflection diagram for the two bending specimens. The deflections for each specimen were so similar that only one curve appears.

The question now to be considered is whether or not this measured load-deflection diagram conforms with the tension stress-strain diagram (assuming that this diagram also holds for direct compression). Assume also that a plane section before bending remains plane after bending. Since no direct shear is involved this assumption is quite justified.

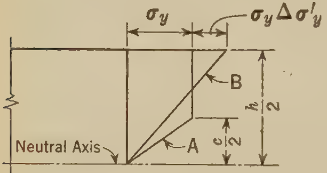


FIG. 11

If a load is applied which is larger than necessary to bring the extreme fibers to the tension yield-point stress, the stress diagram for pure bending must be either as diagram A, Fig. 11, which is based on the idealized tension stress-strain diagram, or as diagram B which assumes an increase of the yield point. In either case the moment of the stresses about the neutral axis for a given load must be the same. The moment about the neutral axis according to diagram A, Fig. 11, is

$$M_1 = 2\left[b\left(\frac{h}{2}-\frac{c}{2}\right)\tfrac{1}{2}\left(\frac{h}{2}+\frac{c}{2}\right)+\tfrac{1}{2}\,b\,\frac{c}{2}\,\frac{2}{3}\,\frac{c}{2}\right]\sigma_y = \frac{b}{4}\left(h^2-\frac{1}{3}c^2\right)\sigma_y \quad \text{. . (3a)}$$

The moment about the neutral axis according to diagram B, Fig. 11, is

$$M_2 = (1+\Delta\sigma_y)\,\sigma_y\,Z = (1+\Delta\sigma_y)\,\sigma_y\,\frac{b\,h^2}{6} \quad \text{. . . . . (3b)}$$

in which Z is the section modulus and $\Delta\sigma_y$ is a factor indicating how much the yield point is exceeded (expressed as a ratio). Setting these moments equal to each other, $c = h\sqrt{1-2\,\Delta\sigma_y}$; or

$$c/2 = h/2\,\sqrt{1-2\,\Delta\sigma_y} \quad \text{. . . . . (4)}$$

Because the moment is constant over the part of the beam considered, the curvature is circular. If diagram A, Fig. 11, governs, the radius of curvature is

$$\rho_1 = \frac{0.5\,c}{\epsilon} = \frac{c\,E}{2\,\sigma_y} \quad \text{. . . . . (5a)}$$

in which ϵ is the unit strain at a distance $c/2$ from the neutral axis. If diagram B governs,

$$\rho_2 = \frac{E\,I}{M_2} = \frac{E\,I}{(1+\Delta\sigma_y)\,\sigma_y\,Z} = \frac{E\,h}{2\,(1+\Delta\sigma_y)\,\sigma_y} \quad \text{. . . . . (5b)}$$

and

$$\frac{\rho_2}{\rho_1} = \frac{h}{(1+\Delta\sigma_y)\,c} = \frac{1}{(1+\Delta\sigma_y)\,\sqrt{1-2\,\Delta\sigma_y}} \quad \text{. . . . . (6)}$$

using Eq. 4. Because of the large radius of curvature as compared with the length of the specimen involved, the ratio of the deflections is inversely proportionate to the radii of curvature, that is

$$\frac{\delta_1}{\delta_2} = \frac{1}{(1 + \Delta\sigma_y)\sqrt{1 - 2\,\Delta\sigma_y}} \quad \text{(7)}$$

in which δ_1 is the midpoint deflection according to diagram A and δ_2 is the same for diagram B. For as far as diagram B governs, the load-deflection line will be a straight line; but, if diagram A governs, the load-deflection line will curve toward the horizontal as the load is increased after the tension yield point has been reached in the extreme fibers.

Referring to Fig. 10, the load in bending at which the extreme fibers reach the yield point in tension is 3,300 lb as indicated by the line $\Delta\sigma_y = 0$. If diagram A, Fig. 11, governed, the load-deflection line would cease to be a straight line above this value and would diverge from the straight line for various values of $\Delta\sigma_y$ according to Eq. 7. Values of the load and the ratio of δ_1 to δ_2 are given in Table 4 for various values of $\Delta\sigma_y$. The curve resulting from these tabulated values is shown in Fig. 10 marked "Load-Deflection Diagram; Old Theory." The load-deflection diagram according to Mr. Kuntze (3) is shown by the dotted line marked "Kuntze." The Kuntze "new theory" assumes a sudden yielding at a unit stress 41.4% higher than the tension yield point, for a rectangular section.

TABLE 4.—VARIATION OF δ_1/δ_2

Description	Values of $\Delta\sigma_y$										
	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
Load	3,308	3,480	3,640	3,805	3,976	4,140	4,300	4,465	4,630	4,800	4,962
δ_1/δ_2	1.000	1.004	1.016	1.040	1.077	1.132	1.218	1.350	1.597	2.180	∞

Now consider the load-deflection diagram (measured). It continues as a straight line up to a load of 4,000 lb, point B′, Fig. 10, giving an extreme fiber stress of 47,500 lb per sq in. This should be defined as the proportional limit in bending. Note that it is 25% higher than the tension yield point. Actually, this value of 47,500 lb per sq in. should be compared with the tension proportional limit. Realizing that this value is never very definite it still does not seem amiss to make an extimate of its value. Referring to the tension stress-strain diagram, a value of 33,000 lb per sq in. (point B, Fig. 10) seems about right. Then the ratio of the proportional limit in bending to the proportional limit in tension would be 47,500/33,000 = 1.44, or an increase of 44%.

For values of the load in bending greater than 4,000 lb the measured load-deflection diagram diverges from the straight line but is much closer to the straight line than to the load-deflection curve based on the "old theory." For a value of $\Delta\sigma_y = 0.30$ the divergence from the straight line according to Table 4 should be 21.8%. It is only 3%. For $\Delta\sigma_y = 0.40$ it should be 59.7% whereas it is actually only 14.5%.

Since the load-deflection diagram of Mr. Kuntze is based on the idealized stress-strain diagram, it was to be expected that the load deflection (measured) should diverge from the straight line at some point below A′ (Fig. 10) in order to conform with the fact that the actual stress-strain diagram begins to curve somewhat below point A.

Thus these latter tests, made without reference to Lueders lines, tend to confirm the fact of an increase in the yield point over that determined by the ordinary axial tension test when the stress is not uniformly distributed.

CONCLUSIONS

The data presented in this paper show that the ordinary axial tension test of mild and structural steel does not give information as to the behavior of the material under nonuniform stress conditions. The stress at which yield occurs is higher under nonuniform stress and is dependent on the kind of stress distribution and also on the form of the cross section under stress.

This fact tends to minimize the importance of stress concentrations since the stress distribution over an entire cross section, rather than the stress at an isolated point must be considered in order to determine when yielding will occur. Mr. Kuntze (3) reports that in tension tests of a flat bar with a round hole through it, the stress near the hole may reach a value from 2.3 to 2.7 times the tension yield-point stress before there is evidence of yield.

Failure due to fatigue stresses is not considered in this paper and, therefore, the paper is confined to cases of static loads or cases where the number of repetitions of loading is relatively small. Nevertheless, in tests by K. Kloppel (5) in the loading of a beam to the tension yield point in the extreme fibers, the load was repeated 700,000 times "without showing any signs of premature breakage through fatigue," and "the permanent deflections were practically zero."

APPENDIX. BIBLIOGRAPHY

(1) **"Grundsätzliches zur Plastizitätstheorie,"** by Josef Fritsche, *Der Stahlbau*, Vol. 9, 1936, p. 65.

(2) **"Die Fliessgrenze bei behinderter Formänderung,"** by A. Thum and F. Wunderlich, *Forschung auf dem Gebiete des Ingenieurwesens*, Vol. 3, 1932, p. 261.

(3) **"Ermittlung des Einflusses ungleichförmiger Spannungen und Querschnitte auf die Streckgrenze,"** by W. Kuntze, *Der Stahlbau*, Vol. 6, Heft 7, 1933, p. 49.

(4) **"Importance of the Ductility of Steel for Calculating and Dimensioning Steel Structural Work, Especially When Statically Indeterminate,"** Preliminary Rept., International Assn. for Bridge and Structural Eng., 2d Cong., 1936.

(5) **"Importance of the Ductility of Steel for Calculating and Dimensioning Steel Structural Work, Especially When Statically Indeterminate,"** Final Rept., International Assn. for Bridge and Structural Eng., 2d Cong., Berlin, 1936.

(6) "Die Beitrag zur Frage der tatsächlichen Tragfähigkeit einfacher und durchlaufenden Balkenträger aus Baustahl St. 37 aus Holz," by H. Maier-Leibnitz, *Die Bautechnik*, Vol. 6, January, 1928, pp. 11 and 27.

(7) "Ductility as a Base for Design-Computation of Steel Bridges and Structures Instead of Proportionality of Stress and Strain," by W. C. Kist, *La Technique des Travaux*, Liége, September, 1930.

(8) "Über die Bemessung Statisch unbestimmter Stahltragwerke unter Berücksichtigung des elastich-plastischen Verhaltens des Baustoffes," by Hans Bleich, *Der Bauingenieur*, Vol. 13, 1932, p. 261.

(9) "Rechnerische Untersuchungen über die Grösse des Fliessbereiches in stählernen Durchlaufbalken unter Berucksichtigung des Momentenausgleiches," by Felix Kann, *Der Stahlbau*, Vol. 5, 1932, p. 105; *Berichtigung*, 1932, p. 160

(10) "Über die Auswirkung der Selbsthilfe des Baustahls in rahmenartigen Stabwerken," by Karl Girkmann, *Der Stahlbau*, Vol. 5, 1932, p. 121.

(11) "Über die Erhöhung der Fliessgrenze in prismatischen Balken aus Baustahl," by E. Chwalla, *ibid*, Vol. 6, 1933, pp. 149 and 153.

(12) "Die Fliessgrenze bei behinderter Formänderung," by W. Prager, *Forschung aus dem Gebiete des Ingenieurwessens*, Vol. 4, 1933, p. 95.

(13) "Tragfähigkeitsversuche an einem durchlaufenden Fachwerkbalken aus Stahl," by G. Grüning and E. Kohl, *Der Bauingenieur*, Vol. 14, 1933, p. 67.

(14) "Principes d'application de la plasticité au calcul des constructions métalliques hyperstatiques," by Artémy S. Joukoff, *L'Ossature Métallique*, Vol. 3, 1934, p. 379.

(15) "Die Bemessung unvollkommen eingespannter Stahl I-Deckenträger unter Berucksichtigung der plastischen Formanderungen," by G. V. Kazinczy, Vol. 2, Band der Abhandlungen der internationalen Vereinigung für Brückenbau und Hochbau, Zurich, 1934.

(16) "Über die Abhängigkeit des Fliessbeiginns von Spannungs-verteilung und Werkstoff," by E. Siebel and H. F. Vieregge, *Mitteilungen aus dem Kaiser-Wilhelm-Institute für Eisenforschung*, Vol. 16, 1934.

(17) "Neuzeitliche Festigkeitsfragen," by W. Kuntze, *Der Stahlbau*, Vol. 8, 1935, p. 9.

(18) "Beitrag zum Traglastverfahren," by F. Stüssi and C. F. Kollbrunner, *Die Bautechnik*, Vol. 13, 1935, p. 264.

(19) "Frage der tatsächlichen Tragfähigkeit durchlaufenden vollwandiger Balken aus Baustahl mit drei Feldern," by H. Maier-Leibnitz, *Der Stahlbau*, Vol. 9, 1936, p. 153.

(20) "Schichtenweises Fliessen in Balken aus Baustahl," by C. Kollbrunner, *Zurich Memoires*—Internationale Vereinigung für Bruckenbau und Hochbau, 1935.

(21) "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., Pt. II, 1930, p. 708.

(22) "Failure Theories of Materials Subjected to Combined Stresses," by Joseph Marin, *Transactions*, ASCE, Vol. 101, 1936, p. 1170.

(23) **"Strength of Aircraft Elements,"** by Army-Navy-Civil Committee on Aircraft Design Criteria, 22 October, 1943, p. 1–18.

(24) **"Theory of Elastic Stability,"** by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 489.

(25) **"Steel Construction,"** A.I.S.C., 4th Ed., 1942, p. 275.

(26) **"The Strength of Timber,"** by Thomas Barlow, Woolwich Arsenal, London, England, 1814.

(27) **"A History of the Theory of Elasticity,"** by I. Todhunter and K. Pearson, Univ. Press, Cambridge, England, 1886.

(28) *Journal*, A.C.I., December, 1914, p. 28.

(29) *Proceedings*, A.C.I., Vol. 33, 1937, p. 498–1.

(30) *Transactions*, ASCE, Vol. LXXXIII, 1919–1920, p. 1667.

(31) **"Steel Column Research,"** Final Report of the Special Committee, *Transactions*, ASCE, Vol. 98, 1933, p. 1376.

(32) **"Theory of Limit Design,"** by J. A. Van den Broek, *Transactions*, ASCE, Vol. 105, 1940, pp. 638 and 673.

(33) **"Column Formula for Materials of Variable Modulus,"** by J. A. Van den Broek, *The Engineering Journal*, Vol. 28, 1945, p. 772.

(34) **"Results of Experiments on Metallic Beams Bent Beyond the Elastic Limit,"** by E. Volterra, *Journal*, Inst. C. E., Vol. 20, 1942–1943, p. 1. (*a*) p. 349.

(35) **"Mécanique des Solides Isotropes au delá du Domain Elastique,"** by W. Prager, Mémorial des Sciences Mathématiques, Pscicule LXXXVIII, Gauthier-Villars, Paris, 1937, p. 59.

(36) **"Further Tests on Beams and Portals,"** by J. F. Baker and J. W. Roderick, 2d Interim Rept., *Transactions*, Inst. of Welding, Vol. 3, No. 2, 1940, p. 83.

(37) **"Röntgenographische Spannungsmessungen bei Uberschreiten der Fliessgrenze an Biegestäben aus Flusstahl,"** by F. Bollenrath and E. Schiedt, *Zeitschrift*, V.D.I., Vol. 82, 1938, p. 1094. (*a*) p. 1098.

(38) **"X-Ray Diffraction Study of Notch-Bend Test,"** by J. T. Norton, D. Rosenthal, and S. B. Maloof, *Welding Journal*, May, 1946, p. 269-s. (*a*) p. 2745.

(39) **"Critical Review of the Weld Stress Problem,"** by J. T. Norton, D. Rosenthal, and S. Maloof, *ibid.*, p. 277-s.

(40) **"Über die röntgenographische Messung elastischer Spannungen,"** by H. Moeller and J. Barbers, *Mitteilungen aus dem Kaiser-Wilhelm-Institute für Eisenforschung*, Vol. 16, 1934, p. 21.

(41) **"Röntgenographische Untersuchung über Spannungsverteilung und Überspannung in Flusstahl,"** by H. Moeller and J. Barbers, *ibid.*, Vol. 17, 1935, p. 157.

(42) **"Die Fliessgrenze bei Biegebeanspruchung,"** by F. Rinagl, *Zeitschrift*, V.D.I., Vol. 80, 1936, p. 1199.

(43) **"Über die Fliessgrenze bei Zug- und Biegebeanspruchung,"** by F. Rinagl, *Bauingenieur*, Vol. 17, 1936, p. 431.

(44) **"Schichtenweises Fliessen in Balken aus Baustahl,"** by C. F. Kollbrunner, *Memoirs*, International Assn. for Bridge & Structural Eng., Vol. 3, 1935, p. 222

(45) **"An Experimental Investigation of the Strength of Seven Portal Frames,"** by J. F. Baker and J. W. Roderick, 1st Interim Rept., *Transactions*, Inst. of Welding, Vol. 1, No. 4, 1938, p. 206.

(46) **"The Yield of Mild Steel with Particular Reference to the Effect of Size of Specimen,"** by J. L. M. Morrison, *Journal and Proceedings*, Inst. of Mech. Eng., Vol. 142, No. 3, 1943, pp. 192–223.

(47) **"The Yield Point and Initial Stages of Plastic Strain in Mild Steel Subjected to Uniform and Non-Uniform Stress Distribution,"** by G. Cook, *Philosophical Transactions*, Royal Soc. of London, Vol. A230, 1931, p. 103.

(48) **"On the Yield Point of Mild Steel,"** by Fujio Nakanishi, Rept., Aeronautical Research Inst., Tokyo Imperial Univ., Vol. 6, 1931, p. 81.

(49) **"Raised Yield Strength in Bend Tests,"** by G. Brewer, *Metal Progress*, December, 1945.

(50) **"The Effect of Non-Uniform Distribution of Stress on Yield Strength of Steel,"** by D. Morkovin and O. Sidebottom, *Bulletin*, Eng. Experiment Station, Univ. of Illinois, Urbana.

(51) **"Ueber den Zusammenhang der Biegungselastizität des GuBeisens mit seiner Zug-und Druckelastizität,"** by H. Herbert, Mitteilungen uber Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Vol. 89, pp. 42–44.

(52) **"Strength of Materials,"** by S. Timoshenko, D. Van Nostrand Co., New York, N. Y., 2d Ed., Pt. II, 1941, p. 370.

(53) **"Physical Properties of Four Low-Alloy High-Strength Steels,"** by W. L. Collins and T. J. Dolan, *Proceedings*, A.S.T.M., Vol. 38, Pt. II, 1938, p. 175.

(54) **"Transition from the Elastic to the Plastic State in Mild Steel,"** by A. Robertson and G. Cook, *Proceedings*, Royal Soc. of London, Series A, Vol. 88, 1913, p. 462.

(55) **"Some Factors Affecting the Yield Point in Mild Steel,"** by G. Cook, Inst. of Engrs. and Shipbuilders in Scotland, February, 1938.

(56) **"Further Tests on the Effect of Time on Testing,"** by H. Quinney, *Engineer*, Vol. 161, 1936, p. 669.

(57) **"Yield Limits and Characteristic Deflection Lines,"** by F. Rinagl, Preliminary Publication, 2d Cong., International Assn. for Bridge and Structural Eng., Berlin, 1936.

(58) **"The Behaviour of Stanchions Bent in Single Curvature,"** by J. F. Baker and J. W. Roderick, 3d Interim Rept., *Transactions*, Inst. of Welding, Vol. 5, No. 3, 1942.

(59) **"The Behaviour of Stanchions Bent in Single Curvature,"** by J. F. Baker and J. W. Roderick, 4th Interim Rept., Research Rept., British Welding Research Assn., 1944.

(60) **"Einfluss der Federung der Zerreissmaschine auf das Spannungs-Dehnungs-Schaubild,"** by W. Späth, *Archiv für das Eisenhüttenwesen, No. 6*, December, 1935, pp. 277–283.

(61) *Journal of Applied Mechanics*, A.S.M.E., June, 1939, p. A-89.

(62) *Bulletin No. 106*, A.S.T.M., October, 1940, pp. 31–34.

DISCUSSION

F. P. Shearwood,[3] M. ASCE.—In commenting on this paper, it should be kept in mind that the true benefit of such a thesis will be realized through the promotion of investigation into the practical and useful application of the findings of the theories. The paper illustrates the fact that "yield" is a most useful and safe quality of steel and an important factor in bringing into play all resistances that oppose deformation (failure). It also indicates that overstrained fibers can be deformed slightly without endangering the structure, provided that other less strained fibers are brought into resistance. Ductility has undoubtedly been the safety valve of commercial designing in cases where poor detailing, eccentric connections, etc., would have caused failure if the elastic factor was the only deformation to be relied upon; but the ductile yield causes redistribution of the stress and so failure is avoided.

In designing or considering continuous spans, and other indeterminate structures as well as the cross bending of beams, modern practice demands that the structures be limited to the specified unit stress in their highest strained path when calculated by the elastic theory. It neglects the fact that, should this path or fiber be strained beyond its elastic limit, the other paths will then exert an increasing proportion of the required extra resistance. This proportion should be about in the ratio of the strain after and before passing the elastic limit—a fairly big ratio.

Investigation of old structures, abused structures, arc welding, badly designed structures, or other weak constructions frequently demonstrates that the yielding properties of steel must have been utilized to prevent failure. This evidence should warrant further investigation leading to the prudent use of this yielding property in the designing of indeterminate steel structures.

The manufacturing of steel and iron is made possible by this property of ductility which is constantly utilized during construction. In fact, much of the fabrication of steel would be impossible without this valuable characteristic of yielding. It is inconsistent to assume that yielding of steel is harmless during fabrication or when it relieves the excessive overstressing from faulty detailing and at the same time to prohibit counting on it when designing structures with more than one path of resistance.

It is interesting, and of much consequence, to determine the exact point where yielding will occur; however, the importance of such information lies in practical application to proportioning steel structures.

Edwin H. Gaylord,[4] Assoc. M. ASCE.—The tests described in this paper are an important contribution to the problem of the behavior to yield in pure flexure and in eccentric tension. It is worth noting that the unit bending stress at which the load deflection curve begins to deviate from the straight line bears the same ratio to the proportional limit in axial tension, as does the unit bending stress at the appearance of Lueders lines to the unit stress in axial tension at the appearance of Lueders lines (Figs. 5 and 10). This phenomenon seems to indicate that the behavior of the extreme fiber in bending, as predicted by the assumption of linear variation of stress, is the same as that of the fiber in axial tension, except for extension of the proportional limit and yield point.

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The writer presents an analysis of the data in this paper on the basis of an assumption analogous to that made in the strain-energy theory for combined stresses at a point (21)(22). For a bar of unit length, the elastic strain-energy equations for axial tension and for pure bending are, respectively:

$$U_a = \frac{1}{2}\frac{P^2}{A\,E} \qquad \text{(8a)}$$

and

$$U_b = \frac{1}{2}\frac{M^2}{E\,I} \qquad \text{(8b)}$$

In Eqs. 8, let $\frac{P}{A} = \sigma_a$, the unit stress in axial tension; and $\frac{M\,c}{I} = \sigma_b$, the extreme fiber stress in bending. Then:

$$U_a = \frac{P^2}{2\,A\,E} = \left(\frac{P}{A}\right)^2 \frac{A}{2\,E} = \frac{\sigma^2_a\,A}{2\,E} \qquad \text{(9a)}$$

and

$$U_b = \frac{M^2}{2\,E\,I} = \left(\frac{M\,c}{I}\right)^2 \frac{I}{2\,c^2\,E} = \frac{\sigma^2_b\,I}{2\,c^2\,E} \qquad \text{(9b)}$$

If the unit stress at the yield point in axial tension is σ_{ay}, the yield-point stress in the extreme fiber for a rectangular section in bending is $\sqrt{2}\,\sigma_{ay}$, as computed by the method of W. Kuntze (3), and, using the relation $I/c^2 = b\,h/3$, Eq. 9b becomes:

$$U_b = \frac{1}{3}\frac{\sigma^2_{ay}\,b\,h}{E} \qquad \text{(10a)}$$

For a rectangular bar of the same cross section at the yield point in axial tension, Eq. 9a gives:

$$U_a = \frac{1}{2}\frac{\sigma^2_{ay}\,b\,h}{E} \qquad \text{(10b)}$$

If the shape of the cross section is a symmetrical diamond, the Kuntze method indicates an extreme fiber stress of precisely 2 σ_{ay} at the yield point in bending. If b and h are taken as the width and depth, respectively, $A = b\,h/2$ and $I/c^2 = \frac{b\,h^3/48}{h^2/4} = b\,h/12$; and, from Eqs. 9,

$$U_b = \frac{1}{6}\frac{\sigma^2_{ay}\,b\,h}{E} \qquad \text{(11a)}$$

and

$$U_a = \frac{1}{4}\frac{\sigma^2_{ay}\,b\,h}{E} \qquad \text{(11b)}$$

Based on Eqs. 10 and 11, the assumption can be made that, at the yield point in pure bending, the strain energy in a given length of bar is equal to two thirds of the strain energy of a bar of the same length and cross section at the yield point in axial tension. The yield point is defined as that point at which Lueders lines appear. Then, from Eqs. 9, if σ_{by} is the extreme fiber unit stress in bending at the yield point:

$$\frac{1}{2}\,\frac{\sigma^2_{by}\,I}{c^2\,E} = \frac{1}{3}\,\frac{\sigma^2_{ay}\,A}{E} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12a)$$

and, since $I = A\,r^2$,

$$\sigma_{by} = \sqrt{\frac{2}{3}}\,\frac{c}{r}\,\sigma_{ay} = 0.816\,\frac{c}{r}\,\sigma_{ay} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12b)$$

It is obvious that Eq. 12*b* will check the Kuntze method for the rectangular and diamond cross sections precisely. Table 5 gives a comparison of the results of Mr. Kuntze's method and the results of Eq. 12*b* for a circular section, a tube, and a few wide-flange sections. For some of the wide-flange sections, the bending yield stress, as calculated by Eq. 12*b*, is indicated as being slightly less than the yield-point stress in axial tension. It is undoubtedly true that the two-thirds factor in Eq. 12*b* is not constant, but dependent on the shape, and that it approaches unity as *r* approaches *c*. Only in this way could the equation check the hypothetical beam in which all the material is concentrated along two lines parallel to, and equidistant from, the neutral axis; and such a section would surely be expected to yield at the yield stress for axial tension. Nevertheless, Cols. 4 and 5, Table 5, agree very well. In fact, by changing the factor 0.816 to 0.85, Eq. 12*b* will predict the bending yield, for the sections which the writer has checked, within ± 4% of the values obtained by the Kuntze method.

TABLE 5.—COMPARISON OF EQ. 12*b* WITH THE KUNTZE METHOD

Section (1)	*c* (2)	*r* (3)	σ_{by}/σ_{ay}: Eq. 12*b* (4)	σ_{by}/σ_{ay}: Kuntze (5)	Col. 4 / Col. 5 (6)
Circular	*R*[(a)]	*R*/2	1.632	1.644	0.993
Standard 6-in. pipe	3.3125	2.25	1.202	1.207	0.996
36WF230	17.94	14.88	0.984	1.047	0.940
36WF150	17.92	14.29	1.024	1.039	0.986
18WF47	8.95	7.30	1.001	1.041	0.961
8WF17	4.00	3.36	0.972	1.050	0.926

(*a*) *R* equals radius.

Eq. 12*b* provides an extremely simple method of calculating the yield stress in bending and, in so far as the two have been compared, gives substantially the same results as does the Kuntze method. Should subsequent tests establish the increase in bending yield as being reliable enough to warrant a change in design procedure, and also establish the strain-energy hypothesis, choice of a section to resist bending can be made without using Eq. 12*b* directly. If the yield stress as given by Eq. 12*b* is equated to the stress calculated from $\sigma = \frac{M\,c}{I}$,

$$\sqrt{\frac{2}{3}}\,\frac{I}{r} = \frac{M}{\sigma_{ay}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

In Eq. 13, the quantity $\sqrt{\frac{2}{3}\frac{I}{r}}$ is the section modulus. If a specification working stress is used in Eq. 13 instead of σ_{ay}, a uniform factor of safety with respect to yield would result for all cross sections with two axes of symmetry, regardless of shape. An extensive series of tests on both small and large sections would seem profitable, to try to establish firmly the apparent increase in the bending yield-point stress. Tests on sections which are unsymmetrical about the principal axis normal to the plane of bending would also be valuable, to determine the effect of this factor on the increase in bending-yield stress.

In extending the strain-energy equation to the case of eccentric tension, at least two assumptions are possible. One is expressed by the equation,

$$\frac{\sigma^2_b\, I}{c^2\, E} = k^2_1 \left(\frac{\sigma^2_{ay}\, A}{E} - \frac{\sigma^2_a\, A}{E}\right) \qquad (14a)$$

The factor k^2_1 may be variable, but it must have the value 2/3 for $\sigma_b = 0$, to agree with Eq. 12*a*. Another possible assumption is expressed by

$$\frac{\sigma^2_b\, I}{c^2\, E} + \frac{\sigma^2_a\, A}{E} = \frac{k^2_2\, \sigma^2_{ay}\, A}{E} \qquad (14b)$$

In Eq. 14*b*, k^2_2 is definitely variable, since it must have the value 2/3 for $\sigma_a = 0$, and the value unity for $\sigma_b = 0$.

To obtain an expression for the maximum yield stress, the relation,

$$\frac{\sigma_b}{\sigma_a} = \frac{M\, c/I}{P/A} = \frac{e\, c}{r^2} \qquad (15a)$$

can be used to eliminate σ_b from Eq. 14*a*; thus:

$$\left(\frac{e}{r}\right)^2 \sigma^2_a + k^2_1\, \sigma^2_a = k^2_1\, \sigma^2_{ay} \qquad (15b)$$

from which

$$\sigma_a = \frac{k_1\, \sigma_{ay}}{\sqrt{k^2_1 + (e/r)^2}} \qquad (16a)$$

Similarly, from Eq. 14*b*,

$$\sigma_a = \frac{k_2\, \sigma_{ay}}{\sqrt{1 + (e/r)^2}} \qquad (16b)$$

The maximum stress σ_{my} is given by

$$\sigma_{my} = \sigma_a + \sigma_b = \sigma_a\, (1 + e\, c/r^2) \qquad (17)$$

and, therefore, from Eqs. 16:

$$\sigma_{my} = k_1 \frac{1 + e\, c/r^2}{\sqrt{k^2_1 + (e/r)^2}}\, \sigma_{ay} = \frac{1 + e\, c/r^2}{\sqrt{1 + (1/k^2_1)(e/r)^2}}\, \sigma_{ay} \qquad (18a)$$

and

$$\sigma_{my} = k_2 \frac{1 + e\, c/r^2}{\sqrt{1 + (e/r)^2}}\sigma_{ay} \qquad (18b)$$

For the specimens in Table 2, $e = 0.3$ in.; c/r^2 ($= 6/h$ for a rectangular section) $= 6/1.800 = 10/3$; and $r^2 = h^2/12 = 0.27$. Assuming an increase in yield stress of about 41%, or, say, $\sigma_{my} = \sqrt{2}\, \sigma_{ay}$, as is indicated for specimens III and IV of Table 3, and, substituting these values in Eqs. 16, $k^2{}_1 = 1/3$ and $k^2{}_2 \doteq 2/3$.

Since $k^2{}_1$ has the values 2/3 for pure bending and 1/3 for the eccentric tension specimens, it seems reasonable to assume that it will approach zero with e. This variation can be expressed very simply by the equation,

$$k^2{}_1 = \frac{2}{3}\,\frac{a\,x}{a\,x + 1} \quad \text{(19a)}$$

in which the constant a depends on the choice of the independent variable x. The variation of $k^2{}_1$ might be assumed to be with e, or with the dimensionless quantities e/r, e/h, $e\,c/r^2$, etc. It seems appropriate, however, to assume the variation to be with $e\,c/r^2$, since this quantity is the ratio of σ_b to σ_a. With this assumption, Eq. 19a reduces to

$$k^2{}_1 = \frac{2}{3}\,\frac{e\,c/r^2}{e\,c/r^2 + 1} \quad \text{(19b)}$$

in which $a = 1$ was calculated on the assumption that $k^2{}_1 = 1/3$ when $e\,c/r^2 = 1$ (the values of these variables for the specimens of Table 3). The simplicity of Eq. 19b is encouraging, and the relation seems even more plausible when $e\,c/r^2$ is replaced by its equivalent σ_b/σ_a. This substitution leads to

$$k^2{}_1 = \frac{2}{3}\,\frac{\sigma_b}{\sigma_a + \sigma_b} \quad \text{(19c)}$$

Since the value of $k^2{}_2$ is the same for the specimens of Table 3 as it is for pure bending, and yet must approach unity as e approaches zero, it is more difficult to assume a variation of $k^2{}_2$ without further data.

In seeking a possible explanation for the larger increase in yield stress for specimens I and II of Table 3, it might be assumed that there was an accidental eccentricity across the thickness of the specimen, since a constant eccentricity would have a greater effect on the thinner specimens. Eq. 18a is easily extended for eccentricities in two directions, and results in:

$$\sigma_{my} = \frac{1 + (e\,c/r^2)_1 + (e\,c/r^2)_2}{\sqrt{1 + (1/k^2{}_1)[(e/r)^2{}_1 + (e/r)^2{}_2]}}\,\sigma_{ay} \quad \text{(20)}$$

The value of $k^2{}_1$ can be assumed to be given by the equation

$$k^2{}_1 = \frac{2}{3}\,\frac{(e\,c/r^2)_1 + (e\,c/r^2)_2}{(e\,c/r^2)_1 + (e\,c/r^2)_2 + 1} \quad \text{(21a)}$$

or

$$k^2{}_1 = \frac{2}{3}\,\frac{\sigma_{b_1} + \sigma_{b_2}}{\sigma_a + \sigma_{b_1} + \sigma_{b_2}} \quad \text{(21b)}$$

In Eqs. 20 and 21, the subscripts denote the principal axes of the bar.

Assuming that there is an accidental eccentricity of 0.001 in. in the direction of the thickness for each specimen of Table 3, Eqs. 20 and 21a yield: For specimen I—

$$\sigma_{my} = 1.54\ \sigma_{ay} \quad \ldots\ldots\ldots (22a)$$

for specimen II—

$$\sigma_{my} = 1.47\ \sigma_{ay} \quad \ldots\ldots\ldots (22b)$$

for specimen III—

$$\sigma_{my} = 1.44\ \sigma_{ay} \quad \ldots\ldots\ldots (22c)$$

and, for specimen IV—

$$\sigma_{my} = 1.43\ \sigma_{ay} \quad \ldots\ldots\ldots (22d)$$

The agreement with the increases shown in Table 3, Col. 6, is surprisingly good. Perhaps Mr. Peterson will state in his closing discussion whether such an accidental eccentricity might have existed.

It will be of interest to investigate Eq. 18*a* for maximum or minimum values. Differentiating the equation with respect to *e*, and remembering that $k^2{}_1$ is also a function of *e*, with $\frac{d(k^2{}_1)}{de}$ to be obtained from Eq. 19*b*; a maximum or a minimum for the value of σ_{my} is found to occur for the value of *e* given by

$$e = \frac{4\,c}{3} - \frac{r^2}{c} \quad \ldots\ldots\ldots (23)$$

For a rectangular section, Eq. 23 gives $e = h/2$. Then $k^2{}_1 = 1/2$ and $\sigma_{my} = (4/\sqrt{7})\ \sigma_{ay} = 1.51\ \sigma_{ay}$. This is evidently a maximum. The result is perhaps disappointing, as one might expect the increase in yield to be greatest for pure bending. However, considering that Mr. Peterson's tests show the same increase for eccentric tension with $e = h/6$ as for pure bending ($e \rightarrow \infty$) and that there can be no increase for $e = 0$, the existence of a maximum value seems plausible. For a circular section, Eq. 23 gives $e = 13\,R/12$. Then $k^2{}_1 = 13/24$ and $\sigma_{my} = 1.71\ \sigma_{ay}$. For an 18WF47 the maximum value occurs for $e = 5.98$ in. and is $1.15\ \sigma_{ay}$.

The increase in yield stress for eccentric tension could be determined by another method. It is common practice in aeronautical structural engineering to use so-called stress-ratio curves for combined stresses (23)(24). The specification of the American Institute of Steel Construction for combined axial and bending stress (25), expressed by the inequality $f_a/F_a + f_b/F_b < 1$, is an example. Although there are insufficient data from which to determine stress-ratio equations for the problem in Mr. Peterson's paper, it is interesting to observe that, for a rectangular section, the equation:

$$\frac{\sigma_b}{\sigma_{by}} = 1 - \left(\frac{\sigma_a}{\sigma_{ay}}\right)^2 \quad \ldots\ldots\ldots (24)$$

checks specimens III and IV of Table 3. For specimen IV, $\frac{\sigma_a}{\sigma_{ay}} = \frac{31{,}750}{45{,}000} = 0.706$. Substituting this value in Eq. 24, $\frac{\sigma_b}{\sigma_{by}} = 0.498$. The actual value of $\frac{\sigma_b}{\sigma_{by}}$ from Table 3 is $\frac{31{,}750}{45{,}000\sqrt{2}} = 0.499$.

The equations developed in this discussion are admittedly speculative, and are based on incomplete data. However, granting the possibility of the accidental eccentricity of 0.001 in. previously discussed, the results seem sufficiently good to justify further verification and development.

L. J. MENSCH,[5] M. ASCE.—The related phenomena of plasticity, ductility, time yield, elastic failure, and fatigue are of great interest to manufacturers of products made from high and low yield-point steel, aluminum and its alloys, copper, gold and silver and other metals, plastics, and especially those of wood, cast iron, and reinforced concrete. The aforementioned phenomena lead to serious deviations from the behavior of materials as visualized by Hooke's law which is the foundation of the modern theory of elasticity.

The problems created by these deviations have been the subject of extensive and costly research during the last two hundred years by large users of the various materials. The first important series of tests known to the writer was made by the French government around 1750 on timber, followed about 50 years later by more thorough tests by the British Admiralty (26).

In the latter tests, Thomas Barlow subjected a great number of full-sized beams as used in ships to uniform loads (with cannon balls) and noted deflections and ultimate loads, when the beams were simply supported. Then he fixed similar sized beams by cast-iron shoes in heavy masonry walls and loaded them to failure. According to the theory of elasticity, the fixed beams should have carried a load 50% greater than the simply-supported beams; actually, without question or doubt, they carried 100% more. In modern terminology it would be stated that failure occurred under a bending moment of $w L^2/16$. Beams fixed at one end only failed under a bending moment of $w L^2/10.8$. Mr. Barlow also tested beams of triangular sections, both with the edge up and the edge down, simply supported and fixed, and found very serious deviations from the results as taught by the theory of elasticity. He discovered similar discrepancies when testing beams of metals and glass, and further referred to tests by Thomas Young (Young's modulus of elasticity) on cast iron, where even greater deviations occurred. Mr. Barlow boldly asserted that the theory of elasticity assumed properties of materials not to be found in nature and led to underestimating the real strength of beams.

Teachers of the theory of elasticity did not pay much attention to Mr. Barlow and the most important series of tests which were ever made up to that time, and one famous work on the theory of elasticity (27) refers to the speculations of Mr. Barlow as the "nadir" of English thought on applied mechanics.

In the nineteenth century the fact that the theory of elasticity underestimated the strength of materials in most cases was the subject of reports by the laboratories of Hodgekinson in Manchester, England; by Sir Thomas Fairbairn also of Manchester; by David Kirkaldy and Son of London, England; by Professor Tresca of Paris, France, and by his pupil A. Considere; and by A. Kick of Prague, Czechoslovakia. No attention was paid by the teachers of the theory of elasticity to all these reports, since the teachers were too much intrigued by the elegance of their methods and too much encouraged by the success of their pupils in the construction of bridges and halls of unprecedented spans. It is, of course, true that the theory of elasticity leads to safe results; but, as Mr. Barlow and others showed, more economical designs may be obtained by considering the deviations of the materials from behavior like that specified by Hooke's law under high stresses.

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This phase of the strength of materials has been treated in papers and discussions before the Society in the last 75 years. Although practical engineers greatly benefited by these papers, teachers did not. In December, 1914, the writer published a working theory of the strength of reinforced concrete beams based on conditions near the ultimate load (28)(29). A similar theory for steel structures and other materials was published in 1918 (30). Not much attention was paid to these and other papers by the writer on this subject; but, when the German engineers in the early 1930's found that no work could go ahead in that impoverished land unless fewer materials were used than formerly, a furious campaign was started in German literature because of a supposed discovery that a theory based on conditions of ultimate load would justify higher stresses on materials of construction.

The author of this paper derived his inspiration from these overseas sources and imitated their crude tests (afflicted with an error of about 20% due to friction, as can be noted by comparison with the tests made by the Special Committee of the Society on Column Research (31), at Madison, Wis.).

Nothing new is offered in this paper and it may be of value only to call the attention of teachers to the insufficiency of the theory of elasticity to describe the strength of materials accurately.

George Winter,[6] M. ASCE.—The question raised in this paper is of considerable interest in connection with modern tendencies toward what is variously called "limit design," "ultimate design," or "plastic design" (32). The application of this method to statically determinate and indeterminate problems of flexure is essentially predicated on the "old theory" of plastic stress distribution as given in Fig. 1. Problems of column buckling beyond the elastic range are also analyzed fundamentally by the same concept with good experimental confirmation (33). If the author's opinion, that this type of stress distribution does not actually occur, were confirmed by sufficient evidence, substantial revision of present ideas on ultimate design of steel structures would become necessary.

The problem is difficult to approach experimentally, chiefly because, beyond the yield point, the ordinary kind of strain measurements can no longer be translated into stress. Even if the strain distribution were investigated carefully, therefore, no conclusion could be reached on the absolute magnitude of the stresses in the "overstrained" parts of the beam.

The difficulty can be overcome to some extent by X-ray stress measurements. This method measures changes in distances of lattice planes in the crystals, but is not affected by relative shifts of various parts of the crystals due to "sliding-yielding." Therefore, it measures that part of the strain which is proportional to stress. The accuracy of this method, however, is generally of the order of from 5% to 10% of the yield point of mild steels. It has not attained the same degree of dependability as strain measurements, therefore, although effects of the order of 40% of the yield point, as indicated in the paper, are easily detected by this method.

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In addition to such X-ray measurements the actual stress distribution can be inferred by indirect methods, such as the occurrence of Lueders lines, permanent distortion, magnitude of curvature and deflection, and similar secondary effects.

That the situation, fundamentally, is not so simple as usually assumed can be recognized from the following consideration: For steels in the elastic range Poisson's ratio is from about 0.25 to 0.30. On the other hand, in the plastic range Poisson's ratio should be that of a liquid—that is, 0.5. Consequently, at the transition from the elastic to the plastic range, the lateral deformation would have to increase suddenly and discontinuously by from 60% to 100% of its maximum elastic value. In other words, the cross section of a rectangular beam stressed partly into the plastic range should be of the shape shown (exaggerated) in Fig. 12(*b*). Such a shape can develop only if the shear resistance in the outer parts of a-a is completely overcome. Actually, a shape more like that of Fig. 12(*c*) must be expected. It is clear that this shape causes shear stresses at and around section a-a such that at least part of the beam above a-a is subject to lateral compression, and part of the beam below section a-a is subject to lateral tension, in addition to the primary, longitudinal compression stress. At and near the supposed boundary of the elastic zone a two-dimensional state of stress, instead of the assumed uniaxial condition, is obtained. Assuming the yield condition to be defined by the distortion energy theory, these lateral stresses cannot fail to affect the yield point of the material in that zone.

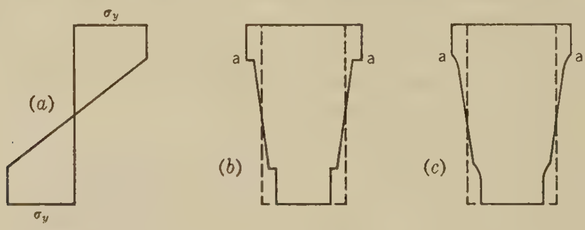

Fig. 12.—Distribution of Longitudinal Stress and of Lateral Strain in Beams Stressed Beyond the Yield Point

From this consideration alone it is evident that the "old theory" can be expected to represent only a reasonable approximation to the actual condition. In addition, Fig. 12 suggests another possible experimental approach for investigation of this problem, namely—the determination of Poisson's ratio by test. It is relatively simple to make simultaneous longitudinal and transverse strain measurements on the top surface of beams of any cross section, for example, by means of electric-resistance gage rosettes. If properly evaluated, the stage at which Poisson's ratio increases from its elastic value and, finally, reaches 0.5 (most likely) would give at least some indication of the actual condition of the metal—plastic or elastic.

A review of existing evidence, alongside the author's interesting tests, is rather perplexing; but it shows, at least, that the situation is not so simple as indicated in the paper. In the following survey of available information, the writer purposely omitted investigations based on the appearance of Lueders lines as evidence of yielding. These lines seem to be a satisfactory indication of the occurrence of strains in the specimen (beam) of magnitude equal to that which, at the yield point, occurs in a tension specimen. However, in the case of nonuniform stress distribution, the strains in "overstressed" zones adjacent to parts that are stressed in the elastic range are necessarily of the order of magnitude of elastic strains. It is doubtful whether macroscopic Lueders lines can be detected when yield strains are of such small amount, although it is likely that slip bands in individual crystals could be observed in such zones by microscopic inspection.

In an extensive investigation (34) E. Volterra obtained interesting experimental evidence on the behavior of a great number of "overstressed" steel beams. Although the report of this work is somewhat sketchy and contains minor errors (34*a*) it presents broad evidence that W. Kuntze's and W. Prager's (35) ideas are not confirmed by test. (Messrs. Kuntze and Prager have held that there is a sharply increased yield point in the "overstressed" parts of a beam; that there is purely elastic behavior far beyond the tensile yield-point stress; and that, subsequently, there is a sudden breakdown of elasticity in the entire cross section.) On the other hand, Mr. Volterra's tests also present evidence that, in the "elasto-plastic range" (that is, at loads larger than those causing tensile yield-point stress in the outer fiber, but below the ultimate), the curvatures and deflections are smaller than those predicted by the "old theory."

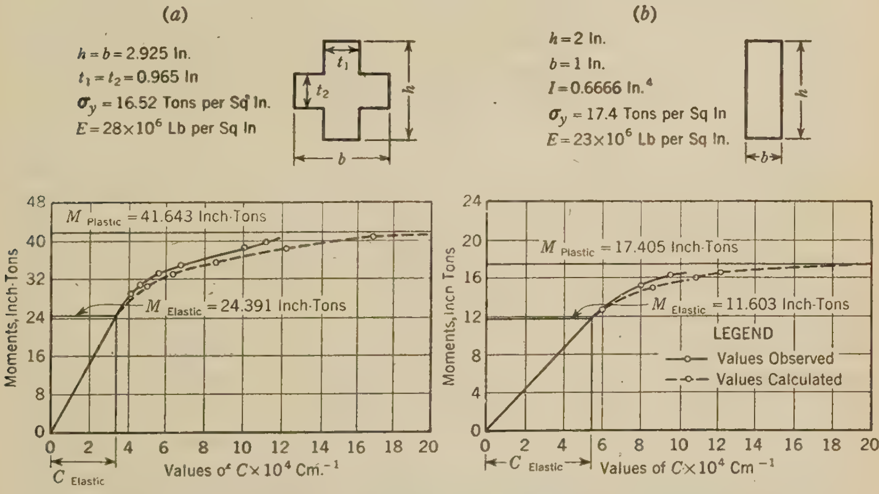

FIG. 13.—LOAD-CURVATURE DIAGRAMS FOR TWO STEEL BEAMS ACCORDING TO E. VOLTERRA

Both these facts are well illustrated by two curvature-bending-moment diagrams (Fig. 13) taken from the Volterra paper. Mr. Volterra attributes the difference between measured and computed curvature to the effect of strain-hardening, whereas, in a discussion of that paper J. F. Baker, Assoc. M. ASCE, (34*a*), holds the upper yield point to be responsible for that difference, a factor which is generally neglected in the "old theory." In the writer's opinion the phenomenon depicted in Fig. 12 is likely to represent at least a contributing factor to this limitation of deformations to values below those expected from the "old theory."

Similar results were obtained by Professor Baker and J. W. Roderick in a very important investigation on welded rigid frames (36). Fig. 14, a load-deflection diagram from this investigation, shows rather close agreement between experimental values and those computed by the "old theory," again exhibiting a somewhat sharper curvature of the load-deflection curve than predicted theoretically.

Whereas measurements of deflections, curvatures, and observations of Lueders lines give only indirect evidence of the actual stress distribution in a beam—X-ray measurements, within their limits of accuracy, enable the investigator to determine the magnitude of stress directly, in the elastic and in the plastic range. In this connection F. Bollenrath and F. Schiedt (37) observed (see Table 6) that in the period from 1932 to 1938 successive investigators found less and less difference between yield points in bending and in simple tension.

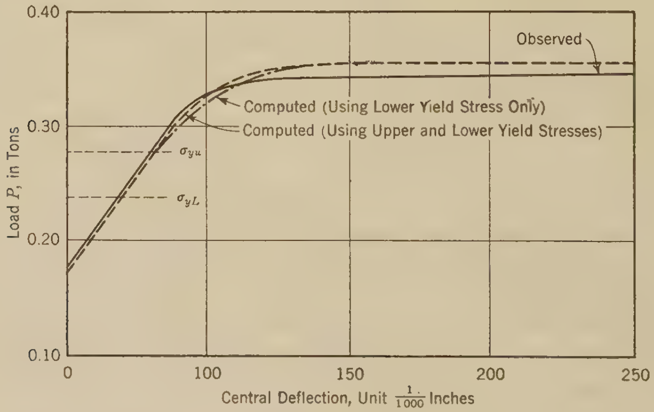

FIG. 14.—LOAD-DEFLECTION DIAGRAM FOR A STEEL BEAM ACCORDING TO J. F. BAKER AND J. W. RODERICK

These tests relate to rectangular sections, although the last of these investigations was extended to triangular and other shapes. Messrs. Bollenrath and Schiedt stated their conclusion as follows (37*a*): "Independent of the shape of the cross-section no increase of the yield point does occur in bending as compared with that determined from a tension test."

Similar conclusions are reported in papers recently published in the United States by J. T. Norton, D. Rosenthal, and S. B. Maloof (38)(39). In the first of these investigations (38) X-ray stress measurements were made in notch-bend tests—that is, on specimens that have steeper stress gradients than ordinary, prismatic beams and, consequently, in which the supposed increase in yield point should be larger than in such beams. However, in the words of Messrs. Norton, Rosenthal, and Maloof (38*a*) "* * * the X-rays reveal no increase of yield point as a result of notching or steep stress gradient."

TABLE 6.—COMPARISON OF PERCENTAGE INCREASES IN YIELD-POINT STRESS

Investigators	Year	Increase	Criterion
A. Thum and F. Wunderlich (2)	1932	35 to 40	Lueders lines
H. Moeller and J. Barbers (40)	1934	40	X-ray
E. Siebel and F. H. Vieregge (16)	1934	28	Beam deflection
H. Moeller and J. Barbers (41)	1935	13	X-ray
F. Rinagl (42)(43)	1936	0	Residual strains
F. Bollenrath and E. Schiedt (37)	1938	0	X-ray

It cannot be stated that these few quotations shed any definite or final light on the actual process of yielding in nonuniformly stressed members. A much more extensive review of available evidence than can be discussed in this brief contribution convinced the writer that the actual details of the process depend, to some extent, on type, treatment, and structure of the steel, and other influences. That yielding is not a process of continuous flow, but that it occurs in discontinuous, although minute, layers also appears (44) well established. The dimensions of these layers, as compared with those of the member or those of the part of the member, subject to nonuniform stress, also may influence the over-all behavior beyond the elastic range.

However, the structural engineer is little concerned with these details. In deciding the rather vital question of practical reliability of the newly proposed methods of limit design, plastic design, or ultimate design, he is chiefly interested in final results—that is, whether or not carrying capacities and deformations determined by these methods agree with test results. In this respect it is noteworthy that the "old theory," as well as the theories of Messrs. Kuntze, Prager, and others, agrees closely in reflecting the influence of cross-sectional shape on ultimate moment, and that these computed ultimate moments are extensively verified by test. Also noteworthy is the fact that deflections and curvatures measured by different investigators were always found to be either equal to, or somewhat smaller than predicted by, the "old theory." Hence, this very simple and practical theory seems to be well confirmed with regard to ultimate strength, but errs slightly on the conservative side with respect to deformations beyond the elastic limit. In the field of statically indeterminate structures, where this design method is particularly appropriate, the degree of agreement of test results with values computed by the "old theory," for seven-portal model steel frames (45), is evident from the following:

Portal No.	Failure Load, in Tons Computed	 Observed
F.3	1.54	1.60
F.4	1.45	1.64
F.5	1.54	1.62
F.6	3.08	2.98
F.8	2.49	2.79
F.9	1.58	1.64
F.10	1.65	1.75

With one exception, discrepancies do not exceed 6%; in the one case of lesser agreement (F.8), the "old theory" errs on the conservative side by about 11%.

Despite the fact, therefore, that the details of the yielding process are probably not so simple as frequently assumed, it is this writer's opinion that the "old theory," by present evidence, represents a simple and reliable tool for structural analysis.

Acknowledgment.—Some of the quotations cited in this discussion were brought to the writer's attention by N. M. Newmark, and Bruce G. Johnston, Members, ASCE.

O. SIDEBOTTOM,[7] ESQ., AND T. J. DOLAN,[8] ESQ.—Structural designers usually have assumed that the material in a ductile metal member subjected to uniaxial stress starts to yield (or attains a small measurable plastic deformation) at a given stress, regardless of the existence of a stress gradient on the cross section of the member. In recent years, however, the results of several investigators (see for instance, J. L. M. Morrison (46), G. Cook (47), Fujio Nakanishi (48), and G. Brewer (49)) and the tests reported by the author have been interpreted as showing that yielding is suppressed in the presence of a stress gradient and hence that an "elastic" peak stress can be built up which is greater than the yield point developed by the material under a uniform stress.

The finding of the author (see "Conclusions") that "The stress at which yield occurs is higher under nonuniform stress * * *" than that found from axial tensile tests, in which the stress is uniform, does not agree with the conclusions reached in studies in 1946 by D. Morkovin and O. Sidebottom (50), and by J. T. Norton, D. Rosenthal, and S. B. Maloof (38). It seems desirable, therefore, that a review be made of the fundamental structural actions involved in a beam to clarify some of the divergent conclusions of various investigators. The writers feel that the conclusions of the author create false impressions which, if not reinterpreted, may lead to unsafe design practices. The material presented in this discussion is based on the work of Messrs. Morkovin and

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Sidebottom whose study included a re-examination of the data of several previous investigators.

The reported changes in yield point for a given metal under different test conditions, involving stress gradients in uniaxial stress, may be attributed to two factors: (*a*) Difficulties in detecting the load at which yielding started in the most stressed fibers, resulting in erroneous interpretation of the test data; and (*b*) the varying stress levels at which the upper yield-point phenomenon may be exhibited by certain mild steels under different test conditions. The following discussion will deal mainly with a study of item (*b*) since the writers believe the material in the members tested by Mr. Peterson exhibited an upper yield point. However, item (*a*) will be considered first in order to facilitate the explanation of item (*b*).

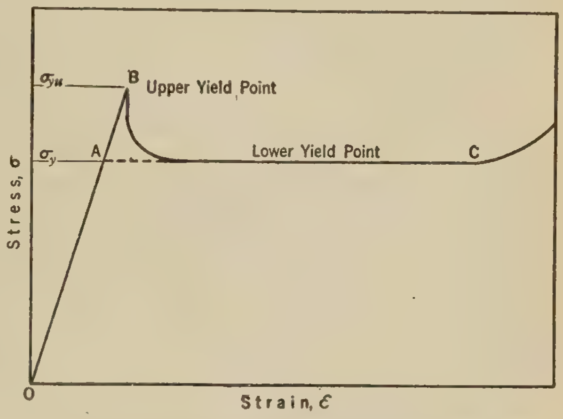

Fig. 15.—Stress-Strain Diagrams for Tension Specimens

First, examine the resisting moments that would be developed by a beam if each fiber exhibited exactly the same stress-strain relationship as that determined for a tension (or compression) specimen. By assuming that, in a beam, plane sections remain plane, and the tensile and compressive stress-strain diagrams of a material are identical and similar to curve OAC in Fig. 15, it is possible to develop mathemathical equations based on equilibrium conditions relating the bending moment in the beam to the strains, ϵ, developed in the extreme fibers for various cross-sectional shapes. The curves shown in Fig. 16 have been so derived (50), and have been plotted in terms of the dimensionless ratios M/M_y and ϵ/ϵ_y to facilitate comparison for the five cross sections illustrated. In this discussion the symbols M_y, σ_y, and ϵ_y will designate the bending moment, stress, and strain, respectively, corresponding to the lower yield point of the material; M and ϵ represent the bending moment in the test part and the strain in the extreme fiber, respectively, developing at any given stage of the test.

As the relative volume of material in the extreme fibers is decreased by modifying the shape of the beam (see Fig. 16), there is a tendency for the departure from the initial straight line to become less abrupt. This leads to an illusion that the proportional limit has been raised. Actually, however, all curves begin to depart from the straight line when the ratio $\frac{M}{M_y} = 1$ is exceeded. Although all curves may represent identical material, the bending moment at which the deviation from linearity becomes a definite measurable amount (as evidenced also by distortion of the member as a whole) is a function of the shape of the beam. Fundamentally, the resisting moment developed by a beam and plotted in a moment-strain curve represents an integrated sum of the resisting moments developed by all fibers of the member. Thus, although yielding in a beam starts in the outer fibers at the same stress, the potential load-carrying capacity of fibers beneath the surface makes it much more difficult to detect initial yielding experimentally than is the case in a simple tension member.

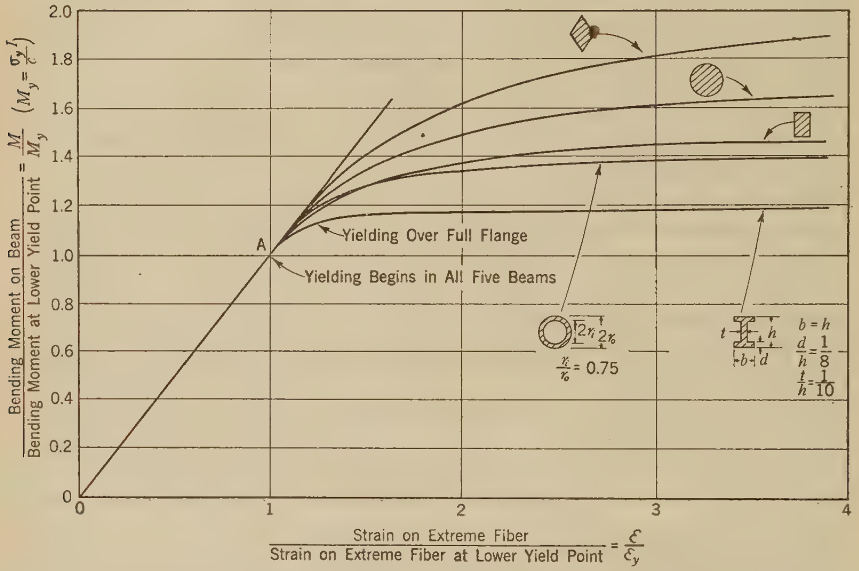

FIG. 16.—THEORETICAL MOMENT-STRAIN DIAGRAMS FOR METAL WITH A DEFINITE YIELD POINT, σ_y (STRESS-STRAIN DIAGRAM SIMILAR TO CURVE OAC IN FIG. 15)

Comparisons of experimental moment-strain data for rectangular beams obtained by Messrs. Morkovin and Sidebottom (50) with theoretical curves of the type shown in Fig. 16, are presented in Fig. 17. In Fig. 17(*b*) good agreement is seen to exist between the moment-strain diagram predicted by the "old theory" and the actual test data for two beams made of an annealed high-carbon steel. In analyzing the data shown in Fig. 17(*a*), it was found that for two of the mild-steel beams initial yielding began at the lower yield point as determined from tensile tests; whereas, the third mild-steel beam (shown by solid circles) exhibited an upper yield point approximately 10% greater than the tensile (lower) yield point. As plastic action progressed, subsequent yielding of the mild-steel beams occurred at a stress equal to the lower yield point. (Analyses of these data in terms of stress were facilitated by use of equations developed by H. Herbert (51).) Thus, for cases in which an upper yield point was not exhibited, the stress gradient occurring in a beam did not raise the yield point of the material. For materials with a high upper yield point the theoretical curves of Fig. 16 must be modified to provide for this phenomenon.

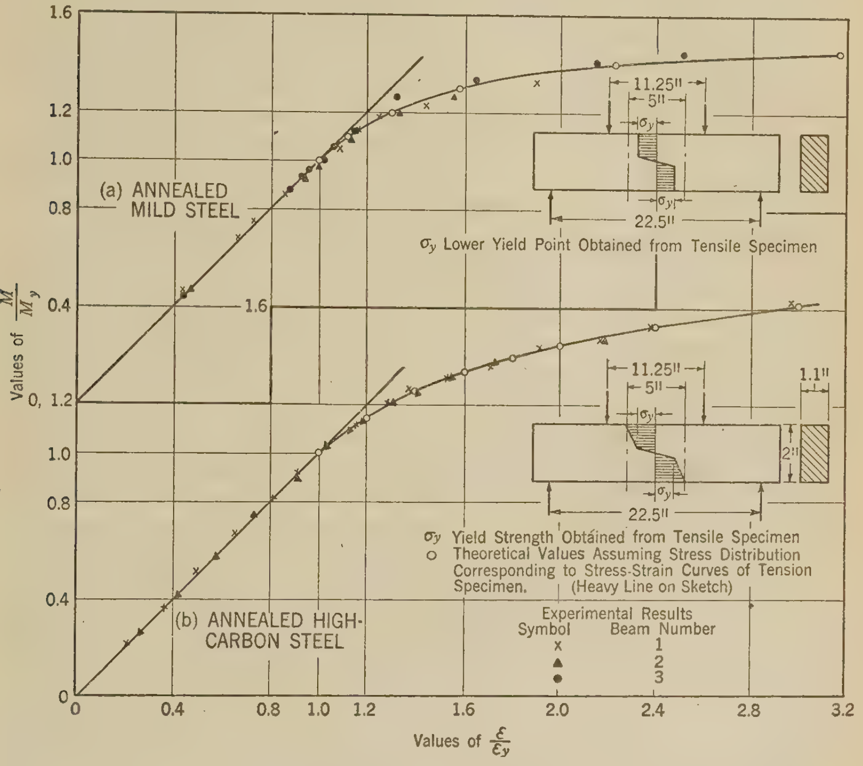

FIG. 17.—COMPARISON OF THEORETICAL AND EXPERIMENTAL MOMENT-STRAIN DIAGRAMS FOR RECTANGULAR BEAMS

The occurrence of a high upper yield point in carefully conducted tensile tests is a phenomenon long associated with mild steel and is a factor which, if present, will modify the behavior of a beam in the early stages of yielding. If special precautions are taken in tensile tests, the stress-strain diagram of the material may be of the shape OBC as shown in Fig. 15, in which an upper yield point, σ_{yu}, is exhibited. The magnitude of the upper yield point is extremely variable and unpredictable depending upon such factors as (*a*) the rigidity of the testing machines; (*b*) the alinement of the specimen in the grips; (*c*) the radius of fillets on the specimen; and (*d*) inherent stress raisers such as surface roughness, residual stresses, or discontinuities in the material. If the effects of these variables are minimized successfully, the magnitude of the upper yield point may be considerably greater than that of the lower yield point, σ_y. For instance, G. Cook (47) has obtained values for upper yield point as large as 1.56 σ_y.

If a mild-steel tension specimen can be tested under such favorable conditions that an upper yield point will be exhibited, it is reasonable to assume that the material in the most stressed fibers of a mild-steel beam, tested under similar conditions, will exhibit an upper yield point. Furthermore, the magnitude of the upper yield point exhibited by the most stressed fibers of the beam will be larger, in most cases, than the value obtained from the average stress in a tensile test. In the foregoing paragraph four factors were listed which tend to lower the upper yield point in a tension specimen. None of these factors (except perhaps inherent stress raisers) has as pronounced an effect in lowering the upper yield point in a beam as it has in a standard tension specimen.

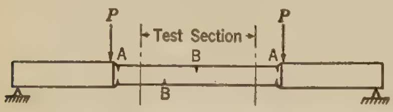

Fig. 18.—Yielding in Test Specimen

If the bar shown in Fig. 18 is subjected to an axial tensile load, there are no available understressed fibers to carry the load by redistribution of the stress once yielding is initiated. Thus, the stress concentrations existing at the fillets cause the average stress at which yielding starts in the tension member to be lower than the maximum upper yield point actually developed at the fillets. Subsequent yielding in the tension specimen will progress by branching of the plastic zones throughout the body of the test section at a stress equal to that of the lower yield point on the material. Hence, tensile test data as generally obtained do not furnish an accurate measure of the maximum upper yield point developed in the material, and the results are more likely to indicate a value close to the lower yield point. Conversely, in a beam, the redistribution of stress caused by initial yielding in the regions of stress concentration (at point A, Fig. 18) enables the beam to carry greater loads before the plastic zones progress by branching out, or before they start to develop in the test section or part. Several investigators have interpreted this to mean that higher stresses are developed in the beam than in a tensile specimen before yielding is initiated, whereas the increase is mainly due to inherent differences in test condition and lack of knowledge of the exact stresses existing at fillets, load points, or other stress raisers.

The effect of an upper yield point on the behavior of a beam can best be shown by considering the moment-strain diagrams for a rectangular beam shown in Fig. 19. If the inherent stress concentrations are of sufficient magnitude so that none of the material in the most stressed fibers of the beam finds it possible to exhibit an upper yield point, initial yielding will begin at the bending moment M_y corresponding to the lower yield point $\sigma_y \left(\frac{M}{M_y} = 1\right.$ as shown at point A in Fig. 19) and the moment-strain diagram will follow the theoretical curve OADB which is the same as that shown in Fig. 16. On the other hand, if the inherent stress concentrations are sufficiently small the moment-strain diagram will remain linear until the stress in the most stressed fibers reaches the upper yield point of the material. For instance, if the upper yield point is equal to 1.3 σ_y, initial yielding will start at a bending moment of $\frac{M}{M_y} = 1.3$ as shown at point C in Fig. 19. Unlike the tensile stress-strain diagram in which there is a sudden drop at the beginning of yielding from an upper yield point to the lower yield point, the moment-strain diagram shows no drop when initial yielding occurs at the upper yield point of $1.3\,\sigma_y$ although the stress in the most stressed fibers of the beam in the yielded region drops to a value equal to the lower yield point. In Fig. 11, if the author's $\sigma_y + \sigma_y\,\Delta\sigma'_y$ is replaced by the upper yield point, σ_{yu}, the stress distribution may be visualized as changing from that shown as B to the type shown as A; the yielded zone penetrates into the beam, thereby forcing the understressed fibers to carry more load and thus increasing the resisting moment to the value of the applied moment. The shape of the moment-strain diagram after initial yielding will depend upon the magnitude and distribution of the inherent stress raisers along the most stressed fibers of the beam.

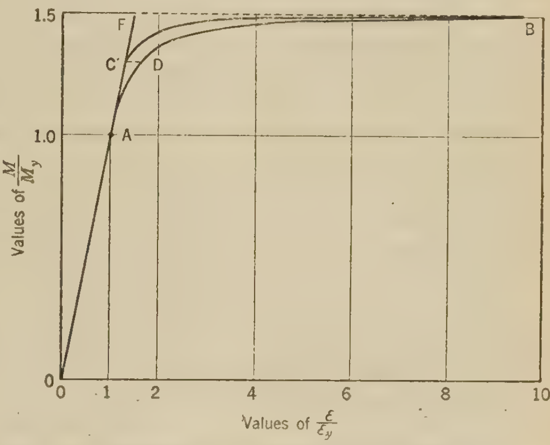

FIG. 19.—MOMENT-STRAIN DIAGRAMS FOR RECTANGULAR BEAMS

The original yielding in a beam usually appears heterogeneously in wedge-shaped zones in regions of localized stress such as at the fillets at point A in Fig. 18. As the load is increased, yielding at a higher load (but at the same stress) will occur in zones B within the test section. Since the strains in the beam are usually measured over a long gage length, localized yielding in the test section may not cause a sudden increase in the measured average strain. Thus, the moment-strain diagram shown in Fig. 19 departs gradually from a straight line at point C. As the applied moment is increased to some value greater than $1.3\ M_y$, the plastic wedges penetrate to greater depth, thereby increasing the resisting moment; the maximum stress at a section some distance from the plastic region increases correspondingly to a value greater than $1.3\,\sigma_y$ or until the upper yield point of the material in that part of the beam is reached. The moment-strain diagram for the beam will follow some relationship such as curve OCB in Fig. 19. In any case the moment-strain diagram will be bounded by the curves OCDB and OFB. The line FB at the bending moment of $\frac{M}{M_y} = 1.5$ is the horizontal asymptote to the theoretical moment-strain diagram and may be considered as the limiting moment for a rectangular beam in which the upper yield point is less than $1.5\,\sigma_y$. This same magnitude for limiting bending moment for a rectangular beam with a definite yield point is also derived by S. Timoshenko (52).

The curves in Figs. 16 and 19 are a development of what the author refers to as the "old theory," as contrasted to the "new theory" attributed to W.

Kuntze (which is not strictly a theory but an approximate empirical relationship). The author's discussions of ultimate resisting moment, M_u, in connection with these two theories, seem irrelevant since (as shown by Fig. 16 and the author's test data) initial yielding of a beam occurs (and large plastic deformations are developed) long before the ultimate bending moment is reached. In the tests of Messrs. Morkovin and Sidebottom (see Fig. 17(*a*)), the limiting bending moments closely approximated the value of 1.5 M_y predicted by the "old theory" for a rectangular beam. The data for the author's beam in Fig. 10 also approach a horizontal asymptote at a limiting bending moment approximately 1.5 times the bending moment corresponding to the lower yield point.

The curves in Figs. 16 and 19 also illustrate the effect of the shape of cross section on the magnitude of the maximum upper yield point that may be exhibited without instability in a beam. Initial yielding at an upper yield point of 1.2 σ_y in the beam of circular cross section (Fig. 16) probably would not cause a pronounced deviation from linearity in the moment-strain diagram. However, initial yielding at the same upper yield point in the I-beam would cause immediate general yielding throughout the entire depth and a breakdown of the beam, since a bending moment of 1.2 M_y lies above the limiting resisting moment for that shape.

The author's tensile specimen shown in Fig. 8(*a*) would not be adequate to determine the existence of a high upper yield point, although there is some evidence in Fig. 10 that an upper yield point occurred. The writers believe, therefore, that for the moment-strain diagram in Fig. 10 the upper yield point of the material was approximately 20% greater than the lower yield point, and only a value of the lower yield point was obtained from the tensile test. Similarly, in connection with the data shown in Fig. 5 the author states that the Lueders lines "* * * spread quite rapidly down the sides of the specimen * * * with very little increase in the load." This is an indication in itself that an unstable condition existed in which a high upper yield point was developed. Once yielding was initiated in the test section at the upper yield point, the stress on the extreme fiber dropped to the lower yield point and it was necessary for the plastic zone to progress deeply before the resisting moment balanced the unchanged bending moment. Lueders lines would generally develop slowly as loads were increased if the material in the beam did not exhibit an upper yield point.

The appearance of Lueders lines on the surface of a beam is not a sensitive criterion of initial yielding since it involves observation of plastic flow in a considerable part of a member. For beams (particularly those having cross sections with a minimum of material on the extreme fibers such as the circular and rhombic sections in Fig. 16), the small volume of metal subjected to the peak stress makes it doubtful whether strain markings would become visible until yielding had progressed to an appreciable depth below the surface fibers.

The author's reference to the fatigue test by K. Kloppel (5), in which a beam withstood a large number of cycles of stress equal to the yield point, should not be interpreted as evidence that the yield point is raised in a bend test. No correlation exists between values of endurance limit and yield strength; hence, there would be no reason to suspect that a change in endurance limit would accompany an increase in elastic strength (even if it did occur in a bend test). In fact, in axial tension fatigue test W. L. Collins, Assoc. M. ASCE, and T. J. Dolan (53) found that for five mild steels (subjected to stress cycles varying from zero to a maximum) the endurance limits were about the same as, or somewhat higher than, their yield points. Thus, one would also expect beams made of these same metals to withstand large numbers of cycles of a stress equal to the yield point without evidence of fracture even though some plastic yielding may occur.

In a study of the stresses developed around a notch in a beam subjected to pure bending (38), X-ray diffraction patterns were used to determine the stress at which yielding was initiated. In these tests a sharp stress gradient was present—resulting in a stress concentration factor of about 2½ at the notch; any effects of delayed yielding resulting in higher elastic stresses should be even more pronounced under these conditions than in the tests reported by the author. However, Mr. Norton concludes that:

> "* * * the stress concentration and the stress gradient resulting from bending have no effect on the initiation of plastic flow in the immediate vicinity of the notch. There the state of stress is uni-axial and plastic flow sets in at the yield stress for pure tension."

For many applications of beams made of ductile steel, considerable yielding of the outer fibers can occur without causing excessive curvature or deflection of the member; hence, for some uses the investigators may infer that the beam is not structurally damaged as a whole. Certain beams may resist loads greater than those required to start yielding in the extreme fibers. It should be recognized that the increased strength is not the result of "higher elastic stresses" but rather that, because of the redistribution of stress, the understressed part can offer greater resistance to the load without permitting sufficient plastic distortion to the member as a whole, to constitute appreciable structural damage. The curves in Fig. 16 indicate the relative extent to which beams of various shapes may be overloaded (stressed beyond the yield point) before appreciable damage is apparent by their relative deviations from a linear load-strain relationship. The plastic deformations of the metal in the extreme fibers corresponding to these small deviations from linearity are rather large, however, as indicated roughly by the percentage by which the ratio ϵ/ϵ_y exceeds unity. This fact might require serious consideration in certain applications where danger of instability because of localized buckling may be aggravated by the plastic deformation of the most stressed fibers.

In conclusion, the basic behavior of flexural members as visualized by the writers may be summarized as follows:

(*a*) The materials that do not exhibit an upper yield point, the actual stress necessary to cause initial yielding of the most stressed fibers of a member subjected to a nonuniform stress distribution (such as that which occurs in a beam) is the same as that required to cause yielding under a uniform stress as obtained in a standard tensile test specimen.

(*b*) For materials exhibiting an upper yield point, the stress gradient that occurs in a beam makes it possible to obtain, before yielding occurs, a computed stress somewhat larger than the yield point that is ordinarily determined as the average stress in a tension specimen. This is true since the magnitude of the upper yield point as computed from the tension test is markedly decreased by unfavorable test conditions, whereas these test conditions do not have as pronounced an effect in decreasing the apparent upper yield point in a beam.

(*c*) The nonuniform distribution of stress that occurs in a beam enables the member to resist static loads considerably greater than those required to start yielding of the extreme fibers. As indicated in Fig. 16, the shape of the cross section of the beam will determine the extent to which plastic deformation of the extreme fibers may occur before structural damage is evidenced by appreciable distortion of the member as a whole.

J. F. Baker,[9] Assoc. M. ASCE, and J. W. Roderick,[10] Esq.—The main contention in this paper is that the upper yield stress for mild steel subjected to nonuniform stress distributions is much higher than that for an axially loaded tension specimen. Mr. Peterson has conducted tests in pure bending and eccentric tension in which the first signs of yielding, as indicated by the appearance of Lueders lines, represent stresses much in excess of the upper yield stress recorded in the usual tension test. This result is contrary to the conclusions reached by A. Robertson (54), G. Cook (55), H. Quinney (56), and J. L. M. Morrison (46), all of whom have shown that, in a tension test on annealed mild steel, where adequate precautions are taken to insure axial loading and where the specimen is tested in a machine designed to record upper and lower yield points, the upper yield stress observed is frequently more than 40% greater than the lower value—showing good agreement with the results of tests in pure bending. As confirmation, the diagram shown in Fig. 20 for a specimen of black mild steel was obtained on a Quinney autographic testing machine (56) in the Engineering Laboratory at Cambridge, England. The steel had received no particular heat treatment and was in the condition "as received"; but despite this the upper yield point was 44% higher than the lower value.

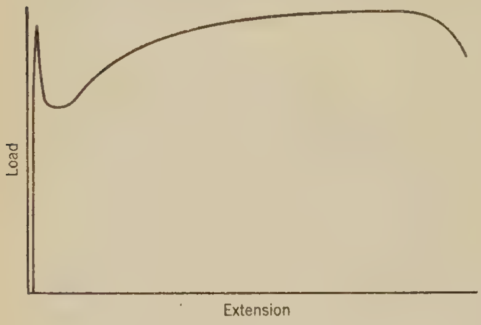

Fig. 20

However, in testing a number of tension specimens cut from any carefully heat treated bar, it will usually be found that, whereas the lower yield stresses are remarkably constant, the upper values will often vary considerably, with no evidence of it at all in some specimens. As Mr. Morrison (46) has shown, however, by using a number of extensometers around the circumference of the tension specimen, even the slightest eccentricity can mask the true value of the upper yield stress. When it is realized that these eccentricities can arise not only from the mechanical shortcomings of the loading gear but also from slight variations in properties across the section of the specimen, inconsistent values of upper yield stress as recorded by over-all extension are not unexpected.

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[9] Prof. of Mech. Science, Univ. of Cambridge, Eng. Laboratory, Cambridge, England.

[10] Asst. Director of Research, Univ. of Cambridge, Cambridge, England.

It would appear from the stress-strain diagrams in Fig. 10 that no special precautions were taken to insure axial loading in the tension test and also, since no reference is made to the type of testing machine employed, that the lower yield point was presumably obtained by rebalancing the normal lever type machine. If this were the case, it is very doubtful whether either the upper or lower yield points recorded represent the true properties of the material. The load-deflection curve for the beam test is compared with curves based on what are described as the "old theory" and the "new theory" of plastic failure, neither of which give good agreement with the beam test results. In England the former theory is now generally accepted, although where the material has an upper yield point the stress distribution for a partially plastic section of a member subjected to bending is taken to be of the form shown in Fig. 21 as opposed to that given in Fig. 1(*d*). The distribution in Fig. 21 is that established by Messrs. Robertson and Cook (54). The "new theory" referred to by the author is that stated by W. Kuntze (3) which, while it gives reasonable results in isolated cases, cannot be claimed to have any true fundamental basis. One cannot but agree with the criticism of this and other theories offered by F. Rinagl (57):

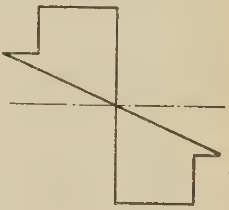

FIG. 21

> "Kuntze's theory, whereby yielding in the extreme fibres is supposed to set in only when the tensile yield limit has been attained in a certain layer—the so called centre of resistance—in the interior of the beam, cannot be accepted either. The 'centre of resistance' is a purely geometrical illusion and has no physical significance. The approximate coincidence with the results of Thum and Wunderlich's tests is only incidental and vanishes as soon as the tests are properly interpreted."

Most continental writers think of W. Prager's theory (12), which assumes that failure of a member subjected to pure bending is instantaneous upon the attainment of the upper yield stress at the extreme fiber, as the "new theory."

The writers have made considerable use of the "old theory" in studying the behavior of rigid welded steel frames tested to collapse (36)(45)(58)(59). At the beginning of this investigation tests were made on beams to verify the theory; but, although the results obtained were satisfactory, it was realized that better agreement between the yield points in tension and bending might have been obtained if more attention had been paid to rate of loading. Another series of beam tests is in progress taking due account of this factor, and a typical load deflection curve is shown in Fig. 22. When the upper yield stress is neglected as in the version of the "old theory" referred to by Mr. Peterson, there is considerable difference between the observed and calculated load deflection curves. On the other hand, using the stress distribution of Fig. 21 and an upper yield stress corresponding to the load at which the observed load deflection curve deviates from the straight line, exceedingly good agreement is obtained. The yield stresses determined from the beam test were: Lower yield stress, 13.46 tons per sq in. and upper yield stress, 19.05 tons per sq in. The average lower yield stress from a number of tension tests was 13.84 tons per sq in.

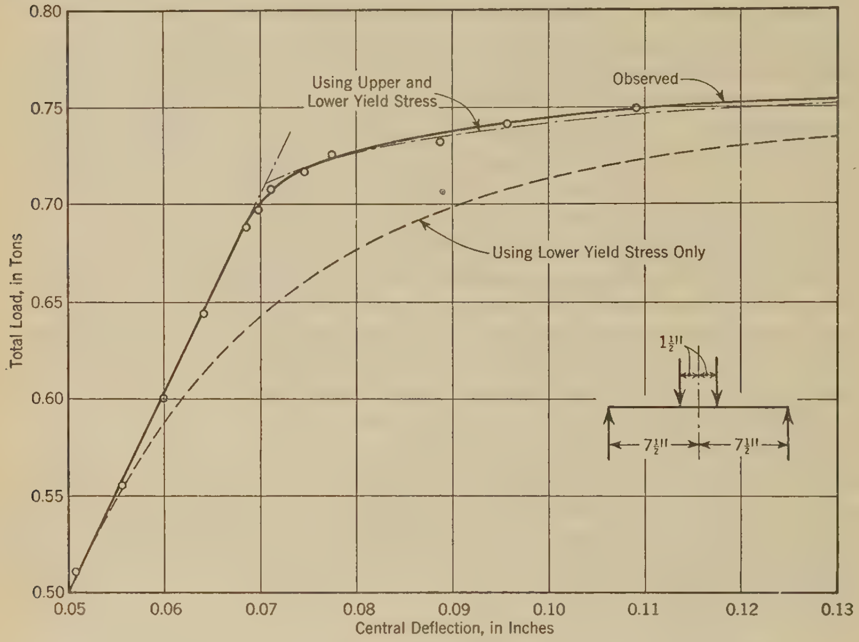

FIG. 22

R. K. BERNHARD,[11] M. ASCE.—The "effect of stress distribution on yield points" described mainly from the point of view of the material itself is presented in this paper. It might be of interest to treat the subject more dynamically (60), discussing the change of equilibrium between specimen and testing machine or the effect of the natural frequency of test machines on the accuracy of indication in high-speed tests with respect to the determination of yield points. (The writer has contributed to this subject twice before (61)(62).)

In high-speed testing of essentially the same material, different results may be obtained depending upon whether a massive or a light testing machine is used. Furthermore, the stress-strain diagrams of certain materials indicate near the yield point peculiar forms, which in some cases may not be dependent upon the qualities of the tested material alone.

Static System.—Any testing machine with a hydraulic cylinder may be considered as a system composed of masses and springs. Masses constitute the frame of the machine (the crosshead, etc.) whereas springs with a definite elastic constant represent the test specimen and the hydraulic medium. At any moment there must be equilibrium between the summation of the elastic forces of the springs and the load on the ram of the machine.

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[11] Prof. of Eng. Mechanics, Rutgers Univ., New Brunswick, N. J.

In the case of a testing machine having a low elastic constant, a very small decrease of load will result with the increase of strain; a high elastic constant, on the other hand, will result in a considerable decrease in load with increase of strain. When plastic flow ceases, equilibrium between the two elastic forces (specimen and pressure medium) may be restored and the point of equilibrium again rises as soon as the pressure pump starts to force new liquid into the cylinder. It must be borne in mind, however, that, with decreasing tension in the specimen, the plastic flow will also cease.

Any rapid flow gradually changes into some kind of creeping flow before equilibrium is finally achieved. Hence, in certain cases of high-speed testing such drop in the load-deformation diagram, due to plastic flow, may be independent of the quality of the tested materials.

Dynamic System.—The plastic flow or any pressure changes in the cylinder may take place very suddenly and the indicating device of the testing machine must follow immediately these rapid changes in both coordinates—that is, load and deformation. It is well known that as soon as rapid changes come into consideration the accuracy of any indicating device depends on the natural period of the entire vibrating system.

Any fluctuations during the test having a frequency of more than one tenth of the frequency of the machine will cause incorrect indications, assuming, for simplicity, that only vibrations of a sine form with a small damping factor come into consideration. This indicates, also, that different values for the upper and lower yield points, independent of the quality of the tested material and due to rapid changes of plastic flow near these yield points, may be found in certain cases of high-speed testing.

Effect Upon Yield-Point Determinations.—The surplus in tension at the upper yield point is a type of delayed phenomenon. Once plastic flow of the material in the specimen has started, the further shape of the diagram depends, at least to a certain amount, upon the character of the test machine. If the elastic constant of the test machine is low, any lower yield point will be suppressed, as the elongation of the test specimen produced by plastic flow means no decrease in tension for the testing machine itself. A very high elastic constant of the testing machine, however, will produce the well-known "leaping effect" fluctuations.

A similar result will be produced if the test specimen is long and has a low elastic constant, thus causing a heavy drop in the diagram. Consequently, a very short and rigid specimen may indicate no lower yield point at all.

Summary.—In the light of the foregoing comment the conclusions are reached that the upper and lower yield points are also associated with the characteristics of the testing machine and, to a certain degree, are independent of the quality of the material, and that in specific cases of high-speed testing the form of the specimen may be secondary.

F. G. Eric Peterson,[12] Assoc. M. ASCE.—As stated by Mr. Shearwood, it is true that the value of the thesis presented "* * * will be realized through the promotion of investigation into the practical and useful application of the findings * * *."

The general formula (Eq. 12*b*) developed by Professor Gaylord, which gives a relation between the tension yield point and the bending yield point for various shapes of cross section, is quite ingenious and appears to give quite accurate results. It should be observed that, in shapes where a large proportion of the area of a cross section is far from the neutral axis (for example, wide-flange beams), the effect is quite small, whereas in cross sections with a large proportion of the area close to the neutral axis (rectangular, cylindrical, diamond shape, etc.) the bending yield point is appreciably greater than the tension yield point. The accidental eccentricity of 0.001 in. mentioned by Professor Gaylord may have existed. It would be difficult to avoid such a small eccentricity.

In the presentation of this paper the writer has not concerned himself with the behavior of the metal after the plastic state has been reached. In other words, he has visualized the final designed structure as acting elastically throughout (after using a suitable factor of safety) without being tied to a definite maximum stress. The relative dimensions of the structure have a definite bearing on the final maximum stress that will result. One structure may have a maximum stress of 30,000 lb per sq in., whereas another may have a maximum stress of 24,000 lb per sq in.; both may have the same factor of safety (N) as defined by the ratio of the ultimate failure load to the working load:

$$N = \frac{L_u}{L_w} \quad \text{..........................(25)}$$

Various structures investigated by the writer, using a factor of safety of $N = 2$, had final working stresses varying from about 22,000 lb per sq in. to about 30,000 lb per sq in.; but such stresses were always appreciably below even the tension yield point.

As a consequence of the study of limit design, the American Institute of Steel Construction has adopted a specification allowing a 20% increase in stress for continuous beams. Although this is probably safe, it is not logical, since, as mentioned herein, a given factor of safety based on ultimate failure loads does not result in the same final maximum stress for different structures.

Mr. Mensch's statement that for fixed beams with uniform load "* * * it would be stated that failure occurred under a bending moment of $w\,L^2/16$ * * *" is logical. This merely means that once the end cross sections have reached their ultimate resisting moments the beam acts as a simple beam until the center cross section has reached the same ultimate resisting moment. The end moment plus the center moment must equal $1/8\ w\,l^2$.

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[12] Civ. Engr., Oakland, Calif.

As stated by Professor Winter, "The problem is difficult to approach experimentally, chiefly because * * * strain measurements can no longer be translated into stress." However, consider the load-deflection diagram of a beam in bending: The first part of that diagram for steel is straight (OB′, Fig. 10) and no other stress distribution than that shown in Fig. 2(*b*) can possibly be justified—that is, a straight-line stress distribution must exist.

Professor Winter states that in a "* * * cross section of a rectangular beam stressed partly into the plastic range * * * a shape more like that of Fig. 12(*c*) must be expected." The writer agrees with this statement but wishes to emphasize that this shape must be accompanied by the appearance of deformation wedges or Lueders lines. Professor Winter also states:

> "It is doubtful whether macroscopic Lueders lines can be detected when yield strains are of such small amount, although it is likely that slip bands in individual crystals could be observed in such zones by microscopic inspection."

In this connection it should be noted that the appearance of Lueders lines is not a gradual process but that they appear quite suddenly—to an appreciable depth at once, afterward expanding in a series of jumps as reported by C. F. Kollbrunner (44). Thus, the writer concludes that the process of initial yield is immediately apparent through the appearance of macroscopic Lueders lines.

Although the writer is not very familiar with the X-ray method of stress measurement, it is commonly agreed that it is not very accurate (possibly 10% in error). What seems more important in the present discussion is that this method sheds no light on the phenomenon of the upper yield point. It may be that the results obtained by the writer and others are directly associated with the existence of an upper yield point—that is, the upper yield point is the controlling factor when the stress is nonuniform and no fibers show any evidence of yield until the extreme fiber has reached the upper yield point.

Referring to Fig. 21 by Messrs. Baker and Roderick, it is quite evident that no yield would occur in any fiber until the extreme fiber reached the upper yield point. Thus, the writer is quite ready to agree with Messrs. Baker and Roderick that the lower yield point is not necessarily raised but that the yield is governed by the upper yield point. Fig. 22 proves this contention. It is possible that the value of the upper yield point is affected by the shape of the cross section being tested.

An important question presents itself to the writer. In the usual commercial tension test the yield point that is reported is the lower yield point. Should this value be used in design, particularly where nonuniform stress is to result? If this lower yield point is the one that is always supplied by the commercial testing laboratories, it does not seem much amiss to refer to a "raised yield point" in bending or other nonuniform stress; or, perhaps, it should be given another designation. The usual testing machine allows the load to decrease because of the yielding of the specimen (especially in machines of a high elastic constant, as suggested by Professor Bernhard), thus exhibiting a lower yield point. Under field conditions of loading, the load is not decreased as the structure yields. It appears to the writer that it would be better if commercial testing machines were of a type that would not allow the load to decrease since (in general) this would conform more closely to the conditions which a designed structure undergoes in actual use.

The testing machine used by the writer in his tests was a 60,000-lb mechanical pendulum machine with a direct-reading dial indicator. The speed of testing for bending was about 0.02 in. per min, whereas for the tension tests it was well under 0.01 in. per min.

The writer deeply appreciates the time and effort expended by various writers in contributing discussions on this paper.

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Paper No. 2326

STRENGTH OF BEAMS AS DETERMINED BY LATERAL BUCKLING

BY KARL DE VRIES,[1] M. ASCE

WITH DISCUSSION BY MESSRS. GEORGE WINTER, DAVID B. HALL, THEODORE R. HIGGINS, NEIL VAN EENAM, H. N. HILL, H. D. HUSSEY, H. G. BRAMELD, EDWIN H. GAYLORD, OLIVER G. JULIAN, AND KARL DE VRIES.

SYNOPSIS

All usual specifications for the design of beams provide for reducing the allowable unit stress of the compression flange as the unsupported span length increases, to guard against lateral buckling. Such provisions, for the sake of simplicity, relate this reduction only to the ratio of unsupported beam length to flange width, thus neglecting the obvious influence of (*a*) the horizontal moment of inertia of the compression flange; and (*b*) the restraint afforded by the tension flange, which decreases as the depth of the beam increases.

By an extension and an evaluation of the treatment of this problem given by S. Timoshenko,[2] more nearly rational formulas are developed. Typical formulas are proposed for inclusion in specifications, and additional formulas for special investigations are derived and their application is described.

Professor Timoshenko's basic formulas have been accepted without repeating their derivations. He has also tabulated certain values[3] for beam strengths, which are practically identical with the corresponding values of this paper; but a greater range of beam strengths is given in this paper than is presented by Professor Timoshenko. In referring to the Timoshenko formulas, some repetition is unavoidable. Since Professor Timoshenko has not shown the steps for computing the beam strengths and since they will be of interest for the reader who wishes to check the work done here, they are given in Appendix I with the necessary condensation.

Notation.—The letter symbols in this paper are defined where they first appear, in the text or illustrations, and are assembled alphabetically, for con-

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NOTE.—Published in September, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Bethlehem, Pa.

[2] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 239, Chapter V.

[3] *Ibid.*, pp. 267, 268, and 272, Tables 22, 23, 25, and 26.

venience of reference, in Appendix II. Discussers are requested to adapt their notation to this form.

PART I. SIMPLE BEAMS

CONCENTRATED LOAD AT CENTER

It is assumed that the ends of the simple beam shown in Fig. 1 can rotate freely with respect to the x-axis and the y-axis, but that rotation with respect to the z-axis is prevented. In calculating the critical value of the load P, Fig. 1, Professor Timoshenko assumes that a small lateral buckling has occurred and, from the differential equations of equilibrium, determines the magnitude of the smallest load required to keep the beam in this slightly buckled form:[4,5]

$$\frac{P^2}{4\,B}\int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz = C \int_0^{l/2} \left(\frac{d\beta}{dz}\right)^2 dz + \frac{B\,d^2}{4}\int_0^{l/2} \left(\frac{d^2\beta}{dz^2}\right)^2 dz \;\ldots\ldots(1)$$

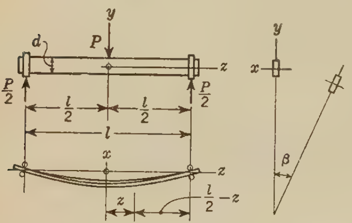

FIG. 1.—CONCENTRATED LOAD AT CENTER

Eq. 1 expresses the relationship between a concentrated load P applied at the centroid of the beam at midspan; and β, the angular deflection of the beam at any point at a distance z from midspan, in terms of two constants B and C which in turn depend on the properties of the beam. The values of these constants are found as follows:[6] B is the transverse flexural rigidity of the beam, and is equal to $E\,I_y$; C is its torsional rigidity and is equal to $\frac{E\,I_y\,d^2}{4\,a^2}$; E is the modulus of elasticity; I_y is the moment of inertia about the y-axis; and a is a torsional bending constant, values of which, for beams, are listed in manuals.[7]

It is necessary to introduce a correction term in Eq. 1 to allow for the effect of loading not at the centroid but at the top or bottom flange. This correction is $+\frac{P\,d}{4}\,\beta^2_o$ for top flange loading and $-\frac{P\,d}{4}\,\beta^2_o$ for bottom flange loading, in which β_o is the angle of twist of the beam at its center line.

With this modification, substituting the foregoing constants, Eq. 1 yields the basic formula:

$$\frac{P^2}{4\,E\,I_y}\int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz \pm \frac{P\,d}{4}\,\beta^2_o - \frac{E\,I_y}{4}\,\frac{l^2}{a^2}\,\frac{d^2}{l^2}\int_0^{l/2} \left(\frac{d\beta}{dz}\right)^2 dz - \frac{E\,I_y}{4}\,d^2\int_0^{l/2} \left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \;\ldots\ldots\ldots(2)$$

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[4] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 251, Fig. 146.

[5] *Ibid.*, p. 266, Eq. (*e*).

[6] *Ibid.*, pp. 240–258.

[7] "Bethlehem Manual of Steel Construction," *Catalogue S-47*, Bethlehem Steel Co., Bethlehem, Pa., 1934, p. 285, Table II.

Eq. 2 is a quadratic equation with respect to P, which can readily be solved into:

$$P = \frac{E I_y d}{2} \times \frac{\mp \beta^2_o + \sqrt{\beta^4_o + 4 \int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz \left[\frac{l^2}{a^2} \frac{1}{l^2} \int_0^{l/2} \left(\frac{d\beta}{dz}\right)^2 dz + \int_0^{l/2} \left(\frac{d^2\beta}{dz^2}\right)^2 dz\right]}}{\int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz} \quad (3)$$

The least value of P, as determined by Eq. 3, is the critical load under which beam failure by flange buckling may be expected for these loading conditions. For a concentrated load applied at the top flange, $-\beta^2_o$ is used; applied at the bottom flange, $+\beta^2_o$ is used; and, applied at the centroid, since the correction factor $\pm \frac{P\,d}{4} \beta^2_o$ disappears, β^2_o and β^4_o are omitted. Eq. 3 is not readily soluble; and, to reduce it to a more usable form, the unit stress in the beam for critical load (f) is determined as explained in Appendix I. This unit stress is expressed as:

$$f = \frac{I_y\, d^2}{I_x\, a^2} E \frac{a^2}{l^2} k \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

For any beam of given length the values in Eq. 4 are constant, with the exception of the factor k which will vary for different loading conditions. The values of k are as shown in Table 1(a).

TABLE 1.—Values of k (See Eqs. 21, 36, 43, and 50, Respectively) for Use in Solving Eq. 4

$\frac{l}{a}$	Simple Beams						(c) Two-Bay Beams			
	(a) Concentrated Load, Eq. 21			(b) Uniform Load, Eq. 36			Concentrated load, Eq. 43	Uniform Load, Eq. 50		
	Top flange	Centroid	Bottom flange	Top flange	Centroid	Bottom flange		Top flange	Centroid	Bottom flange
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	2.525	3.982	6.272	2.269	3.310	4.824	5.411	3.499	3.917	4.381
3	3.127	4.649	6.885	2.788	3.860	5.341	6.287	4.142	4.560	5.018
4	3.863	5.440	7.632	3.418	4.518	5.969	7.331	4.910	5.329	5.780
5	4.687	6.310	8.469	4.121	5.244	6.669	8.476	5.757	6.174	6.620
6	5.568	7.232	9.365	4.872	6.013	7.417	9.686	6.651	7.069	7.509
7	6.490	8.187	10.30	5.655	6.810	8.197	10.94	7.579	7.995	8.432
8	7.439	9.164	11.26	6.459	7.626	8.999	12.22	8.529	8.944	9.378
9	8.409	10.16	12.24	7.280	8.456	9.817	13.52	9.494	9.909	10.34
10	9.393	11.16	13.24	8.113	9.296	10.65	14.84	10.47	10.89	11.31
11	10.39	12.18	14.24	8.954	10.14	11.49	16.17	11.46	11.87	12.30
12	11.39	13.20	15.26	9.802	11.00	12.33	17.51	12.45	12.86	13.29
13	12.41	14.22	16.28	10.65	11.85	13.18	18.86	13.45	13.86	14.29
14	13.43	15.25	17.30	11.51	12.72	14.04	20.21	14.45	14.86	15.29

Uniformly Distributed Load

In a manner similar to that described under the heading, "Concentrated Load at Center," values of f, the critical unit stress, for a load uniformly distributed on the beam, may be determined. The equation for f is identical with Eq. 4, but the values of k for values of l/a from 2 to 14 are not the same as those for a beam with concentrated load (see Table 1(b)).

CRITICAL STRESS FOR ROLLED BEAMS

A comparison of the k-factors in Tables 1(a) and 1(b) indicates that a uniformly distributed load applied at the top flange gives the lowest critical stress; and, since this loading also represents the most usual case, or nearly so, it alone has been used in the following study. Critical stresses for uniformly distributed load applied at the top flange were computed for all "wide-flange" (WF) sections from 36 in. to 10 in. deep, which ordinarily are classified as beams, and also for all "light beams" 12 in. and 10 in. deep—a total of 132 beams.[8] First, the $\frac{I_y\, d^2}{I_x\, a^2}$-factors of Eq. 4 were computed for each beam, and then, using the k-values of Table 1, Col. 5, with their corresponding values of $E\,\frac{a^2}{l^2}$, the critical stresses could be tabulated, omitting, however, any case in which the resulting critical stress would exceed 60,000 lb per sq in. and any case in which l/b (b being the width of the beam flange) would exceed 100. The complete tabulation of all the beams is filed with the Engineering Societies Library in New York, N. Y. The tabulation for five 14-in. sections is given in Table 2(a), Cols. 2 to 6, and is typical of the work.

In computing the values shown in Table 2, it is assumed that in Eq. 4 the modulus of elasticity E = 30,000,000 lb per sq in.; and that the values of a and $10{,}000\,\frac{I_y\, d^2}{I_x\, a^2}$ are as follows:

14-in. WF	a (in.)	$10{,}000\,\frac{I_y\, d^2}{I_x\, a^2}$
87 lb.	104.0	65.54
78 lb.	81.86	71.71
61 lb.	74.32	58.59
43 lb.	69.04	41.27
34 lb.	64.28	29.79

For the section 14WF43, and a beam length of $5\,a = 5 \times 69.04$ in. = 345.2 in., for example, the critical stress is $\frac{41.27}{10{,}000} \times 30{,}000{,}000 \times \frac{1}{25} \times 4.121 = 20{,}410$ lb per sq in.

Entries in Table 2 below the horizontal line represent beams that would be excluded from use under the frequently found arbitrary restriction that l/b shall not exceed 40, yet these beams are found in the present analysis to possess considerable capacity.

To be able to express the critical stresses in terms of readily obtained variables (that is, the properties of the beam sections shown in handbooks), trials of various combinations of the variables that contribute most to the strength of

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[8] "Bethlehem Manual of Steel Construction," *Catalogue S-47*, Bethlehem Steel Co., Bethlehem, Pa., 1934, pp. 146 to 158 and p. 167.

TABLE 2.—CRITICAL STRESS, f, COMPUTED FROM EQ. 4, FOR FIVE 14-IN. WIDE-FLANGE (WF) BEAMS (POUNDS PER SQUARE INCH)

$\frac{l}{a}$	LOAD APPLIED AT THE TOP FLANGE					LOAD APPLIED AT THE CENTROID					LOAD APPLIED AT THE BOTTOM FLANGE				
	87-lb beam	78-lb beam	61-lb beam	43-lb beam	34-lb beam	87-lb beam	78-lb beam	61-lb beam	43-lb beam	34-lb beam	87-lb beam	78-lb beam	61-lb beam	43-lb beam	34-lb beam
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
							(*a*) UNIFORMLY DISTRIBUTED LOADS								
2	....	....	....	....	50,700	....	....	....	....	....	....	....	....	....	....
3	....	....	54,440	38,340	27,680	....	....	....	53,100	38,330	....	....	....	....	53,030
4	42,000	45,960	37,550	26,450	19,090	53,530	....	49,640	34,960	25,240	....	....	....	46,190	33,340
5	32,410	35,460	28,970	20,410	14,730	41,240	45,130	36,870	25,970	18,750	52,450	57,380	46,880	33,020	23,840
6	26,610	29,110	23,790	16,760	12,090	32,840	35,930	29,360	20,680	14,930	40,510	44,320	36,210	25,510	18,410
7	22,690	24,830	20,280	14,290	10,310	27,320	29,900	24,430	17,210	12,420	32,890	35,980	29,400	20,710	14,950
8	19,850	21,710	17,740	12,500	9,020	23,430	25,640	20,950	14,750	10,650	27,640	30,250	24,710	17,410	12,570
9	17,670	19,330	15,800	11,130	8,030	20,530	22,460	18,350	12,930	9,330	23,830	26,070	21,300	15,010	10,830
10	15,950	17,450	14,260	10,050	7,250	18,280	20,000	16,340	11,510	8,310	20,930	22,900	18,710	13,180	9,510
11	14,550	15,920	13,010	9,160	....	16,480	18,040	14,740	10,380	....	18,670	20,420	16,690	11,750	....
12	13,380	14,640	11,960	....	....	15,020	16,430	13,420	....	....	16,840	18,420	15,050	....	....
13	12,390	13,560	11,080	....	....	13,790	15,090	12,330	....	....	15,340	16,780	13,710	....	....
14	11,550	12,640	....	....	....	12,750	13,950	....	....	....	14,080	15,410	....	....	....
							(*b*) CONCENTRATED LOADS								
2	....	....	....	....	56,420	....	....	....	....	....	....	....	....	....	....
3	....	....	....	43,020	31,050	....	....	....	....	46,170	....	....	....	59,060	42,630
4	47,470	51,940	42,440	29,890	21,580	....	....	59,760	42,090	30,380	....	....	....	41,940	30,280
5	36,860	40,330	32,950	23,210	16,750	49,630	54,300	44,360	31,250	22,560	....	....	59,550	32,210	23,250
6	30,410	33,270	27,190	19,150	13,820	39,500	43,220	35,310	24,870	17,950	51,150	55,960	45,720	26,020	18,790
7	26,040	28,490	23,280	16,400	11,840	32,850	35,940	29,370	20,680	14,930	41,330	45,220	36,950	21,790	15,730
8	22,850	25,010	20,430	14,390	10,390	28,160	30,810	25,170	17,730	12,800	34,600	37,860	30,930	18,720	13,510
9	20,410	22,330	18,240	12,850	9,280	24,660	26,980	22.040	15,530	11,210	29,720	32,520	26,570	16,390	11,830
10	18,470	20,210	16,510	11,630	8,390	21,940	24,010	19,620	13,820	9,970	26,030	28,480	23,270	14,570	....
11	16,880	18,470	15,090	10,630	....	19,790	21,650	17,690	12,460	....	23,140	25,320	20,690	....	....
12	15,560	17,020	13,910	....	....	18,020	19,710	16,110	....	....	20,840	22,800	18,630	....	....
13	14,430	15,790	12,900	....	....	16,540	18,100	14,790	....	....	18,940	20,720	16,930	....	....
14	13,470	14,740	....	....	....	15,300	16,740	....	....	....	17,360	18,990	....	....	....

beams were made. Finally, it was found that, if the critical stress, as ordinate, were plotted against the dimension ratio—

$$\frac{l\,d}{b\,t} \qquad \text{(5)}$$

—as abscissa, the resulting group of points would fall in such a pattern that a curve expressing the relationship could readily be plotted. Values of d, b, and t for rolled sections are tabulated in standard handbooks.

For each of the 132 beams, therefore, the ratio $\frac{l\,d}{b\,t}$ was computed. First the quantity $\frac{a\,d}{b\,t}$ was computed from the handbook values of these four factors, and then this value was multiplied by values of l/a from 2 to 14, to give a value of $\frac{l\,d}{b\,t}$ for each value of l/a.

A part of the resulting tabulation embracing only the five sections selected for Table 2, is given in Table 3. Only those $\frac{l\,d}{b\,t}$-values are shown for which critical stresses are given in Table 2. The horizontal dividing lines in the two tables are the same.

TABLE 3.—RATIO $\frac{l\,d}{b\,t}$, FOR THE BEAMS IN TABLE 2

l/a	14 WF 87	14 WF 78	14 WF 61	14 WF 43	14 WF 34
$\frac{a\,d}{b\,t}$	146.0	133.6	160.8	223.6	294.3
2	....	....	....	....	589
3	....	....	482	671	883
4	584	534	643	894	1,177
5	730	668	804	1,118	1,472
6	876	802	965	1,342	1,766
7	1,022	935	1,126	1,565	2,060
8	1,168	1,069	1,286	1,789	2,354
9	1,314	1,202	1,447	2,012	2,649
10	1,460	1,336	1,608	2,236	2,943
11	1,606	1,470	1,769	2,460	....
12	1,752	1,603	1,930	....	....
13	1,898	1,737	2,090	....	....
14	2,044	1,870	....	....	....

The observations in Tables 2 and 3 for uniformly distributed load were then plotted in Fig. 2; values of $\frac{l\,d}{b\,t}$ were plotted as abscissas and the corresponding critical stresses f were plotted as ordinates. Round dots represent the five 14-in. sections, of the other points, × represents entries for all other beam sections for which l/b is less than 40; and +, those for which l/b is 40 or more. Not every entry can be shown, as in some instances they superimpose. It is obvious that the grouping of entries gives a good pattern and that a curve for a practical formula can easily be obtained from it. The formula for critical stress (Eq. 4) has the modulus of elasticity, E, as one of its variables, but there

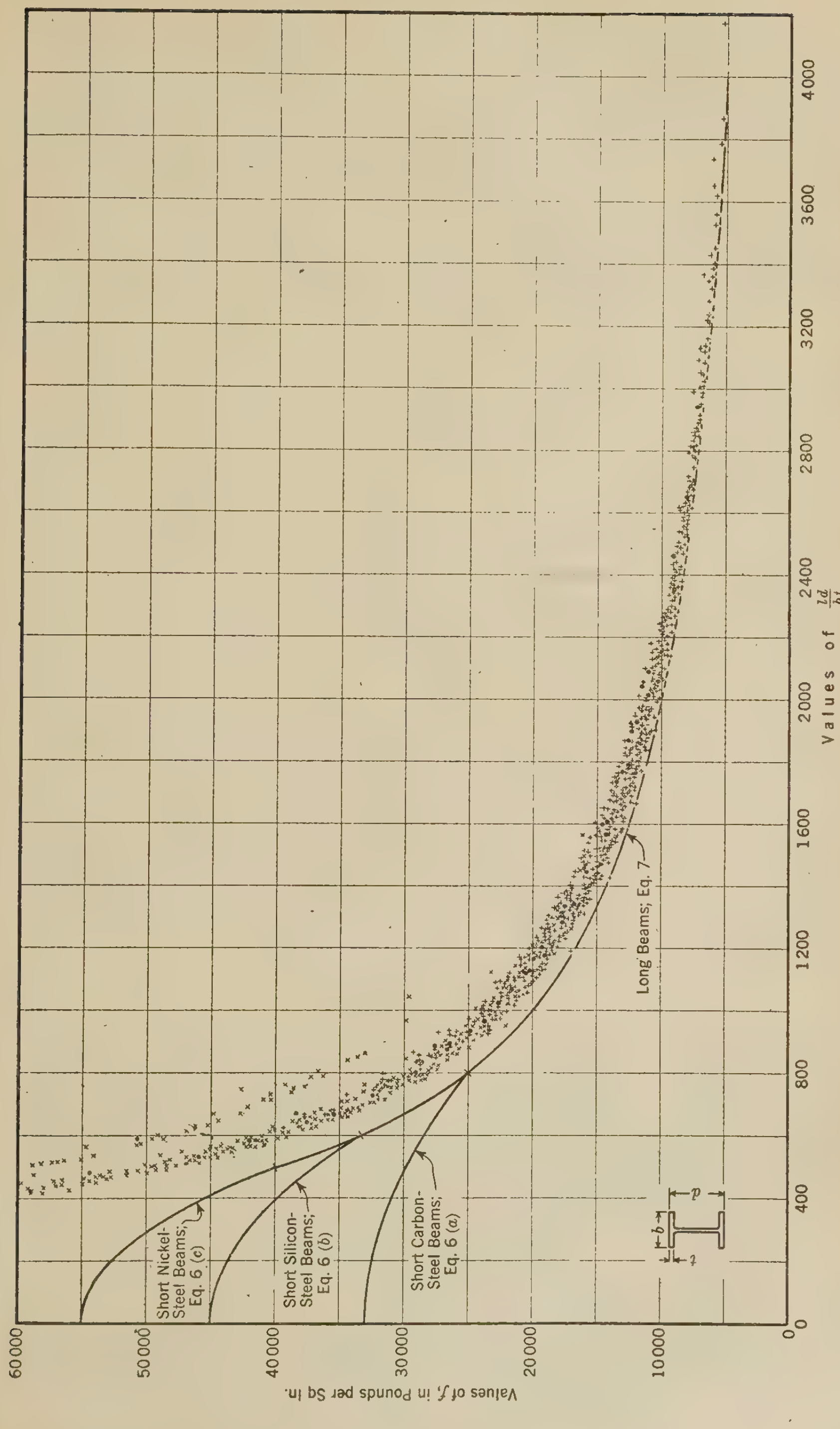

FIG. 2.—CRITICAL STRESSES COMPUTED FOR A SIMPLE BEAM, WITH A UNIFORMLY DISTRIBUTED LOAD AT THE TOP FLANGE, AND CURVES FOR THE FAILURE FORMULAS EQS. 6 AND 7

is no term representing the yield point strength of the steel. Therefore, as with the Euler curve for columns, the actual curve for critical stress must be cut at the yield point strength of the steel under consideration. For higher $\frac{l\,d}{b\,t}$-values (for long beams) then, the same formula should apply to all structural steels since E for all is close to 30,000,000 lb per sq in. For lower $\frac{l\,d}{b\,t}$-values (for short beams), the yield point of each of the three common structural steels is made the upper limit, and transition curves are inscribed.

Failure formulas for short beams (see Fig. 2) are: For carbon steel—

$$33{,}000 - 0.0125\left(\frac{l\,d}{b\,t}\right)^2 \qquad (6a)$$

for silicon steel—

$$45{,}000 - 0.0324\left(\frac{l\,d}{b\,t}\right)^2 \qquad (6b)$$

and, for nickel steel—

$$55{,}000 - 0.06\left(\frac{l\,d}{b\,t}\right)^2 \qquad (6c)$$

For long beams, in carbon, silicon, and nickel steel alike:

$$\frac{20{,}000{,}000}{\dfrac{l\,d}{b\,t}} \qquad (7)$$

These formulas are safe for beams which are laterally unsupported between their ends and where l is the span length; and also for beams with two or more panels—that is, beams which are prevented from rotation at one or more intermediate points and whose panel length is used as l, the unsupported length of the compression flange (as will be shown subsequently). The case of the uniformly loaded top flange has been made the basic example, since it results in lower critical stresses than the other five loading conditions, as can be seen from the k-values of Table 1.

WORKING FORMULAS

Working formulas can be written directly from the formulas for critical stress (failure formulas, Eqs. 6 and 7). Suppose, for instance, that for steel railway and highway structures the unit stresses usually specified for beams of zero length be retained and that safety factors comparable to those now in force are applied, then working formulas for short beams, corresponding to Eqs. 6, might be chosen: For carbon steel—

$$18{,}000 - 0.006\left(\frac{l\,d}{b\,t}\right)^2 \qquad (8a)$$

for silicon steel—

$$24{,}000 - 0.014\left(\frac{l\,d}{b\,t}\right)^2 \qquad (8b)$$

and, for nickel steel—

$$30{,}000 - 0.024\left(\frac{l\,d}{b\,t}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots \text{(8}c\text{)}$$

For long beams, of carbon, silicon, or nickel steel, the formula corresponding to Eq. 7 might be:

$$\frac{12{,}000{,}000}{\dfrac{l\,d}{b\,t}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(9)}$$

Eqs. 8 and 9 have been plotted in Fig. 3 with transition points at $\frac{l\,d}{b\,t} = 1{,}000$, 700, and 500, respectively.

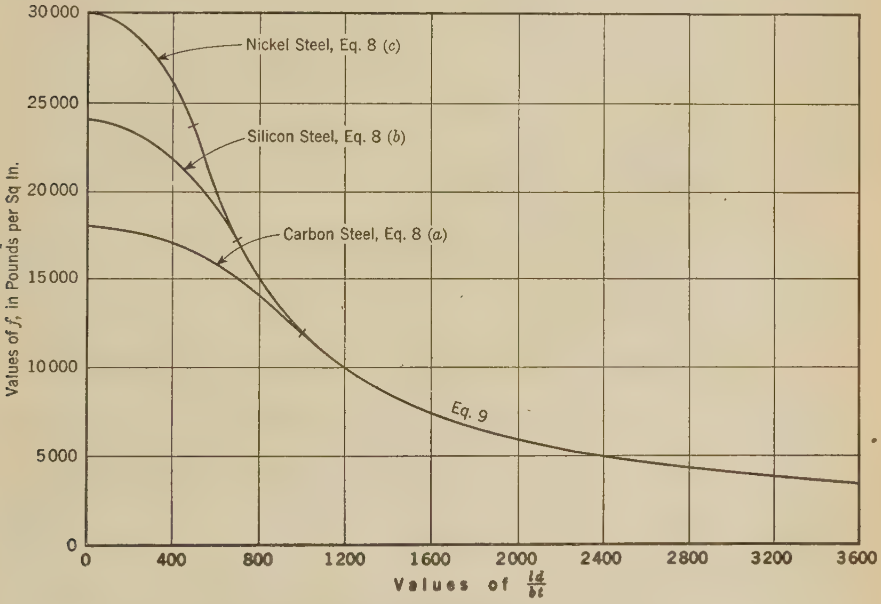

Fig. 3.—Proposed Working Formulas (Eqs. 8 and 9)

To simplify the use of these formulas, it is recommended that the $\frac{d}{b\,t}$-values be listed with the properties of beam sections in future editions of handbooks. Handbook tabulations of allowable stresses, giving "unit stresses" and "ratios" for converting allowable loads on laterally supported beams to allowable loads on laterally unsupported beams, based on the l/b-ratio, could be replaced by tabulations of allowable stresses based on the $\frac{l\,d}{b\,t}$-ratio.

THE FIVE OTHER TYPES OF LOADING

Working formulas of the type of Eqs. 8 and 9, which are based on the failure formulas Eqs. 6 and 7, are proposed for incorporation in specifications. They

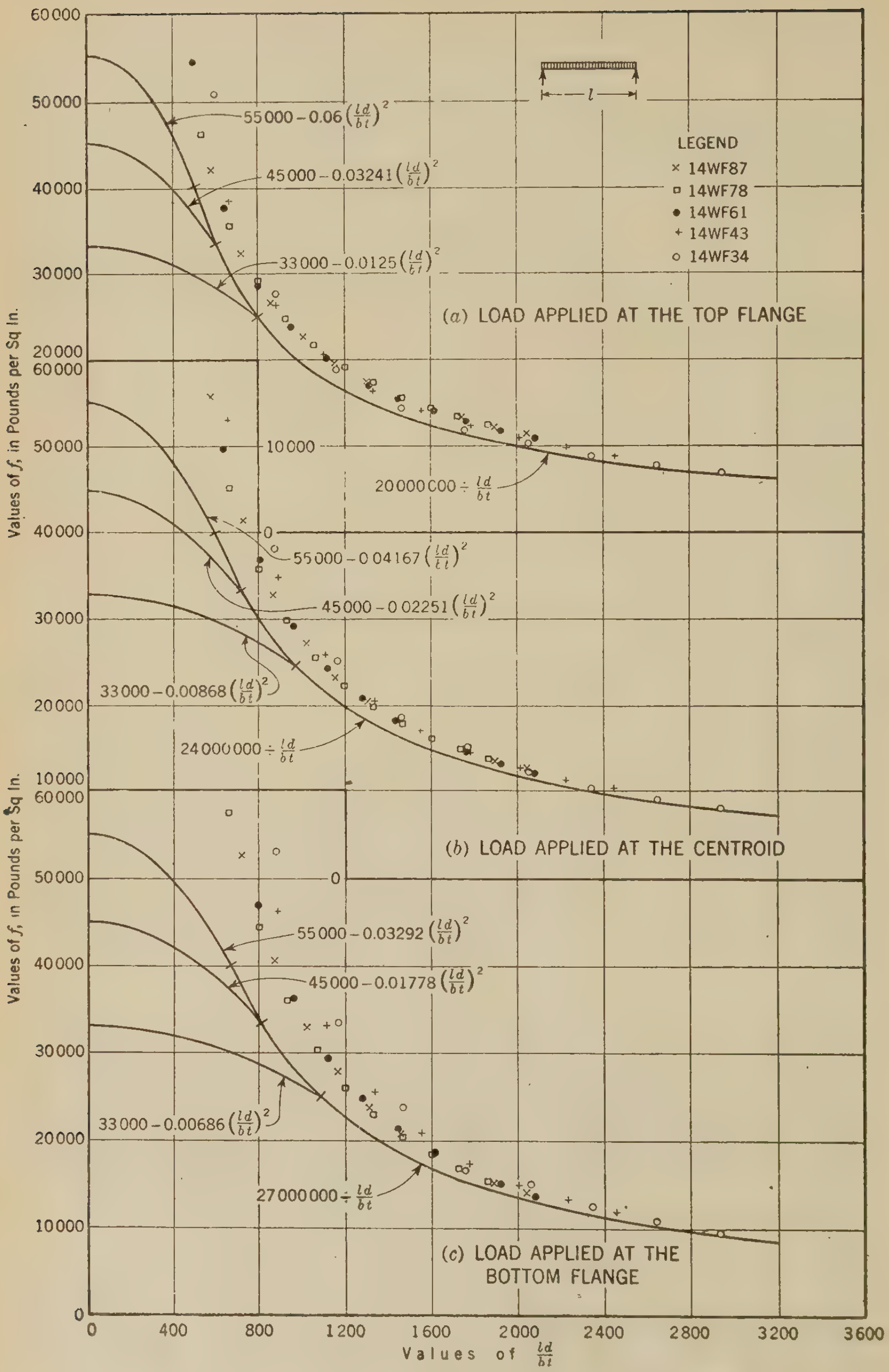

Fig. 4.—Critical Stresses for a Simple Beam, Uniformly Loaded

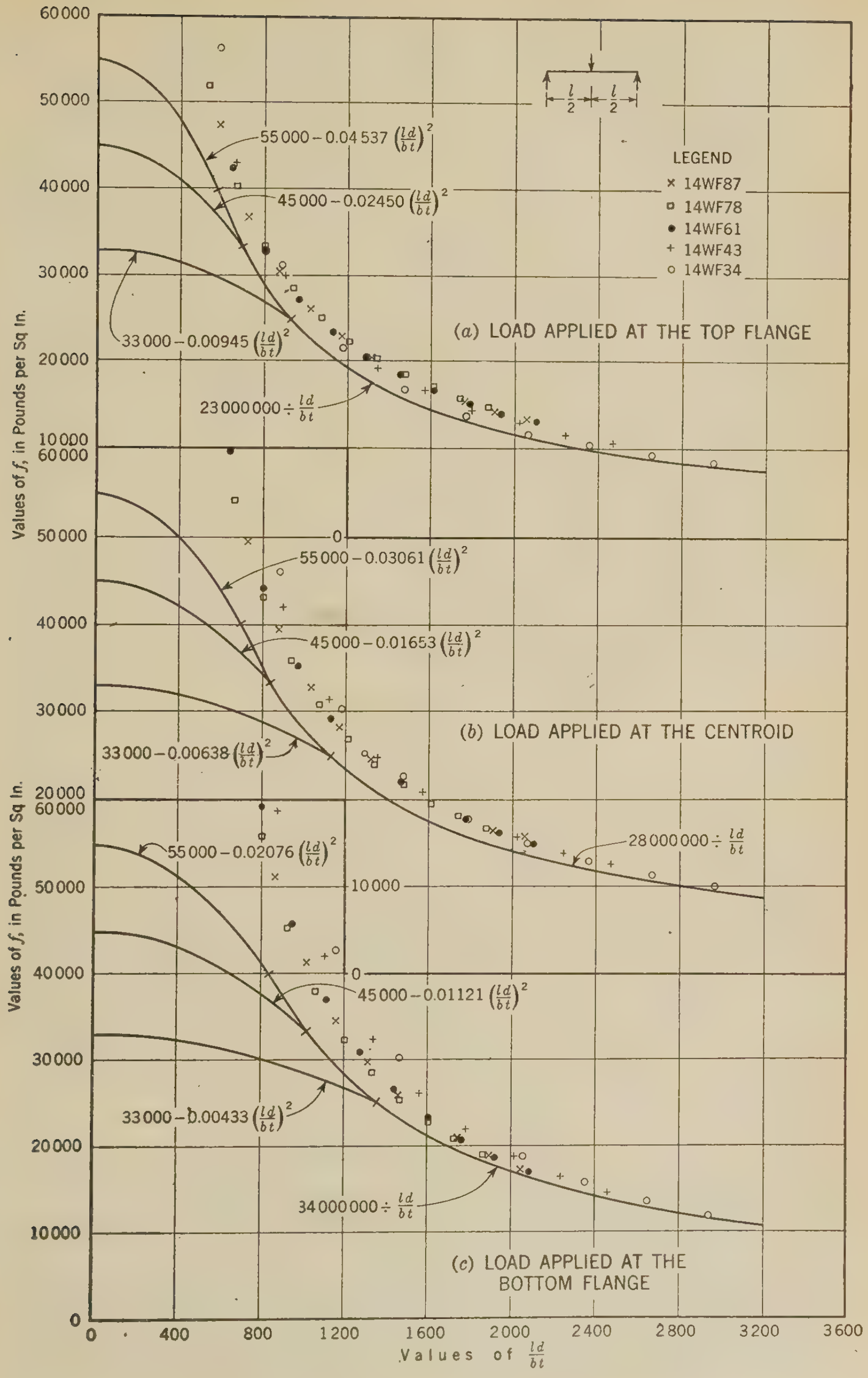

Fig. 5.—Critical Stresses for a Simple Beam, Concentrated Load

are safe for the most severe of the six types of loading considered—the other five types will be of interest for investigators and for special problems. In erection, for example, where long beams frequently have to be picked, and placed temporarily without intermediate lateral support, the weight of the beam itself is the only loading—that is, the condition is that of a uniform load at the centroid. This condition will allow the use of higher failure stress and, therefore, also higher working stress. Other special loadings, which might represent a combination of any of the six loadings, or an intermediate case, would require interpolation.

The five 14-in. wide-flange sections, which had been chosen for Tables 2 and 3, and for which the critical stresses are included in Fig. 2, seem to be characteristic of the entire group of 132 beams; and these sections only will be used in the derivation of failure curves for the remaining five types of loading. The five beams selected all have depths of 14 in. and flange widths varying from $14\frac{1}{2}$ in. to $6\frac{3}{4}$ in.

Using Eq. 4 with the k-values for the five loading conditions from Table 1, the critical stresses are computed and entered in Table 2.

If the $\frac{l\,d}{b\,t}$-values of Table 3 again are taken as abscissas and the critical stresses f in Table 2 are taken as ordinates, curves similar to those in Fig. 2 result. These are shown in Figs. 4 and 5. Fig. 4(a) is a repetition of Fig. 2, showing the calculated critical stresses for the five typical beams used. The failure formulas are listed in Table 4.

PART II. TWO-BAY BEAMS

For a beam with lateral restraint at the middle cross section and a concentrated load P applied at that point, and also for the case of a uniformly loaded beam, the k-values, presented in Table 1(c) have been computed.

In the derivation of the failure formulas, Eqs. 6 and 7, and the working formulas, Eqs. 8 and 9, the k-values of Table 1(b), Col. 5, for a beam loaded with a uniform load at the top flange were used since they were smaller than the k-values for any other loading condition considered. Comparison of the tabulated k-values of Table 1, Col. 5, with those of Table 1(c) indicates that any error induced by using the unsupported length of the beam in the proposed failure and working formulas is on the safe side. Computations were also made for a beam with lateral restraint at the third points, and at the quarter points and center. The results consistently indicated the safety of the assumption that l is the unsupported length.

PART III. PLATE GIRDERS

The foregoing close investigation of rolled beams has been facilitated through the use of handbook values for the torsional bending constant a, the formulas for which had been set up and tested by Inge Lyse and Bruce G. Johnston,[9] Members, ASCE. Torsion formulas for plate girders do not seem to be available, and the following investigation of plate girders, of necessity, will be an

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[9] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 857.

TABLE 4.—FAILURE FORMULAS FOR SIMPLE BEAMS

Load at: (1)	Steel (2)	Short beams (see Figs. 4 and 5) (3)	Transition $\frac{l\,d}{b\,t}$ (4)	Transition f (5)	Long beams (see Figs. 4 and 5) (6)
		(a) UNIFORMLY DISTRIBUTED LOAD			
Top flange........	Carbon	$33{,}000—0.01250\left(\frac{l\,d}{b\,t}\right)^2$	800	25,000	$\frac{20{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000—0.03241\left(\frac{l\,d}{b\,t}\right)^2$	600	33,333	
	Nickel	$55{,}000—0.06000\left(\frac{l\,d}{b\,t}\right)^2$	500	40,000	
Centroid.........	Carbon	$33{,}000—0.00868\left(\frac{l\,d}{b\,t}\right)^2$	960	25,000	$\frac{24{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000—0.02251\left(\frac{l\,d}{b\,t}\right)^2$	720	33,333	
	Nickel	$55{,}000—0.04167\left(\frac{l\,d}{b\,t}\right)^2$	600	40,000	
Bottom flange.....	Carbon	$33{,}000—0.00686\left(\frac{l\,d}{b\,t}\right)^2$	1,080	25,000	$\frac{27{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000—0.01778\left(\frac{l\,d}{b\,t}\right)^2$	810	33,333	
	Nickel	$55{,}000—0.03292\left(\frac{l\,d}{b\,t}\right)^2$	675	40,000	
		(b) CONCENTRATED LOAD AT THE CENTER OF THE SPAN			
Top flange........	Carbon	$33{,}000—0.00945\left(\frac{l\,d}{b\,t}\right)^2$	920	25,000	$\frac{23{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000—0.02450\left(\frac{l\,d}{b\,t}\right)^2$	690	33,333	
	Nickel	$55{,}000—0.04537\left(\frac{l\,d}{b\,t}\right)^2$	575	40,000	
Centroid.........	Carbon	$33{,}000—0.00638\left(\frac{l\,d}{b\,t}\right)^2$	1,120	25,000	$\frac{28{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000—0.01653\left(\frac{l\,d}{b\,t}\right)^2$	840	33,333	
	Nickel	$55{,}000—0.03061\left(\frac{l\,d}{b\,t}\right)^2$	700	40,000	
Bottom flange.....	Carbon	$33{,}000—0.00433\left(\frac{l\,d}{b\,t}\right)^2$	1,360	25,000	$\frac{34{,}000{,}000}{\frac{l\,d}{b\,t}}$
	Silicon	$45{,}000—0.01121\left(\frac{l\,d}{b\,t}\right)^2$	1,020	33,333	
	Nickel	$55{,}000—0.02076\left(\frac{l\,d}{b\,t}\right)^2$	850	40,000	

approximation only. Consider a plate girder with a section such as that in Fig. 6(*a*), for which the moments of inertia are given in Table 5. Assuming that the torsional formulas for rolled beams apply also to plate girders, and that the flange rivets join the flange segments effectively, the torsion constant K

can be computed thus:[10]

$$
\begin{aligned}
2 \times 0.333 \times 13 \times 3^3 &= 234.0\\
2 \times 0.333 \times 5.25 \times 2.5^3 &= 54.7\\
8 \times 0.333 \times 3.5 \times 1^3 &= 9.3\\
8 \times 0.333 \times 1.75 \times 1^3 &= 4.7\\
1 \times 0.333 \times 88 \times 0.5^3 &= 3.7\\
2 \times 0.109 \times 2.15 &= 46.9\\
16 \times 0.10504 \times 1^4 &= -1.7\\
\hline
\text{Torsion constant } K \text{ (in.}^4) &= 351.6
\end{aligned}
$$

TABLE 5.—MOMENT OF INERTIA, PLATE GIRDER IN FIG. 6(*a*)

Section shapes	Number of pieces	Dimensions (inches)	MOMENT OF INERTIA (IN.^{4})	
			I_x	I_y
Outer cover plates..........	Two	20 by 1	107,126	1,333
Inner cover plates..........	Two	20 by 1	103,026	1,333
Angles....................	Four	8 by 8 by 1	137,906	768
Web.......................	One	100 by ½	41,667	1
Total..................	....	....	389,725	3,435

In a report published in 1941, Ingvald E. Madsen, Jun. ASCE, assumes[11] that the torsional constant of a riveted girder should be one third of the *K*-value for an identical solid section, because of rivet slip. If this assumption were used in the foregoing computation, it would reduce the *K*-value of a solid section (which has been computed separately and found to be 428 in.^{4}) to 142.7 in.^{4}, and is thus considerably less than the foregoing value of 351.6 in.^{4} The writer believes that the effect of rivet slip on the small specimens tested by Mr. Madsen is much greater than on full-size plate girders, and he has therefore rejected Mr. Madsen's assumption in favor of the foregoing computations.

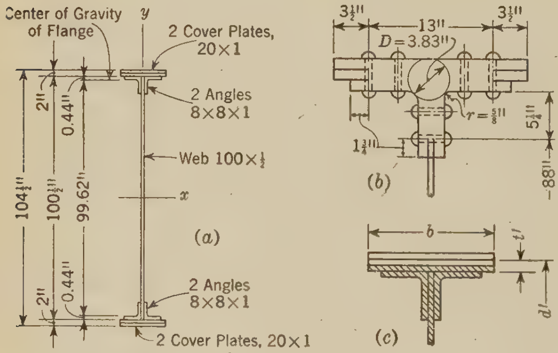

FIG. 6

To determine the torsional bending constant *a*, the formula—[12]

$$a = 0.806\, d_e \sqrt{\frac{I_y}{K}} \qquad \text{(10)}$$

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[10] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 865, Eq. 21.

[11] "Report of Crane Girder Tests," by I. Madsen, *Iron and Steel Engineer*, November, 1941, Section 4, p. 68.

[12] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 869.

—gives $a = 0.806 \times 99.62 \times \sqrt{\frac{3{,}435}{351.6}} = 251.0$ in. and the value of $10{,}000 \frac{I_y d^2}{I_x a^2}$ is $10{,}000 \times \frac{3{,}435 \times 104.5^2}{389{,}725 \times 251.0^2} = 15.28$. In Eq. 10, d_c is the depth of the plate girder between flange centroids.

Since the proposed specification formulas Eqs. 8 and 9 involve the thickness of the flange, t, an equivalent thickness of the flange must be evaluated in the case of plate girders. For rolled beams, t is tabulated in handbooks, but can also be expressed by the formula:

$$t = \frac{6\, I_y}{b^3} \qquad \text{(11)}$$

If b is assumed to be the maximum flange width of the plate girder, Eq. 11 could be used to determine the equivalent average flange thickness of the plate girder of Fig. 6, for which, then, $t = \frac{6 \times 3{,}435}{20^3} = 2.576$ in., and the ratio $\frac{a\, d}{b\, t} = \frac{251.0 \times 104.5}{20 \times 2.576} = 509.1$.

Table 6(*a*) has been compiled from these values. In Table 6, Col. 2 gives the failure stresses of the plate girder up to $\frac{l}{b} = \frac{l}{a} \times \frac{a}{b} = \frac{8 \times 251.0}{20} = 100.4$, for a uniformly distributed load applied at the top flange computed as for Table 2(*a*); Col. 3, the $\frac{l\, d}{b\, t}$-ratio derived from the $\frac{a\, d}{b\, t}$-values; Col. 4, the values of the failure curve of Eq. 7; and Col. 5, the ratio of failure stress to failure curve. Table 6(*a*) shows that the failure stresses of Eq. 7 will be exceeded for $\frac{l\, d}{b\, t}$-ratios greater than about 2,000, or a laterally unsupported girder length of $l = \frac{l\, d}{b\, t} \times \frac{b\, t}{d} = 2{,}000 \frac{20 \times 2.576}{104.5} = 986$ in., or 82.2 ft.

This small overstress for long lengths seems to be immaterial; but, since close investigations and tests are not available, it is proposed to use, instead of Eq. 11, the following formula for the computations of average flange thicknesses of plate girders:

$$t = \frac{5\, I_y}{b^3} \qquad \text{(12)}$$

in which I_y is the moment of inertia of the girder about the y-axis; and b is the maximum flange width. Table 6(*b*) has been prepared to show the results of using Eq. 12 instead of Eq. 11. If the compression flange of a plate girder is different from the tension flange, twice the moment of inertia of the compression flange about the y-axis should be used as I_y in Eq. 12.

Long plate girders usually have cover plates of various lengths, and an average moment of inertia I_y of the girder will have to be computed to express its lateral stiffness. If the cover plates are computed to follow a parabolic moment diagram, with a minimum total cover plate thickness of t (min) at the ends, and a maximum total cover plate thickness t (max) in the middle, then the effective average thickness t' (Fig. 6(c)) will be

$$t' = t(\min) + \tfrac{2}{3}\,[t(\max) - t(\min)] \ldots\ldots\ldots\ldots\ldots (13)$$

This effective average flange thickness t' will have to be used for determining I_y in Eq. 12; and, with the effective depth d' (Fig. 6(c)), the $\frac{l\,d}{b\,t}$-ratio can be established.

TABLE 6.—COMPARISON OF FAILURE STRESS WITH THE FAILURE CURVE FOR THE PLATE GIRDER IN FIG. 6(a), USING DIFFERENT EQUIVALENT FLANGE THICKNESSES, t

$\frac{l}{a}$ (1)	Failure stress, f (2)	(a) $t = 6\,I_y/b^3$ (Eq. 11)			(b) $t = 5\,I_y/b^3$ (Eq. 12)		
		$\frac{l\,d}{b\,t}$ (3)	$\frac{20{,}000{,}000}{\frac{l\,d}{b\,t}}$ (4)	$\frac{\text{Col. 2}}{\text{Col. 4}}$ (5)	$\frac{l\,d}{b\,t}$ (6)	$\frac{20{,}000{,}000}{\frac{l\,d}{b\,t}}$ (7)	$\frac{\text{Col. 2}}{\text{Col. 7}}$ (8)
2	26,000	1,018	19,650	1.323	1,222	16,370	1.588
3	14,200	1,527	13,100	1.084	1,832	10,920	1.300
4	9,800	2,036	9,820	0.998	2,443	8,190	1.197
5	7,560	2,546	7,860	0.962	3,054	6,550	1.154
6	6,200	3,055	6,550	0.947	3,665	5,460	1.136
7	5,290	3,564	5,610	0.943	4,276	4,680	1.130
8	4,630	4,073	4,910	0.943	4,886	4,090	1.132

The computation of the effective average flange thickness would not be necessary for conditions which are generally encountered. Such a computation applies and is of consequence only for long unsupported lengths—as, for example, for the investigation of erection stresses of plate girders temporarily unsupported laterally. It should be noted that the average flange thickness of Eq. 12 should be used for all compound sections, such as cover-plated rolled beams, since a condition results which is similar to that in the plate girder which has been investigated.

PART IV. CONCLUSION

By investigating the strength of steel beams in lateral buckling, it has been shown that the critical stresses of rolled beams can be grouped to follow a good pattern (Fig. 2) and that working formulas can be derived from them. Working formulas (Eqs. 8 and 9) are proposed for inclusion in specifications. For plate girders and other compound sections it is proposed that the same formulas be used with l equal to the unsupported length of the compression flange, d equal to the depth of the section, b equal to the maximum compression flange width, and an average compression flange thickness (see Eq. 12) of $t = 5\,I_y/b^3$. Additional failure formulas for special loading conditions are derived and the treatment of long plate girders is discussed.

For all computations it is assumed that the beams at their supports are held in vertical positions. Beams with eccentrically applied loads are torsional problems and have not been included in this paper.

Commonly used beam sections and plate girders have been discussed. Channels, zees, angles, and other unsymmetrical sections are seldom used as long beams, and it is proposed to allow the same unit stress for them as for symmetrical sections. The deviation of stresses of the proposed formulas from those of other beam formulas is negligible for short beams, and these unsymmetrical sections do not deserve special consideration.

ACKNOWLEDGMENT

The writer wishes to express his appreciation to Jonathan Jones and W. H. Jameson, Members, ASCE, for their assistance in preparing this paper.

APPENDIX I—DERIVATION OF *k*-VALUES

SIMPLE BEAMS; CONCENTRATED LOAD AT CENTER

The solution of Eq. 3 involves the evaluation of quantities of the angle of twist β. This can most readily be done by expressing β as a series:[13]

$$\beta = a_1 \cos \frac{\pi z}{l} + a_3 \cos \frac{3 \pi z}{l} + \cdots \tag{14}$$

Only the first two terms of the series are of sufficient magnitude to be of importance. Introducing $y = \frac{a_3}{a_1}$, the following values are obtained:

$$\beta^2_o = a^2_1 (1 + y)^2 \tag{15a}$$

$$\int_0^{l/2} \beta^2 \left(\frac{l}{2} - z\right)^2 dz = a^2_1 l^3 \frac{1}{8} \left(\frac{1}{6} y^2 + \frac{1}{9 \pi^2} y^2 + \frac{5}{2 \pi^2} y + \frac{1}{6} + \frac{1}{\pi^2}\right) \tag{15b}$$

$$\int_0^{l/2} \left(\frac{d\beta}{dz}\right)^2 dz = a^2_1 \frac{1}{l} \frac{\pi^2}{4} (1 + 9 y^2) \tag{15c}$$

and

$$\int_0^{l/2} \left(\frac{d^2\beta}{dz^2}\right)^2 dz = a^2_1 \frac{1}{l^3} \frac{\pi^4}{4} (1 + 81 y^2) \tag{15d}$$

Substituting Eqs. 15 in Eq. 3:

$$P = \frac{4 E I_y d}{l^3} \left\{ \mp (1 + y)^2 + \sqrt{(1 + y)^4 + \frac{\pi^2}{8} \left[\frac{y^2}{6} + \frac{y^2}{9 \pi^2} + \frac{5 y}{2 \pi^2} + \frac{1}{6} + \frac{1}{\pi^2}\right] \times \left[\frac{l^2}{a^2} (1 + 9 y^2) + \pi^2 (1 + 81 y^2)\right]} \right\} \div \left[\frac{y^2}{6} + \frac{y^2}{9 \pi^2} + \frac{5y}{2 \pi^2} + \frac{1}{6} + \frac{1}{\pi^2}\right] \tag{16}$$

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[13] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 266, Eq. (*f*).

Eq. 16 will now be transformed into an equation in unit stress f, such that:

$$f = \frac{P\,l}{4} \times \frac{d}{2\,I_x} \quad \text{.......................} \quad (17)$$

for any beam, in which I_x is the moment of inertia about the x-axis. If Eq. 17 is written:

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{2}\,\frac{P\,l^3}{4\,E\,I_y\,d} \quad \text{.................} \quad (18)$$

and, if also the substitution—

$$A = \frac{1}{6}\,y^2 + \frac{1}{9\,\pi^2}\,y^2 + \frac{5}{2\,\pi^2}\,y + \frac{1}{6} + \frac{1}{\pi^2} \quad \text{..............} \quad (19)$$

—is made, the unit stress becomes

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{2\,A}$$

$$\times \left\{ \mp (1 + y)^2 + \sqrt{(1 + y)^4 + \frac{\pi^2}{8}\,A \left[\frac{l^2}{a^2}\,(1 + 9\,y^2) + \pi^2\,(1 + 81\,y^2) \right]} \right\} \quad \text{..} \quad (20)$$

or in its general form, Eq. 4, in which k varies because of the introduction of $\frac{P\,d}{4}\,\beta^2_o$ in Eq. 2. For a concentrated load applied, respectively: At the top flange—

$$k = \frac{1}{2\,A} \left\{ -\,(1 + y)^2 \right.$$

$$\left. + \sqrt{(1 + y)^4 + \frac{\pi^2}{8}\,A \left[\frac{l^2}{a^2}\,(1 + 9\,y^2) + \pi^2\,(1 + 81\,y^2) \right]} \right\} \quad \text{..} \quad (21a)$$

at the centroid—

$$k = \frac{\pi}{4\,A} \sqrt{\frac{A}{2} \left[\frac{l^2}{a^2}\,(1 + 9\,y^2) + \pi^2\,(1 + 81\,y^2) \right]} \quad \text{.........} \quad (21b)$$

and, at the bottom flange—

$$k = \frac{1}{2\,A} \left\{ (1 + y)^2 \right.$$

$$\left. + \sqrt{(1 + y)^4 + \frac{\pi^2}{8}\,A \left[\frac{l^2}{a^2}\,(1 + 9\,y^2) + \pi^2\,(1 + 81\,y^2) \right]} \right\} \quad \text{..} \quad (21c)$$

The unit stress determined from Eq. 4 is a variable depending on the value of k. Since the basic problem is to determine the least value of f which will cause the beam to buckle (that is, "the critical stress"), it is necessary to determine, for each of a series of possible ratios l/a the value of y that will make the value of k a minimum. These critical values of y have been computed, and are given in Table 7(a). The corresponding computed values of k for these critical values of y have been tabulated in Table 1(a).

SIMPLE BEAMS; UNIFORMLY DISTRIBUTED LOAD

Assuming a uniformly distributed load of w per linear unit, as in Fig. 7(a), equations similar to those for concentrated load can be set up. For work done by the external forces, assumed to be applied at the centroid, a method analogous to that used by Professor Timoshenko[14] can be used. Consider an element of the longitudinal axis at point D and consider the cross section mn at that point fixed. Then, because of bending of this element in the (ξ, ζ)-plane, the end of the beam, where the reaction $(= w\,l/2)$ is applied, describes an infinitely small arc,

TABLE 7.—CRITICAL VALUES OF y FOR USE IN COMPUTATIONS OF k (SEE TABLE 1)

α	SIMPLE BEAMS						(c) TWO-BAY BEAMS			
	(a) Concentrated Load, Eq. 21			(b) Uniform Load, Eq. 36			Concentrated Load, Eq. 43	Uniform Load, Eq. 50		
	Top flange	Centroid	Bottom flange	Top flange	Centroid	Bottom flange		Top flange	Centroid	Bottom flange
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	0.014	0.008	−0.005	0.002	0.005	0.010	−0.052	−0.030	−0.036	−0.046
3	0.016	0.010	−0.004	0.004	0.006	0.012	−0.066	−0.038	−0.046	−0.054
4	0.020	0.014	−0.001	0.005	0.008	0.014	−0.080	−0.046	−0.054	−0.064
5	0.024	0.016	0.002	0.006	0.010	0.016	−0.092	−0.056	−0.064	−0.072
6	0.028	0.020	0.005	0.008	0.012	0.018	−0.104	−0.064	−0.072	−0.080
7	0.032	0.022	0.008	0.010	0.014	0.021	−0.116	−0.072	−0.080	−0.088
8	0.036	0.026	0.012	0.012	0.016	0.023	−0.126	−0.078	−0.086	−0.094
9	0.038	0.028	0.015	0.014	0.018	0.024	−0.132	−0.084	−0.090	−0.098
10	0.042	0.031	0.018	0.015	0.020	0.026	−0.140	−0.088	−0.096	−0.102
11	0.044	0.034	0.020	0.016	0.022	0.028	−0.146	−0.092	−0.098	−0.106
12	0.046	0.036	0.022	0.018	0.023	0.029	−0.148	−0.096	−0.102	−0.108
13	0.048	0.038	0.026	0.020	0.024	0.030	−0.152	−0.098	−0.104	−0.110
14	0.048	0.040	0.028	0.021	0.025	0.031	−0.156	−0.102	−0.106	−0.112

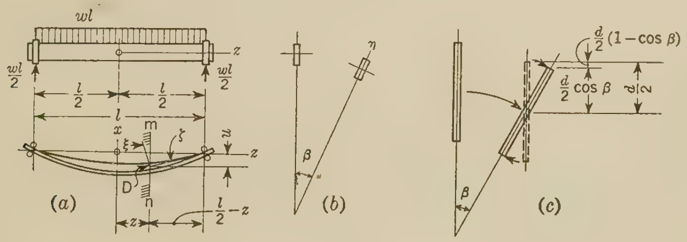

FIG. 7

$$ds = \frac{d^2u}{dz^2}\left(\frac{l}{2} - z\right)dz \qquad (22a)$$

in the (ξ, ζ)-plane, the vertical component of which is

$$ds_v = \beta\,\frac{d^2u}{dz^2}\left(\frac{l}{2} - z\right)dz \qquad (22b)$$

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[14] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 253.

In Eqs. 22, u is the deflection of any element. The uniform load between point D and the right-hand end of the beam can be considered concentrated at its center of gravity, at a distance $\frac{1}{2}(l/2 - z)$ from point D, and the vertical component of movement, due to the bending of the element at point D, is

$$ds_D = \beta \frac{d^2u}{dz^2} \frac{1}{2} \left(\frac{l}{2} - z \right) dz \quad \text{(22c)}$$

The work done by the external forces to the right of point D, due to bending of the element at point D caused by beam buckling, is

$$V_D = \frac{wl}{2} \beta \frac{d^2u}{dz^2} \left(\frac{l}{2} - z \right) dz - w \left(\frac{l}{2} - z \right) \beta \frac{d^2u}{dz^2} \frac{1}{2} \left(\frac{l}{2} - z \right) dz$$

$$= \frac{w}{2} \beta \frac{d^2u}{dz^2} \left(\frac{l^2}{4} - z^2 \right) dz \quad \text{(23)}$$

and the work done on the full length of the beam is twice the summation of these forces from 0 to $l/2$:

$$V_1 = w \int_0^{l/2} \beta \frac{d^2u}{dz^2} \left(\frac{l^2}{4} - z^2 \right) dz \quad \text{(24)}$$

If the load is applied at the top or bottom flanges, instead of along the centroid of the beam, the external work done by the load is increased or decreased by the movement of the flanges caused by the beam rotation.[15] Considering the beam of Fig. 7(a) as a simple beam between its reactions, an element at the point of load application at the top or the bottom of the beam (Fig. 7(c)) is either lowered or raised a distance $\frac{d}{2}(1 - \cos \beta)$ or approximately $\frac{d}{4}\beta^2$. The work done by the element then is $\pm \frac{wd}{4} \beta^2 \, dz$, and the work done along the full length of the beam,

$$V_2 = \pm \frac{wd}{2} \int_0^{l/2} \beta^2 \, dz \quad \text{(25)}$$

The strain energy of lateral bending, torsion, and flange twist can be expressed in the same form as for a beam with concentrated load in the middle,[16] thus:

$$w \int_0^{l/2} \beta \frac{d^2u}{dz^2} \left(\frac{l^2}{4} - z^2 \right) dz \pm \frac{wd}{2} \int_0^{l/2} \beta^2 \, dz$$

$$= B \int_0^{l/2} \left(\frac{d^2u}{dz^2} \right)^2 dz + C \int_0^{l/2} \left(\frac{d\beta}{dz} \right)^2 dz + \frac{Bd^2}{4} \int_0^{l/2} \left(\frac{d^2\beta}{dz^2} \right)^2 dz \quad \text{(26)}$$

Equations corresponding to those for a concentrated load, given by Professor Timoshenko[17] can be written for a uniformly distributed load, for which the vertical moment is

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[15] "The Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 254.

[16] *Ibid.*, p. 266, Eq. (d).

[17] *Ibid.*, p. 251.

$$M_x = -\frac{w\,l}{2}\left(\frac{l}{2} - z\right) + \frac{w}{2}\left(\frac{l}{2} - z\right)^2 = -\frac{w}{2}\left(\frac{l^2}{4} - z^2\right) \quad \ldots .(27a)$$

This moment, projected to the η-axis, is

$$M_\eta = \frac{w}{2}\left(\frac{l^2}{4} - z^2\right)\beta \quad \ldots\ldots .(27b)$$

and the differential equation for the buckled beam is

$$B\,\frac{d^2u}{dz^2} = \frac{w}{2}\left(\frac{l^2}{4} - z^2\right)\beta \quad \ldots\ldots .(27c)$$

Using the value of $\frac{d^2u}{dz^2}$ of Eq. 27*c*, the equation for uniformly distributed load, corresponding to Eq. 2, will be:

$$\frac{w^2}{4\,E\,I_y}\int_0^{l/2} \beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz \pm \frac{w\,d}{2}\int_0^{l/2} \beta^2\,dz - \frac{E\,I_y}{4}\,\frac{l^2}{a^2}\,\frac{d^2}{l^2}\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz - \frac{E\,I_y}{4}\,d^2\int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \quad ..(28)$$

This equation may be solved for the loading $w\,l$:

$$w\,l = E\,I_y\,dl\left\{\mp\int_0^{l/2}\beta^2\,dz + \sqrt{\int_0^{l/2}(\beta^2\,dz)^2 + \int_0^{l/2}\beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz\left[\frac{l^2}{a^2}\,\frac{1}{l^2}\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz + \int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz^2\right]}\right\} \div \left[\int_0^{l/2}\beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz\right] \quad ..(29)$$

With the same definitions for β and y as for concentrated load and the additional solutions for the integrals—

$$\int_0^{l/2}\beta^2\,dz = a^2{}_1\,\frac{l}{4}\,(1 + y^2) \quad \ldots\ldots .(30a)$$

and

$$\int_0^{l/2}\beta^2\left(\frac{l^2}{4} - z^2\right)^2 dz = a^2{}_1\,l^5\,\frac{1}{8}\left(\frac{1}{15}\,y^2 + \frac{1}{27\,\pi^4}\,y^2 + \frac{45}{8\,\pi^4}\,y + \frac{1}{15} + \frac{3}{\pi^4}\right) \quad ..(30b)$$

—the loading may be expressed thus:

$$w\,l = \frac{2\,E\,I_y\,d}{l^3}\left\{\mp(1+y^2)\right.$$

$$+\sqrt{(1+y^2)^2+\frac{\pi^2}{2}\left[\frac{y^2}{15}+\frac{y^2}{27\,\pi^4}+\frac{45\,y}{8\,\pi^4}+\frac{1}{15}+\frac{3}{\pi^4}\right]}$$

$$\overline{\times\left[\frac{l^2}{a^2}(1+9\,y^2)+\pi^2(1+81\,y^2)\right]}\left.\vphantom{\frac{1}{1}}\right\}\div\left[\frac{y^2}{15}+\frac{y^2}{27\,\pi^4}\right.$$

$$\left.+\frac{45\,y}{8\,\pi^4}+\frac{1}{15}+\frac{3}{\pi^4}\right]\ldots.(31)$$

Eq. 31 may be transformed into an equation in unit stress f, by a process similar to that used in the case of concentrated load (Eq. 17):

$$f = \frac{w\,l^2}{8}\times\frac{d}{2\,I_x}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

or

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{8}\,\frac{w\,l\,l^3}{2\,E\,I_y\,d}\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

and, with the substitution—

$$A = \frac{1}{15}\,y^2+\frac{1}{27\,\pi^4}\,y^2+\frac{45}{8\,\pi^4}\,y+\frac{1}{15}+\frac{3}{\pi^4}\ldots\ldots\ldots\ldots(34)$$

—the unit stress becomes (compare Eq. 20):

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{8\,A}\left\{\mp(1+y^2)\right.$$

$$\left.+\sqrt{(1+y^2)^2+\frac{\pi^2}{2}\,A\left[\frac{l^2}{a^2}(1+9\,y^2)+\pi^2(1+81\,y^2)\right]}\right\}..(35)$$

Eq. 35 has the general form of Eq. 4. Values of k are, for a uniformly distributed load applied, respectively: At the top flange,

$$k = \frac{1}{8\,A}\left\{-(1+y^2)\right.$$

$$\left.+\sqrt{(1+y^2)^2+\frac{\pi^2}{2}\,A\left[\frac{l^2}{a^2}(1+9\,y^2)+\pi^2(1+81\,y^2)\right]}\right\}\ldots.(36a)$$

at the centroid,

$$k = \frac{\pi}{8\,A}\sqrt{\frac{A}{2}\left[\frac{l^2}{a^2}(1+9\,y^2)+\pi^2(1+81\,y^2)\right]}\ldots\ldots\ldots(36b)$$

and, at the bottom flange,

$$k = \frac{1}{8\,A}\left\{(1 + y^2) + \sqrt{(1 + y^2)^2 + \frac{\pi^2}{2}\,A\left[\frac{l^2}{a^2}(1 + 9\,y^2) + \pi^2(1 + 81\,y^2)\right]}\right\} \dots\dots (36c)$$

To determine the least value of f in Eq. 4, the critical values of y for a given ratio of l/a have been computed, and are listed in Table 7(b). The corresponding computed values of k for these critical values of y are given in Table 1(b).

Two-Bay Beams; Concentrated Load at Center

For a beam with lateral restraint at the middle cross section and with a load P applied at that point (Fig. 8), an equation similar to Eq. 2 can be written:

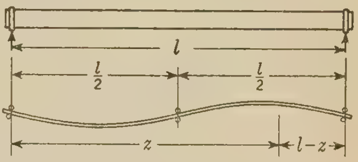

Fig. 8

$$\frac{P^2}{4\,E\,I_y}\int_{l/2}^{l}\beta^2\,(l - z)^2\,dz - \frac{E\,I_y}{8}\,\frac{l^2\,d^2}{a^2\,l^2}\int_0^l\left(\frac{d\beta}{dz}\right)^2 dz - \frac{E\,I_y}{8}\,d^2\int_0^l\left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \dots\dots (37)$$

Note that the term in Eq. 2, which takes into account the external work done by the load P due to twisting of the middle cross section, disappears for Eq. 37. Solved for P, Eq. 37 gives

$$P = E\,I_y\,d\,\sqrt{\frac{\dfrac{l^2}{a^2}\,\dfrac{1}{l^2}\displaystyle\int_0^l\left(\frac{d\beta}{dz}\right)^2 dz + \int_0^l\left(\frac{d^2\beta}{dz^2}\right)^2 dz}{2\displaystyle\int_{l/2}^{l}\beta^2\,(l - z)^2\,dz}} \dots\dots\dots\dots (38)$$

in which

$$\beta = a_2 \sin\frac{2\,\pi\,z}{l} + a_4 \sin\frac{4\,\pi\,z}{l} \dots\dots\dots\dots (39a)$$

and

$$y = \frac{a_4}{a_2} \dots\dots\dots\dots (39b)$$

The integrals in Eq. 38 are solved thus:

$$\int_0^l\left(\frac{d\beta}{dz}\right)^2 dz = a_2^2\,\frac{1}{l}\,2\,\pi^2\,(1 + 4\,y^2) \dots\dots\dots\dots (40a)$$

$$\int_0^l\left(\frac{d^2\beta}{dz^2}\right)^2 dz = a_2^2\,\frac{1}{l^3}\,8\,\pi^4\,(1 + 16\,y^2) \dots\dots\dots\dots (40b)$$

and

$$\int_{l/2}^{l} \beta^2 (l - z)^2 \, dz = a^2{}_2 \, l^3 \frac{1}{2} \left(\frac{1}{24} y^2 - \frac{5}{64 \pi^2} y^2 - \frac{4}{9 \pi^2} y + \frac{1}{24} - \frac{1}{16 \pi^2} \right) \ldots (40c)$$

which values, in Eq. 38, yield:

$$P = \frac{E I_y d}{l^3} 2 \pi \sqrt{\frac{\frac{l^2}{a^2} (1 + 4 y^2) + 4 \pi^2 (1 + 16 y^2)}{2 \left(\frac{1}{24} y^2 - \frac{5}{64 \pi^2} y^2 - \frac{4}{9 \pi^2} y + \frac{1}{24} - \frac{1}{16 \pi^2} \right)}} \ldots (41)$$

and, therefore,

$$f = \frac{I_y d^2}{I_x a^2} E \frac{a^2}{l^2} \frac{\pi}{4} \sqrt{\frac{\frac{l^2}{a^2} (1 + 4 y^2) + 4 \pi^2 (1 + 16 y^2)}{2 \left(\frac{1}{24} y^2 - \frac{5}{64 \pi^2} y^2 - \frac{4}{9 \pi^2} y + \frac{1}{24} - \frac{1}{16 \pi^2} \right)}} \ldots (42)$$

To express the unit stress, f, in terms of the unsupported length of the beam instead of the full length of the beam, as in Eq. 42, l in Eq. 42 should be multiplied by 2. The value of f is then expressed by Eq. 4 in which

$$k = \frac{\pi}{8} \sqrt{\frac{\frac{l^2}{a^2} (1 + 4 y^2) + \pi^2 (1 + 16 y^2)}{2 \left(\frac{1}{24} y^2 - \frac{5}{64 \pi^2} y^2 - \frac{4}{9 \pi^2} y + \frac{1}{24} - \frac{1}{16 \pi^2} \right)}} \ldots \ldots (43)$$

in which l equals the unsupported, or half, length of beam. The critical values of y and of k as determined by Eq. 43 have been computed and are tabulated in Tables 7(c) and 1(c).

TWO-BAY BEAMS; UNIFORMLY DISTRIBUTED LOAD

In the case of a uniformly loaded beam with lateral restraint at the middle cross section (Fig. 8), by the method previously used for the laterally unsupported beam, an equation similar to Eq. 28 can be written:

$$\frac{w^2}{8 E I_y} \int_0^l \beta^2 z^2 (l - z)^2 \, dz \pm \frac{w d}{4} \int_0^l \beta^2 \, dz$$

$$- \frac{E I_y}{8} \frac{l^2 d^2}{a^2 l^2} \int_0^l \left(\frac{d\beta}{dz} \right)^2 dz - \frac{E I_y}{8} d^2 \int_0^l \left(\frac{d^2\beta}{dz^2} \right)^2 dz = 0 \ldots (44)$$

Eq. 44 may be solved for the loading $w l$:

$$w l = E I_y d l \left\{ \mp \int_0^l \beta^2 \, dz \right.$$

$$\left. + \sqrt{\int_0^l (\beta^2 \, dz)^2 + \int_0^l \beta^2 z^2 (l - z)^2 \, dz \left[\frac{l^2}{a^2} \frac{1}{l^2} \int_0^l \left(\frac{d\beta}{dz} \right)^2 dz + \int_0^l \left(\frac{d^2\beta}{dz^2} \right)^2 dz \right]} \right\}$$

$$\div \left[\int_0^l \beta^2 z^2 (l - z)^2 \, dz \right] \ldots (45)$$

With solutions for integrals, not solved before:

$$\int_0^l \beta^2\,dz = a^2{}_2\,l\,\tfrac{1}{2}\,(1 + y^2) \ldots\ldots\ldots\ldots\ldots \quad (46a)$$

and

$$\int_0^l \beta^2 z^2 (l - z)^2\,dz = a^2{}_2\,l^5\,\frac{1}{8}\left(\frac{2}{15}\,y^2 - \frac{1}{2\,\pi^2}\,y^2 + \frac{63}{128\,\pi^4}\,y^2 - \frac{320}{27\,\pi^4}\,y + \frac{2}{15} + \frac{3}{8\,\pi^4}\right) \ldots \quad (46b)$$

Eq. 45 becomes

$$w\,l = \frac{4\,E\,I_y\,d}{l^3}\left\{\mp (1 + y^2) + \sqrt{(1 + y^2)^2 + \pi^2\left[\frac{2\,y^2}{15} - \frac{y^2}{2\,\pi^2} + \frac{63\,y^2}{128\,\pi^4} - \frac{320\,y}{27\,\pi^4} + \frac{2}{15} + \frac{3}{8\,\pi^4}\right] \times \left[\frac{l^2}{a^2}(1 + 4\,y^2) + 4\,\pi^2\,(1 + 16\,y^2)\right]}\right\}$$
$$\div \left[\frac{2\,y^2}{15} - \frac{y^2}{2\,\pi^2} + \frac{63\,y^2}{128\,\pi^4} - \frac{320\,y}{27\,\pi^4} + \frac{2}{15} + \frac{3}{8\,\pi^4}\right] \ldots \quad (47)$$

and, with—

$$A = \frac{2}{15}\,y^2 - \frac{1}{2\,\pi^2}\,y^2 + \frac{63}{128\,\pi^4}\,y^2 - \frac{320}{27\,\pi^4}\,y + \frac{2}{15} + \frac{3}{8\,\pi^4} \ldots\ldots \quad (48)$$

—the formula for stress becomes:

$$f = \frac{I_y\,d^2}{I_x\,a^2}\,E\,\frac{a^2}{l^2}\,\frac{1}{4\,A}\left\{\mp (1 + y^2) + \sqrt{(1 + y^2)^2 + \pi^2\,A\left[\frac{l^2}{a^2}(1 + 4\,y^2) + 4\,\pi^2\,(1 + 16\,y^2)\right]}\right\} \ldots \quad (49)$$

If each l in Eq. 49 is multiplied by 2, to express the unit stress, f, in terms of the unsupported length instead of the full length of the beam, as was done for the concentrated load (Eq. 42), then the value of f for uniformly distributed load is expressed by Eq. 4 in which, with the load at the top flange,

$$k = \frac{1}{16\,A}\left\{-\,(1 + y^2) + \sqrt{(1 + y^2)^2 + 4\,\pi^2\,A\left[\frac{l^2}{a^2}(1 + 4\,y^2) + \pi^2\,(1 + 16\,y^2)\right]}\right\} \ldots \quad (50a)$$

at the centroid,

$$k = \frac{\pi}{8\,A}\sqrt{A\left[\frac{l^2}{a^2}(1 + 4\,y^2) + \pi^2(1 + 16\,y^2)\right]} \quad\ldots\ldots\ldots(50b)$$

and, at the bottom flange,

$$k = \frac{1}{16\,A}\left\{(1 + y^2) + \sqrt{(1 + y^2)^2 + 4\,\pi^2\,A\left[\frac{l^2}{a^2}(1 + 4\,y^2) + \pi^2(1 + 16\,y^2)\right]}\right\} \quad\ldots(50c)$$

The critical values of y and the corresponding values of k have been computed and are given in Tables 7(*c*) and 1(*c*).

APPENDIX II. NOTATION

The following letter symbols, used in the paper and in its discussions, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932. As nearly as possible, consistent with standard practice, they also follow the nomenclature used by Professor Timoshenko:[2]

A = a substitution factor; for simple beams with a concentrated load at the center, A is defined by Eq. 19; for simple beams with a uniformly distributed load, A is defined by Eq. 34; and, for a two-bay beam with a uniformly distributed load, A is defined by Eq. 48;
a = a torsional bending constant; also where defined, a_1, a_2, a_3, a_4 = maximum ordinates of sine curves (Eqs. 14 and 39);
B = transverse flexural rigidity of a beam = $E\,I_y$;
b = width of compression flange of beam or girder;
C = torsional rigidity of beam = $E\,I_y\,d^2/(4\,a^2)$;
D = diameter of an inscribed circle in a flange section;
d = depth of a beam or girder:
    d' = effective depth of a plate girder (see Fig. 6);
    d_c = depth of a plate girder between flange centroids;
E = modulus of elasticity;
f = unit stress;
I = moment of inertia about an axis denoted by an appropriate subscript;
K = a torsion constant;

k = a substitution factor; for concentrated loads on a simple beam, k is defined by Eqs. 21; for uniformly distributed loads on a simple beam, k is defined by Eqs. 36; for a concentrated load on a two-bay beam k is defined by Eq. 43; and, for uniformly distributed loads on a two-bay beam, k is defined by Eqs. 50;

l = length; in general, l denotes "the laterally unsupported length of the compression flange," with exceptions as described in the text;

M = moment, with subscripts indicating the plane of reference;

P = a concentrated load;

r = radius; r_f = fillet radius;

s = length along any arc; ds = an increment of s, with appropriate subscript;

t = thickness of compression flange:
- t' = effective average thickness of cover plates (Eq. 14);
- t(max) = maximum cover-plate thickness;
- t(min) = minimum cover-plate thickness;

u = deflection of any element;

V = work, with subscripts as defined in the text;

w = load per unit length of span;

x = a coordinate;

y = a coordinate; also, where defined, $y = \frac{a_3}{a_1}$ and $y = \frac{a_4}{a_2}$;

z = a coordinate;

β = angular deflection of a beam at any point; β_o = angle of twist of the beam at its center line;

ζ = a coordinate in the (ξ, ζ, η)-system;

η = a coordinate in the (ξ, ζ, η)-system; and

ξ = a coordinate in the (ξ, ζ, η)-system.

DISCUSSION

George Winter,[18] M. ASCE.—It has been shown repeatedly and convincingly that the generally accepted slenderness ratio for unsupported beams, l/b, is by no means a real criterion for their lateral strength.[19] The great merit of this paper is that the author has succeeded in developing a sufficiently simple set of formulas for inclusion in design specifications, all depending on the compact parameter $\frac{l\,d}{b\,t}$. These formulas (Eqs. 6 to 8), however, are not as general as may appear from the paper, and sometimes they result in greatly misleading and uneconomical values. Furthermore, no rational justification is given in the paper for this particular parameter, except that it happens to fit rather satisfactorily the stress magnitudes obtained from the exact formulas.

It is the intent of this discussion to show in a rigorous manner: (*a*) Why it is this particular parameter and no other which gives satisfactory results for the author's beams; and (*b*) the limitations of the proposed formulas. Finally, for beams to which Mr. de Vries' equations are found not to be applicable, another simple formula is proposed to supplement those given in the paper.

Expressions for critical loads and stresses of unsupported beams have been published before. In particular, the writer,[20] in collaboration with the late Lt. R. K. Schrader,[21] Jun. ASCE, published in 1941 a set of expressions for critical loads and moments, which not only covers Mr. de Vries' cases but is more general and also deals with additional types of loading. The only difference in Mr. de Vries' treatment is that he accounts for the second term in the Fourier expansion of the elastic curve, whereas the writer was satisfied with the first term. The negligible influence of this second term can be noted not only in the close agreement of the values in both papers, but also in the fact that the values for y in Table 7 are all rather small as compared with unity.

In a subsequent paper the writer gave the following formula for the buckling stress of such beams:

$$f = \frac{E\,\pi^2}{2\left(\frac{l}{d}\right)^2}\sqrt{\left(\frac{I_y}{2\,I_x}\right)^2 + \frac{K\,I_y}{2(1+\nu)\,I^2_x}\left(\frac{l}{\pi\,d}\right)^2} \quad \ldots\ldots\ldots\ldots(51)$$

in which ν is Poisson's ratio and K is the torsional constant of the beams.[22] He also showed that Eq. 51, derived for pure bending, can be applied satisfactorily to most practical cases of transverse loading[23]—a finding which is identical with Mr. de Vries' analysis of various loading conditions.

The reason for, and the limitations of, the author's parameter, $\frac{l\,d}{b\,t}$, can be derived rigorously from Eq. 51. The buckling stress depends on the two quantities appearing under the radical. It so happens, however, that for most rolled and built-up sections the first term is negligibly small as compared with the second. For example, for the three I-beams (24I120, 3I5.7, and 12I55), with $l/d = 30$, the values of these two terms are, respectively, 0.00020 and 0.0037, 0.0085 and 0.068, and 0.00073 and 0.009. Therefore, in a practical design formula, the first term in Eq. 51 is negligible for such beams, particularly since both are included under the radical. Neglecting this term, the following simplification can be introduced:

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[19] "Strength of Slender Beams," by George Winter, *Transactions*, ASCE, Vol. 109, 1944, p. 1321.

[20] "Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending," by George Winter, *ibid.*, Vol. 108, 1943, p. 247.

[21] *Ibid.*, p. 261.

[22] "Strength of Slender Beams," by George Winter, *Transactions*, ASCE, Vol. 109, 1944, p. 1334, Eq. 41.

[23] *Ibid.*, pp. 1339 and 1340.

In view of the usual shape of rolled and built-up I-beams, the web contributes very little to I_x, I_y', and K. Consequently, the following approximate expressions can be used for these three quantities—

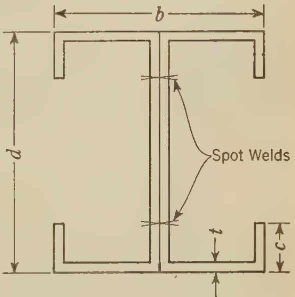
$$I_x = 2\,b\,t\left(\frac{d}{2}\right)^2; \qquad I_y = \frac{2\,t\,b^3}{12}; \qquad \text{and} \quad K = \tfrac{2}{3}\,b\,t^3 \ldots\ldots\ldots (52)$$

Substitution of these values in Eq. 51, with the first term under the radical equal to zero, results in

$$f = \frac{19{,}500{,}000}{\dfrac{l\,d}{b\,t}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

Eq. 53 differs from Eq. 7 by only 2.5%.

It is thereby established that Mr. de Vries' approximate formula is rigorously justified provided that the first term under the radical in Eq. 51 is negligible as compared with the second term. This condition will be true for all beams whose flange thickness is not too small as compared to the width (to about $b/t = 20$). For beams with still thinner flanges the torsional constant K (which is proportional to t^3) decreases rapidly to such an extent that the second of these two terms approaches the order of magnitude of the first. For example (to consider an extreme case) for 6WF15.5, with $l = 20$ ft, the magnitude of these two terms, in order, is 0.030 and 0.071. Although the first term is still seen to be smaller than the second, it is no longer of negligible magnitude. Indeed, the buckling stress from Eq. 51, for $l = 20$ ft, is 27,000 lb per sq in., whereas Eq. 7 results in only 21,800 lb per sq in. Beams of this character, however, are used infrequently except for relatively small structures with light loading.



Beam	t	d	b	c
A	0.075	6.0	5.0	0.75
B	0.050	8.0	3.0	0.60

FIG. 9—LIGHT-GAGE STEEL BEAMS

However, for structures of this kind new types of sections are coming into increasingly widespread use. Such members are fabricated from sheet steel by cold-forming and, usually, by spot-welding.[24] Sections of this type are manufactured at present by a number of specialized firms in that field and, in the near future, will become available commercially on a wide scale. Sheet steel I-beams are frequently made up from two C-channels spot-welded back to back as shown in Fig. 9. In such beams the thickness-width ratio of the flange is ordinarily of the order of b/t = from 25 to 100. It must be expected, therefore, that in such beams the first of the two terms in Eq. 51 is no longer of negligible magnitude. Indeed, for the two beams whose dimensions are given in Fig. 9, with $l/d = 30$, the values of the two terms, in order, are 0.019 and 0.0034 and 0.00049 and 0.00018. The situation here is the reverse of that found for rolled sections; and, consequently, Eq. 7, if applied to these beams, must result in stresses which err very much on the low (conservative) side.

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[24] "Light-Gage Steel for Peacetime Building," by Milton Male, *Engineering News-Record*, October 18, 1945, p. 525.

Since the simple parameter $\frac{l\,d}{b\,t}$ cannot be used for such light-gage I-beams, it will naturally be asked how lateral strength should be determined in the design of sections of this kind. For this purpose Eq. 51 can be used as it is. Most designers, however, will consider it too cumbersome for everyday use. To arrive at a sufficiently simple expression, essentially the same kind of simplification can be used as was employed for rolled sections. It will be noted from the foregoing data that, for light-gage beams, the first term under the radical in Eq. 51 is several times larger than the second. It appears, therefore, that a formula which discards this second term, gives a result not too different from the accurate value of Eq. 51.

(The situation for beam A (Fig. 9) is much more favorable in this respect than that for beam B. However, sections of such extreme shape as beam B will scarcely be suitable for beams, since the low ratio of b/d makes such use very uneconomical. Beam B was chosen as an extreme case to show the maximum discrepancies to be expected from the formula that follows, whereas beam A is probably rather close to average dimension of practical beams.)

For light-gage beams, then, if the second term of Eq. 51 is discarded, the simple formula for the buckling stress is obtained:

$$f = \frac{74{,}000{,}000}{\left(\frac{l}{d}\right)^2} \frac{I_y}{I_x} \qquad \text{(54a)}$$

All quantities in Eq. 54a are those usually tabulated for commercial sections.

For the two beams in Fig. 9, for $l/d = 30$, the following values of the buckling stress are obtained:

Eq.	Beam A	Beam B
51	24,600	4,300
7	6,900	1,900
54	22,500	3,700

It is clear that the author's values cannot be used for such light-gage beams. The writer's values, although somewhat conservative, can be considered accurate enough for design use.

In the "Specifications for the Design Of Light Gage Structural Members"[25] of the American Iron and Steel Institute, Eq. 54*a* was used to develop design requirements for nonbraced light-gage beams. For purposes of simplification, Eq. 54*a* can also be written in the following form:

$$f = \frac{74{,}000{,}000}{\left(\frac{l}{d}\right)^2}\left(\frac{r_y}{r_x}\right)^2 = \frac{74{,}000{,}000}{\left(\frac{l}{r_y}\right)^2}\left(\frac{d}{r_x}\right)^2 \qquad (54b)$$

in which r_x and r_y are the radii of inertia of the section. It was found that for light-gage beams of economical dimensions the ratio d/r_x is almost constant and equal to 2.5 or slightly above that value. By substituting this value for d/r_x in Eq. 54*b*, the following, extremely simple formula for the buckling stress of light-gage beams of the type of Fig. 1 is obtained:

$$f = \frac{463{,}000{,}000}{\left(\frac{l}{r_y}\right)^2} \qquad (54c)$$

It is Eq. 54*c*, with a suitable factor of safety, which was used in the specifications of the American Iron and Steel Institute.

Although general indications concerning the range of applicability of these two formulas have been discussed herein, doubt may sometimes arise as to which of the two to use. In such a case it is a safe and somewhat conservative procedure to compute the buckling stresses by both approximate formulas (Eqs. 7 and 54) and to use the larger of the two values. The stress so obtained will always err on the conservative side, but by not more than about 30%. Indeed, both formulas give values which are always smaller than the exact stress (Eq. 51) since they neglect positive terms. The maximum error can easily be established for the worst case—that is, when both terms under the square root are equal. In that case either formula results in a stress which is $\sqrt{0.5}$, or 0.707, times the exact stress determined by Eq. 51. Any other ratio of these two terms, therefore, will cause the error of the larger of the two stresses to be less than 30%.

Both these investigations are strictly limited to I-shaped beams. Box beams or U-beams should not be designed by these formulas, since, for such beams, the equations give values that are often in error by hundreds of percentage points. Such members must be analyzed by substantially different methods. Current design specifications are rather vague in this respect and the author would have done well to indicate a clear limitation to I-beams in the title.

These remarks are in no way intended to detract from the value of this paper which, at long last, gives a usable formula for standard types of beams. On the contrary, the writer, despite his development of relatively simple rigorous equations, did not reduce them to their simplest possible approximate form. The author, on the other hand, although handicapped by his somewhat unnecessarily complex mathematics, developed a usable formula by persistent trial and error.

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[25] "Specifications for the Design of Light Gage Structural Members," A.I.S.I., April, 1946.

These comments, therefore, are merely intended to show the rigorous basis of the author's formulas, and to supplement his fundamental Eq. 7 by an equally simple one (Eq. 54*a*), which is valid for sections to which Mr. de Vries' formulas cannot be applied.

(In the numerical data presented in this discussion, the torsional constants were computed by the approximate formula:

$$K = (\tfrac{1}{3})\, \Sigma w_n\, t^3_n \qquad (55)$$

in which w_n and t_n are, respectively, the widths and thicknesses of the component rectangles of the section. Slide-rule accuracy was held to be sufficient for these merely illustrative figures.)

David B. Hall,[26] Assoc. M. ASCE.—Some of the beams analyzed in this paper have fairly high ratios of I_y to I_x. Since the formulas of the paper are all derived on the assumption that this ratio is negligibly small, the question arises as to the amount of error introduced. It turns out, however, that, in practice, these beams can never buckle within the elastic range. Hence an increase in stability which, in an elastic structure could permit a considerable increase in the buckling load, would increase the load only slightly in this case. As an illustration, the 14WF87 beam, with a 14.5-in. flange may be considered. Referring to Table 2, it is observed that the lowest value of l/a for which the critical stress f was small enough to be recorded was 4. Since the value of a is 104 in., this would give a length of 4 × 104, or 416 in., which is approximately thirty times the depth of the beam. To prevent excessive deflection, the maximum length permitted in practically any type of structure would be considerably less than this. It may thus be concluded that questions of refinement of the theoretical buckling equations in this region are subordinate to the broader problem of plastic buckling, which the writer is not prepared to discuss.

The academic problem of such a beam, long enough to buckle in the elastic range (about 700 in. for a 14 × 14.5 beam) may still be of interest. The modified buckling load depends on the ratio I_y/I_x and also upon a relation between the torsional stiffness (including torsion bending) and I_x, but for a long I-beam the latter relation is of negligible importance. For any type of loading applied along the centroid of the beam the approximate correct buckling load can then be found by multiplying the buckling load that was previously found by the methods of the paper, by a factor,

$$F = \frac{1}{\sqrt{1 - \frac{I_y}{I_x}}} \qquad (56)$$

For the 14WF87 beam $F = 1.25$. Eq. 56 is easily obtained by considerations of energy. Assuming any section rotated through an angle β, and acted on by a moment M about the x-axis, the moments about the two principal axes of the section are $M \cos \beta$ and $M \sin \beta$, and the components of the curvature in the (yz)-plane are $\frac{M}{E\,I_x}\cos^2\beta$ and $\frac{M}{E\,I_y}\sin^2\beta$. Since $\cos^2\beta = 1 - \sin^2\beta$,

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[26] Stress Analyst, The Glenn L. Martin Co., Baltimore, Md.

$$\text{The total curvature of an element} = \frac{M}{E\,I_x} + \frac{M}{E\,I_y}\,(1 - I_y/I_x)\sin^2\beta \ldots (57)$$

The first term in Eq. 57 represents the curvature in the stable state, and the second represents the additional curvature in the buckled state. In this latter term $\sin^2\beta$ may be replaced by β^2, and it is found that the expression $\frac{1 - I_y/I_x}{E\,I_y}$ takes the place of $\frac{1}{E\,I_y}$ in expressions for both internal and external energy, and ultimately comes out in the form of Eq. 56.

A second question concerns the selection of loading and end support conditions. Several conditions have been treated in the paper. One interesting and very simple case (that of a simple beam acted on by pure end couples) was omitted.

A common type of bridge structure consists of a pair of main carrying beams or girders braced together by floor beams. The weight of the floor and moving traffic is delivered to the main beams at the floor-beam connections. Thus a single panel of such a beam corresponds very closely to the ideal beam. The condition of simply supported ends follows from the freedom of alternate panels to buckle in opposite directions, although it is modified somewhat by the restraint offered by the floor beams and bracing.

Beams in which principal loads are applied between supporting points will ordinarily present a more complicated problem than that treated in the paper, since structures of this kind (for example, a crane runway beam) would almost certainly be subject to lateral as well as vertical loading.

The case of a simple beam acted on by end couples is easier to treat than any of those considered in the paper. The coefficient k in Eq. 4 can be computed directly instead of by trial.[27] A few typical values are:

l/a	k
1	2.92
2	3.41
10	8.2

Comparison with values in Table 1 indicates that this condition is about eight tenths as severe as the case of a beam uniformly loaded along the top flange, when l/a is 1 and that it is practically identical to it when l/a is 10. It would appear logical to choose this as the standard loading condition, treating the other cases as special.

In the paper, the case of a long girder picked up at the ends during erection was cited as an example of the case of a simply supported beam loaded along the center line. Such a beam, supported by ordinary slings, is prevented from rotating about a point in the vicinity of the top flange only by its own weight, and presents an entirely different problem. For equilibrium in the slightly buckled state, the product of the height from center of load to the eye of the sling by the end rotation β must be equal to the average lateral deflection. An equilibrium solution, however, would be difficult except in the case where the torsional rigidity of the beam is assumed infinite. In this case the angle β is constant throughout the length of the beam. The lateral load is $w\,\beta$. The average lateral deflection, obtained by integrating the deflection curve, is found to be $\frac{w\,l^4\,\beta}{120\,E\,I_y}$. The height from load to the eye of sling may be conservatively taken as $d/2$, making the offset $\frac{\beta\,d}{2}$. Then:

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[27] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 261, Eq. 159.

$$\frac{w\,l^4\,\beta}{120\,E\,I_y} = \frac{\beta\,d}{2} \quad \text{(58a)}$$

and

$$\frac{w\,l^4}{E\,I_y\,d} = 60 \quad \text{(58b)}$$

By properly combining Eqs. 58*b*, 32, and 4, there follows the relation,

$$k = 3.75 \quad \text{(58c)}$$

which represents a limiting value approached by k as l/a increases indefinitely. This value of k contrasts strongly with the indefinitely increasing value of k in Table 1, for cases in which end rotation is prevented.

An approximate solution can be obtained by the energy method. An equation very similar to Eq. 28 may be derived, replacing the term from Eq. 25 (for lowering a load distributed along a flange) by the expression,

$$V_2 = -\frac{\beta^2_{z=0.5l}}{2}\,w\,l\,\frac{d}{2} \quad \text{(59)}$$

to represent the work done in raising the entire load when the ends rotate. The new equation becomes:

$$\frac{w^2}{4\,E\,I_y}\int_0^{l/2}\beta^2\left(\frac{l^2}{4}-z^2\right)dz - \frac{w\,l\,d}{4}\,\beta^2_{z=0.5l} - \frac{E\,I_y}{4}\,\frac{l^2}{a^2}\,\frac{d^2}{l^2}\int_0^{l/2}\left(\frac{d\beta}{dz}\right)^2 dz$$
$$- \frac{E\,I_y}{4}\,d^2\int_0^{l/2}\left(\frac{d^2\beta}{dz^2}\right)^2 dz = 0 \quad \text{(60)}$$

A reasonably satisfactory representation of β as a function of z should be:

$$\beta = a_0 + a_1\cos\frac{\pi\,z}{l} = a_0\left(1 + m\cos\frac{\pi\,z}{l}\right) \quad \text{(61)}$$

Substitution of Eq. 61 and its derivatives in Eq. 60 yields:

$$\frac{w^2\,l^5}{4\,E\,I_y}\left[\frac{1}{60} + m\left(-\frac{4}{\pi^3}+\frac{48}{\pi^5}\right) + m^2\left(\frac{1}{120}+\frac{3}{8\,\pi^4}\right)\right]$$
$$-\frac{w\,l\,d}{4} - \frac{E\,I_y}{4}\,\frac{l^2}{a^2}\,\frac{d^2}{l^2}\,\frac{m^2\,\pi^2}{4\,l} - \frac{E\,I_y}{4}\,d^2\,\frac{m^2\,\pi^4}{4\,l^3} = 0 \quad \text{(62)}$$

Let:

$$\frac{w\,l^4}{E\,I_y\,d} = X \quad \text{(63)}$$

Then Eq. 62 reduces to:

$$X^2\left[\frac{1}{60}+m\left(-\frac{4}{\pi^3}+\frac{48}{\pi^5}\right)+m^2\left(\frac{1}{120}+\frac{3}{8\,\pi^4}\right)\right] - X - \frac{\pi^2\,m^2}{4}\left(\frac{l^2}{a^2}+\pi^2\right)=0 \ldots\ldots\ldots\ldots(64)$$

Eq. 64 has been solved approximately for a few typical values of l/a, with the following results:

l/a	X	k
2	17.85	1.12
8	27.2	1.70
∞	60	3.75

The last line, it will be noted, checks with Eqs. 58*b* and 58*c*.

This analysis shows that the handling of a long flexible girder during erection is likely to require special consideration, and also indicates that one possible way to strengthen such a girder is by devising a special lifting rig with a high center of rotation.

THEODORE R. HIGGINS,[28] M. ASCE.—The author is to be complimented for his method of attack, which consisted, first, of solving the problem as rationally as practicable in view of the mathematical complexities and, second, in transforming the results into simple functions of published beam properties. There is obvious logic in a formula which considers, in addition to the length and width of the beam as embraced in formulas used heretofore, also the thickness of the flanges (which makes for lateral stability) and the beam depth (which militates against it). It is equally obvious that the four quantities, l, d, b, and t, appear in the author's formula in the numerator and denominator, respectively, as they properly should to express their influence upon lateral and torsional stability. The form of the author's formula is, accordingly, logical.

It is interesting to note, from the quantitative standpoint, that this formula operates to reduce the rated capacity of deep or narrow beams and to increase the rated capacity of shallow beams, as compared to their capacity when rated by previous specification formulas involving only l and b.

Although the case for Eq. 7, pertaining to long laterally unsupported beams, has convincingly been developed by the author, both analytically and statistically, the supporting argument for Eqs. 6*a*, 6*b*, and 6*c* is lacking. These equations are presented merely as affording "transition curves" with yield point stress as their upper limits. The analogy between the Euler curve for long columns, and Eq. 7, as applied to long laterally unsupported beams, is readily apparent. It does not follow directly, however, that a curve for short beams needs to simulate that for short columns as given by the secant formula.

As will be perceived by dividing 20,000,000 by 600, the critical bending stress in a beam having a $\frac{l\,d}{b\,t}$-value of 600 is not less than 33,333 lb per sq in., even under the most severe of the six different types of loading considered. Thus, the usefulness of a further reduction, amounting to $0.0125 \times (600)^2 = 4{,}500$ lb per sq in., is not apparent in applying the results of the analysis to design problems.

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From a practical standpoint, it is highly desirable to limit the use of variable working stresses—which impose an added burden on the designer—as much as possible, consistent with sound practices.

For these reasons the American Institute of Steel Construction (A.I.S.C.) Committee on Specifications has seen fit, in revising the Standard Specifications,[29] to recommend the use of Eq. 9 for all values of $\frac{l\,d}{b\,t}$ greater than 600, the point where the equation affords a working stress of 20,000 lb per sq in. For values of $\frac{l\,d}{b\,t}$ less than 600, the basic 20,000 lb per sq in. bending stress governs.

On this basis, for most rolled wide flange beams there is a rather substantial length up to which no reduction in capacity need be made because of the lack of lateral support. For instance, for a beam 36 in. deep by $16\frac{1}{2}$ in. wide at 300 lb per lin ft, this length is 38 ft. Accordingly, in the Fifth (1946) Edition of the A.I.S.C. manual on "Steel Construction" in the tables of safe loads on beams, the constant value $\frac{d}{b\,t}$, and the length (L_u) up to which the absence of lateral support does not reduce the allowable load, are given for each beam size. For any given beam, the only variable in Eq. 9 is the factor l and, when $l > L_u$,

$$f = \frac{20{,}000\, L_u}{l} \qquad \text{(65)}$$

Although Eq. 9 determines the allowable unit stress once the beam has been selected, it requires (as did the previous l/b formula) the selection of a trial beam. To avoid this step, since three beam dimensions affect the allowable stress instead of one, the A.I.S.C. manual now provides four new charts from which proper beams can be selected without any "cut and try." The charts are so arranged that, entering on the left with the desired bending moment and at the bottom with the unsupported span length, and proceeding across and up, respectively, to the intersection point thus found, the curves for all beams which satisfy the new formula lie above and to the right of this point.

In applying Eq. 9 to beams where the $\frac{l\,d}{b\,t}$-value requires a reduction from the 20,000 lb per sq in. basic working stress, investigation of vertical deflection for all ordinary conditions is unnecessary. As the laterally unsupported span length increases, the reduction in allowable unit stress operates to an extent such that the vertical deflection will always be within the usual limits prescribed for a fully loaded, fully supported beam of the same span.

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[29] "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," A.I.S.C., New York, N. Y., 1946.

The author has wisely called attention to the fact that the working formula (Eq. 9) is safe for the most severe of the six types of loading considered, and that in investigating special problems the designer would do well to consider the expressions derived for the other five conditions. For the ordinary run of work a single rule, safe for all conditions of loading, will suffice and will not prove wasteful of material. In studies of this kind, however, there is a natural tendency to lose sight of the information pertaining to special cases; and to ascribe to the single recommendation, intended to cover the general case, a sanctity not intended by the originator. These same considerations would suggest that the user of Eq. 9, in attacking problems involving unusual profiles, would do well to study carefully the discussion of some of these problems contained in the paper.

NEIL VAN EENAM,[30] M. ASCE.—Two separate phases of the problem of lateral buckling of beams and girders are treated in this paper. In the first part, additional values of the factor k are derived for various types and conditions of loading which supplement the factors m derived by Professor Timoshenko.[2] The derivation of these additional factors k, which can readily be converted to the Timoshenko factors m, is a valuable contribution. This part of the paper will be appreciated by those who prefer original sources for data on the solution of this type of problem. It is unfortunate, however, that for braced beams of two bays, the symbol l denotes the full length of the beam between end supports in Fig. 8 and in Eqs. 42 and 49, whereas in Table 1 the symbol l denotes the unsupported length of the compression flange, which in this case is one half of the full length of the beam. This may lead to some confusion.

Simplified formulas, both for critical stresses and for working stresses, are proposed in the second part of the paper. These formulas employ the dimension ratio $\frac{l\,d}{b\,t}$. In Figs. 10 and 11, results obtained by these simplified formulas are compared with those obtained by more exact methods. An analysis of the principal source of variation in results follows the comparison.

Curves ABCD of Figs. 10 and 11, representing the more exact values for the type of loading shown, were computed according to Professor Timoshenko's method.[31] However, identical results would have been obtained by using Eq. 4 of the paper. The torsion constants K entering into the more exact formulas may be accurately determined for rolled beams.[9] In riveted plate girders, there may be considerable slip between the various parts comprising the section, and there is uncertainty as to the proper reduction to be made on this account. The tests made by I. E. Madsen,[11] Jun. ASCE, appear to give the best data available, and it was decided to follow his recommendation that

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[30] Highway Bridge Engr., U. S. Public Roads Administration, Washington, D. C.

[31] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 268, Eq. 164, and p. 278.

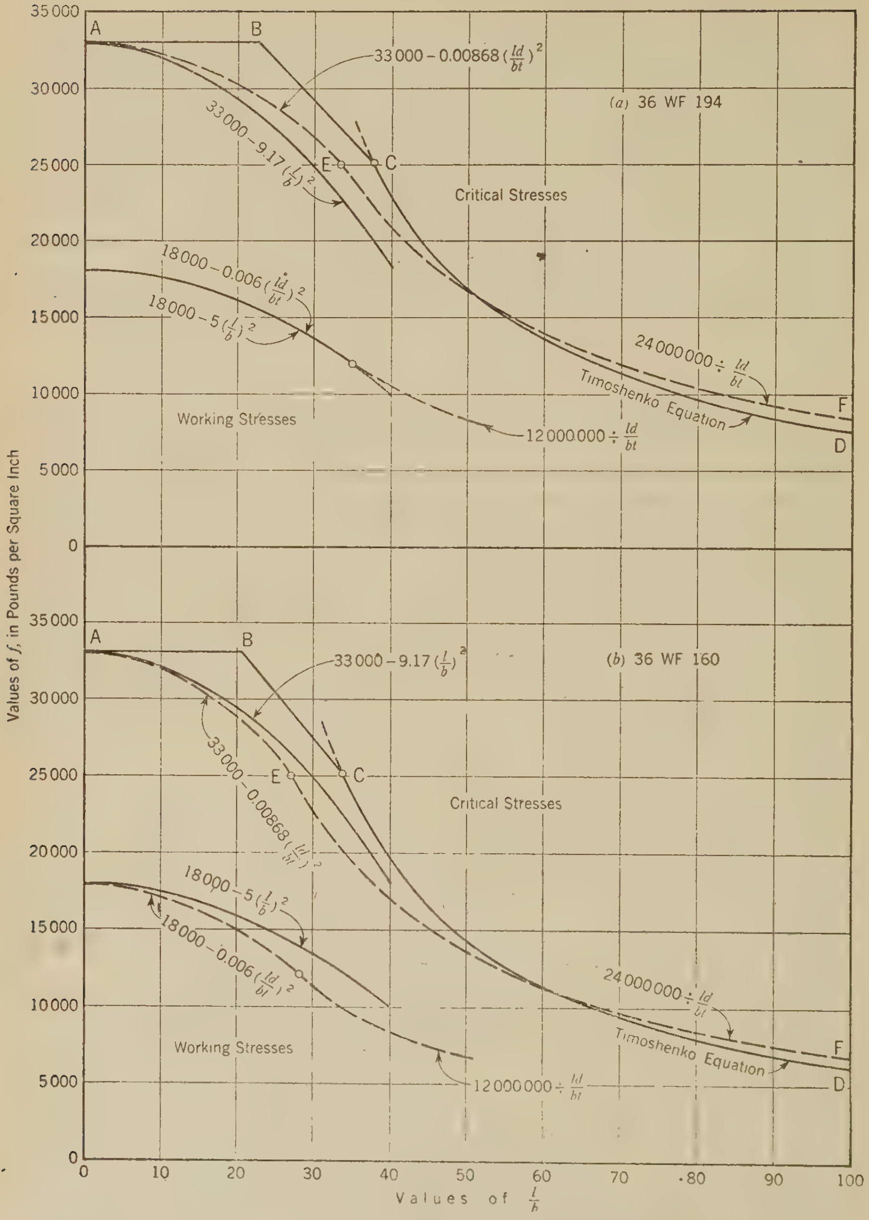

Fig. 10.—Comparison of Formulas for Critical and Working Stresses in Typical Rolled Beams, Uniform Load Applied at the Centroid

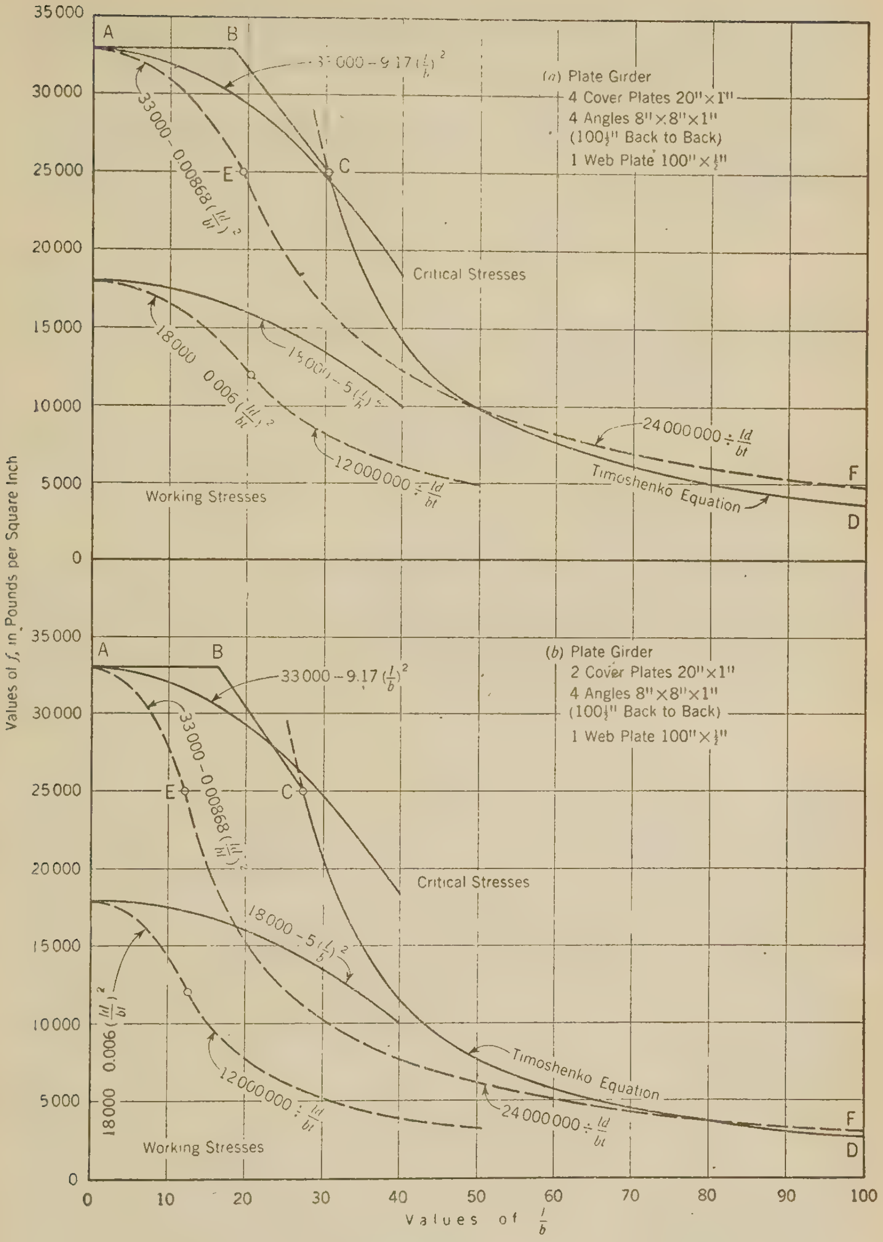

Fig. 11.—Comparison of Formulas for Critical and Working Stresses in Typical Plate Girders, Uniform Load Applied at the Centroid

the values of K for riveted plate girders be taken equal to one third those for identical monolithic sections. Curves ABCD of Figs. 11(*a*) and 11(*b*) may therefore be slightly on the conservative side. There is need of further tests to confirm Mr. Madsen's findings.

Curves AEF of Figs. 10 and 11 were computed by the formulas of the paper, using constants consistent with those given in Table 4(*a*) for carbon steel and loads applied on the centroids of the sections. Values derived from:

$$f = 33{,}000 - 9.17\left(\frac{l}{b}\right)^2 \dots\dots\dots\dots (66)$$

are also included. Eq. 66 was derived by multiplying the commonly used formula for carbon steel working stresses,

$$f = 18{,}000 - 5\left(\frac{l}{b}\right)^2 \dots\dots\dots\dots (67)$$

by the safety factor $\frac{33{,}000}{18{,}000} = 1.83$.

The plate girder of Fig. 11(*a*) is of the same section as that used for illustrative purposes in Part III of the paper. A similar section, but with two cover plates instead of four, is included in Fig. 11(*b*).

In all cases, both of rolled beams and of plate girders, at the higher ratios of $\frac{l}{b}$, Mr. de Vries' formulas yield results very close to those given by Professor Timoshenko. At the lower ratios of $\frac{l}{b}$, however, values proposed by Mr. de Vries are considerably less than those by Professor Timoshenko. Furthermore, the transition curves AE appear to drop off too rapidly, for the points E should practically coincide with the points C on the Timoshenko curves.

The reasons for this characteristic property of the de Vries formulas can best be shown by a mathematical analysis. The notation used in the paper will be adhered to, but the list of symbols given in Appendix II should be supplemented by the following: m equals a factor whose value is dependent upon the type of loading; μ equals Poisson's ratio, whose value for steel is 0.30; and a is a torsion bending constant equal to:

$$a = \frac{d}{2}\sqrt{\frac{2(1+\mu)\,I_y}{K}} \dots\dots\dots\dots (68)$$

The general equation for the critical value of the bending moment at which buckling occurs is:[2]

$$M = \frac{m\,\sqrt{B\,C}}{l} \dots\dots\dots\dots (69)$$

It is convenient to consider a beam subjected to pure bending, since for pure bending the value of the factor m is simply:[32] $m = \pi \sqrt{1 + \pi^2 \frac{a^2}{l^2}}$.

Making this substitution for m, the value of the critical compressive fiber stress is:

$$f = \frac{M\,y}{I} = \frac{d}{2\,I_x}\,\pi \sqrt{1 + \pi^2 \frac{a^2}{l^2}}\,\frac{\sqrt{B\,C}}{l} \ldots\ldots\ldots\ldots\ldots (70)$$

Eq. 70 may be expanded and written:

$$f = \frac{E}{2}\left(\frac{\pi\,d}{l}\right)^2 \sqrt{\left(\frac{l}{\pi\,d\,I_x}\right)^2 \frac{I_y\quad K}{2\,(1 + \mu)} + \left(\frac{I_y}{2\,I_x}\right)^2} \ldots\ldots\ldots (71)$$

If the second term under the radical is dropped, Eq. 71 becomes:

$$f = \frac{E\,\pi\,d}{2\,l\,I_x} \sqrt{\frac{I_y\,K}{2\,(1 + \mu)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (72)$$

Consider now a symmetrical rolled beam section, consisting of a web plate and two flanges. In such a section, the effect of the web on the moments of inertia I_x and I_y and on the torsion constant K is small and may be neglected. Approximately, then,

$$I_x = \frac{b\,t\,d^2}{2}; \qquad I_y = \frac{b^3\,t}{6}; \qquad \text{and} \qquad K = \frac{2\,b\,t^3}{3} \ldots\ldots\ldots (73)$$

Substituting these values in Eq. 72 and also evaluating E, μ, and π,

$$f = \frac{19{,}483{,}000}{\dfrac{l\,d}{b\,t}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

Eq. 74 is of the type proposed in the paper. It should be recalled that, in its derivation, the second term under the radical in Eq. 71 was dropped. By comparing curves (*b*) and (*c*) of Fig. 12, the effect of dropping this term may readily be seen. The curves were computed for a 36WF160 beam. Curve (*a*) represents the critical stresses in a beam loaded uniformly along the centroid of the section over its entire length. In curve (*b*), critical stresses are shown for a beam subjected to pure bending. The ordinates of curve (*b*) were computed by Eq. 71. The ordinates of curve (*c*) were computed by Eq. 74. There is fairly close agreement between curves (*a*) and (*b*), indicating that the error introduced by assuming pure bending is not great. Curves (*b*) and (*c*) agree closely at the higher ratios of l/b, but curve (*c*) is considerably below curve (*b*) at the lower ratios of l/b.

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[32] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 259, Chapter V.

Applying Eq. 71 to the solution of the 36WF160 beam, the following comparisons may be made:

$$\frac{l}{b} = 20; \qquad f = 3{,}331{,}100$$

$$\times \sqrt{0.00005854 + 0.00019994} = 53{,}500$$

$$\frac{l}{b} = 100; \qquad f = \quad 133{,}250$$

$$\times \sqrt{0.00146344 + 0.00019994} = \quad 5{,}430$$

For $l/b = 20$, the value of the second term under the radical is more than three times that of the first term, whereas for $l/b = 100$, the value of the second term is less than one seventh that of the first. Obviously for short beams (low values of l/b), the second term may not be ignored, as was done in Eqs. 72 and 74.

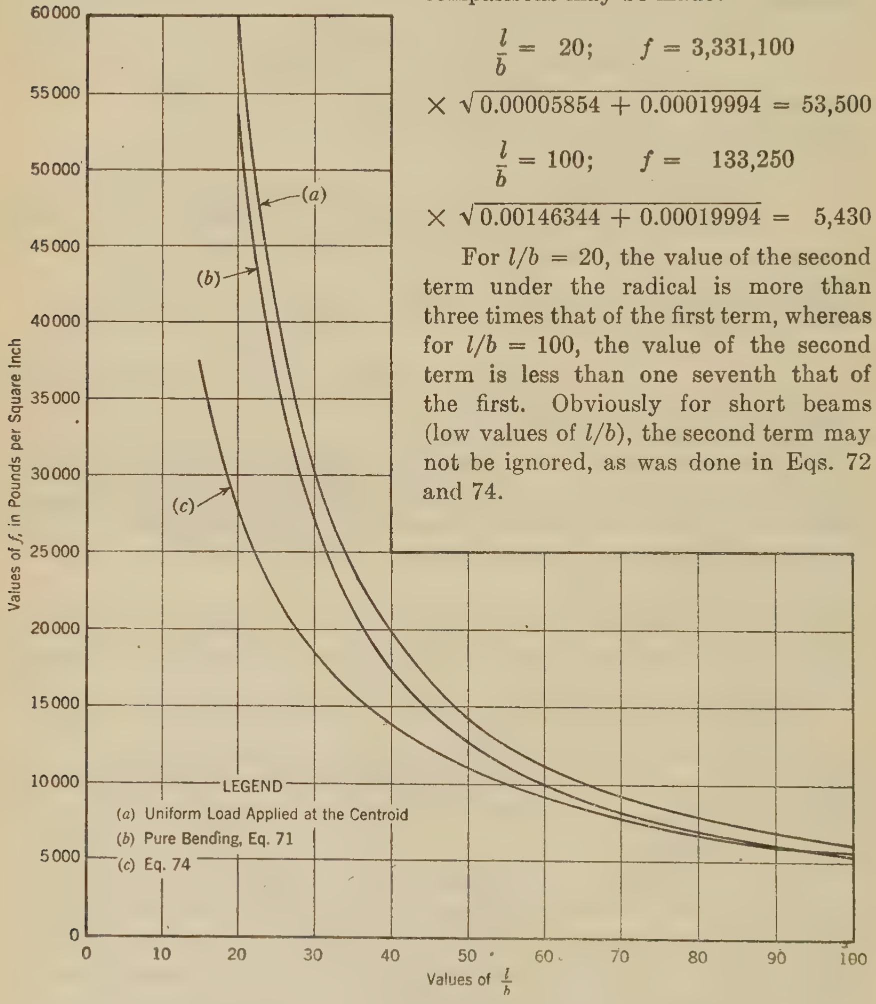

FIG. 12.—COMPARISON OF CRITICAL STRESS FORMULAS AS APPLIED TO 36WF160 BEAM

The foregoing analysis applies particularly to symmetrical, rolled sections consisting of one web plate and two flanges. For built-up girders, consisting of flange angles, cover plates, and web plate, the approximations of Eq. 73 are no longer valid, since a large part of the flange area is concentrated in the flange angles. For this reason, the de Vries formulas do not accurately reflect the effect on the critical stresses of making minor changes in the cross section of a girder. For example, in Figs. 11(*a*) and 11(*b*), by comparing the curves ABCD, the reduction in critical stress due to the omission of the two cover plates is seen to vary between 20% and 25%. On the other hand, curves AEF, representing the de Vries formulas, indicate a constant reduction of 37.5% for all values of l/b greater than 20.

The formulas proposed in the paper are particularly applicable to rolled beams with high ratios of unsupported flange length to flange width, but they should not be applied to beams with low ratios of unsupported flange length to flange width.

H. N. Hill,[33] Assoc. M. ASCE.—Inadequacy of the ratio of unsupported span length to flange width $\left(\frac{l}{b}\right)$ as a criterion of the stability of a compression flange against lateral buckling has long been recognized. As early as 1924 S. Timoshenko[34] indicated this inadequacy and presented theoretical solutions for the problem of lateral buckling of I-beams under various loading conditions. There remained, however, the task of reducing the complicated theoretical solutions to some simplified form that could be conveniently applied by the designing engineer and yet that would retain a satisfactory degree of accuracy. The significant contribution of the present paper lies in the development of a simple parameter relating the stability of the compression flange to the dimensions of an I-beam, with sufficient accuracy for many desgin purposes.

The price of simplicity in design formulas is generally a restriction in the field of application and the acceptance of some sacrifice in accuracy. There can be no doubt that the simple design formulas proposed by the author are a vast improvement over the usual formulas involving only l/b. Moreover, the author has demonstrated, by comparison with theoretical solutions, that, for a wide group of I-beams, the simple formulas have a degree of accuracy sufficient for many design needs. It may be well, however, to examine the limitations of these simple formulas, which can best be done by examining the steps necessary to derive the simple formulas on a rational basis.

The buckling stress in the compressive flange of a symmetrical I-beam, with loads applied at the neutral axis, can be expressed as:

$$f_{cr} = R \sqrt{\frac{I_y K d^2}{I^2_x l^2} + T \frac{I^2_y d^4}{I^2_x l^4}} \qquad \text{(75)}$$

in which

$$R = \frac{c}{2} \frac{E}{\sqrt{2(1 + \mu)}} \qquad \text{(76)}$$

$$T = \frac{2(1 + \mu) \pi^2}{4} \qquad \text{(77)}$$

$$I^2_y = \frac{b^6 t^2}{36}; \quad I^2_x = U \frac{b^2 t^2 d^4}{4} \qquad \text{(78)}$$

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[33] Asst. Chf. Engr., Design Div. of Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[34] "Beams Without Lateral Support," by S. Timoshenko, *Transactions*, ASCE, Vol. LXXXVII, 1924, p. 1247.

and

$$K = W \frac{2\,b\,t^3}{3} \quad \text{.........................(79)}$$

In Eqs. 76 through 79, c is a coefficient dependent on the nature of loading and restraint; μ is Poisson's ratio; U is equal to $\left(1 + \frac{t_w\,d}{6\,b\,t}\right)^2$; W is equal to $1 + \frac{t^3_w\,d}{2\,b\,t^3}$; and t_w is the thickness of the beam or girder web.

By substitution in Eq. 75,

$$f_{cr} = R \sqrt{\frac{4}{9}\,\frac{W}{U}\,\frac{b^2\,t^2}{d^2\,l^2} + \frac{T}{9}\,\frac{1}{U}\,\frac{b^4}{l^4}} \quad \text{.................(80)}$$

For steel beams, in which $E = 29{,}000{,}000$ and $\mu = 0.28$, $R = 9{,}060{,}000\,c$ and $T = 6.31$. To evaluate U and W, twenty-three I-sections representing extremes in wide flange beams, light beams, joists, and American standard beams have been considered. The limiting values obtained are $0.75 < \frac{W}{U} < 1.07$ and $0.44 < \frac{1}{U} < 0.81$. Eq. 80 can now be written:

$$f_{cr} = 9{,}060{,}000\,c \sqrt{C_1 \left(\frac{b}{l}\right)^2 \left(\frac{t}{d}\right)^2 + C_2 \left(\frac{b}{l}\right)^4} \quad \text{..........(81)}$$

in which $0.33 < C_1 < 0.48$ and $0.31 < C_2 < 0.57$.

An equation similar in form to Eq. 7 can be obtained by neglecting the second term under the radical in Eq. 81, which then becomes

$$f_{cr} = \frac{9{,}060{,}000\,c\,\sqrt{C_1}}{\frac{l\,d}{b\,t}} \quad \text{......................(82)}$$

For a uniformly distributed load at the neutral axis and the ends of the unsupported length simply supported against lateral deflection, c is equal to 3.54.[35] Comparing Eq. 82 with the equation for long beams for case (*b*), Fig. 4, $32{,}100{,}000\,\sqrt{C_1} = 24{,}000{,}000$; or $\sqrt{C_1} = 0.748$. According to the limiting values for C_1 in Eq. 81, for the group of I-sections considered, $\sqrt{C_1}$ may be as low as 0.575. Consequently, for beams having extremely large values of l/b (in which case the second term under the radical in Eq. 81 is truly negligible), the author's equation can give buckling stress values as much as 30% higher than the theoretically correct values. This situation can only obtain in extremely long spans of narrow flange beams, which will buckle at very low stresses, and is very probably of no importance in ordinary design problems. It may be well to keep this limitation in mind, however, in certain handling and erection problems involving long narrow beams.

The inaccuracies introduced by neglecting the second term under the radical of Eq. 81 will depend on the unsupported span length of the beam. Since the second term varies inversely as l^4 and the first term varies inversely as l^2, it is apparent that the importance of the second term increases as the span length decreases. This accounts for the increased discrepancy between the plotted points and the curves (within the region of elastic action) as l decreases in the plots of Figs. 2, 4, and 5. The curves become increasingly conservative as the span length decreases. Again the discrepancies, as indicated by the plots in Figs. 2; 4, and 5, do not constitute a valid objection to the use of the proposed formulas for general design of standard solid section I-beams of structural grade steels. When dealing with high-strength alloy steels and high-strength lightweight alloys of aluminum and magnesium, however, relatively short beams can buckle at stresses within the elastic range. In such cases, theoretical buckling stress values may be more than 50% greater than the values obtained by simplified formulas like Eq. 7. In structures in which these materials are used, lightweight is generally of primary importance, and the introduction of refinements in design is warranted to secure maximum weight saving. Such refinements in design usually entail some sacrifice in simplicity of design formulas.

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[35] "The Lateral Instability of Deep Rectangular Beams," by C. Dumont and H. N. Hill, *Technical Note No. 601*, National Advisory Committee for Aeronautics, Washington, D. C., 1937.

Possibly the earliest instance of the inclusion of a rational treatment of the lateral buckling problem in a set of design rules is the "Structural Aluminum Handbook,"[36] published in 1938. The treatment is based on a direct application of Eq. 75, for the case of a beam under pure bending, with the ends of the laterally unsupported length simply supported against lateral deflection. The value of c in Eq. 76 for this case is π. (Values of c for numerous other conditions of loading—on the neutral axis—and restraint are available.[35] The fact that the author has been able to represent the cases of flange loading by the same type of formula as that applicable to loading on the neutral axis, and with about the same degree of accuracy, suggests that the cases of flange loading might also be adequately represented by Eq. 75, by substitution of the proper value for c.) Since the equivalent slenderness ratio method is used throughout this handbook to handle buckling in the plastic range, the buckling parameter is expressed in terms of an equivalent radius of gyration ρ:

$$\rho = \sqrt{\frac{0.2}{S_c}\sqrt{I_y(K\,l^2 + 13.1\,I_f\,d^2)}} \qquad (83)$$

in which S_c is the section modulus about the axis normal to the web (compression side); and I_f is the moment of inertia of the compression flange about an axis through the centroid of the flange, parallel to the web ($I_f = \frac{I_y}{2}$ for symmetrical I-sections—the term I_f being introduced to make the equation applicable to I-sections with unequal flanges).

Although the above equation may have a formidable appearance to a designer accustomed to the utmost simplicity in design formulas, the quantities involved are handbook values, or, in the case of I_f, are easily computed, and the mathematical manipulations involved can be readily and rapidly performed with the aid of a slide rule. When designing lightweight structures in high-strength and low-modulus materials, the appreciable saving in weight that may be realized by the use of the more complicated Eq. 83 will easily outweigh the slight increase in effort required for its application.

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[36] "Structural Aluminum Handbook," Aluminum Co. of America, 1938.

The author is to be highly commended for having developed the simple buckling parameter $\frac{l\,d}{d\,t}$ for I-beams and for having proposed a set of simple design formulas which should adequately handle most design problems concerned with the lateral buckling of solid I-beams of structural grade steels. The writer feels, however, that it is desirable that the limitations of design formulas based on this parameter be recognized, and it is important that such formulas shall not be considered as providing adequate design methods for all problems involving the lateral buckling of beams.

H. D. HUSSEY,[37] M. ASCE.—There has been a need for a formula that takes account of the torsional property of the beam. The basic theory of slender beams has been published for many years, but it has remained for Mr. de Vries to demonstrate how it can be put into simple form.

Eq. 9 appears unduly conservative for use as a general design formula. The author has considered several types of loading but he has omitted any reference to a beam under pure bending, which is the most general form of beam loading. Loadings covered by the author should be considered only as special cases. A formula will be developed, in this discussion, for a beam under pure bending.

When an abstract of this paper was read at the Annual Meeting of the Society in 1945, the following beam formulas were in general use: American Institute of Steel Construction specifications for buildings—

$$f_a = \frac{22{,}500}{1 + \frac{l^2}{1{,}800\,b^2}} \quad \text{(maximum 20,000)} \ldots\ldots\ldots\ldots (84a)$$

and American Railway Engineering Association and American Association of State Highway Officials specifications for bridges—

$$f_a = 18{,}000 - 5\,\frac{l^2}{b^2} \ldots\ldots\ldots\ldots\ldots\ldots (84b)$$

in which f_a is the maximum allowable compressive unit stress in bending. When Eq. 9 is written in the form—

$$f_a = \frac{12{,}000{,}000\;t/d}{\frac{l}{b}} \ldots\ldots\ldots\ldots\ldots\ldots (84c)$$

—the stresses f_a can be plotted as ordinates against the corresponding ratio l/b as abscissas. Each beam will thus be represented by a separate graph, depending upon the value of the ratio t/d, as shown in Fig. 13. Curves for Eq. 84*c* in Fig. 13 represent the following beam sections: (*a*) 12 B 14 J ($t/d = 0.0188$); (*b*) 12 B 16½ L ($t/d = 0.0224$); (*c*) 30WF108 ($t/d = 0.0255$); and (*d*) 36WF230 ($t/d = 0.0351$). There are more than forty rolled beams with values of t/d less than 0.0351. These include practically all the popular lightweight sections of each depth. It will be noted that the strength of these forty beams, when calculated by Eq. 84*c*, is considerably less than when calculated by Eq. 84*a*. The strength of many of these beams falls far below that represented by Eq. 84*b*, in spite of the fact that their strength is derived from a basic unit stress of 20,000 and that Eq. 84*b* is based on 18,000.

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[37] Designing Engr., Am. Bridge Co., New York, N. Y.

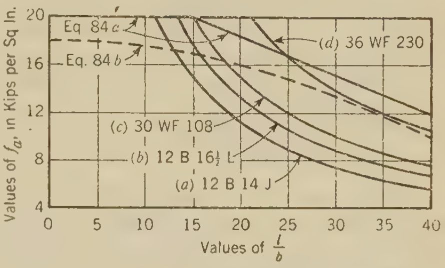

FIG. 13.—ALLOWABLE STRESSES FOR A SIMPLE BEAM, AS COMPUTED BY EQS. 84

For example, the solution of Eq. 84*a*, for an assumed ratio $l/b = 30$, yields a value of $f_a = 15{,}000$. In contrast with this, the corresponding value of f_a for the following beams, when calculated by Eq. 84*c*, is:

Section	$_a$
30WF108	10,200
12 B 16½ L	8,960
12 B 14 J	7,520

These examples show that the allowable stresses for many beams, when derived from Eq. 84*a*, were actually at, or near, the critical stresses as determined by Eq. 9.

It is interesting to analyze the loads and moments on typical beams, as is done in the following problems:

Example 1.—The theory of lateral buckling of beams, as illustrated by Fig. 1, is based on a simply supported beam in which the moment diagram varies from zero at the two ends to a maximum value at the center. When the unsupported length of the beam is less than the beam span, the moment diagram for this part of the span is different from that for the entire beam in having moments at the two ends.

Example 2.—Assume that a beam supports two equal concentrated loads symmetrically spaced on the beam, and that the loads divide the beam into three unsupported parts. When the three parts are isolated it is found that the middle part is under pure bending and each of the two end lengths has a moment diagram which varies from zero at one end to a maximum at the other.

Example 3.—Next assume that the beam in Fig. 1, which supports a single concentrated load at its center, is so long that it is held laterally at the third points, dividing the beam into three unsupported lengths. Assume, also, that the concentrated load is perfectly free to move transversely with the beam as the beam deflects sidewise, as shown in Fig. 1. The middle third of this beam has a maximum moment at its center, but there are moments at the two ends of this part that are two thirds as great as the maximum moment.

Example 4.—If the load on the beam in Example 3 were uniform, instead of concentrated, the moment at the two ends of the middle third would be nearly 90% of the maximum moment. It is assumed, of course, that the uniform load is perfectly free to move transversely with the beam as the beam deflects sidewise. If this beam were divided into more than three parts, the moments at the ends of the middle part would be more than 90% of the maximum moment.

Floor beams in railroad bridges which have symmetrically placed loads, fall in the same class as the beam in Example 2. The middle of the beam, which sustains the greatest stress, is under pure bending. Deck plate girders and stringers in the ordinary railroad bridge are in the class with the beam in Example 4, which has a uniform load. A long deck girder is divided into many panels by the lateral system. If one panel near the center of this girder is isolated, moments will be found at the ends of this panel that are nearly as great as the maximum moment in the panel. The uniform load on the top flange of the girder produces very little change in moment throughout the length of the panel. The principal effect of this top flange load is to contribute to the moments at the two ends of the panel. This girder, therefore, should be considered as a beam under pure bending and not as a beam under top flange loading.

One of the principal considerations in this problem is the nature of the load supported by the beam. To apply this beam theory to a practical problem, the load must be perfectly free to move transversely with the beam as the beam deflects sidewise. If the load is not free to move sidewise, the problem is changed radically and the compression flange is greatly strengthened. For example, if the load in Fig. 1 is applied at the top flange of the beam and the load is perfectly free to move sidewise, the strength of the beam is proportional to the values of the factor k as given in Col. 2, Table 1. However, if this load is not free to move sidewise, the problem is transformed into the problem illustrated in Fig. 8, and the strength of the beam is proportional to the factor k as given in Col. 8, Table 1. The strength of the latter beam is much greater than that of the former.

The number of beams in actual structures, which support loads that are perfectly free to move sidewise, is so small that such beams should be considered as special cases. A trolley beam, for instance, which supports a free concentrated load at the bottom flange, is 50% to 150% stronger than the beam used by the author in developing his proposed Eq. 9.

It is apparent from Eq. 4 that there are three fundamental variables in the solution of the beam problem—length, section, and type of loading. In attempting to present a simple formula the author has included only the first two of these variables in the parameter, $l\,d/(b\,t)$, of Eq. 9. He has covered the third variable, the type of loading, by using a single constant—12,000,000—which produces a formula that is safe for the most severe loading, irrespective of the frequency of its occurrence.

S. Timoshenko[38] has demonstrated the theory of beams subjected to pure bending. The value of the critical moment for this type of loading may be expressed as follows:

$$M_{cr} = \frac{\pi^2 E\, I_y}{l^2}\frac{d}{2}\sqrt{1 + \frac{K\, l^2}{6.4\, I_y\, d^2}} \text{ (assuming } \nu = 0.30) \ldots\ldots (85)$$

in which ν is Poisson's ratio. The value of the critical buckling unit stress (for a symmetrical section) is:

$$f = \frac{\pi^2 E}{4\left(\frac{l\, r_x}{r_y\, d}\right)^2}\sqrt{1 + \frac{K\, l^2}{6.4\, I_y\, d^2}} \ldots\ldots (86)$$

in which r_x and r_y are the radii of gyration about the x-axis and y-axis, respectively.

Eq. 86 may be simplified by the use of two close approximations. First, the ratio $\frac{d}{r_x}$ for rolled beam sections is nearly constant and equal to 2.45. Second, the torsion constant K can be represented for rolled beam sections by the approximate formula:

$$K = 0.30\, A_s\, t^2 \ldots\ldots (87)$$

in which A_s is the sectional area of the beam and t is the thickness of the flanges. (Eq. 87 is exact for many sections. Since K is under the radical in Eq. 86, any error due to the use of Eq. 87 has only a small effect on the value of the critical stress.)

Making use of Eq. 87 and assuming that $\left(\frac{d}{r_x}\right)^2 = 6$, Eq. 86 becomes:

$$f = \frac{444{,}000{,}000}{\left(\frac{l}{r_y}\right)^2}\sqrt{1 + \frac{1}{22}\left(\frac{l\, t}{r_y\, d}\right)^2} \ldots\ldots (88a)$$

The critical buckling stress for rolled beams may be written:

$$f = \frac{444{,}000{,}000}{\left(n\frac{l}{r_y}\right)^2} \ldots\ldots (88b)$$

in which

$$n^2 = \frac{1}{\sqrt{1 + \frac{1}{22}\left(\frac{l\, t}{r_y\, d}\right)^2}} \ldots\ldots (89)$$

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[38] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 259.

Numerical values of n are as follows:

$\frac{lt}{r_y d}$	n
0	1.00
1	0.99
2	0.96
3	0.92
4	0.87
5	0.83
6	0.79

$\frac{lt}{r_y d}$	n
8	0.71
10	0.65
12	0.60
14	0.56
16	0.53
18	0.50
20	0.48
24	0.44

A formula for working stresses can be obtained by dividing Eq. 88*b* by an appropriate factor of safety. Using a factor of safety of 1.65, for instance, the working formula becomes:

$$f_a = \frac{270{,}000{,}000}{\left(n\frac{l}{r_y}\right)^2} \qquad \text{(90}a\text{)}$$

Eq. 90*a* may be expressed in a very simple form as follows:

$$f_a = \left[\frac{16{,}400}{n\frac{l}{r_y}}\right]^2 \qquad \text{(90}b\text{)}$$

Eq. 90*b* avoids the use of large numbers that are involved in the solution of Eq. 90*a*.

By Eq. 90*b*, with $l/b = 30$, $l/r_y = 153$, $\frac{l\,t}{r_y\,d} = 3.9$, and $n = 0.875$, the allowable unit stress for the 30WF108 beam is 15,000. This is in sharp contrast with the value of 10,200 determined by Eq. 9 and agrees exactly with the value of 15,000 obtained by the use of Eq. 84*a*. Values of the coefficient k in Eq. 4, for a beam subject to pure bending, are as follows:

$\frac{l}{a}$	k
2	2.926
3	3.413
4	3.996
5	4.640
6	5.322
7	6.029

$\frac{l}{a}$	k
8	6.753
9	7.490
10	8.236
11	8.988
12	9.746
13	10.51
14	11.27

A comparison of these values of k with those given in Table 1 reveals that a beam supporting a uniform load at the centroid of the section is 13% stronger, and when supporting a concentrated load at its centroid it is 36% stronger, than it is under pure bending.

The author mentions the erection problem of picking long beams at the ends, as a condition of a uniform load at the centroid. It will be recognized that a long beam hanging in erection slings is not the same problem as the beam shown in Fig. 1. The ends of the beam in Fig. 1 are held in a vertical position against rotation. When a beam is picked off the ground, however, the ends can rotate as the beam buckles laterally. It is hoped that the author will extend his investigation to cover this particular condition.

This problem is of such interest to erection engineers that it deserves further study. One can compute the span length at which a simple-span beam will buckle from its own weight, when the ends are blocked so that they cannot rotate. Using Eq. 85, and increasing it by 13% to make it applicable to a uniform load, will give the critical moment at which such a beam will buckle. Substituting $\frac{w\,l^2}{8}$ for the moment produces an equation from which the desired length can be determined.

When solving Eq. 85 for long beams, the numerical value of the second term under the radical sign becomes so great as compared to unity, that the latter may be neglected. A solution of Eq. 85 then becomes:

$$M_{cr} = \frac{\pi^2 E\,I_y}{5\,l}\sqrt{\frac{K}{I_y}} \ldots\ldots\ldots\ldots (91)$$

Increasing Eq. 91 by 13%, and substituting $M_{cr} = \frac{w\,l^2}{8}$ and $K = 0.30\,A_s\,t^2$, leads to the following equation for rolled beams:

$$l^3 = 9.77\,\frac{E\,A_s\,t\,r_y}{w} \ldots\ldots\ldots\ldots (92a)$$

in which w equals the weight of the beam per inch. Noting that, for steel beams, $w/A_s = 0.2833$ lb (the weight of 1 cu in. of steel), the following simple equation can be written:

$$l = 1{,}016\,\sqrt[3]{r_y\,t}\ \text{(inches)} \ldots\ldots\ldots\ldots (92b)$$

Eq. 92*b* gives the length of a simple span rolled beam which will buckle from its own weight. For example, l = 110 ft for a 36WF150, and 98 ft for a 30WF108 beam.

If the foregoing simple-span beam is picked up at the ends, the web blocking has been removed and the ends can rotate as the beam deflects laterally. While hanging in the slings, the centers of the two supports will lie in the vertical plane which passes through the center of gravity of the beam. If the twisting of the beam is neglected (assuming an infinite torsional rigidity) and lateral deflection only is considered, the following formula can be derived:

$$l^4 = 120\,\frac{E\,I_y\,h}{w} \ldots\ldots\ldots\ldots (92c)$$

in which h is the distance from the support to the centroid of the beam, in the plane of the web. If the slings are attached to the top flange of the beam, h

may be assumed equal to $d/2$. For this case Eq. 92c becomes:

$$l^4 = 60 \frac{E\, I_y\, d}{w} \dotfill (92d)$$

Introducing the value of $w/A_s = 0.2833$ for steel beams, Eq. 92d reduces to:

$$l = 282 \sqrt[4]{r^2_y\, d} \text{ (inches)} \dotfill (92e)$$

Solving Eq. 92e for the beams considered in connection with Eq. 92b a length of 89 ft is determined for a 36WF150 beam, and 79 ft for a 30WF108 beam.

Eq. 92e gives the length at which a steel beam will buckle when hanging in slings at the ends, assuming that the torsional rigidity of the beam is infinite. If the true torsional rigidity of the beam were considered, the correct length should be less than that found by Eq. 92e.

Acknowledgment.—The writer wishes to acknowledge the assistance given to him by George E. Howe and C. W. Wixom, Members, ASCE, in the preparation of this discussion. The former was the first to indicate the extremely low unit stresses permitted by Eq. 9 for L-beams and J-beams, as compared with those derived from Eq. 84a. The latter suggested the form of Eq. 90b.

H. G. Brameld,[39] Esq.—An excellent case is made for approximate formulas for critical and working buckling stresses, in so far as rolled steel joists are concerned. Unfortunately, agreement with the fundamental equation (Eq. 4) is not good in the case of plate girders with thin flanges. Startling results were obtained when the equations were applied to the girders of a bridge in which the writer is interested.

The girders vary in depth from 86 in. to 170 in. and are braced laterally at intervals of 22 ft. Their properties are as follows:

Case I.—Depth: 86 in. Make-up: Two flange plates, 18 in. by $\frac{3}{8}$ in.; four flange angles, 8 in. by 8 in. by $\frac{5}{8}$ in.; and one web plate, 84 in. by $\frac{3}{8}$ in. For this section, $I_y = 818$ in.^{4}; $I_x = 108{,}000$ in.^{4}; and the torsion constant = 26.6. The equivalent thickness of the compression flange is found to be $\frac{6\, I_y}{b^3} = 0.84$ in. From Eq. 7, $f = \dfrac{20{,}000{,}000}{\dfrac{l\, d}{b\, t}} = \dfrac{20{,}000{,}000}{\dfrac{264 \times 86}{18 \times 0.84}} = 13{,}400$ lb per sq in. However, by extrapolation of k in Table 1, with $a = 0.806\, d \sqrt{\dfrac{I_y}{k}} = 380$ in., and $k = 2.9$, Eq. 4 yields $f = \dfrac{818 \times 86^2}{108{,}000 \times 380^2} \times 29{,}000{,}000 \times \dfrac{380^2}{264^2} \times 2.9 = 67{,}000$ lb per sq in., or just five times the approximate result.

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[39] Designing Engr., Bridge Board, Brisbane, Queensland, Australia.

Case II.—Depth: 170 in. Make-up: One web plate, 168 in. by $\frac{3}{8}$ in., with flanges the same as in case I. By Eq. 7, f = 6,750 lb per sq in. and, by Eq. 4, f = 48,000 lb per sq in.

Case III.—Depth: 86 in. Make-up: Six flange plates, 18 in. by $\frac{3}{8}$ in.; four flange angles, 8 in. by 8 in. by $\frac{5}{8}$ in.; and one web plate, 84 in. by $\frac{3}{8}$ in. By Eq. 7, f = 21,000 lb per sq in. and, by Eq. 4, f = 90,000 lb per sq in.

Case IV.—Depth: 170 in. Make-up: One web plate, 168 in. by $\frac{3}{8}$ in., with flanges the same as in case III. By Eq. 7, f = 10,700 lb per sq in. and, by Eq. 4, f = 64,000 lb per sq in.

These girder flanges when treated as struts were found to have a safe stress between 16,000 lb per sq in. and 17,000 lb per sq in., consistent with an intrinsic eccentricity of 0.45 in. and an extreme fiber stress of 20,000 lb per sq in. It is apparent, therefore, that the equation for safe working stresses cannot reasonably be applied to deep plate girders and that this fact should be noted in codes using the new formula.

EDWIN H. GAYLORD,[40] ASSOC. M. ASCE.—In a previous paper on the strength of beams as determined by lateral buckling,[41] it was suggested that the critical stress for a simply supported beam subjected to pure bending could be used as a design criterion for most cases of transverse loading. From the formula for the critical bending moment in pure flexure,[27] it can be shown that the critical stress is expressed by Eq. 4 with the factor k given by:

$$k = \frac{\pi^2}{4}\sqrt{1 + \frac{1}{\pi^2}\left(\frac{l}{a}\right)^2} \qquad (93)$$

Eq. 7 can be obtained from Eq. 4 by substituting approximate values for I_x, I_y, and the torsion constant K, using the value of k given by Eq. 93, with the first term under the radical omitted so that,

$$k = \frac{\pi\, l}{4\, a} \qquad (94)$$

Values of k computed from Eqs. 93 and 94 are given in Cols. 3 and 5, respectively, Table 8. In the same table, Col. 2 gives values of k for a uniform load applied at the top flange. These values are taken from Col. 5, Table 1, except as noted. Granting that a design formula should be based on the case of a uniform load applied at the top flange, the errors involved in using the k-values of Cols. 3 and 5 are indicated in Cols. 4 and 6, Table 8. For small values of l/a, both approximate values of k are considerably in error.

Since Eq. 93 overestimates values of k for small l/a-ratios, whereas Eq. 94 underestimates them, it is evident that a better value might be obtained by arbitrarily reducing the first term under the radical of Eq. 93. Col. 7, Table

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[40] Prof. and Chairman, Dept. of Civ. Eng., Ohio Univ., Athens, Ohio.

[41] "Strength of Slender Beams," by George Winter, *Transactions*, ASCE, Vol. 109, 1944, p. 1339.

8, gives values of k computed from the equation:

$$k = \frac{\pi^2}{4}\sqrt{0.50 + \frac{1}{\pi^2}\left(\frac{l}{a}\right)^2} \quad \text{.........} (95)$$

Col. 8 shows that these values check very well with those of Col. 2, even for the extreme values of l/a which were not included in Table 1.

TABLE 8.—VALUES OF k

$\frac{l}{a}$ (1)	Uniform load at top flange (2)	Pure bending Eq. 93 (3)	$\frac{\text{Col. 3}}{\text{Col. 2}}$ (4)	Eq. 94 (5)	$\frac{\text{Col. 5}}{\text{Col. 2}}$ (6)	Eq. 95 (7)	$\frac{\text{Col. 7}}{\text{Col. 2}}$ (8)	Eq. 97 (9)	$\frac{\text{Col. 9}}{\text{Col. 2}}$ (10)
0.6325..	1.84[a]	2.52	1.370	0.497	0.270	1.81	0.984	1.80	0.978
2.......	2.269	2.925	1.289	1.571	0.692	2.348	1.035	2.258	0.995
3.......	2.788	3.412	1.224	2.356	0.845	2.932	1.052	2.814	1.009
4.......	3.418	3.995	1.169	3.142	0.919	3.594	1.052	3.485	1.020
5.......	4.121	4.638	1.125	3.927	0.953	4.297	1.043	4.202	1.020
6.......	4.872	5.319	1.092	4.712	0.967	5.025	1.031	4.942	1.014
7.......	5.655	6.026	1.066	5.498	0.972	5.768	1.020	5.694	1.007
8.......	6.459	6.750	1.045	6.283	0.973	6.521	1.010	6.455	0.999
9.......	7.280	7.487	1.028	7.069	0.971	7.281	1.000	7.221	0.992
10.......	8.113	8.232	1.015	7.854	0.968	8.045	0.992	7.991	0.985
11.......	8.954	8.985	1.003	8.639	0.965	8.814	0.984	8.764	0.979
12.......	9.802	9.742	0.994	9.425	0.962	9.585	0.978	9.539	0.973
13.......	10.65	10.50	0.986	10.21	0.959	10.36	0.973	10.32	0.969
14.......	11.51	11.27	0.979	11.00	0.956	11.13	0.967	11.09	0.964
20.......	16.7[a]	15.90	0.952	15.71	0.941	15.80	0.946	15.78	0.945

[a] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, pp. 268 and 269, Table 23 and Eq. (*h*).

Eq. 95 can be put into more convenient form by using the first two terms of the binomial expansion for $\sqrt{1 + x}$. If $x \leqq 1$, the error of this approximation does not exceed +6%. The equation may be written:

$$k = 0.707\,\frac{\pi^2}{4}\sqrt{1 + 0.203\left(\frac{l}{a}\right)^2} \quad \text{.........} (96a)$$

and also,

$$k = \frac{\pi}{4}\frac{l}{a}\sqrt{1 + 4.93\left(\frac{a}{l}\right)^2} \quad \text{.........} (96b)$$

From these equations:

$$k = 1.745\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \quad \text{.........} (97a)$$

$$k = \frac{\pi}{4}\frac{l}{a}\left[1 + 1.75\left(\frac{a}{l}\right)^2\right] \quad \text{.........} (97b)$$

The coefficient of the second term in the brackets has been reduced from its binomial expansion value. This was done because the effect of this term is greatest in the region where the k-values of Col. 7, Table 8, are most in excess of the values of Col. 2. The equations give approximately equal values when

$l/a = 2$. Eq. 97*a* is to be used if $l/a < 2$, and Eq. 97*b* if $l/a \geqq 2$. Col. 9, Table 8, gives values of k computed from Eqs. 97; and Col. 10 compares these values with those of Col. 2.

If the values of k from Eqs. 97 are substituted in Eq. 4, with $E = 30 \times 10^6$ lb per sq in.,

$$f = \frac{52.3 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \text{ if } l/a < 2 \ldots\ldots\ldots (98a)$$

$$= \frac{23.6 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left(\frac{l}{a} + 1.75\frac{a}{l}\right) \text{ if } l/a \geqq 2 \ldots\ldots\ldots (98b)$$

Applying the factor of safety used by the author to Eqs. 98, formulas for design are:

$$f = \frac{31.5 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \text{ if } l/a < 2 \ldots\ldots\ldots (99a)$$

$$f = \frac{14 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left(\frac{l}{a} + 1.75\frac{a}{l}\right) \text{ if } l/a \geqq 2 \ldots\ldots\ldots (99b)$$

Except for the factor a, these equations are in terms of common properties of beams. They are relatively simple to use if tables of the factor a are available.[8] A limit for l/a beyond which the error in the use of Eq. 9 is inconsequential can be established from Table 8. If it be desired that the error not exceed 5%, then the approximate k of Col. 5 should not be used if $l/a < 5$. The error will not exceed 8% if $l/a > 4$. (This is exclusive of errors inherent in approximations to I_x, I_y, and K, through which Eq. 7 is derived from Eq. 4.)

If tables of the factor a are not available, the ratio l/a may be computed from the equation:

$$\frac{l}{a} = 1.24\frac{l}{d}\sqrt{\frac{K}{I_y}} \ldots\ldots\ldots (100)$$

This equation was derived from the relation $C = \frac{E\, I_y\, d^2}{4\, a^2}$, with Poisson's ratio equal to 0.3. The torsion constant K can be computed with sufficient accuracy in most cases by means of the familiar equation $K = \frac{1}{3}\Sigma b\, t^3$, or, if desired, by a more accurate procedure described elsewhere.[10]

Eqs. 99 may be presented in a different form by calculating I_x, I_y, and K terms of the beam flanges only, that is, $I_x = \frac{b\, t\, d^2}{2}$, $I_y = \frac{t\, b^3}{6}$, and $K = \frac{2\, b\, t^3}{3}$. However, the equations are first written in the following form, by factoring

l/a from the brackets and parentheses, and substituting from Eq. 100:

$$f = \frac{39 \times 10^6}{\dfrac{I_x}{\sqrt{K I_y}} \dfrac{l}{d}} \left(\frac{a}{l} + 0.075 \frac{l}{a} \right) \text{ if } l/a < 2 \quad \text{(101}a\text{)}$$

$$f = \frac{17.5 \times 10^6}{\dfrac{I_x}{\sqrt{K I_y}} \dfrac{l}{d}} \left[1 + 1.75 \left(\frac{a}{l} \right)^2 \right] \text{ if } l/a \geqq 2 \quad \text{(101}b\text{)}$$

When the substitutions for I_x, I_y, and K are made,

$$f = \frac{26 \times 10^6}{\dfrac{l\,d}{b\,t}} \left(\frac{a}{l} + 0.075 \frac{l}{a} \right) \text{ if } l/a < 2 \quad \text{(102}a\text{)}$$

$$f = \frac{12 \times 10^6}{\dfrac{l\,d}{b\,t}} \left[1 + 1.75 \left(\frac{a}{l} \right)^2 \right] \text{ if } l/a \geqq 2 \quad \text{(102}b\text{)}$$

The l/a-ratios in the brackets and parentheses of Eqs. 102 are not expressed in terms of l, d, b, and t because of large errors that could result. The reason for the errors is found in Eq. 100. The value of K expressed in terms of the flanges only may be in error by as much as 50%, whereas the value of I_y is practically unaffected by omitting web moment of inertia. In the substitutions outside the brackets, the error in K is compensated for by an error in I_x. For a 12 B 16.5[42] section, $K = 0.052$ in.^{4} and $I_x = 77.5$ in.^{4} if the flanges only are considered; $K = 0.098$ in.^{4} and $I_x = 105$ in.^{4} if the web is included. The ratio $I_x/\sqrt{K}$ for the true values is 336, and for the approximate values, 340. Calculations for several beams chosen at random showed variations of as much as 8%.

If values of the factor a, or K, are not available, their calculation may be avoided by writing Eq. 100 as $\dfrac{l}{a} = 1.24 \dfrac{l}{d} \dfrac{I_x}{I_x} \sqrt{\dfrac{K}{I_y}}$ and substituting the approximate values for K, I_y, and the I_x in the denominator. This procedure introduces the same compensating errors mentioned previously, and results in:

$$\frac{l}{a} = 5 \frac{l}{d} \frac{I_x}{b^2 d^2} \quad \text{(103)}$$

This value of l/a may be substituted in Eq. 102b to obtain:

$$f = \frac{12 \times 10^6}{\dfrac{l\,b}{d\,t}} \left[1 + 0.07 \left(\frac{b^2 d^2/I_x}{l/d} \right)^2 \right] \text{ if } \frac{l}{d} \geqq \frac{0.3\, b^2 d^2}{I_x} \quad \text{(104)}$$

Eq. 104 should be particularly useful, since it contains only common properties of sections. After selecting a beam according to Eq. 9, the designer need calculate only the second term in the brackets to determine the percentage by which the allowable stress, as given by Eq. 9, should be increased. Eq. 102*a* is not written with the l/a-ratio expressed in terms of Eq. 103, because the result is cumbersome. However, for $l/a = 1.5$, the value of f given by Eq. 102*b* exceeds by only 5% the value given by Eq. 102*a*, so that Eq. 102*b*, or Eq. 104, probably can be safely used for $l/a \geqq 1.5$. The l/d limitation stated after Eq. 104 is based on this assumption. Thus, Eq. 104 will be applicable to the design of practically any rolled shape or plate girder, and Eq. 99*a* or its equivalent, Eq. 102*a*, may be used for light-gage beams.

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[42] "Steel Construction Manual," A.I.S.C., 5th Ed., 1947, p. 24.

A simple criterion to determine whether the second term in the brackets of Eq. 104 is significant can be established. As stated previously, if $l/a > 5$, Eq. 9 will err by about 5%. Therefore, from Eq. 103,

$$\frac{l}{d} \geqq \frac{b^2 d^2}{I_x} \qquad \text{(105)}$$

is an index of the accuracy of Eq. 9.

One other convenient form for Eqs. 98 will be given. The radius of gyration for the axis perpendicular to the web of the I-section and the WF-section varies from about 0.375 d to 0.425 d. If an average value of $r_x = 0.41\ d$ is assumed, formulas for the critical stress for the I-section and the WF-section are:

$$f = \frac{310 \times 10^6}{\left(\dfrac{l}{r_y}\right)^2}\left[1 + 0.075\left(\frac{l}{a}\right)^2\right] \text{ if } l/a < 2 \qquad \text{(106}a\text{)}$$

$$f = \frac{140 \times 10^6}{\left(\dfrac{l}{r_y}\right)^2}\left(\frac{l}{a} + 1.75\,\frac{a}{l}\right) \text{ if } l/a \geqq 2 \qquad \text{(106}b\text{)}$$

These equations are particularly simple in form. However, they may overestimate values of f given by Eqs. 99 by as much as 10%, and may underestimate them by as much as 15%.

The following examples illustrate the use of the equations in this discussion. It is assumed that the maximum allowable bending unit stress is 20,000 lb per sq in. Since the American Institute of Steel Construction has adopted Eq. 9 in its 1947 design specifications,[43] diagrams based on that equation[44] can be used for these problems. The data given do not include the moment due to the weight of the beam.

Example 1.—A simply supported beam on a span of 55 ft is to resist a moment of 450 kip-ft. The diagrams referred to indicate that a 30WF190 section is required. For this section, $a = 121.9$ and $l/a = 5.41$, Therefore, the allowable stress is predicted with sufficient accuracy by Eq. 9. Using Eq. 105, $\frac{l}{d} = 22$ and $\frac{b^2 d^2}{I_x} = 23.3$, which indicates that Eq. 9 is in error by somewhat over 5%. Instead of checking with Eq. 105, the second term in the brackets of Eq. 104 may be calculated. It is found to be 0.078, which shows that Eq. 9 underestimates the allowable stress by about 7%.

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[43] "Steel Construction Manual," A.I.S.C., 5th Ed., 1947, p. 286.

[44] *Ibid.*, p. 203.

Example 2.—A simply supported beam on a span of 24 ft is to resist a moment of 400 kip-ft. The diagram shows that 24WF110 is the lightest section that can be used. For this section, $l/a = 288/108 = 2.67$, and, since the diagram also shows that the allowable stress was reduced for buckling, the section may be unnecessarily large. Using Eq. 105, $\frac{l}{d} = 12$ and $\frac{b^2\,d^2}{I_x} = 25.5$.

The allowable stress for a 24WF100 section is given by Eqs. 99*b*, 102*b*, and 104 as 21,100, 20,800, and 22,200 lb per sq in., respectively. From Eq. 106*b*, with the factor of safety used to obtain Eq. 99*b*, $f = 22{,}300$ lb per sq in. According to Eq. 9, however, $f = 16{,}100$ lb per sq in. Since the more nearly correct values exceed the stated maximum allowable of 20,000 lb per sq in., the allowable moment is $\frac{20 \times 249}{12} = 415$ kip-ft. Because the moment due to its weight is only 7.2 kip-ft, the 24WF100 is satisfactory. Thus, in this example, a few minutes of the designer's time saves 240 lb of steel.

Example 3.—A simply supported beam on a span of 18 ft is to resist a moment of 490 kip-ft. From the diagram, a 30WF124 or a 24WF120 section may be used. The diagram shows that the allowable stress for 24WF120 is greater than 20,000 lb per sq in., and there is no need to investigate the 24 × 12-group further. However, the allowable stress for 30WF124, as given by Eq. 9, is below 20,000 lb per sq in.; and it may be underestimated. The ratio $l/a = 216/105.1 = 2.06$ shows this to be the case; or, using Eq. 105, since $\frac{l}{d} = 7.2$ and $\frac{b^2\,d^2}{I_x} = 18.8$, the allowable stress is seen to be underestimated considerably. For a 30WF108 section, the allowable stress as given by Eqs. 99*b*, 102*b*, 104, and 106*b* is 22,500, 22,600, 24,400, and 21,300 lb per sq in., respectively. According to Eq. 9, $f = 14{,}900$ lb per sq in. Then $M = \frac{20 \times 299}{12} = 498$ kip-ft. The moment due to the weight of the beam is 4.4 kip-ft. In this example, the weight saving is 10% compared to section 24WF120, and 13% compared to 30WF124.

The foregoing examples have been chosen to show extreme differences; it is true that there are many cases in which comparable savings could not be made because of the necessarily limited range of standard sections. It has been assumed that Eqs. 98 apply as long as the critical stress is less than the yield point (assumed to be about 33,000 lb per sq in.). Strictly speaking, the equations are not correct if the critical stress exceeds the proportional limit. Transition curves to accompany the equations in this discussion could undoubtedly be developed, but the writer did not attempt them. The author's transition equations are subject to errors comparable to those in Eq. 7. The examples might give the impression that Eq. 7 errs only when the critical stress is in the region of the proportional limit of carbon steel. Two illustrations will show that this is not the case. The critical stress for a 12-in. 14-lb joist[42] on a span of 12 ft is 10,400 lb per sq in. according to Eq. 7, whereas Eq. 98*b* gives a critical stress of 13,900 lb per sq in. For an 8WF17 section on a span of 15 ft, Eqs. 7 and 98*b* give 22,400 and 27,000 lb per sq in., respectively. It is pertinent to observe that the difference between the two equations is even more significant for beams of high-yield steel.

Since there are some types of loading which subject a beam to pure flexure, or at least approximately so, formulas for the critical stress for this case may be useful. Substitution of the value of k from Eq. 93, as given by the first two terms of the binomial expansion in Eq. 4, leads to the following formulas for the bending critical stress in pure bending:

$$f = \frac{74 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left[1 + 0.04\left(\frac{l}{a}\right)^2\right] \text{ if } \frac{l}{a} < 3 \ldots\ldots\ldots\ldots (107a)$$

$$f = \frac{23.5 \times 10^6}{\frac{I_x}{I_y}\left(\frac{l}{d}\right)^2}\left(\frac{l}{a} + 4\,\frac{a}{l}\right) \text{ if } \frac{l}{a} \geqq 3 \ldots\ldots\ldots\ldots (107b)$$

The coefficient of the second term in the brackets and parentheses has been reduced from the value obtained through the binomial expansion. For $0 \leqq l/a \rightarrow \infty$, the equations give values of f which differ by a maximum of -2% from the values computed with k as given by Eq. 93. The second term in the brackets of Eq. 107a may be omitted if $l/a < 1$; and the corresponding term in Eq. 107b may be omitted if $l/a > 10$. The maximum error resulting from these omissions is about -5%. The value of l/a may be computed from Eq. 100 or Eq. 103 if tables of the factor a are not available.

Mr. de Vries is to be commended for having presented a design formula for beams subject to lateral buckling which, although eminently simple in form, is far superior to previous formulas. In most cases it predicts the critical stress with a good degree of accuracy. The writer has shown how the designer may determine the precision of the formula for any case, and easily obtain a better estimate when necessary. Furthermore, since either Eq. 99b or 102b is simpler to use than is Eq. 104, the writer suggests that the American Institute of Steel Construction list in future editions of its manual values of the factor a not only because it is useful in the problem of lateral buckling of beams, but also because it is necessary in the analysis of torsion.

OLIVER G. JULIAN,[45] M. ASCE.—The formulas given in this paper are the acme of simplicity and far more rational than most specification requirements. With the notable exceptions of members—(a) which are unflanged, asymmetrical, or have boxed cross sections; (b) those for which $\sqrt{1 - I_y/I_x}$ is appreciably different from unity; (c) those with limber webs such that cross sections may distort in their planes so that the effective torsional rigidity of the member is not wholly effective; and (d) those with thin wide flanges such as are made from sheet metal—the author's formulas should give results which are fairly reasonable and conservative. It should be especially emphasized that, as plainly stated[46] by Professor Timoshenko, the Timoshenko formulas (with which Mr. de Vries compares his formulas) were derived for deep narrow members such as I-beams rather than wide-flanged members and do not take account of relative horizontal to vertical stiffness.

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[45] Chf. Structural Engr., Jackson and Moreland, Cons. Engrs., Boston, Mass.

Notation.—In addition to the notation introduced by the author, the following listed letter symbols are used in this discussion:

$a^2_1 = \frac{E\,I_y}{G\,K}\left(\frac{h}{2}\right)^2$;

G = modulus of rigidity;

H = distance of the load above the shear center; if the load is below the shear center, H is negative;

h = distance between flange centroids;

j = ratio of the product of l and the critical transverse loading force to the critical moment; for pure flexure j equals unity;

m = a factor;

n = a factor;

P_{ey} = Euler's critical load for failure of a pin-ended column by flexure about a principal axis y;

P_w = Wagner's critical load for failure of a column by torsion; thus —

$$P_w = \frac{1}{J^2}\left[G\,K + P_{ey}\left(\frac{h}{2}\right)^2\right] = \frac{G\,K}{J^2}\left[1 + \pi^2\left(\frac{a_1}{l}\right)^2\right] \ldots\ldots (108)$$

$\bar{r}_y$ = radius of gyration about a principal axis y;

S = section modulus about the x-axis; and

J = polar moment of inertia.

The earliest work on the elastic stability of beams with which the writer is acquainted is that of A. G. M. Michell,[47] published in 1899. Based on geometrical relations, on the differential equations of equilibrium, and on the assumption that the flexural rigidity about the y-axis may be of the same order as, or much smaller than, that about the x-axis whereas the torsional rigidity about the longitudinal z-axis is small as compared to the flexural rigidity about the x-axis, Mr. Michell derives expressions defining the critical loads for symmetrical prismatic and cylindrical beams, subjected to various types of loads including the case in which opposite forces are applied parallel to the axis at symmetrical points in the plane of the greatest rigidity. Rearranged, Mr. Michell's expression defining the first mode of instability for the latter case is

$$M^2 = \left(\frac{\pi^2}{l^2} - \frac{P}{E\,I_y}\right)\frac{E\,I_y\,G\,K}{1 - I_y/I_x} \ldots\ldots\ldots\ldots\ldots (109)$$

in which P and M represent the concomitant critical axial load and moment, respectively. If I_y/I_x is neglected, Eq. 109 is identical with the result[48] given by Professor Timoshenko for a similar case in which I_y is considered as of a smaller order than I_x. For pure flexure and in terms of critical fiber stress, Eq. 109 becomes

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[46] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 239 et seq.; also see section headings 49 et seq.

[47] "Elastic Stability of Long Beams Under Transverse Forces," by A. G. M. Michell, *The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science*, Vol. XLVIII, 5th Series, 1899, p. 298.

[48] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 243, Eq. (*h*).

$$f^2 = \frac{\pi^2 \, d^2 \, E \, I_y \, G \, K}{4 \, l^2 \, I^2_x \, (1 - I_y/I_x)} \ldots\ldots\ldots\ldots (110)$$

Neglecting I_y/I_x and using the following rough approximations—$\pi^2 = 10$; $I_x = \frac{d^2}{2} \, b \, t$; $I_y = \frac{b^3 \, t}{6}$; $K = \frac{2}{3} \, b \, t^3$; and $E \, G = 360 \times 10^{12}$—Eq. 110 yields the author's basic Eq. 7.

The rationalization of the author's basic equation from the Michell theory involves:

1. Considering the member as subjected to equal opposed end couples rather than to transverse loads.

2. Neglecting the flexural rigidity of the flanges about the y-axis of the beam, which amounts to equating P_w to $\frac{G \, K}{J^2}$ or equating a_1 to zero. For short beams this may lead to ultraconservative design.

3. Neglecting the torsional rigidity of fillets and the web, and using the value of K applicable to extremely wide thin rectangles. The first may lead to errors in K of 25% or more on the conservative side, whereas the latter errs on the daring side.[49,50]

4. Neglecting I_y/I_x as compared to unity. For many members in common use, this may lead to errors of 25% or more on the conservative side.

5. Assuming that the beam flanges and web act as a unit without appreciable warping of cross sections in their planes.[20]

6. Assuming that the areas of the flanges are fully effective in resisting flexure. In the case of thin wide flanges this may lead to errors of dangerous magnitude.[51]

7. Assuming that the cross section of the member is symmetrical or has point symmetry, as for a **Z**-bar.

The sum total of the errors that may be introduced by assumptions 1 to 7 may be far from negligible.

For a flanged beam, the formula corresponding to Michell's solution for a symmetrical prismatic member subjected to end couples is:

$$M^2 = \frac{\pi^2 \, E \, I_y \, G \, K \left[1 + \pi^2 \left(\frac{a_1}{l} \right)^2 \right]}{l^2 \, (1 - I_y/I_x)} = \frac{J^2 \, P_{ey} \, P_w}{1 - I_y/I_x} \ldots\ldots (111a)$$

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[49] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 248, "Table of Constants for Torsion of a Rectangular Bar."

[50] "Structural Beams in Torsion," by Inge Lyse and Bruce G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 861, Eq. 6.

[51] "Strength of Thin Steel Compression Flanges," by George Winter, *ibid.*, Vol. 112, 1947, p. 527.

For purposes of comparison it will be found convenient to write Eq. 111*a* as:

$$M^2 = n^2 \frac{\pi^2}{l^2} E\, I_y\, G\, K \left[1 + \pi^2 \left(\frac{a_1}{l}\right)^2\right] = \frac{m^2\, E\, I_y\, G\, K}{l^2} \quad \text{.....(111}b\text{)}$$

in which

$$n^2 = \frac{1}{1 - I_y/I_x} \quad \text{.....(112}a\text{)}$$

and

$$m^2 = n^2\, \pi^2 \left[1 + \pi^2 \left(\frac{a_1}{l}\right)^2\right] \quad \text{.....(112}b\text{)}$$

A number of values of *n* and *m* are shown in Fig. 14(*a*). If the torsional rigidity of the member is negligible[20] (as may be the case if the web is thin and deep) so that cross sections may deform readily in their planes, Eq. 111*a*, using the value of a_1 given in the notation of this discussion, reduces to:

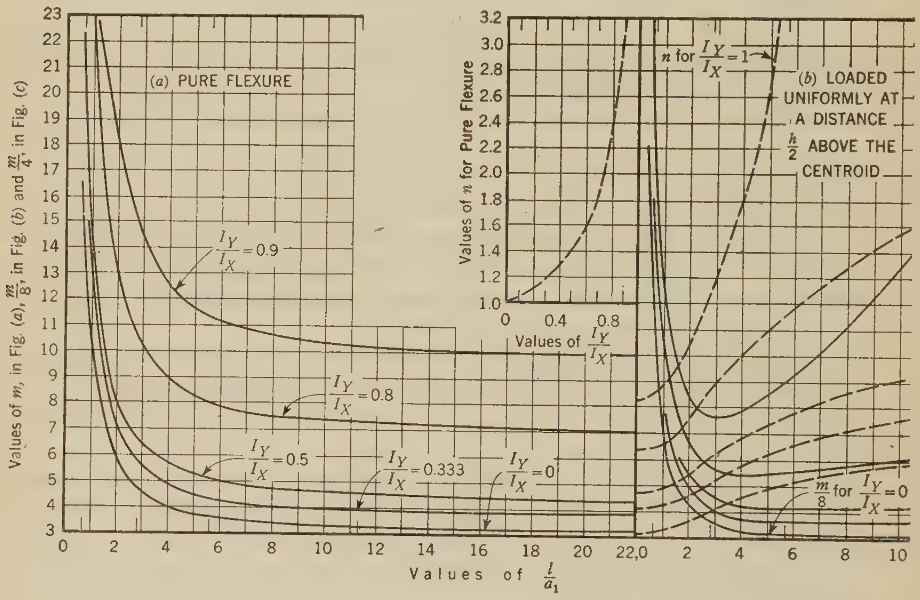


FIG. 14.—VALUES OF *n* AND *m* FOR A

$$M^2 = P^2_{ey} \left(\frac{h}{2}\right)^2 \frac{1}{1 - I_y/I_x} \quad \text{.....(113)}$$

For a symmetrical, uniformly loaded, simple beam with the load rigidly attached to the member a distance *H* above the centroidal axis an approximate solution corresponding to Eq. 111*a* is:

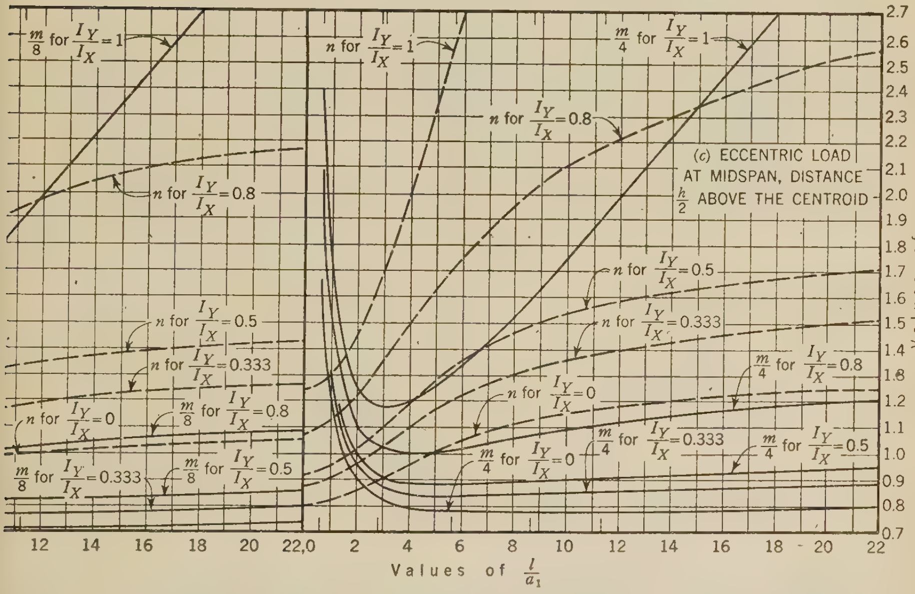
$$M = 1.13 \frac{\pi \sqrt{E\, I_y\, G\, K}}{l\,(1 - I_y/I_x)} \left\{ \sqrt{\left[1 + \pi^2 \left(\frac{a_1}{l}\right)^2\right] \left(1 - \frac{I_y}{I_x}\right) + 2.07 \left(\frac{a_1}{l}\right)^2 \frac{H^2}{(h/2)^2}} - 1.43 \frac{a_1}{l} \frac{H}{h/2} \right\} \quad \text{..(114)}$$

This solution is derived by considering the exchange of energy that occurs while the beam is twisting and buckling, account being taken of the deformation about the x-axis as well as that about the y-axis and the z-axis. Eq. 114 can be written:

$$M^2 = \frac{n^2\,\pi^2}{l^2}\,E\,I_y\,G\,K\left[1+\pi^2\left(\frac{a_1}{l}\right)^2\right] = \left(\frac{m}{8\,l}\right)^2 E\,I_y\,G\,K \ldots\ldots(115)$$

in which

$$n = \frac{1.13}{1 - I_y/I_x}\left[\sqrt{1-\frac{I_y}{I_x}+\frac{2.07\,\dfrac{H^2}{(h/2)^2}}{\pi^2+(l/a_1)^2}} - \frac{1.43\,\dfrac{H}{h/2}}{\sqrt{\pi^2+(l/a_1)^2}}\right]\ldots(116a)$$



SIMPLE SYMMETRICAL FLANGED BEAM

and

$$m = \frac{28.4}{1 - I_y/I_x}\left\{\sqrt{\left[1+\pi^2\left(\frac{a_1}{l}\right)^2\right]\left(1-\frac{I_y}{I_x}\right)+2.07\left(\frac{a_1}{l}\right)^2\frac{H^2}{(h/2)^2}} - 1.43\,\frac{a_1\,H}{l\,h/2}\right\}\ldots(116b)$$

A number of values of n and $\frac{m}{8}$ are shown in Fig. 14(b). From a comparison of these values and those pertaining to pure flexure, it will be noted that, when $\frac{l}{a_1}$ is small, the critical moment for this case is considerably less than that for a similar beam subjected to pure flexure. If I_y/I_x is small, the beams are about on a par when $l/a_1 = 11.3$; this value of l/a_1 increases as I_y/I_x increases. When l/a_1 is large, the critical moment for the uniformly loaded beam is somewhat more than the corresponding moment for a similar beam subjected to pure flexure. Equating h to d Eq. 115 yields the following approximate expression for critical fiber stress:

$$f = \frac{I_y\, d^2\, E}{I_x\, l^2}\,\frac{n\,\pi}{4}\sqrt{\pi^2 + \left(\frac{l}{a_1}\right)^2} \ldots\ldots\ldots\ldots (117)$$

Eq. 117 is identical to Eq. 4 if the insignificant terms (those beyond the first) in the Fourier series used by the author are neglected and I_y/I_x is equated to zero. As K approaches zero, the value of M given by Eq. 115 approaches:

$$M = \frac{0.565\, P_{ey}\, h}{1 - I_y/I_x}\left[\sqrt{1 - \frac{I_y}{I_x} + 0.21\,\frac{H^2}{(h/2)^2}} - 0.46\,\frac{H}{h/2}\right] \ldots\ldots (118)$$

For a symmetrical simple beam subjected to a concentrated load at mid-span, an approximate solution corresponding to Eq. 114 is:

$$M = 1.35\,\frac{\pi\,\sqrt{E\, I_y\, G\, K}}{l\,(1 - I_y/I_x)}\left\{\sqrt{\left[1 + \pi^2\left(\frac{a_1}{l}\right)^2\right]\left(1 - \frac{I_y}{I_x}\right) + 3.03\left(\frac{a_1}{l}\right)^2\frac{H^2}{(h/2)^2}} - 1.74\,\frac{a_1}{l}\,\frac{H}{h/2}\right\} \ldots (119)$$

which can be written

$$M^2 = \frac{n^2\,\pi^2}{l^2}\, E\, I_y\, G\, K\left[1 + \pi^2\left(\frac{a_1}{l}\right)^2\right] = \left(\frac{m}{4\,l}\right)^2 E\, I_y\, G\, K \ldots\ldots (120)$$

in which

$$n = \frac{1.35}{1 - I_y/I_x}\left[\sqrt{1 - \frac{I_y}{I_x} + \frac{3.03\,\frac{H^2}{(h/2)^2}}{\pi^2 + (l/a_1)^2}} - \frac{1.74\,\frac{H}{h/2}}{\sqrt{\pi^2 + (l/a_1)^2}}\right] \ldots (121a)$$

and

$$m = \frac{16.93}{1 - I_y/I_x}\left\{\sqrt{\left[1 + \pi^2\left(\frac{a_1}{l}\right)^2\right]\left(1 - \frac{I_y}{I_x}\right) + 3.03\left(\frac{a_1}{l}\right)^2\frac{H^2}{(h/2)^2}} - 1.74\,\frac{a_1}{l}\,\frac{H}{h/2}\right\} \ldots (121b)$$

A number of values of n and $m/4$ are shown in Fig. 14(c). As K approaches zero, the value of M given by Eq. 119 approaches:

$$M = \frac{0.675\, P_{ey}\, h}{1 - I_y/I_x}\left[\sqrt{1 - \frac{I_y}{I_x} + 0.31\,\frac{H^2}{(h/2)^2}} - 0.56\,\frac{H}{h/2}\right] \ldots\ldots (122)$$

Flexural members are often loaded so that the moment between points of restraint, as regards rotation about their longitudinal axes, is substantially constant. For this case, if the loading is uniformly distributed, a safe but reasonable approximation of the critical moment can be obtained by using Eq. 114 with unity substituted for the constant 1.13. If the loading is concentrated midway between points of torsional restraint, Eq. 119 applies approximately provided 1.00 is substituted for the constant 1.35. If the loading is concentrated at points of torsional restraint, the formulas applicable to pure flexure apply without modification.

The author does not mention cantilever beams. As far as the writer knows, a satisfactory solution of the problem pertaining to a flanged cantilever in which the effective torsional constant is not negligible has not been published to date. Safe, but rough, approximations of the critical moment pertaining to such cantilevers subjected to concentrated terminal loads, can be obtained from Eq. 119, for simple beams, provided 1.27 is substituted for the constant 1.35.

Various cases in which the load is placed at a considerable distance above or below the centroidal axis have been analyzed[52] by H. S. Richmond by investigating the differential equations of equilibrium.

TABLE 9.—EFFECT OF TYPES OF LOADING AND END CONDITIONS

Case	Type of loading	End conditions	n
1	Pure bending	....	1.00
2	Concentrated load at midspan	Fixed ends[a,b]	1.06
3	Uniformly distributed load	Simply supported	1.13
	Concentrated Load at:		
4	The free end	Cantilever	1.27
5	Midspan	Simply supported	1.35
6	0.35 times span length	Simply supported	1.38
7	0.15 times span length	Simply supported	1.53
8	Pure bending	Fixed ends[b]	2.00
9	Uniformly distributed load	Cantilever	2.04

[a] Ends fixed against rotation about the x-axis. [b] Ends fixed against rotation about the y-axis.

The marked effect of various types of loading and end conditions on the value of the critical moment may be further appraised by a glance at Table 9, which pertains to deep narrow prismatic members. The values of n are those involved in the formula:

$$M = \frac{n\,\pi}{l}\sqrt{E\,I_y\,G\,K} \qquad \text{(123)}$$

The supported ends of all members are considered fixed as regards rotation about a longitudinal axis but, unless otherwise stated, are free to rotate about the x-axis and the y-axis. In all cases, the loading is on a level with the centroidal axis. Table 9 was compiled from data given by John Prescott[53] and by Professor Timoshenko.[2]

It appears unfortunate that the author, in his notation, designated l as "the laterally unsupported length of the compression flange." A better designation for l would appear to be "length of member between points of fixity as regards rotation about a longitudinal axis." This usage has been adopted in this discussion It should be emphasized that the author's equations, those given in the references, and in this discussion are based on the ends of the member being fixed as regards torque about a longitudinal axis. Ample provision for such restraint is most important but is often neglected in practice.

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[52] "Elastic Equilibrium in the Theory of Structures," by H. S. Richmond, *Transactions*, ASCE, Vol. 94, 1930, p. 845.

[53] "Applied Elasticity," by John Prescott, Longmans, Green & Co., London, 1924, p. 499.

The columns of free-standing, two-dimensional, frames cannot be said to fix the ends of the beams which are component parts of the frame; the columns will flex normal to the plane of the frame and will thereby allow the ends of the beams to rotate about longitudinal axes. For such cases the stability of the frame as a unit should be considered.

The analysis under discussion assumes that the loading is perfectly free to rotate and move transversely. Quite often this is not so in practice (or tests); frequently the loading is so attached to the member as to stabilize it as regards torque and transverse movement.

The writer has grave doubts regarding the safety of the author's proposal (last paragraph under "Conclusions") to allow the same unit stresses in asymmetrical sections as are used for symmetrical sections; nor can he agree that asymmetrical sections are seldom used as long beams. Towers are often built entirely of angles, all of which are subject to flexure due to eccentric end conditions. It is not clear how the author's formulas, or other formulas based on the torsional rigidity of symmetrical sections, can be applied rationally to such a section as an angle or a T-section. If the horizontal leg (flange) is in compression, b and t may be taken from its cross-sectional dimensions, but it cannot be said that the compression flange is supported by the tension flange; the latter is not present. Also, it is evident that an angle or tee has considerably less torsional rigidity than an I-beam or H-section, the flanges of which have values of b and t identical with those of the angle or tee. H. N. Hill,[54] Assoc. M. ASCE, and George Winter,[20] M. ASCE, have given formulas applicable to asymmetrical I-beams of such proportions that $\sqrt{1 - I_y/I_x}$ is approximately equal to unity. J. N. Goodier has given[55] formulas applicable to deep asymmetrical members subjected to axial load as well as flexure.

Many members, such as crane girders, are ordinarily subjected to flexure about both principal axes, together with torque about a longitudinal axis. This case is not covered by the author. C. O. Dohrenwend, Assoc. M. ASCE, has analyzed[56] this problem for the special case in which a prismatic member is subjected to pure flexure about both principal axes, together with an applied torque at midspan, I_y and K being assumed small as compared to I_x.

The author's method of analyzing the stability of a plate girder appears to contain a series of judgment adjustments for which he no doubt has ample empirical justification. It is hoped that, in his closing discussion, he will explain fully his reasons for these adjustments.

Although included in the scope of the title, the author's equations are obviously not meant to apply to light-gage structural members made of sheet metal of such dimensions that the entire section is not fully effective in resisting torsion and flexure. Such members (made of steel) have been analyzed by Professor Winter using semi-empirical methods.[51] The results of this analysis have been incorporated, in abbreviated form, in specifications of the American Iron and Steel Institute (A.I.S.I.)[25] issued in April, 1946.

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[54] "The Lateral Instability of Unsymmetrical I-Beams," by H. N. Hill, *Journal of the Aeronautical Sciences*, Vol. 9, No. 5, 1942.

[55] "Flexural-Torsional Buckling of Bars of Open Section," by J. N. Goodier, *Bulletin No. 28*, Cornell Univ. Eng. Experiment Station, Ithaca, N. Y., 1942.

[56] "Action of Deep Beams Under Combined Vertical, Lateral and Torsional Loads," by C. O. Dohrenwend, *Journal of Applied Mechanics*, September, 1941, p. A-130.

Flexural members are often subjected to a combination of at least three types of loading—end couples, distributed loads, and concentrated loads. As has been indicated, the critical moments for each of these loadings may differ considerably. It can be shown,[53,57] that a sufficient (as distinguished from a necessary) condition of stability is defined by:

$$\sum \frac{M}{M_{cr}} = 1 \dots\dots\dots\dots\dots\dots\dots\dots (124)$$

in which the terms in the numerators pertain to the actual loading, whereas those in the denominators pertain to the critical condition for that type of loading acting alone. This simple rule will give results that are ordinarily reasonable although somewhat on the safe side.

The author, quite rightly, introduces transition curves between the proportional limit and the yield-point strength of the material. The validity of these transition curves might be supported by assuming initial imperfections, as has been done by Professor Winter in deriving secant formulas for beams,[19] or by taking the moduli E and G as variables above the proportional limit, and building up a mathematical argument on these premises.

For conventional, symmetrical steel members, subjected to flexure about the principal x-axis, the writer suggests the use of the following in lieu of the author's basic Eq. 7:

$$f = \frac{56.1 \times 10^6}{S\,l}\, n \sqrt{I_y\,K + \frac{6.5\,I^2_y\,h^2}{l^2}} = \frac{17.86 \times 10^6}{S\,l}\,\frac{m}{j}\sqrt{I_y\,K} \dots (125)$$

in which E and G have been taken as 29×10^6 and 11×10^6, respectively. Eq. 125 can be applied to other materials such as the structural aluminum alloys by suitable modifications of these moduli. The most important values of n and m/j encountered in practice might well be tabulated and exhibited graphically in standard handbooks, together with values of K, a_1, $I_y\,K$, and $I^2_y\,h^2\;1/s\sqrt{I_y\,R}$ for each section as has been done for K in the Structural Aluminum Handbook.[58]

By using the approximate values of I_x, I_y, and K given after Eq. 110 and equating h to d, Eq. 125 can be written in the roughly approximate form:

$$f = \frac{18.7 \times 10^6}{l\,d/b\,t}\, n \sqrt{1 + 1.62\,\frac{b^2\,d^2}{t^2\,l^2}} \dots\dots\dots\dots (126)$$

If n is taken as unity (its value for pure flexure when I_y/I_x is negligible) and if the second term in the radical is neglected, Eq. 126 is substantially the author's basic Eq. 7 (the difference occurs on account of differences in assumed

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[57] "Rayleigh's Principle," by G. Temple and W. G. Bickley, Oxford Univ. Press, England, 1933, pp. 31 and 41.

[58] "Structural Aluminum Handbook," Aluminum Co. of America, New Kensington, Pa., 1945.

values of the elastic moduli). However, unity, in many cases, is a poor approximation for n and the second term in the radical may be far from negligible; in fact, for the shorter members, this term is the predominating term. This accounts for the increasing divergence between the author's formula and others as l decreases.

In Fig. 15 a comparison is made, for a 14-in., 142-lb wide-flanged beam, between the author's basic Eq. 7 and Eq. 125, for two cases frequently encountered in practice—(*a*) pure flexure; and (*b*) a uniformly distributed load on the top flange, $H = d/2$. Arbitrary transition curves between the approximate proportional limits and the yield points for each of the classes of steel mentioned by the author have been drawn in this figure. For purposes of ready comparison, the loci of the following formulas have also been shown:

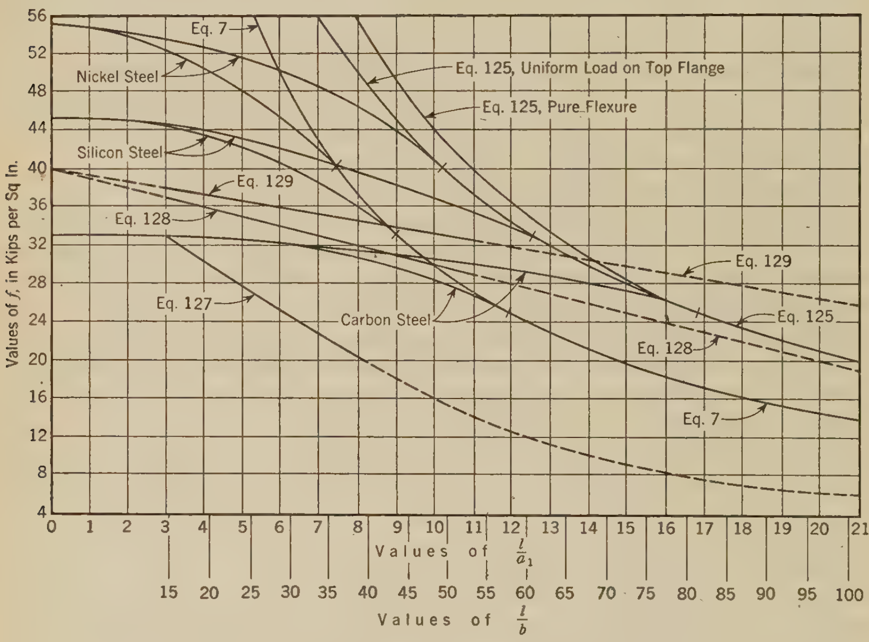

FIG. 15.—COMPARISON OF FORMULAS FOR A 14WF142 BEAM

$$f = \frac{37{,}200}{1 + \frac{(l/b)^2}{1{,}800}} \qquad (127)$$

$$f = 40{,}000 - 60\frac{l}{\bar{r}_y} \approx 40{,}000 - 208\frac{l}{b} \qquad (128)$$

and

$$f = 40{,}000 - 40\frac{l}{\bar{r}_y} \approx 40{,}000 - 139\frac{l}{b} \qquad (129)$$

Eq. 127 will be recognized as the A.I.S.C. specification (1936) formula[59] multiplied by 1.65, which is the safety factor implied therein. Eqs. 128 and 129 between $\frac{l}{r_y} = 50$ and 190 $\left(\frac{l}{b}\text{ between 15 and 55 approximately}\right)$ represent the results of four sets of tests on I-beams and H-beams reported[60] by Herbert F. Moore in 1913. Eq. 128 applies to pure flexure, whereas Eq. 129 applies to the uniformly distributed load case (without taking account of the effect of H). The parts of the loci of these equations which are beyond the somewhat limited range of the tests are shown dotted in Fig. 15. These test results appear to indicate clearly that transition curves such as those introduced by the author are fully warranted.

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[59] "Specifications for the Design Fabrication and Erection of Structural Steel for Buildings," A.I.S.C., 1936.

All the beam formulas given in this discussion pertain to critical (ultimate) conditions. In practice, these formulas should be divided by an appropriate safety factor; or, preferably, the loads should be multiplied by a suitable factor of overload. The latter procedure is especially preferable if the member under consideration is subjected to a combination of flexural and axial loads, in which case the principle of superposition obviously does not apply.

Mr. de Vries is to be commended for having performed a series of tedious calculations most precisely and for again raising an important, but somewhat controversial, subject for discussion. It is unfortunate that test results over an extended range of member sizes, shapes, lengths, loading, and end conditions, with which the author's results and other calculations may be compared, apparently are not available. The writer regrets that space limitations do not permit discussing the paper in detail or to the extent it fully warrants.

In addition to the references cited by the author, and in discussion, several others are especially worthy of note.[61,62,63,64,65,66,67] No attempt has been made to make this list complete.

The writer is indebted to James Adam, Jr., for aid in preparing the illustrations in this discussion.

KARL DE VRIES,[68] M. ASCE.—The general tenor of the discussions has been to the effect that the writer's proposed working, or specification, formula, Eq. 9, is reasonable for most conditions, but that it falls down for very short, slender beams and for plate girders. The writer, in this closing discussion, will attempt to show wherein the discussers have made approximations which he considers incorrect, or have used assumed loadings or conditions which practically cannot exist. The writer also proposes modification of specification requirements to permit the use of the full basic unit stress, without reduction, for the case of braced beams.

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[60] "The Strength of I-Beams in Flexure," by Herbert F. Moore, *Bulletin No. 68*, Univ. of Illinois Urbana, 1913.

[61] "Problems Concerning Elastic Stability in Structures," by S. Timoshenko, *Transactions*, ASCE, Vol. 94, 1930, p. 1000.

[62] "The Torsional Effect of Transverse Bending Loads on Channel Beams," by F. B. Seely, W. F. Putnam, and Wm. L. Schwalbe, *Bulletin No. 211*, Univ. of Illinois, Urbana, 1930.

[63] "Elastic Instability of Members Having Sections Common in Aircraft Construction," by G. W. Trayer and H. W. March, *Report No. 382*, National Advisory Committee for Aeronautics, Washington, D. C., 1931.

[64] "Strength of Light I-Beams," by Milo S. Ketchum and Jasper O. Draffin, *Bulletin No. 241*, Univ. of Illinois, Urbana, 1932.

[65] "The Lateral Stability of Equal-Flanged Aluminum-Alloy I-Beams Subjected to Pure Bending," by C. Dumont and H. N. Hill, *Technical Note No. 770*, National Advisory Committee for Aeronautics, Washington, D. C., 1940.

[66] "Strength of Materials," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., Vol. 2, 2d Ed., 1941, p. 229.

[67] "Lateral Buckling of I-Section Column with Eccentric End Loads in Plane of Web," by Bruce Johnston, *Journal of Applied Mechanics*, December, 1941, p. A-176.

[68] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Bethlehem, Pa.

The writer believes firmly that Eq. 4 is the best and simplest formula for beams in buckling; and substitutions of approximate values for its ratio of moments of inertia and simplifications of its k-values, listed in Table 1, will lead to approximated or distorted results, which should not be compared critically with the true values determined from Eq. 4. Formulas for beams in pure bending are generally used by the discussers. They are simpler than those for most other loadings, but, if other formulas are available, they do not deserve this consideration, as will be proved in this discussion.

TABLE 10.—CRITICAL STRESSES FROM EQ. 4, FAILURE FOR UNIFORM CENTROID LOADS AND FOR

				LOAD AT CENTROID			LOAD AT TOP	
$\frac{l}{a}$	l (ft)	$\frac{l}{b}$	$\frac{l\,d}{b\,t}$	f_{cc} (Eq. 4)	$24 \times 10^6 \div \frac{l\,d}{b\,t}$	$\frac{\text{Col. 5}}{\text{Col. 6}}$	f_{ct} (Eq. 4)	$20 \times 10^6 \div \frac{l\,d}{b\,t}$ (Eq. 7)
				Pounds per Square Inch			Pounds per Square Inch	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				36WF194				
4	36.6	36.3	1,050	27,400	22,860	1.20	20,730	19,050
5	45.8	45.4	1,313	20,360	18,280	1.11	16,000	15,230
6	55.0	54.4	1,576	16,210	15,230	1.06	13,130	12,690
7	64.1	63.5	1,838	13,490	13,060	1.03	11,200	10,880
8	73.3	72.6	2,101	11,560	11,420	1.01	9,790	9,520
9	82.4	81.6	2,363	10,130	10,160	1.00	8,720	8,460
10	91.6	90.7	2,626	9,020	9,140	0.99	7,870	7,620
11	100.7	99.8	2,889	8,130	8,310	0.98	7,180	6,920
				36WF160				
3	32.2	32.2	1,135	28,500	21,150	1.35	20,580	17,620
4	42.9	42.9	1,513	18,760	15,860	1.18	14,200	13,220
5	53.6	53.6	1,891	13,940	12,690	1.10	10,950	10,580
6	64.3	64.3	2,269	11,100	10,580	1.05	8,990	8,810
7	75.0	75.0	2,647	9,240	9,070	1.02	7,670	7,560
8	85.7	85.7	3,026	7,920	7,930	1.00	6,710	6,610
9	96.4	96.4	3,404	6,940	7,050	0.98	5,970	5,880

The simple beam, with its own weight uniformly distributed along its centroid, will have its lowest critical stress if its carried load is uniformly distributed along the top flange (see k-values in Table 1). For relatively short beams the weight of the beam will be a small part of the total load; but as span lengths increase, the weight of the beam will form an increasing portion of the total load until the beam is capable of supporting only its own weight. Uniform top flange and centroid loads, then, should be the ones considered for writing a specification formula of the type of Eq. 9. Mr. Van Eenam finds disagreement on centroid load stresses for two 36WF beams, but his computations are based on approximate values for I_x, I_y, and k. Since the true values are known, the critical stresses for centroid loading have been computed accurately by the writer and are given in Table 10. The values thus determined show that, for these two beams, consistent safety factors for the most unfavorable loading exist. The 36WF194 and 36WF160 sections have the torsional bending constants a = 109.9 in. and a = 128.6 in. respectively.[69] Their critical stresses for uniform centroid and top flange loads are computed in Cols. 5 and 8, Table 10; the corresponding failure stresses computed from the formulas of Table 4 are given in Cols. 6 and 9; and the ratios of critical stress to the corresponding failure stress are shown in Cols. 7 and 10. Col. 11, Table 10, gives the working stress from the proposed specification formula Eq. 9; Col. 12 shows the unit stress resulting from the weight of the beam, w, which is 194 and 160 lb per lin ft, respectively; Col. 13, Table 10, lists the unit stresses that are available

FORMULAS FROM TABLE 4, AND SAFETY FACTORS FROM EQ. 130
UNIFORM TOP FLANGE LOADS, ON A SIMPLE BEAM

FLANGE								
$\frac{\text{Col. 8}}{\text{Col. 9}}$	$12 \times 10^6 \div \frac{l\,d}{b\,t}$	f_c $\frac{12\,w\,l^2}{8} \times \frac{d}{2\,I_x}$	Col. 11 minus Col. 12 (f_t)	$\frac{f_c}{f_{cc}}$ $\frac{\text{Col. 12}}{\text{Col. 5}}$	$\frac{f_t}{f_{ct}}$ $\frac{\text{Col. 13}}{\text{Col. 8}}$	Col. 14 plus Col. 15	n $\frac{1}{\text{Col. 16}}$ (Eq. 130)	$\frac{l}{a}$
	Pounds per Square Inch							
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
				36WF194				
1.09	11,429	588	10,841	0.021	0.523	0.544	1.84	4
1.05	9,139	920	8,219	0.045	0.514	0.559	1.79	5
1.03	7,614	1,324	6,290	0.082	0.479	0.561	1.78	6
1.03	6,529	1,802	4,727	0.134	0.422	0.556	1.80	7
1.03	5,712	2,354	3,358	0.204	0.343	0.547	1.83	8
1.03	5,078	2,979	2,099	0.294	0.241	0.535	1.87	9
1.03	4,570	3,678	892	0.408	0.113	0.521	1.92	10
1.04	4,154	4,451	..	..	..	..	..	11
				36WF160				
1.17	10,573	459	10,114	0.016	0.491	0.507	1.97	3
1.07	7,931	815	7,116	0.043	0.501	0.544	1.84	4
1.03	6,346	1,274	5,072	0.091	0.463	0.554	1.81	5
1.02	5,289	1,834	3,455	0.165	0.384	0.549	1.82	6
1.02	4,533	2,497	2,036	0.270	0.265	0.535	1.87	7
1.02	3,966	3,261	705	0.412	0.105	0.517	1.93	8
1.02	3,525	4,127	..	..	..	..	..	9

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[69] "Bethlehem Manual of Steel Construction," *Catalogue S-47*, Bethlehem Steel Co., Bethlehem, Pa., 1934, p. 285.

for top flange load. The load at the centroid causes the unit stress f_c (Col. 12), but f_{cc} (Col. 5) would be critical; the load at the top flange causes the unit stress f_t (Col. 13) and f_{ct} (Col. 8) would be critical; the critical stress of the beam is reached when both stresses, f_c and f_t, are increased n times, or when

$$\left(\frac{f_c}{f_{cc}} + \frac{f_t}{f_{ct}}\right) n = 1 \quad \ldots\ldots\ldots\ldots (130)$$

in which n is the least factor of safety which the working formula, Eq. 9, has with the critical stress, Eq. 4; n is computed in Cols. 14 to 17. Col. 10, Table 10, gives the close relationship between Eq. 7 and Eq. 4, and Col. 17 shows the nearly constant safety factor regardless of l/a. Since all rolled beams show critical stresses well above the proportional limit for $l/a = 2$ (see also Table 2), similar results will be obtained for all rolled beams, and only special deep sections with thin flanges, having useful lengths below the limit $l/a = 2$, show excessively high safety factors.

To apply Eq. 4 to a thin-flanged deep section with computable a, it was assumed that a rolled beam was cut in the middle of the web, and that 2-ft, 4-ft, and 6-ft additional webs were inserted and welded between the beam halves. It was found for these thin-flanged deep beams that the l/a-ratio for long lengths is larger than 2 and that therefore good agreement of Eqs. 4 and 7 is maintained; but that, for short lengths and increasing depth, Eq. 4 gives critical stresses well above those indicated by Eq. 7. This fault of Eq. 7 has been emphasized by several discussers, especially by Mr. Brameld; but, for reasons explained hereinafter, the writer proposes that the full basic unit working stress be used for such cases.

The treatment of riveted plate girders in the paper has been criticized and preference has been expressed for adherence to Mr. Madsen's[11] findings. Mr. Madsen tested two riveted girders of very small size and thin flanges in torsion and recommended that a K-value equal to one third of that of the identical solid section be used; the writer proposes the modification of Eq. 11, which expresses the properties of the solid section, to Eq. 12. The use of Eq. 12 for thin-flanged girders has the same effect as Mr. Madsen proposes, but the reduction for heavy-flanged girders is less by Eq. 12, and should be less. The writer believes this reduction is necessary not so much for rivet slip, but mostly for the overestimated contribution of the vertical flanges to the K-value of the solid section. In the example of the laminated section of Fig. 6 and its accompanying computations of K, and not considering the effect of the inscribed circle (46.9 in.^{4}), the vertical flange (54.7 in.^{4}) adds about 16% to the total value of K, and will add a similar percentage to the solid K. In a similar example, if the same flange angles are used and the thickness of the cover plates is reduced, the contribution of the horizontal flange (234.0 in.^{4}) is reduced as the cube of its thickness, whereas the vertical flange, which should have a minor influence on torsional stiffness, appears unreduced and has a major influence on the value of K. The same percentage reduction of K for the solid section should not hold for both examples; and it is found that in applying Eqs. 7 and 12 the stresses of the thin-flanged girder correspond to the use of one third, or less, of the K-value of the solid section in Eqs. 4 and 10, and that for heavier flanged girders the used fraction of K of the solid section is more than one third. The heavy-flanged riveted plate girder, then, showed the same full agreement of Eq. 7 with Eq. 4 as rolled beams and heavy-flanged deep beams; and thin-flanged riveted plate girders showed the same gap between Eqs. 7 and 4 as the corresponding thin-flanged deep beams.

Rolled sections, deep beams, and plate girders with intermediate lateral supports between vertical supports, generally, can be divided into two types. If they are of the deck type, the main load is uniformly distributed along the top flange; since specifications require that the compression flange be stayed by lateral bracing and by cross frames at short intervals, the laterally unsupported lengths are short, and the short beam formulas, Eqs. 8, will limit their working stresses in most cases. If it appears from their properties, however, that they will fall in the range of long beams, then Eq. 7 gives, for them, critical stresses that are too low. This is true for two reasons: First, there is the gap between Eqs. 4 and 7 for short deep beams and plate girders with thin flanges; and, second, at $l/a = 2$, for example, a k-factor of 2.269 for simple beams has been used to compute Eq. 4, whereas, actually, $k = 3.499$ for two-bay beams and is larger for these multiple-bay beams. If they are of the through type with bottom flange bracing and brackets to support the top flange, the laterally unsupported lengths are larger, but limited, since the lengths between vertical supports themselves are limited by specifications; the main load is delivered, say, through floor beams and is a concentrated load at these points; and instead of $k = 2.269$ for $l/a = 2$, as used in Eq. 4 to establish Eq. 7, $k = 5.411$ for two-bay beams and is larger for multiple-bay beams. It seems, then, that no reduction of the basic bending stress is required for beams with intermediate lateral supports, regardless of the unsupported length between the points of attachment of these supports, provided the lengths are within usual specification requirements.

All this adds up to the recommendation that Eqs. 8 and 9 be used for specification purposes if the compression flange of the beam has no intermediate lateral supports between its vertical supports, and that the basic bending stress (for carbon steel, 18,000 lb per sq in. for the American Railway Engineering Association (AREA) and 20,000 lb per sq in. for the American Institute of Steel Construction (AISC)) be used if intermediate lateral supports are provided; t in Eqs. 8 and 9 for plate girders and other compound sections is determined from Eq. 12. It is not proposed to abandon the provision of most specifications that the gross section of the compression flange be not less than the gross section of the tension flange.

Eq. 9 is offered to replace the highly inaccurate l/b-formulas in common use and is conservative for loadings other than the one for which it is derived. For special cases, if specifications do not have to be followed strictly, capacities can be computed from Eq. 4, using the proper k-value from Table 1. The general formulas for k are quite complicated, but Eqs. 21 and 36, which are of most interest, can be simplified by dropping y (see Table 7) and formulas will result which, for simple beams, are correct within a fraction of 1%. For example, for uniform top flange load on a simple beam Eq. 36*a* reduces to

$$k = 1.2825 \left(\sqrt{0.481 \frac{l^2}{a^2} + 5.747} - 1 \right) \ldots\ldots\ldots\ldots (131)$$

and for uniform centroid load on a simple beam Eq. 36*b* reduces to

$$k = 0.8895 \sqrt{\frac{l^2}{a^2} + 9.87} \ldots\ldots\ldots\ldots\ldots\ldots (132)$$

and the critical stress may be computed from Eq. 4 since all other terms in that equation are known. A simple method, if a is known as for rolled beams and if l/a is larger than 2, is to increase the working formula, Eq. 9, in the ratio of the proper k-factor of Table 1, Cols. 2, 3, 4, 6, or 7, to that of Col. 5.

The erection problem of picking beams has been mentioned in the paper and Messrs. Hall and Hussey take exception to the statement made. The writer feels that this problem is important enough for a separate paper and beyond the scope of this one.

Professor Winter claims that the writer's formulas do not apply to beams for which $b/t > 20$, and reaches this conclusion from a formula for pure bending. In computing the properties of the beams of Fig. 9, he substitutes actual flange thicknesses in Eq. 7 instead of those determined from Eq. 11, which in this case is preferable to Eq. 12. When t as determined from Eq. 11 is used, close agreement between his and the writer's values results.

Mr. Hall suggests that some wide beams, as the 14WF87 section, fall in the plastic range; the writer's formulas place them there and take full recognition of this fact. Mr. Hall also discusses beams with high $\frac{I_y}{I_x}$-ratios and agrees that it is unnecessary to use his proposed correction factor of Eq. 56. If the correction factor for beams with top flange loading were derived, it would be smaller than the value determined by Eq. 56, and would so complicate Eq. 4 that a general solution of that equation would be impossible. His example of a beam in pure bending, consisting of a single panel of a through plate girder bridge cannot be considered as typical of this condition; it is a multiple-bay beam with load application at the panel points, and gives k-values different from those shown by Mr. Hall.

Mr. Higgins misses in the paper a supporting argument for Eqs. 6. For the steels considered, Col. 5, Table 4, gives proportional limits which are the best the writer could establish, and above which the computed critical stresses do not indicate true stresses. Stresses corresponding to a reduced modulus of elasticity beyond the proportional limit have not been computed; instead, arbitrary transition curves between yield point and proportional limit have been drawn. If the AISC committee on specifications[29] prefers to maintain the basic bending stress up to the intersection with the long beam formula, Eq. 9, then it seems that it introduces a safe and worthwhile simplification, in spite of a slightly lower safety factor which might exist for a short range.

Mr. Hill uses Eq. 75 for centroid load and after making substitutions concludes that the writer's formula for very long lengths may give values 30%

higher than the theoretically correct values. The writer uses Eq. 4 with k of Eq. 36b for the same loading and believes it to be more nearly theoretically correct than Mr. Hill's, since the approximations in Eq. 4, considering rolled beams, are entirely negligible. If also, within the practical range of long spans, all beams computed by Eq. 4 check with Eq. 7, then there seems to be no reason why Eq. 7 should give erroneous results. Mr. Hill states that the writer's formulas become increasingly conservative as the span decreases; the writer has already discussed this for the case of top flange loads, and has proved that it leads to a constant safety factor (see Table 10); for other loads this might need consideration especially if high-alloy materials are used.

Mr. Hussey compares the stresses of the proposed formula, Eq. 9, with those of the old AISC specification formula and finds some rolled beams considerably reduced in their allowable stresses; he fails to state that most rolled beams will have their allowable stresses increased. The proposed formula tends to allot to the beam its allowable unit stress according to its computed buckling strength; and if a beam in the past has been designed for its critical stress this practice does not have to be continued if it is found to be faulty. Mr. Hussey prefers the investigation of a single bay of a multiple-bay beam; the writer believes that it is extremely complicated to simulate the loading on such a separate bay and rather considers the multiple-bay beam. Instead of groups of concentrated and partial uniform loads on simple beams, the extreme cases of a concentrated load at the center and uniform load over the full length of the beam are considered in Table 1, Cols. 2 to 7.

Mr. Brameld presents some very unusual cases of laterally braced thin-flanged deep plate girders; the writer believes that Mr. Brameld will agree with him on the proposed treatment of such girders. To compute girder flanges as struts is equivalent to applying the l/b-rule and should be avoided.

Professor Gaylord starts with the pure bending formula and, correcting it for top flange load, derives Eq. 104, which is possibly closer to theory, but more involved and as limited in range as Eq. 9. Professor Gaylord's Eq. 102a will apply when $l/a < 2$. It is applicable to thin-flanged deep beams with known or computable torsional bending constant a, but for these rare cases the writer would prefer Eq. 4 with Eqs. 131 and 132, respectively. Examples 1 to 3 indicate conservatism of the proposed formula; Table 10, nevertheless, shows that the apparent understress of Eq. 9 for top flange load is false, and that there is really a nearly constant safety factor for the beam.

Mr. Julian gives a thorough investigation of the problem and proposes, instead of Eq. 7, his Eq. 125, the factors of which, if charted or listed in handbooks for rolled beams, would not be available for other sections. The formula is written for pure bending, and the n-values of Table 9 will not make it usable for other loadings, since n, which should be an equivalent of ratios of k, is not as general as indicated in Table 9, but varies with the length of the beam, as can be seen from Table 1. Eq. 7 is not derived from a formula for pure bending, as is Eq. 109, but is derived as an observed approximation to the plotted results of a formula for top flange loading. The limitations stated by Mr. Julian as necessary to justify passing from Eq. 109 to Eq. 7 do not exist, because no such method of approach is in question. Eq. 12, as proposed in the paper for compound sections, can also be applied to box sections. In computing cantilever beams, the writer uses twice the cantilever as the laterally unsupported length.

The writer wishes to thank the discussers for the interest they have shown in the subject of the paper; he appreciates the many new ideas they have expressed and which have led the writer to additional investigations and to conclusions incorporated in this discussion.

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RELIEF WELLS FOR DAMS AND LEVEES

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With Discussions by Messrs. Henry C. Barksdale, Willard J. Turnbull, Glennon Gilboy, W. A. Wall and C. A. Stone, John R. Charles, Horace A. Johnson, P. C. Rutledge, H. H. Roberts and Carter V. Johnson, Frank E. Fahlquist, Harry R. Cedergren, John S. McNown, Reginald A. Barron, Preston T. Bennett, Charles I. Mansur, S. J. Johnson, Kenneth S. Lane, and T. A. Middlebrooks and William H. Jervis.

Synopsis

Since 1930 a large number of flood control dams and levees have been constructed. Many of them are located on pervious foundations—a practice which would have been considered unsafe in the 1920's. In addition, many of the structures are founded on alluvial deposits which usually grade from fine materials near the ground surface to coarser materials in lower portions of the strata. As a result, the lower parts of the deposits forming the foundations are much more pervious than the upper parts, and deep drainage facilities are necessary for relieving high uplift pressures downstream from the structures to prevent serious and alarming boils and seepage. The writers have studied this problem over a period of years and have developed certain design criteria for systems of relief wells which provide deep drainage. Application of these principles has been quite successful and there has been a rather large demand for information on relief well design. This paper will outline the general method of design, together with the theoretical and empirical background, so that it can be used with full knowledge of its limitations as well as of its advantages.

Danger of Uncontrolled Underseepage

The detrimental effect of the outcropping of underseepage at the toe of dams and levees has long been recognized as a major problem in design. This seepage is most dangerous when it develops in the form of uncontrolled boils

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at the toe. The uplift pressure beneath concrete structures, low weirs, and masonry dams on sand and the piping of sand have been the subjects of much study. One form of solution consists of relief wells, which, reputedly, were first used by Indian engineers of the Punjab Irrigation District in Punjab, India. Considerable difficulty had been experienced in the prevention of piping beneath low concrete weirs constructed in connection with irrigation projects. After a comprehensive study of seepage pressures, it was found that excessive uplift existed beneath the aprons and that high hydraulic gradients were present near the escape gradients. The first and obvious attempt at a solution of the problem was the installation of sheet-pile cutoffs; but, because of the excessive depth of the sand formation and because of possible leakage through the sheet piling itself, this method was not always effective. After considerable experimentation both in the field and in the laboratory, the method of treatment finally adopted was the relief well, which allowed exit of the water downstream from the weir in such a manner that the uplift pressures and escape gradients were reduced to safe values. It is believed, however, that the application of relief wells to earth dams and levees is new and that, except in unusual cases, these wells will reduce, satisfactorily, the uplift pressures downstream from dams, and on the land side of levees, by providing a controlled exit for the ground-water flow.

In the usual dangerous condition of seepage that may develop, the stratification is such that the over-all horizontal permeability of the foundation is considerably in excess of the vertical permeability. Therefore, it is easier for the water to flow horizontally, and hydrostatic pressures are built up in the foundation of the structure. Although this condition exists most frequently where a relatively impervious top stratum overlies a very pervious foundation, it also exists where the foundation is composed entirely of sands and gravels. Because of the stratification of more pervious and less pervious materials in all natural sedimentary deposits, it is normal for the horizontal permeability to be greater than the vertical permeability. The conclusion that must be drawn, therefore, is that all pervious foundations on which hydraulic structures are built are susceptible to the development of detrimental boils or uncontrolled seepage at the land-side toe.

In a great number of cases on low dams and levees, the relatively impervious material near the ground surface is sufficiently thick to prevent the development of dangerous boils. Theoretically, the thickness of the top stratum necessary to prevent boils represents an effective weight equal to the uplift pressure at the toe. If no seepage is assumed and therefore no loss of head through the foundation, the effective weight of the top stratum, to provide satisfactory protection against boils, must be equal to the total head. However, this case represents an impossible situation, and a reduction in head from the river side to the land side of from 25% to 50% is not unusual. Thus, a top stratum with an effective weight equal to from 50% to 75% of the net head would prevent boils.

Typical conditions conducive to the development of dangerous underseepage conditions are shown in Fig. 1. The relatively pervious stratum is usually stratified horizontally so that its horizontal permeability is greater than its vertical permeability and may vary considerably in thickness. The relatively impervious stratum may be of variable thickness, or it may be nonexistent. Where it is not present, local nonuniformities in the pervious layer are often such that they can cause boils and piping to occur through local concentrations of flow.

All boils are considered potentially dangerous, regardless of size or volume of flow. The discharge from boils is uncontrolled and there is always danger of piping or "quickening" of the foundation material which will result in sufficient movement of material to cause a complete collapse of the structure. However, areas that may probably be considered the most dangerous are those which, although they do not seep actively at a low head, still have an insufficient top stratum to withstand the uplift produced by a higher head. Under such conditions a sudden rupture of the top stratum is possible which can lead to extremely violent boiling and even to rapid ultimate failure of the dam or levee. In one case known to the writers, a boil broke out under a levee, removing foundation material in sufficient quantity to cause the levee to settle several feet. This incident occurred during the night and numerous guards and patrols on the levee did not discover it until the next morning.

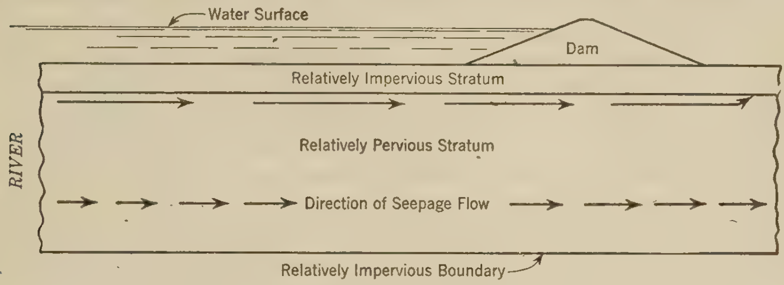
Water Surface
Dam
Relatively Impervious Stratum
RIVER
Relatively Pervious Stratum
Direction of Seepage Flow
Relatively Impervious Boundary

FIG. 1.—TYPICAL CONDITIONS CONDUCIVE TO THE DEVELOPMENT OF UNDERSEEPAGE

Another example of the action of a violent boil is the failure of a flood wall during the 1943 flood. In this case the foundation consisted of a relatively impervious top stratum underlain by clean sand. There was no indication of dangerous underseepage until seven o'clock on the morning of the failure. At this time a small boil broke out approximately 10 ft from the wall. This boil was "sacked" (that is, confined by sand bags), and directly thereafter another boil broke out approximately 40 ft from the wall. While the second boil was being sacked, a third boil broke out 100 ft from the wall. The third boil was quite violent and destroyed a small shed under which it had broken out. Observers stated that within a few minutes the ground between the boil and the wall seemed to disintegrate and a section of the wall moved out. It is interesting to note that the flood wall involved had only approximately 6 ft of water against it and that the average hydraulic gradient in the foundation was approximately 1 on 16. Such a value for average gradient has often been considered safe even with uncontrolled outlet, but this case indicates that the average hydraulic gradient is not a true criterion of safety.

DESIGN OF RELIEF WELL SYSTEMS

In view of the potential danger of boils, a rather large research program on the relief of uplift pressure was undertaken by the Mississippi River Commission, the Vicksburg Engineer District, and the United States Waterways

Experiment Station, at Vicksburg, Miss. The research consisted largely of hydraulic models and electric models, both for designing well systems as a whole and for checking well inflow losses. These models yielded certain information concerning relief well design which has been successfully applied to several earth dams and which has been augmented by various experimental observations in the field.

In the design of a well system operating under gravity flow, there are certain fundamental considerations. First, the design is subject to the usual uncertainties involved in all seepage problems. These uncertainties lie in the determination of the boundaries of the seepage flow and in the accurate determination of the effective coefficient of permeability of the pervious layer to be drained. In the usual case, determination of the limits of the pervious layer (which may be complicated by the existence of upstream borrow pits or by the presence of lenticular impervious deposits), the solution often cannot be obtained by a reasonably economical series of studies. If this is true, estimation of seepage quantity and thus of the number and size of wells is problematical. However, the well system is very flexible and more and larger wells can be added at any time if the first installation proves inadequate. The second uncertainty, of course, is one that can generally be solved by sufficient exploration. Several procedures for sampling pervious materials below the water table have been developed, but these are expensive and can be used economically to only a limited extent. Both these considerations require the application of judgment based on experience to obtain reasonable solutions in complicated geological formations.

In view of these conditions, a theoretical design for a simple case will be developed only for the purpose of showing the relative effect of the variables involved. The effect of these variables can then be used as an aid in adjudging the proper solution of more complicated problems.

The first consideration is that there is a definite upper limit to the quantity of water which can be produced by a gravity flow system. This quantity (Q_M), which will be designated as the "carrying capacity" of the pervious layer, can be described algebraically according to Darcy's law as follows:

$$Q_M = \frac{k\,h\,A}{l} \qquad (1)$$

in which k is the effective coefficient of permeability of the pervious stratum; h is the head at source of seepage; l is the length of seepage path measured in direction of flow; and A is the effective area of pervious stratum at right angles to the direction of flow.

To illustrate this quantity, it can be assumed that a condition exists similar to that shown in Fig. 1. If it is assumed that a free outflow face is introduced at the downstream toe of the dam, that a free inflow face exists at the river, and that the top stratum is impervious, the carrying capacity per foot of dam (q_M) then becomes:

$$q_M = \frac{k\,h\,d}{l} \qquad (2)$$

in which d is the depth of pervious stratum. Therefore, Eq. 2 is representative of complete drainage relief and a drainage system should be designed to carry this quantity plus a reasonable safety factor. Thus, the problem of relief wells is different from the problem usually encountered in dewatering excavations, for instance, where close spacing of a number of short well points or the installation of an additional pump at a lower elevation can be used to lower the ground-water table further by increasing the capacity of the wells. For relief wells to be installed effectively, they must be so placed and so designed that they will either give the desired percentage of relief or furnish the desired percentage of the carrying capacity of the pervious layer without the use of pumps or without excessive outflow loss of head when flowing free.

For the case illustrated in Fig. 1, the purpose of a relief well installation should be to reduce uplift pressure to a safe value. The concept of uplift in the top stratum is somewhat different from that of uplift beneath masonry structures. Consider that a column of the top stratum of unit area is placed in a container such that a head can be applied to its base as shown in Fig. 2 and, furthermore, assume that there is no friction between the soil and the container. Let H be the height of the soil column; h_1 be the head at base of soil column; h_2 be the head at top of soil column; i be the hydraulic gradient; i_c be the "critical" hydraulic gradient (that gradient at which soil column will start to move); γ_w be the unit weight of water; and γ_s be the submerged unit weight of soil. The upward force (p_w) exerted by the water is equal to:

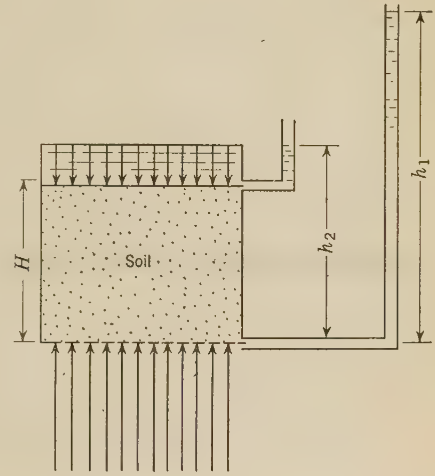

FIG. 2

$$p_w = (h_1 - h_2)\,\gamma_w \quad (3)$$

The resisting force (p_s) of the soil unsupported at the sides of the container is:

$$p_s = \gamma_s H \quad (4)$$

The hydraulic gradient is:

$$i = \frac{h_1 - h_2}{H} \quad (5)$$

and the critical gradient from Eqs. 3 and 4 becomes:

$$i_c = \frac{h_1 - h_2}{H} = \frac{\gamma_s}{\gamma_w} \quad (6)$$

The design should therefore be made with the idea of so reducing h_1 that the critical gradient will not be exceeded. Normally, values for the critical gradient vary from 0.7 to 1.0, depending on the submerged unit weight of the material under consideration, and it may be that design for a safety factor of 1.5 against developing the critical gradient is satisfactory.

For illustration, the case in Fig. 1 will be considered. This case represents a condition that might be expected along the Mississippi River levees. An infinite length of dam or levee and an infinite line drive source can be assumed without too much error, since the river and levee are parallel to each other, roughly. This condition is illustrated in Fig. 3. The line of wells is considered to be of infinite length. The pervious stratum is likewise assumed to be of infinite extent landward (that is, away from the river), with the pervious stratum completely saturated and the top stratum impervious, so that full riverside uplift acts beneath the land-side top stratum. The last assumption is severe, as the pressure reduction due to seepage, which was previously described, has not been considered. Under these conditions, no seepage is occurring to the landward and a fully static condition exists. To simplify the problem further, it will be assumed that the pervious layer is homogeneous and isotropic. The design of the wells considered subsequently will be such as to prevent the development of the critical hydraulic gradient through the top stratum.

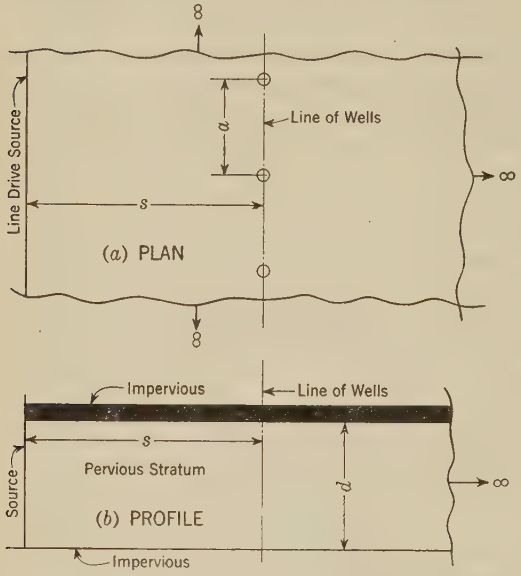

Fig. 3.—Infinite Line Drive Source with an Infinite Parallel Line of Wells

This well system has been studied by other investigators[3] in so far as wells with strainers completely penetrating the pervious stratum are concerned. If the well is regarded as merely a hole with no clogging on the sides (due to the screen soil contact) and with no outflow friction or velocity head loss, a mathematical solution of the flow characteristics of the system, including head midway between wells and behind the line of wells and the quantity of flow from the wells, can be computed.

Morris Muskat gives a solution of this condition but the form is general with respect to the fluid under consideration and the porous medium. Therefore, it is necessary to convert Mr. Muskat's equation into a more suitable expression for use in studying the seepage of water through sand.

Mr. Muskat gives the following equation[4] (converted to the notation of this paper):

$$p = h_s + c \log_e \frac{\cosh j\,(y - s) - \cos j\,x}{\cosh j\,(y + s) - \cos j\,x} \qquad \text{.............} (7a)$$

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[3] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1937.

[4] *Ibid.*, p. 527, Eq. 6.

in which, to simplify typography, $j = \frac{2\pi}{a}$. Furthermore, when a is the well spacing, r_w is the radius of a well, μ is the viscosity of the fluid; and $\frac{K}{\mu}$ is the permeability coefficient k, $\frac{r_w}{a} \ll 1$ and $\frac{2s}{a} \geqq 1$:

$$Q = \frac{\frac{1}{\mu} 2\pi K \Delta p}{\log_e \frac{e^{js}}{j r_w}} \qquad (7b)$$

in which p is the pressure at any point (x, y) and s is the distance from line drive to line of wells. In Eq. 7a, c is a flux coefficient defined as:

$$c = \frac{Q\mu}{4\pi K} \qquad (7c)$$

For these formulas the x-axis ($y = 0$) corresponds to the line drive and the y-axis ($x = 0$) passes through a well. The head h_s is the uniform pressure maintained at the "line drive," and h_w is the pressure at the well, such that $p = h_s - h_w$. The thickness, d, of the sand stratum is unity.

For a point in the line of wells midway between two of the wells $y = s$ and $x = \frac{a}{2} \pm n\,a$, in which n signifies a number in a series (n = 0, 1, 2, 3, 4, etc.). Only one point ($x = \frac{a}{2}$ and $y = s$) will be considered, since the pressure is the same at all such midpoints. Substituting the values for x and y, and letting p equal the unit pressure at point $\left(\frac{a}{2}, s\right)$, Eq. 7a becomes:

$$p = h_s + c \log_e \frac{\cosh 0 - \cos \pi}{\cosh (2js) - \cos \pi} = h_s + c \log_e \frac{2}{\cosh (2js) + 1} \qquad (8)$$

By equating Q from Eqs. 7b and 7c and substituting $h_s - h_w$ for p:

$$c = \frac{h_s - h_w}{2 \log_e \frac{e^{js}}{j r_w}} \qquad (9)$$

Therefore,

$$p = h_s + \frac{h_s - h_w}{2 \log_e \frac{e^{js}}{j r_w}} \log_e \frac{2}{1 + \cosh 2js} \qquad (10)$$

and

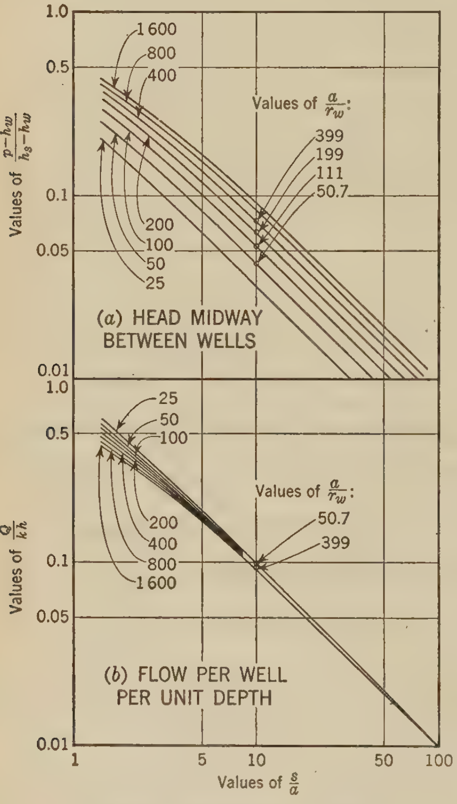
$$\frac{p - h_w}{h_s - h_w} = 1 + \frac{\log_e \frac{2}{1 + \cosh 2js}}{2 \log_e \frac{e^{js}}{j r_w}} \qquad (11)$$

Working with Eq. 7b directly, when $k = \frac{K}{\mu}$ and $h = h_s - h_w$ (since $p = h_s - h_w$):

$$Q = \frac{2\pi k h}{\log_e \frac{e^{js}}{j r_w}} \qquad (12)$$

Curves representing Eqs. 8 to 12 have been reproduced in Fig. 4. With these charts it is possible to determine the flow per well, and the head midway between wells, for an infinite line of wells completely penetrating a pervious stratum fed from a line source parallel to the line of wells. If the effective diameter and the spacing for the wells is assumed and the seepage boundaries and the net head are known, the seepage quantity and the pressure midway between the wells may be found; but, as previously stated, these determinations hold true only for the cases where the actual well points completely penetrate the pervious stratum.

This is a usable solution for such wells. However, if the pervious stratum is very deep, the cost of penetrating it completely with a well strainer is large, particularly as the size of well increases. Larger strainers are very expensive, as is the cost of drilling large wells. Therefore, it was deemed necessary to study partly penetrating well systems. In this connection, the term "percentage of penetration" will be used, referring not to the total percentage of thickness of pervious stratum through which both strainer and riser pipe pass, but only to the percentage of the thickness penetrated by the strainer alone.



(*a*) HEAD MIDWAY BETWEEN WELLS

(*b*) FLOW PER WELL PER UNIT DEPTH

FIG. 4.—CHARTS FOR THE SOLUTION OF EQS. 8 TO 12

No exact mathematical solution is available for studying partly penetrating wells. A series of hydraulic model tests performed at the U. S. Waterways Experiment Station, augmented by electric analogy model tests performed by the Vicksburg Engineer District Laboratory, have yielded empirical data from which design curves were developed. A brief description of the two methods of analysis is pertinent. Hydraulic seepage models are constructed to the same horizontal and vertical scale, and the permeability coefficients of the various materials are so selected that the same ratio of coefficient of permeability is maintained between the various parts of the model as exists between the various similar parts of the prototype. Usually the actual values of the coefficient of permeability chosen are somewhat higher for the model than for the prototype to speed up the action of the model and to reduce capillary effects, which are greatly out of scale in the model. Hydraulic models are subject to several disadvantages such as the capillary effect, high cost, and the clogging of the voids in the model material by air in solution in the water. Also, a fairly large model must be constructed and the degree of control over the model is sacrificed for size. However, the models have a very definite value in indicating, qualitatively, the effect of various drainage systems; and they give a better visual picture of seepage conditions than any other possible type of study. In addition, the models allow a range of coefficient of permeability which cannot be reached in an electric analogy model. Electric analogy models are based on the fact that Ohm's law governing the flow of electricity and Darcy's law governing the laminar flow of seepage are analogous. Since this condition exists, the flow characteristics of both the seepage and the electricity are the same, and similar results can be obtained from hydraulic models and electric analogy models. A description of the hydraulic electric analogy has been given by L. F. Harza,[5] M. ASCE, and others. Electric models have the advantages of being relatively inexpensive and quickly constructed. They are small and can be accurately controlled. However, attempts by the writers to vary the permeability coefficient over a wide range have not been successful. Furthermore, the only seepage systems that can be readily modeled are those completely enclosed by impervious material or those without a free seepage surface.

Therefore, it is probable that a combination of a few hydraulic models with a larger number of electric models for seepage problems of this type forms the best basis for study, and such an approach was used in the investigations.

Several hydraulic models run at the U. S. Waterways Experiment Station illustrate the effect of partly penetrating wells and well spacing under the aforementioned conditions. However, in this case the conditions simulated consisted of a pervious layer that was more pervious toward the bottom. The top stratum was 10 ft thick with a coefficient of permeability of 0.005×10^{-4} cm per sec. The underlying pervious stratum consisted of two layers—the upper one 18 ft thick with a coefficient of permeability of 220×10^{-4} cm per sec and the lower one 22 ft thick with a coefficient of permeability of $3,700 \times 10^{-4}$ cm per sec.

The ratio of permeabilities in this case is about 1 : 415,000. The riverside or upstream end of the two pervious strata was exposed directly to the head on the river side of the levee by a pervious bulkhead, arranged so as to simulate the existence of the river at a distance of about 650 ft from the center line of the levee. An impervious bulkhead was provided on the land side or downstream at a distance simulating 1,270 ft from the river as shown in Fig. 5. The levee simulated was 38 ft high and the maximum head was 37 ft above the land-side toe. The cross section was that of a Mississippi River levee. The length of levee simulated, measured along the levee center line, was 190 ft.

The first experiment (Fig. 5(*a*)) was run with no special treatment for control of underseepage. Without drainage, it was found that practically the full head applied on the river side of the levee acted as uplift under the land-side top stratum. This condition is to be expected in the model, since, without

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[5] "Uplift and Seepage Under Dams on Sand," by L. F. Harza, *Transactions*, ASCE, Vol. 100, 1935, p. 1352.

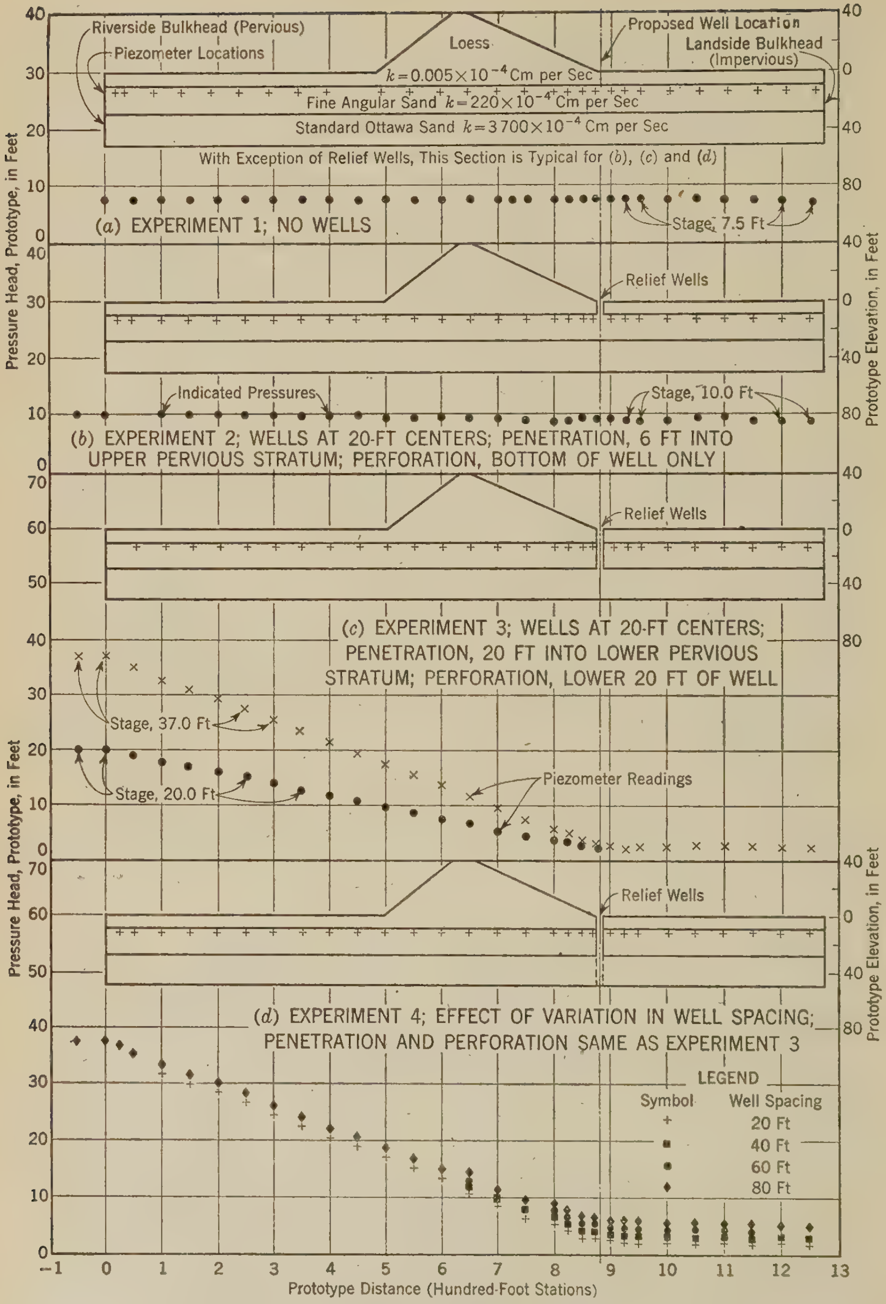

Fig. 5.—Distribution of Hydrostatic Pressure in Hydraulic Seepage Model

movement of water through the foundation material, there can be no loss of head—the pressure drop being caused by the friction loss resulting from flow of seepage water through the sand. The pressure in the model had not appreciably diminished at a distance of 600 ft from the levee. Thus, more leakage would occur in the field and some reduction in pressure would therefore be expected in the prototype. The maximum head shown is 7.5 ft. This head was the highest that could be built up, as a 10-ft head created boils, indicating a critical gradient in the model of approximately 1.

The second experiment utilized wells placed as shown in Fig. 5(*b*). As would be expected, the wells had little effect on the uplift pressure, because they did not penetrate the most pervious material or have sufficient area of well screen.

A third experiment utilized wells spaced at intervals of 20 ft along the land-side toe of the levee. The wells extended through the upper pervious stratum and the effective well point pierced 20 ft of the lowermost pervious stratum. The results of this experiment are shown in Fig. 5(*c*). The major conclusion that may be drawn from this experiment is that a properly designed system of wells will relieve the uplift pressure effectively. If 10 ft of uplift pressure damages the land-side top stratum, this critical pressure corresponding to 10 ft of uplift has been moved back well under the levee by the wells and almost the full height of the levee section is available to resist it. Such a condition is safe. Increasing the percentage of penetration greatly reduced uplift.

A fourth experiment (Fig. 5(*d*)) was run to demonstrate the relatively small effect of varying the well spacing within reasonable limits. In this case the head was kept constant at 37 ft and a safe value for uplift was obtained by using 20-ft, 40-ft, 60-ft, and 80-ft well spacing. The effect of the well spacing on uplift pressure will be discussed subsequently. The effect of varying the spacing is quite small. Wells spaced on 20-ft centers gave 93% relief whereas those on 80-ft centers gave 86% relief.

An attempt was also made independently in the Vicksburg Engineer District Laboratory to obtain similar information from electric analogy models. In the case of the electric models, a homogeneous pervious stratum was used. The first model was set up to simulate the condition computed by Mr. Muskat for 100% penetration. An examination of Fig. 4 will show how closely the model checked curves obtained from the Muskat formulas. Therefore, it was assumed that this check verified the use of the models and the case of partial penetration of wells was studied in the remaining models.

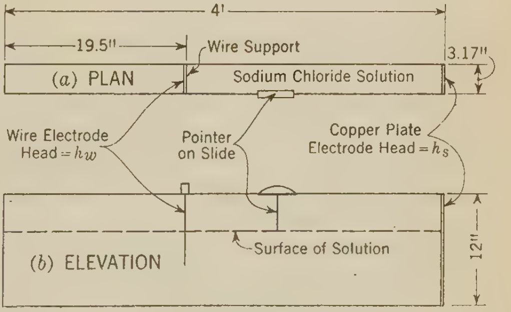

FIG. 6.—SKETCH OF ELECTRIC MODEL SETUP

A diagram showing how the models were set up is shown in Fig. 6. The models consisted of a small wooden flume, 3.17 in. wide, 12 in. deep, and 48 in.

long. The inflow region was represented by a solid sheet of copper in one end of the trough and the well by a copper wire, sized to scale, and a series of tests made varying the wire size and the position of the wire to represent values of s/a from 3 to 10 and values of a/r_w from about 50 to 500. Provision was made for adjusting the depth of penetration of the wire. The pervious stratum was simulated by filling the flume to the desired depth with a sodium chloride solution.

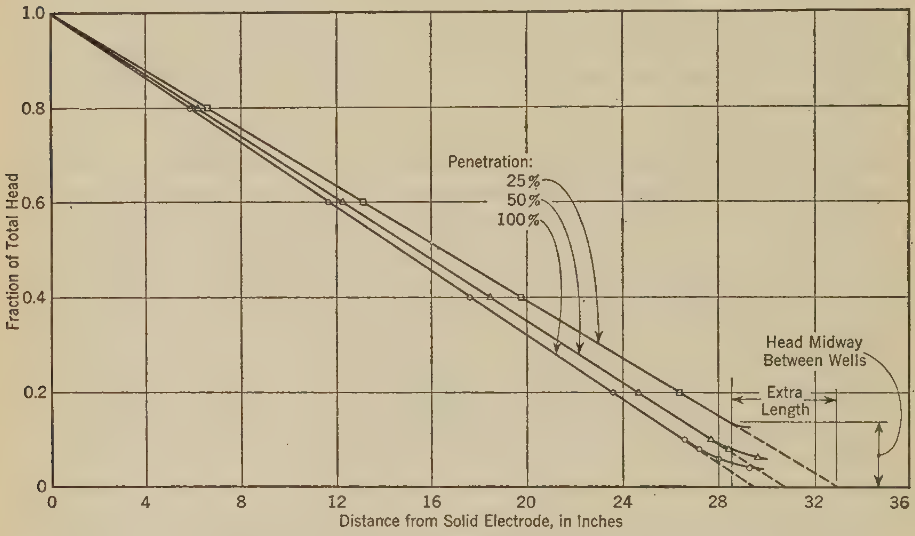

Fig. 7.—Representative Electric Model Test Results; Partial Penetration of Wells; Sodium Chloride Solution Resistance

A plot of typical data obtained from an electric analogy model is shown in Fig. 7. Since the model represents the part of the seepage system drained by one well, it is evident that the line along which the data were taken corresponds to a line at the surface of the pervious stratum midway between two wells and perpendicular to the line of wells and the line drive source. Therefore, the head midway between the wells can be read directly from the curves as a fraction of the head, or voltage on the model. Since the head decreases uniformly with distance over a major part of the system, the flow is uniform in that part of the system and the total flow per well can be calculated by using the total cross section of the flow and the gradient given by the straight-line part of the head distribution.

To interpret the model studies conveniently, a term called "extra length" is introduced. This term refers to the resistance to flow into the wells. If it is considered that a seepage system is flowing out through a vertical outflow face which offers no resistance to the escape of water, the most effective possible pressure relief is produced. If any other system of outflow is introduced, the system has acquired more total resistance to flow which may be expressed as an additional length of pervious stratum. This additional length will be designated as "extra length" and can be determined from the model test results as shown in Fig. 7. Also, the head midway between wells can be found in a similar manner.

The important facts that can be gained from a study of the data are that: (1) The length of the strainer has more effect on the effectiveness of the well system than any other factor; (2) the seepage quantity falls off rapidly; and (3) the head midway between wells rises rapidly as the penetration drops below 25%. An examination of Table 1, which gives results of a numerical example worked out by the writers, shows plainly that well size and well spacing have only minor over-all effects within the limits studied, provided the wells are large enough to carry off the seepage quantity. The curves in Fig. 8, which were plotted from the electric model studies, will readily yield head midway between wells and seepage quantity from a line well system producing from a

TABLE 1.—TYPICAL NUMERICAL RESULTS OF DESIGN COMPUTATIONS
(25% Penetration)

Plan No.	Nominal size of well (in.)	Effective well radius (ft)	Well spacing (ft)	Head[a] (ft)	Seepage quantity[b]	Pipe discharge friction loss (ft)
1	8	0.279	25	11.4	0.095	Negligible
2	8	0.279	33⅓	11.7	0.094	Negligible
3	8	0.279	50	11.9	0.093	Negligible
4	8	0.279	75	13.0	0.091	Negligible
5	6	0.195	25	11.5	0.095	Negligible
6	6	0.195	33⅓	11.6	0.094	Negligible
7	6	0.195	50	12.5	0.093	Negligible
8	6	0.195	75	13.0	0.090	Negligible
9	4	0.104	25	11.6	0.094	0.036
10	4	0.104	33⅓	11.6	0.094	0.066
11	4	0.104	50	12.4	0.093	0.144

[a] Midway between wells. [b] In cubic feet per second for stations 100 ft apart.

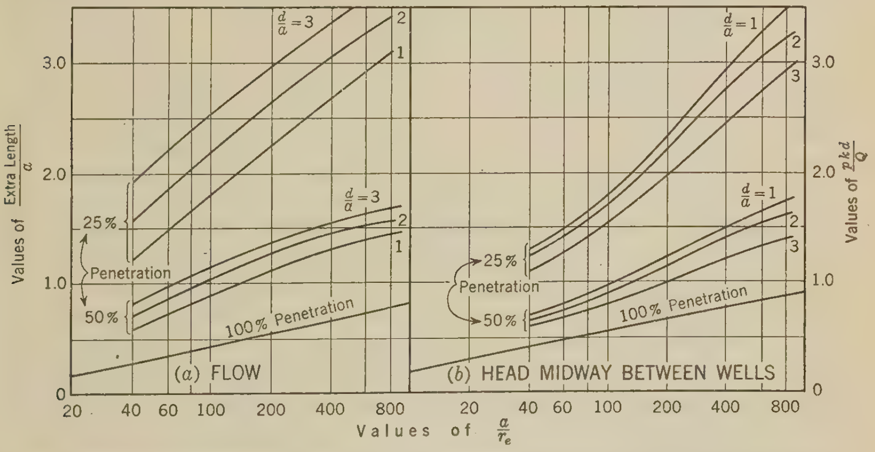

FIG. 8.—CHART FOR FLOW AND HEAD MIDWAY BETWEEN WELLS FOR AN INFINITE LINE OF WELLS TAPPING A PERVIOUS STRATUM FED BY A "LINE DRIVE" PARALLEL TO THE LINE OF WELLS

"line drive" source for any assumed spacing and for 25%, 50%, and 100% penetration. The curves for flow in Fig. 8(a) are used first. From an assumed well spacing and well radius $\left(\frac{a}{r_e}\right)$, a value of $\frac{\text{(extra length)}}{a}$ can be found (r_e is the effective radius). From this the extra length and Q can be computed, using the formula,

$$Q = \frac{k\,h\,a\,d}{s + \text{(extra length)}} \qquad \text{(13)}$$

Then the curves in Fig. 8(b) are entered with the same value of $\frac{a}{r_e}$, and a solution for p is possible. It should be noted, however, that the only criterion is the effect of frictionless wells on a closed seepage system. No consideration has been given to leakage through the top stratum, to losses due to inflow into the well, or to losses due to outflow pipe friction. These factors are of utmost importance, especially when the seepage quantity is large.

The well design curves can be applied in a great number of cases by utilizing the well design in only one part of the seepage system. It can be assumed that there is no further inflow of seepage to the pervious stratum between the line of wells and the upstream toe of the levee or possibly the upstream toe of the core of a dam. A reduced head can be utilized, obtained from the ratio of total resistance to flow on either side of this point, and the assumptions made in developing the curves will then apply quite accurately. The reduced head will depend, of course, on the upstream soil conditions such as width and thickness of blanket and the upstream dimensions of the pervious layer. In considering the system as a whole, the outflow resistance of the wells, the flow between the line drive and the assumed point, the flow between the assumed point and the wells, and the upstream part of the seepage system must all be considered. If this is done, a reliable design can be obtained. Although head midway between wells and seepage quantity can be estimated from the curves in Fig. 8, only in very simple cases can the curves be used directly. Seepage quantities may be considerably in error due to the uncertainties of assumptions as to effective permeability of the stratum and its boundaries as previously described.

When a well problem is to be solved, reference to such curves might lead to the conclusion that 2-in. wells will carry a very large quantity of water. It may be found, however, that this quantity of seepage would entail extremely high friction loss in escaping from the well. Since such head losses must be added to the pressure beneath the top stratum, and since they reduce the flow to be expected from the wells, they may greatly impair relief effectiveness. Under conditions of extremely high permeability coefficients in the pervious stratum, the outflow loss will therefore be the controlling factor in the design and must be so considered, regardless of the results given by the curves concerning the size of well required.

From the foregoing discussion it is evident that the design of the well system is a complicated problem, but one that can be solved reasonably well if judgment is used. However, design by trying various well sizes and spacings can be made on installations of existing structures about which information is not available in sufficient detail for theoretical study—because a system that is too small will be of some help and the flexible nature of well systems allows additions to be made as required.

Installation of Relief Wells

Whenever possible, well systems should be installed before danger develops, but they may be placed as they were at the Fort Peck Dam in Montana with considerable head against the dam or levee. The wells are placed in cased drill holes. The casing is withdrawn from around the well point and the riser pipe, and the hole is backfilled. Installation of the wells with water behind the dam or levee, however, should be done only with the greatest care and only in cases of extreme emergency when other methods cannot be used.

The part of the riser pipe which penetrates the top stratum should be backfilled and tamped with extreme care. A mixture of sand with about 10% bentonite makes an excellent backfill material and is believed generally preferable to local material since the mix can be used in the form of moist balls to cut off any flow of water into the hole and the mixture can then be tamped in successfully in the dry.

Examples of the Use of Relief Wells

Various methods of controlling underseepage are in general use—trench or sheet-pile cutoffs, upstream blankets, and various types of gravel-toe drains. All these methods have been used with success and have proved satisfactory where foundation conditions were favorable. However, these methods are not effective or economical in a large number of cases, and it is preferable under suitable conditions to install relief wells to provide the necessary pressure relief. Where the loss of water is not an item and where surface drainage of the effluent can be readily achieved, relief wells are considered more economical and more generally satisfactory than any of the foregoing methods. However, in some cases, it is advisable to employ other methods in conjunction with the relief wells, particularly for drainage of the dam or levee embankment itself.

The most outstanding example of the use of relief wells is the Fort Peck Dam. The foundation consisted of an extremely tight, practically impervious stratum of clay overlying very pervious sands and gravels.[6] Although a sheet-pile cutoff was driven to shale, sufficient leakage occurred through the pile cutoff to develop high hydrostatic pressures at the downstream toe that produced a head of 45 ft above the natural ground surface. This uplift pressure was first observed in piezometers installed in the pervious sand and gravel foundation. The first surface evidence of the high hydrostatic pressure came in the form of high discharge from an old well casing that had been left in place. It was recognized that this high uplift pressure was dangerous. Therefore, consideration was given to methods of effecting its release. A board of consultants, consisting of Joel D. Justin, W. H. McAlpine, and Arthur Casagrande, all Members, ASCE, and T. A. Middlebrooks, Assoc. M. ASCE, made a complete study of the problem and recommended the immediate installation of relief wells 100 ft outside the toe on 50-ft centers. Since it was considered imperative that the installation be made as quickly as possible, 4-in. and 6-in. well casings, available at the site, were slotted with a cutting torch and installed in the pervious stratum with solid pipe extending to the surface. To afford some relief over the full length of the affected area, the wells were first spaced on approximately 250-ft centers. After intermediate wells were installed, making the spacing 125 ft, it was evident that satisfactory relief could probably be provided at this spacing rather than at the originally proposed 50-ft spacing.

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[6] "Fort Peck Slide," by T. A. Middlebrooks, *Transactions*, ASCE, Vol. 107, 1942, p. 723.

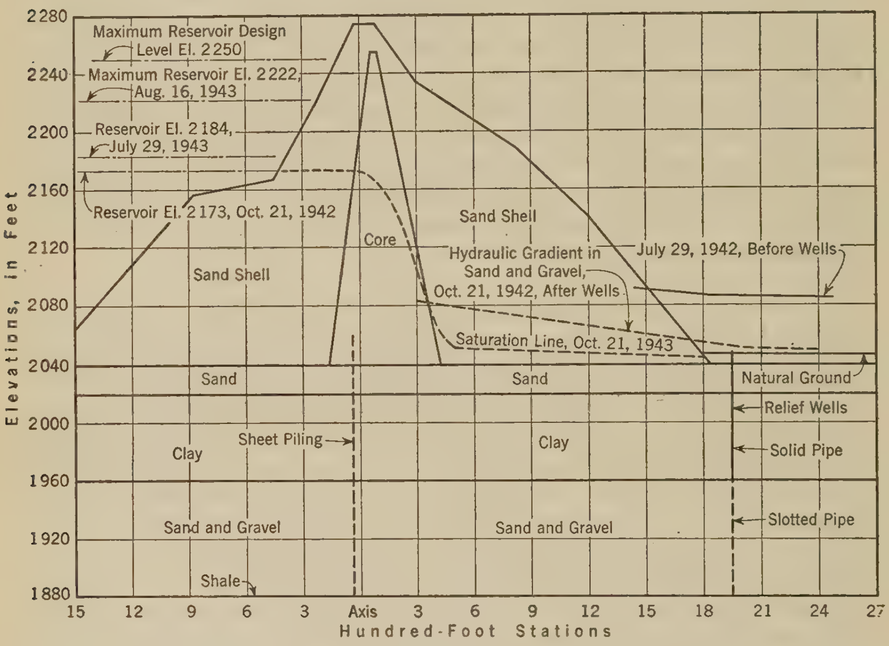
Maximum Reservoir Design Level El. 2250
Maximum Reservoir El. 2222, Aug. 16, 1943
Reservoir El. 2184, July 29, 1943
Reservoir El. 2173, Oct. 21, 1942
Sand Shell
Core
Sand Shell
Hydraulic Gradient in Sand and Gravel, Oct. 21, 1942, After Wells
July 29, 1942, Before Wells
Saturation Line, Oct. 21, 1943
Natural Ground
Sand
Sand
Relief Wells
Clay
Sheet Piling
Clay
Solid Pipe
Sand and Gravel
Sand and Gravel
Slotted Pipe
Shale
Elevations, in Feet
2280
2240
2200
2160
2120
2080
2040
2000
1960
1920
1880
15
12
9
6
3
Axis
3
6
9
12
15
18
21
24
27
Hundred-Foot Stations

FIG. 9.—RELIEF-WELL INSTALLATION, FORT PECK DAM IN MONTANA

This theory was shown to be true after an extensive testing program, and the spacing was left at 125 ft. The results obtained from this installation are shown in Fig. 9. Special attention is called to the facts that the hydrostatic pressure at the toe was reduced from approximately 45 ft to about 5 ft and that the total flow from all twenty wells averaged around 10 cu ft per sec. It is definitely concluded that the installation is entirely satisfactory.

The foundation for the Great Salt Plains Dam in Oklahoma consisted of approximately 30 ft of pervious sand, varying from fairly fine sand near the surface to medium and coarse sand immediately above the rock. In the design of the structure, it was considered adequate to control the saturation line in the structure and to relieve the seepage, partly, in the foundation by a horizontal drain installed at about the midpoint of the downstream slope. Because of the development of small boils at the toe when the reservoir head reached approximatley 20 ft (maximum 60 ft), it was evident that the foundation seepage was by-passing the drain in the dam, since the horizontal permeability of the lower stratum was much greater than the vertical permeability. To relieve this situation, relief wells were installed at the downstream toe. Although the maximum head has not yet been experienced, the functioning of the wells at the lower heads indicates entirely satisfactory performance.

Foundation conditions at Sardis Dam in Mississippi consisted of a deep deposit of sand with irregular horizontal strata of more impervious material. In the design of the dam, a relatively impervious blanket was extended from the core 1,000 ft upstream; and a natural blanket, except in the narrow river channel, extended for an unlimited distance in the reservoir. In addition, the top stratum of less pervious material was excavated beneath the downstream section of the dam and backfilled with more pervious material connecting into a horizontal drain. The first filling of the reservoir showed considerable seepage, as was to be expected. However, the existence of hydrostatic pressures downstream from the dam was evidenced by small boils at the toe and by the increase in flow into sides of the dredge borrow area downstream. Relief wells were installed along the downstream toe and have been in operation for several years. In addition to the regular relief well system on this dam, a large number of different types of wells were installed; and their effectiveness, as evidenced by their flow quantities, was studied. It was discovered that porous concrete pipe and even wooden pipe surrounded by suitable gravel filters could be used satisfactorily for this type of installation. It was found that large gravel-filled wells are inefficient.

The installation on the Arkabutla Dam in Mississippi is believed to be the first installation of relief wells included in the original design of a dam. This dam was designed and constructed by the Vicksburg Engineer District of the U. S. Engineer Department and the board of consultants on the dam were Joel D. Justin, L. F. Harza, W. H. McAlpine, and the late O. N. Floyd, all Members, ASCE, and Glennon Gilboy, Assoc. M. ASCE. The wells were installed on 25-ft centers and were approximately 100 ft inside the downstream toe. The foundation of Arkabutla Dam consisted of approximately 30 ft of impervious redeposit loess over a very pervious sand and gravel deposit. The reservoir has not been filled to a high elevation, but low levels indicate that the performance of the wells is entirely satisfactory. In this case, the excavation of an adequate drainage trench would involve excavation from about 30 ft to 40 ft below the ground-water table. The cost was estimated at approximately $300,000. However, the well installation, the cost of which was about $27,000, is apparently serving the same purpose adequately.

An example of an installation for a levee foundation is that at Lawrenceburg, Ind. The levee foundation in the vicinity of Lawrenceburg consists of an overburden of relatively impervious material overlying a deep stratum of water-bearing sand and gravel. The levee practically rings the town; and, as is to be expected, considerable underseepage occurred even at moderately low river stages. When the old levee was enlarged, a system of drainage wells was installed along the inland toe of the new levee. It was the purpose of these wells to insure the stability of the levee and not to relieve all uplift pressures and underseepage completely. This system has been subjected to two moderately high floods and has been generally satisfactory. Seepage did occur in low areas in the town and at some places along the levee toe, and expansion of the well system will probably be required to provide positive control during higher river stages. At this place, and at other locations where the valley deposit is extremely deep and exposed directly to the river, it may be impossible to take out enough water to relieve all uplift pressures completely and to prevent outcropping of seepage on the land side of the levee. However, it is believed that, in most cases of this type, sufficient water can be released to insure stability of the levee against dangerous boils.

CONCLUSIONS

The writers do not recommend relief wells as the most economical and satisfactory drainage system for all dams and levees, but it does appear that they have proved themselves to have definite possibilities and to be especially suited for certain drainage conditions. However, since it takes some pressure (which is represented by the head midway between and behind the wells) to make them operate, their discharge must be placed below the downstream ground surface, to avoid underseepage completely. Likewise, it will probably be only in unusual cases that the wells can be used for drainage of the dam or levee section. However, they can be combined with section drainage which will result in a more economical system. Also, it is doubtful that wells can be made to stop large or channelized boils in areas of thin top stratum, unless they are pumped, or unless they discharge some distance below the ground surface. However, in spite of their limitations, they should prove very effective and economical where deep drainage is required, and in many cases they can be used with great economy in conjunction with the other types of drainage systems. Experience has shown that large wells (6-in. minimum inside diameter) and deep penetration, full where practical, are necessary for highly efficient operation.

ACKNOWLEDGMENTS

The assistance of the personnel of the U. S. Waterways Experiment Station and the Vicksburg Engineer District in working up the data on which the design of wells has been based is gratefully acknowledged—especially the contributions of W. M. Snyder, Jr., and A. R. Bourquard, of the Vicksburg Engineer District Laboratory, who did a large part of the work on electrical analogy models and those of W. J. Turnbull, M. ASCE, C. I. Mansur, Jun. ASCE, and William R. Perret of the U. S. Waterways Experiment Station, who did a great deal of research on development of new types of well points for the Sardis Dam installation and who conducted the hydraulic model experiments. Acknowledgment is also made to Brig.-Gen. Max C. Tyler, M. ASCE, formerly President of the Mississippi River Commission, to Gerard H. Matthes, Hon. M. ASCE, formerly Director of the U. S. Waterways Experiment Station, and to Maj.-Gen. Raymond G. Moses, formerly District Engineer, Vicksburg Engineer District, under whose general direction a large part of the experimental work was performed.

DISCUSSION

Henry C. Barksdale,[7] Assoc. M. ASCE.—It seems to the writer that Messrs. Middlebrooks and Jervis are to be commended highly for their careful and intensive studies and for the clarity with which the results have been presented. The note of moderation that permeates the whole paper and prefaces the presentation of the theoretical design is particularly gratifying. Experience with the development of ground-water supplies and studies of their characteristics lead the writer to believe that the uniform conditions and homogeneity of the aquifer, which must be assumed to apply any theoretical methods to the solution of such problems, is the exception rather than the rule. Sound engineering judgment is of prime importance in the application of theoretical equations to ground-water conditions. The following comments are intended to be supplementary rather than critical.

Reference is made in the paper to the difficulty of obtaining accurate samples of material at great depths for the determination of the coefficient of permeability. This difficulty is one that has long confronted students of ground-water hydrology. No inexpensive method of obtaining satisfactory samples has been devised. However, the results of pumping tests for determining the coefficient of permeability are being used increasingly. The several methods of determining permeability from pumping tests have been discussed fully by L. K. Wenzel,[8] Assoc. M. ASCE. They depend upon careful observation of the effect of pumping one well upon the water levels in other wells near by. The coefficient of permeability thus determined automatically averages the irregularities of the formation. It is suggested that, in structures of major importance, test wells and pumping tests on them would be a reliable and not unduly expensive method of determining the permeability of the materials beneath the foundation. Some of the test wells conceivably might later be used as relief wells.

Since relief wells must usually depend for their effectiveness upon natural gravity flow, the reduction of losses of head in the structure of the well itself is, as the authors state, a most important consideration. Both the loss in the well casing and the entrance loss through the screen are important. It is recognized that the rate of flow into a relief well would be slower than that into a well pumped for water supply. The losses would therefore be smaller. Nevertheless, experience with wells for water supply has shown that, in the modern gravel-walled type of well, the entrance losses are less than in wells not so constructed. In formations composed of mixed sand and gravels, the gravel wall may be built up by adequately developing the well. If, however, the formation is uniform and fine grained, it may be advantageous to introduce gravel around the screen at the time the well is constructed.

It seems to the writer that relief wells along a levee might present some special maintenance problems. There would be no flow through them over long periods of time. It is conceivable, therefore, that some of the wells in the system might become so clogged by corrosion or by the growth of organisms in the water that they might not be effective when needed in times of flood. Maintenance of the well system would, therefore, become an important part of the maintenance of the levee. Each well should be pumped and tested periodically to assure its continued effectiveness.

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[7] Hydr. Engr., U. S. Geological Survey, Trenton, N. J.

[8] "Methods for Determining Permeability of Water-Bearing Materials," by L. K. Wenzel, *Water-Supply Paper No. 887*, U.S.G.S., Govt. Printing Office, Washington, D. C., 1942.

Where relief wells are an essential part of a dam, the problem of their maintenance becomes especially important. There is usually a more or less constant head, so that the relief wells will flow continually. It would, therefore, be unnecessary to pump them to determine their continued efficiency if arrangements are made to measure the flow from them periodically. The public and in fact many engineers are inclined to assume that the design of a dam more or less permanently assures its safety. Where the safety of the dam may depend upon the effectiveness of relief wells, the need for competent and continuous engineering supervision of the structure becomes imperative, and the responsibility to the public is greatly increased. Under favorable conditions the life of a properly constructed well might be as much as 40 or 50 years depending largely on the character of the water and the type of materials used in the well. Under less favorable conditions the well might fail within a decade or so. In any event sustained vigilance would be necessary to assure the continued safety of the structure.

In conclusion, the writer believes that this paper deserves the widest publicity and discussion. This relatively new method of increasing the safety of dams and levees should be called to the attention of everyone concerned with the design and maintenance of such structures. In presenting this paper the authors are rendering a distinct service to the profession.

WILLARD J. TURNBULL,[9] M. ASCE.—Information and design data on drainage wells have been presented in an interesting manner in this paper. Drainage wells have been used on structures of any degree of importance only in comparatively recent times, and this particular engineering feature should prove of more and greater importance as times goes on with reference to the relief of detrimental substratum pressures beneath and downstream from dams, levees, miscellaneous embankments, and other types of engineering structures. In the not distant future drainage wells probably will be incorporated as a regular feature in the design of all engineering structures built on pervious foundations which are subject to dangerous exit pressures of seepage water.

The question often arises, particularly with reference to structures already built, as to whether drainage wells are needed for the relief of excessive substratum pressures. An effective method in demonstrating the need of a drainage system is by an installation of piezometers. The piezometer installation should be of such magnitude that true gradients of horizontal and vertical pressures can be obtained. Once the true picture of the pressure gradient is known and the actual horizontal and vertical soil profile established, then the engineer will have sufficient data to indicate whether a pressure relief system of some type is needed.

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[9] Chf., Embankment, Foundation & Pavement Div., U. S. Waterways Experiment Station, Vicksburg, Miss.

The authors give a very complete discussion of the critical escape pressure gradients from which boils may result, and state that a pressure head loss of from 25% to 50% is not unusual between the upstream and downstream sides. It is desirable to stress that, in some exceptional instances where the top stratum is quite pervious, the pressure head loss may even be as high as from 70% to 80%. If such a condition exists, it is quite obvious that seepage losses under the structure and through the top stratum behind the structure may dissipate dangerous heads with the result that sand boils could not occur. In these particular cases drainage wells would not be feasible since their effectiveness would be relatively small. Drainage wells also might not be feasible in the case of an extremely pervious and thick foundation where the amount of water from the pervious stratum necessary for adequate pressure relief would require a drainage well system beyond all economical limits of construction. In general, on all ordinary pervious foundations with relatively tight top strata, a properly designed drainage system can be entirely effective and feasible, and in many cases may be a necessity to adequately insure the safety of the structure.

In some instances objections may be raised concerning the additional water that is brought to the surface of the ground by a system of drainage wells. It is true, of course, that any system of drainage brings more water to the ground surface and results in additional drainage requirements. In many cases additional drainage requirements can be easily and economically met. However, in some other instances, particularly behind long levees, the drainage problem may be serious from the practical and economical viewpoint and may have to be handled as an independent project. In these latter cases the increased cost of the well system due to drainage will have to be carefully weighed against the reduced hazards attributed to the drainage well system. As a matter of interest, particularly from the psychological angle, the writer believes that if possible all water from drainage wells should be released into a header pipe beneath the ground surface and collection and disposal should be made in a given sump area. An alternate procedure is to have a properly constructed collection ditch which will carry the water to the disposal sump.

The writer would very much like to indicate a difference between the terms "pressure relief" and "seepage relief." In numerous instances these terms have been used interchangeably although actually there is a vast difference between them. To secure pressure relief—that is, for prevention of boils—an escape pressure gradient of less than 0.6 is certainly adequate in all cases. However, to secure complete seepage relief, a pressure escape gradient of zero is necessary. These facts amply demonstrate why a drainage well system which would give adequate pressure relief may still show considerable free surface seepage. In this connection, the writer wishes to refer to a field experimental installation of pressure relief wells at Sardis Dam at Sardis, Miss. In this installation numerous types of commercial and improvised drainage wells were used. The following is a list of these wells: Improvised—(1) porous concrete (gravel aggregate), bevel joint; (2) porous concrete (slag aggregate), lap joint; (3) cement-asbestos drilled with $\frac{1}{8}$-in. holes on $\frac{1}{2}$-in. centers; (4) square wooden pipe with $\frac{1}{4}$-in. holes on 2-in. centers; (5) perforated concrete pipe with twelve $\frac{1}{4}$-in. holes per foot of pipe; (6) perforated clay pipe with thirty $\frac{5}{8}$-in. holes per foot of pipe; (7) 10-in. steel casing perforated with $\frac{3}{16}$-in. holes on 3-in. by 5-in. centers filled with sand-gravel; (8) 10-in. steel casing perforated as in item (7) with a perforated collection galvanized sheet-iron pipe in the center of the casing surrounded with gravel filter; (9) perforated galvanized sheet-iron tubes with $\frac{3}{16}$-in. holes on $\frac{3}{4}$-in. centers; and commercial—(10) perforated steel pipe with porous cemented gravel filter; and (11) seven commercial well screens with various type slot openings. In all installations of either commercial or improvised wells, a filter was used around the well if the slot opening was of such size that a filter was necessary to prevent the entrance of the foundation sands into the well. In practically all cases the improvised wells have functioned equally as well as the commercially produced products with the noticeable exception of the 10-in. well which was backfilled on the inside with pervious gravel. In this particular case the frictional resistance of the water through the gravel was such that an excessive head loss resulted and in consequence very little water was delivered by the well and very little pressure relief was accomplished.

The U. S. Waterways Experiment Station at Vicksburg, Miss., had previously done considerable work on the design of gravel filters and had arrived at the general criterion that the ratio of the 15% size of the filter material to the 85% size of the foundation material should not be greater than 5. Laboratory tests in a specially designed pressure tank indicated that this ratio might range from 2 to 5, preferably being around 3 to 4. Furthermore, these special tests showed that the ratio of the 85% size of the filter gravel to the diameter of the screen opening should be greater than 1. All filter gravels used in the placement of the Sardis experimental wells were designed on the preceding assumptions and in every case the filters proved to be entirely successful even though in some cases the thickness of the filter around the pipe was not more than 3 in. The necessity for the ratio of the 85% size of foundation sands or filter (as the case may be) to the slot opening (being greater than 1) was demonstrated by the installation of two metal slot wells without filters. In one case this ratio was 1.1 and in the other 0.7. In the former instance the infiltration of sand into the well did not occur, whereas in the latter instance the well became completely clogged with sand.

In the following paragraphs one of two drainage well installations made along Mississippi River levees is described and discussed briefly. At the site of the installation, excessive underseepage accompanied by numerous small-sized to medium-sized sand boils was observed during the 1937 high water. The sand boils were not considered dangerous and only a few were sacked. Heavy underseepage occurred as far as 1,000 ft to the land side of the levee whereas the boil area was confined mainly within 100 ft of the levee toe. A top stratum of relatively impervious silts and silty clays covers most of the area with a thickness ranging from 4 ft to 40 ft. On one low ridge the surface soil consists of a sandy silt which is relatively pervious, and the earliest and heaviest seepage occurred on this ridge. The top stratum is underlain by a fine sand stratum which is quite pervious, and this in turn is underlain by strata of medium to coarse sands and sand gravel, all of which are highly pervious. The average thickness of the top stratum along the line of wells is about 10 ft. The river is about 1,800 ft away. However, large borrow pits exist in front of the levee, some areas of which extend into the fine sand stratum. This fact caused difficulty in arriving at a proper assumption of the over-all average distance of the source of the water from the line of wells. The permeability of the pervious strata was based on laboratory tests obtained on disturbed samples of the material in question. The laboratory tests were used because of the lack of more adequate field information on the in-place permeability of the pervious material.

The installation as made was for test purposes. Consequently, the length of the screen was varied to agree with 10%, 20%, and 30% penetrations of the pervious strata. Every fourth well had a 10% penetration screen at the 20% depth. A minimum clear, inside diameter of 2½ in. was chosen for the well screen and riser pipe. On the basis of the assumptions for permeability and length of flow path and well diameter, a well spacing of 50 ft was used. This spacing did not agree, of course, for the various penetrations but represented an average spacing. The wells used were nonmetallic and consisted of screen, riser, and discharge sections. The screen section was composed of a 4-in. inside diameter porous concrete pipe which served as a filter for a core of perforated clay tile, 2½ in. in inside diameter. The riser section was of solid clay tile, 3 in. in inside diameter, which was connected to a solid clay discharge pipe by a T-section. The riser pipe was carefully sealed through the top stratum by a tamped sand-bentonite mixture. Space does not permit further detailed description of the wells. A surface collection ditch was dug near and parallel with the line of the wells for disposal of the water.

A system of piezometers was installed. The system consisted of a single line of piezometers in the line of wells with several lines running transverse both to the land side and river side of the levee.

During the spring flood season of 1943 a relatively low head of about 7.5 ft was experienced against the levee at the location of the wells. A very brief summary of the test findings follows:

a. The piezometers in the line of wells with all wells closed indicated that about 70% of the river-side head was dissipated by underseepage alone and that with the wells open an additional decrease in head of 10% was obtained. These data indicate that the top stratum at the site of the test installation was too pervious to demonstrate the effectiveness of a well system adequately.

b. Test data indicated that the well discharges obtained were about twice those expected for the given head, thus indicating either that the assumed coefficient of permeability was not great enough or that the effective distance to the source of the water chosen was too great. These facts indicate the necessity of designing a well system with a fairly large factor of safety.

c. Test data indicated that the friction losses obtained in the well, from 2½ in. to 3 in. in inside diameter, were too great for the quantity of water produced and that larger clear diameters were necessary.

d. The nonmetallic type of well as installed is highly efficient in the production of water but needs further thought and study to insure that the wells do not develop faults caused by the many joints which are not positively connected.

e. Test data indicated that the 10% penetration well was relatively ineffective but that, if the wall were placed at the 20% penetration depth, it would produce about as much water as the full 20% penetration well. The 30% penetration well is appreciably more effective than the others, showing that a minimum penetration of at least 25% may be desirable in most cases.

f. Test data were not conclusive because of the relatively low head; higher heads are needed for final and conclusive data.

g. Test data indicated that the lack of information on actual field in-place permeabilities is serious and every effort should be made to determine this value. (An excellent method of determining the field permeability is by a large-scale field pumping test and alternate field procedures are the dye or electric methods; however, in general, determination of permeabilities by pumping water into a stratum is not recommended.)

h. The outlet of drainage wells should be as low as possible.

The writer wishes to emphasize a few features of the design and installation of drainage wells. Extreme care should be taken to secure a tight backfill around the well through the top stratum material. This measure is obviously necessary because the higher pressure of the deeper and usually more pervious strata will be brought directly to the surface through the medium of the well and should the well in any manner become plugged, there may be material danger of piping around the well. Thus, every precaution should be taken so that a drainage well is not plugged in any way while in operation. Another feature of extreme importance in the design of drainage wells is adequate insurance that the diameter of the drainage well is sufficiently great that friction and velocity head losses are maintained at a minimum. It is quite possible that the difference between an effective drainage system and one that is ineffective might be a difference of 1 in. in diameter of the wells. To insure the proper functioning of a drainage well system, it is very essential to remember that a properly designed system may be rendered ineffective by a poor installation. The authors have placed needed emphasis on the importance of obtaining the best information possible concerning (1) proper evaluation of the over-all coefficient of permeability of the pervious strata, and (2) the effective distance between the source of the water and the line of wells.

GLENNON GILBOY,[10] ASSOC. M. ASCE.—A neat summary of the present state of development, both theoretical and practical, of well relief systems is presented in this paper. Although much remains to be learned, such systems have already proved to be of great practical value, and probably they will have an increasingly wider range of application.

Compared with rock toes, gravel drains, and other surface relief measures, the drainage well has outstandingly useful characteristics. Not only does it serve to penetrate a thick impervious blanket at much less expense than trenching, but it also serves, in the pervious stratum itself, to intercept a whole succession of layers, thus providing relief where needed and compensating for the inferior drainage characteristics of stratified deposits in the vertical direction.

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[10] Cons. Engr., South Lincoln, Mass.

The importance of these effects can readily be visualized by considering, as an extreme case, a pervious deposit composed of thin layers of sand separated by layers of tar paper. It is obvious that a surface drain would afford practically no relief, whereas a well, punching through the layers of tar paper, would be very effective. The formation of a boil would correspond to a successive tearing of the tar paper layers, beginning at some weak point, as a result of the horizontally transmitted pressures. The surface drain would not prevent this, whereas a properly designed well system would reduce the pressures so that the progressive rupture could never get a start.

Natural deposits do not normally reach this extreme, but the effect is present to some degree in practically all sedimentary materials. Therefore, the applicability of theoretical analyses and model tests based on homogeneous and isotropic media is limited. As guides such analyses and tests are useful, but it is too much to expect that their results will be reflected quantitatively in the prototype. Thus, although wells are peculiarly adapted to drainage of stratified deposits, the existence of stratification throws a considerable element of uncertainty into the design. It is proper to recognize this condition at the outset and to be prepared to modify the installation if the first trial proves inadequate. Fortunately, as the authors state, well systems are naturally flexible, so that the cut-and-try method is not as much of a drawback as it is in other types of drainage installations.

The authors speak of controlled versus uncontrolled drainage. This concept is of prime importance. In certain cases it is desirable, and often necessary, to make a dam as nearly watertight as possible; but there are many instances in which a high degree of watertightness not only is expensive to obtain, but also is not in the least necessary for the functioning of the structure. In structures intended merely to retard, rather than completely stop, the passage of water, a design which contemplates substantial seepage losses can be made entirely adequate by proper drainage control.

Control of seepage through the body of the dam itself is usually provided by a rock toe, a horizontal drainage blanket, or other suitable method. The factors governing the flow pattern and the quantity of seepage are reasonably well understood, so that the drainage system can be planned intelligently. Good modern practice in this respect, however, is by no means universal. Although the subject has already been well reviewed, within the past five years a flood control dam, with normally dry reservoir, has been constructed with a 1-on-3 upstream slope and a 1-on-6 downstream slope—the avowed purpose of the latter being to keep the line of saturation covered up. The fact that this queer concept is still extant shows the need for continued emphasis on the subject of sound drainage control.

Control of foundation seepage is every bit as important, and is frequently much more complicated. Sometimes it is possible to combine the control systems, draining the foundation into the same element which drains the dam.

Sinking an underdrain through an impervious natural blanket and connecting it to the rock toe of the dam is a good example of this practice. On the other hand, there are many cases in which this procedure is impractical, or, if adopted, would prove ineffective. In this region, the drainage well comes into its own. The well should be considered, therefore, not merely as an adjunct or a corrective measure, but as an element in the design, coordinated with the other elements of the drainage system. The Arkabutla installation is an important advance in this field, and there is every reason to believe that future designs will benefit from a similar approach.

The factual material presented by the authors on existing installations is interesting and instructive. The writer has no comment to make, except to suggest that, for those cases which have not as yet been subjected to extreme operating conditions, the authors present a follow-up when those conditions arise, so that the profession may be informed of the final adequacy of the installations and of the additional corrective measures, if any, which the extreme conditions may require.

W. A. WALL,[11] ESQ., AND C. A. STONE,[12] ESQ.—An interesting and comprehensive discussion of the theoretical principles of underseepage control and the relief of subsurface pressures by the use of relief wells is presented in this paper. The writers, having been concerned with the theoretical design of relief well systems for a number of projects, have used these design principles, and those presented by Preston T. Bennett,[13] M. ASCE, as the "Modified Muskat-Jervis Well-Spacing Formula." The formulas thus applied give results that are in close agreement; and the Bennett method simplifies, to some extent, the assumptions that must be made to permit a mathematical solution. In both cases, however, assumptions must be made for the well spacing, well radius, and well losses; only by repeated trials can a theoretically ideal combination of these variables be found which will provide the required subsurface pressure relief most economically.

To simplify the design computations and to facilitate the determination of the most economical design of a well system, the writers suggest a well-spacing chart for the design of pressure relief well systems. Fig. 10 is a further modification of the "Modified Muskat-Jervis Well-Spacing Formula." The intent and purpose of this chart is to provide a reasonably accurate method utilizing fewer initial assumptions in determining the optimum well system design. The writers were unable to eliminate all the assumptions; however, those that must be made are of minor significance, and the chart provides a rapid method for solving for the optimum well spacing for a given set of conditions.

In the usual problems of relief well design, particularly where the design is made in conjunction with the design of a levee or a dam, the maximum permissible head midway between wells is the prime factor governing the design.

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[11] Engr., Office, Dist. Engr., Kansas City Dist., Kansas City, Mo.

[12] Engr., Office, Dist. Engr., Kansas City Dist., Kansas City, Mo.

[13] "Comments on the Design of Relief Wells," by P. T. Bennett, Rept. on Conference on Control of Underseepage, U. S. Waterways Experiment Station, Vicksburg, Miss., April 1, 1945.

In addition to the head midway between wells, the following factors are known, can be fixed, or must be assumed regardless of the analysis used: (1) The maximum elevation of the impounded water; (2) the maximum allowable pressure in the line of wells; (3) the elevation of the well discharge; (4) the length of path of flow from the source to the line of wells; and (5) well losses. The well losses, which are dependent on the quantity of discharge, size, and type of construction, must be assumed. In properly designed wells, these losses (consisting of the well-screen entrance loss, friction loss through screen riser and casing, and velocity head loss), generally, are of a magnitude of less than 1 ft of head. Except in unusual circumstances this value should not be exceeded and can therefore be assumed for the initial computations.

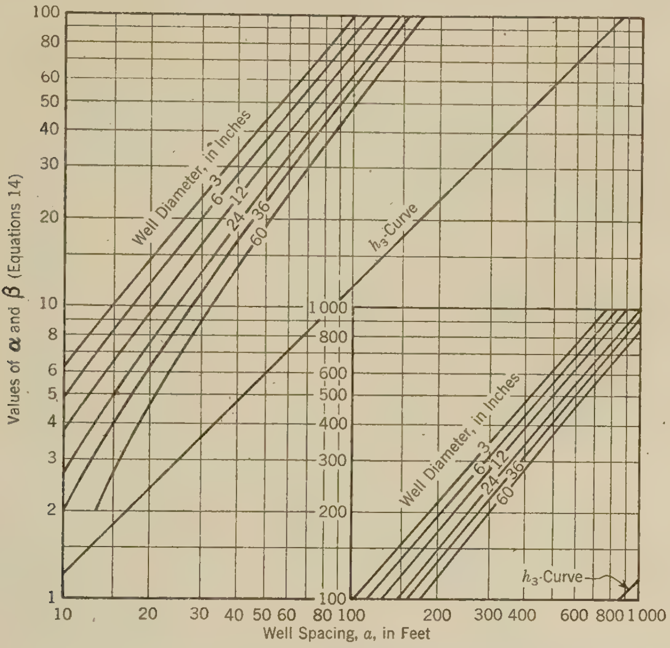

FIG. 10.—WELL-SPACING CHART

Referring to Fig. 11, let the difference in head h_m between the midpoint of wells and the well be expressed as

$$h_m = \alpha \frac{h_r}{l} \qquad 14a)$$

or

$$\alpha = \frac{h_m\, l}{h_r} \qquad (14b)$$

Furthermore, let the difference in head h_3 between the midpoint of wells and the average in the plane of wells be expressed as

$$h_3 = \beta \frac{h_r}{l} \qquad (14c)$$

By entering Fig. 10 with value α, obtained by solving Eq 14*b*, the required well spacing a can be determined for the well size selected, and the value of β can be determined from the h_3-curve for that well spacing.

After solving Eq. 14*c* for h_3, the rate of discharge, Q, per well is obtained by the formula:

$$Q = \frac{k\,a\,d}{l}\,(h_r + h_3) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

A check is then made to determine the accuracy of the assumption made for the well losses, and corrections are made if required. With a little practice, the value can usually be approximated within a few tenths of a foot.

Generally, a slight increase in the well size may be justified in the judgment of the designer to compensate for the decrease in efficiency of the well over a period of years; but an arbitrary decrease of well spacing to compensate for inaccuracies in the coefficient of permeability cannot be justified, since the well-spacing requirement is not a function of the permeability. The maximum well spacing should not exceed the distance from the source to the line of wells, or *l*-distance. This is particularly true where large wells are used with a relatively short distance to the source. This is not in conformity with Messrs. Middlebrooks and Jervis who have indicated that the maximum well spacing should not exceed twice that distance.

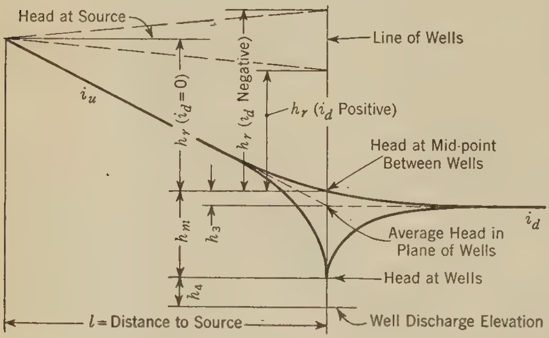

FIG. 11

Fully penetrating wells should be used if economically feasible, as most natural formations present extremely wide variations in permeability from the top to bottom of the stratum. The percentage of penetration of a well should be in proportion to the carrying capacity of the pervious stratum rather than to the thickness of the pervious stratum. Since the permeability usually increases with depth, the economy of the partly penetrating well may be more hypothetical than real. Designs based on partly penetrating wells must, therefore, be very carefully considered on the basis of comprehensive underground data or on actual pumping tests on installed wells.

The theory of pressure relief wells is based on the concept that the well is "a hole with no clogging on the sides." It is apparent that, with present construction methods, an installed well is not 100% efficient. Data pertaining to the relationship of actual production or efficiency of an installed well to the production or efficiency of the theoretical well are very limited or nonexistent. However, since the most open types of well screen have less than 50% open area, it appears that the entrance face area of the well is reduced more than 50% when the well screen is placed in the well to support the formation. Although the entrance face area may be increased somewhat by surging the well to remove the fines, or by placing a gravel filter around the well screen, it is doubtful whether the effective entrance face area of the installed wells is more than 50% of that of the theoretical well. For this reason it is considered that the installed well diameter should be at least three times greater than the theoretical diameter. Although the excess well diameter will provide for subsequent loss of efficiency from clogging and encrustation, it will also produce more water than is required for pressure relief. In those instances where excess well discharge is of consequence from the standpoint of pumping or conservation, an orifice or valve may be placed in the well for reducing the flow to that actually required to produce the desired pressure reduction.

Since mathematical design formulas for relief well systems are based on idealized assumptions as to the uniformity of the pervious stratum and blanket, and the determination of permeability, which is at best only a good approximation, the results from the formulas should not be considered as an exact design but rather as an approximation to aid the judgment of the designer. With this thought in mind it is the writers' opinion that reasonably accurate results quickly computed are more desirable than mathematically exact solutions achieved through repeated trials. Also the writers wish to emphasize that generally wells of large effective radius are necessary to provide sufficient entrance face face area, and that well screen and riser losses should be kept to a minimum.

JOHN R. CHARLES,[14] ASSOC. M. ASCE.—This carefully prepared report on a relatively new use for wells is commendable. Wells to supply water are as old as history; more recently wells have been designed to return water to the ground; and now wells are being designed for the relief of pressure under dams and levees.

It is interesting to note that the problems concerning determinations of permeability and other factors of water source, which increase specific capacity of the well units and provide materials to assure long life with low maintenance expense, have occurred with each of the foregoing type of well. In the modern water-supply well, the outer casings are sealed with cement grout under pressure to prevent seepage down around the casing. Bronze screens and gravel walls reduce the friction loss of entrance and increase specific capacity; outer and inner casings are constructed with welded joints throughout—all in order that the units will have the desired capacity and long life.

At the first introduction of the return well, it was thought that all these refinements might be superfluous; the first return wells were homemade affairs which quickly clogged up, and permitted the return water to escape up around the outer casings. It was soon found that construction equal to that used for the water-supply well was required to provide a constant operating return well.

Similarly, it will be found that relief wells should be constructed in a manner equal to present-day methods accepted for the construction of water-supply wells and return wells. The use of an outer casing sealed with cement grout under pressure would be superior to casing sealed with bentonite and sand mix. Bronze screens will be found necessary to assure continued life and permanent use of the relief wells; the gravel wall is generally accepted as a means of increasing specific capacity and reducing the number of wells required.

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[14] Dist. Engr., Layne-New York Co., Inc., Pittsburgh, Pa.

In 1945, three experimental wells were completed for the Corps of Engineers, U. S. Army, in the Missouri River Valley, to test the type of well best suited for the relief of pressure under levees. Relative specific capacities (in gallons per minute per foot of drawdown) were as follows:

Type	Capacity
Gravel wall (screen C)	224
"Natural gravel pack" (screen C)	68.3
Gravel wall, double cased (screen L)	360

These three wells were located 150 ft on centers in a line along the levee and were provided with 12-in. screens in each case. This test would indicate the variation in specific capacity between the gravel wall well and the so-called "natural gravel pack" as well as between various types of screens.

Relief wells should be constructed in order to provide the highest measure of efficiency in relieving water pressure. They should also be constructed so that there will be the minimum of maintenance and tested at regular intervals. As with most wells, it is to be expected that measures of standard service and cleaning will be required to keep these units in service, and ready for use. Assuredly there would be no time to clean relief wells during the flood period when the relief of pressure is most desperately needed.

Messrs. Middlebrooks and Jervis have cited the uncertainties concerning the boundaries of the seepage flow and the coefficient of permeability. Under the heading, "Design of Relief Well Systems," the authors state: "Both these considerations require the application of judgment based on experience to obtain reasonable solutions in complicated geological formations." Similar experience with water-supply wells proves the accuracy of this statement.

It is possible that some part of the pressure behind a levee may be caused by the increase of water in the natural underground formations. Heavy rainfall on the sides of the valley would result in a ground-water peak moving toward the river. The weight of the levee would help to confine this peak, and the effect of the river rising faster than the ground water would also increase the pressure immediately outside the levee. The combination of these two pressures at the line outside the levee might account, in some part, for the greater-than-anticipated pressure in this area. Similarly, the increase of natural underground water following heavy rainfall, and consequent increase of natural underground pressure, might account for the greater flows beneath dams than would be indicated by the hydrostatic pressure of the water in the reservoir.

The writer can visualize dual-purpose wells constructed to relieve pressure on dams and levees and also to supply this water, which has been filtered and purified by passage through the sands, for use in water-supply systems.

Horace A. Johnson,[15] Assoc. M. ASCE.—It has been assumed in this paper that an infinite line source is paralleled by an infinite line of wells. This is a nearly true condition in many levees and simplifies the problem by making it two dimensional. However, in the design of an earth dam across a V-shaped or U-shaped valley this assumption may be considerably different from the true condition. The purpose of this discussion is to show that by use of the electrical analogy, and by constructing a three-dimensional model, the foregoing assumption may be dispensed with in certain cases, thus broadening the field of application. The authors discuss the use of the electrical analogy, but confine the discussion to a two-dimensional model. In many cases it is practically as easy to construct a three-dimensional model as it is to construct a two-dimensional model and a more exact representation and solution are obtained.

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[15] Engr. (Civ.), U. S. Engr. Office, Sacramento, Calif.

As a case in point, consider a pervious stratum with a very low dip, downstream about 2°, and striking approximately at right angles to the stream bed, which outcrops about 3,000 ft above the proposed dam. At the downstream toe of the dam in the stream bed this stratum is overlain by 100 ft of practically impervious material. With a depth of water of 125 ft in the reservoir, or a head of 225 ft to the bottom of the impervious stratum, the weight of this overlying stratum gives a factor of safety of less than 1 against uplift, assuming no loss in head through flow in the pervious stratum and no flow in the impervious stratum. Fig. 12 illustrates the conditions existing in the example. Permeability of the stratum A was determined by well tests to be 50 ft per day with a hydraulic gradient of 1. Stratum B is very impervious as attested by the fact that an artesian head of approximately 30 ft above river water level exists in stratum A. Stratum B is assumed to be absolutely impervious in the analogue.

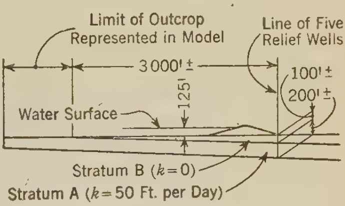
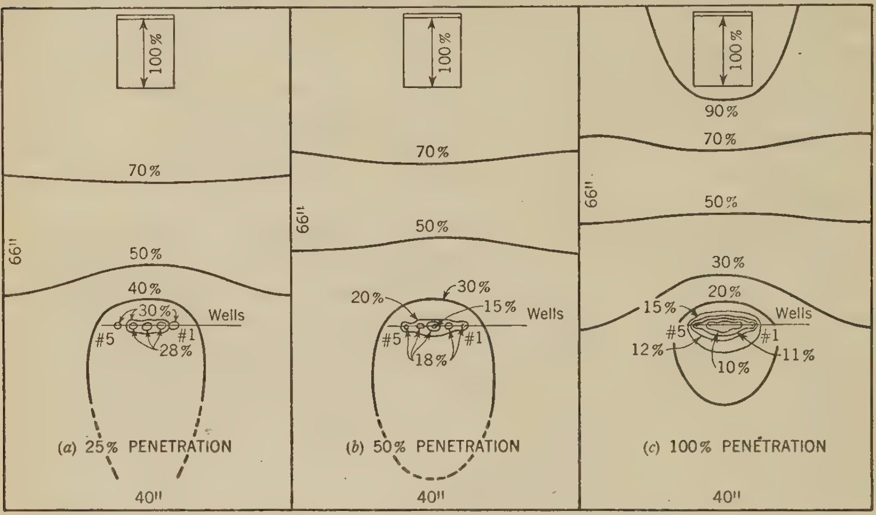


FIG. 12

With these conditions existing it is necessary to reduce the pressure beneath the stream bed and up the abutment slopes until there is sufficient material overlying stratum A to resist the uplift. Five wells spaced at 200-ft centers were tried and gave satisfactory results in pressure reduction.

An electrical analogue was set up on a scale of 1 in. equals 100 ft with 2 in. of water representing the 200 ft of stratum A. The bottom of stratum B was originally represented by the bottom of the electrical analogy tank. The model was thus built upside down, with the wells projecting up from the bottom of the electrical analogy tank. It was found more convenient to rebuild the model right side up for ease in adjusting the well penetration. The wells were represented by No. 22 gage copper wire, which is 0.025 in. in diameter. The source of inflow was represented by a copper plate 8 in. by 12 in. with 2 in. of the 12-in. dimension bent at 90°. The 8-in. by 10-in. dimension was placed at the water surface level with the 2-in. leg projecting down to the bottom of the tank. The area of this plate represented the area of the outcrop of stratum A in the reservoir.

Although the boundary conditions are not exactly represented because of the limitation of the size of the tank (40 in. by 66 in.), the effect of the imposed boundaries can be estimated by building the model at a smaller scale. In this case another model on a scale of 1 in. equals 200 ft was constructed and tested. Results of this test indicated that the effect of the artificial boundaries on the larger model (1 in. equals 100 ft) were very small.



100%
70%
50%
40%
30%
Wells
#5
#1
28%
66"
(a) 25% PENETRATION
40"
100%
70%
50%
20%
30%
15%
Wells
#5
#1
18%
66"
(b) 50% PENETRATION
40"
100%
90%
70%
50%
30%
20%
15%
Wells
#5
#1
12%
11%
10%
66"
(c) 100% PENETRATION
40"

FIG. 13.—EQUIPOTENTIAL LINES FOR DIFFERENT DEGREES OF WELL PENETRATION

Results of the electrical analogy tests for three different well penetrations of 50 ft, 100 ft, and 200 ft, representing 25%, 50%, and 100% penetration of the 200-ft stratum, are presented in Fig. 13. In these tests the source was at 100% potential and the wells at 0% potential. The other lines marked with percentage figures represent the loci of points having those percentage potentials.

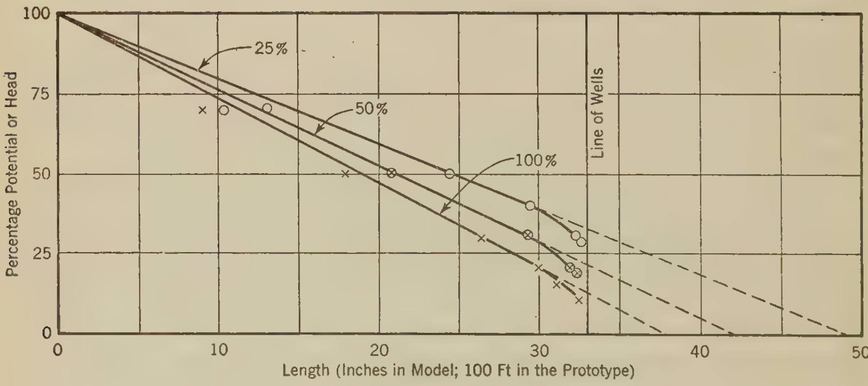


FIG. 14

In order to provide a rough comparison with Fig. 7, Fig. 14 is a plot of the potentials along an axis through the center well and the center of the source.

As would be expected, because of the finite number of wells, the "extra length" is considerably greater than that obtained by the authors, but the effect of penetration is relatively about the same.

A further difficulty with a finite number of wells is the computation of flow from each well. The computation of the total flow from all the wells can readily be made by sketching in the flow lines on the potential net. However, it is very difficult to extend this method to an individual well. The flow in individual wells, therefore, was obtained by calibrating the electrolyte and measuring the flow of current into each well. This was done by running individual wires to each well, maintaining all wells at zero potential, and reading the current for each well in turn. Total flow was also measured by measuring the current in all wells simultaneously. The results of current measurements and conversion to flow of water in Table 2 appear to be very satisfactory. The total flow was also checked by sketching the flow net.

TABLE 2.—FLOW MEASUREMENTS[a]

Well No. (Fig. 13)	(a) 25% PENETRATION; $k' V d' = 140$			(b) 50% PENETRATION; $k' V d' = 157$			(c) 100% PENETRATION; $k' V d' = 19.6$		
	I (1)	Q (2)	% (3)	I (1)	Q (2)	% (3)	I (1)	Q (2)	% (3)
1........	15.7	1.62	103.5	21.1	1.95	107.8	3.1	2.28	114.0
2........	14.9	1.54	98.5	18.6	1.71	94.5	2.5	1.84	92.0
3........	14.3	1.48	94.5	18.3	1.69	93.3	2.4	1.77	88.5
4........	14.8	1.53	98.0	18.9	1.74	96.2	2.5	1.84	92.0
5........	15.9	1.64	105.0	21.1	1.95	107.8	3.1	2.28	114.0
All.......	75.5	7.80	....	100.0	9.21	....	13.8	10.2	....

[a] $k = 50$ ft per day; $h = 125$ ft; $d = 200$ ft; $k\,h\,d = 1{,}250{,}000$ cu ft per day or 14.47 cu ft per sec; I (see Cols. 1) = measured flow of electrical current; Q (see Cols. 2) $= I \frac{k\,h\,d}{k'\,V\,d'}$ in cubic feet per second (see Eq. 16); and the values in Cols. 3 represent the well flow compared to the average, rated as 100.

The flow of current in the model is easily converted to water flow in the prototype since

$$\frac{Q}{I} = \frac{k\,h\,d}{k'\,V\,d'} \quad \text{(16)}$$

in which Q is water flow in cubic feet per second; k is the effective coefficient of permeability of the pervious stratum; h is the head in feet at the source of seepage; d is the depth in feet of the pervious stratum; I is the current flow in the model; k' is the electrical conductivity of the electrolyte; V is the voltage impressed on the model; and d' is the depth of electrolyte in the model. The product $k'\,V\,d'$ can be determined directly without breaking it down into component quantities by calibrating the electrolyte in a tray of known dimensions. No absolute values of the current flow I, in amperes, were obtained in these experiments; relative values only were obtained.

Two factors proved to be troublesome in the model testing for current flow. The first of these was discovered when a calibration of the electrolyte from a cylindrical surface to a well did not check the calibration using a rectangular tray. The apparent explanation was that the effective diameter of the model well was considerably smaller than the actual diameter of the wire representing the well. The effective diameter of the well can be computed from the formula:

$$I = \frac{2\,\pi\,k'\,d'\,V}{\log_e\left(\dfrac{r_c}{r_w}\right)} \quad \text{(17)}$$

in which r_c is the inside radius of the calibrating cylinder, and r_w is the effective radius of the model well. All the factors except r_w are known or can be measured; so r_w can be determined. Eq. 17 is analogous to the formula for flow of water into a single well.[16]

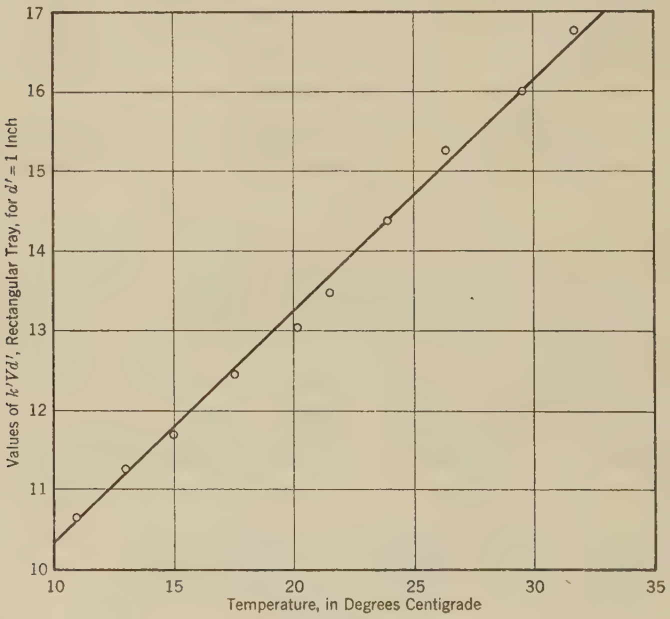


FIG. 15

A further series of tests, run to determine the effective diameter of various sizes of electrodes at several different current concentrations, showed that the effective diameter was only from 5% to 10% less than the actual diameter. Since this represents a difference of actual current flow into the electrode of only 2% or 3%, it was decided to ignore the effect of decrease in diameter as insignificant.

The second factor that must be watched in the measurement of current is the temperature of the electrolyte. It was found that a very few degrees of difference in temperature had an appreciable effect on the current flow. It is necessary, therefore, to calibrate the electrolyte at the temperature at which the experiment is run. Fig. 15 shows the effect of temperature on the conductivity of the electrolyte used which was tap water of unknown analysis.

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[16] "Methods for Determining Permeability of Water-Bearing Materials," *Water-Supply Paper No. 887*, U.S.G.S., U. S. Govt. Printing Office, Washington, D. C., 1942, p. 79.

P. C. RUTLEDGE,[17] Assoc. M. ASCE.—An outstanding feature of this paper is a prudent use of rather complex theoretical analyses. The authors have emphasized the uncertainties in the field conditions and the necessity for tempering with judgment a design based on their analyses. They claim as a significant advantage the flexibility of relief wells because modifications of an installation can easily be made to meet conditions disclosed by field use for the control of underseepage.

One question which the writer pondered after studying this excellent paper was: "On what can one base one's judgment of a relief well design?" Engineering judgment should be based on accumulated experience and field observations, or on an orderly method of estimating the effects of the several variables involved. Both bases for judgment seem to be lacking for this problem. In particular, the uncertainties in the character and the extent of the pervious soil layer drained by relief wells have effects that cannot be evaluated directly by judgment. Briefly, these uncertainties about the actual pervious soil layer are five:

(*a*) Degree and effects of stratification;
(*b*) Effects of lenticular deposits of silt and clay;
(*c*) Effective depth of the entire layer;
(*d*) Effective distance from the source to the wells; and
(*e*) Effective permeability of the layer.

It may even be argued that these uncertainties invalidate the analyses made by the authors. The writer does not agree with such an argument, however, and will endeavor to show in this discussion that a rational basis for judgment can be formulated and that the effects of the uncertainties previously listed are not serious.

Neglecting for the moment the uncertainties in the pervious soil layer and assuming, in its place, a homogeneous layer of measurable dimensions, the variables that affect the performance of drainage wells are:

1. Length of the pervious layer, s, in feet;
2. Depth of the pervious layer, d, in feet;
3. Effective coefficient of permeability of the pervious layer, k, in feet per minute;
4. Spacing of the wells, a, in feet;
5. Percentage penetration of the wells into the pervious layer;
6. Difference in head between the source and the discharge point, Δh, in feet;
7. Effective radius of the well, r_w, in feet; and
8. Frictional head loss in the well screen and riser pipe.

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[17] Prof., Civ. Eng., The Technological Inst., Northwestern Univ., Evanston, Ill.

The inflow into a well is determined by the first seven variables when the difference in head is that between the source and the point of inflow. The effective radius of the well is rarely the actual pipe radius. It may be larger if the well pipe is surrounded by a filter or smaller for a screened well in contact with the soil. The radius of the well pipe governs frictional head losses for flow up the pipe but otherwise has nothing to do with the well inflow.

Eight variables, all of which definitely affect performance, are too many to think about as clearly and concisely as one must in the exercise of judgment. To simplify the problem the authors have introduced two very useful terms:

(1) "Carrying capacity" or maximum possible discharge of the pervious layer under any given conditions; and
(2) "Extra length" which relates well discharge to the "carrying capacity."

The writer will use these two terms to simplify the problem to a greater extent. The authors have shown that the carrying capacity, Q_M, and the discharge of one well in a line of wells, Q_a, both in cubic feet per minute, can be expressed as follows:

$$Q_M = \frac{k\,\Delta h\,a\,d}{s} \qquad (18a)$$

and

$$Q_a = \frac{k\,\Delta h\,a\,d}{s + \text{extra length}} \qquad (18b)$$

Further simplification is obtained if Eqs. 18 are written in terms of

$$\mathbf{W} = \frac{Q}{k\,\Delta h\,d} \qquad (19)$$

in which **W** is a dimensionless number which might be called the "well production number." It can readily be shown that the solution of every problem of steady flow into a well can be expressed in terms of **W** and that, for every problem, **W** depends only on geometrical considerations. Thus, the production number for the "carrying capacity" is

$$\mathbf{W}_M = \frac{a}{s} \qquad (20a)$$

For a single well completely penetrating a homogeneous pervious layer fed by an infinite line source, the production number is

$$\mathbf{W}_1 = \frac{2\,\pi}{\log_e\left(\frac{2\,s}{r_w}\right)} \qquad (20b)$$

For a single well on the perpendicular bisector of a line source of finite length, a, the production number is

$$\mathbf{W}_{1a} = \frac{2\,\pi}{\log_e\left[\frac{2\,s}{r_w}\left(1 + \frac{4\,s^2}{a^2}\right)\right]} \qquad (20c)$$

For one well in a line of wells spaced at a distance, a, on centers and completely penetrating a pervious layer fed by an infinite line source, the production number is

$$W_a = \frac{a}{s + \frac{a}{2\pi}\log_e \frac{a}{2\pi r_w}} = \frac{a}{s + \text{extra length}} \quad \text{..........} \quad (21a)$$

From Eq. 21a it is obvious that the extra length for fully penetrating wells is

$$\text{Extra length} = \frac{a}{2\pi}\log_e \frac{a}{2\pi r_w} \quad \text{................} \quad (21b)$$

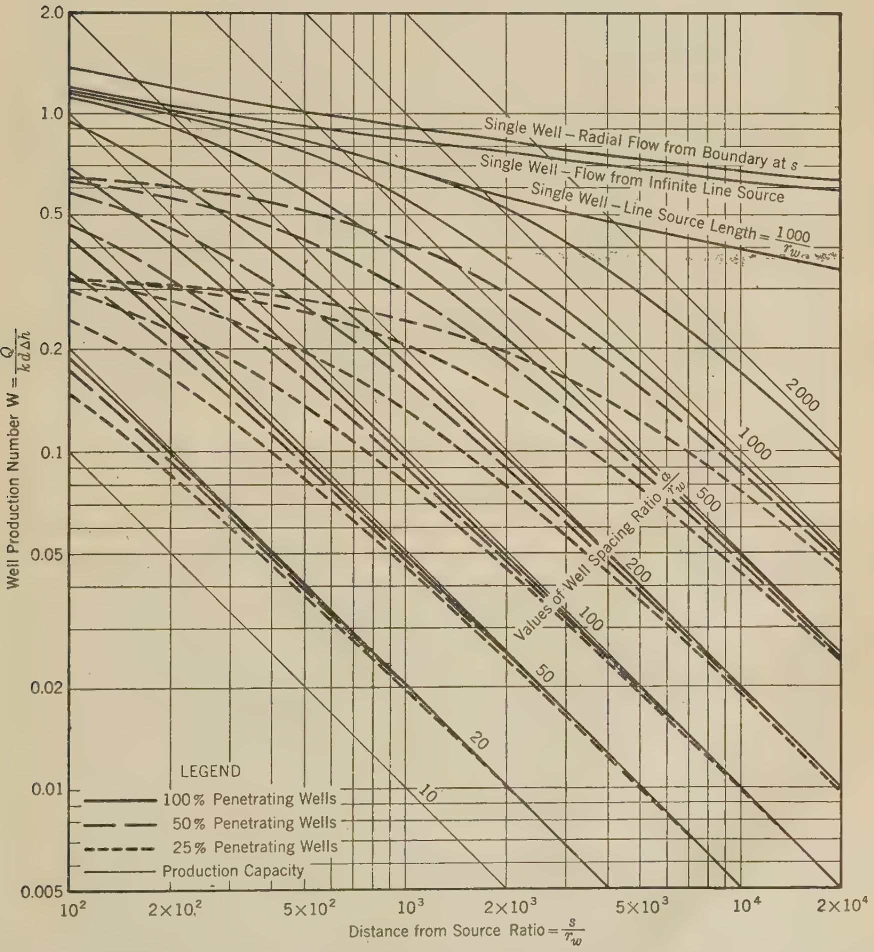

FIG. 16.—WELL PRODUCTION NUMBERS; FOR STEADY FLOW ONLY

The basic solutions for the foregoing cases, and for many others, have been presented elsewhere[18] by M. Muskat. Fig. 16 shows values of the well production numbers for various values of s and a expressed as ratios of the effective well diameter. This chart shows the dimensional conditions under which fully penetrating wells in a line produce practically the equivalent of the carrying capacity of the pervious layer. As the well spacing is increased, the productions of individual wells in a line approach, but do not equal, the production of a single well fed by an infinite line source or by a line source with a length of the order of 1,000 ft. The production numbers of single wells are indicative of performance but are not numerically significant. The reason is that the solutions for single wells assume that the well has been flowing long enough to have affected the heads in the entire pervious layer. Even with very pervious materials, this condition is not reached within any practical period of time.

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[18] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1937, pp. 178, 191, and 529.

The disadvantages of Fig. 16 are that a comparison of well production to the carrying capacity is more significant than the actual well production numbers and that, as the authors have stated, wells do not fully penetrate the pervious layer in most field installations. For fully penetrating wells the ratio of well production to carrying capacity is

$$\frac{W\,a}{W_M} = \frac{Q\,a}{Q_M} = \frac{\frac{s}{r_w}}{\frac{s}{r_w} + \frac{a}{2\,\pi\,r_w}\log_e\frac{a}{2\,\pi\,r_w}} \quad\text{..............}\quad (22a)$$

This ratio might be called the "relative production." For partly penetrating wells the relative production can be computed from the experimental curves in Fig. 8:

$$\frac{W_{ap}}{W_M} = \frac{Q_{ap}}{Q_M} = \frac{\frac{s}{r_w}}{\frac{s}{r_w} + \frac{a}{r_w} \times \frac{\text{extra length}}{a}} \quad\text{..............}\quad (22b)$$

in which the extra lengths are taken from the experimental curves for various values of $\frac{a}{r_w}$ and of percentage penetration. An analysis of fully and partly penetrating wells, made by replotting the experimental data in Fig. 8, indicates that, within the limits of accuracy required by any practical applications, well spacing and degree of penetration are directly compensating variables in the relative production. Thus relative production depends on a term of the form:

$$\frac{a}{r_w}\,\frac{100}{\%\text{ penetration}} = \text{inflow length} \quad\text{...............}\quad (23)$$

In other words, the inflow lengths of well systems with 100% penetrating wells 80 ft on centers, with 50% penetrating wells 40 ft on centers, or with 25% penetrating wells 20 ft on centers are identical. Fig. 17 is a plot of relative production in terms of s/r_w and inflow lengths. This one chart summarizes completely the performance of wells penetrating homogeneous pervious layers of measurable dimensions.

Uses of Fig. 17.—The purpose of relief wells is to reduce to safe values the hydrostatic pressures in underlying pervious soil layers on the land side of levees and the downstream side of dams. A continuous vertical discharge face which completely intersects the pervious layer, the condition assumed for the carrying capacity, limits the head at and on the landward side of the discharge face to the discharge head. The carrying capacity and the discharge of a line of wells are affected in exactly the same way by stratification, lenticular inclusions, and uncertain effective permeability of the pervious layer. Therefore, the relative production is not affected by these uncertainties in the character of the pervious layer and, if the relative production of a line of drainage wells is 100%, they are doing the same job as a continuous discharge face. If the relative production is 90%, computations from the test data in the paper indicate that midway between wells the maximum head in excess of the discharge head will not exceed 20% of the total change in head for any combination of well spacing and percentage penetration. Since the test data presented do not include penetrations less than 25%, this and all other conclusions in regard to effects of partial penetration may not be valid for penetrations less than this value.

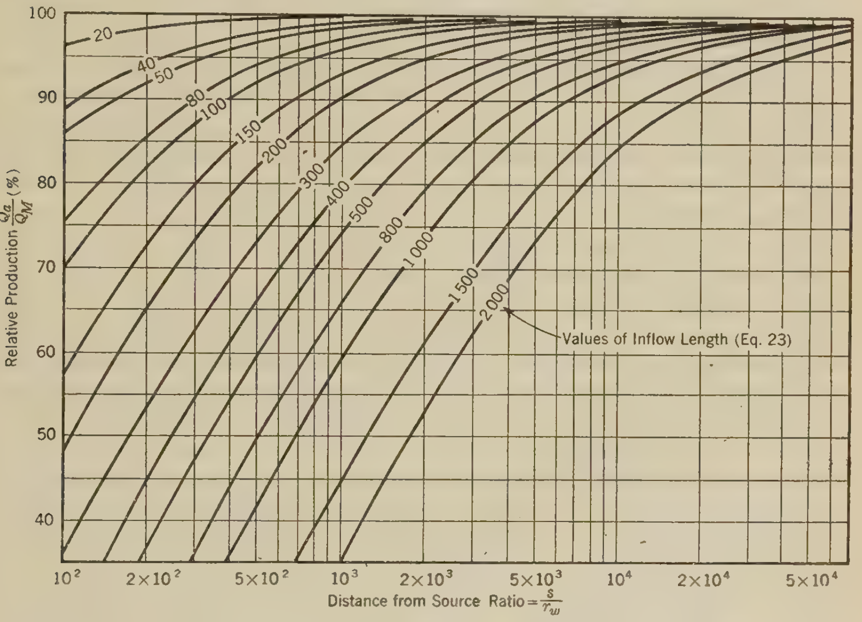

FIG. 17.—RELATIVE PRODUCTION OF RELIEF WELLS FOR DAMS AND LEVEES

Assume that s is 1,200 ft and that wells with an effective radius of 0.25 ft will be used. Taking a relative production of 90% as a design value, the inflow length from Fig. 17 is 800. Since the inflow length is expressed in terms of effective well radius, the ratio of well spacing to degree of penetration is 200. Therefore, 100% penetrating wells spaced 200 ft on centers or 50% penetrating wells spaced 100 ft on centers, or 25% penetrating wells spaced 50 ft on centers could be used. From Fig. 8 the percentage of excess head midway between wells for these three possible designs will be 13.5%, 10.5%, and 8.3% of the total head change, respectively. Next assume that 50% penetrating wells spaced 100 ft on centers are selected for the design but that the depth of the pervious layer actually is twice that determined from field explorations and used in design. The result will be that relative production is 81% instead of 90% and the excess head is 19% instead of 10.5% of the total change in head. Similarly, if the actual effective length of the pervious layer were only 600 ft instead of 1,200 ft for the same design, the relative production would be reduced from 90% to 82% and the excess head would again be increased to 19% of the total change. In the rather remote case that 100% errors on the unsafe side are made in both the depth and the effective length of the pervious layer, the relative production would be reduced to 67% and the excess head would be 31% of the total change.

It should be apparent from this numerical example that extreme errors in determinations of the dimensions of the pervious layer do not impair seriously the effectiveness of a design based on the erroneous dimensions. This fact, coupled with the fact that use of the relative production automatically eliminates the effects of stratification, lenticular inclusions, and uncertain effective permeability of the pervious layer, should answer any objections to relief well designs based on the uncertainties of the pervious layer. In addition, it has been shown by A. F. Samsioe[19] and A. Casagrande[20] that the effects of stratification can be compensated for and the problem reduced to one of flow through a homogeneous, isotropic material, by reducing true dimensions parallel to the stratification to effective dimensions by a factor $\sqrt{\frac{k\ (\text{min})}{k\ (\text{max})}}$. Therefore, the effect of stratification on partly penetrating wells cannot be greater than that of a change in effective length of the previous layer, by the ratio $\sqrt{\frac{k\ (\text{min})}{k\ (\text{max})}}$.

One word of caution is necessary, however. These conclusions are valid for the primary purpose of relief wells, which is to eliminate dangerous hydrostatic pressures due to underseepage. They do not apply to the actual discharge quantities from the wells. The discharge quantities will vary directly with every uncertainty in the pervious layer. Therefore, variations of 1,000% from the anticipated discharge quantities should not be considered unusual. Since the discharge quantity governs the frictional head loss in upward flow through the well riser pipe, a liberal factor of safety should be used in selecting the size of the well pipe.

Summary.—In conclusion, the writer believes that the authors have made a significant contribution in their development of the use of drainage wells for the relief of dangerous pressures due to underseepage. Their conclusions are valid and they have justly advocated the use of drainage wells only for the specific problem of relieving pressures in underlying pervious layers. Drainage wells usually will have little or no effect on seepage which actually passes through a dam or levee. However, for the specific problem to which drainage wells apply, the authors have been modest in their claims and, if anything, have underestimated the flexibility of the system.

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[19] "Einfluss von Rohrbrunnen auf die Bewegung des Grundwassers," by A. F. Samsioe, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 11, No. 2, 1931, pp. 124–135.

[20] "Seepage Through Dams," by A. Casagrande, *Journal*, New England Water Works Assn., Vol. 51, No. 2, June, 1937, pp. 151–154.

H. H. Roberts,[21] Esq., and Carter V. Johnson,[22] Esq.—The pressure-relief well system at the Fort Peck Dam is cited by the authors as being the most outstanding example of the use of such wells. A brief statement regarding the Fort Peck installation and its effectiveness is also given in the paper under discussion.

Information that has been obtained during the period that the relief wells have been in operation at the Fort Peck Dam verifies the effectiveness of this method of relieving excessive subsurface pressure in the critical area immediately downstream from a large dam. Where data from field investigations for a proposed dam indicate the possibility of underseepage, under similar conditions, consideration should be given in the design stage to the immediate installation of a few pressure-relief wells for the purpose of obtaining accurate information for the design and installation of a complete relief-well system. The information obtainable from the initial wells taken during the period that the reservoir is being filled, could be used for the final design based on the criteria and methods given in the paper. This procedure should reduce to a considerable extent the need for extensive underground explorations and laboratory tests of the materials that would be required if the permanent system were to be designed in its final form along with the plans for the dam.

As a matter of interest the equations and charts given in the paper have been applied to the well system at the Fort Peck Dam, using the heads and gradients that existed on July 29, 1946 (highest elevation of water in the Fort Peck Reservoir to that date, 2,232.25 ft), the relief well system discharge, and available data regarding the pervious strata. Correlation of the computed characteristics of the system with the actual was very good.

Data collected since the installation of the pressure-relief wells show that the total discharge of the well system is proportional to the reservoir elevation; it has also been observed that the water elevation in the river downstream from the dam has a material effect on the well discharge. Piezometer pipes both upstream and downstream from the line of wells furnish evidence of the change in well discharge due to fluctuating reservoir and tailwater elevations. Piezometer pipes 50 ft upstream from the line of wells are used to measure the average hydrostatic pressure in the well area; and since the installation of the well system this hydrostatic pressure has never indicated a head in excess of 7 ft above the average ground surface. Maximum and minimum discharges of 12.35 cu ft per sec and 8.5 cu ft per sec, respectively, have been observed for a reservoir fluctuation of 62 ft in which time the reservoir reached an elevation only 18 ft below the maximum level. It has been estimated that the well system discharges from 75% to 85% of the total amount of seepage water that approaches the system. The remainder of the seepage water continues downstream, confined within the pervious strata by the thick overlying layer of clay. Consequently, an average hydrostatic pressure can be allowed downstream from the dam which will produce a head that extends above the ground surface without creating a wet and boggy condition.

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The present well system at Fort Peck is in excess of the capacity actually needed to relieve the pressure to the required level. This allows a factor of safety for well failures which have occurred due to deterioration of the screen and casing. Several wells have failed because of such deterioration since the original installation; and additional wells have been installed with a more permanent type of screen and casing to assure adequate capacity should too many of the original wells fail. When an individual well fails, the total discharge of the system does not change appreciably and the load is absorbed by all the wells in inverse proportion to their distance from the failed well. When a new well is added to the system the flow in the other wells is reduced in the same manner. Individual wells vary considerably in discharge. The factors that have the greatest effect on the discharge are the effective head producing discharge (which can be varied for individual wells by changing the elevation of flow line), the length of screen, and the type of material which the individual well taps. Since the well system is adequate and the discharge is produced by a very small head, the wells are sensitive to small changes.

In the design of a well system certain fundamental factors enter into the problem; and a thorough understanding of these is necessary in order that a reasonable design may be developed. Furthermore, the effectiveness of the equations and charts for an actual design would depend, of course, on the accuracy of the basic data. The equations and charts in the paper can be used to illustrate the effect of these factors on a system of wells by assuming different conditions and solving for the result. As previously stated, when checking the equations and charts with observed data at Fort Peck, the results checked very well with the actual. This check indicates that, if the fundamental data and tests are reliable, an adequate system of relief wells can be designed by the methods outlined. However, wherever practicable, it is considered desirable actually to install a minimum number of wells and to utilize data from their operation during the initial filling of the reservoir to complete the final design.

Frank E. Fahlquist,[23] M. ASCE.—The design of underground drainage facilities for partial yet adequate control of underseepage pressures is presented by the authors. The paper summarizes the very valuable data of experimentation and experience that was accumulated for several years by the United States Engineer Department. Although the authors have presented the important features of relief well design, it is believed they could also contribute valuable additional data and discussion concerning the fabrication and installation of wells. In particular, comments on the several available types of well intakes, such as metal screens, concrete pipe with porous wall, and perforated tile would be appreciated. Comments as to the preparation and the stabilization of ground surrounding the well intakes, either by development methods or by placement of graded filter materials, would be in order.

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[23] Cons. Engr. and Geologist, Riverside, R. I.

The authors' explanation of Fig. 1 can be further illustrated by the application of two formulas showing the relations of permeability in stratified materials. In general, the permeability in an unstratified or homogeneous deposit is the same in all directions. Geologically, there can be exceptions to this rule but they are not important. If, in an otherwise homogeneous deposit, stratifications of less permeability occur, the permeability in a direction that is across the bedding is much less than that in a direction parallel to the bedding, as shown by the two equations,[24]

$$k_H = \frac{1}{H}\,(k_1\,d_1 + k_2\,d_2 + k_3\,d_3 + \cdots + k_n\,d_n) \ldots\ldots\ldots (24a)$$

and

$$k_V = \frac{H}{\dfrac{d_1}{k_1} + \dfrac{d_2}{k_2} + \dfrac{d_3}{k_3} + \cdots + \dfrac{d_n}{k_n}} \ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

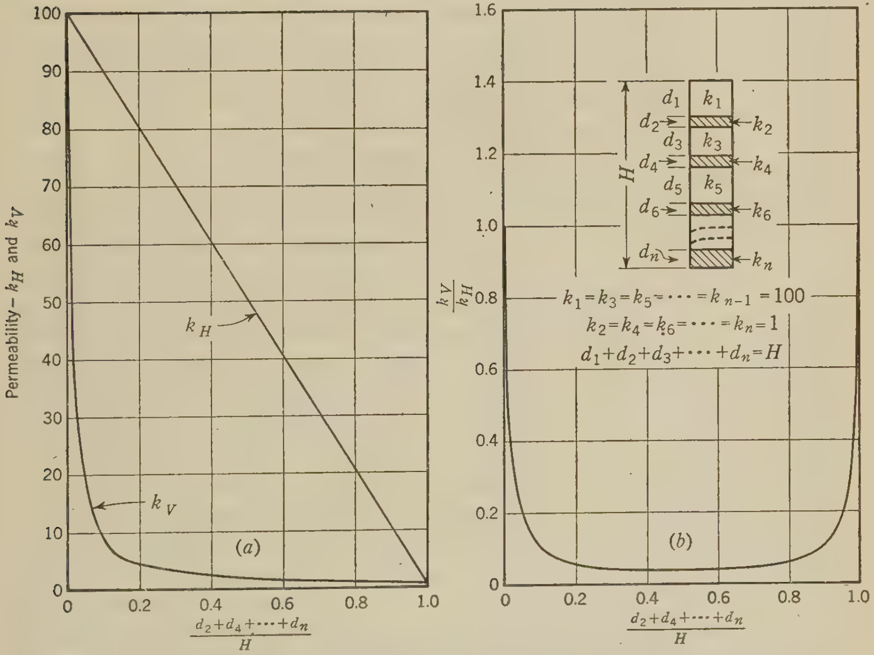

FIG. 18.—HORIZONTAL AND VERTICAL PERMEABILITY IN A PERMEABLE SEDIMENT CONTAINING THIN STRATIFICATIONS OF RELATIVELY IMPERMEABLE MATERIAL

in which k_H is the permeability parallel to bedding or horizontal direction; k_V is the permeability perpendicular to bedding or vertical direction; k_1, k_2, k_3, $\cdots$ k_n represent the permeability of individual layers; d_1, d_2, d_3, $\cdots$ d_n represent the thickness of individual layers; and $H = d_1 + d_2 + d_3 + \cdots + d_n$ is the total thickness.

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[24] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943; pp. 243–244.

The relations expressed in Eqs. 24 are shown graphically in Fig. 18. In Fig. 18 the aggregate thickness of relatively impermeable stratifications is indicated by $d_2 + d_4 + \cdots + d_n$, and the fractional part of these thicknesses to the total thickness is $\frac{d_2 + d_4 + \cdots + d_n}{H}$. If the permeability of the pervious beds is assumed to be alike (for demonstration purposes) and 100 times that of the impervious stratifications, also assumed to be alike, the impermeable occurrences of only a fractional part of the total thickness will cause a very appreciable drop in the k_V-value. In the k_H-value there is only a very small reduction, whereas a large reduction is noted in the ratio, $\frac{k_V}{k_H}$. It does not matter whether the impermeable component occurs as a single thick unit, or in numerous thin stratifications. It is this phenomenon, as indicated by the authors, that produces the inadequate vertical drainage which causes flotation of the overburden with consequent piping and formation of boils. In constructing relief wells, the impermeable stratifications or layers are pierced, and facilities are provided for the escape of ground water that is confined in the permeable beds.

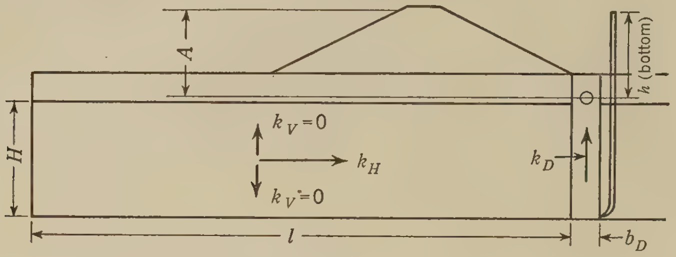

Fig. 19.—Factors Involved in the Derivation of Eqs. 25

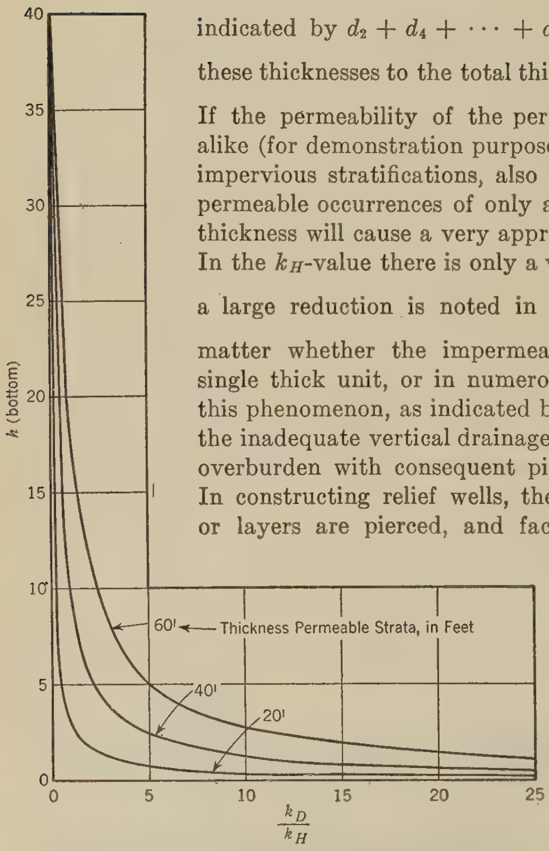

Fig. 20.—Relation of Piezometric Pressure, h (bottom) at Underside of Permeable Strata to Required Ratio of Permeabilities of Drainage Zone and Foundation

The average over-all permeability of a bedded sediment is related to the degree of stratification; the amount of fine particles mixed with coarser particles, such as sand and gravel; and the density or porosity of the various beds. The last two factors can be demonstrated easily in the laboratory, and the effect of the first factor has been discussed. Considering these factors and also the manner in which sediments, either fluvial or glacial in origin, are frequently found in nature, the writer has investigated the theoretical factors involved, and the construction methods required, for creating a continuous and deep drainage zone, in the underground, for the interception of seepage.

The method would involve systematic jetting in a vertical zone, generally less than 10 ft wide, in which, by controlling the jetting process, stratifications could be disrupted, fines floated and washed away, and the entire zone rendered more porous than the adjacent foundation materials. It would not be applicable to all materials or conditions—for example, it would not be practicable to attempt such relief in strata in which sand was predominant. However, there are situations where, because of geologic processes, the deposits are of stratified sand and gravel with varying proportions of fines (silt and clay), in dispersed and concentrated occurrence. For conditions such as these, the relation of the several factors, shown in Fig. 19, can be expressed as:

$$\frac{h\text{ (bottom)}}{A} = 1 - \left(\frac{2\,e^{-\beta H}}{1 + e^{-2\beta H}}\right) \qquad (25a)$$

$$i_0 = \beta\, H \left(\frac{1 - e^{-2\beta H}}{1 + e^{-2\beta H}}\right) \qquad (25b)$$

and

$$\beta = \sqrt{\frac{k_H}{k_D}\,\frac{1}{l\,b_D}} \qquad (25c)$$

in which h (bottom) is the piezometric pressure at the bottom or underside of the pervious strata; i_0 is the hydraulic gradient at the elevation of the drain pipe or ditch; A is the head acting; H is the thickness of the pervious strata; e is the base of Naperian logarithms = 2.71828; k_H is the horizontal permeability of the pervious strata (vertical permeability assumed to be zero); k_D is the permeability of the drainage zone; b_D is the width of the drainage zone; β is a number defined by Eq. 25c; and l is the length of the seepage path. For a convenient solution of Eqs. 25, curves of $\beta\, H$ versus h (bottom) A, and $\beta\, H$ versus $i_0\, H/A$ should be prepared.

To consider a hypothetical case, assume $A = 40$ ft; $l = 300$ ft; and $b_D = 8$ ft. The relation between the piezometric pressure, h (bottom), at the underside of the permeable strata to the ratio, k_D/k_H, is demonstrated by the curves in Fig. 20. A change in permeability of the pervious foundation strata, within the narrow drainage zone, from $k_D = 1$ to $k_D = 4$, produces a very appreciable decrease in the piezometric pressure acting at the bottom or underside of the pervious strata. The practicability of producing such a change in permeability, of course, is the important uncertainty. To the writer's knowledge, such a construction has never been attempted and consequently there is not an experience record by which to judge its feasibility; but by reasoning alone it would appear that, in certain types of material (as previously discussed), a moderate change from $k_D = 1$ to $k_D = 3$ could be produced. The jetting operation would be accomplished by from four to six jets, held rigidly at the same spacing, with provisions for the flexible control of pressure and flow, and operated simultaneously.

The curves in Fig. 21 can be used for determining the width of drainage zone, b_D, required for a length, l, from 100 ft to 1,000 ft. The ratio of permeability of drainage zone to pervious foundation strata is assumed to be unity. If a greater change in permeability is believed possible, the width of zone required is less—for example, if the permeability of the zone is doubled, the width may be reduced by one half.

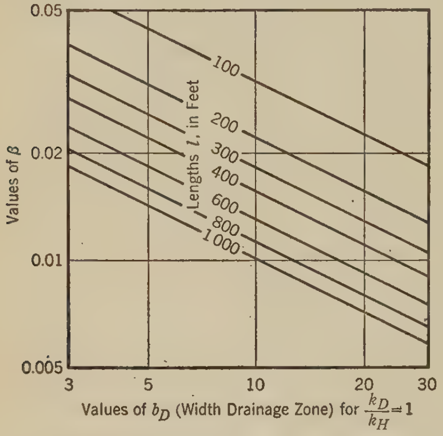

Fig. 21.—Required Width of Drainage Zone

As a further illustration, consider a typical problem of an earth dam, for which $A = 100$ ft; $H = 40$ ft; and $l = 800$ ft. At the downstream toe there is a trench 8 ft deep, and the drainage zone is constructed in such a manner that the h (bottom) at the underside of the pervious stratum is 8 ft. Therefore, h (bottom)$/A = 8/100 = 0.08$. From Eqs. 25, or design curves based on them, $\beta H = 0.41$ and $\beta = \beta H/H = 0.41/40 = 0.01$. By reference to Fig. 21, for $\beta = 0.01$ and $l = 800$ ft: $b_D = 12$ ft when $k_D/k_H = 1.0$, and 6 ft when $k_D/k_H = 2.0$.

Considering both the theoretical and practical aspects of the continuous and deep drainage zone, it is obvious that:

(*a*) All seepage may be intercepted;

(*b*) The zone can be varied in width to take care of varying amounts of seepage;

(*c*) The zone is formed in place by modifying the foundation materials; and

(*d*) The drainage zone can be easily incorporated in other drainage control facilities.

The several uncertainties and disadvantages are as follows:

1. Feasibility of construction depends on the character of foundation materials—the most suitable primary materials being sand-gravel mixtures having dispersed silt and clay, or silt and clay interstratifications;

2. The change of permeability in a drainage zone, relative to adjacent foundations, would be difficult to determine, but the problem is not insuperable; and

3. The cost may be greater, and yet may be less, than that for relief well installations, depending on the number of wells required, and the necessity for placing filters, or stabilizing the ground around well intakes.

Acknowledgment.—The differential equation leading to Eqs. 25 was developed and solved by R. A. Barron, Jun. ASCE.

Harry R. Cedergren,[25] Assoc. M. ASCE.—The subject of relief wells, in this paper, is presented in a straightforward, yet reserved, manner. The statement (under the heading, "Design of Relief Well Systems") that "* * * the well system is very flexible and more and larger wells can be added at any time if the first installation proves inadequate" is one of the keys to the potential value of relief well systems.

During 1946 the writer was engaged in design studies for a 25-mile system of levees along the Columbia River in Washington. Explorations of proposed sites for levees to protect agricultural and industrial areas revealed extremely heterogeneous deposits of gravelly materials underlying a relatively impervious blanket of fine-grained soils. The overlying blanket varied in thickness from less than 1 ft in isolated areas to 15 ft or 20 ft in others, and averaged about 8 ft. The gravelly formations ranged from well-graded, relatively impervious, silty sandy gravel to extremely open-graded strata of 0.5 in. (+) to 4-in. (+) material with only a trace of fines. The open-graded strata generally did not exceed 1 ft or 2 ft in thickness, and occurred in lenses or pockets of limited extent. The individual lenses or pockets were more or less interconnected, and permitted a relatively free passage of water in the horizontal direction. Underlying the gravelly formations at depths of from 30 ft to 60 ft below the ground surface were beds of clay that were indicated by incomplete explorations to be impervious and continuous.

Complicating the problem, much of the area to be protected was under irrigation, which produced a substantial flow of water toward the river. This flow added to potential construction problems and necessitated even greater seepage protection than would otherwise have been necessary. Where the impervious bed was from 30 ft to 40 ft deep, the use of an impervious cutoff, in combination with shallow drains for the removal of runoff, seemed the most promising solution. Blanketing the riverbank and beaches with relatively impervious soil was considered as the primary control in some areas. In other places, the use of deep relief wells as the principal controls appeared to be the most economical and adequate solution. One of the principal advantages of relief wells on this project is their flexibility. Prediction of the probable seepage quantities, even with the aid of considerable explorations and extensive field and laboratory permeability tests, could be nothing more than a reasonably good guess. Consequently, it was essential that the drainage, or relief system, be of such type that it could be "tailored" to fit the needs of the job as those needs were finally determined by observation of the prototype.

A second advantage of relief wells on this project is their effectiveness in relieving hydrostatic pressures in highly stratified deposits. Shallow drains in such formations are relatively ineffective, since even a thin impervious stratum separating the bottom of a drain from pervious underlying strata can seriously reduce the efficiency of the drain. However, wells drilled into the pervious strata provide a direct means of escape for seepage and can reduce uplift pressures in the foundation substantially.

Should an initial relief well installation not be adequate to control seepage during maximum stages of river or reservoir, this inadequacy frequently can be predicted by piezometric observations during low and intermediate stages. Such a possibility was demonstrated by the authors under the heading, "Examples of the Use of Relief Wells."

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[25] Cons. Engr., O. J. Porter & Co., Sacramento, Calif.

Relief wells should be constructed with considerable care in accordance with specifications prepared by competent soil engineers. When used on important structures, in which their continued functioning over a long period is required, each well should be accessible for cleaning and repairing. If wells are installed beneath an embankment section, vertical riser pipes should extend above the embankment to provide access to the wells. With this provision, and with adequate attention to their design and construction, relief wells can be used with confidence in the most important structure. Periodic observation of seepage pressures within and beneath important earth dams and levees, and requent observation of the effluent of relief wells and drains, should be routine matters for attendants.

John S. McNown,[26] Assoc. M. ASCE.—For some time the writer has been interested in the application of mathematical methods to practical problems of fluid flow, and has contended that many existing mathematical treatments would find wider application if presented in a less obscure manner. It is important from the standpoint of clarity that symbols, wherever used, be clearly defined and used in an entirely consistent manner. Moreover, since the algebraic representation as applied to practical problems is merely a symbolic shorthand, the relationship represented by an equation should be expressed in words wherever the significance of the equation is not readily apparent.

The application of the methods of hydrodynamics to relief well design presented by the authors is an interesting analytical problem leading to worthwhile results. However, the development of the basic equations is exceedingly difficult for the reader to appraise and follow because (1) the explanation of principles underlying the development is far from complete, (2) the symbology is thoroughly confusing, and (3) several inaccurate or inconsistent statements have been included. Since the writer feels that the results obtained by the authors are significant, it seems worthwhile to clarify parts of the development and to reconcile some of the troublesome inconsistencies.

In any development of a highly mathematical nature, it is not the rigorous algebraic transformations which interest the practical-minded engineer, but rather the accuracy of the underlying assumptions and the acceptability of approximations so often essential to the obtainment of useful results. From this viewpoint, the omission of any discussion of the rather surprising analogy which permits the use of hydrodynamic theory for certain problems of viscous motion is regrettable. As presented, the material leaves the reader with no method of evaluating the usefulness of the final equations. Much of the mathematical development is unquestionably beyond the scope of the paper and, furthermore, has been presented elsewhere; nevertheless, the authors might fully as well omit the mathematics and present only the final equations, once they have omitted the initial steps leading to Eq. 7*a*.

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[26] Research Engr., Iowa Inst. of Hydr. Research, and Asst. Prof., Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

In deriving this basic equation, it is necessary to define and to utilize briefly the concept of velocity potential in relating the flow of a frictionless fluid into mathematical sinks and the laminar flow of a real fluid into relief wells. A velocity potential φ is usually defined as a quantity whose negative derivative in any direction is the component of velocity in that direction—that is:

$$-\frac{\partial \phi}{\partial x} = V_x \qquad (26a)$$

Furthermore, several writers have shown[3,27,28] that the Navier-Stokes equations of viscous motion for flow with negligible acceleration reduce to the form:

$$-\frac{\partial h}{\partial x} = C\,V_x \qquad (26b)$$

in which h is the piezometric head and C is a dimensional constant of proportionality.

A comparison of Eqs. 26 demonstrates the analogy between these two types of motion and indicates that the piezometric head (multiplied by a proportionality factor) will serve as the velocity potential for flow through granular media. Because of this accidental parallel between two widely differing types of motion, many of the results and methods of hydrodynamics may be applied to problems of flow through porous media. Flow into a single well has its exact counterpart in the flow into a mathematical sink, and a number of wells can be studied by utilizing expressions already developed for flow into a comparable number of sinks similarly arranged. Although flow from a river into a line of wells is closely approximated by the corresponding expression for flow into a line of sinks, the condition of a straight line of uniform pressure (the edge of the river) may be exactly reproduced by placing a symmetrical line of sources or supply wells an equal distance the other side of the straight line of uniform pressure. The mathematical expression for the infinite lines of sources and sinks is initially quite complex, but it may be partly simplified to a usable form corresponding to Eq. 7*a*. Since piezometric heads are of more direct usefulness than pressure intensities, Eq. 7*a* has been rewritten to give the piezometric head h at the point with general coordinates (x,y) in the following form:

$$h = h_s + c \log_e \frac{\cosh j(y - s) - \cos j\,x}{\cosh j(y + s) - \cos j\,x} \qquad (27)$$

The numerator of the fraction in Eq. 27 is a mathematical simplification giving the sum of the velocity potentials for the sinks along $y = s$ and the denominator for the sources at $y = -s$, s being therefore the distance between the edge of the river and the line of wells. The piezometric head along the edge of the river is designated by h_s. The x-axis coincides with the line of symmetry (the edge of the river), and the y-axis passes through one of the wells. The substitution $j = 2\,\pi/a$, in which a is the well spacing, has again been used to simplify typography. The coefficient c, which reflects both the quantity of flow into

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[27] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1938.

[28] "Hydraulik," by Phillipp Forchheimer, B. G. Teubner, Leipzig and Berlin, 1930.

each well and the characteristics of the medium and the fluid, may be defined as

$$c = \frac{\mu Q}{4 \pi k \gamma} \qquad (28)$$

in which the dynamic viscosity μ characterizes the fluid resistance; and Q is the rate of discharge into each well (the vertical depth of the stratum being assumed as unity). The permeability of the medium k is defined by Mr. Muskat[29] as "the volume of a fluid of unit viscosity passing through a unit cross section of the medium in a unit time under the action of a unit pressure gradient" and is entirely determined by the geometry of the medium. The value of k reflects the ability of the medium to transmit fluid much as a coefficient of heat transfer reflects ability to transmit heat. The unit weight of the liquid, γ, has been introduced to balance the units of Eq. 27. These equations differ somewhat from Eqs. 7 because the writer found it impossible to follow the conflicting definitions and units and still to arrive at a correct equation.

Together with these fundamental relationships, a restatement of the problem in view is essential. Expressions are sought relating (*a*) the well discharge to the drop in head between the river and the well, and (*b*) the head midway between adjacent wells to that at the wells. It is necessary, therefore, to evaluate from Eq. 27 the piezometric head at the edge of the well and midway between wells in terms of known or measurable quantities. If the radius of the well r_w is assumed to be small relative to the well spacing (as it is in any practical case), the lines of constant pressure in the region of the well are essentially circular, and the piezometric head at the well may be evaluated at any point around the periphery. The pressure cannot be evaluated at the center of the well, because the medium is discontinuous in this region in direct contrast to the assumption underlying the derivation. Therefore, if the coordinates (r_w, s) are substituted in Eq. 27, an expression is obtained for h_w, the head in the well:

$$h_w = h_s + c \log_e \frac{1 - \cos j r_w}{\cosh 2 j s - \cos j r_w} \qquad (29a)$$

In the numerator of the fraction $j r_w$ is small, so that $\cos j r_w$ may, with good approximation, be replaced by the first two terms of the equivalent series expansion, $1 - \frac{1}{2}(j r_w)^2$. Also, since $2 j s$ is considerably greater than unity, $\cosh 2 j s$ may be replaced by $\frac{1}{2} e^{2js}$ and $\cos j r_w$ may be neglected in the denominator. With these readily permissible simplifications,

$$h_s - h_w = 2 c (j s - \log_e j r_w) \qquad (29b)$$

If the coordinates $(a/2, s)$, corresponding to a point midway between the wells at $(0, s)$ and (a, s), are substituted in Eq. 27, the piezometric head at this point can also be determined. In the absence of consistent notation, the symbol h_m will be assigned to this head, which can be referred to either the head at the river or the head in the wells by using Eqs. 27 and 29—that is:

$$h_s - h_m = 2 c (j s - \log_e 2) \qquad (30a)$$

or

$$h_m - h_w = 2 c \log_e \frac{2}{j r_w} \qquad (30b)$$

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[29] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co. ,Inc., New York, N. Y., 1st Ed., 1937, p. 71.

Finally, from Eqs. 29*b* and 30*b* the ratio presented as the ordinate scale in Fig. 4(*a*) can be quite simply expressed as follows:

$$\frac{h_m - h_w}{h_s - h_w} = \frac{\log_e (j\, r_w/2)}{\log_e j\, r_w - j\, s} \qquad (31a)$$

For purposes of computation, Eq. 31*a* can be simplified even further by substituting the value of *j* and changing to decimal logarithms:

$$\frac{h_m - h_w}{h_s - h_w} = \frac{\log_{10}\left(\dfrac{a}{\pi\, r_w}\right)}{2.73\,\dfrac{s}{a} + \log_{10}\left(\dfrac{a}{2\,\pi\, r_w}\right)} \qquad (31b)$$

Also, the combination of Eqs. 28 and 29 yields a relationship between the drop in head from the river to the well and the discharge per well:

$$Q = \frac{2\,\pi\, k\,(h_s - h_w)\,\gamma}{\mu\,(j\, s - \log_e j\, r_w)} \qquad (32)$$

Eqs. 31 and 32 correspond to Eqs. 11 and 12, but are presented in consistent units and in a more readily usable form.

The foregoing development has been included in some detail, in part because the parallel treatment by the authors lacked a systematic and readily understandable order, but primarily because throughout their development the authors have consistently departed from good practice in use of symbols. Although the results of the analysis as embodied in Fig. 4 are correct, it is typical of the confusion confronting the reader that the same quantity is designated as h in one part of Fig. 4 and as $h_s - h_w$ in the other. In fact, four different symbols were used at various points in the authors' development to represent this difference in head—namely, p, Δp, h, and $h_s - h_w$. The symbol p, furthermore, was assigned four distinct meanings, denoting the piezometric head at the point (x,y) in Eq. 7*a*, the difference in head $h_s - h_w$ a few lines thereafter, the unit pressure (actually piezometric head) at the midpoint between the wells in Eq. 8, and the difference $h_m - h_w$ in Fig. 8(*b*). As a result, the reader is forced to use intuitive as well as logical reasoning in following the authors' development.

Although the essential results given by the authors have proved to be correct, several omissions and points of inconsistency further detract from the presentation. It is difficult to obtain a satisfactory interpretation of Table 1 in the absence of specific values for s, h_s, and h_w.

The definition of the quantity "extra length," given as part of the discussion of Fig. 7, is not compatible with its representation in Figs. 7 and 8 and Eq. 13. Neither theory nor electric analogy are suitable for evaluating the effect of a well screen. As presented, this length term is the difference between the length of the direct path from the river to the line of wells and an effective length obtained by dividing the total drop in head $h_s - h_w$ by the piezometric gradient evaluated some distance away from the line of wells. Introduction of this residual quantity leads to the useful plot (Fig. 8(*a*)) which is independent of the variable s (or s/a). However, since Fig. 8(*a*) yields only the extra length and Eq. 13 is necessary for the evaluation of Q, the word "flow" in the caption of Fig. 8 seems inappropriate. If this extra length is designated as b, it may be defined in relative form as

$$\frac{b}{a} = \frac{1}{2\,\pi} \log_e \frac{a}{2\,\pi\,r_w} \quad \text{(33)}$$

which corresponds to the line for 100% penetration in Fig. 8(a). The equation for the related curve in Fig. 8(b) can be written in a like manner if it is noted that the value p in the ordinate scale is intended to signify the difference in head $h_m - h_w$ and that the porosity factor includes not only the effect of the porous material but the viscosity and unit weight of the water as well. If the writer's terminology and Mr. Muskat's definition of porosity are used, Eqs. 28 and 30b can be combined to represent the 100% line in Fig. 8(b)—that is:

$$\frac{(h_m - h_w)\,k\,d}{Q\,\gamma\,\mu} = \frac{1}{2\,\pi} \log_e \frac{a}{\pi\,r_w} \quad \text{(34)}$$

It has been assumed that the effective radius of the well r_e is interchangeable with r_w in the absence of a definition of r_e or a relationship between the two.

The foregoing discussion and the restatement of the analytical part of the authors' paper were undertaken in an attempt to improve and correct the form of presentation. It is hoped that, in so doing, this section of the general discussion has been brought up to the high level of the remainder of the paper.

Reginald A. Barron,[30] Jun. ASCE.—Practical and theoretical requirements for the design of relief wells have been presented in this paper. Of necessity both these requirements must be considered in a design. The general case presented by the authors consists of a semi-infinite pervious layer of uniform thickness covered by an absolutely impervious blanket. In nature the blanket is always slightly pervious and in some cases, such as at Sardis Dam, Mississippi, the downstream blanket is of finite length.

The writer has obtained a mathematical solution for this case that meets the boundary conditions as shown in Fig. 22(b). The hydraulic head h_{xy} measured from a datum plane is

$$h_{xy} = h_p - (h_p - h_T)\,\frac{y}{L} - \bar{h}_{xy} \quad \text{(35)}$$

in which

$$\bar{h}_{xy} = q \sum_{n=-\infty}^{n=+\infty} \log_e \left[\frac{\cosh \frac{\pi}{L}\,(n\,a - x) - \cos \frac{\pi}{L}\,(y + s)}{\cosh \frac{\pi}{L}\,(n\,a - x) - \cos \frac{\pi}{L}\,(y - s)} \right] \quad \text{(36)}$$

in which n takes on all integral values between $-\infty$ and $+\infty$ including zero; and

$$q = \frac{\bar{h}_w}{\log_e \left(\dfrac{1 - \cos \dfrac{2\,\pi\,s}{L}}{1 - \cos \dfrac{\pi\,r_w}{L}} \right) + 2 \displaystyle\sum_{n=1,\,2,\,3,\,\dots}^{n=\infty} \log_e \left(\dfrac{\cosh \dfrac{\pi\,n\,a}{L} - \cos \dfrac{2\,\pi\,s}{L}}{\cosh \dfrac{\pi\,n\,a}{L} - 1} \right)} \quad \text{(37)}$$

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[30] Engr., Embankment and Foundation Branch, U. S. Waterways Experiment Station, Corps of Engrs., Vicksburg, Miss.

Approximating $\dfrac{1}{1 - \cos\dfrac{\pi\, r_w}{L}}$ as $\left(\dfrac{2}{\pi^2}\,\dfrac{L^2}{r^2_w}\right)$, Eq. 37 may be rewritten as:

$$q = \frac{\bar{h}_w}{f\left(\dfrac{L}{r_w}\right) + f\left(\dfrac{s}{L}\right) + F\left(\dfrac{s}{L}, \dfrac{\pi\, a}{L}\right)} \ldots\ldots\ldots\ldots\ldots (38)$$

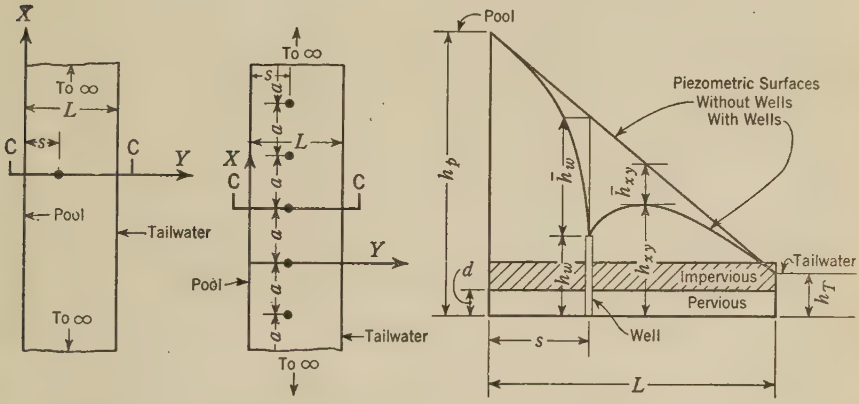
in which

$$f\left(\frac{L}{r_w}\right) = \log_e\left(\frac{2}{\pi^2}\,\frac{L^2}{r^2_w}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

$$f\left(\frac{s}{L}\right) = \log_e\left(1 - \cos\frac{2\,\pi\, s}{L}\right) \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

and

$$F\left(\frac{s}{L}, \frac{\pi\, a}{L}\right) = 2 \sum_{n=1,2,3,\ldots}^{n=\infty} \log_e\left[\frac{\cosh\dfrac{\pi\, n\, a}{L} - \cos\dfrac{2\,\pi\, s}{L}}{\cosh\dfrac{\pi\, n\, a}{L} - 1}\right] \ldots\ldots (41)$$


X
To ∞
L
C
s
C
Y
Pool
Tailwater
To ∞
To ∞
s
a
a
L
X
C
C
a
Y
a
Pool
a
a
Tailwater
To ∞
Pool
Piezometric Surfaces
Without Wells
With Wells
h_p
$\bar{h}_w$
$\bar{h}_{xy}$
h_{xy}
h_w
d
Impervious
Pervious
Tailwater
h_T
Well
s
L

(*a*) FOR SINGLE WELL (*b*) INFINITE LINE OF EQUALLY SPACED WELLS (*c*) SECTION C–C

FIG. 22.—SINGLE WELL AND INFINITE LINE OF EQUALLY SPACED WELLS FOR CASE OF BLANKET WITH FINITE WIDTH

The individual well discharge is

$$Q = 4\,\pi\, k\, q\, d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and the head acting on the well is

$$\bar{h}_w = h_p - (h_p - h_T)\,\frac{s}{L} - h_w \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

The head above the well discharge midway between wells is $\bar{h}_w - \bar{h}_m$. Of these two values, $\bar{h}_m$ is expressed as:

$$\bar{h}_m = q \sum_{n=-\infty}^{n=+\infty} \log_e \left[\frac{\cosh \frac{\pi a}{L} (n - \frac{1}{2}) - \cos \frac{2 \pi s}{L}}{\cosh \frac{\pi a}{L} (n - \frac{1}{2}) - 1} \right] \ldots\ldots\ldots (44)$$

in which n takes on all integral values between $-\infty$ and $+\infty$ including zero. The head $\bar{h}_w - \bar{h}_m$ may then be written as

$$\bar{h}_w - \bar{h}_m = \left[1 - \frac{f\left(\frac{s}{L}, \frac{\pi a}{L}\right)}{f\left(\frac{L}{r_w}\right) + f\left(\frac{s}{L}\right) + F\left(\frac{s}{L}, \frac{\pi a}{L}\right)} \right] \bar{h}_w \ldots\ldots (45)$$

In Eq. 45, $f\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ is the quantity within the braces of Eq. 44. Curves for Eqs. 39, 40, and 41 and the quantity $f\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ in Eq. 45 are given in Figs. 23, 24, and 25. For $\frac{a}{L} = \infty$, the values within the summation signs of Eqs. 36, 37, 41, and 44 are zero except for $n = 0$ and the case reduces to that of a single well as shown in Fig. 22(a). The value $\bar{h}_{xy}$ is then

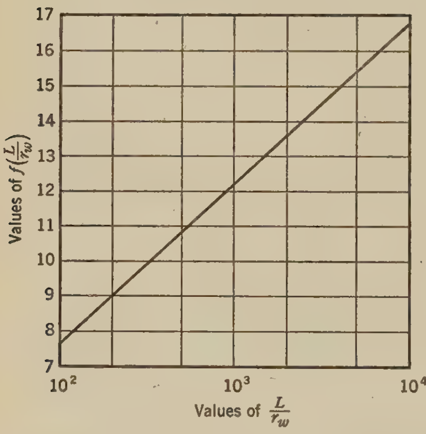


FIG. 23.—CURVE FOR THE SOLUTION OF EQ. 39

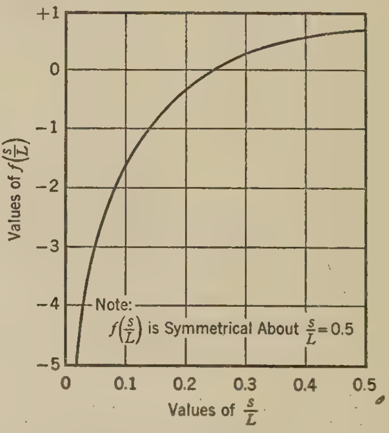


FIG. 24.—CURVE FOR THE SOLUTION OF EQ. 40

$$\bar{h}_{xy} = q \log_e \left[\frac{\cosh \frac{\pi x}{L} - \cos \frac{\pi}{L} (y + s)}{\cosh \frac{\pi x}{L} - \cos \frac{\pi}{L} (y - s)} \right] \ldots\ldots\ldots\ldots\ldots (46)$$

and q may be determined by use of Eqs. 39 and 40 (Eq. 41 being equal to zero).

The average gradients to the well for the case of equal pool and tailwater may be approximated as follows by equating the well flow to that flowing to a continuous slot at s:

$$Q = 4\,\pi\,k\,q\,d = k\,h'\,a\left(\frac{1}{s} + \frac{1}{L - s}\right)d \quad \text{(47)}$$

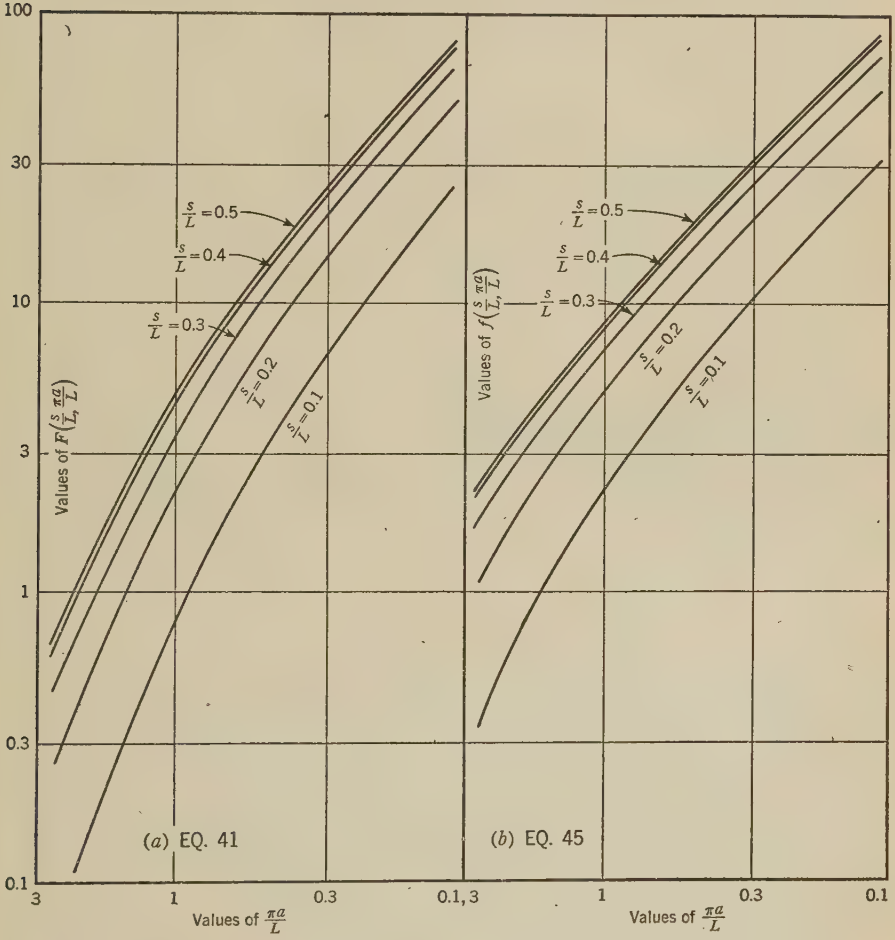


FIG. 25.—CURVES FOR THE DETERMINATION OF $F\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ IN EQ. 41 AND $f\left(\frac{s}{L}, \frac{\pi a}{L}\right)$ IN EQ. 45

The effective head for the slot is

$$h' = 4\,\pi\,q\,\frac{s\,(L - s)}{a\,L} \quad \text{(48)}$$

The gradient out of the pool is then

$$g_p = \frac{h'}{s} = 4\,\pi\,q\,\frac{L - s}{a\,L} \quad \text{(49)}$$

whereas the gradient from the tailwater is

$$g_t = \frac{h'}{L - s} = 4\,\pi\,q\,\frac{s}{a\,L} \qquad (50)$$

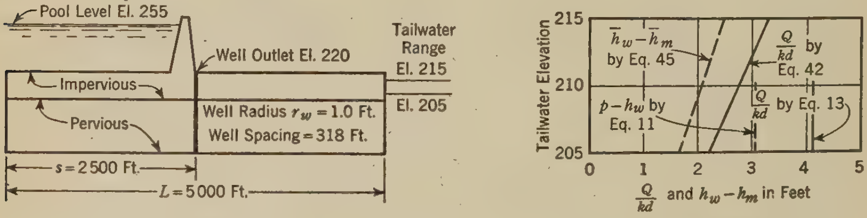
For the case in which pool and tailwater are not at the same elevation:

$$g_p = \frac{h_p - h_T}{L} + 4\,\pi\,q\,\frac{L - s}{a\,L} \qquad (51)$$

and

$$g_t = \frac{h_p - h_T}{L} - 4\,\pi\,q\,\frac{s}{a\,L} \qquad (52)$$

It is readily apparent that the well discharge and the uplift midway between the wells for the foregoing case is not a function of the head difference ($h_p - h_w$). Variation of the tailwater elevation for constant pool elevation has an important effect on well discharge observed at the Sardis Dam relief wells. An example is shown in Fig. 26. The physical data are given in Fig. 26(a) and the result in Fig. 26(b). As should be expected, the relief measure furnished by the tailwater outlet reduces both the well discharge and the uplift pressure midway between wells as compared with those obtained by using Eq. 11 and 13. As the value of $\frac{s}{L}$ becomes smaller these differences become smaller. It is thus apparent that, for a project having excessive foundation seepage pressures and also a downstream outlet, the effectiveness of relief wells should be determined as outlined in this discussion rather than by the methods given by the authors.


Pool Level El. 255
Well Outlet El. 220
Tailwater Range El. 215
El. 205
Impervious
Pervious
Well Radius r_w = 1.0 Ft.
Well Spacing = 318 Ft.
s = 2500 Ft.
L = 5000 Ft.
Tailwater Elevation
215
210
205
0
1
2
3
4
5
$\bar{h}_w - \bar{h}_m$ by Eq. 45
$\frac{Q}{kd}$ by Eq. 42
$p - h_w$ by Eq. 11
$\frac{Q}{kd}$ by Eq. 13
$\frac{Q}{kd}$ and $h_w - h_m$ in Feet

(a) PHYSICAL DATA (b) RESULTS

FIG. 26

PRESTON T. BENNETT,[31] M. ASCE.—This study of pressure relief is admirably balanced between a general discussion of basic principles involved, and the specific analytical methods that may be used in the application of these principles to the design of an installation.

With regard to the uncertainties concerning the boundaries of flow, permeability coefficients, and the effects of anisotropic permeability, it is believed that extensive field investigations of these factors may or may not be justifiable, depending on the type of installation. If the wells are to be installed at the toe of a dam, flexibility of the system can be insured by placing them in an accessible location, and by omitting any elaborate collection system with a fixed discharge capacity. As the authors have stated in the paper, any deficiency in the original installation can be remedied by the addition of more wells when, and if, actual operation shows it to be necessary.

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[31] Senior Engr., Corps of Engrs., Omaha Dist., Omaha, Nebr.

If, on the other hand, a deep subdrainage system is being considered for prevention of boils behind a levee, the inherent flexibility of vertical well drainage cannot always be fully utilized. Deep drainage would seldom be considered in connection with a levee unless the levee protected highly developed urban or industrial areas, where discharge of the well flow on the landside surface would not be permissible. In such a situation, design of the pressure-relief system involves the determination of the required capacity of the collection and disposal system, and the cost of these facilities may justify fairly expensive field studies.

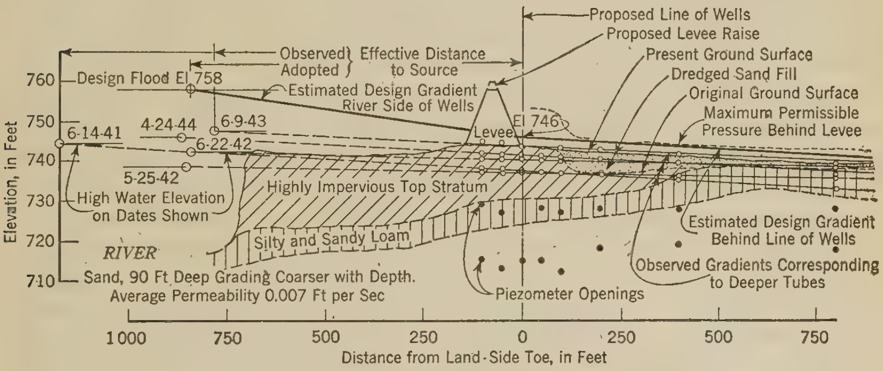

Proposed Line of Wells
Proposed Levee Raise
Present Ground Surface
Dredged Sand Fill
Original Ground Surface
Maximum Permissible Pressure Behind Levee
Observed
Adopted
Effective Distance to Source
Design Flood El 758
Estimated Design Gradient River Side of Wells
Levee
El 746
6-14-41
4-24-44
6-9-43
6-22-42
5-25-42
High Water Elevation on Dates Shown
Highly Impervious Top Stratum
Silty and Sandy Loam
RIVER
Sand, 90 Ft Deep Grading Coarser with Depth. Average Permeability 0.007 Ft per Sec
Piezometer Openings
Estimated Design Gradient Behind Line of Wells
Observed Gradients Corresponding to Deeper Tubes
760
750
740
730
720
710
Elevation, in Feet
1000
750
500
250
0
250
500
750
Distance from Land-Side Toe, in Feet

FIG. 27.—SECTION THROUGH FAIRFAX LEVEE AT KANSAS CITY, KANS., AT PIEZOMETER INSTALLATION

Two types of field investigations are suggested. First, to determine approximately the boundaries of flow, liberal use may be made of piezometers installed in the pervious substratum. Interpretation of piezometer observations is complicated because ordinarily they reflect nonsteady states of flow resulting from fluctuations of river stage. However, if the period of observations includes several stages of sufficient height and duration to fill or nearly fill underground storage, the field observations can be of great direct value in design. Fig. 27 indicates how the effective distance to the source of flow may be determined by piezometer observations. It shows data obtained by an installation at Kansas City, Kans., behind a levee constructed by the Corps of Engineers. The general relationship between the foundation pressure gradient, the rate of rise of the river, and the duration of the high stage may also be studied by use of observed data of the type shown in Fig. 27.

For a fairly accurate estimate of foundation permeabilities, deep well pumping tests are considered indispensable. They furnish directly the information needed in design, which is the quantity of water that will move through the entire foundation depth under a given gradient. Existing water-supply wells may be available for this purpose, or it may be necessary to install wells as a part of the test. In the latter case, the test can be used to evaluate the performance of the type of well screen proposed for the system being designed.

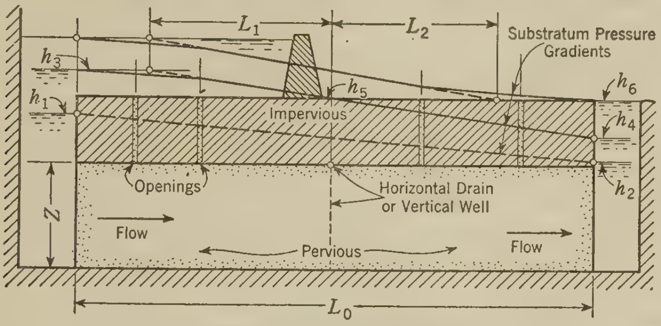
It is also suggested that well pumping tests afford the only means of evaluating the factor of partial penetration in a stratified formation. The authors have shown, by model tests with an isotropic medium, that the percentage of penetration is more important than well size or spacing. In an actual foundation, penetration undoubtedly is of even greater importance, but it cannot be measured solely in terms of screen length versus sand depth, without reference to the comparative water-bearing capacities of the strata penetrated. A series of pumping tests at one location, in which the penetration of the screen is varied by successively filling and plugging off lower sections, will give directly a correction factor for penetration which can be applied to full penetration formulas. The applicability of this factor to other locations would of course be questionable, and should be checked by borings throughout the proposed system for comparison of the degree of stratification at various points.

Considering the difficulty in determining the flow boundaries and permeabilities involved in an actual foundation, and consequent inability to use accurate constants in design formulas, it may seem inappropriate to suggest modifications to the analyses presented. Certainly no greater numerical accuracy will be obtained. However, further discussion of theoretical design methods is not inconsistent with the authors' presentation of theory as an aid to judgment in arriving at a reasonable solution for an actual problem.

The writer has had frequent occasion during the past five years to apply the authors' methods to studies of proposed well installations for levees at several locations on the Missouri River, and to compare theoretical with actual well performance as exemplified by the installation at Fort Peck Dam in Montana. As a result of these studies, the writer proposed in 1944[13] a modification of the well-spacing formulas developed by the U. S. Waterways Experiment Station (Vicksburg, Miss.). The modification is, in most cases, of minor importance in its effect on numerical results of design computation; but it is fundamental in so far as it concerns the relation between the wells (or any other deep drainage system) and the over-all flow pattern in the foundation. Since the deviation of the modified formula has been published elsewhere,[13] this discussion will be concerned principally with its more general aspects.

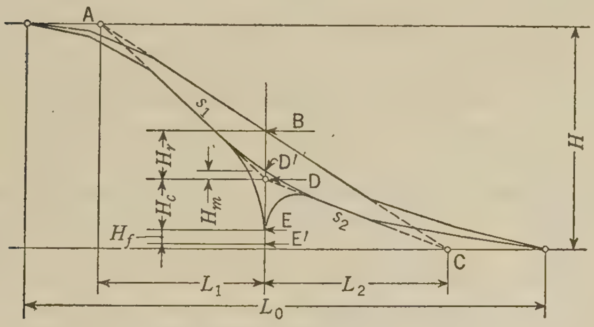
The only basic difference between the writer's analysis and that of Messrs. Middlebrooks and Jervis is that a regional flow landward from the well system is introduced. Perhaps the omission of this flow from the early studies was due to the use of models as shown in Fig. 5, which had an impervious bulkhead a short distance landward from the wells. In an actual installation, the pressure head in the region of the wells that causes them to flow ordinarily also will cause a landward flow away from the wells. This flow may or may not be important in any given case, but should be included in any general theoretical solution.

To simplify the discussion, a very simple model with definite dimensions will be substituted for the actual problem. Fig. 28 represents such a model, and shows the notation used in this discussion. It represents a pervious medium of depth Z and length L_0, covered by an impervious layer which has a few openings filled with pervious material. These openings in the model represent places in the prototype where inflow or outflow from the pervious sand is relatively easy, as, for example, in borrow pits which partly expose the sand. They may also represent seepage through a surface blanket of low permeability. In this case the foundation pressure profile would become a smooth curve which, theoretically, can be determined analytically.[32] Water flows through this system from the reservoir head, h_1, on the left to tailwater head, h_2, on the right. If both headwater and tailwater are below the top of the impervious cover, the effective length of the path of flow is L_0, and the pressure gradient is the straight line connecting h_1 and h_2 on the diagram. Assume that the headwater is raised to h_3, covering the impervious top stratum and causing inflow into the pervious lower stratum through the small openings. The pressure gradient is now broken, becoming steeper below each opening in proportion to the amount of seepage admitted. At this stage, the effective upstream length of the flow path could be determined by measuring the pressure gradient at the toe with several piezometers and dividing the head $(h_3 - h_5)$



L_1
L_2
Substratum Pressure Gradients
h_3
h_1
h_5
h_6
h_4
Impervious
Z
Openings
Horizontal Drain or Vertical Well
h_2
Flow
Pervious
Flow
L_0

(*a*) SIMPLIFIED MODEL SECTION



A
s_1
B
H
H_r
D'
D
H_c
H_m
E
s_2
H_f
E'
C
L_1
L_2
L_0

(*b*) FOUNDATION PRESSURE PROFILES

FIG. 28.—SIMPLIFIED MODEL SECTION (*a*), WITH FOUNDATION PRESSURE PROFILES FOR ASSUMED CONDITIONS

by the observed gradient. Such a procedure would be identical with that shown by Fig. 27 for an actual piezometer installation.

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[32] "The Effect of Blankets on Seepage through Pervious Foundations," by Preston T. Bennett, *Transactions*, ASCE, Vol. 111, 1946, p. 215.

The space below h_6 in the model represents underground storage in the prototype. As this storage fills up, seepage through the impervious cover begins at the outlets nearest the toe, and progresses landward. When the storage is exhausted, the effective land-side length of the model is established as L_2. Dimensions corresponding to L_1 and L_2 of the model have been established by piezometric observations at Fort Peck Dam, and have been used for fairly accurate predictions of relief well performance for all combinations of headwater and tailwater.[33] In a levee, where the "tailwater" elevation cannot be definitely established, and where filling of underground storage involves driving out compressible air from the foundation, the writer knows of no case where L_2 has been satisfactorily determined by piezometer observations. Field observations show a variation in L_2, just as it varies in the model, Fig. 28, before a steady state is established. In the absence of observational data, the distance L_2 may be estimated roughly from theoretical blanket formulas.[32]

After the equivalent paths of flow upstream and downstream from the proposed line of wells have been determined, the basic over-all flow patterns, without drainage relief, may be represented very simply by the foundation pressure profile, ABC of Fig. 28(*b*). Referring to the simplified model, and not the prototype, let it now be supposed that a narrow, fully penetrating slot is introduced into the pervious medium at point B, and that water is allowed to flow out of the slot through a standpipe with a variable discharge elevation represented in Fig. 28 by D. Assuming no head loss in the slot or standpipe, the former profile ABC is broken into two tangents AD and DC whose slopes, s_1 and s_2, are directly proportional to the flows approaching and leaving the pressure-relief slot. The flow from the drainage slot is, therefore:

$$Q_d = k_h \, Z \, (s_1 - s_2) \ldots\ldots\ldots\ldots (53a)$$

or, in terms of the pressure reduction H_r,

$$Q_d = k_h \, Z \, H_r \left(\frac{L_1 + L_2}{L_1 \, L_2} \right) \ldots\ldots\ldots\ldots (53b)$$

These equations represent the performance of a hypothetical perfect drain, where no loss of head within the drain is involved. Any type of actual drain will involve a loss of head due to concentration of the flow into a drainage surface smaller than the sectional area of the pervious stratum. This loss of head is represented by H_c in Fig. 28(*b*). Suppose that the model flume, Fig. 28(*a*), is provided either with a vertical well or a horizontal drain as indicated by the dotted lines, and that the discharge elevation of the drain outlet is lowered to some elevation E′ (Fig. 28(*b*)) at which the drain discharge is equal to the open slot discharge given in Eqs. 53. The total friction loss in the system is H_f, and the head at the drain surface within the stratum is at elevation E.

The profile AD′C represents pressures along a straight line parallel to the axis of the model flume, and at the maximum distance from the drain. Such a line would be midway between vertical wells, or at the bottom of the stratum if a horizontal drain were used. The profile AEC similarly represents horizontal flow gradients along a straight line passing through the drain. Beyond the region of influence of the wells (which from well formulas is not very extensive) the flow is evenly distributed and the pressure profiles AD′C and AEC coincide with ADC for most of their length. Near the drain, water moves laterally from the most remote path to the path through the drain, flattening the profile AD′C and making AEC steeper. Between the extremes AD′C and AEC there are an infinite number of intermediate horizontal flow gradients, but at any plane upstream from the drain the total horizontal flow is:

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[33] "Pressure Relief Wells at Fort Peck Dam," by F. B. Slichter, Rept. on Conference on Control of Underseepage, U. S. Waterways Experiment Station, Vicksburg, Miss., April 1, 1945.

$$Q_1 = k_h \, Z \, s_1 \dots\dots\dots\dots\dots\dots\dots\dots (54)$$

and, consequently, the average horizontal gradient over the entire area of flow remains s_1. Similarly, the average horizontal gradient at any point downstream from the wells is s_2; and it follows that the average head in the vertical section through the drain is at D, exactly equal to the head in a fully penetrating open slot discharging an equal quantity.

This statement may seem so obvious that the foregoing discussion is unnecessary, but it expresses a basic idea which can be used to simplify the design of any deep pressure-relief system. The recommended procedure is briefly as follows:

(*a*) Establish the permissible subsurface pressure, D, using the criteria given by the authors. If any field comparisons of subsurface pressure with the actual occurrence of boils is available, such data should also be used.

(*b*) Estimate L_1 and L_2, or s_1 and s_2, using actual field observations if possible, by the general methods mentioned earlier in the discussion.

(*c*) Estimate the coefficient of transmissibility ($Z \, k_h$) as closely as possible, preferably using the Thiem tests if the cost of collection and disposal of the drain discharge is an important factor in the problem. At this stage, without going into final details of the type of drain to be used, the pressure relief H_r and the total drainage discharge Q_d can be determined, establishing the general features and cost of a collection and disposal system. It is quite possible that at this stage the economic feasibility of deep drainage can be determined.

(*d*) The location and grade of the collection system will establish the outlet elevations E′ along the drainage system, and the last step of the design procedure becomes the choice of a drain type, size, and penetration which will discharge the required quantity Q_d within the available head ($H_e + H_f$).

If the pervious stratum is deep and has the normal degree of stratification, vertical wells appear to be the best type of drain. A horizontal drain may prove to be more economical if the pervious stratum is thin and homogeneous. In either case, well formulas similar to those given by the authors can be used.

From Morris Muskat's[34] basic solution of multiple-well problems, it has been shown that the head loss H_e can be expressed as:

$$H_e = (s_1 - s_2) \, F \, P \dots\dots\dots\dots\dots\dots\dots (55)$$

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[34] "The Flow of Homogeneous Fluids through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1937, Chapter IX.

in which

$$F = \frac{a}{2\pi}\log_e \frac{a}{2\pi r_w} \qquad (56)$$

and P is a well penetration factor.

Since F is equal to the "extra length" factor used by the authors, it is evident that Eqs. 53 and 55 are equivalent to Eq. 13 when $s_2 = 0$. Thus, for the special case of no flow past the wells, the only difference involved is that the authors use the maximum pressure between wells as the permissible design value, whereas the writer suggests that the average pressure in the plane of the wells be used. For fully penetrating wells, the difference between maximum and average pressures, D′–D (Fig. 28(*b*)), is

$$H_m = a\,(s_1 - s_2)\,\frac{\log_e 2}{2\pi} \doteq 0.11\,a\,(s_1 - s_2) \qquad (57)$$

or

$$H_m = \frac{0.59\,H_e}{\log_e \dfrac{a}{2\pi r_w}} \qquad (58)$$

The difference between average and maximum pressures is ordinarily so small that either may be used in determining allowable substratum pressures. The writer's preference for the average value is based solely on the fact that it is the "PI" of the tangent gradients s_1 and s_2 which fix the relationship between the drains and the remaining parts of the flow system.

For homogeneous strata, the factor P can be determined from the experimental data given by the authors. For stratified foundations, it should be determined by field tests in which the penetration is actually varied within the limits being considered. In the latter case, the factor would be:

$$P = \frac{\text{well drawdown for partial penetration, discharge } Q_w}{\text{well drawdown for full penetration, discharge } Q_w} \qquad (59)$$

Fig. 28 suggests a method for defining the efficiency of a deep drainage system. If the system is designed so that the friction losses (H_f) are negligible, as recommended by the authors, the head E can be lowered to the tailwater elevation C. In this case, the efficiency of the system may be defined in terms of the ratio of head reduction to the head without drainage. It can be shown that the drain efficiency is:

$$\eta_d = \frac{H_r}{H_r + H_e} = \frac{L_2}{L_2 + F\,P} \qquad (60a)$$

If L_2 is infinite, as assumed by the authors,

$$\eta_d = \frac{H_r}{H} = \frac{L_1}{L_1 + F\,P} \qquad (60b)$$

Fig. 29 shows curves which may be used to determine readily the efficiency of a fully penetrating well system if the penetration factor P is known.

Well formulas may be used to evaluate a horizontal drain at the top of the pervious stratum by transposing vertical and horizontal dimensions in the plane of the wells, and considering the horizontal drain to have half the discharge capacity of wells spaced a distance of $2\,Z$. If, as usually happens, the stratum is anisotropic, the equivalent well spacing used should be transformed to $2\,Z\,\sqrt{k_h/k_v}$, and the corresponding permeability $\sqrt{k_h\,k_v}$ should be used in computing the quantities of flow.

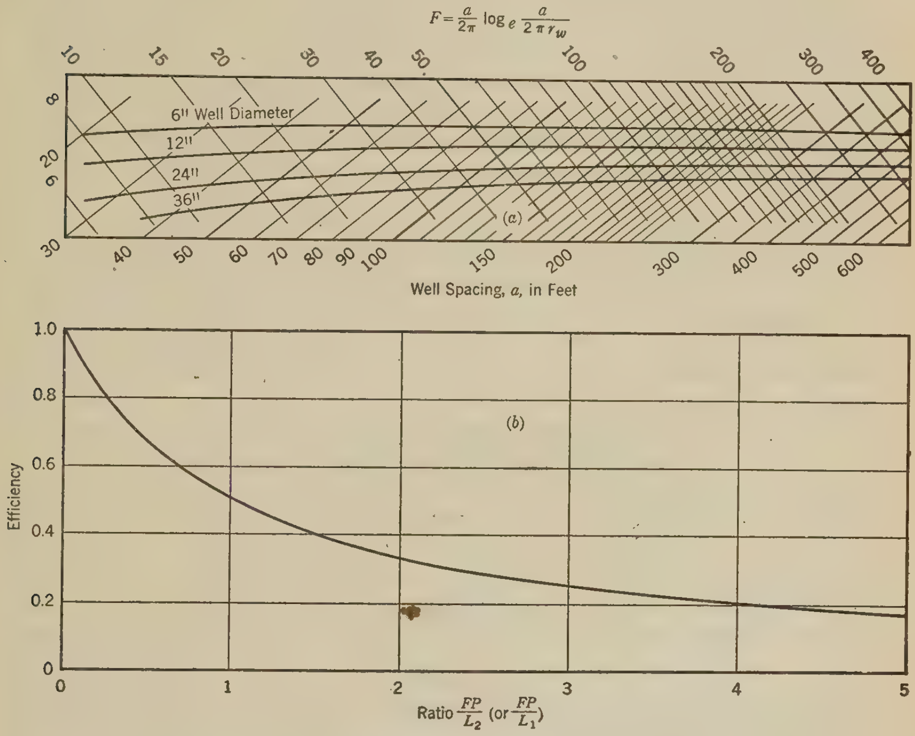

FIG. 29.—CURVES FOR DETERMINATION OF RELIEF WELL EFFICIENCY WHEN PENETRATION FACTOR P IS KNOWN

Eqs. 53 are directly applicable to drainage systems in which a continuous deep drainage zone might be used instead of vertical wells. It should be possible, by model studies or by analytic methods, to evaluate a factor for this type of drain which would be analagous to $F\,P$ in Eqs. 60, thus permitting a comparison in the efficiency of the two types.

It has been proposed to construct continuous deep drainage zones by excavation and backfill with highly pervious material, or by increasing the permeability of the sand by jetting. The writer has no knowledge on which to base any definite conclusion as to the practicability of materially increasing the permeability of the drainage zone. It would appear, however, that washing out any considerable part of the fines would be a very costly and difficult operation. If the jetting should succeed in breaking up the stratification without removing the fines, the maximum possible value of k_d would be the mean permeability of the original materials, or

$$K_d\ (\text{max}) \approx \sqrt{k_h\, k_v} < k_h \ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Even if a considerable volume of fines were removed, as would be necessary to make k_d greater than $\sqrt{k_h\, k_v}$, it is difficult to imagine a treatment that would make k_d equal to k_h. It appears that the drainage zone would still be anisotropic, with a horizontal permeability k_{dh} less than k_h, and a vertical permeability k_{dv} greater than k_v. If it is conceded that some flow landward from the drain always occurs in the prototype, and thereby gives some natural pressure relief, the possible decrease from k_h to k_{dh} within the drainage zone would form a slight obstruction to horizontal flow across the drain, and therefore would offset to some extent the pressure relief gained by increasing k_v to k_{dv}.

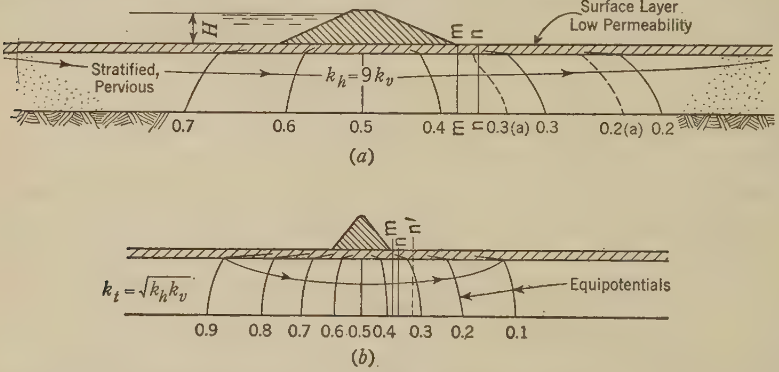

FIG. 30.—COMPARATIVE FLOW NETS FOR DAM OR LEVEE ON PERVIOUS FOUNDATION WITH $k_h = 9\,k_v$ FOR
(a) Natural Section
(b) Transformed Section

For the possible case $k_{dh} = k_{dv} = \sqrt{k_h\, k_v}$, the effectiveness of the drainage zone may be estimated by inspection of a flow net sketch. Fig. 30 represents a dam or levee on a pervious foundation in which $k_h = 9\,k_v$. The lines m-m and n-n represent limits of a potential drainage zone. If Fig. 30(b), all horizontal dimensions have been reduced in the ratio $\sqrt{k_v/k_h} = \frac{1}{3}$, and a flow net sketched in. This net has been replotted in Fig. 30(a) as shown by the solid equipotential lines, representing conditions without the drain. Foundation pressures in the region of the proposed drain appear to range from 35% to 38% of the total head. If the permeability of the drainage zone were altered to $\sqrt{k_h/k_v}$ the transformation in Fig. 30(b) would be accomplished by reducing all distances to the left of m-m and to the right of n-n, but leaving the interval between unchanged in scale, as shown by the line n′-n′ in the figure. Transposing the net back to the undistorted section gives the modified flow net indicated by dotted lines wherever it differs from the original. The pressure in the disturbed zone now appears to be from 30% to 36% of H, and the efficiency as previously defined would be about 15%. Actually, some additional pressure relief not indicated by the flow net would be gained by removal of the top stratum resistance, but the relief due to jetting alone is fairly well represented by the sketches.

Charles I. Mansur,[35] Jun. ASCE.—The importance of the control of substratum pressures and seepage beneath hydraulic structures on pervious foundations, as outlined by the authors, cannot be overemphasized. Uncontrolled substratum pressure or seepage has frequently been a direct or contributing cause of the failure of structures that must withstand differential hydrostatic pressures. In addition to the use of pressure-relief wells on the land side of levees and dams as discussed in the paper, wells may also be used to provide pressure relief and drainage beneath other types of structures on pervious foundations. Some examples are stilling basins for outlet and spillway structures, locks, dry docks, weirs, floodgates, and drainage structures.

The primary function of a system of drainage wells is to provide pressure relief and drainage for water under hydrostatic pressure in a pervious substratum. If a drainage well is to operate efficiently as a means of relieving substratum pressures, it must be designed to: (*a*) Offer little resistance to the flow of water into and out of the well; (*b*) prevent infiltration of sand into the well after initial pumping; (*c*) resist the deteriorative action of the water and soil; and (*d*) withstand the earth pressure.

The material for the screen and riser pipe is a special item, which should be selected for each installation. Some of the factors that must be considered in the selection of a material are: (1) Corrosiveness of the soil and water; (2) expected operation of the wells—whether intermittent or continuous; (3) method of installation; (4) size and depth of well; (5) durability under water and alternate wetting and drying; (6) crushing strength; and (7) types of joints for joining the various sections. If practicable, all joints should be flush and smooth with the outside diameter of the pipe. A variety of materials which may be used as drainage wells are: Brass, wrought iron, aluminum, cast iron, galvanized iron, plastic, cement asbestos, bituminous fiber, concrete, clay tile, porous concrete, and wood. None of these materials should be used without first ascertaining whether they will withstand the corrosive action of the soil and water in which they will be placed. General experience has indicated that certain materials, such as brass, stainless steel, cast iron, and special metal alloys, clay tile, and wood, are to be preferred for drainage wells.

The wells, including screen, riser, and discharge pipe, should have an inside diameter which will permit maximum design flow without a head loss (friction and velocity) of more than $\frac{1}{4}$ ft to 1 ft of water, depending on the residual pressure permissible in the design. In no instance should the inside diameter be less than 2.5 in. From hydraulic considerations the screen section of wells up to 6 in. in diameter should have at least twenty-five holes or ten slots with an open area of at least 3 sq in. to 6 sq in. per lin ft of screen. For design purposes it is the writer's belief, based on test data, that the screen openings should be equal to, or less than, the 70% size of the foundation sand or filter gravel.

The joints in the screen and riser sections of the well should be designed and constructed, where practicable, in such a manner that they will support the weight of the pipe as it is lowered into the hole. When not practicable to construct the joints in this manner, the riser should be supported, while being lowered into the casing, by some means which will insure that the joints will not open up. Where the screen and riser pipe consists of numerous loose joints (as, for example, with clay or concrete pipe) the top and bottom of the well should be joined with a brass or wrought-iron rod to eliminate or minimize the opening of any of the joints. All joints should have an overlap of at least ½ in., and should be coated with a good quality asphaltic roofing cement or equivalent plastic.

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Experience indicates that wooden pipe is one of the best materials for pressure-relief wells, providing the screen and riser pipe are submerged continuously. The screen of wooden wells may be perforated either with circular holes or slots. It is desirable that circular holes be seared smooth with a hot iron prior to installation. Wooden pipe may be slotted with a small circular saw. Various widths of slot may be obtained by using one or more circular saw blades.

Where metal pipes are used for relief wells, all parts of the screen, riser pipe, and discharge pipe should be of the same kind of metal to minimize corrosion by electrolysis. The results of field and laboratory tests indicate relatively little difference in the discharge efficiency of metallic well screens of the same diameter that have not had an opportunity to corrode or clog. Laboratory tests, using a circular well-tank testing device, on six different new 3-in. commercial well screens, showed well-screen entrance losses of about 0.2 ft to 0.5 ft of water for a discharge of 5 gal per min per lin ft of well screen. A uniform, fine foundation sand was used in these tests.

Where filter gravels are to be placed around the screen part of the well, the perforations should be equal to, or less than, the 70% size of the filter gravel. Gravel filters around a perforated screen pipe should have a thickness of at least 4 in., measured radially from the outer circumference of the pipe. The filter material should be a washed gravel or a crushed stone composed of hard, tough, and durable particles. The gradation of the filter material will depend on the foundation sand being drained. The gravel should be of a size and gradation that will prevent "inwash" of any appreciable quantities of sand into the well, and also of a size and gradation which will be retained by the screen section of the well. The filter should also be considerably more pervious than the foundation sand. These filter requirements may be met by a filter which conforms to the following criteria;

$$\frac{15\%\text{ size of filter}}{85\%\text{ size of foundation}} < 4 \text{ to } 5 < \frac{15\%\text{ size of filter}}{15\%\text{ size of foundation}}$$

In addition to meeting these criteria, filters should be graded uniformly without any lack or excess of particles of any particular size. To minimize segregation during placement under water, the filter should be as uniform as possible and yet meet the aforementioned criteria.

There are many pervious foundations that are sufficiently well graded and which contain sufficient gravel to permit the installation of perforated well pipe without an artificial filter. Following installation, such wells should be "developed" by pumping and surging so as to build up a pervious filter around the well screen. The perforations in the screen for a well of this type should be of sufficient size to permit the inwash of sand within the immediate vicinity of the well and yet to hold back the larger gravel particles, which thus build up a natural filter.

Porous concrete pipe may also be used as a filter around perforated pipe screens. If porous concrete pipe is used as a filter, it should be fitted closely around the inner pipe and should have a wall thickness of at least 1 in., and preferably greater. It should drain the foundation material freely without inwash of sand (this should be checked by laboratory tests prior to installation). The 70% size of aggregate used in the manufacture of the pipe should be greater than the perforations of the inner pipe.

The efficiency of various types of perforated pipes, with pervious gravel filters surrounding them, is illustrated by the results of tests in a well tank, using a fine, uniform foundation sand (see Table 3).

TABLE 3.—RELATIVE EFFICIENCY OF PERFORATED PIPE (FLOW, 5 GAL PER MIN PER LIN FT)

WELL PIPE		PERFORATIONS		Filter	HEAD LOSS, IN FEET OF WATER	
Inside diameter, in inches (1)	Material (2)	Size and shape (in.) (3)	No. per linear foot (4)	(5)	Screen (6)	Screen and filter (7)
2½	Clay tile	$\frac{3}{16}\ \phi$	48	Porous concrete[a]	....	0.3 to 0.5
6	Wood	$\frac{3}{8}\ \phi$[b]	36	Gravel[c]	0.12	0.28
6	Wood	$\frac{5}{16}$ by 2½[b]	6	Gravel[c]	0.18	0.24

[a] 4-in. inside diameter pipe, 1-in. wall thickness. [b] Holes and slots seared with a hot iron. [c] Uniformly graded gravel from No. 20 sieve to ½-in., filter 8-in. thick.

The casing for drainage wells may be installed by various methods; but the method used should be one which will not cause any excessive disturbance of the foundation sand outside the casing. Where filter materials are to be placed around the well screen, the bottom foot of the hole should be filled with the filter gravel prior to placing the well. The screen should then be properly centered in the casing and held securely in position while the filter is being placed and compacted, and while the casing is being withdrawn.

The filter material should be placed in small increments, so as to permit thorough "rodding" and not to interfere with the removal of the casing. To obtain a continuous filter, each increment of it should be compacted by rodding immediately after placing, and again after each increment of casing has been withdrawn. The casing should be withdrawn in an amount approximately equal to the depth of increment of filter placed. The alternate placing of filter and withdrawing of casing should be continued until the filter has been placed to the elevation of the top of the screen section and the casing has been withdrawn. The riser pipe should then be cut off at the elevation indicated for its termination, capped, and connected to the collector line or ditch in an approved manner. As pointed out by the authors the space around the riser pipe through the top stratum must be sealed carefully.

Promptly on completion of the installation of a well it should be pumped until a clear stream of water is obtained. Sometimes it is desirable to surge the well. When a well continues to produce sand it should be abandoned and a new well constructed near by. Abandoned wells should be plugged, and the riser pipe removed where possible.

Under the heading, "Design of Relief Well Systems," the authors state that a safety factor of 1.5 as applied to the critical gradient, $i_c = \frac{\gamma_s}{\gamma_w}$, may be satisfactory. The adequacy of this factor of safety depends, to a considerable extent, on the completeness of the field exploration of the foundation and top stratum. As illustrated by the authors, sand boils on the land side of dams and levees are the result of excessive hydrostatic pressures in a pervious substratum, which offers communication of pressure and seepage from the river or from borrow pits excavated to sand on the river side of the structure. If this subterranean pressure creates a hydraulic gradient sufficient to cause "flotation" of the soil in cracks, holes, or other thin or weak spots in the top stratum, sand boils will appear. The gradient required to cause such flotation may be considerably less than that theoretically required to cause flotation of the entire top stratum. As the foundation and top strata are usually nonhomogeneous in character, erosional seepage tends to concentrate in paths of least resistance to flow, causing sand boils to appear at localized spots instead of causing the entire foundation or top stratum to become "quick." Because of the foregoing reasons, effect of time on well-screen efficiency, and the other uncertainties of well design, a factor of safety of 2.0 may be warranted. If the flow from the wells is more than necessary to provide adequate pressure reduction, the wells can always be throttled down by valves, orifices, or other means; however, in some cases, conditions or time may not permit the installation of additional wells if the original design is not adequate.

At various times, there has been considerable discussion on the increase of seepage on the land side of levees, occasioned by the installation of pressure-relief wells. The primary purpose of pressure-relief wells is to reduce the hydrostatic pressure on the land side, or the downstream side, of levees or dams. By so doing, the hydraulic gradient from the river or reservoir toward the land side is increased, of course, which therefore increases the landward flow a certain amount. The writer knows of no field observations regarding the amount of such increase of land-side seepage; but the following example, based on the theory of critical gradient as presented by the authors, and on a study of seepage models by the writer, indicates the approximate increase of landward seepage that would be caused by drainage wells. Begin with four assumptions: A water height, H, of 30 ft; a top stratum depth of 8 ft; a critical gradient, i, of 0.9; and a saturated foundation.

The flow through a pervious stratum beneath a levee is directly proportional to the head drop from the river side to the land side. The maximum hydrostatic pressure that can exist land side of the levee would be $h_2 = 8 \times 0.9 = 7.2$ ft. (Any substratum pressure greater than this would raise the land-side blanket.) Therefore, the net hydrostatic head causing landward flow would be $\Delta H = 30 - 7.2 = 22.8$ ft. The flow resulting from this head drop will emerge somewhere landward of the levee as general seepage, or as flow from sand boils. If a sufficient number of drainage wells were installed to lower the residual pressure on the land side to, say, $h'_2 = 1.0$ ft, the landward flow would be that corresponding to a pressure drop of $\Delta H' = 30 - 1 = 29$ ft. In other words, the drainage wells would increase the landward flow by $\left(\frac{29.0 - 22.8}{22.8} \times 100\right) = 27\%$. The total flow that must be cared for by the surface drainage system land side of levees, during flood time, is composed of surface runoff occasioned by rainfall, general seepage, and the additional flow that would result from the installation of some drainage system. On the basis of the foregoing computations, it may be seen that the additional flow resulting from drainage wells would be relatively small in most instances, considering the total flow that must normally be cared for by the surface drainage system.

The authors have very thoroughly covered the theoretical design of relief wells. Proper design of well spacing, diameter, and penetration is of primary importance in a well system. Of equal importance in the design of a relief well system is the design of the individual wells and their installation because, unless the wells function properly, the best of theoretical designs will not accomplish their purpose. Therefore, some additional discussion of this aspect of well design and installation was considered warranted.

S. J. JOHNSON,[36] JUN. ASCE.—In bridging the gap between the mathematician and the engineer in a practical manner, the authors have rendered a service to the engineering profession. The design of relief well systems has been greatly simplified by the design curves in this paper, without which the solution of the fundamental design equations becomes a time-consuming operation. Relief well systems are assuming such importance that the presentation of this paper is indeed timely.

The mathematical development presented by the authors is for purely artesian flow. A formula similar to Eq. 7*a* has been developed by Phillipp Forchheimer[28] for gravity flow in a homogeneous soil, based upon J. Dupuit's assumption that the gradient on any vertical is constant and equal to the slope of the seepage surface in the flow direction. The depth of saturation h at any point is given by:

$$h^2_s - h^2 = \frac{Q_g}{2\,\pi\,k} \log_e \left[\frac{\cosh j\,(y + s) - \cos j\,x}{\cosh j\,(y - s) - \cos j\,x}\right] \qquad (62)$$

in which h_s is the depth of saturation at the line-drive source; Q_g, the gravity flow discharge out of each well, is:

$$Q_g = \frac{2\,\pi\,k\,(h^2_s - h^2_w)}{2 \log_e \dfrac{e^{js}}{j\,r_w}} \qquad (63)$$

The writer has had occasion to use the gravity equations in studies for a dam that rests on a pervious foundation possessing a natural impervious top layer.

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[36] Engr., U. S. Waterways Experiment Station, Corps of Engrs., Vicksburg, Miss.

Over the downstream part of the foundation, this top layer had been removed to permit foundation drainage, which was hindered by foundation stratifications by-passing part of the underseepage. This condition created undesirable uplift pressures at and beyond the downstream toe of the dam. To study the effect of pressure-relief wells, the gravity discharge, Q_g, was equated to the artesian seepage, Q_a, that existed between the upstream end of the natural blanket and the upstream end of the stripped dam foundation, thus:

$$Q_g = \frac{2\,\pi\,k\,(h^2_s - h^2_w)}{2 \log_e \dfrac{e^{js}}{j\,r_w}} = Q_a = \frac{k\,a\,d\,(\Delta H + h_w - h_s)}{L} \quad \ldots\ldots\ldots(64)$$

—to obtain h_s which is the head at the upstream end of the stripped foundation. In Eq. 64 ΔH is the difference in elevation between reservoir pool and tailwater of the well discharge (head losses in the well considered to be negligible), and L is the distance between the upstream end of the natural blanket and the upstream end of the stripped foundation. With h_s thus determined, the head at any desired location may be determined from Eq. 62. For conditions studied, the head h_s is slightly less, and the heads midway between wells are slightly more (as computed by the foregoing method), than those given for similar points by Eq. 7*a*.

An important item, as the authors have indicated, is the determination of the effective distance from the source to the line of wells. Information on this factor can be determined in advance, when the wells are to be installed behind existing dams or levees, by placing piezometers at various distances from the dam or levee on the water side. Another method is to evaluate this distance by mathematical means[32] after first determining the permeability and extent of the permeable stratum and top stratum. This latter method must be used when piezometer observations cannot be made. The effective distance from the source can also be determined for existing installations if the quantity of flow is known, together with the permeability and the dimensions of the foundation soils. These three methods were used by the writer in analyzing data obtained from a drainage well installation at a flood control dam. The methods gave results which were all the same degree of magnitude, although there were some differences, which probably can be explained by the variations in field conditions which are generally present.

The behavior of the toe drainage provisions at one dam was studied by the U. S. Waterways Experiment Station (Vicksburg, Miss.) in January, 1945. The effect of closing the wells on the piezometric pressures in the line of wells was very pronounced, since the head above ground surface increased from about 1.3 ft with the wells in operation to 6.1 ft with the wells closed. Inasmuch as the natural blanket, or top stratum, was only 8 ft thick at the section where these readings were taken, the necessity of maintaining the wells in good operating condition is evident. Tests such as described herein obviously should be made only under closely controlled conditions.

An important point, not considered by the authors, is the reduction of well discharge rates after installation. The Waterways Experiment Station has studied the reduction in well discharge of the Sardis experimental well installation. A study of forty-six wells completed in May, 1942, showed that, assuming the initial output as 100%, after two years the average output of all wells had dropped to 35%. However, surging of the wells at that time increased the average well output to 56% of the original output. The comparisons were made at the same net head. These reductions may be caused by one or both of the following: (1) Silting up of the bottom of the reservoir; and (2) clogging of the well screen or filter. Inasmuch as the reservoir is fed by a muddy river some silting probably occurs, although the total quantity of seepage flow has not decreased materially according to other data available. The general improvement of the wells after surging indicates that the major cause of discharge reduction is screen clogging. Such clogging can be caused by imperfect filter action or by chemical deposition.

The experimental wells were pulled during December, 1946, and the screens revealed corrosion and deposition, especially those with fine screen or mesh openings and those that were composed of iron pipes with brass screens or mesh. The ground water at this site has a low pH-value and a sufficient amount of dissolved CO_2 to render it corrosive. The ratio of the 85% size of the sand or gravel immediately adjacent to the screen to the size of screen opening varied from 1.0 to 1.4 for the nonmetallic pipe; whereas the ratio of the 15% size of the filter gravel to the 85% size of the foundation sand varied from 2.3 to 3.0. Some inwash of sand occurred during surging after initial installation. In view of the foregoing satisfactory ratios and the variations of the well discharges, it is evident that there was either a progressive clogging of the well screen, or filters, or both. The surging in June, 1944, materially increased the flow, and thus it probably partly cleared the screen clogging—but not entirely. In general, it is desirable to keep the entrance velocity through the screen openings as low as possible, since the change in pressure head outside the screen to velocity head in the screen releases CO_2 and other gases, thereby increasing corrosion and incrustation.

The useful life of water-supply wells was the subject of six papers presented at the 1946 Annual Conference of the American Water Works Association, and which have been reported in abbreviated form.[37] These papers presented factual data showing that screen clogging and incrustation are common and that a reduction in well output with time is to be expected to some degree. "The average life of screened wells under all conditions seems to be from 25 to 30 years," according to E. W. Bennison, M. ASCE, and "Sufficient length of experience is not available to forecast the life of gravel-packed wells but it is believed one-half of the value is gone in 15 years," according to A. D. Henderson. Although the conditions surrounding the use of water-supply wells may be different from those surrounding the use of relief wells, the data mentioned in this discussion are at least indicative.

The experiences cited herein emphasize the necessity for designing pressure-relief wells oversize to provide for possible reduction in capacity, and to provide for easy replacement of wells at the expiration of their useful life, regardless of whether that be five or fifty years. From the data and experience to date, it appears necessary to consider carefully the materials of which the wells are to be made, and the ground water and soil conditions at the site.

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[37] *Journal*, A. W. W. A., January, 1947.

In the experimental well system at Sardis Dam (Mississippi) it was found (as the authors have stated) that wooden wells performed satisfactorily. In this connection, it is interesting to record that slotted wooden wells have been used for water supply purposes for many years in Holland, and with satisfactory results. The wells used in that country are made of hard wood. The wooden wells may be obtained with thin slots parallel to the grain, which are about $\frac{1}{16}$ in. wide, 3 in. long, and spaced at intervals of approximately $\frac{1}{2}$ in. around the outside of the pipe. They may also be obtained with slots perpendicular to the grain but inclined upward, which would tend to minimize the possibility of sand being carried into the well.

One item, which the writer believes to be especially important, is the determination of the discharge elevation of the wells. In general, it is highly desirable that this elevation be as low as possible. Considerable care should be given to this decision, as it may mean the difference between a system which is entirely satisfactory and one which is not.

Gravel filters artificially placed around relief wells are necessary if an adequate natural filter is not developed by pumping and surging of the well. The relative merits of these two types of filter are somewhat controversial; however, the use of one or the other is considered essential.

The writers are to be congratulated for bringing the design of relief well systems to the attention of engineers in a timely and well-prepared paper.

KENNETH S. LANE,[38] Assoc. M. ASCE.—The authors have presented a worthwhile and timely assembly of theoretical considerations and experimental data which form the basis of design of relief wells. Such wells are among the most flexible of the various types of drains used to relieve seepage pressure by tapping pervious foundation strata, in accordance with the principle that control of seepage pressure rather than seepage quantity is the main factor in avoiding underseepage failures—an important principle which the authors have done well to emphasize. It is rare when uncertainties do not exist about a pervious foundation, especially in regard to chance arrangement of minor geologic layers of widely different permeability, which greatly influence seepage behavior. Hence, it is generally necessary to test the effect of several plausible assumptions of foundation conditions. In certain cases the straightforward charts of the paper will quickly furnish results to guide judgment. For several other common foundation cases, reference could well be made to formulas developed by Preston T. Bennett[32] and William H. Jervis, Members, ASCE, and Reginald A. Barron,[32] Jun. ASCE. These formulas give solutions for the head lost in seepage beneath the structure, and thus indicate whether the resulting seepage pressures in the zone of seepage emergence are likely to be sufficient to cause flotation, warranting relief well or other drainage treatment.

For the condition of flotation (when upward seepage pressure approaches, or equals, downward soil weight), two effects are prone to occur: (1) Emerging seepage concentrated in boils of moderate to large size tending to develop into piping which progresses as an underground channel toward the water source; or (2) flow dispersed over the emergence zone of upward seepage, causing a quick condition over a substantial area, which is often accompanied by many tiny to small boils. In stressing the fact that boils of all sizes are dangerous, the authors give two examples, of the first effect (piping), and also mention the second effect—a general quickening of the foundation. In the latter state the soil behaves as a heavy liquid and has only minor shear strength; a mat of vegetation or root-bound topsoil will generally weave under foot, and seepage is likely to escape from many small boils.

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The writer recalls one instance near West Springfield, Mass., where more than 1,000 small boils were counted in a reach of less than 1,000 ft behind a levee, which was then subjected to about two thirds of the design head. Fortunately, the flood was of short duration and seepage reduced the effective head by building up tailwater so rapidly that the levee escaped failure. Subsequently, a relief well system was designed to meet this condition.

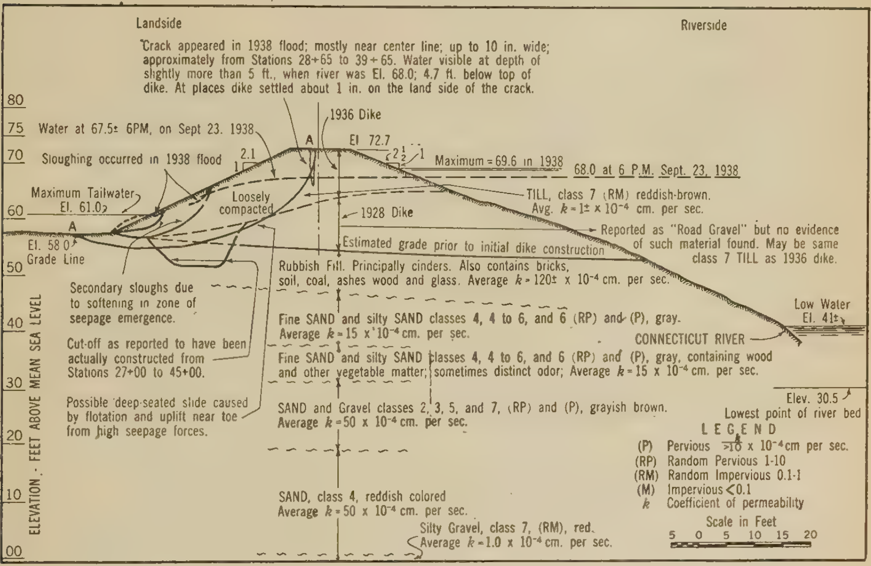
Landside
Riverside
Crack appeared in 1938 flood; mostly near center line; up to 10 in. wide; approximately from Stations 28+65 to 39+65. Water visible at depth of slightly more than 5 ft., when river was El. 68.0; 4.7 ft. below top of dike. At places dike settled about 1 in. on the land side of the crack.
80
75
70
60
50
40
30
20
10
00
ELEVATION - FEET ABOVE MEAN SEA LEVEL
Water at 67.5± 6PM, on Sept 23. 1938
Sloughing occurred in 1938 flood
2.1
1
1936 Dike
A
El 72.7
2½
1
Maximum = 69.6 in 1938
68.0 at 6 P.M. Sept. 23, 1938
Maximum Tailwater El. 61.0
Loosely compacted
TILL, class 7 (RM) reddish-brown. Avg. k = 1± x 10⁻⁴ cm. per sec.
1928 Dike
A
El. 58.0
Grade Line
Reported as "Road Gravel" but no evidence of such material found. May be same class 7 TILL as 1936 dike.
Estimated grade prior to initial dike construction
Rubbish Fill. Principally cinders. Also contains bricks, soil, coal, ashes wood and glass. Average k = 120± x 10⁻⁴ cm. per sec.
Secondary sloughs due to softening in zone of seepage emergence.
Fine SAND and silty SAND classes 4, 4 to 6, and 6 (RP) and (P), gray. Average k = 15 x 10⁻⁴ cm. per sec.
Low Water El. 41±
CONNECTICUT RIVER
Cut-off as reported to have been actually constructed from Stations 27+00 to 45+00.
Fine SAND and silty SAND classes 4, 4 to 6, and 6 (RP) and (P), gray, containing wood and other vegetable matter; sometimes distinct odor; Average k = 15 x 10⁻⁴ cm. per sec.
Possible deep-seated slide caused by flotation and uplift near toe from high seepage forces.
SAND and Gravel classes 2, 3, 5, and 7, (RP) and (P), grayish brown. Average k = 50 x 10⁻⁴ cm. per sec.
Elev. 30.5
Lowest point of river bed
LEGEND
(P) Pervious >10 x 10⁻⁴ cm per sec.
(RP) Random Pervious 1-10
(RM) Random Impervious 0.1-1
(M) Impervious <0.1
k Coefficient of permeability
SAND, class 4, reddish colored Average k = 50 x 10⁻⁴ cm. per sec.
Silty Gravel, class 7, (RM), red. Average k = 1.0 x 10⁻⁴ cm. per sec.
Scale in Feet
5 0 5 10 15 20

FIG. 31

With such a general quick condition there is real danger that slides may develop on the downstream or land-side slope of the water barrier embankment, from loss of toe support, as the quick condition results in a drastic loss of shear strength of the foundation soil adjacent to the embankment toe. If the slide, or succession of slides, extends back to intersect the water surface, it is followed immediately by overtopping, and a crevasse results.

A partial example of dangerous loss of toe support is the near failure of Springdale Dike during the September, 1938, flood. This levee was built by local interests along the Connecticut River near Holyoke, Mass. Its section is homogeneous, not specially compacted, and contains a shallow cutoff, strangely located near the land side. Fig. 31 shows foundation conditions and behavior observed, except in the land-side area where tailwater prevented observations. During the flood, electric failure occurred within the pumping station and tailwater was built up rapidly, partly from seepage, but largely from unintentional diversion through an unclosed cross connection from a sewer communicating with the river. As an explanation of surface evidence observed, line AA in Fig. 31 has been drawn as a possible surface of a slide of the land-side slope which may have been in progress until accidental creation of a tailwater pond lowered the effective head, and very likely was a major factor in averting complete failure. To no small degree the unfortunate location of a partial cutoff at the land side, instead of at the river side, would contribute toward such a slide by increasing seepage pressure in the land-side part of the levee to nearly full headwater pressure—shown as having occurred from the water level observed in the center-line crack. Although the Springdale Dike foundation might be treated with relief wells, the reconstruction design included a continuous pipe drain (extending from 4 ft to 10 ft into the pervious upper layer of fill), which the writer considers as more positive than intermittent relief wells for foundations not too highly stratified, and where the most pervious important foundation layers lie at such depth that they can readily be tapped by a drain.

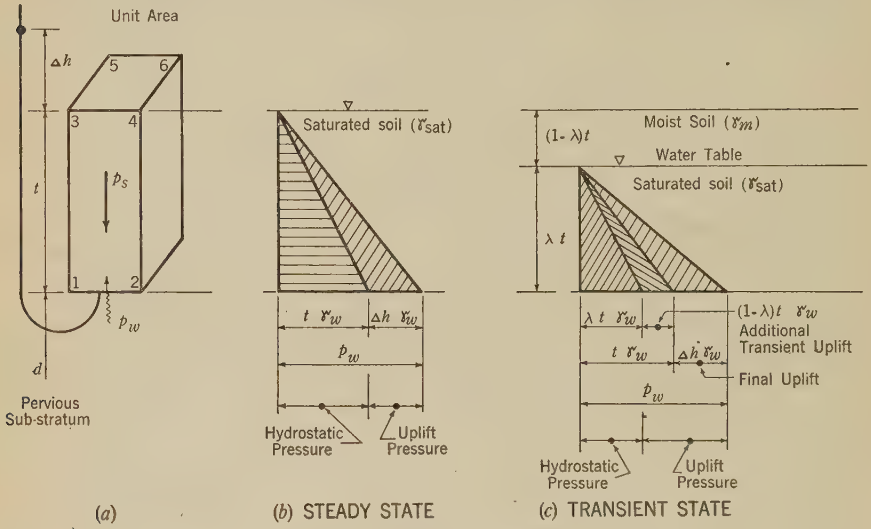
Unit Area
Δh
5
6
3
4
t
p_s
1
2
p_w
d
Pervious Sub-stratum
(a)
Saturated soil (γ_{sat})
$t\ \gamma_w$
$\Delta h\ \gamma_w$
p_w
Hydrostatic Pressure
Uplift Pressure
(b) STEADY STATE
Moist Soil (γ_m)
$(1-\lambda)t$
Water Table
Saturated soil (γ_{sat})
$\lambda\ t$
$\lambda\ t\ \gamma_w$
$(1-\lambda)t\ \gamma_w$
Additional Transient Uplift
$t\ \gamma_w$
$\Delta h\ \gamma_w$
Final Uplift
p_w
Hydrostatic Pressure
Uplift Pressure
(c) TRANSIENT STATE

FIG. 32

As indicated by the authors' examples, most of the present relief well installations have been placed behind existing structures as repair measures. From successes in such usage, relief wells are now being designed increasingly as part of initial construction. To aid judgment in determining when pressure-relief treatment is initially needed, it is of interest to investigate the equations of flotation in more detail than is permitted by Eqs. 3 to 6. Figs. 32(*a*) and 32(*b*) show the steady state case usually considered for investigating flotation of a top stratum which is saturated, with the water table at the surface. Seepage is flowing upward and Δh represents the piezometric pressure or uplift head acting at the base of the top stratum, as estimated from computing head lost in seepage beneath the structure from solutions given by the authors and by others.[32] For the state of flotation, the total upward pressure—

$$p_w = t\,\gamma_w + \Delta h\,\gamma_w \qquad (65a)$$

—equals the total downward pressure—

$$p_s = t\,\gamma_{bs} + t\,\gamma_w \qquad (65b)$$

—assuming no resistance on the sides of the parallelepiped 1–2–3–4–5–6, Fig. 32(*a*). This assumption is substantially correct when flotation occurs, because the soil then acts as a heavy liquid with no shear strength. Accordingly:

$$t\,\gamma_{bs} + t\,\gamma_w = t\,\gamma_w + \Delta h\,\gamma_w \qquad (66a)$$

or

$$t\,\gamma_{bs} = \Delta h\,\gamma_w \qquad (66b)$$

Flotation occurs when the buoyant soil weight is equaled by the uplift. In Eqs. 65 and 66 γ_{bs} is the buoyant or submerged soil unit weight and γ_w is the unit weight of water. When Δh becomes equal to Δh_c, sufficient to create flotation, the gradient, $i = \frac{\Delta h}{t}$, equals the critical gradient, i_c:

$$i_c = \frac{\Delta h_c}{t} = \frac{\gamma_{bs}}{\gamma_w} \qquad (67)$$

which is similar to Eq. 6 and indicates that i_c, being dependent only on the soil density, represents a property of the soil in its natural state.

The factor of safety against flotation is generally expressed as the ratio of unbalanced forces acting (herein termed apparent factor of safety, $(\mathrm{FS})_a$):

$$(\mathrm{FS})_a = \frac{\text{buoyant weight}}{\text{uplift}} = \frac{t\,\gamma_{bs}}{\Delta h\,\gamma_w} = \frac{1}{i}\frac{\gamma_{bs}}{\gamma_w} = \frac{i_c}{i} \qquad (68)$$

For strict correctness it is considered the factor of safety should be expressed as a ratio of total forces acting (herein termed true factor of safety $(\overline{\mathrm{FS}})$):

$$(\overline{\mathrm{FS}}) = \frac{t\,\gamma_{bs} + t\,\gamma_w}{t\,\gamma_w + \Delta h\,\gamma_w} = \frac{\frac{\gamma_{bs}}{\gamma_w} + 1}{1 + \frac{\Delta h}{t}} = \frac{1 + i_c}{1 + i} \qquad (69)$$

Eqs. 68 and 69 are compared in Fig. 33, which shows that the function $\frac{1 + i_c}{1 + i}$ is not a sensitive one. It has a definite maximum in the vicinity of $(\overline{\mathrm{FS}}) = 2$ and has the further disadvantage of not varying directly with Δh which is related directly to total head acting. Since it eliminates these disadvantages, the function $\frac{i_c}{i} = (\mathrm{FS})_a$ is considered the flotation factor of safety in usual practice, with which the writer concurs.

The authors suggest (in the text following Eq. 6) that "it may be that design for a safety factor of 1.5 against developing the critical gradient is satisfactory." If the authors intend this to be computed as $(\overline{FS}) = 1.5$, it becomes of the order of $(FS)_a = 3.0$ when converted in Fig. 33. However, if the authors intended this to be obtained from the usual equation, $(FS)_a = \frac{i_c}{i} = 1.5$, then the writer is inclined to feel that the suggested value is low for dams, although probably reasonable for most levees except those protecting property of particular importance or value. To a considerable extent the choice of a numerical value for a design objective should depend on the severity of the assumptions used because, in the analysis of flotation, a factor of safety often represents a factor of ignorance to allow for the great influence on seepage behavior caused by discontinuous stratifications of widely different permeability. Fortunately, for decidedly stratified foundations a lower factor of ignorance is applicable to a relief well than to a usual drain treatment because, except for a drain of comparable depth, the relief well treatment is the more positive. Once it has been decided to use relief wells in a levee design, it will often be found that the additional cost of increasing safety to the desired objective by a moderate decrease in spacing is not great.

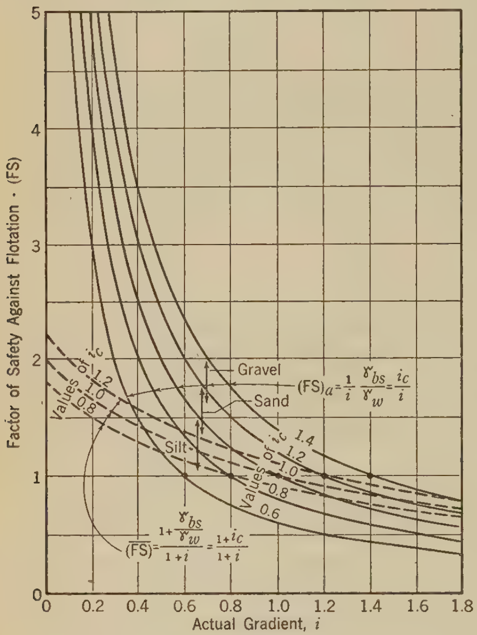

Fig. 33

The most difficult problem is to decide if wells are needed. In studying relief well applications to several levees around built-up areas of moderate property value, the writer has found the following a convenient approach which is advanced here more to stimulate thought than as a proposal for general application. Two flood heights are considered—(1) design flood and (2) top-of-levee flood. The latter equals design flood plus freeboard. If analysis reveals conditions that are considered moderately dangerous for the design flood, and decidedly so for the top-of-levee flood, then pressure-relief treatment is recommended. For less severity, consideration is given to taking a calculated risk by initially omitting extensive treatment and providing for later installation if needed. As the authors mention, the relief well is a particularly flexible tool for such later installation. Once pressure-relief treatment is adopted, an attempt is made to secure definite safety for the design flood and at least a slight margin of safety for the top-of-levee flood.

Greater conservatism is felt warranted in the case of dams, including consideration of the possibility that partial quickening of the downstream foundation with attendant reduction in shear strength may begin at a gradient below that computed as the critical from Eq. 67. The writer is inclined to the belief that this is a definite possibility from occasional qualitative observations of a quicksand device. In several performances the soil column parted near the middle; the top part lifted a matter of several inches and remained suspended while seepage emerged under a gradient somewhat below a probable value of i_c. Rather similar behavior also has been reported by others.[39] To explore the possibility of such behavior in nature, consider the transient case shown in Fig. 32(*c*) with tailwater rising toward the surface from an initial position of the water table below depth *t*. When tailwater is rising from point 1 to point 3, the top stratum will tend to lift when the total upward pressure—

$$p_w = \lambda\, t\, \gamma_w + (1 - \lambda)\, t\, \gamma_w + \Delta h\, \gamma_w \ldots\ldots\ldots\ldots\ldots (70a)$$

—equals the total downward pressure exerted by the partly saturated soil—

$$p'_s = \lambda\, t\, \gamma_{bs} + \lambda\, t\, \gamma_w + (1 - \lambda)\, t\, \gamma_m \ldots\ldots\ldots\ldots\ldots (70b)$$

—assuming that the side friction can be neglected around the periphery 3–4–5–6. Although this assumption is too conservative for a small area as indicated in Fig. 32(*a*) (since at least the moist soil above the water table retains substantial shear strength), it becomes reasonable for large areas where the peripheral friction is small in comparison with the weight, p'_s.

For the condition of lifting, equating $p_w = p'_s$ from Eqs. 70*a* and 70*b* and rearranging terms:

$$\underbrace{\underbrace{\Delta h\, \gamma_w}_{\substack{\text{Final}\\ \text{uplift}}}}_{\text{Final uplift}} \;=\; \underbrace{\underbrace{\lambda\, t\, \gamma_{bs}}_{\substack{\text{buoyant}\\ \text{soil}}} + \underbrace{(1 - \lambda)\, t\, \gamma_m}_{\substack{\text{moist}\\ \text{soil}}} - \underbrace{(1 - \lambda)\, t\, \gamma_w}_{\substack{\text{additional}\\ \text{transient uplift}}}}_{\text{net downward pressure}} \ldots\ldots (71)$$

Defining $(i_c)_m$ as the lifting gradient at the time that lifting of the partly moist top stratum begins:

$$(i_c)_m = \frac{\Delta h_{cm}}{t} = \lambda \frac{\gamma_{bs}}{\gamma_w} + (1 - \lambda) \frac{\gamma_m}{\gamma_w} - (1 - \lambda) \ldots\ldots\ldots\ldots (72a)$$

Let

$$\beta = \frac{\gamma_m}{\gamma_w} = \frac{S\,(\gamma_{sat} - \gamma_d) + \gamma_d}{\gamma_w} \ldots\ldots\ldots\ldots\ldots\ldots (72b)$$

in which *S* denotes percentage saturation. Making necessary substitutions of β from Eq. 72*b* and i_c from Eq. 67:

$$(i_c)_m = \lambda\, i_c + (1 - \lambda)\, \beta - (1 - \lambda) \ldots\ldots\ldots\ldots\ldots (72c)$$

or

$$(i_c)_m = \lambda\, [i_c - (\beta - 1)] + (\beta - 1) \ldots\ldots\ldots\ldots\ldots (72d)$$

Expressing the apparent factor of safety against lifting, $(FS)'_a$, as the ratio of net downward pressure to final uplift (see Eq. 71) and using the same substitutions as in Eq. 72*c*:

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[39] "Cofferdams," by Lazarus White and Edmond A. Prentis, Columbia Univ. Press, New York, N. Y., 1940, p. 13.

$$(\mathrm{FS})'_a = \frac{1}{i}\left[\lambda\, i_c + (1-\lambda)\,\beta - (1-\lambda)\right] = \frac{(i_c)_m}{i} \qquad (73)$$

in which $i = \dfrac{\Delta h}{t}$ is for the final condition when the water table has reached the surface as in Fig. 32(*b*).

An expression can also be derived for true factor of safety against lifting which (omitting the algebra) results in:

$$(\overline{\mathrm{FS}})' = \frac{\text{total downward}}{\text{total upward}} = \frac{p'_s}{p_w} = \frac{1 + (i_c)_m}{1 + i} \qquad (74)$$

Eqs. 73 and 74 are similar in form to Eqs. 68 and 69; and, using $(i_c)_m$ for i_c, then are solved easily from the curves in Fig. 33.

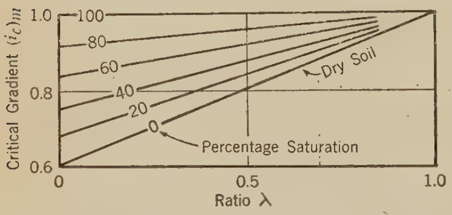

FIG. 34.—EFFECT OF SATURATION, WITH CONSTANT DENSITY ($i_c = 1.00$; $e = 0.65$; $\gamma_d = 100$; AND $G = 2.65$)

Eq. 72*c* shows that $(i_c)_m$ varies directly with λ for a given soil and is a minimum for $\lambda = 0$. It is always less than i_c for less than full saturation until $\lambda = 1$, whereupon $(i_c)_m = i_c$. With other variables held constant, $(i_c)_m$ varies directly with β and is a minimum for dry soil. Fig. 34 shows this characteristic for a typical loose sand ($i_c = 1.00$) where the value of $(i_c)_m$ is reduced nearly one half for a dry soil and for $\lambda = 0$. Fig. 35 shows the effect of changes in density with constant saturation, drawn for a common specific gravity of $G = 2.65$ and saturation of $S = 70\%$.

The computation of the lifting gradient, $(i_c)_m$, as appreciably less than i_c, is believed to explain the parting and lifting in the quicksand device, which was generally observed at the first beginning of upward flow when the soil was dry to moist. After seepage emerged for some time and was then shut off, successive resumptions of upward seepage did not again cause lifting, presumably as the soil was then saturated and $(i_c)_m$ equaled i_c for all values of λ according to Fig. 34. The quicksand device used was not adapted to close measurement of actual gradients; hence verification of Eq. 72*d* is as yet qualitative rather than quantitative.

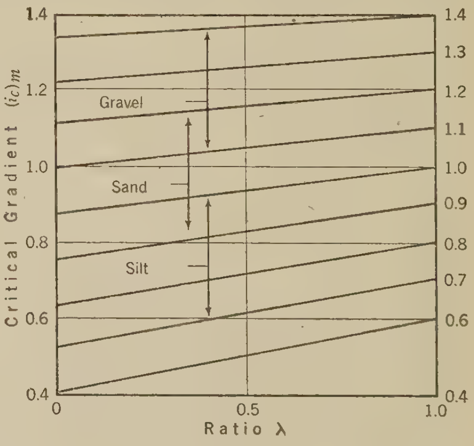

FIG. 35.—EFFECT OF DENSITY, WITH CONSTANT SATURATION ($S = 70\%$ AND $G = 2.65$)

In nature, the case of moist soil with a rising water table is a transient state. Since piping is not likely to start working backward from the seepage outlet until the steady state has been attained and has established that outlet, most flotation problems should be analyzed for the gradient i_c in the steady state. However, a few cases are visualized where analysis on the basis of the lower gradient $(i_c)_m$ may be of considerable importance. Consider a top stratum of clay, tight and impervious, underlain by a material of such greater permeability that the seepage from it can accumulate rapidly to fill such gaps as might be formed by lifting of the top stratum. With the common case of a sheet pile cofferdam, braced at the top and toed into above top stratum, it is easy to visualize loss of toe support by lifting of the top stratum sufficiently for failure by rotation around the top brace. Another example is that of a dam or high levee, with such a critical condition of downstream slope stability that even a small loss of toe support would be undesirable. Pending the assembly of quantitative data on the possible reduction in shear strength at gradients below i_c, the writer has attempted to allow for this by considering $(i_c)_m$ as the point where reduction starts. For one important dam so analyzed, the result was the inclusion of a few relief wells to relieve seepage pressure in a pervious stratum buried about 130 ft deep under soft varved silt in order to insure against any reduction in strength of the already weak silt foundation.

T. A. Middlebrooks,[40] Assoc. M. ASCE, and William H. Jervis,[41] M. ASCE.—The excellent discussions of this paper have added immensely to the knowledge of relief well design and will be most helpful to engineers who are confronted with this problem.

Mr. Barksdale emphasized the need for careful investigations. In-place permeability pumping tests, mentioned by Mr. Barksdale, are essential for large jobs, where maintenance pumping cost will be excessive. However, in most cases, it is cheaper to install a few additional wells as an added precaution against low discharge estimates than it is to conduct expensive pumping tests. Head loss in the casing and screen, and clogging of wells due to growth, are most important factors, all of which justify larger wells and periodic checks.

Mr. Turnbull's prediction that relief wells will be used on all important engineering structures on pervious foundations is a good possibility as soon as the costs of well installations are lowered by improved techniques. His comment that the foundation may be too pervious for relief wells to remove sufficient water to reduce the pressure to a safe value is a possibility, but a remote one. Collector systems are preferred in most cases; however, they are expensive and should be omitted where it is practicable to do so. The Mississippi River levee installations referred to were experimental and woefully underdesigned. There is no doubt that these installations would have been satisfactory if wells of 6-in. diameter or larger had been used, and if the wells had penetrated deeper into the water-bearing strata. Tight packing around the well is costly and it is not important as long as no provision is made for closing the well during operation.

Mr. Gilboy has emphasized that a wide difference between vertical and horizontal permeability is present in all deposits. This difference is large even in seemingly uniform deposits of sand. His comments on controlled versus uncontrolled seepage are important. A boil serves the same purpose as a relief well; but it is uncontrolled and dangerous.

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[40] Head Engr., Corps of Engrs., Chf., Soil Mechanics, Geology and Geophysical Section, Washington, D. C.

[41] Cons. Engr., O. J. Porter & Co., Sacramento, Calif.

Fully penetrating wells, as shown by Messrs. Wall and Stone, are definitely preferred. Wall spacing no greater than the distance to the source, as they propose, appears reasonable and installation of wells greater than the theoretical is certainly desirable.

The adoption of grouted outer seals, as proposed by Mr. Charles, is definitely preferred to the use of bentonite wherever either is needed. His suggestion of dual-purpose wells is a definite possibility in some locations, such as wells for industrial and irrigation purposes.

Electrical analogy models will certainly be helpful in analyzing dam problems as described by Horace A. Johnson. Fig. 13 clearly indicates the advantage of full penetration over partial penetration.

Professor Rutledge has made an excellent presentation of the variables and the effect of these variables on the design. He further emphasizes the importance of being certain that the well has adequate diameter to reduce friction losses to a minimum. H. H. Roberts and Carter V. Johnson have offered further data on the Fort Peck installation. Their suggestion to install a minimum number of wells initially is sound, and further emphasizes the flexibility of the well system provided the wells installed are large enough and deep enough. The failures mentioned involved the deterioration of the slotted black metal pipe used as an emergency installation during World War II. Recently, permanent installations were completed at Sardis and Fort Peck dams, using a slotted wood pipe.

Mr. Fahlquist has requested data on fabrication and installation of wells. Although experience with water well installations has been most helpful in selecting the general type of design, there is still much to be done in the selection of a well screen that can be depended upon for long life. Experience with relief wells has shown it to be a most serious problem. Usually the water carried by the pervious strata where pressure relief is required is much more highly charged with active salt than water-supply wells in the same area. At Sardis Dam metal well screens, which were giving excellent service in water-supply wells, failed within two years when operated as relief wells. The seepage water was highly acid, whereas in the water-supply wells it was neutral. At Fort Peck the ground water in the pervious strata was highly alkaline and was found unsatisfactory for water-supply purposes before the dam was built. The slotted black pipe used in the temporary installation during World War II started failing because of complete deterioration of the metal in about two years. At both of these dams slotted wood pipes have been used in the recent "permanent" installations. The writers recommend that only noncorrosive materials, such as wood, chemical resistant vitrified clay, stainless steel, monel metal, and other highly resistant metals should be used. Jetting to create continuous drainage is a definite possibility and should be further investigated. In this case the most difficult problem is to obtain a trench wide enough to discharge the required quantity of seepage.

Mr. Cedergren has properly emphasized the importance of exercising considerable care in constructing, and observing the functioning of, relief wells. In connection with the latter, the United States Corps of Engineers has a special research project on relief well design which includes detailed observations of all major relief well installations. A current meter has been designed for measuring the velocity in relief wells by lowering it into the casing. In this manner the total discharge of the well can be quickly obtained; and, in addition, by reading the velocity at different elevations, the quantity of seepage from the various strata can be determined.

Professor McNown has revealed one of the most troublesome factors in the presentation of mathematical solutions—that is, the lack of uniform symbols. The writers regret that the symbols used were not familiar to Professor McNown. Mr. Barron has presented an excellent discussion in which he has obtained a mathematical solution where a blanket of finite length is present. He has modified this solution for seepage through the blanket, which is an important factor in some cases.

Mr. Bennett's discussion should be especially helpful to engineers engaged in the design of relief well systems. He indicates the value of piezometer readings wherever they are available. Field pumping tests, of course, are highly desirable in the case mentioned, and in other cases where the area is highly developed and a large discharge is expected. However, pumping tests are expensive and must be conducted with experienced personnel and extreme care, or the results will be misleading. Where the quantity of seepage is not a major factor, it is usually cheaper to install additional wells than to conduct pumping tests. Modification of the formulas as presented by Mr. Bennett is certainly desirable where there is a definite gradient of flow away from the wells; but this is not often an important factor. As stated, jetting to form an outlet, as proposed by Mr. Fahlquist, may form a block for the more pervious strata and make seepage conditions in levee sections worse.

Mr. Mansur has stressed the importance of the type of well screen used in an installation. He places brass and cast iron in the preferred class. In some cases the use of these two materials may be highly questionable. His minimum diameter of 2.5 in. is considered entirely too small. It is the writers' opinion that this minimum should be 6 in. Quite correctly, Mr. Mansur emphasizes the danger of using dissimilar metals because of the rapid corrosion caused by electrolysis. Surging of relief wells is an essential part of well installation even though the well is gravel packed. Mr. Mansur has shown that wells behind levees will not increase the quantity of seepage by an exorbitant amount. The 27% increase given is reasonable where the pressure head at the land-side toe is reduced to 1.0 ft. Generally this great reduction of pressure is not necessary. Experience with large boils along the Mississippi River levees shows conclusively that a moderate increase in the normal seepage flow will prevent the occurrence of dangerous boils. During high water stages, the writers have seen numerous cases in which one or two large boils were giving adequate pressure release to an extensive area. However, when these boils were sacked (a ring levee built around them) to prevent movement of fines, other dangerous boils occurred in the vicinity. In these cases one or two large relief wells (which, in effect, are controlled boils) would give permanent relief to such dangerous areas.

Reduction of well discharge rates after installation is a most important factor as shown by S. J. Johnson. The wells referred to by Mr. Johnson were too small and high velocities were a contributing factor in the reduction of flow. This reduction has not been evident on large adequately designed wells. However, clogging is another factor in favor of large-diameter wells and provisions for surging as a maintenance operation. The clogging of water-supply wells is more common because of the high drawdown heads which result in high entrance velocities.

Mr. Lane has added further to the mass of recorded experience in giving a specific example in which the seepage at the toe of a levee has proved dangerous. His discussion of the problem of determining where wells are needed should be most helpful.

Since mathematical design formulas for relief well systems are based on idealized asumptions as to the uniformity of the pervious stratum and blanket and the determination of permeability (which, at best, is only a good approximation), the results from the formulas should not be considered as an exact design but rather as an approximation to aid the judgment of the designer. With this thought in mind, it is the writers' opinion, that reasonably accurate results, quickly computed, are more desirable than mathematically exact solutions, achieved through repeated trials. Also, the writers wish to emphasize that, in general, wells of large effective radius are necessary to provide sufficient entrance face area and to keep well-screen and riser losses at a minimum.

The writers wish to express their appreciation for the most valuable discussions and to express their regrets that they did not have time to compare the various modifications of design with the procedure proposed in the paper, and with the actual installations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2328

THE ENGINEER IN BUILDING FOR PEACE

ANNUAL PRESIDENTIAL ADDRESS

By E. M. Hastings,[1] President, ASCE

Past-President J. C. Stevens,[2] in his Annual Address in 1945, expressed the view that "The engineer will play a most important rôle in the revitalization of the world for peacetime pursuits." How many of the members of our Society today remember that most important statement? I am going to take it up and press forward with it now, two years after it was spoken, in an endeavor to direct attention to the tasks which to my mind are the most important that have ever been given to any people.

The day is here when the creative mind of the engineer must be used for the revitalization of the world if we are to work our way out of the confusion that is almost universal. All too long has the engineering mind been content to channel itself in narrow and, in many respects, selfish ways. There has been much broadening in our public expressions and efforts, reflected to a degree in the activities of the Society in recent years and given impetus in the actions taken by the Society today. We are concerning ourselves about many things outside the purely technical field and gaining for ourselves a large and respectful following in the areas where our profession should make us the leaders.

The mind of the engineer is trained to weigh carefully the important and fundamental elements that inherently are found at the beginning of any great engineering project. Should not that mind be used in leadership to formulate policies for the guidance of men in the pursuit of enduring peace?

Is it necessary that we wait for power politics, so-called diplomacy, or the might of certain nations to declare themselves for this or that solution before we go to work toward the building of the new and better world for which we so recently fought and for which we made so many sacrifices?

Our profession has contributed so many magnificent accomplishments conceived, designed, and constructed for the comfort and welfare of mankind. That same profession also has been used for so many devices that have been concocted to destroy and finally to wipe from the very earth large areas and unnumbered people. Now that we can see the magnitude of the destruction wrought by World War II, should we not face the future with a determination that such things shall not happen again?

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[1] Chf. Engr., Richmond, R. F. & P. R. R., Richmond, Va.

[2] "Private Enterprise," by J. C. Stevens, *Transactions*, ASCE, Vol. 110, 1945, p. 1605.

Although we have destroyed great cities and have in our hands the knowledge and means to destroy greater areas and wipe out nations, please bear in mind that we have one thing that cannot be destroyed, and that is a national ideal. Ideals are indestructible, they will rise out of the debris and ashes of a war-torn world and point the minds and energy of men toward nobler and higher purposes than ever have been known before. That is what we in this nation possess—the ideals of liberty and freedom. They form the rock upon which our nation is founded and have been the banner that has guided us through the trials and dark days of the past; they have been the torch that has lighted the way to victory and today they constitute a searchlight pointing the way to the revitalization of the world.

We engineers here in the United States of America cannot be content to wait for political diplomats to solve the problems; we must put our talents to work and enter wholeheartedly into the tasks of democracy that are now before us.

What is democracy? An American schoolgirl was asked this question. Her answer was: "It is what we have and are." Can you find a better definition than this? Look at what we have and are. I believe that our success as a nation stems from that which, through the years, we have set up in the United States. I am a believer in private enterprise and can find no hope for the world in the philosophy of a Karl Marx. I am a believer in universal military training. We should make our position so strong, so just, and so tempered with realistic generosity that we would gain the respect of the world. Such respect cannot be purchased from foreign nations with dollars.

Let the engineering profession then enter the affairs of the state, the nation, and the world as a profession that creates and revitalizes the factors for a world of peace. With all our resources, our marvelous skill for the production of material things, our creative genius for new and better achievments, our great might, and our power to command respect, it cannot be that we will be willing to let this year of opportunity pass with little or nothing accomplished to that end.

We of the engineering profession need to have for ourselves the inward resources that our young and virile America had when it fought for and won its independence and which our leaders have had in their ensuing struggles to maintain it. If we ever hope to persuade other nations to our way of life we must first recapture for ourselves the zeal, the courage, the eloquence, and the inner spirit of the great men of our history who made it possible. Never for a moment dare we relax our vigilance. Always both from without and within the United States, there are forces at work to destroy it. David Hume said, "It is seldom that liberty of any kind is lost all at once." Our minds must not weakly capitulate to the cynicism which exalts national selfishness, and we must cling hopefully and faithfully to the ideals of our past, so that we shall not lose that precious liberty bit by bit.

I have dealt up to this point with engineers in building for peace—generally and idealistically. In addition, the American Society of Civil Engineers and its membership, as a part of the engineering profession and as the oldest of the

Founder Societies should lead in bringing forcefully before the people of the Unites States and the people of the world the knowledge that engineers must play a most important rôle in the revitalization of the world for peacetime pursuits and economy. Let us no longer hold ourselves within the narrow confines of a purely technical life and of purely technical thinking. Let our members provide the leadership and the financial backing within our Society so that its professional activities will be effective. Let us strike out vigorously into the tasks that are here for us to do in a manner that shall win for us the admiration and the respect of diplomats and of statesmen, as well as of the general public. We can do it. Just because we have not done it before to any great extent is no reason why we should not now resolve to take our rightful place in the thinking and in the building of a new era of peace. As long as we adhere to American ideals, remember that we stand upon the firmest foundation of any nation in the world. We can try, and we can succeed.

MEMOIRS OF DECEASED MEMBERS

GEORGE HERRICK DUGGAN, Hon. M. ASCE[1]

DIED OCTOBER 8, 1946

Although national professional societies in America are young as compared with the European branches of engineering associations, the men who promoted the dissemination of scientific knowledge among early members are, in the nature of things, rapidly passing away. The death of George Herrick Duggan, under the tragic circumstances of an unforeseen motor accident at the age of eighty-four years, is a contingency of this nature which deprived the Society of one of its most valued Honorary Members.

George Herrick Duggan was born in Toronto, Ont., Canada, on September 6, 1862. He spent his early boyhood in Europe on the French Riviera, where his father died and was buried, but returned to Toronto as soon as he was old enough to attend Upper Canada College. From Upper Canada College, Mr. Duggan matriculated at the School of Practical Science, University of Toronto, now the Faculty of Applied Science and Engineering, University of Toronto, from which he was graduated in 1883 at the head of his class in ten subjects out of eleven. The eleventh subject was an essay. That summer he had a job and was able to spend only three days on the essay; yet he received 97%, whereas another chap who had spent the entire six months on the essay received 100%. At Toronto University he specialized in engineering under Prof. John Galbraith. By April, 1884, he had completed the postgraduate course and was awarded the degree of Civil Engineer.

Mr. Duggan's first professional undertaking, in the summer of 1884, was in the Rocky Mountain Division of the Canadian Pacific Railway Company engaged on construction work. For six months he was on the staff of the late Walter A. Doane,[2] M. ASCE, at the summit of the Rockies, designing wooden bridges, tunnels, etc. The year 1885 found Mr. Duggan on railroad location and construction on the west slope of the Selkirks, at the time that the late James Ross, M. ASCE, was the manager for that section of the Canadian Pacific Railway Company.

In 1886 he joined the engineering department of the Dominion Bridge Company which was under the direction of the late Job Abbott,[3] M. ASCE, and in 1891 was appointed chief engineer of that company. He held this position for ten consecutive years and was responsible for the design and construction of most of the important bridges built in Canada during that period—a period which so rapidly developed the science of steel construction. Among others built at that time under his supervision was the interprovincial bridge at Ottawa. In 1901 he was called upon to go to Sydney, Nova Scotia, as assistant to the president of the Dominion Iron and Steel Company and of the Dominion Coal Company and in 1904 was appointed general manager of the latter.

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[1] Memoir prepared by Frederick P. Shearwood, M. ASCE.

[2] For memoir, see *Transactions*, ASCE, Vol. 105, 1940, p. 1830.

[3] For memoir, *ibid.*, Vol. XXXVI, December, 1896, p. 538.

Mr. Duggan returned to the Dominion Bridge Company in 1910 as chief engineer when that company was contemplating the preparation of a competitive design of the Quebec Bridge. With this project in view, the St. Lawrence Bridge Company was formed by a combination of engineers from the Dominion Bridge Company and from the Canadian Bridge Company, with G. H. Duggan as chief engineer. The Quebec Bridge was finally designed and built under Mr. Duggan's supervision.

In 1912 he was appointed general manager of the Dominion Bridge Company, becoming vice-president in 1917, and president in the following year. He relinquished the presidency in 1936 but remained with the company as chairman of the board of directors, which office he held until his death. During the period of his association and largely through his influence, this organization grew from a small fabricating shop into a country-wide, important industry, manufacturing a diversity of steel structures, in the production of which thousands of men are employed and hundreds of engineers are engaged and selected for important executive positions.

Mr. Duggan was also president of the Dominion Engineering Company and of several other organizations allied with the Dominion Bridge Company. The long list of his directorships at the time of his death included the Shawinigan Water and Power Company, the Steel Company of Canada, the Montreal Trust Company, and the Wabasso Cotton Company. He was a former vice-president of the Royal Bank of Canada. In each and all these positions his engineering ability was frequently sought and depended on as of great assistance to his confrères in all their undertakings. He also accomplished much in the service of many professional societies—for example, he served his own Engineering Institute of Canada (which he joined in 1888) as councillor, vice-president, and president (1916); and he was made an honorary life member in 1937 when he presented the Institute with a medal and a prize to be awarded annually which now bears his name. In 1931 the Sir John Kennedy Medal was awarded him by the Institute for great services to his profession.

In 1936 this society conferred upon him an Honorary Membership. He was already a member of the Institute of Civil Engineers of Great Britain, a vice-president of the Canadian Institute of Mining and Metallurgy, and an honorary life member of the Canadian Enginering Standards Association.

The Canadian universities also honored Mr. Duggan: McGill University of Montreal, Quebec, and Queens University of Kingston, Ontario, awarded him the degree of Doctor of Laws; and Toronto University conferred on him the degree of Doctor of Science *Honoris Causa.*

Although the profession of engineering was his chief interest and occupation, Dr. Duggan's devotion to his favorite hobby and sport was remarkable enough to gain fame for him on two continents. He sailed and designed sailing yachts with exceptional success in many international races, notably those held for the challenge and defense of the Seawanhaka International Cup, which he brought to Canada in 1895. For six years, in spite of the fact that the very best that the professional designers of the United States could produce was lined up against him, his designs and his racing were always successful.

One might think that such a richly endowed mind might portray an austere character, aloof from his associates in his profession and sport, but such was not the case. Many members of the Society will remember him as a genial host, modest and quiet in manner, although his eyes often twinkled while he enjoyed the humor of his fellow engineers. His hospitality in his own home was proverbial among hosts of friends.

On October 13, 1888, he was married to Mildred Stevenson in Montreal. Dr. Duggan is survived by a daughter, Margaret, wife of Senator, the Honorable Adrian Knatchbull-Hugessen, his two sons, both graduates in engineering at McGill University, having given their lives for their country in World War I. He also left three grandsons, Edward Herrick, Andrew, and James; and a granddaughter, Mary Hugessen.

Dr. Duggan was elected a Member on October 2, 1895, and an Honorary Member on October 11, 1936.

ROBERT HOFFMANN, Hon. M. ASCE[1]

DIED MARCH 2, 1946

Robert Hoffmann, the son of Robert and Marie (Treiber) Hoffmann, was born in Cleveland, Ohio, on December 16, 1865.

Mr. Hoffmann received his elementary education in Brownell and Central high schools in Cleveland, and was graduated from Hiram College, at Hiram, Ohio, in 1885, with the degree of Bachelor of Science. Later he entered the Case School of Applied Science at Cleveland, from which he received the degrees of Bachelor of Science in Civil Engineering, in 1893, and of Civil Engineer, in 1896. During his undergraduate days at Case, he was elected to Tau Beta Pi and Sigma Xi. In addition to these academic honors, and in recognition of his "invaluable service to the community," Case School of Applied Science, in 1930, also conferred upon him the honorary degree of Doctor of Engineering.

Immediately following graduation from Case, in 1893, Mr. Hoffmann was employed by the City of Cleveland as a rodman in the paving department, and continued in the service of the city until his death. His advancement in the engineering department was rapid, and, in 1904, he was appointed assistant chief engineer. Three years later, he became chief engineer and held this position until 1930 although his title was changed to commissioner and chief engineer of the Division of Engineering and Construction. In 1930, in an effort to relieve him of much of the detail work in his department, Mr. Hoffmann was appointed consulting engineer on public works. In 1930, he was also appointed a member of a Special Engineering Commission which reported upon "The Future Development of the Cleveland Water Supply System." In 1939 and 1940, as a member of an advisory board of engineers, he assisted in the preparation of reports on "Sewage Rates" and "Proper Water Rates for Cleveland and Its Suburbs."

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[1] Memoir prepared by the following committee: Joseph W. Ellms, Wendell P. Brown, and William L. Havens, Members, ASCE.

For more than half a century, Mr. Hoffmann rendered distinguished public service to the City of Cleveland. Over a span of years matched by few men, and because of character, integrity, and devotion, he held his position, independent of politics, throughout successive administrations. Largely because of his foresight and skill in planning and constructing new streets, pavements, sewers and sewage treatment works, bridges, grade-crossing eliminations, and river and harbor improvements—with expenditures estimated at more than $117,000,000—the city was able to make proper provisions for its rapid development. Illustrative of the esteem in which he was held, and of the public confidence in his honesty, is the fact that, during the construction of the Division Avenue Filtration Plant—which had been designed by another city department—following a dispute between the contractor and the city, he was chosen, by both parties, to arbitrate the issue. The contractor "was willing to let his interest rest on the intelligence and fairness of Mr. Hoffmann."

Behind professional attainment was the man himself. At City Hall, he was distinguished for his modesty, patience, and forbearance. Among those of his own organization he was known affectionately as "The Chief" and his capacity for detail was regarded as amazing. He was a quiet man, intensely interested in his work, and those who knew him well realized that he had a keen sense of humor. Upon one occasion when City Council, in a fit of patriotic fervor, had acted to change the name of Hoffmann Avenue, because of its Teutonic origin, Mr. Hoffmann wrote: "No engineering objections," in approving the legislation and returning it to the Council.

Some twenty years ago, in a press interview, Mr. Hoffmann summarized his own ideas of public service, as applied to younger men, as follows:

> "Like every other occupation, it holds an appeal for certain natures. The youngster is impetuous, impatient. He chafes under delay. Action and rapid advancement is craved. That's why so few young men have a desire for public jobs.
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> "Patience is the prime requirement for the man who would work for the public. Endless red tape hampers his action. He must wait and wait, seemingly eternally. That red tape has its purpose. A man in private life is careful of his own interests. Serving the public he is inclined to be careless. His mistakes cost him nothing personally. That's why he is bound up with red tape.
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> "Public jobs offer wonderful opportunity for men of vision, imagination, and energy. It is within their power to perform services for the community far beyond the reach of the average citizen.

> "It is from the performance of such services that their satisfaction must come. Pay, as a rule, is not great. Advancement is not rapid. But for a man with an aptitude for such work, social in its nature as it is, the chances for a contented career are unexcelled. Public service provides a good living and it opens the door to information that will widen the outlook upon the world's life.

A former city councilman once identified Mr. Hoffmann to an inquiring press reporter in a single sentence, stating "That is Bob Hoffmann, the city engineer, and a man who comes as close to being entirely honest as you'll ever find any man ever being."

Modest and taciturn, he carried his honors lightly. In 1926 the Cleveland Chamber of Commerce awarded him its distinguished service medal in recognition of his contributions to the growth of Cleveland. The citation on this award read:

> "Undisturbed and uninfluenced by partisan considerations or changes of administration, he has steadily pursued a high policy of intelligent, loyal, and effective public service, and has both won the confidence of the people and contributed to the physical reconstruction of Cleveland to make the city more adequate for, and more responsive to, its increasing responsibilities and opportunities."

In 1945, the City Council passed a resolution extending to him the heartiest congratulations of the City of Cleveland, upon the occasion of his eightieth birthday. Perhaps the best tangible evidence of the esteem in which he was held is contained in the following resolution of the City Council, adopted March 4, 1946:

> "Whereas, the death of Robert Hoffmann has closed a career of public service to the city of Cleveland commencing in 1893 and paralleling the period of the growth and development of the city from a small struggling community to its present metropolitan status; and
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> "Whereas, the contribution of Robert Hoffmann to the welfare of this city is inestimable and much of its development is the result of his foresight and skill in planning and constructing the improvements which have so long been taken for granted; and
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> "Whereas, the engineering profession, the community and civic organizations and this council have bestowed honors upon Robert Hoffmann during his long and distinguished public career, and in so doing they have honored themselves and the city of Cleveland; and
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> "Whereas, the unimpeachable integrity, the unquestioned ability and tireless energy of Robert Hoffmann commanded the respect of all with whom he came in contact, and his innate modesty, his infinite patience, his unfailing kindliness, and his courteous and considerate manner, won and kept for him a host of friends and endeared him to his associates; now, therefore,
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> "Be it resolved that this council pauses to pay this final tribute of respect to the memory of Robert Hoffmann, to acknowledge the debt in which his life and services have left this, his native city, to bear testimony to his worth as a citizen and as an individual, and to take just pride in the splendid example of his distinguished public service; to mourn his death and to extend to his bereaved widow the sincere sympathy and condolences of the city of Cleveland; and the clerk of council be and he hereby is requested to transmit a certfied copy of this resolution to Mrs. Robert Hoffmann."

On his vacations Mr. Hoffmann liked to study the engineering achievements of other cities. He found recreation in the Professional Men's Club, a variegated lecture and discussion group of Cleveland. Symphony music was also a delight to him.

He was a member of the Board of Trustees of Hiram College; the Corporation of the Case School of Applied Science; numerous technical organizations—including the American Public Works Association; American City Planning Institute; International Association of Navigation Congresses; Cleveland Engineering Society (past-president and honorary member); and national, state, and county societies of professional engineers. He served as a Director of the American Society of Civil Engineers from 1932 to 1934. He was also a member of the Cleveland Chamber of Commerce, the City Club of Cleveland, and the Cleveland Athletic Club.

Robert Hoffmann was a distinguished engineer, a faithful public servant, and, withal, a gentleman whom everyone was proud to call a friend.

On September 3, 1903, Robert Hoffmann was married to Martha W. Muerman in Cleveland. He is survived by his widow.

Mr. Hoffmann was elected an Associate Member of the American Society of Civil Engineers on June 5, 1901; a Member on September 6, 1904; and an Honorary Member on October 11, 1936.

WILLIAM ANDREW ALLEN, M. ASCE[1]

DIED JUNE 27, 1946

William Andrew Allen was born in New York, N. Y., on March 1, 1868, of Irish parentage.

He received his early education in the public schools of New York, from which he was graduated in 1883, and later attended the College of the City of New York. From 1888 to 1891, he attended the Mechanics Institute of the General Society of Mechanics and Tradesmen in New York, receiving a certificate of merit for three years' attendance. In the years from 1891 to 1895, he attended Cooper Union Institute, also in New York, from which he received the degree of Bachelor of Science in 1899 and the degree of Civil Engineer in 1900. From 1888 to 1893, while still a student, he was employed by H. M. Smith and Sons, architects and builders, in New York.

For a period of thirty-six years, from 1896 to 1931, Mr. Allen was employed by the American Smelting and Refining Company and the Guggenheim interests, where he had full charge of construction engineering for steel, brick, and concrete buildings; railroads; railroad trestles; steam and electric power plants; and white lead plants and shot towers. His work also consisted in digging oil wells; coal mining; sinking shafts; building tunnels through rock;

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[1] Memoir prepared by Sidney H. Carpenter, Secy., General Society of Mechanics and Tradesmen, New York, N. Y.

and erecting entire town sites of one hundred and fifty houses—complete with stores, churches, and schools and serviced for sewerage, water, and electricity. These operations took place in California; Bayonne, N. J.; Garfield, Utah; Ely, Nev.; Eccles, W. Va.; East Hillside, Mont.; Omaha, Nebr.; Federal, Ill.; El Paso, Tex.; Pueblo and Leadville in Colorado; and Baltimore, Md.

From the fifth to the eleventh of October, 1918, during the explosion of the munitions plant at Morgan, N. J., Mr. Allen was in command of 1,200 men of various military organizations, for six consecutive days and nights, with headquarters at the city hall in Perth Amboy, N. J.

Mr. Allen was initiated as a member of the General Society of Mechanics and Tradesmen of the City of New York on December 19, 1923. He was a veteran of the Seventh Regiment, New York National Guard, and a member of the Cooper Union Alumni Asociation; the Washington Lodge No. 35 of the F. and A. M. in Elizabeth, N. J.; the New Jersey Consistory, thirty-second degree, of Jersey City, N. J.; and the Islam Temple of the Ancient Arabic Order of Nobles of the Mystic Shrine in San Francisco, Calif. In Perth Amboy he belonged to the Raritan Yacht Club, the International Rotary Club, and the Simpson Methodist Episcopal Church.

Mr. Allen was a man of fine purpose, and his word was his bond. To him, a promise meant that it must be fulfilled. He inherited honesty, integrity, and loyalty. He had many friends, because they found him to be ever faithful.

On August 29, 1931, in Annapolis, Md., he was married to Alva Adelaide Brandt. He is survived by his widow and a son, William McKinley Allen.

Mr. Allen was elected an Associate Member of the American Society of Civil Engineers on February 6, 1901, and a Member on September 6, 1904. He became a Life Member in January, 1936.

URIE NELSON ARTHUR, M. ASCE[1]

Died September 13, 1945

Urie Nelson Arthur, the son of John V. and Elizabeth (Clark) Arthur, was born on April 24, 1870, in Erie, Pa.

He obtained his education in the public schools of Erie; Erie Academy; Edinboro State Normal School, at Edinboro, Pa.; and Allegheny College, at Meadville, Pa. He received the degree of Bachelor of Science in Civil Engineering from Allegheny College in 1894. He was a member of the Phi Kappa Psi fraternity.

Mr. Arthur's first professional endeavors were in the railroad field. Beginning as rodman, he was employed by the Pennsylvania Railroad at Erie and at Renovo, Pa., from 1895 to 1898. For the next three years he served as assistant engineer for the Pittsburgh and Lake Erie Railroad at McKeesport, Pa. When plans were being prepared for routing the Wabash Railroad through Pittsburgh, Pa., Mr. Arthur served as assistant engineer under the late George T. Barnsley, M. ASCE,[2] from 1901 to 1904, in charge of much of the construction between Pittsburgh and Jewett, Ohio. His next professional work was with the Western Maryland Railroad Company, as division engineer in charge of two hundred miles of surveying and location in western Pennsylvania and West Virginia.

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[1] Memoir prepared by Louis P. Blum and Ray S. Quick, Members, ASCE.

His long service with the City of Pittsburgh began in 1907, when he became assistant engineer in the Bureau of Surveys; he was promoted to principal assistant engineer in 1910, becoming chief engineer in 1916. During this period, he planned and supervised the collection of an extensive history of every street in Pittsburgh and the annexed territory. This work, involving abundant research in old municipal records, has proved of great value and has been consulted extensively by municipal engineers, title insurance examiners, and others interested in early land titles. During Mr. Arthur's term as chief of the Bureau of Surveys, he supervised the establishment of a system of precise bench marks and survey monuments in a large area of the city.

When the Pittsburgh City Planning Commission was formed with the late Morris Knowles,[3] M. ASCE, as chairman, Mr. Arthur became the first chief engineer. The first project of this commission, done under the supervision and direction of Mr. Arthur, was the preparation of zoning ordinances and maps of the entire city area. Although the original ordinance has been amended from time to time and changes of zone boundaries have been revised to fit changing developments, Mr. Arthur's work still forms the fundamental basis of Pittsburgh zoning.

During this time, he was associated with R. H. Randall, M. ASCE, in the topographical survey of Pittsburgh. After Mr. Randall's retirement, the survey was directed by Mr. Arthur—to the extent allowed by councilmanic appropriation. This phase of his work is preserved in a joint publication by Messrs. Randall and Arthur, "The Geodetic and Topographic Survey of Pittsburgh and Allegheny County."[4] After his retirement from the city service in 1936, Mr. Arthur was associated in general engineering and surveying practice with the McCully Engineering Company of Pittsburgh.

He was the ideal public servant: Hard working, painstaking, and clear thinking; inspiring his associates to greater earnestness and zeal; and with little use for the diplomatic expression of opinion. Having made up his mind, as the result of study of an engineering proposition, he asserted his opinions directly and unequivocally, not hesitating to differ with his colleagues and superiors.

On April 26, 1914, Mr. Arthur joined the Smithfield Street Methodist Episcopal Church, under the ministry of Dr. J. T. Pender, by certificate of transfer. He was an active member of the Board of Stewards (later the Board of Trustees) and of the Board of Directors of the Church Union of the Methodist Church, then affiliated with the Smithfield Street Church. When the church was being remodeled during the pastorate of Dr. Daniel L. Marsh, Mr. Arthur was quite interested in the architectural changes and improvement. During his later years he was not quite so active, but he will always be remembered by his church as a Christian gentleman of distinctive and unusual qualities.

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[2] For memoir, see *Transactions*, ASCE, Vol. LXXXVII, 1924, p. 1312.

[3] For memoir, *ibid.*, Vol. 98, 1933, p. 1567.

[4] "The Geodetic and Topographic Survey of Pittsburgh and Allegheny County," Pittsburgh, Pa., December, 1925.

Mr. Arthur was married on September 12, 1907, to May E. Ryan of Renovo. He is survived by his widow and son, Robert L.

Mr. Arthur was elected a Member of the American Society of Civil Engineers on October 1, 1928.

CHARLES DWIGHT AVERY, M. ASCE[1]

Died February 1, 1947

Charles Dwight Avery was born at Galesville, Wis., on July 29, 1877. His parents were Henry N. and Catherine S. (Fowler) Avery.

At an early age he moved with his parents to Minneapolis, Minn., where he received most of his schooling including three years at the University of Minnesota. He then entered the Michigan College of Mines and Technology in Houghton, and was graduated with the degree of Engineer of Mines in 1903. At college he belonged to the Beta Theta Pi fraternity.

From 1903 to 1908 he was engaged in coal and metal mining, principally in Utah, and superintended some construction work in Oregon. In 1908 he was appointed a mineral and Carey Act inspector by the United States General Land Office of the Department of the Interior, with headquarters at Salt Lake City, Utah, and later at Cheyenne, Wyo. He remained with the General Land Office (later the Bureau of Land Management) from 1908 to 1920, doing field work in the states of Utah, Montana, Wyoming, and Colorado. In 1920 Mr. Avery transferred to the United States Geological Survey as a mining engineer, and later was appointed a senior geologist, remaining with the Geological Survey until his death.

While employed by the General Land Office, he examined and reported on the Carey Act irrigation projects, as to the adequacy of the water supply, the facilities for irrigating the land, and the estimated cost. He also reported on the feasibility of proposed projects from an engineering and financial viewpoint.

Mr. Avery's transfer from the General Land Office to the U. S. Geological Survey in February, 1920, antedated by about two weeks the approval, by the late Woodrow Wilson, then President of the United States, of the general leasing law for nonmetalliferous minerals in lands owned by the federal government. To the adoption of sound policies and expeditious procedures in the administration, particularly of the oil and gas provisions of this important legislation, the remaining twenty-seven years of Mr. Avery's public service were devoted. His personal knowledge of men and conditions throughout the states west of the 100th meridian was an invaluable asset to the Geological Survey in the discharge of its responsibilities under the mineral leasing law, and a never failing source of aid and inspiration to his associates. His remarkable record during these twenty-seven years included basic determinations made and reported to administrative agencies on the structural situation and the relation to other holdings of the same applicant, of the lands described in more than 57,000 applications for oil and gas prospecting permits on public lands, and of the more than 35,000 applications for oil and gas leases on such lands. He made determinations of the bona fide potential value for oil and gas of more than 34,500 land parcels affected by conflicting applications for homestead and mineral prospecting rights. All this was accomplished with less than one hundred appeals from his findings in the entire period. This achievement is in itself an eloquent tribute both to his judgment and to his capacity for constructive work and it contributed to the highly successful venture of the government in the field of oil and gas leasing.

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[1] Memoir prepared by Benjamin E. Jones, M. ASCE.

It is only a part, however, of his contribution, which also included most of the spadework involved in defining the "known geologic structure" of producing oil and gas fields containing federal lands, as an administrative basis for distinguishing between land parcels to be leased for oil and gas by competitive bidding, and others that might be leased to the first applicant therefor. The record of 250 outstanding definitions of such structures on January 31, 1947, aggregating more that 4,326,000 acres in nine public land states, is mostly of his making, and constitutes a lasting monument to the utility of his unpublished but invaluable public service.

In the field of conservation he was an ardent advocate, from its inception, of the policy of unit operation for oil and gas fields containing government lands.

He participated actively in the conferences and discussions leading to the amendment of the oil and gas provisions of the federal leasing law in 1930, 1931, and 1935, to authorize and encourage unit or cooperative development of such fields or pools. His only literary contribution in the period of his Geological Survey employ, aside from the thousands to his credit in the files of the Interior Department and its agencies, was as senior author of a paper on this subject, presented at the Dallas (Tex.) meeting of the American Association of Petroleum Geologists on March 23, 1934.[2]

Although Mr. Avery was primarily a mining engineer, his early work involved civil engineering, and it was then he became interested in the Society. He was President of the District of Columbia Section in 1927 and 1928. Throughout his twenty-seven-year residence in Washington, D. C., he was one of the most active members of the Section, and at his death was chairman of a committee engaged in revising the constitution, for which he was largely responsible.

In addition, he belonged to the American Association of Petroleum Geologists and to the Geological Society of Washington, D. C. He was a member of the F. and A. M., the Scottish Rite in Cheyenne, and the Korein Temple Shrine in Rawlins, Wyo.

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[2] "Relationship of Geology to Unit Operation of Oil and Gas Fields, Involving Government Lands," by Charles D. Avery and John C. Miller, *Bulletin*, Am. Assn. of Petroleum Geologists, November, 1934, p. 1454.

On December 6, 1917, he was married to Myra K. Graham of Greeley, Colo., at Storm Lake, Iowa. He is survived by his widow; a daughter, Nancy (Mrs. Charles A. Kuhl); and two grandchildren, Nancy Louise and Barbara Jean Kuhl, of California, Pa.

Mr Avery was elected an Associate Member of the American Society of Civil Engineers on January 2, 1912, and a Member on January 16, 1917. He became a Life Member in January, 1947.

HOMER GAGE BALCOM, M. ASCE[1]

DIED JULY 3, 1938

Homer Gage Balcom, the son and only child of Mahlon Balcom and Frances (Gage) Balcom, was born in Chili, N. Y., on February 16, 1870.

He received his primary education in the grade school at Morton, N. Y., and was graduated from the Brockport (N.Y.) State Normal School. After teaching for two years in Stone Church, Kendall, N. Y., he entered the College of Civil Engineering at Cornell University in Ithaca, N. Y., from which he was graduated with the degree of Civil Engineer in 1897. He was elected to the honorary society of the Sigma Xi.

After being graduated from Cornell, Mr. Balcom entered the employ of the Berlin Iron Bridge Company in Berlin, Conn., as a draftsman. In 1900 when the American Bridge Company was formed, inclusive of the Berlin Iron Bridge Company, he was appointed to one of the designing engineer positions of this new company and, in January, 1902, was put in charge of design for the New York (N.Y.) and Pittsburgh (Pa.) districts.

In 1905 he became structural engineer of Reed and Stem, architects for the Grand Central Railroad Terminal in New York, in charge of steel design for the station and the many terminal buildings. On these projects he mastered the difficult structural and foundation problems and also insulated the building framework against the transmission of vibration from the track and street by independent foundations, vibration absorbing mats in footings, and vertical separation between building and traffic carrying framework.

In 1908 he formed a partnership with Wilton J. Darrow, M. ASCE, and opened an office of consulting engineers in New York. This partnership continued until 1916 when Mr. Darrow retired from the firm. Mr. Balcom continued the office under his own name until a few years before his death, when he was joined by his fellow workers in the firm of H. G. Balcom and Associates.

His practice grew rapidly from the start, and before many years he was recognized as the leading structural engineer, for buildings, in the metropolitan area. A list of the structures for which he designed the steelwork would include buildings in practically all the large cities of the United States and also many in foreign countries.

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[1] Memoir prepared by George E. J. Pistor, M. ASCE.

A few of the more prominent structures are recorded: The Empire State Building, the Chrysler Building, most of the Rockefeller Center buildings, the Waldorf-Astoria Hotel, the Grand Central Terminal buildings, and the Federal Reserve Bank Building, all in New York; the Mellon Art Museum, the Archives Building, and the Department of Commerce Building in Washington, D. C.; the Cathedral of Learning in Pittsburgh; the Library of Louvain University in Louvain, Belgium; the Young Men's Christian Association Building in Jerusalem; and the Devonshire House in London, England. He was responsible for the structural design of buildings costing close to a billion dollars, an outstanding accomplishment.

Mr. Balcom found time and energy to serve the engineering profession on many technical committees, in matters on which he was considered an authority. He was president of the New York Chapter of the New York State Society of Professional Engineers and was earnestly interested in promoting public recognition of the engineering profession.

On January 3, 1912, Mr. Balcom was appointed a member of the Board of Education of Hastings-on-Hudson, N. Y., and served continuously until 1933. From 1921 to 1933, he held the office of president of the board and, as such, exercised a great influence in developing a modern suburban school system for Hastings. His aim was to create a centralized school plant where children from all sections of the village, both industrial and residential, would receive education and training in a democratic way of life.

During World War I he contributed his services as chief structural engineer of the American International Corporation in fabricated ship construction at the Emergency Fleet Corporation Yard at Hog Island, Pa. His knowledge of fabricating shop practices enabled him to design the frames and hull plating of the ten thousand-ton freighters so that the steel could be fabricated both rapidly and economically at scores of fabricating shops throughout the United States with such precision as to permit direct assembly and riveting at the shipyard without fitting and drilling.

Mr. Balcom was always glad and ready to assist his fellow engineer with advice whenever he was approached.

On October 24, 1900, he was married to Gertrude McCrum of East Berlin, Conn. Besides his widow, he is survived by their only child, Gertrude Marie (Mrs. John Moon) and three grandchildren, Barbara Joan Moon, Lois Gage Moon, and John Andrew Moon, Jr.

He belonged to the Cornell Society of Engineers and the American Society for Testing Materials; he was an honorary committeeman of the American Institute of Steel Construction, a director of the New York State Society of Professional Engineers, a past-president of the New York Chapter of that society, and past-master of the Fernbrook Lodge of the F. and A. M.

Mr. Balcom was elected a Member of the American Society of Civil Engineers on April 18, 1916.

WESTERN RADFORD BASCOME, M. ASCE[1]

Died June 14, 1940

Western Radford Bascome, the son of Western and Ellen (Kearny) Bascome, was born in St. Louis, Mo., on April 15, 1867. He came from a long line of American ancestors that reached back to Colonial times. Outstanding among these were: William Nicoll, member of the Council of New York from 1691 to 1698, speaker of the Colonial Assembly from 1702 to 1718, and Attorney General from 1687 to 1690; Matthias Nicoll, secretary of the Province of New York from 1664 to 1680, member of the King's Council from 1667 to 1680, speaker of the Provincial Assembly in 1683, and Mayor of New York in 1672; Jeremias Van Rensselaer, second patroon member and speaker of the Colonial Assembly in 1664; Stephanus Van Corlandt, Colonel of the Kings County Regiment from 1671 to 1693 and twice Mayor of New York; and Olaff Stevensen Van Cortlandt, Colonel of the Burgher Corps in 1649 and the last Burgomaster of New Amsterdam (1655 to 1664) before the English conquest. He was the grandson of Gen. Stephen Watts Kearny, the centenary of whose entry into Santa Fe, N. Mex., was celebrated in 1946 by the issuance of a commemoratory United States postage stamp.

Western Bascome was graduated from Washington University at St. Louis, with the degree of Bachelor of Engineering, in 1888, and the degree of Civil Engineer in 1889. His professional career began immediately after graduation. From June to November, 1889, he was employed by the Department of Bridges in St. Louis, engaged in drafting, designing, and field work.

From November, 1889, to May, 1893, he was with the St. Louis Merchants' Bridge Terminal Railway Company as assistant engineer during the construction of the elevated steel viaduct (about 8,000 ft long) with terminals and connections. This position included office work, drafting, designing, field work, and inspection.

From May, 1893, to May, 1896, he was employed as assistant engineer with the St. Louis Water-Works Extension (under the late M. L. Holman,[2] M. ASCE) in charge of field work on construction of four settling basins (each 400 ft by 700 ft) with connecting conduits, and gate chambers, at Chain of Rocks, Mo. He also did work on foundations and buildings for a high service pumping station at Baden in St. Louis. From May, 1896, to July, 1897, Mr. Bascome engaged in private business with his father in St. Louis.

On July 8, 1897, he entered the Department of Bridges of the City of New York (which became the Department of Plant and Structures in 1916), as assistant engineer on the construction of the Williamsburg Bridge across the East River, engaged on office work, drafting, computing, designing, and also on field work and inspection. From January, 1902, to June, 1903, he inspected the construction of main cables and suspenders, of that bridge. His next position, from June, 1903, to December, 1904, was assistant engineer, in charge of field work and construction of the Vernon Avenue Bridge, across Newtown Creek, between Brooklyn and Long Island City (center span of 172 ft, two side spans of 80 ft each, and about 1,400 ft of masonry and steel approaches).

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[1] Memoir prepared by Theodore Belzner, Affiliate, ASCE.

[2] For memoir, see *Transactions*, ASCE, Vol. 89, 1926, p. 1630.

For a short period in early 1905, Mr. Bascome again took charge of his father's business in St. Louis. Then, in April, 1905, he became assistant engineer on the Manhattan Bridge across the East River in New York City, designing plans for cables, towers, and superstructure, as well as plans for a subway terminal station for the Williamsburg Bridge. From March, 1906, to March, 1907, he was resident engineer in charge of construction of the Manhattan anchorage of Manhattan Bridge and, until June, 1910, was resident engineer in charge of all Brooklyn construction of this bridge, including anchorages, steel towers, side span, one half of main span, and approach. In the latter position, he was also in entire charge of construction of main cables and suspenders for the whole suspended structure.

Mr. Bascome was then assistant to the late E. A. Byrne,[3] M. ASCE, chief engineer in the Department of Plant and Structures, in charge of all correspondence, reports, and general executive work, until August, 1924, when he was transferred to the Department of Public Markets in New York City, as engineer in charge of construction of the Bronx Terminal Market (under the late J. O. Eckersley,[4] M. ASCE, chief engineer of design and construction). From December, 1926, to February, 1927, Mr. Bascome was again in charge of his father's private business.

He then became engineer in charge of the reinforcement and reconstruction of the Pines (highway) Bridge across Croton Lake, a part of the New York City water supply until March, 1931.

On March 18, 1931, he was transferred from the Department of Plant and Structures to the Department of Parks (New York City); and until May, 1933, he was resident engineer in charge of construction of the West Side Express Highway, along Riverside Park from West Seventy-Second Street to the Harlem River. From this date until June 16, 1935, when he retired from the Department of Parks, he was assistant to the chief engineer of parks in charge of general construction work. After his retirement until his death, he was a consulting engineer.

Mr. Bascome held the esteem and affection of his friends and associates, and the respect of all with whom he came in contact. He was a quiet, unassuming man, diplomatic and considerate in his dealings with all who came to know him. During the construction of the Williamsburg Bridge, there developed a lifelong friendship between Mr. Bascome and the late Holton D. Robinson, M. ASCE. In tribute to Mr. Bascome, Isidore Delson, M. ASCE, who had frequent contact with him as engineer of design during the reconstruction of the bridges in the Croton Watershed wrote:

> "He certainly was a gentleman in the true traditional sense of the word with whom courtesy and considerateness were innate, and not assumed mannerisms. In that sense he was not unlike his friend, the late H. D. Robinson."

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[3] For memoir, see *Transactions*, ASCE, Vol. 104, 1939, p. 1890.

[4] For memoir, *ibid.*, Vol. 103, 1938, p. 1785.

Mr. Bascome was a licensed professional engineer and land surveyor of the State of New York. He was also a member of the Society of Colonial Wars. His hobby was photography.

On August 10, 1904, Mr. Bascome was married to Shelley Barriger of Louisville, Ky. He is survived by his widow and a son, Western Radford Bascome, Jr.

Mr. Bascome was elected a Junior of the American Society of Civil Engineers on December 3, 1891; an Associate Member on May 5, 1897; and a Member on September 11, 1917. He became a Life Member in January, 1932.

JAMES EVERETT BESWICK, M. ASCE[1]

Died April 15, 1946

James Everett Beswick was born on May 18, 1882, in Hermon, N. Y., the son of William Everton Beswick and Eva (Kelly) Beswick. His earliest known ancestor was William Beswicke, Alderman of London, who died May 5, 1567, and was buried in St. Lawrence, Poultney, London, England. Mr. Beswick was named James Everton Beswick, but in early youth the family changed his middle name to Everett, by which he was thereafter known. The name, Everton, was carried by eight of his direct ancestors.

His earliest ancestor in America was George Beswick who resided at Wethersfield, Conn., in 1672. From there the family migrated to West Bridgewater, Scituate, and Chesterfield in Massachusetts, and thence to Warrensburg and Morley in New York, where his father was born. His direct ancestor Lt. Everton Beswick of Chesterfield was one of the forty-eight minute men who, under Capt. Robert Webster, responded to the Lexington alarm and marched to that battle on April 21, 1775.

Mr. Beswick's early education was obtained in the grammar and high schools at Hermon. He attended Clarkson College of Technology at Potsdam, N. Y., and was graduated in 1903 with the degree of Bachelor of Science in Civil Engineering.

His first engineering work was with Waring, Chapman and Farquhar, Consulting Engineers of New York, N. Y., where he was engaged on the construction of sewers and sewage disposal systems. This was followed in 1904 by the position of transitman with the Rapid Transit Subway Construction Company on the first subway from New York to Brooklyn, N. Y. From 1905 to 1907, he was with the Rapid Transit Commission of the City of New York as assistant engineer on the construction of the Battery Tunnel to Brooklyn and the Fulton Street Subway in Brooklyn.

In 1907 he transferred to the Board of Water Supply of the City of New York, where he was engaged on the Catskill Aqueduct, on topographic, real estate, rights of way, and alinement surveys, and on both opencut and tunnel construction. Later he was in charge of all work on shaft No. 17 at 42d Street and Sixth Avenue in New York City, including the shaft sinking and the complete tunnel construction.

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[1] Memoir prepared by Charles A. Pohl, M. ASCE.

For three years, from 1913 to 1915, he was with the New York State Highway Commission as special assistant to the late George A. Ricker,[2] M. ASCE, first deputy commissioner, on progress and final inspections of highway contracts and other special and confidential work. For a period he was resident engineer of Erie County, New York, in charge of all construction and maintenance.

From 1915 to 1920, he supervised all field work and construction of a section of subway for the Public Service Commission of New York, First District work, that cost about $3,500,000, and that included three stations with both opencut and twin-tube, shield driven tunnels.

During the years from 1920 to 1922 he was engaged in a comprehensive investigation of all existing power generation and transmission in the State of New York for the late Gen. William Barclay Parsons,[3] Hon. M. ASCE, and under the general supervision of Col. John P. Hogan, Past-President ASCE. This work also included collection of data on stream flow and rainfall and complete studies of stream characteristics and potential water-power developments.

His next work was as resident engineer in charge of construction of sewers and a sewage treatment plant at Rye, N. Y., from 1922 to 1924. He then built many miles of distribution lines, wells, and pumping plants for the Hicksville (Long Island) water district, during 1924–1925, after which he returned to Rye as chief engineer for about five years, from 1925 to 1929, engaging in extensive enlargements of the sewer system, and improvements and enlargements in the disposal system.

He served with the Water Policy Commission of New Jersey, compiling population studies and forecasts with particular reference to future water requirements for many of the most populous areas of the state. Stream studies that included frequency and magnitude probabilities were made, as well as some on water storage and distribution requirements.

He also made many studies of buildings adjacent to construction projects in connection with liability insurance against damage which might arise by reason of construction activities. This was followed by the position of resident engineer for the Public Works Administration in charge of a number of projects mostly in connection with the water supply of the City of New York which had an aggregate cost of about two million dollars.

From 1936 to the time of his death he was assigned to the Bureau of Claims of the Board of Water Supply of the City of New York. He worked in close cooperation with the Law Department in engineering settlements for property and rights-of-way claims arising from the acquisition of lands and rights for water supply purposes. He gave engineering assistance and advice in preparation for the trial of claims including the securing of expert witnesses. He also acted as an engineering expert in many technical cases involving water powers, special manufacturing processes, and similar enterprises affected by the city's acts. His department head said of him:

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[2] For memoir, see *Transactions*, ASCE, Vol. 99, 1934, p. 1501.

[3] For memoir, *ibid.*, Vol. 98, 1933, p. 1485.

> "Of the highest integrity, he was most dependable for an accurate opinion upon an engineering problem with which he was confronted. He never ventured an expression unless he was fully informed of his facts."

Throughout his life Mr. Beswick was an assiduous student of engineering and mathematical subjects. Completely trustworthy in every particular, he was always loyal and helpful to his associates and friends.

He was a licensed professional engineer in New York State, and a member of the Sons of the American Revolution and the F. and A. M.

On July 3, 1915, Mr. Beswick was married to Cecilia Agnes Farren of Albany, N. Y., who survives him.

Mr. Beswick was elected a Junior of the American Society of Civil Engineers on July 1, 1909; an Associate Member on October 1, 1913; and a Member on January 16, 1928.

ROBERT WRIGHT BOYD, M. ASCE[1]

Died March 22, 1946

The career of Robert Wright Boyd as an engineer and builder was in a period of great advancement in the development of modern structural design and in the standardization of building codes. An early interest in the design and use of reinforced concrete as a structural material greatly influenced his future achievements.

Robert Wright Boyd, the son of Robert W. and Mary (Ballantine) Boyd, was born in New York, N. Y., on July 14, 1879. He was graduated from the College of the City of New York with the degree of Bachelor of Science in 1899. His technical education was continued at New York University from which he received the degree of Bachelor of Science in 1900 and the degree of Civil Engineer, in 1901. His scholastic achievements brought him the Duryea Fellowship and the Hoe Engineering Prize. Because of Bob Boyd's sound common sense, pleasing personality, and ability to get along with others, in later life he was able to apply this thorough technical training in a very practical way.

After his graduation in 1901, he was employed by W. F. Whittemore,[2] M. ASCE, a civil engineer of Hoboken, N. J., and, for five years, he gained experience in drafting, surveying, and designing. Later he became general office assistant, supervising in the field construction work on sewers, water supply systems, waterfront development, and railroad track. In 1906, he went with Rudolph P. Miller, M. ASCE, consulting engineer of New York, as principal assistant and later associate, specializing in the design and supervision of construction of structural steel and reinforced concrete buildings and foundations. From 1910 to 1918, Mr. Boyd was in private practice—maintaining his own office in New York City as consulting engineer. His work covered not only the design and supervision of structural steel and reinforced concrete buildings, but also the broader field of special examinations and reports.

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[1] Memoir prepared by George E. Horr, Vice-Pres., Turner Construction Co., New York, N. Y.

[2] For memoir, see *Transactions*, ASCE, Vol. 106, 1941, p. 1696.

By this time, he was recognized as an outstanding authority in reinforced concrete construction, so it was not surprising that in January, 1918, he was made advisory engineer, then assistant head, and finally head of the concrete ship section of the United States Shipping Board Emergency Fleet Corporation. In this capacity he directed, designed, and supervised construction of concrete ships and barges needed by the United States Government to replace the vessels destroyed by the ravages of World War I. When this emergency program was completed, Mr. Boyd joined Turner Construction Company as engineer and later became chief engineer in charge of many important projects. His wide experience made him not only an able engineer but also an administrator and executive of real ability.

In November, 1934, the Turner Construction Company granted him leave of absence to head the depression-born Temporary Emergency Relief Administration of the City of New York. He continued in this work, later becoming assistant executive director and finally director of New York State Employment Service (until May, 1940). These difficult jobs were ably and efficiently handled and his administration was always free of the taint of political influence that so often hinders the efficient operation of government departments. On January 6, 1942, Mr. Boyd returned to Turner Construction Company and gave unstintingly of his great knowledge and ability to help in the execution of a huge war program. He was associated with that company until his death.

With all his many engineering and construction interests Robert Boyd still found time for activities outside his profession. He was for many years president and director of Roxmor Colony, Inc., a community built up by a group of summer home owners in the Catskill Mountains. He was a director of the United States Life Insurance Company, and for a long time an Elder in the First Presbyterian Church in New York. Each year he gave a talk at the New York University School of Engineering on "An Engineer's Relations with the Public." He also belonged to the American Society for Testing Materials, the honorary engineering fraternity Tau Beta Pi, and the Engineers Club of New York.

In both his private and business life, he was admired for his kindness, thoughtfulness, and unfailing willingness to help others. His one great hobby was his family.

On May 29, 1906, in New York he was married to Elsie Grace Bushong. He is survived by his widow; two sons, Robert W., Jr., and John B. Boyd; a daughter, Helen Bushong Boyd (Mrs. Laurence Duggan); seven grandchildren, Stephanie, Laurence, Robert, and Christopher Duggan, and Nancy, Robert W., III, and John Murray Boyd; a brother, William B. Boyd; and a sister, Virginia T. Boyd.

Mr. Boyd was elected an Associate Member of the American Society of Civil Engineers on January 8, 1908, and a Member on June 20, 1922.

WALDO EMERSON BUCK, M. ASCE[1]

Died April 6, 1945

Waldo Emerson Buck was born in Stoneham, Mass., on February 21, 1856. He was graduated from the Massachusetts Institute of Technology at Boston as a civil engineer in the class of 1876.

After following his profession for several years, and after gaining experience as an agent of the Lake Company in Laconia, N. H., he became a special inspector and chief adjuster for the Associated Factory Mutual Fire Insurance Companies on January 1, 1889.

In 1897 Mr. Buck was elected vice-president, secretary, and director of the Worcester Manufacturers Mutual Insurance Company and three years later became president and treasurer of that company. The latter offices were held by him until March 12, 1942, when he retired from active service as president emeritus. He continued as a member of the company's board of directors and as a member of the advisory board of the Boston Manufacturers Mutual Group until February, 1945, when ill health forced him to retire from that service.

Mr. Buck was a director of the United States Envelope Company and of the Merchants and Farmers Mutual Fire Insurance Company. He was a member of the Boston Society of Civil Engineers as well as of this Society.

His life was a full one which, because of his character and ability, enriched the lives of those with whom he came in contact. As a friend and associate he will be greatly missed.

On June 6, 1894, in Woburn, Mass., he was married to Frances Sherwood Jones. He is survived by his widow and a daughter, Eleanor (Mrs. Harry R. Davis), both of Worcester, Mass.

Mr. Buck was elected a Member of the American Society of Civil Engineers on July 3, 1889. He became a Life Member in January, 1924.

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[1] Memoir prepared by Otto F. Hauck, Secy., Boston Mfrs. Mutual Fire Insurance Co., Boston, Mass.

NEIL STANLEY BUCKBEE, M. ASCE[1]

Died April 10, 1946

Neil Stanley Buckbee, the son of William Doane and Kate (Brown) Buckbee, was born on September 21, 1883, at French Mountain, near Glens Falls, N. Y. He was graduated from Dartmouth College, in Hanover, N. H., in 1906, and from the Thayer School of Civil Engineering at Dartmouth in 1907.

Throughout his entire life, after leaving college, Mr. Buckbee practiced the profession in which he was educated in his native state and in the State of Pennsylvania, except for a brief period in Youngstown, Ohio. The first half of his business life was devoted constantly to reinforced concrete design and construction, with manufacturers or contractors in Buffalo, N. Y., Rochester, N. Y., and Niagara Falls, N. Y. The second half of his career was spent as designing engineer and then chief engineer on varied work with Gannett, Fleming, Corddry and Carpenter, Incorporated, of Harrisburg, Pa., an organization which he joined in 1929.

"Buck," as he was called, was a careful and accurate designer, an earnest worker, and a good teacher of the young men under him. He was greatly respected and appreciated by those closely associated with him, but, because of his refusals to attend conventions or to write engineering articles or speeches, he was not as widely known as he would otherwise have been. In his drafting room, he usually had several young Dartmouth men in whom he naturally took a particular interest.

The design and supervision of earth dams became Mr. Buckbee's specialty over the last fifteen years of his life. The largest jobs, those in which he took an active part, were three water storage reservoirs and pipe lines for the Lehigh Coal and Navigation Company and the cities of Harrisburg and Lebanon, Pa.

Mr. Buckbee was married in Canton, Ohio, to Alice May Loud, by whom he had three sons— Donald M., Neil S., Jr., and John A. Buckbee. He is survived by his widow; one son, John A.; four grandchildren; his mother; and two sisters, Mrs. Blanche Townsend and Mrs. William Edmunds. Mr. Buckbee died suddenly of unsuspected heart trouble on April 10, 1946, at the age of sixty-three. He was a licensed professional engineer in the states of New York and Pennsylvania.

During recent years the Buckbees lived in a very attractive house filled with the antiques which he and Mrs. Buckbee had collected over many years. He designed this house to meet their desires and needs and was exceedingly fond of it, as well as of his flower and vegetable gardens.

Mr. Buckbee was elected a Member of the American Society of Civil Engineers on June 16, 1919.

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[1] Memoir prepared by Farley Gannett, M. ASCE.

SAMUEL JEFFERSON CHAPLEAU, M. ASCE[1]

Died February 25, 1947

Samuel Jefferson Chapleau, the son of Maj. S. E. Saint Onge Chapleau and Caroline Kirby (Patten) Chapleau, was born at Atlanta, Ga., on January 1, 1869. Between 1873 and 1917 his father was, successively, secretary of the Department of Public Works of Canada, sheriff of the Northwest Territories, clerk of the Crown in Chancery, and clerk of the Senate.

After the usual elementary education, Samuel Chapleau entered Rensselaer Polytechnic Institute at Troy, N. Y., in 1887, being graduated with honors and the degree of Civil Engineer in 1891.

He began his career, most of which was spent in Canada, in 1892 as assistant to the division engineer, Eastern Division, Canadian Pacific Railway Company in Montreal, Que. Between January and August, 1894, Mr. Chapleau was in private practice in Ottawa, Ont. Then he was appointed engineer in charge of a topographic party on a hydrographic survey of the Fraser River in British Columbia for the Department of Public Works. In 1895 he was a structural draftsman in connection with the construction of the Soulanges Canal in Quebec. He then became assistant engineer on the adjustment of the final estimates for the Canadian Ship Canal at Sault Sainte Marie, Ont. From 1896 to 1900 he was assistant engineer in the Department of Railways and Canals, in charge of design, construction, and estimates on the Soulanges Canal construction.

From June, 1900, to March, 1901, Mr. Chapleau was assistant engineer on the New York State Barge Canal Survey, in charge of the design and estimates of locks and equipment on the proposed 12-foot canal between Buffalo and Albany, N. Y. He then returned to Canada and the Department of Public Works, being appointed engineer in charge of hydrographic investigation of the middle channel of the Saint Lawrence River in Ontario between Kingston and Brockville, and also of surveys and improvements of harbors and channels in the upper Saint Lawrence District. He left the department to become principal assistant engineer of the Department of Marine on the organization of the Canadian Hydrographic Office and was in charge of the district extending from Montreal, Que., to the Great Lakes.

In 1904 he again returned to the Department of Public Works as district engineer in charge of the Nipissing District of the Georgian Bay Ship Canal Survey, remaining until 1908, when he was appointed district engineer of the Upper Saint Lawrence District with headquarters at Ottawa.

In 1922 when this district was merged with the Ottawa District, Mr. Chapleau was promoted to engineer grade two at headquarters. With the reorganization of the headquarters staff in 1937, he became superintending engineer and chairman of the Board of Engineers, which position he held until his retirement from the public service on Jaunary 1, 1939, after more then forty-three years.

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[1] Memoir prepared by Fred G. Smith, Supervising Office Engr., Dept. of Public Works of Canada, Ottawa, Ont., Canada.

Among the outstanding engineering works for which Mr. Chapleau was responsible are the international bridges between Clair, N. B., and Fort Kent, Me.; Edmundston, N. B., and Madawaska, Me.; and Saint Leonard, N. B., and Van Buren, Me.; the Lasalle causeway between Kingston and Barriefield, Ont.; and the two control dams on Lake Nipissing at the head of the French River in Ontario. He also devised and completed the middle channel, 300 feet wide and 16 feet deep at low water, on the Saint Lawrence River between Kingston and Brockville through the Thousand Islands.

As a member of the Board of Consulting Engineers of the International Commission of the River Saint John in New Brunswick, he carried out an investigation of the River Saint John watershed. From 1908 until his retirement in 1939, he had direct control of all matters in connection with the French River watershed, Rainy River, and Lake of the Woods and of all international bridges between New Brunswick and Maine, and was engaged in studies of all engineering projects of major importance undertaken by the Department of Public Works.

Mr. Chapleau became an associate member of the Canadian Society of Civil Engineers in 1896 and attained full membership in 1909. He was chairman of the Ottawa Branch in 1911 and 1912 and councillor of that society in 1913, 1914, and 1915. Early in 1918 when the Canadian Society of Civil Engineers became the Engineering Institute of Canada, the "Institute" became the possessor of a case containing two beavers, one a pure albino. These were the gift of Mr. Chapleau and are highly prized by all members. In his letter to the institute at that time, he stated:

> "Some years ago I had occasion to arrange the distribution of flow in the Rainy River past the Fort Frances, Ontario—International Falls, Minnesota Section, in order that the Canadian Side would receive the benefit of its share, or one-half of all power generated there by the operating Companies, and during my stay at Fort Frances I greatly admired a pair of stuffed beavers, one black and one white, that I thought, if properly arranged together, would look well as a group illustrating 'The First Engineers,' after a picture that I had seen of like nature. I tried to buy the beavers at the time with the idea of sending them to the Main Society at Montreal, but was unable to do so, and expressed my ideas in that regard to Colonel D. C. McKenzie, then Mayor of the town.
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> "About one and a half years afterwards, much to my surprise, Dr. McKenzie sent me down two mounted beavers which are now in the case, asking me to accept them as a gift from himself and other friends of the town, so I turned them over to our Branch with the request that they be sent on to Headquarters.
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> "On enquiring for a mounted white beaver Dr. McKenzie found none were to be had, but learned from an old trapper that there was one on a lake in the Atikokan District, who trapped the same next Winter at Dr. McKenzie's request.
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> "The 141st Battalion, the 'Bull Moosers,' which was raised for Overseas Service by Dr. McKenzie as its Colonel, wished to take these specimens with them to England and present them to the King, but the Doctor preferred to carry out his original intention.

"They were mounted here by Mr. Petch, who does all that kind of work for the Dominion Museum."

As a result of this typical generous act, the institute's crest shows "a beaver at work" which typifies the engineering profession in general and Mr. Chapleau in particular. On October 18, 1935, he became a life member of the institute.

He was a member of Saint John's Anglican Church; Civil Service Lodge, A. F. and A. M.; Carleton Chapter, Royal Arch Masons; Ottawa Preceptory, Knights Templar; and Karnak Temple of the Mystic Shrine.

On October 21, 1897, Mr. Chapleau was married to May Edith Gouin, who predeceased him in 1944. He is survived by a son, Jefferson, and a daughter, Mary Louise (Mrs. L. Clare Moyer).

The funeral service was conducted by the Venerable Archdeacon C. G. Hepburn on February 27, 1947, and interment was at Beechwood Cemetery, Ottawa.

In Mr. Chapleau's death the engineering profession has lost an outstanding and brilliant engineer, a gentleman, and a friend.

His associates have paid him the following tribute:

"Life held you fast—
And how you loved it, too.
You found it good to touch
And see and smell the things of earth.
But came a day when Life,
With wistful fingers, beckoned you away.
You did not halt, nor fear, nor fail,
But straightway followed that strange lead
Into a world so full of life and joy,
That could we call you back with one quick word,
We would keep silent.
Yours is the gain,
And ours the blessing, too.
The world of things unseen, unfelt, unheard
Is strangely friendly now,
Because you walk where we have never trod
And sing the songs, the melodies of God."

Mr. Chapleau was elected an Associate Member of the American Society of Civil Engineers on May 1, 1901, and a Member on January 31, 1905. He became a Life Member in January, 1936.

JOHN ABELL CLEVELAND, M. ASCE[1]

Died April 17, 1946

John Abell Cleveland, the son of Charles and Lizzie (Houston) Cleveland, was born in Linden, Ala., on March 8, 1876. He entered the United States

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[1] Memoir prepared by a Committee of the Miami Section consisting of Lynn Perry, M. ASCE, and Herbert J. Morrison, Assoc. M. ASCE.

Military Academy at West Point, N. Y., in 1898, but resigned three years later to work with Archer Harmon, engineer, of New York, N. Y.

In May, 1901, he became associated with the Guayaquil and Quito Railway in Ecuador, South America, under Mr. Harmon who was then the engineer in charge. Mr. Cleveland began as leveler, topographer, transitman, and in charge of party, surveying bridge sites. From December, 1901, to March, 1902, he was assistant to the bridge engineer and in charge of bridge abutments and erection of steel bridges, and later that year, he was placed in responsible charge on the same work His next position was as acting resident engineer, and in March, 1903, he became resident engineer. By December, 1903, he had advanced from assistant engineer in charge of stadia survey to engineer in charge of assembling and erecting steel bridges. From January, 1904, to October, 1905 (during which time he was granted a seven-month leave of absence), he was resident engineer and also locating engineer. He worked as general roadmaster and inspecting engineer until October, 1906, after which he became resident engineer in charge of all construction. In May, 1907, he was made chief engineer of the railway, and in September, 1907, he held the positions of vice-president and chief engineer. In December, 1907, he became general manager of the entire railway.

The Government of Ecuador employed him, from November, 1909, to September, 1911, as chief engineer in charge of engineering and public works. In October, 1911, he was consulted in connection with a proposed steel custom house for the City of Guayaquil on estimates, specifications, and plans. He then became chief engineer and general director of Ferrocarril de la Costa for the Government of Ecuador serving from October, 1912, to April, 1920. During 1918, he was also consulting engineer on a new line and projected viaduct for the Guayaquil and Quito Railway. For three years, beginning in May, 1920, he was consulting engineer for that railway and other organizations, after which he was on location and in charge of construction of a plantation narrow-gage railway on a banana farm in Ecuador.

Mr. Cleveland returned to the United States in 1919 to place his son in school. He returned again with his family in 1926 to make Miami, Fla., his permanent home. There he continued to engage in an active business and civic life.

He was vice-president of the Filer-Cleveland Company which engaged in mortgage loans and investments, and which subsequently engaged in apartment and hotel property management. He became the first president of the Miami branch of the Mortgage Bankers Association. In various capacities, he served the American Red Cross, the Boy Scouts of America, the Children's Home Society of Florida, and other local civic, religious, and charitable organizations. He was a Thirty-Second Degree Mason and a member of Mahi Temple, Ancient Arabic Order of Nobles of the Mystic Shrine. For fifteen years prior to his death he served as Senior Warden of the Miami Trinity Episcopal Church.

For the last nine years of his life from 1937 to 1946, he was consul ad honorem for the Government of Ecuador at Miami.

Although Mr. Cleveland's career was varied, particularly in his later years, his basic and fundamental background was that of civil engineering. This profession gave to him that incisive, clear cut, and unswerving approach to civic problems for which he was well known in the Miami area. His work was characterized by thoroughness and his judgment, by broad vision and keen insight. During a long and successful career he enjoyed a wide acquaintance both in the United States and in South America. His vast experience as a civil engineer in Latin America was a valuable factor in initiating and developing a smoother and more understandable relationship between the United States and South America. As testified to (*Miami Daily News*) by the community in which he lived,

> "* * * he did much to cement neighborly relationships, and he was always acutely conscious of the things Miami should do, as a community, to fit itself better for the role of bridge between the Americas."

In 1908, Mr. Cleveland married Maria Teresa Valdez of Guayaquil. He is survived by his widow and their four sons, Charles B., John A., Jr., Rafael, and Houston H.

Mr. Cleveland was elected an Associate Member of the American Society of Civil Engineers on March 14, 1916, and a Member on January 17, 1927.

ALBERT SEARS CRANE, M. ASCE[1]

Died August 25, 1946

Albert Sears Crane was born on May 30, 1868, in Addison, N. Y., where he spent his boyhood. His ancestry was mostly English; the first Crane in America was said, by Mr. Crane, to have arrived on the *Mayflower*. A great-grandmother, on his mother's side, Abigail Ayrault, was a French Hugenot who settled in Rhode Island.

He prepared at the Addison Union School for Cornell University, at Ithaca, N. Y., from which he was graduated with the degree of Civil Engineer, in 1891. During one of his summer vacations he was chosen as secretary by the late Rudolph Hering,[2] M. ASCE, whom he accompanied on a study and inspection trip of sewerage systems in Germany and other European countries. Mr. Crane was always particularly proud of this association.

His first job after graduation was as assistant engineer of the City of Newton, Mass., a position he held for four years. Following this employment, he went to Brooklyn, N. Y., as assistant engineer in the department of sewers. While in Brooklyn he became a charter member and was active in the formation of the Brooklyn Engineers Club, and in the establishment of its library.

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[1] Memoir prepared by Henry A. Lardner, Vice-Pres., The J. G. White Engineering Corp., New York, N. Y.

[2] For memoir, see *Transactions*, ASCE, Vol. LXXXVII, 1924, p. 1350.

From 1898 until 1902, he was associated with the power companies at Sault Sainte Marie, Mich., and, during the last two years of residence there, he became chief engineer of the Lake Superior Power Company at Sault Sainte Marie, Ont., Canada. For three years, beginning in 1902, he was principal assistant engineer of the Chicago Drainage Canal, in Illinois, then under construction.

With this background of valuable experience, he was prepared to undertake his major lifework—he became hydraulic engineer with the J. G. White interests in New York, N. Y. During his active association with the White companies, he was a vice-president and director of The J. G. White Engineering Corporation until his retirement in 1928, when he became a valued consultant until his death.

His splendid technical knowledge and skill in design commanded the respect, and his fine personal character the love, of his associates and many friends. He was a man noted for his high integrity. His skill in the design of large earth dams, masonry dams, hydroelectric power stations, and irrigation projects was widely recognized in the profession. His advice was sought both by engineers and by investment bankers.

He never married. He maintained his residence at the Engineers Club, in New York City, continuously, from the time of its completion. In addition to his membership in the American Society of Civil Engineers, he was a member of the American Institute of Electrical Engineers, the Western Society of Engineers, the Boston Society of Engineers, the Brooklyn Engineers Club, and the Cornell and Lawyers clubs of New York.

Mr. Crane was elected a Junior of the American Society of Civil Engineers on September 3, 1895; an Associate Member on May 4, 1898; and a Member on May 1, 1901. He became a Life Member in January, 1933.

FRANK TERENCE DAVIS, M. ASCE[1]

DIED JANUARY 7, 1947

Frank Terence Davis was born in Evanston, Ill., on January 9, 1900. His parents were John J. and Jane Davis.

He received his early schooling in Chicago, Ill., and spent two years at the University of Havana in Cuba, later doing postgraduate work in civil engineering and engineering law at the University of Tennessee in Knoxville.

His first professional engagement was as project engineer for the Standard Construction Company in Havana. He became a partner in the firm in 1930. Mr. Davis' work involved the construction of heavy bridges, docks, and sugar mills. In 1934 he was engaged by the Cerro de Pasco Corporation where he was in charge of a large hydroelectric power station at Malpaso, Peru.

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[1] Memoir prepared by Stewart S. Neff, M. ASCE.

During 1940, he returned to the United States and was engaged by the Federal Housing Administration in Washington, D. C., to supervise several large housing developments in Tennessee and Kentucky. In 1942 Mr. Davis was transferred to the Reconstruction Finance Corporation (R.F.C.), and assigned to responsible charge of a large nickel extraction plant and mining development in Cuba. On completion of this assignment, he was recalled to the United States and appointed division engineer and senior division engineer for plant clearance in the Pittsburgh (Pa.) division of the R.F.C. In 1945 he returned to Washington, D. C., as the principal engineer and administrative assistant in the office of the vice-president of R.F.C., where he was busily engaged until his death.

Affable and of an engaging manner, and with an untiring capacity for work, Mr. Davis had many friends and associates in the United States and the tropical countries. All mourned his passing. His engineering and administrative ability were of the highest standard, and he had the respect and confidence of his associates.

In 1926, in Cuba, he was married to Cleo Gonzales, who survives him.

Mr. Davis was elected a Member of the American Society of Civil Engineers on March 12, 1945.

HENRY DIEVENDORF DEWELL, M. ASCE[1]

Died March 20, 1946

Henry Dievendorf Dewell, the son of William Henry and Maria J. (Dievendorf) Dewell, was born in Springfield, Ohio, on October 24, 1881. He came to Fresno, Calif., with his parents at an early age. There he received his grammar school and high school education, being graduated with honors from Fresno High School, in 1900. His boyhood experiences at Fresno, when that city was in its period of rapid pioneer development and when it was becoming the center of one of the most intensively developed, irrigated areas of the West, made a deep impression on him.

He entered the University of California at Berkeley, in August, 1901. In the following year, however, circumstances required his return to Fresno, where he spent an active year in the lumber industry. During that period he acquired an interest in, and a knowledge of, timber which was later to aid him in becoming an authority on timber construction and an author on the subject.

He resumed his college course in 1903, and was graduated from the College of Civil Engineering of the University of California in 1906. At the time of his graduation, San Francisco, Calif., had been largely destroyed by the earthquake and fire of April 18, 1906. He at once entered into problems of reconstruction as a member of the staff of Howard and Galloway, Architects and

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[1] Memoir prepared by a Committee of the San Francisco Section consisting of Walter L. Huber, Chairman, Austin W. Earl, and the late L. H. Nishkian, Members, ASCE.

Engineers, and, indeed, had a responsible part in many important design problems. When his employers were commissioned supervising architects of the Alaska-Yukon Pacific Exposition at Seattle, Wash., he became the designing structural engineer in their Seattle office, where he designed the frames of some of the principal buildings for the exposition, and some of the buildings for the University of Washington.

He returned to San Francisco in 1909, and, until 1911, was principal assistant engineer for Galloway and Markwart, Consulting Engineers. His work included design of hydroelectric power plants, bridges, water-supply systems, and buildings, and also involved appraisals. From February, 1911, to January, 1912, he was employed by the Oro Electric Corporation, under the supervision of Galloway and Markwart, in direct charge of surveys for reservoirs, conduits, powerhouses, and transmission lines. On completion of this service, he returned to Galloway and Markwart to design a steel drawbridge spanning the Sacramento River.

The next three years were spent with the Panama Pacific International Exposition at San Francisco, where he was, successively, chief structural engineer, assistant superintendent of building construction, and engineer of domestic water supply and distribution. His connection with the exposition gave him wide experience in timber design, a subject which had long interested him. Later he was the author of "Timber Framing,"[2] and was associate editor of the following textbooks edited by George A. Hool and W. S. Kinne: "Handbook of Building Construction," "Structural Members and Connections," and "Steel and Timber Construction."

After 1915 he was engaged in private practice as a consulting engineer with offices in San Francisco. From 1932 to 1940, he was associated with Austin W. Earl, M. ASCE, as Dewell and Earl, Consulting Engineers; and, from 1944 until his death, he was with his son Robert D. Dewell, Assoc. M. ASCE, as Dewell and Dewell, Consulting Engineers.

His practice covered a wide range of important projects that included, among many others, additions to the water supply of the City of Sacramento (Calif.), involving the design and construction of three elevated reinforced-concrete water towers of three-million-gallon capacity and involving some unusual features. His advice on design problems was widely sought, as for instance in connection with the great cableway head tower which was the master feature of the concrete plant at Shasta Dam. Perhaps his best remembered work will be his intensive studies on earthquake resistant construction, which resulted in many assignments of special design problems.

In 1919 and 1920, he lectured at the University of California. He was a member of the California State Board of Registration for Civil Engineers, from 1931 to 1945, and was long president of the board. He served the Seismological Society of America as its treasurer and was a member of the publication committee. He was a member of the Advisory Committee on Vibration Research at Stanford University, Stanford University, Calif.; the Structural Engineers Association of Northern California; the American Concrete Institute; the American Society for Testing Materials; and the National Committee on Wood Utilization. Mr. Dewell was also the northern technical editor of the "Uniform Building Code" (California edition), which was adopted in large measure as "Appendix A" of the "Rules and Regulations" of the Division of Architecture of the California State Department of Public Works, governing the design and construction of school buildings.

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[2] "Timber Framing," by Henry Dievendorf Dewell, Dewey Pub. Co., San Francisco, Calif., 1917.

He was a member of Sigma Xi, Tau Beta Pi, and Chi Epsilon, honorary scholarship societies. In addition he was a member of the Engineers Club of San Francisco and the Commonwealth Club of California.

In Society affairs Mr. Dewell was most active: He served as a Director, from 1925 to 1927, and a Vice-President in 1934 and 1935. He also served on many of the Society's committees and was particularly active as Chairman of the Committee on Local Sections while serving as Director. He was President of the San Francisco Section in 1930.

He was married on June 30, 1909, to Manie Ola Harrell at Fresno. He is survived by his widow; three children, Robert D. Dewell, Henry D. Dewell, Jr., and Jane (Mrs. Frank Reanier); and by his sister, Jessie W. Dewell.

A man of brilliant mind and steadfast purpose, he never spared himself in his professional work, whether it was in his endeavor to provide his clients with the best possible solution to their problems; in his willingness to do his share, and more than his share, of work for the profession as a whole; or in his helpfulness to the younger men of the profession. Although his chief characteristic was earnestness—indeed, he took a very serious view of professional affairs—he had a droll sense of humor which appeared even at times of serious discussion. He was very much respected and liked by all who knew him, both inside and outside the profession. His life was enriched by many loyal friendships.

Mr. Dewell was elected an Associate Member of the American Society of Civil Engineers on May 2, 1911, and a Member on June 12, 1917. He became a Life Member in January, 1946.

GILBERT COLFAX DOBSON, M. ASCE[1]

Died December 29, 1945

Gilbert Colfax Dobson was born in Philadelphia, Pa., on February 27, 1881. He was the son of James Wrigley Dobson and Rachel (Walsh) Dobson, who were natives of England.

James Wrigley Dobson emigrated to America in 1870, when he was nineteen years of age. Finding the United States much to his liking, he returned to England to claim as his bride, Rachel, daughter of John and Margaret Walsh, of Hooley Bridge, Lancashire. The couple was married on June 9, 1874, and, on the same day, they sailed for America on the maiden voyage of the *Pennsylvania,* the first "iron steamship" to cross the Atlantic. With true pioneer courage and faith in their adopted land, they established their home in Philadelphia and reared their sons and daughters in accordance with high Christian ideals. In 1881, the year in which Gilbert Colfax Dobson was born, his father became a naturalized American citizen.

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[1] Memoir prepared by Victor H. Cochrane, M. ASCE.

In 1886 the Dobson family moved to St. Louis, Mo., later settling in the town of Mexico, Mo., where the father was engaged in pioneer work in industrial insurance. His duties in connection with the establishment of district offices necessitated several moves by the family, and Gilbert received his grammar school education in various places. By the time he was ready to enter high school, the family had taken up residence in Columbia, Mo. Here, he demonstrated his ability as a student, and particularly his talent for mathematics, by completing three years of high school work in two years.

About this time because of failing health, his father decided to return to Philadelphia with his family to be near all the relatives who had followed him to America. Gilbert remained in Columbia and there entered the University of Missouri, receiving the degree of Bachelor of Science in Civil Engineering in 1905. (The university awarded him the postgraduate degree of Civil Engineer in 1909.) He supported himself during his college years by working in a real estate office after school hours, which often necessitated his studying most of the night. He started as office boy and, during his last year in school, became a member of the firm. He had charge of rentals, insurance, and instalment purchases, in addition to taking part in other activities of the firm.

In the fall of 1905 he received an appointment for service in the Philippine Islands, and he proceeded to his station in Manila, remaining there until June, 1909. His first assignment was as surveyor to the Court of Land Registration, in which capacity he was responsible for surveys of city and province lands in title investigations. In connection with this work, Mr. Dobson had to develop control methods and prepare instructions for field operations.

In the wilderness remote from adequate library facilities, his ability in mathematics stood him in good stead. He carried on research work on the direct solar observation for azimuth, and handled the coordinate computations for several very large surveys, one having more than five thousand courses. He carried the astronomical triangle through all its differential variations, plotted the probable error curves for all locations and conditions in the Philippines, and complied mathematical tables based on these computations. During the last two years of his stay in the Philippines, Mr. Dobson served as assistant city engineer of Manila, supervising many designs for streets, bridges, and canals. He also had full charge of the emergency drainage of infected swamps in the fight against the Asiatic cholera epidemic.

Mr. Dobson returned to the United States in August, 1909, to accept an appointment with the United States General Land Office in Sante Fe, N. Mex. He served for nearly a year as special agent, in charge of investigation of violations of public land laws. In addition, he prepared a topographic map of 160 square miles of mountain country and mapped by reconnaissance methods several hundred square miles of illegally enclosed land.

In July, 1910, Mr. Dobson accepted an appointment with the Isthmian Canal Commission, and he remained in the Canal Zone during the next five and a half years. Here he had a wide range of experience and great responsibility in the design and construction of locks, dams, buildings, and other structures. He had charge of the field engineering for the Gatun Spillway, including the control gates and appurtenant hydroelectric plant and later of installing the mechanical equipment for the Gatun Locks. For a period of two and a half years he supervised the engineering design of all permanent buildings, and structures costing several million dollars were built from his designs.

On his return to the United States in April, 1916, Mr. Dobson worked for the Kansas City Terminal Railway Company, in Kansas City, Mo. During the following thirteen months he was engaged in making an economic study of column spacing, the rearrangement of tracks, and other factors relating to the design of a five-mile elevated railway through industrial areas and complicated railway yards in Kansas City.

Upon completion of this work in May, 1917, he was commissioned a Captain in the Corps of Engineers, United States Army. He remained in the Army for three and a half years, rising to the rank of Major. Before serving in France with combat troops, the 314th Engineers, he had charge of the construction of the drainage system of Camp Funston at Fort Riley, Kans. Following the Armistice, he was assigned to the post of professor of military science and tactics at the Colorado School of Mines in Golden, where he remained until he resigned his commission in October, 1920, to become a member of a firm in Tulsa, Okla., specializing in special hazard insurance. Here Mr. Dobson made examinations of fire hazards in oil refineries and gasoline plants and investigated methods of reducing fire risks. The firm was dissolved in February, 1925.

Mr. Dobson's next professional engagement was as a representative of the Portland Cement Association, with headquarters in Tulsa, from February, 1925, to July, 1928. He was responsible for the promotion of the use of cement and for the dissemination of data relating to concrete technology. During the next seven months he was a consulting engineer in Tulsa, engaged chiefly in structural design.

The remaining years of Mr. Dobson's professional career were spent in the service of the federal government. From February, 1929, to November, 1935, he was in the United States Engineer Office in Memphis, Tenn., and subsequently, was with the Soil Conservation Service until his retirement on July 31, 1943. During the period of his service with the U. S. Engineer Office in Memphis, he made engineering and economic investigations on the Arkansas River and its tributaries for flood control, navigation, and other uses. He served both as field engineer and specialist in mathematical investigations. His most notable technical contribution during this period was a review of the mathematical processes employed by M. P. du Boys in his "Study of the Regime of the Rhone and the Action Exercised by the Water on an Indefinitely Shifting Bed of Gravel."

His first position in the Soil Conservation Service was as hydraulic engineer of the Sedimentation Section in the Office of Research. He supervised investigations of bed load movements in natural streams, and he guided the design and construction of the experimental stations on the Enoree River near Greenville, S. C., and at other locations. In October, 1936, he became the head of the Sedimentation Section, serving in that capacity until August, 1942, in charge of a wide range of investigations pertaining to reservoir silting, stream and valley sedimentation, and bed load transportation, as well as to related laboratory work. His last year of service was spent at the Greenville station, winding up the work of this project which had been under his supervision from its inception.

Mr. Dobson contributed to the literature on sedimentation and related problems. He collaborated with others in the preparation of papers and reports dealing with sedimentation, published by the United States Department of Agriculture and by the Society. The most notable of these was "Some Principles of Accelerated Stream and Valley Sedimentation." [2]

After Mr. Dobson's retirement from federal service, he returned to his home in Tulsa, and served as a member of a board of consultants appointed to study the sedimentation problems of the river basins comprising the Southwestern District of the U. S. Engineer Corps.

He was greatly esteemed by all who knew him. He was an engineer of the highest principles, and his close friends believed that at least one important move during his career was made to avoid the violation of professional ethics. He was a pleasant and agreeable companion, and his friends and associates gained much from the sound philosophy of life which he represented. His hobby was mathematics; he would sometimes spend most of the night on an intriguing problem, even if it were unrelated to his regular work. Mr. Dobson's associates gained from him many useful mathematical tricks and processes.

He was a member of the American Geophysical Union; the F. and A. M., Thirty-Second Degree; and the Shrine. If he had lived but three days longer his life membership in the Society would have become effective.

On October 26, 1905, at Columbia, he was married to Dee Lake. He is survived by his widow and a son, Gilbert Lake Dobson.

Mr. Dobson was elected an Associate Member of the American Society of Civil Engineers on May 2, 1911, and a Member on March 11, 1919.

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[2] "Some Principles of Accelerated Stream and Valley Sedimentation," by Gilbert Colfax Dobson, Strafford C. Happ, and Gordon Rittenhouse, *Bulletin No. 695*, U.S.D.A., 1940.

JOHN STEPHEN DOYLE, M. ASCE[1]

Died March 11, 1947

John Stephen Doyle, the son of John I. Doyle and Adelaide (Ruff) Doyle, was born in Baltimore, Md., on December 26, 1873. He received his early education in Baltimore at Calvert Hall and Rock Hill College. He was graduated from Johns Hopkins University, also in Baltimore, with the degree of Civil Engineer in 1900 and, subsequently, in 1921, he received the degree of Doctor of Philosophy.

Mr. Doyle was employed by the Baltimore District of the Corps of Engineers from August 2, 1897, to May 16, 1900, as inspector on fortifications and on sea-wall, road, and electrical construction, also designing military structures. From May 17, 1900, to January 15, 1910, he was with the Street Repair Department of the Baltimore City Engineering Department, as assistant engineer. In 1904 he was in charge of wrecking dangerous structures standing after the fire of that year. As engineer of parks, he designed reinforced concrete bridges, two skewed arch bridges, a 167-foot steel truss bridge, and a large bridge over Gwynns Falls. In private practice he designed a highway bridge and timber piers, and was examining engineer and appraiser on masonry bridges, road and culvert work, and other large bridges.

During his tenure of office with the Corps of Engineers, he was responsible for the design and construction of many river and harbor, flood control, and national defense works. He wrote many reports on river and harbor and flood control projects relative to the Chesapeake Bay and its tributaries, and, also, relative to the Susquehanna River Basin in Maryland, Virginia, Pennsylvania, and New York.

In 1910 he returned to the Baltimore District Office, Corps of Engineers, where he remained until his retirement on January 25, 1943. In 1914 he enrolled in the officers' training course at Plattsburg, N. Y., and studied there for two years. During World War I he became District Engineer with the ranks first of Captain and then of Major for the period from June 6, 1918, until January 17, 1919. He always retained his interest in the Reserve Corps, attending annual encampment whenever his duties permitted, and rose to the rank of Lieutenant Colonel in the Engineer Reserve Corps. During the last years of his service with the Corps of Engineers, he was consultant to the District Engineer. He died in St. Petersburg, Fla., where he had gone in November, 1946, to recuperate from an illness.

Mr. Doyle was a strict disciplinarian of both himself and those who worked with him. At the same time he took a great interest in new employees and young men and at all times insisted that they come to him with any problem that might be troublesome. He took great pleasure in showing others how to overcome their difficulties. He was a very capable, energetic man of high character.

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[1] Memoir prepared by C. C. Neher, Assoc. M. ASCE.

Mr. Doyle was married to Mary Frances Doyle in 1905. She died in 1929. He is survived by a son, J. Stephen Doyle, Jr., a special assistant to the Attorney General in Washington, D. C.

Mr. Doyle was elected an Associate Member of the American Society of Civil Engineers on November 7, 1906, and a Member on May 3, 1910. He became a Life Member in January, 1941.

FRANK KEYS DUNCAN, M. ASCE[1]

Died February 20, 1946

Frank Keys Duncan, the son of George H. and Mary Louise (Keys) Duncan, was born at Warren, Baltimore County, Md., on March 10, 1874. He attended the public schools of Baltimore, Md., and the Maryland Institute in Baltimore where he studied mechanical drawing.

Upon graduation he entered the employ of John Laing, a civil engineer. A year later he went to work in the engineering department of the Baltimore and Ohio Railroad Company.

Mr. Duncan's long and diversified career in the engineering service of the City of Baltimore began in 1894, when he was twenty years old, with the Topographical Survey. Three years later, he transferred to the city commissioner's office. His civilian engineering activities were interrupted the following year when he volunteered for naval duty in the Spanish-American War. At the conclusion of the war, he returned to the city commissioner's office. In 1900 he was transferred to the city engineer's office; and, in 1911, he was promoted to office engineer. With the creation of the Paving Commission, in 1911, he became engineer in charge of design and then engineer in charge of construction.

During World War I, almost singlehanded, he conducted the operations of the Paving Commission resigning, in 1921, to enter the contracting business as a member of the firm of P. G. Ligon and Company. The firm's name was later changed to Ligon and Duncan.

After an absence of two years, in 1923, he returned to municipal service as assistant highway engineer. In 1925, when the engineering bureaus of the city were coordinated under the Department of Public Works, headed by the chief engineer of Baltimore, he became the first assistant chief engineer and, in 1938, chief engineer. He retained this, the highest engineering position in the city government, for a little more than a year before retiring to private life.

During his long career of municipal service, in which he served the City of Baltimore almost continuously for forty-five years, Mr. Duncan was at all times a conscientious and energetic public servant. Under his guidance and direction, many public improvements were completed. His ability to analyze and solve a problem was unusual. Possessed of an underlying sense of humor that won him many friends, he nevertheless exhibited uncompromising firmness when the occasion demanded.

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[1] Memoir prepared by George A. Carter, and Nathan L. Smith, Members, ASCE.

Mr. Duncan was married to Lucy Amelia Wright, in Suffolk, Va., on December 10, 1907. He is survived by his widow; five sons, Gerald (by a previous marriage), Donald Wright, Frank Keys, James Robert, and Louis Edward Duncan; two daughters, Dorothy (Mrs. D. M. Muth) and Eleanor (Mrs. C. A. Eisenhauer); four grandchildren; three brothers; and two sisters.

In addition to his membership in the Society, Mr. Duncan was a member of the Engineers Club of Baltimore, the Maryland Association of Engineers, and the Masons.

Mr. Duncan was elected a Member of the American Society of Civil Engineers on August 9, 1920. He became a Life Member in January, 1945.

ROBERT PURL EASLEY, M. ASCE[1]

DIED JUNE 10, 1945

Robert Purl Easley, the son of Robert Alexander and Edith Barbara (Schramm) Easley, was born on January 24, 1887, in Boise, Idaho. At the age of fourteen he moved with his parents to California, where the remainder of his life was spent. He received his high school education in Santa Cruz and San Francisco.

His career as a contractor began in 1906, on small brick work jobs, following the San Francisco earthquake and fire, when he was only nineteen years old. Then for two years he was employed in the rebuilding of San Francisco, as a draftsman and inspector. During this period he also obtained his engineering training in the Humboldt Evening Engineering School in San Francisco. Beginning in 1907, he spent two and a half years on railroad work—much of the time in charge of rail and terminal construction—with the Western Pacific and the Oakland-Antioch railroads, followed by a year and a half on the installation of water and sewer systems in California and Nevada.

In 1913 Mr. Easley entered private practice and for the remainder of his career followed a combination of engineering and contracting. For a part of that period he acted as assistant city engineer for Daly City, Calif., and as city engineer for Antioch, Calif. His contracting and construction work covered the building of city water systems, sewers, and streets; large earthfill dams; and irrigation, drainage, levee, and other reclamation projects. Since 1933, in partnership with L. P. Brassy, he held a contract with the City of San Francisco for disposing of all the city's garbage and refuse by covering in trenches excavated in the tidelands of San Francisco Bay.

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[1] Memoir prepared by Edward Hyatt, M. ASCE.

Unlike most engineers, Mr. Easley took time from his professional activities to devote to public affairs. He was elected to the Assembly of the California State Legislature, serving from 1925 through 1933. During those years he was active in connection with legislation and in other ways affecting California's water problems. He was a member of the joint committee of the senate and assembly, appointed in 1929, which reported to the legislature of 1931 on the water problems of the state. He was vice-chairman of a legislative water resources committee, appointed at the 1931 session, which submitted to the legislature in April, 1932, a "Report of the California Joint Legislative Water Committeee Dealing with the Water Problems of the State of California."

Other public activities included membership on the Water Conservation Committee appointed by James Rolph, Jr., then Governor of California, in 1931, to confer with Herbert Hoover, Hon. M. ASCE, then President of the United States, and representatives of federal agencies on California's State Water Plan; the State Water Plan Association (1933); the Citizens' Committee on Irrigation, Reclamation, and Drainage; the National Reclamation Association (director); and the California State Planning Board.

Mr. Easley was a member of Antioch Lodge No. 175, F. and A. M., of California; Islam Temple of the Shrine, San Francisco; and Ariel Chapter No. 42, Order of the Eastern Star of Antioch. He was Patron of his Eastern Star Chapter in 1927, and Grand Patron of the Order of the Eastern Star, State of California, during 1937–1938.

In 1911, Mr. Easley was married to Jeanette F. Mowbray in San Francisco. He is survived by his widow and a daughter, Roberta (Mrs. W. D. Trewhitt, Jr.).

Mr. Easley was elected an Associate Member of the American Society of Civil Engineers on October 14, 1930, and a Member on April 22, 1935.

WILLIAM CHESTER EMIGH, M. ASCE[1]

Died September 26, 1946

William Chester Emigh was born on April 28, 1888, in Troy, N. Y. His parents were John H. and Emma F. (Allen) Emigh.

He was educated in private and public schools in North Adams, Mass., and was graduated in 1909 as a civil engineer from Rensselaer Polytechnic Institute in Troy. Following his graduation, he was connected with the Cambria Steel Company at Johnstown, N. Y., and with the New York State Department of Health.

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[1] Memoir prepared by Francis S. Friel, M. ASCE.

In 1922 Mr. Emigh became associated with the Bethlehem Steel Company in Coatesville, Pa., as a sanitary engineer, and remained with that company until 1931 when he became city engineer of Coatesville. In South Coatesville, he also served as borough engineer and had charge of all city engineering work including, among other things, the operation of the water works and sewage treatment plants.

In addition to being a member of this Society, Mr. Emigh also belonged to the American Water Works Association, the American Public Works Association, and the Pennsylvania Sewage Works Association, and was a past-president of the Pennsylvania Water Works Association.

Mr. Emigh was prominent and active in the religious and civic affairs of Coatesville. He was an outstanding member of the First Baptist Church and for twenty-four years was a Bible class teacher. In his civic activities, he was a past-president of the Coatesville Rotary Club, and a director in the Coatesville Young Men's Christian Association and the chamber of commerce. He also held membership in the honorary technical societies of Sigma Xi and Tau Beta Pi.

Mr. Emigh was elected an Associate Member of the American Society of Civil Engineers on October 8, 1918, and a Member on May 18, 1942.

JAMES DEARING FAUNTLEROY, M. ASCE[1]

DIED JULY 20, 1943

On the three peninsulas of eastern Virginia between the estuaries of the James and Potomac rivers dwells a community which has few counterparts in other sections of the United States. The restful landscape of low rolling hillsides and rich farm lands has left its benign influence on each generation of its people—descendants of the early settlers of the seventeenth and eighteenth centuries, who were already richly endowed with the gracious manners and sincere courtesy of the folk of old England whence they came. It was in this environment and with this fine heritage that James Dearing Fauntleroy was born and spent his earliest years.

The son of Thomas and Mary Anne (Dearing) Fauntleroy, he was born at Oakenham near Saluda in Middlesex County, Virginia, on April 4, 1869. The old Virginia Fauntleroys are descended from Col. Moore Fauntleroy who came to Virginia in 1643, and settled in Nansemond County. In 1650 he moved into that part of the "Northern Neck" Peninsula which was then included in old Rappahannock County. Other ancestral families were the Keenes, Wallers, Griffins, Corbins, Tayloes, and Terrells.

While still a boy, he moved with his parents to Avoca, not far from Altavista in Campbell County (Virginia), the homestead of his great-great-grandfather, Col. Charles Lynch, of Revolutionary War fame. The present "law" that bears his name is a gross perversion of the fair trial given by Colonel Lynch and his associate justices to the evildoers whom they caught and punished.

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[1] Memoir prepared by William Roy Glidden, M. ASCE.

His youthful days, spent amid the difficult times of the reconstruction era following the "War Between the States," culminated with his graduation from the Virginia Military Institute at Lexington, on July 4, 1888. Later, this same institute was to confer on him its highest academic award, the degree of Civil Engineer, on June 17, 1926, in recognition of his fine service in the profession.

In Mr. Fauntleroy's professional career there were three distinct periods of activity. From the time of his graduation from the Virginia Military Institute to the outbreak of the Spanish-American War, he was finding his way in the engineering field in various minor capacities. The second or middle period began soon after the oubreak of the Spanish-American War when he was commissioned a First Lieutenant and later a Captain in the United States Army. This connection with the United States Government, thus begun, continued for many years, and took him to Cuba and the Philippine Islands. The third or final period found him back in his native land whose government he served, except for a brief interlude, until his retirement in 1939.

After graduation from the Virginia Military Institute, Mr. Fauntleroy's first engagement was with the Atlantic and Danville Railway Company, where he served as rodman, from December, 1889, to the following March. As an instrumentman he worked for the St. Paul (Va.) and Minneapolis (Va.) Land and Improvement Companies until December, 1890. The following year he became secretary and general manager for the Craig Valley Iron and Mining Company at Goshen, Va. An opportunity for exercising independent engineering judgment came when he was employed as assistant engineer for Langhorne and Allen, Contractors, on railroad location and construction in Buckingham County, Virginia, and Washington County, Pennsylvania. He was employed by them from July, 1892, to August, 1893.

Beginning in August, 1893, he was engaged in the Norfolk (Va.) area by the Norfolk and Western Railway Company, and Thomas L. Rosen, Contractor, in work related to engineering. The outbreak of the war with Spain in 1898 proved a turning point in Mr. Fauntleroy's career which influenced the remainder of his life.

Commissioned a First Lieutenant in July, 1898, and later a Captain in the Third Regiment of Engineers, United States Volunteers, he served as an instructor to enlisted personnel in the construction of such works as entrenchments, blockhouses, and temporary bridges. He saw service in Cuba in the Pinar Del Rio area. After the close of hostilities with the Spanish Government, from July, 1899, to June, 1901, he was, successively, First Lieutenant and Captain with the Twenty-Seventh Regiment of Infantry, United States Volunteers. During this interim Captain Fauntleroy was sent to the Philippine Islands where he was destined to remain until March, 1907.

As a public works officer, Captain Fauntleroy made a conspicuous contribution to the improvement of Philippine civilization. His first task was to compile data for the preparation of military maps for the major part of the provinces of Morong and Manila. Later he had charge of the construction of a number of public buildings in the City of Manila, and was also engaged on the construction of provincial highways and bridges. In December, 1904, he became sanitary engineer for the Philippine Islands and, acting in conjunction with the medical inspectors and the city engineer of Manila, he set up an organization for sanitary inspection and prepared plans and specifications for standard designs for sewage disposal which included a system of sewage disposal for Bilibid Prison. He did sanitary work at Santa Cruz in Laguna Province, and at the Culion leper colony, where he also made a study of the water-supply system.

While in the Philippines, Captain Fauntleroy made an exhaustive study of the ancient Spanish road laws and prepared legislation for the general improvement of Philippine highways. He also assisted in the preparation of laws relating to sanitation as he was highly accomplished in the use of the Spanish language and the Tagalog dialect, the principal native tongue of the Filipinos.

Returning to the United States in March, 1907, Captain Fauntleroy became associated with the United States Reclamation Service, first as assistant engineer, and then as engineer. He was in charge of the construction of the Laguna Dam in Arizona, the Bumping Lake Dam in Washington, and the Elephant Butte Dam in New Mexico. He was responsible for the field work on the restoration of the Colorado River to its original channel in Mexico, after its historic break into the Salton Sea in Southern California, reporting to Col. J. A. Ockerson,[2] Past-President, ASCE, whose headquarters were in Yuma, Ariz. For about a year, from August, 1912, to September, 1913, Captain Fauntleroy was general manager for the Goldsboro Construction Company in Laramie, Wyo., being in charge of the construction of the Lake Hattie Storage Dam near by and the appurtenant canal system.

Beginning in February, 1914, and until his retirement in April, 1939, except for a period of two years, from February, 1922, to February, 1924, when he was state highway engineer for the State of Texas, Captain Fauntleroy was prominently associated with the United States Bureau of Public Roads.

When he became state highway engineer in Texas, the matter of matching federal aid with state funds was a serious problem, as there was no highway department with centralized control, and very few counties had funds available. Captain Fauntleroy was instrumental in organizing the State Highway Department and deserves much credit for the early advance of this state in highway development.

From November, 1916, to February, 1922, he was district engineer for the U. S. Bureau of Public Roads, in charge of all work in the states of Arkansas, Louisiana, Oklahoma, and Texas. The remainder of his service with this federal bureau was spent in his native state of Virginia, where as senior highway engineer he was responsible for the administration of federal funds expended on the highways of Virginia.

Tall and strongly built, he was a commanding figure in any gathering; yet, mild and persuasive in manner, he seldom needed to crack the whip of authority. In his public relations he was firm but courteous; in his private life and among his friends his natural proficiency in the social graces was always apparent. He truly exemplified the traditional Virginia gentleman.

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[2] For memoir, see *Transactions*, ASCE, Vol. 88, 1925, p. 1329.

In 1942, a year before he died, Captain Fauntleroy was elected president of the Third United States Volunteer Engineers. The yearbook of this organization, published in that year, contains many fine tributes from his former comrades. Typical quotations from them are: "He is God's nobleman"; "A man of high character"; "I am proud to call him my friend"; "All his men adored him"; "He put us boys through some hard drilling and himself, too"; "Known by all as a straight shooter"; "Exacted discipline from intimate friend"; and "He was the best singer in the regiment." This last quotation was in reference to the Captain's repertoire of old southern songs which he sang with a lusty voice on appropriate occasions. His particularly effective rendering of "De Watermillion Smilin' on de Vine" was something to be remembered.

He was a member of the Society of American Military Engineers, the American Association of Engineers, the University Club of Manila, the Columbia Club of Manila, the Veteran Army of the Philippines, the Third United States Volunteer Engineers Veteran Organization, the Society for the Preservation of Virginia Antiquities, and the Sons of the American Revolution (past-president of the Virginia Chapter). During World War II he was appointed by the Hon. Colgate W. Darden, Jr., then Governor of Virginia, to the state technical committee on civilian defense. As is customary among old Virginia families, he was a strong adherent of the Episcopal Church.

Like a sturdy oak felled by a sudden fury of the elements, Captain Fauntleroy was stricken on the night of July 19, 1943, and the end came peacefully the following morning while he slept. His great and genial heart which had endeared him to all who knew him had failed, its labor ended.

Captain Fauntleroy was married to Frances Hamilton Fox of Campbell County at Evington, Va., on February 9, 1903. Mrs. Fauntleroy died in 1941. Surviving are their three daughters, Mary Hamilton (Mrs. George S. Riggs), Frances Dearing (Mrs. A. K. Phillips), and Martha Lorimer; and a son, James Dearing, Jr.

The yearbook of the Third U. S. Volunteer Engineers, previously referred to, was dedicated to him with this fine tribute:

> "Gallant soldier, engineer of the highest standing in training and experience. An officer whose innate ability, sterling character and commanding presence won complete discipline, obedience and cooperation from his men. He was beloved as a lifelong friend by every man in his Company; it was a privilege and inspiration to know him, ideal American citizen in war and in peace—'Chevalier sans peur et sans reproche.' "

Captain Fauntleroy was elected a Member of the American Society of Civil Engineers on June 5, 1907. He became a Life Member in January, 1940.

DONALD EGBERT FUELLHART, M. ASCE[1]

Died October 23, 1945

Donald Egbert Fuellhart was born in Warren, Pa., on December 12, 1899, the son of John H. and Ida Loretta (Howard) Fuellhart. He entered Ohio State University at Columbus in 1916, but World War I forced him to abandon his studies in 1918.

His career as an engineer began when he left the university to serve as surveyor and draftsman with the United States Navy in the construction of a large training station. He was retained by the Navy after his discharge in 1919, and served a year with the Public Works Department in Newport, R. I.

When his work with the Navy had been completed in 1920, Mr. Fuellhart came to Louisiana to practice civil engineering and land surveying. In 1922 he began his outstanding career in the oil and gas industry in the geological department of the Gulf Refining Company. As a result his experience in land surveying and in construction work led to his appointment in 1924 as parish engineer of Natchitoches Parish, Louisiana. While in this office he designed and constructed many highway bridges, culverts, and dams, in and around Natchitoches.

By 1927 he found the opportunity he had been hoping for with the firm of Brokaw, Dixon and McKee, Engineers and Geologists, in New York, N. Y. Mr. Fuellhart's untiring efforts and love for his profession enabled him to accomplish much during the thirteen years he devoted to this work. He became intensely interested in the production of gas and oil, and his research and investigations resulted in the discovery of much useful information. Three technical publications which he wrote are: "Open Flow Capacity High Pressure Gas Wells,"[2] "Natural Gas Flow Tables,"[3] and "Sub-Surface Disposal Oil Field Brines."[4] He also contributed much toward the development of butane gas systems. In 1931 he designed and directed the construction of a $42,000,000 natural gas transmission system for the Panhandle Eastern Pipe Line Company, and was designing and constructing even larger systems at the time of his death.

In the years from 1935 to 1940, Mr. Fuellhart temporarily put aside his work in the gas and oil industry and once again came to the aid of the State of Louisiana in the capacities of engineer with the Highway Commission, assistant agricultural engineer, and engineer with the Department of Conservation. During this period he did much to improve the roads in Natchitoches Parish; he planned and constructed many fish and quail hatcheries; and he designed and built many bridges and dams in the state parks.

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[1] Memoir prepared by Edward D. Hogg, Jun. ASCE.

[2] *Oil and Gas Journal*, May 9, 1929.

[3] *Ibid.*, January 20, 1939.

[4] *Ibid.*, January 6, 1938.

The people of Louisiana and neighboring states are grateful to Mr. Fuellhart for the many benefits his achievements have given them. He was an active member of the American Association of Petroleum Geologists, and of the Louisiana Engineering Society.

On September 19, 1921, Mr. Fuellhart was married to Marie Aurelia O'Quinn in Natchitoches. He is survived by his widow and a daughter, Kathryn Marie Fuellhart. His only son died on December 16, 1944, of wounds received in the fighting on Mindoro Island.

Mr. Fuellhart was elected an Associate Member of the American Society of Civil Engineers on January 26, 1931, and a Member on July 10, 1944.

EDWARD GILLETTE, Jr., M. ASCE[1]

DIED JANUARY 3, 1936

Edward Gillette, Jr., the son of Edward and Anna Frances (Selby) Gillette, was born on December 14, 1854, in New Haven, Conn. He was a descendant of Jonathan Gillette, who was born in England; settled at Dorchester, Mass., in 1630; and moved to Windsor, Conn., in 1635. Thus, Edward Gillette had a sound basis for his membership in the Sons of the American Revolution and the Order of Founders and Patriots of America.

After attending the New Haven High School, he entered Sheffield Scientific School of Yale University, also in New Haven. He was graduated in 1876 with the degree of Bachelor of Science.

During 1876 and 1877 Mr. Gillette was employed by the U. S. Coast and Geodetic Survey. In 1878 and 1879 he was topographer on the U. S. Geographical Survey West of the 100th Meridian, working in New Mexico and Arizona before any railroads had been constructed in those territories (later states) and in Colorado near Leadville and the Royal Gorge, toward which the Atchison, Topeka and Santa Fe and the Denver and Rio Grand railroads were racing their surveys and construction. With such experience he became locating engineer for the Rio Grande Construction Company in 1880 in Utah, continuing as chief draftsman through 1881 and 1882.

Then for two years he was engineer in charge of additional construction and maintenance of way for the Denver and Rio Grande Western Railway Company. At one time the company had thirty-six engineering parties in the field, equally divided between location and construction. Numerous lines were run in Utah and Nevada. The route finally selected was by way of Walker pass and down the San Joaquin Valley to San Francisco. In 1885, at the age of thirty, he was appointed United States deputy mineral surveyor for Utah and chief engineer of the San Pete Valley Railway.

In December, 1885, Mr. Gillette was employed as locating engineer by the Burlington and Missouri River Railroad in Nebraska (later the Chicago,

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[1] Memoir prepared by Frank T. Darrow, M. ASCE.

Burlington and Quincy Railroad, Lines West) and continued with that road during its rapid expansion until 1892. His work covered hundreds of miles of surveys in Nebraska, Wyoming, South Dakota, and Montana.

In 1892 he opened an office in Sheridan, Wyo., as surveyor and civil engineer; and, from February 18, 1895, to February 18, 1899, he was superintendent of Water Division No. 2 for the State of Wyoming. In 1899 and early 1900 Mr. Gillette located the line of the Chicago, Burlington and Quincy Railroad from Toluca, Mont., in the Crow Indian Reservation, one hundred and twenty-nine miles to Cody, Wyo.—the eastern entrance of Yellowstone Park. This line was built in 1900 and 1901. After this engagement he made a survey for a railroad from Valdez, Alaska, up the Copper River under very difficult conditions. In the fall of 1900 he was made assistant superintendent of the Northern Division of the Chicago, Burlington and Quincy Railroad; and, on December 1, 1902, superintendent of the Sheridan Division at Sheridan, continuing until October 27, 1905.

On January 7, 1907, Mr. Gillette became State Treasurer of Wyoming, serving the full four-year term permitted by law; but he declined repeated importunities to run for other political office. In 1915 and 1916 he was chairman of the Board of Review of the U. S. Reclamation Service for Northern Wyoming and Montana. In 1916 he was appointed a member of the Board of Industrial Preparedness in the United States, representing the Society for the State of Wyoming. He was also Commissioner of Conciliation in the Department of Labor in 1918. In Sheridan, he was an alderman and a member of the board of education.

He died at his home in Sheridan on January 3, 1936, after a period of ill health—although he remained active, and his mind was keen. In regard to his public service, it was stated:

> "Although remaining in the background the Republican party counted him a leader of its ranks. His work throughout his life was of a public character."

He was married on April 10, 1893, in Chicago, Ill., to Hallie O. Coffeen, daughter of the Hon. Henry A. Coffeen, one-time U. S. Congressman from Wyoming. Mrs. Gillette died on January 5, 1937. Mr. Gillette is survived by a daughter, Harriet (Mrs. William L. Kleitz), and a son, Edward Hollister Gillette.

Mr. Gillette was a man of wide and skilled experience in western railway location, considerate of his men, and beloved by them. Genial and affable, he was a good visitor and was held in high esteem by all.

His trip of fifty miles on the ice through the Big Horn River Canyon in Montana and Wyoming during five days in March, 1891, with a prospector as his sole companion, is a thrilling story of hardship as described in *Transactions*.[2] His book "Locating the Iron Trail"[3] is an excellent account of his experiences in New Mexico, Arizona, and Utah before 1885 and on the Chicago, Burlington and Quincy Railroad western lines from 1886 to 1902. It is indeed fortunate that there is such a clear, accurate, and interesting record of the work of the early United States surveyors and railroad locating engineers in the west.

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[2] "The First Trip through Big Horn Cañon," by E. Gillette, *Transactions*, ASCE, Vol. XXV, July–December, 1891, p. 8.

[3] "Locating the Iron Trail," by Edward Gillette, Christopher Pub. House, Boston, Mass., 1925.

Mr. Gillette was elected a Member of the American Society of Civil Engineers on July 3, 1889.

STUART CHAPIN GODFREY, M. ASCE[1]

Died October 19, 1945

Stuart Chapin Godfrey, the son of Charles B. and Cora A. (Chapin) Godfrey, was born in Milford, Mass., on January 1, 1886. He was educated at Phillips Exeter Academy, at Exeter, N. H., the Massachusetts Institute of Technology at Boston, and the United States Military Academy at West Point, N. Y. He was graduated from the military academy in 1909, at the head of his class.

Following his graduation, he began his career as an officer in the Corps of Engineers of the United States Army. By 1912 he had experience as a student officer on the Panama Canal, then under construction, and had served a tour of duty with engineer troops, during which time he initiated the survey of the Fort Sill (Oklahoma) reservation. From 1912 to 1917, he was an instructor and an associate professor of mathematics at the United States Military Academy and spent his summer months on river and harbor assignments on the Ohio River.

From 1917 to 1919, he served as Major, Lieutenant Colonel, and Colonel in the Corps of Engineers, U. S. Army. Before going to France with the 318th Combat Engineer Regiment he was stationed at two engineer officers' training camps. In France he constructed the base depot for the American Expeditionary Forces at Gievres, later serving as Engineer Personnel Officer at Tours and as assistant to the Chief Engineer of the First Army in the closing weeks of the Meuse-Argonne offensive. After the Armistice in 1918, Colonel Godfrey became the Executive Officer for the Chief Engineer of the Army of Occupation in Coblenz, Germany, and finished his assignment as Division Engineer of the Second Division, and Commanding Officer of the Second Engineer Regiment. While in Germany, Colonel Godfrey received a commendation from Gen. John J. Pershing for having constructed a pontoon bridge, 1,440 ft long, across the Rhine River in fifty-eight and a half minutes.

Upon the return of Colonel Godfrey to the United States in 1919, he became the principal assistant engineer at Wilson Dam, Muscle Shoals, Ala., which was the largest concrete dam in the world at that time. Later, as chief of the Finance Division in the Office of the Chief of Engineers, from 1921 to 1923, he was in charge of contracts, accounting, and field civilian personnel.

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[1] Memoir prepared by J. H. Stratton, James B. Newman, Hal H. Hale, and the late J. T. L. McNew, Members, ASCE.

From 1923 to 1925, he was District Engineer of the Boston (Mass.) District, Corps of Engineers, on river and harbor improvements and fortifications along the New England coast, including the Portland (Me.) harbor and the Cape Cod (Mass.) Canal. He initiated construction of a forty-foot channel from Boston Harbor and completed construction of a sixteen-inch battery at Fort Duvall, Little Hog Island, Mass., during this assignment.

Colonel Godfrey was graduated with honors from Command and General Staff School at Fort Leavenworth, Kans., in 1926, and was then assigned to the Memphis (Tenn.) Engineer District, in charge of the navigable channel of the Mississippi River from Cairo, Ill., to the sea. He participated actively in the fighting of the great flood in 1927, rebuilding some of the breached levees by dredging, and assisted in preparing new projects for flood control and navigation. During this civil assignment, he had a prominent part in organizing the Mid-South Section of the American Society of Civil Engineers, and served as its first President.

Colonel Godfrey returned to teaching in 1928, when he was assigned as instructor in tactics and military engineering at the Command and General Staff School, where he served until 1932. Following this assignment he spent the summer touring England, Germany, Switzerland, and France. In the fall he attended the Army War College in Washington, D. C., and was graduated with high honors in 1933. As engineer for the First Corps Area at Boston, from 1933 to 1935, Colonel Godfrey was busy on matters of fortification, troop training, and war plans.

In 1935 he became engineer of the Panama Canal Department and commanding officer of the Eleventh Engineer Regiment. In this capacity he installed the antiaircraft guns and searchlights, and built the first asphalt roads for the U. S. Army in the Canal Zone.

Upon his return to the United States in 1937, Colonel Godfrey became Chief of Operations and Training Section, as well as Executive Officer of the Military Division, Office of the Chief of Engineers. In this capacity he supervised, for the Chief of Engineers, all military engineering training, including troops, officers, and student officers at engineer camps. He also prepared plans for new and expanded engineering units, as well as for their equipment and utilization, which included a new air forces component of engineers to be known as aviation engineers. He assisted in the development of several types of transportable steel landing mats which later were to become invaluable in all theaters of operation. As a representative of the United States State Department, he attended the International Navigation Congress at Brussels, Belgium, in 1939, at the time of the opening of the Albert Canal. In his official capacity with the Chief of Engineers, he cooperated with the agencies of the engineering societies in the United States in planning for civilian defense and supervised the preparation of the first official manual on blackouts.

Beginning in 1941 he became Engineer of General Headquarters, United States Army Air Forces, which was later named the Air Forces Combat Command. On March 9, 1942, he was promoted to Brigadier General. During this period he made exhaustive studies of the problems confronting engineers assigned the mission of supporting an air force in offensive combat. He secured authorization for major changes in organization of engineer aviation units and in their equipment. General Godfrey, more than any other individual, was responsible for the activation and training of additional units which were later to serve with distinction in all theaters of operation. For his accomplishments in this important work he was awarded the Legion of Merit for distinguished service in directing the growth of the aviation engineer component of the U. S. Army Air Forces, his citation reading in part:

> "His vision, devotion to duty and judgment in planning, equipping and training the special engineer units to meet expanding Army Air Forces requirements have been exceptional. His personal contribution to the development of the transportable steel landing mat is an achievement of outstanding note."

As the Air Engineer, U. S. Army Air Forces, on Gen. Henry H. Arnold's staff, he continued the development of aviation engineers and visited every active overseas theater to study airfields and operations of engineer aviation units. General Godfrey was instrumental in setting up aviation engineer replacement training centers and initiated and pushed the formation and development of air-borne engineer aviation units which were unique in the American army. He gave untiring support to the authorization of an Engineer Command, a unit designed to perform all engineer work for an air force in the field. Largely as the results of his efforts, two such commands were activated and operated with signal success in the Mediterranean and European theaters. In addition, the Buildings and Grounds Division, U. S. Army Air Forces, which was the agency responsible for formulating requirements of the three billion dollar program of military air base construction, was placed under the supervision of the air engineer in 1943. Actual construction involved in this program was under the direction of the Chief of Engineers, who proceeded in accordance with the air forces program developed under General Godfrey's supervision.

In December, 1943, General Godfrey became air engineer in the China-Burma-India Theater (CBI) where he supervised for the air forces the planning and construction of United States military airfields. Much of his time was devoted to the program of construction of B-29 bomber bases in China which were built by hand methods in periods as short as three months. From these fields, the second bombing of the Japanese mainland was carried out in June, 1944. As air engineer he planned and supervised the use of air-borne engineers in Burma with Colonel Cochran and General Wingate. At "Broadway," a historic airfield in the Burma campaign, General Godfrey supervised the work of his air-borne engineers and lent encouragement to them in their efforts behind the Japanese lines by his presence during the operation. Later in the capture of the airstrip at Myitkyina, General Godfrey again experienced great satisfaction as his glider-borne engineer aviation units performed miracles in keeping the captured airstrip open, in spite of Japanese efforts and a monsoon, to the contrary. Without these quickly made airfields to accommodate troops and combat aircraft, the Burma campaigns would have been exceedingly difficult, if not impossible.

After the fall of Myitkyina, General Godfrey planned, staffed, and supplied several engineer aviation battalions to organize the Tenth Air Forces Engineer component which took part in another historic achievement involving the transportation of all its personnel, heavy tractors, eight-yard scrapers, shovels, trucks, and other equipment by air from India—as well as the thousands of tons of supplies necessary for the ground forces. This movement, by air, of some fifteen hundred sorties, carrying heavy construction equipment and personnel of sufficient size to build a large number of all-weather airfields, was accomplished during the monsoon in a land where no road or rail access was available. It demonstrated strikingly and uniquely the efficacy of an air force to supply, by air, a forward operation and to bring up its own engineers for advanced airdrome construction. For his outstanding achievements as aviation engineer in the CBI, General Godfrey was awarded the Air Medal for meritorious achievement in aerial flight, and the Oak Leaf Cluster to the Legion of Merit. His citation for the Air Medal read in part:

> "In the performance of his duties as Air Engineer, Gen. Godfrey accumulated over two hundred hours flying time in supervising the layout, design and construction of airfields in the China-Burma-India Theater. On many occasions his missions were accomplished in the face of hazardous flying weather over territory where enemy fire was probable and expected and where landings were made on difficult terrain behind enemy lines."

Early in 1945 General Godfrey returned to the United States and was assigned to duty as Commanding General of Geiger Field, Wash., which was an Engineer Replacement Training Center he had organized when he was on duty in Washington. It was on this assignment and while returning from conferences at Fourth Air Force Headquarters in San Francisco, Calif., that General Godfrey was killed in the crash of a C-45 Army transport on October 19, 1945.

In addition to his engineering activities General Godfrey was long closely associated with the Boy Scouts of America. He served as a scoutmaster at Fort Leavenworth in 1912 and continued his scouting and cubbing activities at West Point, Boston, Washington, D. C., and the Canal Zone. He was a personal friend of Dr. West, the Chief Scout Executive, and he once visited the International Boy Scout Chalet at Kanderstag, Switzerland. In 1936 he organized a Boy Scout camp in Costa Rica. While he was president of the Boy Scout Council in the Canal Zone, he was awarded the Silver Beaver for distinguished service to boyhood.

Although a busy man with his countless calls of duty, General Godfrey found time to serve other interests for the public welfare. He was a member of the Society of American Military Engineers, and received the Gold Medal award of that society in 1939; the National Council, Boy Scouts of America; the Visiting Committee for the Civil Engineering Department of Massachusetts Institute of Technology; and the Army and Navy Club in Washington,

D. C. As a soldier, as an engineer, as a teacher, and as a gentleman, his influence will endure through the years. His personal friends will long remember him as a wise counselor and one who entered into his every assignment with all his heart and soul.

In 1915, in New York, N. Y., he was married to Dorothy Rich who, with his daughter, Dorothy Hope, and two sons, Charles Stuart and Pearce, survives him.

General Godfrey was elected a Junior of the American Society of Civil Engineers on September 5, 1911; an Associate Member on September 12, 1916; and a Member on January 18, 1921.

GEORGE ESTYN GOODWIN, M. ASCE[1]

Died September 15, 1945

George Estyn Goodwin was born on June 4, 1875, in Shelburne, N. H. His parents were Otis F. and Sarah (Hutchinson) Goodwin. He was educated at public and private schools and at Hebron Academy in Hebron, Me., and was graduated from the University of Maine at Orono, in 1901, with the degree of Bachelor of Science in Civil Engineering with honors. In 1910 he received the degree of Civil Engineer from his alma mater.

While performing jobs as timekeeper, assistant foreman, foreman on bridge erection and railway and sewer construction, and assistant locomotive fitter for the Grand Trunk Railway during undergraduate years, he learned firsthand the practical side of construction. This knowledge became the keystone of his career and provided a good background for the future. After graduation he was employed by the Corps of Engineers, United States Army, as inspector of dredging and jetty and breakwater construction in Massachusetts, and was resident engineer for three years in charge of fortifications construction in Boston Harbor.

The spring of 1905 found Mr. Goodwin following other University of Maine graduates out west to work for the United States Reclamation Service, later the Bureau of Reclamation, which was authorized by Congress in 1902. Arriving in Montana, he was placed in immediate charge of design and construction of the Huntley Project. The work involved a main canal and distribution system, pumping plants, and many other structures. Three years later he became project engineer on the Blackfeet Project, where he had supervision of similar work. Those were frontier days in Montana, and many a less hardy and determined individual failed to make the grade.

Mr. Goodwin then became general superintendent in charge of construction of The Dalles-Celilo Canal, a nine-mile navigation canal with five locks which was being built in the Columbia River Gorge by the United States Engineer Department, near The Dalles, Ore. That project was completed in the spring of 1913.

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[1] Memoir prepared by Ray E. Mackenzie, M. ASCE.

He was then placed in charge of sixty miles of difficult highway construction which the Corps of Engineers was undertaking for the National Park Service in Crater Lake National Park, Oregon. This work lasted four years, and received the special attention of the National Park Service. On its completion Mr. Goodwin became associated with the National Park Service as general civil engineer, with headquarters in Washington, D. C. He had general oversight and technical review of all engineering in the national parks and also served in a consulting capacity.

In 1921 Mr. Goodwin was made chief engineer of the National Park Service, serving until his resignation in 1925. The first suspension bridge across the Colorado River in Grand Canyon Park was designed and built under his supervision; the first paving in Yosemite Park was laid under his specifications and direction. The location of the original Rim Road in Crater Lake Park was made by him personally and the famous Swift Current Trail in Glacier Park was located and built by him.

His love of the soil and the outdoors caused him to return to Oregon and settle on a small fruit orchard in Hood River Valley. There he carried on a consulting and general engineering practice, principally in building construction.

Returning to the Portland (Ore.) District, Corps of Engineers, in 1930, he was associated with revetment and levee work and channel changes on the Willamette and Columbia rivers. In 1933 he was in local charge of deepening and improving Willamette Falls Locks, at Oregon City; and, at the initiation of the Bonneville Dam Navigation and Power Project on the Columbia River, he was made resident engineer. Later, during the early part of 1935, Mr. Goodwin was administrative assistant.

With federal interest in flood control and multiple purpose projects so active, his wide and varied background was indispensable and he was assigned to the investigation of several projects in Idaho, Washington, and Oregon. The following two years he devoted to those investigations; in the spring of 1937 he completed investigations of irrigation, drainage, and ground-water resources, covering twenty-two individual irrigation projects involving 660,000 acres of irrigable land for the Willamette Valley Project in Oregon. His devotion to these investigations doubtless caused the break in health which he suffered near the close of the work in 1937. He retired in July, 1938, after completing studies and estimates for enlarging and rebuilding Willamette Falls Locks.

Returning to his beautiful orchard home in Hood River Valley he carried on a small consulting practice until his death in Santa Cruz, Calif.

Those having more than a casual acquaintance with Mr. Goodwin were generally impressed with his dry, calm, wit. Only occasionally was it caustic. His positive personality was equally impressive. Once, in Montana, a big bruiser, who considered himself injured because he had been fired, towered over him, threatening to tear him apart then and there. Mr. Goodwin calmly sat and slowly reduced the ruffian in stature with cutting and pointed remarks. His ego shattered, the man was glad to leave. A loyal friend, Mr. Goodwin was ever interested and inquiring about his many associates, both young and old. Being a good conversationalist, he loved to entertain his friends with talk about his travels and experiences.

Mr. Goodwin was a member of Hood River Lodge No. 105 of the A. F. and A. M., a Thirty-Second Degree Scottish Rite Mason, Oregon Consistory No. 1, and a Universalist. In addition, he also belonged to the honorary fraternities of Kappa Sigma and Phi Kappa Phi. He loved the outdoors ardently; his greatest interests were mountain climbing and fishing. With the late Walter W. Crosby, M. ASCE, he was coauthor of the book, "Highway Location and Surveying."[2]

Mr. Goodwin was married in 1901 to Margaret Wallace but was later divorced. To this union were born Esther W. and Susanna B. Goodwin. In 1922 he was married to Ruth G. Oliver, who survives him.

Mr. Goodwin was elected an Associate Member of the American Society of Civil Engineers on January 8, 1908, and a Member on July 1, 1909. He became a Life Member in January, 1943.

NORMAN BASSETT GURLEY, M. ASCE[1]

DIED FEBRUARY 27, 1947

Norman Bassett Gurley, the son of William Chamberlain and Catherine (Russell) Gurley, was born in Marietta, Ohio, on May 3, 1890. He was educated in the public and high schools of Marietta, in close association with his father, whose scientific achievements in the fields of astronomy and photography as director of the Marietta College Observatory influenced the son to embark upon an engineering career.

Mr. Gurley was first employed by the Parkersburg and Marietta Interurban Railway Company in Parkersburg, W. Va. He began as a rodman in June, 1908, and by May, 1913, was in complete charge of important construction, remaining until February, 1916, when he entered the service of the Baltimore and Ohio Railroad Company. He continued with the Baltimore and Ohio until September, 1918, as assistant geologist engaged in coal mining property and tunnel structure investigations, with headquarters at Morgantown, W. Va., and, beginning in May, 1917, at Wheeling, W. Va. From September, 1918, to May, 1924, he practiced civil and mining engineering at Williamson, W. Va.

In May, 1924, Mr. Gurley was appointed chief engineer of the Red Jacket Consolidated Coal and Coke Company at Red Jacket, Mingo County, W. Va. During the following fifteen years, he designed many large-scale operations and directed the construction of railroad extensions, coal preparation plants, and other facilities in a rapidly expanding and modernizing field. Because of ill health, he retired in July, 1939.

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[2] "Highway Location and Surveying," by Walter W. Crosby and George E. Goodwin, Gillette Pub. Co., Chicago, Ill., 1928.

[1] Memoir prepared by J. N. Wallace, Assoc. M. ASCE.

Mr. Gurley moved to Huntington, W. Va., in May, 1940, and became track materials sales and service engineer for the West Virginia Steel and Manufacturing Company. During this engagement, he wrote a technical article[2] for *Coal Age,* which is considered an important contribution to modern track layout practice.

In May, 1946, he accepted the position of chief engineer of the Lando Coal Corporation and its affiliates, with offices in Huntington. Possibly the heavy strain under which he worked in this capacity contributed to his untimely death, for he was working on plans for the replacement of a large tipple, which had been recently destroyed by fire, when he collapsed on February 27, 1947.

Mr. Gurley was registered professional engineer No. 54 in West Virginia. A leader among men, "Norm," as he was affectionately known to his friends, was active in the affairs of the West Virginia Section of the Society and in the West Virginia Society of Professional Engineers (former president of the Huntington Chapter and, at the time of his death, member of the State Board of Directors).

On June 29, 1916, he was married to Gladys Smith Hupp, in Parkersburg. From that day until he died, she stood by his side, taking pride in his achievements and sharing with him his love of motoring, photography, and music. Mr. Gurley is survived by his widow; a nephew, William B. Horstman, of Chicago, Ill.; and a niece, Mrs. Carl Berger, of Cincinnati, Ohio.

Funeral services were held at Huntington on March 1, 1947, and interment was in the Oak Grove Cemetery at Marietta on March 2, 1947.

Mr. Gurley was elected a Member of the American Society of Civil Engineers on May 20, 1946.

ISAAC HARBY, M. ASCE[1]

DIED OCTOBER 28, 1946

Isaac Harby, the son of Horace and Emma (Solomons) Harby, was born in Sumter, S. C., on February 25, 1873. His grandparents were early settlers in this community dating back to prerevolutionary days. His preliminary education was obtained in the schools of Sumter, and at the Citadel in Charleston, S. C. Along with a great many young men of the South at the time, he went for his college education to Union College in Schenectady, N. Y., from which he was graduated in 1895 with the degree of Bachelor of Science in Engineering. At Union he was a member of Delta Upsilon fraternity.

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[2] "Better Track Through Turnouts as Transition Elements," by Norman B. Gurley. *Coal Age,* January, 1946.

[1] Memoir prepared by Louis W. Abrons, New Rochelle, N. Y.

Upon graduation, from July, 1895, to March, 1896, he was employed as architectural and engineering draftsman by George P. Post and Sons, of New York, N. Y. In March, 1896, he joined the Department of Buildings of the City of New York as special engineer, a position he held until 1900 except for the period of his military service (June, 1898, to February, 1899) as First Sergeant, Company K, First United States Volunteer Engineers, during the Spanish-American War. His work in the Building Department consisted of special examinations of unsafe buildings and in testing strength of various floor systems and various building materials.

In 1900 he was engaged as designer for structural steelwork, first for Brown, Ketchum Iron Works of New York City and later for S. C. Weiskopf,[2] M. ASCE, a consulting engineer of New York City. In the latter part of 1900 he accepted a position with the John A. Roebling's Sons Company of Trenton, N. J., and remained with that company until 1909. Until 1902 he was assistant engineer on the Roebling contract for the installation of the cables and suspenders on the Williamsburg Suspension Bridge spanning the East River in New York City. As a result of his work on the bridge he wrote a paper on "The Footbridge for Building the Cables of the New East River Bridge"[3] for which he was awarded the Collingwood Prize by the Society in 1903. He was also the author of an illustrated article in *Cassiers Magazine,* on "The Cables of the New East River Bridge." From 1902 to 1909 he was engineer in charge of the design and construction of various additions to the Roebling plant at Trenton, and at Roebling, N. J. The work at Roebling consisted of the construction of four hundred and seventeen residences; three hotels; stores; a municipal building; street paving, water, gas, and sewer systems; and sewage disposal works.

From 1909 to 1913 he was engaged on building work first in charge of construction at Forest Hills, N. Y., for the Sage Foundation Homes Company, and then in New York City for the Theresa Hotel and following that for the Lord and Taylor Store on Fifth Avenue. From 1913 to 1914 he was engaged as coordinator of war industries by the United States Labor Department.

In 1915 he joined with two other associates, namely, the late Ludlow L. Melius, Assoc. M. ASCE, who had been a classmate at Union College, and Louis W. Abrons to form the firm of Harby, Abrons and Melius, located in New York City and specializing in building construction. This association lasted until 1929. The company at first acted as general contractors but within a few years decided to limit its construction work to its own investment account. As a general contractor it built the Nurses Home for the Willard Parker Hospital, the Ward Island Power House, and the sewage disposal plant at the foot of Dyckman Street—all in the City of New York. Among the important buildings erected for the company's own investment account were a nine-story apartment building at Forty-four West Tenth Street, a thirteen-story apartment house at Twenty-five Fifth Avenue, office and business buildings at Thirty-three West Sixtieth Street and Two Hundred and Seventy Madison Avenue, and the twenty-five story "Murray

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[2] For memoir, see *Transactions,* ASCE, Vol. 103, 1938, p. 1913.

[3] *Ibid.,* Vol. XLIX, December, 1902, p. 165.

Hill" office building at Fortieth Street and Madison Avenue. All these ventures were successful.

In 1929 Mr. Harby retired from active work to spend four years in Europe with his family. Some of this time he devoted to studying housing conditions abroad. Returning to New York in 1934 he mixed periods of retirement with periods of activity until the beginning of World War II. From 1934 to 1937 he was control engineer and adviser on the construction of large developments for the United States Housing Administration—five hundred and eighty-two houses at Greendale, Milwaukee, Wis., and four hundred and forty-eight family units at South Jamaica, Long Island, N. Y. The beginning of World War II brought him back to active service. From December, 1941, to September, 1946, he was supervising engineer for the Defense Plant Division of the Reconstruction Finance Corporation. He approved plans and the purchase of all production equipment for the projects assigned to him as well as supervised construction. Under his direction were four projects for the General Electric Company totaling $7,500,000 (radio and electronics equipment); a $2,500,000 project with the Adirondack Steel Company and the American Locomotive Company (guns and heavy steam engine work); a $6,500,000 project with the National Lead Company (a railroad and development of a titanium mine); and finally a $1,200,000 project for Lederle Laboratories (penicillin and biological plants).

Isaac Harby was first married in 1900 to May Levin, who died in September, 1920. He is survived by two children by that marriage, Anita and Esther (Mrs. Donald Cook). On November 5, 1921, he was married to Isabel Solomons. Two children by his second marriage, Anne and William, survive him.

Mr. Harby was elected a Junior of the American Society of Civil Engineers on June 6, 1899; an Associate Member on May 6, 1903; and a Member on May 2, 1911.

STEPHEN HARRIS, M. ASCE[1]

DIED JUNE 24, 1946

Stephen Harris was born in Pottsville, Pa., on October 15, 1864. He was the son of Stephen and Catharine (McArthur) Harris. He came from Scotch-Irish stock, his ancestors having settled in the Chester Valley of Pennsylvania about 1747. His father, a civil and mining engineer, was in charge, for the City of Philadelphia, Pa., of valuable coal properties for the Girard Estate. His uncle, Joseph Smith Harris, who had been his father's partner, was president of the Philadelphia and Reading Railway Company from 1893 until 1901. Stephen Harris, Sr., was killed in 1874 in a railroad accident leaving his son, Stephen, then only nine years of age, and three younger children.

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[1] Memoir prepared by Laurence B. Manley, M. ASCE.

Under the influence of this environment, coupled with a natural aptitude for mathematics, it was not strange that the boy should be attracted to the engineering profession; and, after the family had moved to Germantown, Pa., he attended the Germantown Academy and, afterward, the University of Pennsylvania at Philadelphia from which he was graduated in 1886 with the degree of Bachelor of Science in Civil Engineering. He kept himself abreast of the rapidly expanding engineering profession, not only by constant study but also, in its practical application, by actual surveying and construction in the field. Few persons who knew him in his later years as a studious office worker realized the extent of his early outside experience and its great value to him in solving design problems, particularly in his specialty, urban transportation.

After graduation, Mr. Harris was assistant superintendent of the Lehigh Coal and Navigation Company. During the next eighteen years he had numerous engagements on surveys and design work, and was in responsible charge of construction—all this experience leading to more than twenty-two years of subway design in Philadelphia and Camden, N. J. Among these early surveys and reports, the most conspicuous was the work, which he directed between 1897 and 1900, of running a line of duplicate precise levels from the Caribbean Sea to the Pacific Ocean for the Nicaraguan Canal Commission. He also made surveys and reports for the United States Army Engineers on a projected 30-mile canal for the Intracoastal Waterway from Cape Fear to Little River, South Carolina, in 1910–1911; surveys for the Reading Railway Company in connection with the elevation of four and one-half miles of tracks in Philadelphia between Green Street and Wayne Avenue; surveys and some plans for two projected single-track tunnels under the Delaware River for three miles of connecting double-track railway at Camden, in 1909–1910; and surveys for other railroad projects in California and Porto Rico, the latter involving surveys and the design of structures for a projected railroad line ten miles long.

From 1895 to 1897 he was employed on preliminary work on the plans and in charge of the construction of a half-mile section of the Pennsylvania Avenue subway for the Survey Bureau, in connection with the City of Philadelphia grade crossing elimination program. He made the principal drawings and superintended the greater part of the construction for the Upper Roxborough filters from 1900 to 1902. From 1905 to 1908, as assistant engineer for the Philadelphia Rapid Transit Company, he was responsible for the determination of the profiles of the elevated lines and supervised the design of the mile of Market Street Subway east of the City Hall.

On July 10, 1913, at the beginning of the program of municipally constructed rapid transit facilities adopted by the City of Philadelphia, Mr. Harris was transferred from the Highway Bureau of the city, where he had been assistant commissioner of highways for about two years, to the newly organized Department of City Transit. He remained for nearly twenty years until the close of 1932 when all active work was brought to an abrupt close by lack of funds. Until 1924, he was assistant engineer on designs, and after that designing engineer in charge of all plans except those relating to electrical installations.

Among the structures with which he was connected were the Frankford Elevated Railway about 6.2 miles long and about 11.1 miles of subway costing more than one hundred and thirty-seven million dollars, a large part of which were fully equipped for operation in his time. This project consisted, in round numbers, of 6.66 miles of four-track subway and 1.64 miles of two-track subway in Broad Street; 2.32 miles of a branch two-track subway in Ridge Avenue, Eighth Street, and Locust Street; and a short half-mile section of four-track subway in Market Street crossing under the Schuylkill River through the piers of a new bridge constructed as a part of one subway contract. Throughout the work Mr. Harris took a leading part in the determination of the alinement and grades, had full charge of the track layout, and also made many studies and reports on other projected lines.

In 1933 Mr. Harris was made engineer of design on the construction of a two-track branch from the Philadelphia subway system to Camden, about 2.7 miles long—known as the High Speed Rail Transit Line—for which Modjeski, Masters and Case, Incorporated, were the engineers. This line consists of tracks over the Delaware River Bridge, with connecting subways in Philadelphia and Camden (matching, as nearly as possible, those of the Philadelphia system), involving some necessary alterations to the bridge structure.[2] Again, Mr. Harris personally determined the alinement and grades and had complete charge of the preparation of plans and equipment including cars and substation buildings, but not including electrical equipment, the province of the electrical engineer.

The design of a modern rapid transit subway, in city streets bordered by high buildings, completely equipped with tracks, yards, buildings, and cars, constitutes an engineering problem of the first magnitude, calling for the joint efforts of specialists under able direction. The electric power facilities for the Philadelphia and Camden subways did not come under Mr. Harris' jurisdiction while he was designing engineer; but, nevertheless, the successful completion of the design of all other work connected with these subways is an outstanding engineering achievement and marks a fitting climax to his long and honorable career.

Upon the completion of the Camden subway Mr. Harris, then seventy-two years old and in poor health, retired from active engineering work. During the remaining ten years of his life he suffered severely from bronchial asthma and rested quitely, under the ministrations of a devoted wife, at his home in Chestnut Hill, Pa., where he died on June 24, 1946. His life revolved around his work. He enjoyed the outdoors, tramping in the Pocono Mountains (Pennsylvania), and boating on the Muskoka Lakes in Northern Ontario where he spent many vacations seeking relief from the asthma that afflicted him for years; but, in general, his spare time outside home duties and interests was given to study and the perfection of his professional work rather than to the pursuit of hobbies. How well his lifework profited thereby is evidenced in the completeness of all his undertakings. A distinguishing trait of his character was his retiring disposition and a failure fully to appreciate and take advantage of his own capabilities, which led him to shun publicity and, unfortunately, not to receive credit for many successes for which he was primarily responsible. In private life, no one but those whom he often helped to tide over hard times will ever know how generously he could share with those in need.

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[2] "Rapid Transit Begins Across Philadelphia-Camden Bridge," by Laurence B. Manley and Stephen Harris, *Engineering News-Record,* June 4, 1936, p. 811.

Mr. Harris had many friends. He was respected by all, and loved by those who knew him well enough to pierce his shell of reserve and to appreciate his sterling qualities. He held the ethics of his profession high, and his character was above reproach. He was loyal to his superiors, unobtrusively helpful to any one needing aid or advice, and enthusiastic in his praise of good work—all with the quiet courtesy of good breeding which marked his sympathetic regard for the feelings of others.

Mr. Harris was married in 1899 to Agnes Cointat of Turny, France, who, together with two daughters, Eleonore Dubois (Mrs. Frank T. Gucker) and Katharine McArthur (Mrs. Henry Phillips), and four grandchildren, survives him. Until his retirement he was an associate of the American Railway Engineering Association, and was a registered professional engineer in the states of Pennsylvania and New Jersey. He was an active member of the Mount Airy Presbyterian Church for more than forty years.

Mr. Harris was elected a Member of the American Society of Civil Engineers on June 4, 1902. He became a Life Member in January, 1935.

EDWARD HAUPT, M. ASCE[1]

Died February 6, 1947

Edward Haupt was born at Philadelphia, Pa., on March 24, 1869. He was the son of Jacob Benjamin and Mary Elizabeth (Ziegler) Haupt. His earliest ancestor who came to this country was Sebastian Haupt who emigrated from Germany and arrived in Philadelphia in 1738.

Edward Haupt was a descendant of a noted engineering family and added to its reputation. His grandfather, Herman Haupt of Civil War fame, was responsible for the construction and operation of many lines of the Pennsylvania Railroad Company. He was one of the first men to develop the theories and formulas of scientific bridge design which he issued in pamphlet form in 1846, and a development of which was published by D. Appleton and Company in 1851. In 1862 he was placed in charge of the railway and transportation services of the federal armies and was commissioned a Colonel. Later Herman Haupt was made a Brigadier General and, as chief of construction and transportation of the United States military railroads, he had many inventions to his credit and received many honors. He died in 1905.

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[1] Memoir prepared by J. L. McConnell, T. L. Condron, and I. F. Stern, Members, ASCE.

Edward Haupt's father, Jacob Benjamin Haupt, was a mechanical engineer who built the first practical mechanical fire engine used in the United States.

An uncle, Lewis M. Haupt, M. ASCE,[2] was professor of civil engineering at the University of Pennslyvania in Philadelphia, and was the author of a textbook on surveying, which was used as a standard for many years.

His brother, Charles, added an appendix on aerial mapping to this book in 1886. This was among the very first publications on that subject.

Ed Haupt, as he was familiarly known by his associates, attended the Philadelphia public schools and was graduated from the Central High School in 1886. He did not attend college but, on the advice of his uncle, Lewis M. Haupt, gained his vast engineering experience and knowledge in actual practice. From 1886 to 1896 he worked for the Phoenix Iron Company, in the rolling mill and in the drafting room, on the designing and estimating of steel structures.

In 1896 and 1897 he joined the Universal Construction Company, a subsidiary of the Illinois Steel Company, as secretary and later as president in charge of operations. This company designed and fabricated structural steel buildings and bridges, and was among the first promoters of steel car construction. From 1897 to 1904 Mr. Haupt represented various manufacturing companies in Chicago, Ill., among others the Mount Vernon Bridge Company of Mount Vernon, Ohio.

In 1904 and 1905 he was in New Orleans, La., as a partner in the firm of W. W. Bierce, Limited, representing the Cambria Steel Company, handling structural steel. In 1905 he joined Charles L. Strobel,[3] Hon. M. ASCE, as a member of the Strobel Steel Construction Company of Chicago—first as secretary, later as vice-president, and from 1920 to his death as president in executive charge of design and construction of fixed and movable bridges and framed structures all over the United States. His principal responsibility was the design and construction of bascule bridges.

Edward Haupt was a member of the Western Society of Engineers, and of the Chicago Engineers Club (past-president and honorary member). He was director of the Building Construction Employers Association and chairman of the Joint Conference Board to which were referred all disputes among the various building trades. As a member of the arbitration board for the structural steel workers union and their employers, he was instrumental in settling numerous disputes and was highly regarded by both sides. In 1921 he was a member of the citizens' committee sponsoring the "Landis Award."

Although Mr. Haupt was a very busy man, he always found time to serve others in a purely unobtrusive, unselfish manner, and his services were always available. His integrity was unquestioned. He had a wonderful sense of humor and a collection of quaint anecdotes which he related to his intimates but always in a modest way, giving credit for the story to one or other of his numerous friends, who had probably never heard of it before the telling. His favorite recreation was golf which he played at every opportunity.

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[2] For memoir, see *Transactions*, ASCE, Vol. 103, 1938, p. 1801.

[3] For memoir, *ibid.*, Vol. 102, 1937, p. 1491.

In 1894, in Phoenixville, Pa., he was married to Charlotte Maud Wistar who died on September 3, 1943. He is survived by one son, Caspar Wistar Haupt, M. ASCE, who has kept up the family tradition following in his father's footsteps, succeeding him as president of the Strobel Construction Company.

Mr. Haupt was elected an Affiliate of the American Society of Civil Engineers on January 31, 1911, and a Member on October 1, 1928.

JULES JOSEPH HENNEBIQUE, M. ASCE[1]

DIED AUGUST 14, 1944

Jules Joseph Hennebique, the second son of François Hennebique (1842–1921) was born in Brussels, Belgium, in 1873. He attended the technical school in this city and at the age of twenty two joined the engineering office of his father in Paris, France.

François Hennebique, as a building contractor, desired to protect steel girders from the action of fire; he first embedded them in concrete, but, after making many tests, replaced the steel girders with steel rods placed longitudinally and transversely. This was to become the reinforced concrete beam, patented in 1886. He perfected his invention, making extensive tests on girders, slabs, and posts, and in 1898 he established himself as a consulting engineer on reinforced concrete structures, with offices in the main cities of Europe and America. This last measure contributed to the rapid adoption of the Hennebique system of reinforced concrete in many countries.

Jules Hennebique took an active part in this pioneer work and spent several months of the year 1900 in Bilbao, Spain, where a flour mill and a grain elevator were being built. In 1902 the Hennebique System was introduced in the United States; the Hennebique Construction Company was formed with head offices in New York, N. Y., and agencies in Baltimore, Md., Philadelphia, Pa., St. Louis, Mo., Boston, Mass., and Cleveland, Ohio. This concern, with its construction department, acted as engineer for contractors. The manager was R. Baffrey-Hennebique, a son-in-law of F. Hennebique, and Jules Hennebique was active in the construction department.

From its start, the company met with great success, and after a fire test proved successful in the laboratory at Columbia University in New York, many structures were erected. Reinforced concrete was used extensively in buildings, retaining walls, bridges, piers, quay walls, grain elevators, reservoirs, and similar structures. This development took place in less than five years with Mr. Hennebique taking a large part in the improvement of building methods using the new material.

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[1] Memoir prepared by Prof. Gustave Magnel, M. ASCE, and Flamant Hennebique, Head of Bureau Hennebique, Paris, France.

In 1908 he directed the construction of reinforced concrete seashore defenses in British Guiana, and in 1911 he took an active part in the construction of the Brandywine Lighthouse in the Delaware River. This structure is erected on a reinforced concrete caisson built on shore, floated into place, and supported by reinforced concrete piles driven through the bottom.

Among the reinforced concrete structures erected in the United States, the following deserve mention: In 1902—the floors of the McKinley High School in St. Louis, Mo.; the Sheldon House in New York; the reservoir in Fort Greble, R. I., for the War Department; and the Athletic Club Building in Baltimore; and in 1903—the Marine and Engineering Building at the U. S. Naval Academy in Annapolis, Md., and the Salvation Army Citadel in Cleveland.

In later years the many varied structures were to include: The Maverick Cotton Mills in Boston; the Boston Cold Storage building; the Union Cold Storage building in Jersey City, N. J.; a coal breaker for the Delaware, Lackawanna and Western Railroad Company; bridges for the Pennsylvania Railroad Company for the City of Philadelphia and others; piers for the Standard Oil Company in Brooklyn, N. Y., at Atlantic City, N. J., and at Long Branch, N. J.; quay walls for the Navy Department at Key West, Fla., and at Norfolk, Va.; and the National League grandstand in Brooklyn.

In 1914, Mr. Baffrey, the general manager of the Hennebique Construction Company, returned to France to serve in the French Army, leaving Mr. Hennebique to conduct the business. Mr. Baffrey did not return to the United States and Mr. Hennebique continued the management.

In 1904 Mr. Hennebique was married to Aimée Baffrey. He died after a long illness in 1944. A daughter, Yvette, survives.

Mr. Hennebique was elected a Member of the American Society of Civil Engineers on May 12, 1919.

DERWENT GORDON HESLOP, M. ASCE[1]

Died January 28, 1944

Derwent Gordon Heslop, eldest son of Thomas H. B. and Gertrude (Goyder) Heslop, was born at South Shields, Durham, England, on October 30, 1877. He attended the Norwich Grammar School, and by the time he had attained school age his father was surveyor for the County of Norfolk. On leaving school he was sent to the works of Messrs. Aveling and Porter at Rochester, Kent, England, for training in mechanical engineering and after two years became a pupil of the Midland and Great Northern Joint Railway. On completion of his training his great desire was to do constructional engineering work abroad; in the years from 1900 to 1931 he succeeded in seeing much of the world.

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[1] Memoir prepared by Mrs. Derwent G. Heslop, Eastcote, Middlesex, England.

His first appointment in 1900 was to Upper Assam Province, India, where he took charge of mechanical construction on the Tiphook Tea Estate. After a year he became assistant engineer on the Bengal Nagpur Railway and remained on construction and maintenance work for six years. In 1907 he was appointed assistant engineer of railway location in the Republic of Colombia, South America, a position which lasted for two years. The next appointment took him to South China where he worked as resident engineer on construction and design of the Canton Hankow Railway. This employment ended in 1911 when the Chinese Revolution forced him to leave the country. Having relatives in Australia, Mr. Heslop spent the next two years there—first, in the west and, later, in Victoria and the northern provinces as assistant engineer and chief of survey party under the Commonwealth Gevernment.

After a leave in England in 1913, Mr. Heslop was appointed assistant engineer on construction of the Ceylon Government Railways; he remained in that country until 1915 when, realizing that World War I was not going to be of short duration, he returned to England and joined the Royal Engineers. He was with the British Army in France, Egypt, Palestine, Syria, and Anatolia, enlisting as a sapper and retiring with the rank of Major. In France and Palestine he engaged in railway construction.

After the cessation of hostilities in the East and while he was still in military service his most important appointment was as chief engineer of the Bagdad Railway and the Hedjaz Railway. He was also officer in charge of the survey company laying out a pipe line across the desert—which work had to be abandoned because of hostile tribes, but which has since been completed.

After the demobilization in May, 1920, Mr. Heslop was acting chief resident engineer on the Gold Coast in 1921, and from 1923 until 1931, when he left the tropics for the final time, he was employed by the Crown Agents for the British Colonies in Tanganyika—first, rebuilding the Lukonde Bridge and, later, extending the railways. Because of his age he could no longer obtain employment abroad; from 1937 to 1940, he held a temporary appointment on the Great Western Railway in England and was responsible for its air raid shelters at many points on the line between London and Oxford.

During World War I Mr. Heslop wrote several articles for the engineer on the Bagdad Railway, and after he left Tanganyika, at the request of his family and friends, he wrote and published the story of his life, entitled "Through Jungle Bush and Forest," which disclosed some very unique experiences.

Soldiering was always one of Mr. Heslop's greatest interests. Wherever he found himself he always joined the local force if one was available. In later years he became an ardent cinematographer and showed his films of Tanganyika to many schools in England under the auspices of the British Empire Society.

Mr. Heslop is survived by his widow; a daughter, Dorothy Goyder Heslop; and a son, Robert Stanley Gordon Heslop.

Mr. Heslop was elected an Associate Member of the American Society of Civil Engineers on May 2, 1911, and a Member on June 2, 1920.

PHILIP BROCKETT HILL, M. ASCE[1]

Died March 23, 1946

Philip Brockett Hill was born in Brooklyn, N. Y., on April 14, 1877. He was the son of Philip Hamilton and Emma (Tompson) Hill.

He received his early education in the public schools of Nashville, Tenn., his college preparatory training in the Webb School at Bell Buckle, Tenn., and was graduated from Vanderbilt University at Nashville in 1900, with the degree of Bachelor of Engineering. In 1931 his alma mater conferred upon him the professional degree of Civil Engineer at the same exercises during which his daughter, Marion, received the degree of Bachelor of Arts.

Prior to completion of his formal education and for the first two years following graduation, Mr. Hill was employed on the surveys for, and the design of, the Tennessee Central Railroad in and around Nashville and from Nashville to Lebanon, Tenn. In 1902 and 1903 he was with the Walsh and Weidner Boiler Company of Chattanooga, Tenn., as engineer in charge of the drafting room on the design of water tanks and towers and the preparation of all the estimates for structural work and platework and special boilerwork.

In 1903 and 1904 he went to St. Louis as assistant engineer in the Department of Sewers, Water Supply, and Fire Protection for the Louisiana Purchase Exposition. He worked on both design and construction and then, during the exposition, on maintenance of those facilities. Mr. Hill stated, "I had a varied and interesting experience there." Following his work at the exposition he went to Copperhill, Tenn., where he was an engineer with the Tennessee Copper Company, which at that time was doubling the capacity of its plant and adding a sulfuric acid plant to convert the destructive sulfur fumes into a commercial product. Mr. Hill was engineer in charge of the powerhouse extension, and laid out the foundations for the building and machinery and supervised a large part of the construction of the sulfuric acid plant, including a 300-foot chimney, furnaces, and trackwork.

A large part of Mr. Hill's professional work was devoted to practice as a consulting engineer. From 1906 to 1922 he was a partner with Fred M. Lund in the engineering firm of Lund and Hill at Little Rock, Ark. Little Rock was a fast-growing city, and Mr. Hill's firm designed and built many miles of streets and sewers for the City of Little Rock and for neighboring towns. Lund and Hill also designed and supervised the construction of many miles of highways radiating from Little Rock. Other projects handled by Lund and Hill involved the design of special floodway gates for levee work and of pumping plants for low drainage areas in Arkansas and West Tennessee.

From 1923 to 1926 Mr. Hill was engineer for the Continental Rock Asphalt Company in Leitchfield, Ky., a company engaged in mining and marketing rock asphalt for use as a paving material. In 1926 Mr. Hill moved to Florence, Ala., and opened a consulting engineering firm of his own. He stated, "Early in 1926 I chose the Muscle Shoals area as a promising field for engineering practice." His principal work there until 1933 was on land development and municipal improvements.

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[1] Memoir prepared by Roland A. Kampmeier, M. ASCE.

When the Tennessee Valley Authority (TVA) was established in 1933, Mr. Hill was appointed civil engineer at Wilson Dam in Alabama, in charge of transmission line surveys and right of way in the Power Engineering and Construction Department, which position he held until his death in 1946. In 1941 Mr. Hill's office was moved from Wilson Dam to Chattanooga, where he continued in the same capacity. During his thirteen years with the TVA, Mr. Hill's organization surveyed and obtained the right of way for several thousand miles of transmission lines. His work placed him in contact with a large number of landowners throughout the Tennessee Valley and adjacent areas, locating transmission lines across their properties and also making contracts for the clearing of these rights of way. Mr. Hill's work with the TVA was characterized by excellent relationships with these thousands of people; and, because of his fairness and sincere methods of dealing with others, he did much to maintain good will between all parties concerned.

Mr. Hill was an active and faithful member of the Methodist Church wherever he resided and was appointed to the boards of stewards of the churches at Little Rock, Leitchfield, Florence, and Chattanooga. At Florence, he was chairman of the board for several years. He was a member of the Delta Kappa Epsilon college fraternity and of the Masonic Blue Lodge at Little Rock. He was a Thirty-Second Degree Scottish Rite Mason and a member of the Alhambra Shrine Temple at Chattanooga. He was also a member of the American Association of Engineers.

When he resided in Florence and Wilson Dam, he was a member, and at one time president, of the Florence Rotary Club. He was interested and active in alumni work for both Webb School and Vanderbilt University. His work in each of these alumni groups entailed heavy correspondence, but was very gratifying to him, since he always enjoyed hearing from friends and associates of other years.

Mr. Hill was a Christian gentleman in the true sense of both words. His word was his bond. In addition to always being very considerate of the other fellow and having a kind word for and about everyone, he had a wonderful sense of humor and enjoyed a good joke on himself as well as on others. He was indeed young in spirit and was never taken to be his real age. Mr. Hill was very prompt in all his duties and very careful to meet every obligation, both social and business. His thinking was clear, his language was correct and concise, and his work was always done neatly and in an orderly and systematic way. He was very capable with his hands, and created for his home and for his friends many objects reflecting careful and loving workmanship. A talented photographer, he pleased his friends each year with unusual Christmas greeting photographs. He had a valuable collection of old coins and stamps which had interested him a number of years.

In 1908 he was married to Elizabeth Fletcher Fite, daughter of James W. Fite and Marion Fletcher Fite, at Hendersonville, Tenn. He is survived by his widow; his daughter, Marion Dudley Hill of Murfreesboro, Tenn.; and a sister, Edith (Mrs. Marvin S. Enochs), of Jackson, Miss.

On his passing, his local associates in the Society recorded the following expression of their sorrow at his untimely death and of their appreciation of his splendid qualities:

> "With profound sorrow the Chattanooga Sub-Section of the American Society of Civil Engineers records the death of one of its most earnest and loyal members—Philip Brockett Hill. Mr. Hill was for thirty-four years a corporate member of the Society, giving unstintingly of his time and efforts in advancing the affairs of the Society. He was most faithful in attendance at meetings and could be counted on to be present unless prevented by circumstances beyond his control. His friends, neighbors, and the community could count on Philip, too. His counsel and advice were highly valued. His ability as an engineer won the respect of his associates and his warm heart and friendly spirit, the affection of all who knew him."

Mr. Hill was elected an Associate Member of the American Society of Civil Engineers on April 30, 1912, and a Member on September 10, 1918.

THOMAS VICTOR HODGES, M. ASCE[1]

DIED JUNE 12, 1946

Thomas Victor Hodges, the son of Thomas Benton Hodges and Katherine Elizabeth (Dissisway) Hodges, was born in New Orleans, La., on September 5, 1881. He was the youngest of four children, having a sister and two brothers, none of whom survive. His father, a contractor, was killed in an accident before Mr. Hodges was born. The Hodges family returned to Maine, the father's home state; the mother died soon after, and Mr. Hodges was reared by relatives.

He attended elementary schools at Bath, Me., high school at Bridgton, Me., and entered the University of Maine at Orono in 1901, remaining for two years. He found it necessary to leave the university to earn money for completing his education.

His first engineering work, from December, 1903, to April, 1904, as a rodman on surveys for the Hampton Terrace Hotel Company of Augusta, Ga., was followed by five months of work as rodman, instrumentman, and draftsman with the International Paper Company on the construction of a new paper mill at Berlin, N. H. He then entered Swarthmore College in Swarthmore, Pa., from which he was graduated in 1906 with the degree of Bachelor of Arts, with a major in civil engineering.

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[1] Memoir prepared by George S. Beal, M. ASCE.

Mr. Hodges was employed during the summer of 1905, and after graduation from college, until June, 1908, by the American Pipe and Construction Company which was constructing a series of water supply systems for the Pennsylvania Railroad Company along the railroad's lines in Pennsylvania. He was engaged on surveys, design, and construction; among other duties, he was resident engineer on the construction of reservoirs near Latrobe (Pa.) and Mifflintown (Pa.), and engineer and inspector on the laying of fifty miles of cast-iron pipe.

Later in 1908 he was contractor's engineer for the firm of Lewis and Wiley on the Jackson Street regrade, in Seattle, Wash.—an interesting project involving the moving of a large quantity of earth by hydraulic sluicing. For about three years, until 1911, he was draftsman and engineer on surveys and design of highways and highway structures with the Los Angeles County Highway Commission. This was followed by a year with the Department of Public Works of Santo Domingo in the Dominican Republic.

Upon returning to the United States, he spent a year with the American Water Works and Guarantee Company in charge of the installation of mechanical filters at New Castle, Pa., and the construction of an earth dam at Suffolk, Va. Subsequently, he was engaged by the New York Continental Jewell Filtration Company, until October, 1915, building plants at Harrisburg, Ill., and at Albion and Lyons in New York.

Mr. Hodges' next engagement, with the Semet-Solvay Company of Syracuse, N. Y., was a long and interesting one. From 1915 until a reorganization of the company late in 1924, he was safety engineer and insurance manager. In this capacity he was responsible for engineering design and plant operation involving accident prevention, fire prevention, sanitation, ventilation, health, and insurance. In addition, he handled many special investigations and studies for water supplies and sewage disposal as well as studies of labor relations in other industries.

After leaving the Semet-Solvay Company Mr. Hodges had a short engagement as engineer in charge of topographical surveys and property maps for the Laurel Park Estates in Hendersonville, N. C. Then, because of the illness and advancing age of his father-in-law, Joseph M. Cranston, of Philadelphia, Pa., Mr. Hodges assumed management of Mr. Cranston's coal and building material business which was sold in 1927.

Next, in the employ of the Pennsylvania Railroad Company for about two years, he was engineer in charge of: The construction of a water supply system at Conowingo, Md.; the design and construction of a half-mile-long dike on the east side of the Susquehanna River, as an extension of the Conowingo Dam, to protect the railroad; the reconstruction and grouting of the foundation of the Brush Mountain Dam near Altoona, Pa.; and the construction of a mechanical filter plant at Dauphin, Pa.

In 1929 he became resident engineer for the Philadelphia Surburban Water Company on the construction of the Springton Dam on Crum Creek in Delaware County, Pennsylvania. This structure, two thousand feet long and eighty feet high, created a reservoir with a capacity of 3,500,000,000 gallons of water. In addition, he was also responsible for the construction of a new filter plant with a fifteen million-gallon daily capacity at Media, Pa., for the same company.

From January, 1932, until his death in 1946, Mr. Hodges was employed by the Bureau of Waters and the Water and Power Resources Board in the Department of Forests and Waters of the Commonwealth of Pennsylvania. The first two years he worked on the Pymatuning Reservoir project, which was then being built to regulate the flow of the Shenango and Beaver rivers in western Pennsylvania. He had charge of construction of the dam near Jamestown, Pa., and later of a combined highway and railroad embankment, near Linesville, Pa., which also served as an auxiliary dam across an arm of the reservoir.

Later he supervised the design and construction of forty dams of various types, from seven feet to forty feet high, forming lakes up to 250 acres in extent, in recreation areas on state forest lands. As assistant director of the Bureau of Waters, he was responsible for the proper allocation and use of the water resources of the state and assisted in the state supervision of the design, construction, and maintenance of all dams, except those federally owned, in Pennsylvania. At his death he was district engineer in charge of the Schuylkill River desilting project.

A conscientious, capable, and reliable engineer, he was well loved and respected by his superiors, associates, and subordinates. Mr. Hodges was married twice; his first wife was Mary Bernard Cranston to whom he was married in 1912. She died in November, 1917. One child of this marriage, Elizabeth Cranston (Mrs. James A. Murphy), survives. On June 1, 1920, he was married to Katharine Curtin, of the famous Curtin family of Centre County, Pennsylvania, who survives with one son, Thomas Victor Hodges, Jr., a student at Swarthmore College.

In addition to the Society, he belonged to the Engineers Society of Pennsylvania and the Masonic Fraternity. He was a member of the Beta Theta Pi fraternity at the University of Maine and of Book and Key at Swarthmore College.

Mr. Hodges was elected a Member of the American Society of Civil Engineers on February 13, 1940.

NEIL CUMMINGS HOLDREDGE, M. ASCE[1]

DIED OCTOBER 10, 1946

Neil Cummings Holdredge, the son of William Martin and Ida Paulina (Cummings) Holdredge, was born at West Burlington, N. Y., on January 2, 1883. After attending a little red schoolhouse and two high schools, he was graduated from Union College at Schenectady, N. Y., in 1905, with the degree of Bachelor of Engineering. He was a member of Beta Theta Pi fraternity.

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[1] Memoir prepared by Max F. Freund, Assoc. M. ASCE, and Paul H. Bird, Geologist, New York City Board of Water Supply, Ellenville, N. Y.

Immediately following college, he went to work as a structural steel detailer; but soon he realized that his temperament was unsuited to this type of employment. He then secured employment with the Brooklyn Rapid Transit Corporation and rapidly advanced from the position of inspector to assistant engineer on elevated railway construction.

Mr. Holdredge first joined the forces of the Board of Water Supply of New York City, N. Y., in October, 1906, as assistant engineer, and was assigned to the Esopus Division of the Northern Aqueduct Department on the construction of the Rondout pressure tunnel. In 1910 he was promoted to section engineer and the following year was transferred to the City Aqueduct Department, Bronx Division, where he served in the same capacity until November 30, 1915, when he temporarily severed his connection with the Board.

Then, for three years, he was an assistant engineer with the Borough President of Manhattan in charge of sewer construction. From August, 1918, to January, 1919, he was field engineer with the Air Nitrates Corporation on the construction of a nitrate plant at Cincinnati, Ohio.

For the next seventeen years, 1919 to 1936, he was with the North Jersey District Water Supply Commission as assistant chief engineer. In this capacity, he had direct charge of construction for the complete development of a water supply project of one hundred million gallons per day costing $25,000,000. The work included the Wanaque and six other dams together with some twenty miles of pipe line aqueduct, two tunnels, bridges, and the usual amount of highway and railroad relocation. During the last two years of this period he was connected with the operation of the water supply.

From March until September, 1936, he was construction engineer with Greeley and Hansen in charge of a $15,000,000 sewer project in Buffalo, N. Y., but left this position at the call of the New York City Board of Water Supply to serve as division engineer in the Newburgh Division which was then attached to the Watershed Department. After seven months as division engineer, he was made department engineer of the Northern Department, supervising the construction of the Rondout-West Branch tunnel. He retired on June 30, 1944.

The Northern Department and Neil C. Holdredge and the longest continuous tunnel on earth are inseparable in the minds of the men who enjoyed the privilege of calling him "boss" and seeing the "hole" go through. He endeared himself to his associates by his appreciation of sincere effort. He pulled no punches, stated in no uncertain terms what he wanted done, and was eminently fair in his dealings with both his own force and the contracting organizations. His friends will remember a Holdredge institution—the cheer he dispensed at clambakes, at banquets, whenever Board of Water Supply men got together, the best of cheer!

On February 11, 1908, in Guilderland, N. Y., he was married to Emma Veeder Relyea. He is survived by his widow and a daughter, Lois Caroline (Mrs. Halsey Peckworth).

Mr. Holdredge was elected an Associate Member of the American Society of Civil Engineers on October 31, 1911, and a Member on April 17, 1923. He became a Life Member in January, 1946.

PAUL HOLLAND, M. ASCE[1]

Died April 14, 1946

Paul Holland, the son of Martin and Ann (Drew) Holland, was born on July 14, 1895, in Big Rapids, Mich. He received his grammar school education in Big Rapids and attended high school in Grand Rapids, Mich., being graduated from Central High School in that city in 1913. In the fall of that year, he entered the Engineering College of the University of Michigan at Ann Arbor.

Mr. Holland's university education was interrupted by a period of service in World War I, but he returned to the university upon receiving his discharge after the armistice. He graduated with the class of 1919, receiving the degree of Bachelor of Science in Civil Engineering.

Mr. Holland's employment began on November 15, 1919, with the Board of County Road Commissioners of Wayne County, Mich., and he was employed by that organization until his death. He virtually grew up with it, becoming chief designing engineer in the early 1920's. In that capacity, he was in charge of designing the entire Wayne County road system of 2,400 miles of highways and superhighways.

In 1928, the board undertook the development of the mile square Wayne County Airport. Mr. Holland was placed in charge of preparing the designs, plans, and specifications for this project. More recently, and until his death, he was preparing the plans and designs for the expansion of this airport.

Other activities of the board in which Mr. Holland participated included the out-county sewage disposal and water supply systems, the Wayne County system of parks and parkways, and the John C. Lodge Expressway, now ready for construction in Detroit, Mich. His knowledge of national, state, county, and local road affairs made him invaluable to the county in his professional capacity.

Paul Holland was a member of the Highway Research Board of the National Research Council, the Michigan Engineering Society, the Engineering Society of Detroit, and the Michigan Section of the American Society of Civil Engineers, as well as being a registered professional engineer and land surveyor in the State of Michigan.

He was married to Mary Josephine Whalen on February 1, 1922, at Big Rapids. He is survived by his widow; a son, Robert M. Holland; a daughter, Margery Holland; three brothers, Cornelius, Martin, and Thomas Holland; and two sisters, Ellen (Mrs. E. Commons) and Anne Holland.

Mr. Holland was elected a Member of the American Society of Civil Engineers on November 12, 1940.

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[1] Memoir prepared by Joseph W. Gross, Assoc. M. ASCE.

HAROLD RISTINE HOLMES, M. ASCE[1]

Died June 6, 1946

Harold Ristine Holmes was born in Tabriz, Persia (Iran), on April 6, 1885. His parents were Dr. George W. Holmes, a Presbyterian medical missionary, and Eliza (Wisner) Holmes, both of whom were born in the United States. They lived in Persia until Harold reached the age of twelve, when the family returned to the United States.

He attended high school in Wellesley, Mass., and was graduated from Princeton University at Princeton, N. J., in 1908 with the degree of Civil Engineer. For two years following his graduation Mr. Holmes worked as a rodman for the New York State Engineer's Department, running instruments, drafting, and computing in connection with a barge canal construction at Lockport, N. Y. From 1910 to 1911, he was an assistant engineer for the New York Highway Commission in charge of construction work and responsible for some designing.

Mr. Holmes' next position, for a period of five years, was with the New York State Public Service Commission for the First District, New York, N. Y., on subway and elevated rapid transit construction. From 1913 to 1916, he was in responsible charge of construction on route No. 50, subway and elevated rapid transit route in Long Island City, N. Y. He was then employed by Manion Brothers Company, Incorporated, estimating industrial and school buildings; and, for a time, he was a draftsman for the Shepard Electric Crane and Hoist Company.

From May, 1918, to May, 1919, Mr. Holmes was a member of The American-Persian Relief Commission. Later the work and part of the personnel were taken over by the Near-East Relief, which became the Near-East Foundation. Harry Pratt Judson, then president of the University of Chicago, in Chicago, Ill., was head of the commission. Mr. Holmes was selected for this work not only because of his knowledge of the language and customs of the country, but also because of his engineering knowledge and ability, as the commission expected to set up engineering projects as relief work for war refugees.

Upon arrival at Hamadan, Persia, he took over the duties of secretary-treasurer of a relief committee there. This committee purchased food and clothing for many thousands of people in various near-by camps. It also loaned money to responsible refugees in order that they could return to their homes. In September, 1918, Mr. Holmes stated that the price of bread in Hamadan was twelve krans per mann but six months later it was only five krans per mann, thus reflecting the efforts of the relief committee. Some additions to a hospital building were carried out under his direction, and he also organized a group of refugees and had them making quilts and beds for the hospital.

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[1] **Memoir prepared by Lloyd D. Knapp, M. ASCE.**

In December, 1918, he went to Teheran, Persia, to urge the British Legation to pay the outstanding bills of the Refugee Relief Fund. There he organized relief work and was in charge of it until March, 1919. With the assistance of the native police, the various wards of the city were canvassed, and needy families were given weekly cash relief. While in Teheran, he examined a road construction project which he advised against, as available labor would be needed in the spring for agricultural purposes.

Upon his return to New York, he was employed in several capacities until February, 1921, when he was appointed city engineer and building inspector in Lockport. In 1923 he became office engineer for the Long Island Railroad Company, and following this he was an engineer for Consoer, Older and Quinlan of Chicago.

In 1925 he was employed by the Bureau of Sewers of the City of Milwaukee (Wis.), as a sewer designer, and in a short time he was made chief sewer engineer, which position he held until his death. During this period he was responsible for the design of sewers costing approximately twenty million dollars. Among the more important projects on which he was engaged were the twelve-foot-diameter relief tunnel on North Twenty-Sixth Street, the eight-foot-diameter tunnel on West Becher Street, and the seven-foot by ten-foot double box sewer on North Thirty-First Street.

During the first years of his employment in Milwaukee, Mr. Holmes, in cooperation with others, developed a rainfall intensity curve, considered to be somewhat better than the curve then in use. Later he was instrumental in having the city purchase several rain gages and, with the data thus obtained, continued his studies on rainfall. He did a great deal of original work on the subject, largely on his own time, and finally developed still another rainfall intensity curve which is more accurate than either of the others, particularly in the range of high intensities. It is expected that this curve will be adopted for use by the City of Milwaukee. By the development of this curve he made a distinct contribution to the sum of engineering knowledge.

As a student of world affairs, Mr. Holmes was internationally minded and interested in all efforts to bring about world peace. In Milwaukee he was one of the organizers of the Committee to Defend America by Aiding the Allies; in fact, the first meeting was held in his home. In addition, he was active in urging the United States Government to prohibit the sale of oil, scrap iron, and other war supplies to Japan, anticipating the later known results of such acts.

Mr. Holmes was a registered engineer in Wisconsin and a member of the Wisconsin Society of Professional Engineers. He also belonged to the Princeton Engineering Society and the Engineers' Society of Milwaukee.

On September 26, 1911, he was married to Gertrude Oliver Baker in Lockport. He is survived by his widow and a son, George, a chemical engineer. A daughter, Barbara, died in 1945.

Mr. Holmes was elected an Associate Member of the American Society of Civil Engineers on March 16, 1925, and a Member on July 5, 1933.

ARTHUR GRANT HOLT, M. ASCE[1]

Died June 28, 1946

Arthur Grant Holt, the son of Edmund Dyer Holt and Caroline (Warner) Holt, was born in Chatfield, Minn., on August 30, 1864. He was a descendant on his father's side of Nicholas Holt, who came to Massachusetts from England in 1635, and on his mother's side of Andrew Warner, who came in 1632.

Mr. Holt had the advantage of an ancestry that was steeped in good old New England ideals. His mother attended Mount Holyoke College in South Hadley, Mass., and his father was a graduate of Amherst College at Amherst, Mass., from which he received the degrees of both Bachelor of Arts and Master of Arts. Later, Edmund Holt was graduated from the Union Theological Seminary in New York, N. Y.

While Mr. Holt was still very young, his father died, leaving his mother to bring up the children alone. In this situation, she proved her fortitude. When the children were ready to attend college, the family moved to Minneapolis so that the children could enter the University of Minnesota there. Mr. Holt was in the class of 1885, specializing in mathematics. He was a member of Phi Delta Theta fraternity.

He first went to work for an uncle in Moline, Ill.; but, since railroads fascinated him, he decided to enter that field. He obtained employment with the Chicago, Milwaukee and Saint Paul Railway Company (later the Chicago, Milwaukee, Saint Paul and Pacific Railroad Company) in August, 1885, as rodman and draftsman in the office of the division engineer at Minneapolis. In 1888 he became instrumentman and resident engineer on location and construction of new lines near Fergus Falls, Minn., for the Minnesota and Manitoba Railroad Company (later the Great Northern Railway Company). The next year he was with the Duluth, Winnipeg, and Pacific Railway Company on work near Cloquet, Minn.

In 1890 Mr. Holt returned to the Minneapolis office of the Chicago, Milwaukee and Saint Paul Railway Company and gradually took charge of location and construction of new lines, especially in South Dakota. He had charge of the lines from Eureka to Linton (N. Dak.), Woonsocket to Wessington Springs, Armour to Stickney, and Madison to Sioux Falls. In 1905 he was sent to Seattle, Wash., to report to the late W. L. Darling, Hon. M. ASCE, then chief engineer of the Puget Sound lines, who placed him in charge of two parties locating a line through Natches Pass of the Cascade Mountains. In this capacity, Mr. Holt supervised four locating parties through the Snoqualmie Pass. Also included in this work was a reconnaissance survey for a line from Cedar Falls to Everett, Wash.

In 1906 he was appointed division engineer in charge of location and construction of the main line from the Columbia River to Plummer, Idaho, and then of the St. Maries Branch, also in Idaho. After this, Mr. Holt was in Seattle for a short time before going to Spokane, Wash., where he was in charge of location and construction of lines into and through Spokane, including the Coeur d'Alene lines.

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[1] Memoir prepared by T. H. Strate, M. ASCE.

When the line to the coast was finished, he was transferred to Chicago, Ill., in February, 1913, as assistant chief engineer. He held this position until 1932 when he was appointed assistant to the chief engineer, continuing until his retirement in 1938.

Mr. Holt enjoyed a wide acquaintanceship in the engineering profession and with railroad contractors, all of whom had a high regard for him and respected his judgment and advice. He was well-liked and admired by all his associates and will always be remembered for his high ideals.

He belonged to the American Railway Engineering Association. He also was a member of the Presbyterian Church, the Blue Lodge, Knights Templar, and the Shrine in the A. F. and A. M.

Mr. Holt was married on October 11, 1893, to Caroline Warner, of Hatfield, Mass., who died in 1917, and, on January 31, 1924, to Alice Brenne, of Sioux City, Iowa. He is survived by his widow; two daughters, Lois Holt (Mrs. L. H. Schmidt) and Elisabeth Holt (Mrs. L. S. Holler); a son, Arthur Grant Holt, Jr.; and two grandchildren, Elisabeth Jane Holler and John Holt Holler.

He was buried in Chatfield.

Mr. Holt was elected a Member of the American Society of Civil Engineers on April 5, 1910.

JOHN CLAYTON HOYT, M. ASCE[1]

DIED JUNE 21, 1946

John Clayton Hoyt was born in La Fayette, N. Y., on June 10, 1874. He was the son of Newton and Mary (Ford) Hoyt.

The broad foundation of his technical education was laid at the College of Civil Engineering, Cornell University, in Ithaca, N. Y., from which he was graduated with the degree of Civil Engineer in 1897. The new hydraulic laboratory in Fall Creek was then under construction, and his first engagement was as an assistant engineer with the Cornell Hydraulic Laboratory Construction Company. He had a few years of experience with the United States Board of Engineers on Deep Waterways, the Bureau of Yards and Docks of the Navy Department, and the United States Coast and Geodetic Survey before entering the service of the United States Geological Survey in the Water Resources Branch where he succeeded Gerard H. Matthes, Hon. M. ASCE, as chief of the computing section on September 3, 1902.

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[1] Memoir prepared by Charles H. Pierce, M. ASCE.

Stream-gaging work in Alaska was started by Mr. Hoyt in the spring of 1906 when he was detailed to make an investigation of the water resources available for placer mining in the vicinity of Nome on the southern slope of the Seward Peninsula. Two years later, he made a water-power reconnaissance in southeastern Alaska. The results of these two pioneer investigations are described in Geological Survey *Water-Supply Paper No. 196* and *Water-Supply Paper No. 372.*

After being actively connected with the supervision of stream gaging for several years as assistant to F. H. Newell,[2] M. ASCE, and M. O. Leighton, M. ASCE, in 1911 he was placed in charge of the Division of Surface Water, a position he held until December 18, 1930, when he relinquished his administrative duties to act as consultant to the chief hydraulic engineer and to the director. During his long service in the Water Resources Branch, Mr. Hoyt achieved a nation-wide reputation for his knowledge of hydrology and his improvements in equipment and methods for obtaining hydrologic data. Much of the equipment used in measurements of river discharge was originally designed or improved under his direction while he was chief of the Division of Surface Water.

Mr. Hoyt made numerous contributions to engineering literature on subjects relating to hydrology and water resources. He was responsible for the preparation of many official reports and water-supply papers, including "Droughts of 1930–1934," *Water-Supply Paper No. 680,* and "Drought of 1936," *Water-Supply Paper No. 820*—with discussion of the significance of drought in relation to climate. The authoritative text on "River Discharge,"[3] of which he was author jointly with N. C. Grover, M. ASCE, was first published in 1907 and was followed by a second edition in 1912 and by third and fourth editions at two-year intervals. This was, undoubtedly, the best known and the most widely used publication on the subject at that time.

He was a frequent contributor to the publications of the Society, his paper on "Use and Care of the Current Meter as Practiced by the United States Geological Survey"[4] being an outstanding example. He was always active in the affairs of the Society and was influential in organizing the District of Columbia Section. He gave unsparingly of his time to promote the activities of the Section, serving on many committees and in various offices, including that of President. Notwithstanding his failing health and physical weakness, his unfailing interest in Society affairs brought him to the Annual Meeting in New York, N. Y., on January 16–18, 1946, and to the dinner meeting of the District of Columbia Section on January 29. From 1920 to 1922 he served the Society as a Director and, in 1927–1928, as a Vice-President.

Mr. Hoyt was one of the founders of the Washington Society of Engineers, of which he was secretary (1908–1915) and president (1916). In the formation of the American Engineering Council, he served as secretary of the meeting at which the council was organized and throughout the life of the council he acted either as a representative of the Society or of the District of Columbia Section. He was a member of the Washington Academy of Sciences and vice-president in 1917.

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[2] For memoir, see *Transactions,* ASCE, Vol. 98, 1933, p. 1597.

[3] "River Discharge," by John C. Hoyt and Nathan C. Grover, John Wiley & Sons, Inc., New York, N. Y., 1st Ed., 1907.

[4] "The Use and Care of the Current Meter as Practiced by the United States Geological Survey," by John C. Hoyt, *Transactions,* ASCE, Vol. LXVI, March, 1910, p. 70.

His peculiar talents in organizational work and his wide acquaintance with members of the engineering profession were recognized by his appointment as the representative of the Department of the Interior on the American Committee at the World Power Conference in London, England, in 1923; as the official United States delegate at the Fourteenth International Congress of Navigation at Cairo, Egypt, in 1926; and as a delegate to the World Engineering Congress at Tokyo, Japan, in 1929.

For many years Mr. Hoyt served on the board of directors of the Union Trust Company, in Washington, D. C. He was a member of the Cosmos Club, serving the club as chairman of many important committees and as vice-president in 1920 and president in 1921; the Chevy Chase Club; and the National Press Club. He belonged to the honorary society of Sigma Xi.

Mr. Hoyt's interest in the welfare of engineers, especially those in federal employment, was shown by his work on the Engineering Council Committee on Classification and Compensation of Engineers. An article in *Civil Engineering*[5] gives a brief description of the results that were obtained, largely through the efforts of that committee. The interest that he took in everyday affairs and the wide range of his activities were evidenced by the courtesy card given him by the District of Columbia Department of Vehicles and Traffic as an award for his "Traffic Suggestions" to promote safe and sane driving. The award also included membership in the Traffic Advisory Council.

On October 31, 1900, he was married to Jennie Farnham King of Tully, N. Y. He is survived by his widow; a son, Kendall K. Hoyt; and a brother, William Glenn Hoyt, M. ASCE.

John Hoyt possessed strong determination and great perseverance, and he unswervingly followed the paths that he believed to be right and for the greatest good. A marked characteristic was his loyalty to his friends; his friendship was enduring and his memory will long be cherished by those who were associated with him.

Mr. Hoyt was elected an Associate Member of the American Society of Civil Engineers on May 4, 1904, and a Member on June 1, 1909. He became a Life Member in January, 1939.

FRANK DAVID HUTCHINSON, M. ASCE[1]

DIED DECEMBER 18, 1946

Frank David Hutchinson was born on November 25, 1880, at Pine Lake in the Town of Caroga Lake, Fulton County, New York. His father, Frederick

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[5] "Classification and Compensation of Federal Engineering Positions," by John C. Hoyt, *Civil Engineering*, March, 1936, pp. 194–197.

[1] Memoir prepared by Frank D. Hutchinson, Jr., Sales Engr., New York, N. Y.; Theodore C. Tuck, Civ. Engr., New York, N. Y.; and Charles E. Pett and J. H. Lawrence, Assoc. Members, ASCE.

E. Hutchinson, a native of Connecticut, was a lumberman and furniture manufacturer; his paternal grandfather, David Hutchinson, was a western pioneer. His mother, the daughter of John Culver, a farmer, was born in Blandford, Mass. He was named "Frank" after his uncle, Frank, and "David" after his paternal grandfather, but was always known to his friends as "Hutch."

Mr. Hutchinson's early childhood was spent at a logging camp in Pine Lake. When he was nine years old, his father left the lumber business and moved to Keyser, W. Va., to become a partner in a mill for manufacturing furniture. Hutch attended the public schools there and was considered an excellent student—in fact, he was graduated at the age of sixteen. During the years at Keyser, he drove cows for a farmer to earn pin money.

Upon completion of his schooling at Keyser in 1897, he entered West Virginia University in Morgantown, with the idea of becoming a doctor. However, after two years he decided to become a civil engineer and in 1902 was awarded the degree of Bachelor of Science in Civil Engineering. While attending the university, he became a member of the Beta Theta Pi fraternity and was active in its support during his lifetime.

Mr. Hutchinson's first position in the engineering field was with the McClintic Marshall Construction Company in Rankin, Pa., from 1902 to 1903, as draftsman on shop details for mill buildings. On leaving Rankin, he went to Steelton, Pa., where he worked as draftsman and designer in the Erection Department of the Pennsylvania Steel Company, from 1903 to 1904.

In 1904 he began his major lifework with Post and McCord, structural engineers and contractors, on the erection of structural steel in New York, N. Y., and continued with that company for thirty-five years. During this period, Mr. Hutchinson was engineer, in complete charge of preparing shop and erection drawings of the steelwork for many large buildings. These included most of the major electric and steam power plants in New York, as well as many skyscraper office buildings, banks, department stores, pier sheds, schools, and churches. In some cases, his work involved the complete structural design, and, in many others, complicated erection problems. Among the contracts handled by him for Post and McCord were the Empire State Building, 30 Rockefeller Plaza, the Chrysler Building, the Yale Club, the Chase National Bank, and several buildings in the area over the New York Central Railroad tracks about Grand Central Station.

Mr. Hutchinson was twice married, first, to Ellen A. Fowle in 1906, and the second time, to Anne Marie St. Clair in 1922. His son, Frank D. Hutchinson, Jr., survives. His home was in Bronxville, N. Y., where he took particular delight in raising flowers.

In 1940 he was engaged by the Metropolitan Device Corporation of Brooklyn, N. Y., to design the steelwork for industrial plants in the midwest and later was connected with the extensive war work carried on by that company. At the time of his death, he was structural engineer.

Mr. Hutchinson was elected a Member of the American Society of Civil Engineers on November 26, 1918.

THEODORE REED KENDALL, M. ASCE[1]

DIED FEBRUARY 4, 1946

Theodore Reed Kendall was born in Boston, Mass., on April 25, 1890, the son of Joseph Sewall and Bertha (Allen) Kendall. He was educated in Boston elementary schools and in Boston English High School. In 1908 he entered Harvard College, in Cambridge, Mass., and was graduated in 1912 with the degree of Bachelor of Science, and in 1914 received the degree of Master of Civil Engineering from the Harvard Engineering School.

After graduation in 1914, Mr. Kendall studied stanitary engineering projects in France, Germany, and England, and on his return was employed as an assistant to the late Prof. George C. Whipple,[2] M. ASCE, of Harvard University. Under Professor Whipple he was engaged in various sanitary engineering projects, including surveys, mapping and reports, and investigations of stream pollution of the Blackstone River in Massachusetts and Rhode Island, and oyster pollution in Narragansett Bay, R. I. Also, as water works engineer and superintendent, he was in charge of operation and rehabilitation of the Agua Clara Filtration Plant, Gatun, Canal Zone.

In March, 1917, Mr. Kendall entered the publishing field as engineering editor of *The American City,* a career which was destined to continue with distinction for the remainder of his life, except for the interruption of military service in World War I. From January to December, 1918, he served as First Lieutenant with the Sanitary Corps of the United States Army with various assignments in Washington, D. C., and in charge of operation and construction of water works at Petersburg, Va., and Watervliet, N. Y.

In January, 1919, Mr. Kendall returned to his position as engineering editor of *The American City* and also of *The Municipal Index,* and in May, 1920, in addition to his other duties, he became editor of *Contractors' and Engineers' Monthly,* a publication which he helped to found. Later, he withdrew as engineering editor of *The American City,* but continued as its consulting editor until September, 1943.

During the period of more than twenty-five years in which he was associated with the Edgar J. Buttenheim publications, his coverage of engineering projects included field trips and studies of all phases of construction throughout the United States, Canada, and Mexico, writing descriptive articles on water supply, sewage disposal, refuse collection and disposal, street and highway construction and maintenance, bridges, tunnels, foundations, docks, dams, and subways—practically the entire field of the heavy construction industry. His travels and his writings made him a nationally known figure in the engineering and construction world.

In addition to an extremely active life in the professional engineering field, he found time to devote to the activities of technical societies. He served two terms, in 1927–1928 and in 1931–1932, as Director, and one term, in 1928–1930, as Vice-President, of the Metropolitan Section of the Society. He also served for several years on the Metropolitan Section's Committee on Student Guidance and frequently assisted in the work of this committee in the high schools of New York City.

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[1] Memoir prepared by Dean G. Edwards, M. ASCE.

[2] For memoir, see *Transactions,* ASCE, Vol. 88, 1925, p. 1453.

Mr. Kendall was particularly active as a member of the Harvard Engineering Society. He assisted in its organization in 1920, when the society was formed by a consolidation of formerly existing associations of Harvard engineers. He was its first treasurer, serving for two years, then he served as its secretary for seven years, and became its president in 1929–1930.

During his undergraduate days, he had been editor in chief of the *Harvard Engineering Journal.* In 1920, drawing from his rich experience, he started *The Harvard Engineering Society Bulletin* and became its first editor. His keen interest and unselfish devotion to the Harvard Engineering Society will long be remembered by his host of friends.

Mr. Kendall was also a member of the American Road Builders' Association and a director of the association at the time of his death. In addition, he was a member of the American Society of Mechanical Engineers, the Boston Society of Civil Engineers, the Society of American Military Engineers, the American Water Works Association, the New England Water Works Association, and the American Public Works Association.

A resident of South Nyack, N. Y., for many years, he served as village trustee from 1924 to 1926, and was president of the Men's Club of Nyack in 1926–1927. Among his many friends, Mr. Kendall will always be remembered for his delightful personality, his keenness and ability to think and express himself intelligently and forcefully, his unfailing good humor, his ready wit, and his capacity for work.

He was married to Helen Wilbur Brown of Providence, R. I., on January 10, 1917. He is survived by his widow and two daughters, Cynthia Thurston and Bernice Alden Kendall.

Mr. Kendall was elected an Associate Member of the American Society of Civil Engineers on April 25, 1921, and a Member on July 14, 1930.

HENRY KERCHER, M. ASCE[1]

DIED JANUARY 8, 1946

Henry Kercher was born in Germantown, Ohio, on November 12, 1887. He was the son of Alvin C. and Ermina (Fudge) Kercher.

He was educated in the public schools of Germantown and at Ohio State University, in Columbus. From the latter institution he received a degree in civil engineering in 1910.

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[1] Memoir prepared by Warren W. Parks, M. ASCE.

Mr. Kercher started work in his chosen profession immediately upon his graduation, and for two years served as draftsman with the Mount Vernon (Ohio) Bridge Company. He continued in this capacity with the King Bridge Company of Cleveland, Ohio, for a few months, when he was advanced to assistant engineer. His connection with this firm continued until January, 1923. For four years he was rated as chief draftsman, and for the last five years as assistant chief engineer.

During his affiliation with the King Bridge Company, Mr. Kercher had a hand in the design and construction of movable railroad bridges, the Detroit-Superior Bridge at Cleveland, various railroad and highway bridges, traveling cranes, movable bridge machinery, turntables, and mill buildings.

In January, 1923, Mr. Kercher joined the engineering staff of the Cleveland Union Terminals Company. His first assignment was as designer on estimates of cost of overhead bridges, railroad viaducts and structures for electrification of railroads, large retaining walls, and building foundations. The next year he was rated as assistant engineer, preparing estimates and checking designs for high retaining walls and railroad viaduct piers, as well as for the foundations for the Terminal Tower Building which rises 735 ft above the footings. His connection with the Cleveland Terminal continued until its completion in 1930.

In 1930 Mr. Kercher moved to Cincinnati, Ohio, to become chief engineer for the Broadway and Newport Bridge Company of Newport, Ky. This company operates a toll bridge over the Ohio River between Cincinnati and Newport. After serving as chief engineer for one year, he was named vice-president and general manager of this company, and he served in this capacity at the time of his death. The operation of this bridge involved many maintenance problems and public relations in connection with its use for street cars, buses, and private vehicular transportation.

During all his years as a structural designer, involved as they were with heavy responsibilities and complicated problems of design, Mr. Kercher found relief and relaxation through two most interesting hobbies. He took great pleasure in collecting rare coins and had assembled a small but superb collection of early Greek and Roman coins, the former from the period prior to the Hellinistic era, and the latter during the period of the Republic. He specialized in early Athenian coins and many of his specimens from Athens are extremely rare—only a few others are known to be in existence. Many ancient cities are represented in his collections.

In connection with the collection of coins, Mr. Kercher spent about ten years on a comprehensive study based on statistical methods of the determining of weight standards of ancient coins. He gathered information from museums and individuals in Europe and the United States. These calculations were completed, but a book which he planned was never finished. Hundreds of pages of calculations and observations, bound in book form, have been forwarded to the American Numismatic Society in New York, N. Y. These were bound into nine large volumes and are to remain on file after a recapitulation has been completed. A professor at Princeton University at Princeton, N. J., has undertaken to finish the work started by Mr. Kercher.

The other hobby, the pursuit of which gave him much pleasure and material for conversation, was his study of American archeology. He started collecting Indian relics as a boy. "Mill's Archeological Atlas of Ohio" included much material supplied by Mr. Kercher. He had visited and was familiar with practically every site of major importance in southern Ohio. Mr. Kercher personally explored and excavated at the Anderson, Taylor, and Madisonville sites in Ohio, and surveyed and made maps of the Anderson explorations at Fort Ancient. These maps are the property of the Ohio State Archeological and Historical Society.

From this recital of Mr. Kercher's professional activity and his hobbies, it is evident that he never lacked an interesting subject for conversation, and he was always very generous in sharing his experiences and his discoveries.

In addition to all these activities, Mr. Kercher found time to fulfil his duties as a churchman and a useful citizen in the community. He was a member of the Engineering Society of Cincinnati and active in the Cincinnati Section of this Society, having served as President for a term.

Mr. Kercher married Jeannette Good, at Westerville, Ohio, on June 27, 1912. In addition to his widow he is survived by two children, a daughter, Dorothy (Mrs. M. D. Knoop), and a son, Robert A. Kercher.

Mr. Kercher was elected an Associate Member of the American Society of Civil Engineers on April 18, 1916, and a Member on January 16, 1928.

BURR ROBERT KULP, M. ASCE[1]

Died February 27, 1946

Burr Robert Kulp was born in Duncannon, Pa., on December 16, 1883, the son of John David and Alice May (Bowers) Kulp. His ancestors came from Germany and settled in southeastern Pennsylvania early in the eighteenth century. His grandfather's farm was a part of the Gettysburg Battlefield. In 1887 his parents moved to Harrisburg, Pa., and it was in that city that he grew up and went to grammar school and high school. He then attended Rensselaer Polytechnic Institute at Troy, N. Y., where he received the degree of Civil Engineer in 1905, and where he was a member of the Theta Nu Epsilon fraternity.

He was employed by the Chicago and North Western Railway Company in June, 1905, as instrumentman, and spent his entire professional career with that railway. He subsequently held the positions of architectural draftsman, assistant engineer, division engineer, trainmaster, again division engineer, then principal assistant engineer, engineer of maintenance, and chief engineer. He was chief engineer of the Chicago and North Western Railway Company, from May, 1940, and also of the Chicago, St. Paul, Minneapolis and Omaha Railway Company, from April, 1943, until his death.

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[1] Memoir prepared by George W. Hand, M. ASCE.

Mr. Kulp was a member of the American Railway Engineering Association (a director), the Western Society of Engineers, the American Railway Bridge and Building Association, and the Rensselaer Society of Engineers.

The positions he held are indicative of the high quality of engineering skill that he possessed. During his career he planned and constructed may of the railway's important structures and facilities and was responsible for the efficient expenditure of large amounts of money for both construction and maintenance. His professional ability contributed greatly to the success of the railway with which he was associated, and this, together with his winning personality, earned him the respect and warm friendship of his associates.

Mr. Kulp was a trustee of Rensselaer Polytechnic Institute in 1938, 1939, and 1940. He was a communicant of St. Luke's Episcopal Church in Evanston, Ill.

In fraternal matters he was also active, being a member of the F. and A. M., the Knights Templar, and the Shriners. He belonged to the Oriental Consistory at Chicago, Ill. He also belonged to the Union League Club of Chicago and the Evanston Golf Club.

Mr. Kulp was married on February 6, 1906, to Edna Mary Herbst of Chicago, who died in 1927. They had a son, John Herman. On April 1, 1929, he married Gertrude Elizabeth Madden of Sparta, Wis. He is survived by his widow; his son, John Herman; and a brother, George R. Kulp.

He died in his home in Evanston and was buried in Forest Hill Cemetery, Madison, Wis.

Mr. Kulp was elected a Member of the American Society of Civil Engineers on May 18, 1943.

CHARLES SCOTT LANDERS, M. ASCE[1]

DIED JULY 27, 1946

Charles Scott Landers was born in Thurmont, Md., on August 27, 1879, He was the son of John and Harriet (Forman) Landers.

After being graduated in 1900 from the Sheffield Scientific School at Yale University in New Haven, Conn., with the degree of Bachelor of Science, he spent two years as designer and resident engineer on the Guayaquil and Quinto Railway in Ecuador, South America. In 1903 he returned to the Sheffield Scientific School as instructor in engineering and graduate student, receiving the degree of Civil Engineer in 1904, and membership in the Society of Sigma Xi.

When the Hudson River railroad tunnels were projected in 1904, Mr. Landers joined the Pennsylvania Railroad Company as construction engineer. On this project he acquired his first experience in compressed-air subsurface operations. This led to his association with The Foundation Company in 1906 as engineer and superintendent on pneumatic caisson foundations for many early skyscrapers, including the Singer Tower and the Trinity buildings, in New York, N. Y. He then joined the engineering staff of the Board of Water Supply of New York City where he remained for a period of two years as engineer and designer.

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[1] Memoir prepared by Charles R. Krieg, M. ASCE.

In 1911 Mr. Landers entered private practice as a consulting engineer in New York, specializing in pneumatic caisson and other difficult foundation methods. As consultant, designer, or as supervisor for owners, architects, and contractors, he participated in the foundation work for such noteworthy buildings in New York as the Woolworth Building at 233 Broadway, the Bankers Trust at 16 Wall Street, the Equitable Building at 120 Broadway, the W. R. Grace and Company Building at 7 Hanover Square, the Continental Bank Building at 30 Broad Street, the Squibb Building at 745 Fifth Avenue, the United States Federal Office Building at Vesey Street, the buildings at 80 Broad Street and 29 Broadway, and many others.

Similar foundation engineering outside the New York area included the Gulf Oil Building at Pittsburgh, Pa.; the Hackensack River bridges for the Pennsylvania Railroad Company; the L. Bamberger Company addition at Newark, N. J.; the powerhouse for the Ohio Power Company's (American Gas and Electric Company) Windsor Plant at Power, W. Va.; and others.

Mr. Landers' participation in underpinning problems included the Trinity Church tower and a number of buildings on the route of the Nassau Street Extension of the Brooklyn-Manhattan Transit System in lower Manhattan. His efforts, however, were not confined to substructure engineering. A great tennis enthusiast, he designed the tennis stadium of the West Side Tennis Club at Forest Hills, Long Island, of which he was a former president. He also acted as expert witness in many cases for the Corporation Counsel of the City of New York, and his extensive knowledge of substrata conditions in and about New York was solicited by many architects and engineers.

As a civic-minded citizen, he was a member of the Federal Grand Jury Association of the Southern District of New York, and gave much time to its work, particularly during his two-year tenure as president. Mr. Landers was also a member of the Yale Club, the Yale Engineering Society, the Society of Sigma Xi, the "Moles," and the West Side Tennis Club.

He was respected by all who knew him, and his benign and gracious personality will be missed by all his associates. Many young engineers benefited by his advice and assistance in placement in engineering positions. He was truly a humanitarian, and broad in all his thoughts and actions.

On November 10, 1909, Mr. Landers was married to Josephine Schramm in New York. He is survived by his widow and a son, Charles Scott Landers, Jr.

Mr. Landers was elected a Member of the American Society of Civil Engineers on October 10, 1927.

LOUIS EMILE LAURENT, M. ASCE[1]

Died November 5, 1945

Louis Emile Laurent, the son of Eugene J. and Helen A. (Goold) Laurent, was born in Paris, France, on June 30, 1874. He was brought to the United States at an early age by his parents. His family was asociated with some of the leading families of New York, N. Y., such as the Coopers, the Hewitts, and the Havemeyers. He was educated in the public schools of New York, at Cooper Union, also in New York, and at Princeton University, in Princeton, N. J.

From 1897 to 1900 he was employed by the Johnson Foundry and Machine Company in New York and for the next nine years was engaged by the Trenton Iron Company of Trenton, N. J., in the design, and later in the erection, of aerial tramways and mill and mining machinery. In the latter capacity he had charge of the construction of large installations at Bingham Canyon, Utah; Cranbrook and Kimberly, B. C., Canada; Placerville, Calif.; and Velardenia, Durango, Mexico. This work brought him in close touch with the mining industry and developed in him an interest which resulted in his taking part in the organization of the Santa Eduviges Mining Company.

In 1909 Mr. Laurent was made manager and went to Sonora, Mexico, where his company acquired mining property. Under his management the property was developed and the operation was brought to a paying basis when the country was disrupted by the revolution under Francisco Villa. The property was confiscated and the manager was sentenced to be shot. By a ruse he outwitted his captors and escaped. He made his way back to the United States, but with his investments lost.

From 1914 to 1916 he worked for the American Steel and Wire Company at Denver, Colo., as sales engineer on cableways and tramways. In 1917 he entered the employ of the Erie Railroad Company as assistant engineer on the design and installation of heating, refrigeration, and air conditioning facilities.

His ingenuity was put to a test when, in the heat of summer, he was called on by the management to make an immediate installation to cool the six auction rooms at its fruit sale piers in New York. He devised a scheme for pumping ice water through the steam mains and unit heaters. The installation was rushed, and in three weeks he had the temperature in the auction rooms reduced by 8° or 10°, to the satisfaction of all concerned. The idea was original with him, as he had not heard of its use before. It has been used since. In June, 1945, he retired because of ill health.

Mr. Laurent was a member of the New York Engineers Club and of the F. and A. M. He became a citizen of the United States, but the delightful French characteristics of courtesy and chivalry clung to him throughout life.

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[1] Memoir prepared by Charles H. Splitstone, M. ASCE.

His fine qualities of honesty, loyalty, and cooperation claimed the affectionate regard of his associates.

On November 16, 1928, he was married to Deal Jean Bushnee Stephens in Jersey City, N. J. His marriage was a happy one. He is survived by his widow.

Mr. Laurent was elected a Member of the American Society of Civil Engineers on September 9, 1940.

WILLIAM IRA LEE, M. ASCE[1]

DIED JULY 5, 1946

William Ira Lee was born in Greensville County, Virginia, on September 18, 1872. His parents were Samuel Wornum and Rebecca Ellice (Wyche) Lee. He was educated at the Emporia (Va.) Academy before enlisting in the United States Army at the age of twenty one.

After serving in the Army for three years, he began work in 1897 as rodman with the Richmond, Petersburg and Carolina Railroad (later part of the Seaboard Air Line Railway). He continued with this company until January, 1900, and was promoted, successively, to levelman, instrumentman, and resident engineer. Then he was employed by various railroads in the south and southwest as locating engineer and resident engineer on railroad construction.

In the fall of 1907, Mr. Lee first entered the highway construction field with the Virginia Highway Department. He returned to the Winston-Salem Southbound Railway in September, 1909, and remained until April, 1911, as resident engineer on construction.

Mr. Lee returned to the Virginia Highway Department in the spring of 1911 and, except for a few months as county engineer of Mercer County, West Virginia, remained with that department until the outbreak of World War I. He served as resident engineer and was promoted to division engineer at Lynchburg, Va.

He went on active duty as a Major in the Engineer Reserve Corps of the Army on October 11, 1917, and sailed for France on October 29. First, he served with the British Ninth Army Corps for observation and training and, later, was assigned as superintendent of roads and quarries for Base Sector No. 1 at St. Nazaire, France. In March, 1919, he returned to the United States being released from active service on June 18, 1919. After the war Mr. Lee remained in the Engineer Reserve Corps until he was retired because of age. During this period he was promoted to Lieutenant Colonel and Colonel. At the time of his retirement he was commanding officer of the 305th Engineers.

Shortly after returning to the Virginia Highway Department in 1919, he was appointed engineer of plans and surveys. However, in February, 1921, he resigned to enter the contracting business for himself. From 1922 to 1925 he was county engineer for Charles City County, Virginia.

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[1] Memoir prepared by Harold J. Spelman, M. ASCE.

He began work for what was then known as the United States Bureau of Public Roads in July, 1928, on the surveys for the Mount Vernon Memorial Highway. In 1930 he was promoted to the grade of associate highway engineer and was placed in charge, under the district engineer, of grading and paving construction on that important project. Upon completion of that work in August, 1932, he was assigned to head the field office then opened at Gatlinburg, Tenn., and was subsequently promoted to highway engineer and senior highway engineer. Although he was retired because of age in September, 1942, he was immediately returned to service and remained at Gatlinburg until October 1, 1945.

During his thirteen years at Gatlinburg he supervised the surveys, design, and construction of the roads built in the Great Smoky Mountains National Park; the Chickamauga-Chattanooga National Military Park; the Guilford Court House (North Carolina) Park; and thirty miles of the Blue Ridge Parkway, immediately adjacent to the Great Smoky Mountains National Park.

Among the projects planned and built by Colonel Lee during this period was the road from Newfound Gap at El. 5050 on the crest of the "Smokies" to Gatlinburg at El. 1800. This road was described by the late W. T. B. MacCormack, a visiting Australian engineer, in 1937, in part, as follows: "The trip down to Gatlinburg was wonderful—stone bridges, massive stone walls, fine tunnels, and one particular work known as 'Lee's Loop,' a triumph for the designer." "Lee's Loop" is a circular alinement requiring a bridge where the road crosses over itself. This unusual design replaced a bad triple switchback in the old pioneer road.

Another difficult, picturesque, and interesting project located, designed, and built by Colonel Lee was the road along the ridge of the Smoky Mountains from Newfound Gap to Clingman's Dome, climbing from 5,050 feet to 6,200 feet above sea level. This splendid mountain highway is the highest road east of the Mississippi River.

In July, 1904, at Atlanta, Ga., William I. Lee was married to a native of that state, Mattie Cunningham, whom he first met while engaged on railroad work near her home. Mary Ellice Lee of Alexandria, Va., is their only child. Mrs. Lee died on April 30, 1924. On April 6, 1941, Colonel Lee was married to Mildred Comyn Crawford of Waynesville, N. C., who survives him.

The passing of Colonel Lee, at the age of seventy four, some nine months after his retirement from active practice, removed from the highway field one of the few remaining engineers whose early training and experience was on railway location and construction. The "Colonel," as he was called affectionately by his many friends for the last twenty years of his life, was known for his hearty laugh which "shook the rafters." Of powerful physique, six feet tall, he was erect and energetic until past seventy years of age. Affable and pleasant in personality, he could be stern and quick to reprove careless errors in his subordinates. Even in his later years he kept abreast of developments in his chosen field by reading. He encouraged his younger associates to do the same, frequently lending them books from his own not inconsiderable engineering library.

His early experience in railroad location and construction and in highway surveys, plans, and construction contributed to the rounded knowledge as a finished highway engineer that marked his later professional career. He was meticulous in every detail, accurate in his judgment and work. His plans as well as his construction were finished products of quality.

The young men who worked under his direction during the last fifteen years of his professional career obtained a thorough training in engineering fundamentals. Accuracy, careful attention to details, completeness of records of work accomplished, and a high quality of construction were all stressed by Colonel Lee. He was fair and just in his dealings with contractors who performed work under his direction.

He was a member of the American Legion and the Veterans of Foreign Wars, and he belonged to the Episcopal Church.

Colonel Lee was elected a Member of the American Society of Civil Engineers on October 15, 1923.

CONDE BALCOM McCULLOUGH, M. ASCE[1]

Died May 6, 1946

Conde Balcom McCullough, the son of John Black and Lenna (Balcom) McCullough, was born at Redfield, S. Dak., on May 30, 1887. He received his elementary education in the public schools of Fort Dodge, Iowa, and his engineering training at Iowa State College at Ames, from which he received the degree of Bachelor of Science in Civil Engineering in 1910 and the professional degree of Civil Engineer in 1916. Busy as he was with his engineering work, he found the time and energy to complete a course in law at Willamette University in Salem, Ore., receiving the degree of Bachelor of Laws in 1928. He was admitted to the Oregon bar and, although never active in the legal profession, retained his membership until his death. Recognizing his many contributions to engineering literature and his services to the state, the Oregon State College at Corvallis conferred the degree of Doctor of Engineering on him in 1934.

Civil engineering claimed Mr. McCullough's primary interest even in his boyhood. In 1905, prior to entering college, he worked as rodman, chainman, and transitman on railway maintenance and reconstruction for the Illinois Central Railroad Company in Illinois and Indiana. Immediately after graduation, he secured a position as assistant engineer with the March Engineering Company in Des Moines, Iowa. In 1911 he joined the Iowa State Highway Commission at a time when that organization was in its infancy. He served as designing engineer and assistant to Thomas H. MacDonald, Hon. M. ASCE, then state highway engineer of Iowa.

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[1] Memoir prepared by G. S. Paxson, M. ASCE.

Oregon State College brought Mr. McCullough to Oregon in 1916 as assistant professor and later as professor of civil engineering. He remained on the college faculty until 1919, being on leave of absence during part of 1918 when he served as Captain of Engineers in the United States Army.

In May, 1919, he became bridge engineer for the Oregon State Highway Commission. The construction of the modern highway system of Oregon was just getting under way following the end of World War I, and the organization which Mr. McCullough built was new and entirely his. During his sixteen years of service as bridge engineer and later as assistant state highway engineer, he planned and supervised the design and construction of literally thousands of bridges and buildings for the state highway commission. The bridges which he built in Oregon are known throughout the world for their beauty. Among the best known are the steel arches over the Willamette River at Oregon City and Yaquina Bay at Newport; the concrete arches over the Rogue River at Gold Beach; the steel cantilever over Coos Bay at North Bend; and the McLoughlin Bridge over the Clackamas River on United States route No. 99, south of Portland. The McLoughlin Bridge, a series of tied steel arches with concrete approaches, was built in 1933 and was rated by the American Institute of Steel Construction as the most beautiful steel bridge in its class built in that year.

In 1936 Mr. McCullough was granted a leave of absence to serve with the United States Public Roads Administration, supervising the design and construction of bridges on the Inter-American Highway in Central America. During the year and a half which he devoted to this work, major bridges in Honduras, Nicarauga, Guatemala, and Panama were built. His headquarters were at San José, Costa Rica; and, because of the large territory covered and the primitive rail and highway system existing at that time, much of his travel was by air.

Upon his return to Oregon in 1937, he was made assistant state highway engineer and immediately undertook an analysis of the economic features of the highway system. The results of this work—three technical bulletins published by the Oregon State Highway Comission on "The Economics of Highway Planning," "The Determination of Highway System Solvencies," and "An Analysis of the Highway Tax Structure in Oregon"—have received nationwide recognition for their clear and logical presentation of this difficult subject.

Mr. McCullough was the author of several textbooks on structural subjects. They include "Economics of Highway Bridge Types"[2] and "Elastic Arch Bridges."[3] In the latter book he was coauthor with Edward S. Thayer. He also wrote bulletins on "Highway Bridge Surveys"[4] and "Electrical Equipment on Movable Bridges."[5] In recent years he became interested in suspension bridges and was coauthor, with other members of the Oregon State Highway Department, of five technical bulletins on this subject. At the time of his death he was chairman of the subcommittee on Interpretation and Analysis of the National Advisory Board on the Investigation of Suspension Bridges. A few months before his death he published, as coauthor with his son, John R. McCullough, a two-volume treatise entitled "The Engineer at Law," [6] which has had a very favorable reception by the profession because of its clear presentation of the subject.

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[2] "Economics of Highway Bridge Types," by C. B. McCullough, Gillette Pub. Co., Chicago, Ill., 1929.

[3] "Elastic Arch Bridges," by Conde B. McCullough and Edward S. Thayer, John Wiley & Sons, Inc., New York, N. Y., 1931.

[4] "Highway Bridge Surveys," by Conde B. McCullough, *Technical Bulletin No. 55*, U.S.D.A., Washington, D. C., 1928.

[5] "Electrical Equipment on Movable Bridges," by C. B. McCullough, A. L. Gemeny, and W. R. Wickerham, *Technical Bulletin No. 265*, U.S.D.A., Washington, D. C., 1931.

Mr. McCullough felt keenly his civic obligations to his state and community and was always ready to serve on public projects. He had, for several years, been chairman of the Salem City Planning Commission and had given generously of his time and energy to the development of the city.

He belonged to the American Society of Civil Engineers, the American Concrete Institute, the Northwest Society of Highway Engineers, Tau Beta Pi, Sigma Tau, and Delta Theta Phi, the honorary legal fraternity. He was a communicant of the Episcopal church.

Mr. McCullough was married to Marie Roddan at Perry, Iowa, on June 4, 1913. He is survived by his widow and a son, John R. McCullough.

The following tribute to him as a man and an engineer headed the editorial page of a newspaper in his home town:

> "C. B. McCullough was a gallant soul who lived life to the full. He gave much from his great mind and overflowing heart, and from life he derived rich satisfaction in achievement, in generous participation in affairs and hearty companionship. So there is something appropriate in his being called when life for him stood at flood tide—no slow fading of powers on a tardy ebb. But what a void he leaves in his profession, among his professional associates and among the wide host of his friends!"

Mr. McCullough was elected an Associate Member of the American Society of Civil Engineers on March 14, 1916, and a Member on March 5, 1928.

JAMES HENRY STEWART MELVILLE, M. ASCE[1]

Died April 17, 1946

James Henry Stewart Melville was born on May 2, 1878, in Birnam Wood, Jamaica. His family was closely identified with the British West Indies for several generations.

His father, Charles Melville, a clergyman of the Church of England, was rector of the parish of Saint Elizabeth and canon of the Diocese of Jamaica. His paternal grandfather was a doctor, a graduate of the University of Edinburgh in Scotland; his grandmother was the former Eliza Choppin of St. Vincent, British West Indies.

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[6] "The Engineer at Law," by Conde B. McCullough and John R. McCullough, The Collegiate Press, Inc., Ames, Iowa, 1946.

[1] Memoir prepared by George W. Burpee, M. ASCE.

At one time in the early 1860's, his grandfather practiced in New York, N. Y. During that period young Charles, although only fourteen years of age and fired with enthusiasm for the Union cause, attempted to enlist in the Union Army, giving his age as sixteen. As he was large for his age he was about to be accepted when he was discovered by a friend of the family and sent home. He then went to Jamaica to work on an uncle's estate, and was so impressed with the need of the native Jamaicans for help and guidance that he decided to enter the ministry.

James Melville's mother, the daughter of a clergyman, was born in Jamaica. Her father was born in the Barbadoes in the British West Indies, and her mother was the daughter of a clergyman who came to Jamaica from Ireland at the age of two. Since his father, one grandfather, and a great-grandfather were all clergymen, and one grandfather was a physician, it would be natural to expect that James Melville inherited a strong bent toward a life of service to others.

One in a family of ten, having five brothers and four sisters, he was educated at Jamaica College. From 1895 to 1898 he served an apprenticeship as a draftsman with the late W. S. Wortley, engineer and contractor of Kingston, Jamaica. For part of the time, in 1898–1899, he was associated with the Jamaica Government Railway and was assistant to the government inspector of electric tramways during the construction of the tramway system in Kingston. For the next seven years (1899–1906), he was in the office of the Commanding Officer of the Royal Engineers at Up-Park Camp, starting as a draftsman and, finally, becoming chief draftsman.

In 1906 Mr. Melville was attracted by the opportunities which the United States then offered young engineers, and he moved from Jamaica to New York. There he became associated almost immediately with the engineering department of the New York Central Railroad Company, commencing work as a draftsman and, subsequently, becoming assistant engineer under the late Victor Spangberg, engineer on the West Side Improvements in New York.

During the years of his association with that railroad, his work was principally on preliminary plans for the elimination of the freight tracks from the streets in New York, and the incidental layouts for freight terminals and warehouses related thereto. He also participated in valuation studies and in the various investigations which are common to all railroad engineering departments.

In 1913 he was employed as engineer with Westinghouse, Church, Kerr and Company, Engineers and Constructors, of New York, and for the next eight years was engaged on design and construction of many industrial and railroad projects, as well as on numerous engineering investigations. Throughout 1918 and the first half of 1919, he was office engineer on the construction of the United States Nitrate Plant, Number II, at Muscle Shoals, Ala., one of the mammoth undertakings of World War I. In 1921, upon completion of the American Rolling Mill Company's East Works at Middletown, Ohio, on which he was resident engineer, he joined the staff of Messrs. Coverdale and Colpitts, Consulting Engineers of New York. His work with the latter firm embraced a wide variety of activities. These included valuations of railway and industrial properties, investigations and reports on the economic justification of proposed expenditures and the soundness of plans for financing or refinancing, and studies of the economics of mergers or consolidations and related matters—in which banking or investment houses ordinarily desire an independent outside judgment involving engineering knowledge, financial analysis, and business judgment.

During this period Mr. Melville was much in demand as a highway traffic expert, and his firm numbered among the clients to whom he gave personal services: The Port of New York Authority, the Maryland State Roads Commission, the Kentucky Highway Commission, the California Toll Bridge Authority, the Washington State Highway Commission, and many private firms. His function in these cases was to estimate the probable operations of the toll facilities involved in the respective projects and to advise as to toll rate structures.

Mr. Melville retired in 1939 and returned to his old home in Jamaica. In July, 1942, however, responding to a desire to participate actively in the war effort of the United States, he returned to Coverdale and Colpitts for a few months, engaged on the construction of the Basic Magnesium Plant at Las Vegas, Nev. From November, 1942, to the fall of 1945, he served as a project engineer with the Defense Plant Corporation. In April, 1946, he suffered a stroke from which he never fully recovered.

Mr. Melville was married in 1905 to Lillian Mary Latreille. He is survived by his widow; a daughter, Margaret (Mrs. Hugh Haggard), of England; and two sons, Charles and Alexander, both residing in the United States.

Mr. Melville's avocation was the writing of poetry, and, if poetry had been his principal occupation, there is no doubt that he would have achieved a high place in the literary field. Some of his verses were published from time to time in periodicals and newspapers, but his complete works, which he termed "Mea Impedimenta," were circulated only among his friends. His verses entitled "The Fifth of June" were printed on the editorial page of the *New York Times* for June 5, 1917, the registration day for World War 1. His poem, "We," was one of the one hundred poems chosen in the Doran competition and published in "The Spirit of St Louis," to celebrate Charles A. Lindbergh's New York to Paris flight.

Mr. Melville possessed fine judgment and insight and had the faculty of always stimulating those with whom he was associated to do their best. He was modest to an extreme. By those who knew him well, he will be remembered as one who was the epitome of selflessness. He was held in high esteem and affection by those who were fortunate enough to fall within the orbit of his influence. Wherever he found himself, he assumed the responsibility for developing an esprit de corps and a pride in the job. With all his modesty and unselfishness, he had the qualifications of a leader, the judgment, the courage, and the ability to inspire men.

Mr Melville was elected a Member of the American Society of Civil Engineers on August 28, 1922.

HIRAM MILLER, M. ASCE[1]

DIED OCTOBER 14, 1945

Hiram Miller was born on June 15, 1880, in Middlefield, Conn. His father, Waldo Brainard Miller, a farmer, was named Brainard for David Brainard of Haddam, Conn., an apostle to the Indians, whose sister was married to David Miller, Hiram's great-great-grandfather. Thomas Miller, a paternal ancestor, came to Rowley, Mass., from England, in 1638, and moved to Connecticut, in 1650, where he was one of the founders of Middletown. He built the first grist mill at Miller's brook and had numerous descendants. His son, Benjamin, was the first settler in Middlefield and a great friend of the Indians. Benjamin's son, David, built the ancestral home in Middlefield, in 1741, and the house which is in excellent condition and the farm of one hundred acres has been in the family ever since. Hiram Miller's mother was Betsey Jane Chapman, a school teacher, who came from a family of fine character and strong religious background. Her ancestral home was in Westbrook, Conn. Robert Chapman was her immigrant ancestor who settled in Saybrook, Conn. Mr. Miller's parents, Waldo and Betsey Miller, were married in April, 1870, and had five children.

Hiram Miller acquired his early education at a small district school and later at a private school in Middlefield, before he entered the high school at Middletown at the age of fourteen. During this formative period he was fortunate in having excellent teachers who were an inspiration to him in his studies at all times. He showed his appreciation of their interest and attention by responding wholeheartedly, and, when he graduated, by receiving high honors and standing fourth in his class.

During that summer he decided to take up engineering, and with this purpose in view, he entered the Sheffield Scientific School at Yale University, New Haven, Conn., in September, 1898. He received honors for excellence in French and honorable mention in chemistry in his freshman year and as a senior was given two-year honors and honorable mention in sanitary engineering. He also belonged to the Society of Sigma Xi. He was graduated with the degree, then given to graduates in science, of Bachelor of Philosophy. From August, 1901, to July, 1940, Mr. Miller held the following positions:

August, 1901, to May, 1902—rodman and instrumentman for the Mexican International Railroad, in Mexico, and assistant to the resident engineer on construction of a drainage canal (until October, 1902).

October, 1902, to February, 1903—postgraduate work in civil engineering at Yale University.

February, 1903, to August, 1905—transitman and assistant division engineer with the Mexican International Railroad in charge of field parties on preliminary and location surveys, under the division engineer.

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[1] Memoir prepared by F. W. Andros, M. ASCE.

October, 1905, to March, 1906—engineer with A. William Sperry in New Haven, making general surveys, in the city and country.

March, 1906, to November, 1906—rodman and instrumentman for the Pennsylvania Tunnel and Terminal Company in New York, N. Y.

December, 1906, to December, 1907—division engineer with the Mexican International Railroad, in full charge of a locating party on preliminary and location surveys.

January, 1908, to February, 1909—locating engineer with the Bolivia Railway Company in Bolivia; in charge of construction as resident engineer for the La Paz-Oruro line (until December, 1908); then in charge of a locating party on preliminary and location surveys for the Oruro-Cochabamba line.

March, 1909, to June, 1909—traveled in South America and Europe (not engaged in engineering work from July to December).

January, 1910, to March, 1911—city engineer in Middletown and engaged in private practice.

March, 1911, to June, 1912—assistant to the superintendent of the New Haven Water Company, in Connecticut.

June, 1912, to August, 1913—draftsman and designer for the Alabama Power Company in Birmingham, the work involving computations and design of structures such as dams, retaining walls, and reinforced-concrete powerhouse substructure for the hydroelectric development at lock No. 12 on the Coosa River, and also power studies, stream flow, and other hydraulic problems.

October, 1913, to March, 1920—inspector and junior engineer in the U. S. Engineer Department. This included river and harbor surveys, inspection of material, drafting, and report on power developed on the Connecticut River and tributaries in the New London (Conn.) district (until March, 1915); 1915 to 1916—junior engineer in the office of the department engineer in Fort Sam Houston, Texas, in charge of the preparation of special maps of areas in northern Mexico and along the Mexican border; then junior engineer in New London on river and harbor surveys.

1916 to 1920—designer of fortifications and miscellaneous structures.

March, 1920, to March, 1921—resident engineer with Lockwood, Greene and Company; assistant engineer on construction of a new plant for the Hartford Rubber Works in Connecticut (until June, 1920); then in charge of construction of a new mill for the Albany Felt Company in New York, completed in March, 1921.

May, 1921, to May, 1922—draftsman and computer with the U. S. Engineer in New London.

June, 1922, to December, 1922—designer in the hydraulic division on design of hydroelectric developments with the Electric Bond and Share Company in New York.

January, 1923, to February, 1931—assistant hydraulic engineer in charge of design of hydroelectric developments.

April, 1931, to January, 1932—inspector of dredging (Passaic River, N. J.) with the U. S. Engineer Department, Second District, in New York (until

November, 1931); then inspector of borings on a proposed canal across New Jersey, from Raritan Bay to the Delaware River.

November, 1932, to August, 1934—computer with the U. S. Coast and Geodetic Survey in New York.

August, 1934, to August, 1935—designer on hydroelectric developments with the American Gas and Electric Company, in New York.

August, 1935, to January, 1936—junior engineer on flood control surveys in the State of New York; then with the U. S. Engineer Department as chief of party.

March, 1936, to April, 1937—inspector on dredging and flood control studies with U. S. Engineer Office, Second District, in New York; then junior engineer with the first district on flood control studies and design.

April, 1937, to July, 1940—design engineer on design of hydroelectric developments for the Puerto Rico Reconstruction Administration.

Mr. Miller enjoyed many of the usual sports, but was not active in athletics while at Yale. He was of a studious nature and a great reader; naturally, with an active mind and a retentive memory, he absorbed quickly the best there was in the many books, magazines, and papers he read.

He was brought up in the Congregational Church, but later he became a member of the Methodist Church and was regular and faithful in attending the services as well as taking part in the Sunday school, especially if there was a Bible class. His last class was a men's class at Montclair, N. J., led by the minister. He heartily enjoyed the singing on all such occasions.

He was honored and respected by his many friends, for his integrity of character and for his professional loyalty and rigid honesty. Those who were well acquainted with him mourn the loss of a firm friend and a most congenial companion. His death has deprived the profession of a man who represented the finest in engineers. He was a devoted husband and father—a gentleman of the old school, with a most pleasing personality, modest and unassuming at all times. From his Pilgrim ancestors he inherited the traits that are linked to them.

In foreign languages Mr. Miller had a Spanish license. He was a licensed professional engineer of the State of New York and member of the American Association of Engineers.

He was a member of the F. and A. M., Blue Lodge, and of the council in St. John's Lodge, Middletown, and also a life member of the Royal Arch Chapter in New Haven.

On September 20, 1910, he was married in New York City to Minnie Viola Sweet. He is survived by his widow; two sons, Waldo Benedict and Benjamin Thomas; a daughter, Betsey Chapman; two sisters, Sarah Bradley and Grace Eleanor (Mrs. Fred Harris); and a brother, Frederick C.

Mr. Miller was elected an Associate Member of the American Society of Civil Engineers on January 5, 1909, and a Member on January 14, 1929. He became a Life Member in January, 1944.

WILLIAM PRENTISS MORSE, M. ASCE[1]

Died February 21, 1946

William Prentiss Morse, the son of Charles and Lucy (Jennings) Morse, was born in Weston, Mass., on March 29, 1863.

After graduation from high school he studied civil engineering by himself in the evenings. At the age of eighteen he began working as rodman in the city engineer's office in Newton, Mass., and remained in that department until his retirement on July 1, 1935—more than fifty-four years later. During those fifty-four years the population of Newton increased from 17,000 to 66,300, and Mr. Morse took a significant part in the consequent development of the city.

As rodman, instrumentman, and assistant engineer, he worked on such projects as surveys for the establishment of the sewer and surface drainage systems, the grade crossing elimination of the Boston and Albany Railroad lines through the city, the building of Commonwealth Avenue, and, as resident engineer, on the extension of the Newton water supply. He was engaged in the inauguration of an adequate street numbering system for Newton's thirteen hundred streets and of the assessors' block system. He developed comprehensive indexing systems for Newton's record plans and field notes. In 1893 he became principal assistant engineer and on February 4, 1924, was appointed city engineer.

A considerable part of the city engineer's work is in the preparing of board of alderman orders with descriptions of land takings and examination of titles, in figuring and levying street and sewer assessments, and in making all necessary studies and estimates. In addition to this work, Mr. Morse attended practically every meeting of the board of aldermen for more than thirty-five years.

His varied experience gave Mr. Morse an unusually thorough knowledge of his city, a knowledge which prepared him for his later work on the planning board and with the zoning districts of Newton. When he retired as clerk of the planning board, the resolutions adopted stated:

> "Since the Planning Board assumed the functions of a Board of Survey, Mr. Morse has attended all but one of the 122 meetings. The records kept by him as Clerk are models of accuracy and completeness. Owing to his ability as an Engineer and his thorough knowledge of the City of Newton, Mr. Morse has rendered conspicuous service. The effective work performed by the Board since he became a member has been due in no small degree to his ability, fair mindedness, and understanding of the future needs of our City. In dealing with the technical details in connection with the Zoning of the City, the laying out of streets, parks and public improvements and in planning for the future growth and development of Newton, the Board has relied largely upon his knowledge and skill with unfailing satisfaction and gratitude."

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[1] Memoir prepared by Albert Morse, Office Engr., Newton, Mass.

Mr. Morse was parish clerk in St. Mary's Episcopal Church for thirty years, later becoming active in the nearer Second Congregational Church and in the West Newton Men's Club. He derived great pleasure from the meetings and excursions of the Society and of the Boston Engineering Society, becoming a life member of both organizations. After his retirement as city engineer he spent his time compiling a very complete genealogical record of his family.

Throughout his life Mr. Morse was invariably honest, courteous, and thoughtful of others. His friends meant much to him, and he always had a smile and a hearty handclasp for them.

His marriage on March 18, 1896, to Laura Elizabeth Snell of Natick, Mass., was followed by forty-seven years of devoted married life. They had no children.

Mr. Morse was elected a Member of the American Society of Civil Engineers on May 4, 1909.

CHARLES SABIN NICHOLS, M. ASCE[1]

DIED APRIL 20, 1946

Charles Sabin Nichols was born in Plainfield, Iowa, on November 28, 1883. His parents were the Rev. Albert Codon and Martha (Simmerman) Nichols.

He was graduated from the Osage (Iowa) High School in June, 1898, and attended a business college in Cedar Rapids, Iowa, until December of the same year. From December, 1898, until September, 1904, he was in the employ of the Burlington Cedar Rapids and Northern Railway Company, a branch of the Rock Island system, at Cedar Rapids; he was a stenographer and clerk, becoming chief clerk on April 1, 1902. By request, he was transferred to the Maintenance of Way Department as rodman, and soon afterward became instrumentman.

Mr. Nichols entered Iowa State College of Agriculture and Mechanic Arts, at Ames, in September, 1904, and was awarded the degrees of Bachelor of Civil Engineering in 1909 and Civil Engineer in 1914. As his entire living and educational expenses for these years were met by his own energy and effort, he naturally had a number of engagements.

From September, 1904, to June, 1905, he acted as stenographer and draftsman in the office of the Iowa State Highway Commission at Ames. The summer months of 1905 were spent as chief of party on the Meander Lake Bed Surveys for the Iowa State Executive Council. During the academic year, 1905–1906, his spare time was spent in designing culverts and short bridge spans, again for the Iowa State Highway Commission. From June, 1906, to October, 1907, which included one year's absence from college, he was instrumentman and assistant resident engineer for the Minneapolis and Saint Louis

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[1] Memoir prepared by Lynn Perry, M. ASCE, and Herbert J. Morrison, Assoc. M. ASCE.

Railway Company on fifty-three miles of heavy railroad construction in South Dakota, which included the terminal facilities on the Missouri River.

On re-entering college in October, 1907, his spare time, both during and between academic years, was occupied in the design and construction of reinforced concrete bridges for the Iowa State Highway Commission, a contact that he had maintained for two years after his college days.

On March 1, 1911, he was called to act as assistant to Anson Marston, Past-President and Hon. M. ASCE, then Dean of Iowa State College, and in September of the following year he became assistant director of the Iowa State Experiment Station at Ames, where he carried out extensive research, in addition to editing and publishing technical bulletins, largely on structural, hydraulic, and sanitation subjects. In February, 1916, he became professor of hydraulics and sanitary engineering, and directing engineer at the Experiment Station in these fields. Time brought promotions to Professor Nichols, and, in due course, he was engaged in a number of research projects for the Iowa State Board of Health.

In 1920 he was appointed a member of the Iowa Board of Engineering Examiners by the governor and served until he moved from the state. He became a member of the Iowa Zoning and Planning Commission in 1921, serving, similarly, while he was a resident of the state. He was honored, also, as president of the Iowa Engineering Society.

In June, 1925, Mr. Nichols came to Miami, Fla., commissioned to make a comprehensive plan for the collection, treatment, and disposal of sewage. In February, 1926, he was placed in charge of the Department of Sewers and in September of the same year, upon the creation of the Department of Streets and Sewers, was made director. The office of city engineer in Miami was created in August, 1927; Mr. Nichols became the first city engineer, also remaining as director of the Department of Streets and Sewers.

This placed under his direction all the design, construction, operation, and maintenance of bridges, and water supply, docks, harbors, canals, rivers, and municipal railway, in addition to supervision of the building and plumbing codes and their enforcement, and of the municipal surveys. From 1937 to 1939, he was director of public service, after which he again assumed the office of city engineer which he held until his death on April 20, 1946. The construction work which he directed for the City of Miami amounted to more than ten million dollars.

The Rev. Albert Codon Nichols was a native of Vermont, as alumnus of Colgate University in Hamilton, N. Y., and a Baptist minister. His training and his example instilled in his son those outstanding Christian virtues, so noticeable in men of culture and refinement, and endowed him with all the old fashioned virtues one would expect from a home atmosphere of work, scholarship, and service.

Mr. Nichols had no hobby or avocation; his entire professional life was devoted to his work which was in the public service. After more than twenty years of toil and sacrifice, the crowning project of his career, the construction of a complete sewerage system, including treament and disposal works, for the

City of Miami was authorized by a special election on April 16, 1946—just four days before his death.

On March 30, 1910, he was married to Lucretia Mellor of Ames. He is survived by his widow; two daughters, Norma (Mrs. Glenn Coon) of Mediapolis, Iowa, and Martha (Mrs. Earl Chism) of New London, Iowa; and two sons, Sabin A. Nichols of Ames and Marvin Nichols of Miami.

Mr. Nichols was active in Society affairs, having served as President of the Miami Section. He was an Elk, and a member of the Masonic Lodge at Ames, the Acacia fraternity, and the Central Baptist Church in Miami.

Mr. Nichols was elected an Associate Member of the American Society of Civil Engineers on June 18, 1918, and a Member on June 6, 1921.

WALTER PEARL, M. ASCE[1]

DIED FEBRUARY 10, 1946

Walter Pearl was born in Ableman, Sauk County, Wis., on August 30, 1866. He was the son of Stephen and Sarah (Cook) Pearl.

At the age of twenty Mr. Pearl began what was to become a life of activity in surveying and civil engineering that was to last until within a week of his death. His entrance into this field was as axman for the Montezuma Water Supply Company in Colorado. During the next eight years, he rapidly progressed through the positions of chainman, rodman, leveler, transitman, resident engineer, and assistant engineer. This initial period of his professional life was spent in Colorado and his engagements were in the fields of irrigation and water supply. His self-education was coincident with his field experiences.

At the age of twenty eight he participated in the Oklahoma land race, riding into the "strip" and through what was to be the City of Enid, Okla., on the top of a railroad boxcar—riding "through" because the train refused to stop until they arrived at the "railroad town" some miles farther on. He and his pal hastily hired a team and wagon to take them back to Enid but they arrived too late to stake out any of the good lots. They staked out on the best that was left, however, and immediately organized the firm of Pearl and Codding, Civil Engineers, and started a general engineering practice.

When the activity in Oklahoma subsided, Mr. Pearl returned to Colorado and, for the next seventeen years, was engaged in civil engineering projects, mostly in water supply and irrigation. Part of the time he had his private office, but he also held important employee positions such as assistant engineer, Denver Union Water Company; supervising engineer, United States Government; chief engineer, Fort Lyon Canal Company; chief engineer, Raton Water Works Company; chief engineer, Bar J. H. Cattle and Land Company; chief engineer, Sterling Promotion and Irrigation Company; assistant to general manager, Denver Reservoir Irrigation Company; and consulting engineer, Pioneer Construction Company.

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[1] Memoir prepared by Daniel Marsh, Chief Surveyor, Los Angeles County Flood Control Dist., Los Angeles, Calif.

Most of this work was in Colorado but some extended into Wyoming and New Mexico. Mr. Pearl acquired an intimate knowledge of that region of the Rocky Mountains, and he played an active part in the development of mining, irrigation, and railroads. His reminiscences of that period and that locality were most colorful and interesting.

In 1912 Mr. Pearl went to California and, except for a short time as assistant engineer for the United States Reclamation Service at Yakima, Wash., spent the remainder of his life there. His work during this time followed surveying. He was usually chief of party or assistant engineer and, in those capacities, served the California Highway Commission, Los Angeles County Surveyor, American Beet Sugar Company, Southwest Collon Company, and Southern California Edison Company. During the final eleven years of his busy life, he was an engineering aid of the Los Angeles County Flood Control District.

On August 10, 1910, in Denver, he was married to Loma M. Goodwin. He is survived by his widow; a daughter, Stella (Mrs. John Runge); and three grandchildren.

Walter Pearl's name among engineers is chiefly associated with the early development of Colorado and neighboring states. His was a long life of useful engineering work, spanning the period from the preliminary wood and timber structures to the permanent structures of steel and concrete. He made early studies of the duty of water and was author of a technical publication on that subject.

He was a member of Highlands Lodge No. 86, F. and A. M. (Denver Colo.); the Colorado Society of Engineers; and the Engineers and Architects Association of Southern California.

Mr. Pearl was elected a Member of the American Society of Civil Engineers on June 3, 1908. He became a Life Member in January, 1937.

WILLIAM DAVID PENCE, M. ASCE[1]

DIED JUNE 16, 1946

William David Pence, the son of David and Nancy (Hart) Pence, was born in Columbus, Ind., on November 26, 1865. He was educated at the University of Illinois in Urbana, from which he received the degree of Bachelor of Science in Civil Engineering in 1886, and the degree of Civil Engineer in 1895.

His first engineering experience was obtained as a field party assistant in municipal drainage and highway work in Columbus during summer vacations, from 1882 to 1885. From June, 1886, to September, 1887, he was employed as an assistant engineer on surveys and on the design and construction of yards and bridge masonry for the Gulf, Colorado and Santa Fe Railway. For three months, from October to December, 1887, he was roadmaster on the main line from Galveston to Millheim in Texas, and on the Houston branch of the same company, until he became assistant engineer of roadway, bridges, and buildings on 550 miles of this railroad.

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[1] Memoir prepared by W. S. Lacher, M. ASCE.

In September, 1892, Mr. Pence returned to the University of Illinois as an assistant in theoretical and applied mechanics and thereafter, for the following twenty-one years, devoted his time, interests, and energies to engineering education. He was advanced to assistant professor in 1898, but left the University of Illinois a year later to become professor of civil engineering at Purdue University in Lafayette, Ind. There he remained until 1906 when he joined the faculty of the University of Wisconsin in Madison, as professor of railway engineering. While at Madison, he also served as chief engineer on the Wisconsin Railway Commission and the Wisconsin Tax Commission.

Following the enactment of the federal valuation law in 1913 which led to the organization of the Bureau of Valuation by the Interstate Commerce Commission, he was appointed a member of the engineering board which assumed direct responsibility for the appraisal of the physical property of the railroads. The United States was divided by the bureau into five districts of approximately equal size, and Mr. Pence was placed in charge of the Central District, with headquarters in Chicago, Ill. He continued to serve in this capacity until 1921, when a curtailment of the valuation program led to the disbanding of the district organization. Thereafter he engaged in private engineering practice.

Mr. Pence's interests were by no means confined to railway engineering. His curiosity regarding the rather large number of failures of municipal water supply standpipes led to an intensive study into their causes, and the preparation of a monograph on "Standpipe Accidents and Failures in the United States" that appeared in 1885. In 1901 he was awarded the Octave Chanute Medal of the Western Society of Engineers for a paper on thermal expansion in concrete. In 1900 he collaborated with the late Milo S. Ketchum,[2] Hon. M. ASCE, in the preparation of a surveying manual that appeared in four editions. Also as his interests varied in later years, he became the author of papers on structural analysis, regulation of public utilities, terminal air rights over railway property, and other subjects. He retained a keen awareness of the world about him to the end, especially in the current developments in science and their applications in engineering. He was a member of the Indiana Engineering Society (president from 1903 to 1905), the Western Society of Engineers, the Society for the Promotion of Engineering Education, Alpha Tau Omega, Sigma Xi, and Tau Beta Pi. As a charter member of the American Railway Engineering Association, he assisted in the formulation of its publication policies, later serving as editor.

On December 31, 1888, Mr. Pence was married in Columbus, to Charlotte Gaston, who died in 1938. He is survived by three daughters, Ada (Mrs.

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[2] For memoir, see *Transactions*, ASCE, Vol. 102, 1937, p. 1487.

Sumner H. Slichter), Helen Charlotte (Mrs. A. B. J. Wace), and Esther Nancy (Mrs. A. F. Britton).

Mr. Pence was elected a Member of the American Society of Civil Engineers on October 4, 1899.

JAMES EDWARD PIRIE, M. ASCE[1]

DIED JULY 17, 1946

James Edward Pirie, the son of James A. and Scottie Eliza (Roddy) Pirie, was born on January 11, 1881, at Sayers, Tex. He was graduated from Texas Agricultural and Mechanical College at College Station, in 1904, receiving the degree of Bachelor of Science in Civil Engineering.

The professional career of Mr. Pirie extended over a period of forty-two years, and covered a varied type of endeavor. He began his lifework as a special student of operation with the Southern Pacific Railroad Company, in 1903, and worked through the construction, maintenance, and operations divisions as instrumentman, assistant engineer, roadmaster, and chief clerk to trainmaster by 1910.

Then followed a period of service as assistant city engineer of the City of Houston (Tex.), in 1911, after which he was construction superintendent for Street and Born, general contractors, in 1912 and 1913. In 1914 he was locating engineer for the proposed N. V. and R. G. Railroad and, in 1915 and 1916, he followed land surveying and was county engineer of Live Oak County in Texas. Mr. Pirie was employed as instrumentman and resident engineer on railway location and construction for the firm of Howe and Wise at Houston, from January 1, 1917, to May, 1919.

In 1919 Mr. Pirie began his service with the Texas Highway Department as division engineer and thereafter, with the exception of one year (1922–1923), during which time he was city engineer for Ballinger and Winters in Texas, he remained in the employ of this department. From 1923 to 1931, he served as county engineer and resident engineer in Runnels, Concho, Shackelford, and Throckmorton counties in west Texas; and, from 1931 until his death, he served as district engineer of district No. 1 in northeast Texas.

During Mr. Pirie's service with the Texas Highway Department he was actively engaged in the development of the organization and in the formulation of its policies, his wide experience and sound judgment making him a valuable consultant in all matters of public and professional policy. In the spring of 1946, Mr. Pirie received certificates of service and service pins from both the Texas Highway Department and the American Association of State Highway Officials, in recognition of meritorious public service for more than twenty-five years.

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[1] Memoir prepared by F. M. Davis, M. ASCE.

The professional and personal ideals of Mr. Pirie were of the highest quality and he enjoyed the respect and friendship of all who came in contact with him. One of his outstanding traits was his keen sense of fairness. He was possessed with a particularly intense sense of responsibility toward the younger professional engineer, and derived much pleasure from observing the younger man develop and progress in the profession. A well-balanced professional training was the reward of engineers fortunate enough to secure employment in his organization, and Mr. Pirie received his greatest satisfaction when professional or public recognition came to them.

On September 3, 1907, he was married to Jessie Erna Still, of Kemp, Tex. He is survived by his widow; one son, James E. Pirie, Jr.; and two daughters, Betsy and Eugenia Lee Pirie.

Mr. Pirie was elected a Member of the American Society of Civil Engineers on April 3, 1922.

PHILIP WALLIS PRICE, M. ASCE[1]

DIED JULY 22, 1946

Philip Wallis Price was born on October 6, 1879, in Pittsburgh, Pa. He was the eldest son of Charles Bohlen and Florence (Macrum) Price. He received his early education in the Park Institute of Allegheny (Pa.), now North Side, Pittsburgh, and in 1899 received the degree of Civil Engineer from Western University of Pennsylvania, in Pittsburgh, later the University of Pittsburgh.

His father was superintendent of the Allegheny Valley Railway Company, and later of the Allegheny Division of the Pennsylvania Railroad Company. From his father he acquired a love for railroading which lasted throughout his life. During vacations he worked as rodman on surveys for the Pennsylvania Railroad Company in western and central Pennsylvania.

In the ten years that followed his graduation he worked, successively, as rodman, transitman, assistant engineer, irrigation engineer (after passing the civil service examination), and draftsman. During this period he was engaged on five railroad surveys and revisions of alinement; three hydraulic projects, one of which was on the Pittsburgh filtration plant; two location and construction jobs on pipe lines and pumping stations; and one draftsman's job with the American Bridge Company. He was employed on this work in the states of Louisiana, Montana, Ohio, Oklahoma, Oregon, Pennsylvania, Texas, and Washington.

In January, 1909, he entered the Allegheny County engineer's office in Pittsburgh as assistant engineer. The work of the office was varied, with chief emphasis on design, construction, and maintenance of numerous highway bridges; Mr. Price's training and past experience admirably fitted him for this position. In 1924 when this office became the Bureau of Bridges in the Allegheny County Department of Public Works, he was made principal assistant engineer in the Division of Construction.

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[1] Memoir prepared by a Committee of the Pittsburgh Section consisting of Vernon R. Covell, Adelbert A. Henderson, and James C. Jordan, Members, ASCE.

During the next seventeen years Allegheny County entered into the largest highway bridge program in its entire history, involving an expenditure of forty million dollars. Its four rivers, rugged topography with many deep ravines, together with numerous minor streams, makes Allegheny County the bridge builders paradise—nearly every type of bridge being found here. In this program, Mr. Price made location surveys and study plans for major as well as minor bridges and had an intimate part in the field construction of the eleven bridges on the Ohio River Boulevard; and the seventeen major bridges over the rivers and deep ravines that include the Sixth, Seventh, Ninth, and Thirty-First street bridges over the Allegheny River; the Point, Liberty, South Tenth Street, and McKeesport-Duquesne bridges over the Monongahela River; the McKees Rocks and West End-North Side bridges over the Ohio River; the Boston Bridge over the Youghiogheny River; and the Duquesne viaduct and the George Westinghouse Bridge on the Lincoln Highway over deep ravines. He had direct charge of the field construction of the Highland Park Bridge over the Allegheny River.

Mr. Price joined the National Guard in 1910, and during the disturbance on the Mexican border in 1916, was called to active service with the First Field Artillery, United States Army. He was mustered out in December, 1916, but in May, 1917, he re-enlisted as an officer candidate in the U. S. Army and was sent to France with the Third Field Artillery in October. In May, 1919, he received his discharge and resumed his work in the county engineer's office. His interest in military affairs never flagged; he was an active member of the 324th Field Enginers, in which he held a reserve captain's commission until his death. One of his bitterest disappointments was to have been disqualified for active service during World War II, because of physical disabilities.

Mr. Price had strong convictions on the ethics involved in his decisions and actions, and he could not be swerved from what he believed was right. After his retirement in 1941, he devoted much of his time to the many civic organizations in which he was keenly interested.

He was a Life Member of the Society, and Vice-President of the Pittsburgh Section. His other memberships included the Society of American Military Engineers, the Engineers Society of Western Pennsylvania, and the Pittsburgh Chapter of Professional Engineers.

Mr. Price was a director of the Church Club (Episcopalian) of the Diocese of Pittsburgh. He took an active interest in the Civic Club of Allegheny County, the Historical Society of Western Pennsylvania, as well as Biard-Atwood Post of the Veterans of Foriegn Wars, and the East Liberty Post No. 5 of the American Legion.

During the early months of 1945, he was employed by the Allegheny County Conference on Community Development to conduct a survey of traffic conditions in Pittsburgh. This was his last formal employment.

In June, 1919, he was married to Edith Arensberg of Oakmont, Pa. He is survived by his widow; their son, Charles Philip Price; a brother Benjamin M. Price; and two sisters, Eleanor (Mrs. Royal Thomas) and Florence (Mrs. Charles H. Barton).

Mr. Price was elected a Junior of the American Society of Civil Engineers on April 2, 1901; an Associate Member on February 5, 1908; and a Member on October 10, 1927.

OTTO JOHN RAIFFEISEN, M. ASCE[1]

Died October 22, 1946

Otto John Raiffeisen was born in Sedalia, Mo., on October 21, 1887. He was the son of Charles H. and Katherine (Bahrenburg) Raiffeisen, whose parents came to the United States in 1849 and settled in what became the family home south of Sedalia. His preparatory education was obtained in the schools of Sedalia. He was graduated from the University of Missouri at Columbia in 1910 with the degree of Bachelor of Science in Civil Engineering.

From 1910 to 1924, Mr. Raiffeisen was engaged in the structural and mechanical design of steel buildings, bridges, and other steel structures for the Kansas City (Mo.) Structural Steel Company and the Fort Pitt Bridge Works in Canonsburg, Pa. In addition, he designed copper smelting and iron ore concentrating plants and equipment for the Arizona Copper Company in Clifton, Ariz., and for S. Shapiro, consulting mining engineer, in New York, N. Y., gaining broad experience in these fields of engineering. In June, 1924, he joined the engineering department of the Gulf Oil Corporation, then located in Port Arthur, Tex., as designer of steel structures, vessels, stills, and other construction associated with a major oil refinery. In 1929, when the Gulf engineering department moved to Pittsburgh, Pa., Mr. Raiffeisen was transferred there as supervisor of design of pressure and vacuum vessels, stills, heat exchange units, steel and concrete structures, wharves, etc.

He was a man of quiet, but strong, personality, who found his greatest enjoyment in his home, and in the company of his many friends. His end came suddenly from a heart attack, while driving home from his office with friends.

The record and ability of Mr. Raiffeisen stand as a tribute to his family, school, and profession, and the benefits of his personality, training, and experience, which were generously shared with his associates, will long be remembered by those who owe much to his friendly guidance. Mr. Raiffeisen was a former member of the American Institute of Mining and Metallurgical Engineers.

On March 18, 1916, in St. Joseph, Mo., he was married to Marguerite A. Jackson. He is survived by his widow; two sons, Charles Jackson and Otto John, Jr.; and one grandson.

Mr. Raiffeisen was elected a Member of the American Society of Civil Engineers on January 11, 1943.

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[1] Memoir prepared by J. G. Glasgow, M. ASCE.

MELVILLE EMERSON REED, M. ASCE[1]

Died October 12, 1946

Melville Emerson Reed, the son of William B. Reed and Amanda (Bunnell) Reed, was born in 1865 in Hastings, Minn. After preparation in the public schools of Hastings, he entered the University of Minnesota, at Minneapolis, in 1884, from which he received the degree of Bachelor of Science in Civil Engineering in 1888.

During his first year after college, he was employed as a transitman by the Saint Paul Water Works; and, on leaving that position in 1889, he served as an assistant engineer on the Pacific Extension of the Great Northern Railway. With the exception of one engagement during 1895 as assistant engineer with the United States Engineer Corps on river and harbor construction in Wisconsin, Mr. Reed remained with the Great Northern Railway Company until 1909, serving, successively, as assistant engineer on the construction of the Cavalier Branch in North Dakota and on the west portal of the Cascade Tunnel. From 1899 until 1902 he was employed as engineer in charge of location and construction of this tunnel. For the next seven years he was in charge of location and construction for the Great Northern Railway Company and the Northern Pacific Railway Company in Montana and Washington.

After serving as project engineer for the United States Reclamation Service in charge of the Huntley Project in Montana during 1909, Mr. Reed became chief engineer of the Oregon Electric Railway Company and, on completion of that engagement, maintained a consulting engineering office in Portland, Ore., from 1911 to 1922. During World War I, he was inspector for the Emergency Fleet Corporation on the construction of wooden hulls in the Portland area.

Following World War I until 1931, Mr. Reed was engaged on bridge construction—for the first seven years as principal assistant engineer and resident engineer under Gustav Lindenthal,[2] Hon. M. ASCE, on the location and construction of the Burnside, Sellwood, and Ross Island bridges and on the reconstruction of the Broadway Bridge, all of which are on the Willamette River at Portland. Mr. Reed then served for two years as bridge engineer for Multnomah County, Oregon, which position he left to become engineer examiner and inspector for the Public Works Administration in charge of some fourteen projects, including bridges, buildings, and water works in Oregon and Idaho.

When these projects were completed in 1939, Mr. Reed reopened his consulting engineering office in Portland; but, with the advent of World War II and the rapid development of shipbuilding on the west coast, he was employed by the Henry Kaiser Swan Island Yards as assistant engineer on the construction of 16,000–ton tankers, one hundred and forty seven of which were launched from the ways of this plant.

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[1] Memoir prepared by Ben S. Morrow, Assoc. M. ASCE.

[2] For memoir, see *Transactions*, ASCE, Vol. 105, 1940, p. 1790.

Mr. Reed was an engineer of the highest ethical and professional standards. Throughout his entire career it was his constant effort to advance the standing of the engineering profession and to bring to the general public a proper appreciation of the importance of the part played by the engineer in the development of the social and economic life of the United States. He was an active and interested member of the Portland Section of the Society and served as President in 1921. He was a registered professional engineer in Oregon.

In 1908 Mr. Reed was married to Maud Messner of St. Paul, Minn., who, with their daughter, Josephine (Mrs. Rudolph Eiswick), and a granddaughter, Susan Eiswick, survives him.

Mr. Reed was elected a Member of the American Society of Civil Engineers on March 6, 1901. He became a Life Member in January, 1936.

ROBERT LEMUEL SACKETT, M. ASCE[1]

Died October 6, 1946

Robert Lemuel Sackett, the son of Lemuel Miller and Emily Lucinda (Cole) Sackett, was born at Mount Clemens, Mich., on December 2, 1867. He was reared in a Quaker home, and throughout his life exhibited the gentleness and understanding, combined with unfailing principles of self-discipline and personal rectitude that arose from this environment.

He attended high school in Mount Clemens, and pursued his engineering education at the University of Michigan in Ann Arbor, where he was awarded the degree of Bachelor of Science in Civil Engineering in 1891. Later he was awarded the degrees of Civil Engineer (1896) and Doctor of Engineering (1937) by the same institution.

From 1891 to 1907, Mr. Sackett taught mathematics and astronomy at Earlham College in Richmond, Ind., and for the next nine years he was professor of sanitary and hydraulic engineering at Purdue University in Lafayette, Ind. In 1915 he assumed the deanship of engineering at The Pennsylvania State College at State College, where he remained until his retirement in 1937.

During the period of his academic career, he served also as a practicing consulting engineer in various capacities, in Michigan, Indiana, and Pennsylvania. After his retirement, Dean Sackett moved to New York, N. Y., where he continued his interest in professional and educational matters. He served the American Society of Mechanical Engineers as assistant to the secretary during World War II, and the Engineers' Council for Professional Development as chairman of the Committee on Student Selection and Guidance, and also in the capacity of an informal adviser on a great many matters incidental to the activities of this organization.

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[1] Memoir prepared by H. P. Hammond, M. ASCE.

In 1938 Dean Sackett was awarded the Lamme Medal, the chief honor of the Society for the Promotion of Engineering Education (American Society for Engineering Education). He was a fellow of the American Association for the Advancement of Science, and its vice-president during 1928 and 1940. Dean Sackett was a member of this society as well as of the American Society of Mechanical Engineers, and of the Society for the Promotion of Engineering Education, of which he was president in 1927. Among his fraternities and societies were Sigma Xi, Tau Beta Pi, Phi Kappa Phi, and Phi Gamma Delta.

Throughout his life, Dean Sackett was interested in the development of young people. This interest displayed itself not only in his work as a teacher and dean, but also in his many activities with the Engineers' Council for Professional Development. Although he was a man of strict self-discipline and insisted on the highest ethical standards among his students and colleagues, he was, nevertheless, a man of warm sympathies and understanding of others. To the end he remained active, serving on boards, committees, and commissions, and giving freely of his time and mental vigor to the numerous activities he assumed after his retirement. He was a leader who furnished, both by precept and example, a pattern which could be followed by his students, his colleagues, and his associates. He never outgrew a fresh interest in the problems of young men, and always remained youthful in his thinking.

Dean Sackett was a man of hobbies, not in the trivial sense, but in the development of his own character and interests. He took up painting, as an outgrowth of professional work in his early life, and developed this interest to the point of creditable amateur proficiency in water colors. He was attracted to sailboating at a fairly early age, and pursued that hobby with keen interest. While at Pennsylvania State College he was instrumental in developing instruction in fine arts. In a word, he was a man of broad interests and human understanding, who lived his life with the guiding principle, not only of the highest personal ideals, but of helping others to attain his own standards.

On July 22, 1896, he was married to Mary Lyon Coggeshall in Fountain City, Ind. He is survived by a son, Ralph L. Sackett, professor of economics at the University of Miami in Coral Gables, Fla.

Dean Sackett was elected an Associate Member of the American Society of Civil Engineers on February 6, 1907, and a Member on January 18, 1916. He became a Life Member in January, 1938.

HENRY JENNESS SAUNDERS, M. ASCE[1]

Died August 14, 1943

Henry Jenness Saunders, the son of Smith and Emma (Jenness) Saunders, was born in Mount Pleasant, Iowa, on January 14, 1880. He was the elder of two sons. When he was about six years old, the family moved to Council Bluffs, Iowa, where his father was engaged in the grocery business. Henry's business training began early. During his elementary school years he spent his spare time helping his father in the store, performing the chores which only a very energetic boy could accomplish. In addition to his duties around the store, he had a morning paper route, delivering papers to his home community on the family horse. One of his early ambitions was to have a college education, and most of his boyhood earnings went into a fund for that purpose.

When Henry entered high school he lacked weight and size to compete in athletics. However, this did not prevent him from trying. At the first field meet held after his admission to the Council Bluffs High School, he displayed his contempt for odds, and entered the mile run against a field of older and more experienced competitors. He finished last. The humiliation aroused his determination to prove he really could run. Through perseverance and careful training, before he finished high school, he ran the hundred-yard dash in ten and two-fifths seconds, and won the Iowa State Championship. His speed enabled him to play football, and he made an enviable reputation locally, playing halfback.

During his summer vacations he worked for the Union Pacific Railroad Company as chainman, rodman, and instrumentman. He continued working for the railroad for a year after he finished high school. He then enrolled at the University of Wisconsin, in Madison, and received the degree of Bachelor of Science in Civil Engineering in 1903.

The year after his graduation he was assistant engineer for the Union Pacific Railroad in charge of some fifteen miles of construction. From 1905 to 1912, he was employed by various organizations including the United States Reclamation Service, and the consulting engineering firms of Ford, Bacon and Davis of New York, N. Y., and the Arnold Company of Chicago, Ill. In 1910 he received the degree of Civil Engineer from the University of Wisconsin.

In 1912 he opened a consulting engineer's office at Valier, Mont., specializing in municipal work, irrigation problems, and highway bridge design. In July, 1914, he entered the service of the Interstate Commerce Commission, Division of Valuation, Pacific District, with offices in San Francisco, Calif. He was in charge of computing, tabulating, and assembling data inventoried in connection with the valuations of the railroads in that district. In 1921 the San Francisco office was discontinued, and Mr. Saunders was transferred to the Washington, D. C., office, as an assistant to the supervising engineer of the

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[1] Memoir prepared by F. W. Amadon, Assoc. M. ASCE, and F. J. Leonard, Associate Engr., Civil, Interstate Commerce Comm., Washington, D. C.

Bureau of Valuation. In that capacity he was in charge of the cost branch, and presented testimony at the valuation hearings supporting the bureau's methods and engineering reports. In all, he appeared in more than one hundred cases and did much pioneer work in preparing exhibits and testimony for the bureau. In 1928 he opened a consulting office in the Transportation Building in Washington, D. C.

The practice established by Henry J. Saunders was of unique character. He was technically a well-trained engineer, and possessed a comprehensive knowledge of accounting. He correlated the two subjects and applied them to the economic problems of transportation facilities and public utilities. Rates were his specialty—transportation rates charged by railroads, steamships, motor buses, trucking concerns, and air lines; also rates charged the consumers by utilities producing gas and electricity. He was often on the consumers' side and represented them before regulatory commissions and in federal and state courts.

Many of the cases in which he appeared included an appraisal or valuation of facilities. In 1938 he made an appraisal of the famous Gandy Bridge in Florida. In some of his important cases he represented Swift and Company, Armour and Company, the Chicago Union Stock Yards and Transit Company, the Koppers Company, the National Coal Association, the Anchor Coal Company, and various other coal operators' associations.

During the fifteen years of his practice in Washington, D. C., among his clients were: The War Department; Tennessee Valley Authority; and state commissions in Virginia, Florida, North Carolina, South Carolina, and the District of Columbia. He represented the Southern Governors' Rate Conference from its beginning to August 1, 1943.

A short time before his death he demonstrated that he had not lost the fighting spirit displayed during his high school days. He had been engaged to testify in an important case that came to hearing when he was at his home fatally ill. Several taxicabs of lawyers drove to his residence and took his testimony there. He weathered the ordeal and was in a jubilant mood at the close of the hearing.

Henry Saunders was a generous, tolerant man, and very considerate of his associates. Friends of his early youth were still intimate friends at the close of his life, despite the fact that he had traveled far from home. Those who knew him best loved him most, and to them his death marks the passing of a Christian gentleman.

He was a member of the Presbyterian Church, the Association of Interstate Commerce Commission Practitioners, and the National Aeronautical Association, and chairman of the Public Utilities and Transportation Committee of the Washington Board of Trade. He belonged to the University Club of Washington, D. C.; In-Com-Co Club of the Interstate Commerce Commission; Delta Tau Delta fraternity; and the Masons. He lectured on valuation subjects before the Washington Society of Engineers and the engineering students of the University of Wisconsin and George Washington University, in Washington, D. C., and was the author of a number of articles on the same subject appearing in technical magazines.

In January, 1904, he was married to Virginia Hayner in Madison. They had two children, Barbara (Mrs. Allen J. Hovermale), and Henry J. Saunders, Jr. He is survived by his widow and children.

Mr. Saunders was elected an Associate Member of the American Society of Civil Engineers on May 3, 1910, and a Member on May 5, 1915.

OLIVER JAY SCHIEBER, M. ASCE[1]

Died June 23, 1944

Oliver Jay Schieber was born in the City of Bucyrus, Crawford County, Ohio, on August 28, 1888, the son of John A. and Della May (Schell) Schieber.

Oliver Schieber's paternal ancestry stems from Wuerttemberg, Germany. His great-great-grandfather, Christian Schieber, participated in Napoleon's march into Russia, and during the disastrous retreat from Moscow in 1812–1813 he died from exposure at the age of about forty-two years. Johann Gottlieb Schieber (April 23, 1794–August 18, 1869), son of Christian, was born near Stuttgart, and in 1818 was married to Christina Magdalena Brose (April 22, 1799–April 25, 1889). In the spring of 1832 the Schieber and Brose families, with other related families, left their home communities and came by sailing vessel to the port of New York, N. Y., thence by the Hudson River to Albany, N. Y.; by the newly opened Erie Canal to Buffalo, N. Y.; by boat to Sandusky, Ohio; and thence overland to Bucyrus. The country thereabout was then virgin forest, and these early settlers had to clear the land of timber before real farming could begin. Johann Gottlieb brought his wife and several children with the party. His son, Gottlieb (September 11, 1823–June 18, 1865), Oliver's grandfather, was then a boy of nine, and Christina's father, Andreas Brose, was leader of the party.

Gottlieb Schieber was married, in Bucyrus, to Christina Heckenlaible (1827–1895), daughter of Rev. Johann Heckenlaible, a Lutheran minister, and Margaret Loeffler, who had come from Wuerttemberg in 1826.

Oliver's maternal ancestry stemmed from the Rhineland. Early in the eighteenth century Jacob Schell and his wife, with other Palatinates, fled from their homes and the oppression of the armies of Louis of France, to London, England. In 1709 Queen Anne assisted these Palatinates to reach the colonies in America. With them Jacob and his wife reached the port of New York early in 1710 and settled in Schoharie, N. Y. Their son, Peter Schell, with others, accepted in 1719 a grant of land from the governor of Pennsylvania, in the Tulpehocken region, which later became the heart of the Pennsylvania-Dutch area. Peter's son, also called Peter, had a son, Peter, III, born in 1760 in this region. Peter Schell, III, as a youngster pioneered in what was then a remote part of Pennsylvania—Franklin County—where he was married to a

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[1] Memoir prepared by Walter E. Jessup, M. ASCE, with the assistance of Homer J. Schieber, Los Angeles, Calif.

Scotch-Irish lass, Eleanore. They settled at Springfield Furnace, then in Huntingdon County. Peter, III, served in the concluding stages of the Revolutionary War, and in 1814 at the age of fifty four enlisted in the War of 1812. He died in Markleysburg, Pa., in 1849.

A son of Peter, III, and Eleanore, James Schell (January 14, 1799–December 6, 1854) and his wife, Ellen Benton, moved from Pennsylvania to Crawford County, in 1846, where they occupied a farm near Bucyrus. Ellen Benton's father, Jonathan Benton, a member of the six-foot, and more, tall King's Guards, came to America in 1781 from England with his bride to escape military prosecution because of his unauthorized marriage, and settled near Reading, Pa. There he enrolled in the Pennsylvania Militia, and so it was that he served in both the English and Colonial armies during the Revolutionary War.

A son of James and Ellen (Benton) Schell, Johnson Schell (February 7, 1826–April 17, 1890), was married to Maria Treusch (September 3, 1841–March 12, 1917), near Bucyrus. Maria's father, Adam Treusch, and her mother, Elizabeth Schmucker, had landed in Baltimore, Md., from Hesse-Darmstadt in 1830 to settle in York, Pa. In 1843 Adam Treusch and his family moved to Bucyrus, where his brother, Louis Ludwig Treusch, great-grandfather of Wendell Wilkie, had settled in 1835. Adam Treusch died in Bucyrus in 1846, and his widow, Elizabeth, was married, in 1847, to a neighbor, Daniel Barlitt (July, 1788–December 11, 1892), who achieved some measure of fame by living to the age of 104 years when he died just six days after the death of his wife. Daniel's grandfather, Jacob Barlitt, was a member of the personal bodyguard of George Washington, and as a boy, Daniel saw George Washington on several occasions. Although Oliver Schieber was but four years old when Daniel Barlitt died, it is recorded that Oliver sat on old Daniel's knee and heard him tell about having seen George Washington.

Oliver's father, John A. Schieber (August 31, 1855–February 7, 1914), son of Gottlieb Schieber and Christina Heckenlaible, was married to Della May Schell (who was born on May 6, 1862), the daughter of Johnson Schell and Maria Treusch, in Bucyrus. She had taught at a country school near Bucyrus. To regain his health John Schieber, with his wife and three children, moved to Los Angeles, Calif., in 1899. There he operated grocery stores in downtown Los Angeles, and Oliver was entered in the Olive Street Grammar School.

Oliver was graduated from the Los Angeles High School in the class of 1906, and then earned his way through college by working during vacations and while in school. Always interested in athletics, he taught physical culture and coached athletic teams in public schools. He was an accomplished gymnast, a good swimmer, and an ardent golfer.

In 1906 he entered the University of Southern California in Los Angeles, specializing in science, mathematics, and surveying. The school year of 1909–1910, he spent in the practical application of his preliminary college work by working in the engineering department of the Southern Pacific Railroad Company in Los Angeles as a rodman.

By now, being sure that civil engineering was to be his chosen field, Mr. Schieber entered the University of Wisconsin at Madison, in 1910, where he became an honor student during his two years of specialization in hydraulics, hydrology, and water-power engineering under the guidance of Daniel W. Mead, Past-President and Hon. M. ASCE. He was elected to Tau Beta Pi, and after completing the requirements for the five-year course leading to the degree of Civil Engineer was graduated in the spring of 1912.

Immediately on graduation, he obtained a position with the Knoxville Power Company, a subsidiary of the Aluminum Company of America, at Alcoa, Tenn., where he was engaged on topographic surveys for the then proposed Calderwood Dam.

Returning to his adopted state in 1913, Mr. Schieber was engaged by the Stone and Webster Corporation of Boston, Mass., as chief of party on the construction of two high-head hydroelectric power plants in the Sierra Nevada at Big Creek, Calif., which were being built for the Pacific Light and Power Corporation of Los Angeles. This was a high-pressure job; construction went on day and night, twenty-four hours a day, seven days a week. There, Oliver Schieber got his introduction and invaluable training for the construction engineering of western water and power projects that was to become his lifework. He was engaged on the field engineering for powerhouse and penstock construction, and the location and construction of access roads to the project. When the initial Big Creek project was completed in December, 1913, he left Stone and Webster and continued with the Pacific Light and Power Corporation until August, 1914.

The next three years, from 1915 to 1917, Mr. Schieber spent on the Salt River project of the United States Reclamation Service with headquarters in Phoenix, Ariz. There he had charge of the location and construction of irrigation canals and structures; he conducted the first silt survey of Roosevelt Reservoir and was resident engineer on the intricate work of concreting the spillways of Roosevelt Dam. He also had charge of the maintenance and operation of the canal system of the Salt River project and, ultimately, was named construction engineer of the project.

When the United States entered World War I, Mr. Schieber volunteered his services to the Corps of Engineers of the United States Army. He was accepted, and in January, 1918, entered active duty as a First Lieutenant of Engineers. After a training period, he was assigned to the 602d Engineers and embarked for France. In France, Lieutenant Schieber was engaged on the construction of roads, bridges, ammunition dumps, and other vital army construction work. He participated in two major engagements, Saint-Mihiel and the Meuse-Argonne. After the armistice on November 11, 1918, his unit was assigned to the Army of Occupation of Germany. While there, Lieutenant Schieber, who commanded Company C, was recommended for promotion to Captain.

In July, 1919, after his release from army duty, the U. S. Reclamation Service again sought his experienced services. This time it was on the King Hill project in Idaho, where, for the next year, he was assistant engineer on the design of irrigation structures, and then was appointed division engineer of construction of the project.

In 1917 the Big Creek hydroelectric development on which Oliver Schieber had been engaged previously, together with all its other properties, was acquired from the Pacific Light and Power Corporation by the Southern California Edison Company which served Los Angeles and other Southern California communities with electricity for light and power. The company had begun an extensive development of the power available in 5,500 ft of the fall of the waters of the San Joaquin River and its tributary, Big Creek.

In August, 1920, Oliver Schieber accepted a position on this construction in the high Sierra, with headquarters at Big Creek. Here again he demonstrated his willingness to accept responsibility, his dependability, and his capacity for hard work. For the next four and a half years he was engaged, successively, as production engineer, estimating engineer, and assistant resident engineer of the project. The major work after 1917 involved driving twenty-three miles of hard-rock tunnel; the building of two major concrete dams—Shaver Lake at El. 5370, and Florence Lake at El. 7229; the construction of three major hydroelectric power stations with penstocks and turbines for heads of 730, 830, and 2,418 ft, respectively; and the installation of additional power generating equipment in two other previously built power stations. For the recreation of the relatively isolated construction staff of the company at Big Creek, Oliver Schieber initiated, designed, and built a nine-hole golf course on a mountain meadow near Shaver Lake, which is now under the water of the reservoir.

In March, 1925, he was appointed technical assistant to E. R. Davis, manager of construction of the company, and was transferred to the Los Angeles headquarters. There he was engaged on the preparation of estimates and budgets, making layouts for construction plants, and arranging for the purchase of materials and equipment for the construction of the company's developments, which included tunnels, pipe lines, penstocks, powerhouses, transmission lines, and transformer stations.

By 1931, the City of Los Angeles had joined with other communities in Southern California to form the Metropolitan Water District for the purpose of constructing an aqueduct to bring sixteen hundred cubic feet per second of water from the Colorado River to Southern California. The district had selected the late Frank E. Weymouth,[2] Hon. M. ASCE, as its chief engineer. Oliver Schieber's experience was needed on this work, and he accepted an appointment as senior engineer with the district. The 233-mile route of the aqueduct is in mountainous and desert country, and most of it was inaccessible by roads. Since the construction was to be by contract, to avoid duplication of effort by contractors and to remove some of the contingencies due to inaccessibility, and other bidding hazards, the district built district camps, housing facilities, field headquarters, and roads, and drilled water wells along the route for the use of the contractors who were successful bidders on the various sections of the work. Oliver Schieber designed the district camps, field headquarters, and the water-supply facilities, and then superintended their construction.

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[2] For memoir, see *Transactions*, ASCE, Vol. 107, 1942, p. 1712.

After contract construction got under way, and for the succeeding three years, he was resident engineer in charge of construction on division No. 6 of the aqueduct, which included eight and a half miles of fifteen-foot-diameter hard-rock tunnel; six and a half miles of fifteen-foot concrete conduit; and nine miles of twelve-foot-diameter reinforced-concrete siphon pipe. On completion of division No. 6, he was moved to division No. 4, as division engineer in charge of engineering, where for the next eight months he handled the lining of twelve miles of sixteen-foot-diameter tunnel and the construction of ten twelve-foot-diameter reinforced-concrete inverted siphons, having an aggregate length of two miles. He was then made division engineer in charge of construction of division No. 4. This included the lining and pressure grouting of eight miles of tunnel by force account, the contract construction of three and one-fourth miles of sixteen-foot-diameter concrete conduit, and the laying of a half mile of reinforced-concrete inverted siphon pipe. By the end of 1938, when this two hundred million dollar Colorado River aqueduct was nearing completion, Oliver Schieber severed his connection with the Metropolitan Water District.

He became construction superintendent for the Federal Works Projects Administration in Southern California. During 1939 and 1940 he had charge of fifteen hundred men, engaged on the construction of two miles of twelve-foot-diameter storm drain tunnel through the Los Angeles business district, and one and a half miles of the Arroyo Seco storm channel, which has a capacity of twenty-five thousand cubic feet per second. This channel is an integral part of the Arroyo Seco Parkway development, connecting Los Angeles and Pasadena, Calif.

In May, 1940, he returned to the Southern California Edison Company, Limited, where, until his death, he directed and supervised a group of engineers engaged in various investigations and economic analyses. Among the projects on which he was engaged was the accumulation of data for presentation to a federal appraisal board in the determination of damages to be paid by the United States Government to the Edison Securities Company, a subsidiary of the Southern California Edison Company, for the acquisition of water rights appurtenant to the Herminghaus Ranch lands in central California. He made a comprehensive survey and study of the hydrology of the entire Big Creek hydroelectric system, including a determination of the feasibility of additional reservoir capacity at the headwaters of the San Joaquin River, and a determination of the effect of storage by the Southern California Edison Company of the waters of the San Joaquin River upon the operations of Friant Reservoir of the central California project then about to be built by the U. S. Bureau of Reclamation.

Mr. Schieber prepared cost data in connection with Federal Power Commission proceedings concerning the cost of Federal Projects 67 and 120 of the Big Creek—San Joaquin River hydroelectric system. He prepared plans, specifications, and cost estimates for major maintenance work on several of the company's dams and hydraulic structures. These included repairs to the Ward Tunnel, the thirteen-mile hard-rock tunnel on the construction of which he had been engaged from 1920 to 1925; the upstream backfill of Florence Lake Dam and two of the Huntington Lake dams; and repairs to the top of Shaver Lake Dam. He worked on several projects in connection with the company's Long Beach steam plant, including the general subsidence of the plant occurring after the construction of the graving dock at the Terminal Island Naval Base; and the replacement and valuation for compensation purposes of two steam generator units which were seized by the United States for installation in Russia.

On January 21, 1924, Mr. Schieber was married to Mildred Gray Bulfinch of Los Angeles who had been admitted to the practice of law in the State of California. After their marriage they lived at Big Creek until he was transferred to Los Angeles in 1925. In 1928 they built a home at 1751 Windsor Road in San Marino, Calif., which continued to be their home regardless of where Mr. Schieber's professional work took him. Mr. Schieber died on June 23, 1944, at the Huntington Hospital, Pasadena, after a brief illness. His family and his home were a source of pride and happiness to him. He is survived by his widow; his daughter, Margery G.; and a son, Norman B.—of San Marino; and his mother, Mrs. John A. Schieber; his brother, Homer J.; and his sister, Ella—all of whom live in Los Angeles.

In his youthful days Mr. Schieber took pleasure in mountain climbing, and in his more mature years found recreation in exploring by motor with his family the many remote and inaccessible regions of the West.

Oliver Schieber's interest in his profession never lagged. He was a registered civil engineer in the State of California. In Society affairs, he was very active, serving for many years on the Local Qualification Committee of the Los Angeles Section, and contributing his time and talent to preparing papers for Society and Section meetings.

His solid, substantial, character, based on straight thinking and clean living, was outstanding. Among his associates in the several organizations that claimed his services, Oliver Schieber was loved as few men are loved. His tireless energy and constant striving for the worthwhile things of life were an inspiration to them. He bore no men malice, nor they him. His passing has meant the loss of a sincere friend to all who knew him.

Mr. Schieber was elected an Associate Member of the American Society of Civil Engineers on April 25, 1921, and a Member on July 31, 1933.

SYDNEY ABRAM SHUBIN, M. ASCE[1]

DIED SEPTEMBER 18, 1946

Sydney Abram Shubin was born on May 14, 1890, in New York, N. Y. He was the son of Jacob and Anna (Sobel) Shubin. After preparation in the public grade and high schools of New York, he entered Cooper Union Institute, also in New York, in 1911, from which he received the degree of Bachelor of Science in Civil Engineering in 1915 and the degree of Civil Engineer in 1926.

For two years after graduation he did drafting, designing, and estimating work for the Buckingham Steel Company in Brooklyn, N. Y., the Lackawanna Steel Company in Buffalo, N. Y., and the Bollinger-Andrews Company in Verona, Pa. This was followed by four years of work (from 1917 to 1921) with the Heyl and Patterson Company in Pittsburgh, Pa., on checking, designing, and estimating work for coal tipples, conveyors, and bunkers. This period also included two six-month engagements with the McClintic-Marshall Company in Pittsburgh, on design and checking of details and connections on long-span highway and railroad bridges, and on buildings. From 1922 to 1924, he was employed by the J. Harold Rapp Company in Pittsburgh, engaged in laying out and checking work in industrial plants, office buildings, and theaters, which included a theater and market house for the City of Memphis, Tenn.

This ten years of training and experience, after graduation, in the design and related work connected with many kinds of steel structures, served as an excellent foundation for the work in which Mr. Shubin spent the remainder of his life. In April, 1924, he was employed by the County of Allegheny, Pennsylvania, in its newly organized Department of Public Works, where he became assistant design engineer in the Division of Design, Bureau of Bridges. From 1924 to 1932, the Allegheny County Department of Public Works carried out a large program, including the design and construction of major and minor bridges, the cost of which was provided by a forty million dollar bond issue for bridges alone, as well as a similarly large expenditure for road construction. Mr. Shubin shared with his associates an intimate and responsible part in this large program, during which were built such structures as the McKees Rocks and the West End-North Side bridges over the Ohio River; the Point, Liberty, South Tenth Street, McKeesport-Duquesne, and Glassport-Clairton bridges over the Monongahela River; the Sixth, Seventh, Ninth, and Thirty-First street bridges over the Allegheny River; numerous ravine bridges on the Ohio River and the Allegheny River boulevards; and the monumental George Westinghouse Bridge on U. S. route No. 30 at East Pittsburgh, Pa., as well as many minor structures.

Mr. Shubin's particular responsibility in connection with this program was the design of the beautiful South Tenth Street (Monongahela River) crossing, a parallel wire cable suspension bridge, with gravity anchorages and two river piers, having a 725-ft main span and side spans of 275 ft, completed at a cost of nearly two million dollars.

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[1] Memoir prepared by a Committee of the Pittsburgh Section consisting of Harry G. Appel, Clarke K. Harvey, and George S. Richardson, Members ASCE, and Ernest Tessitor, Assoc. M. ASCE.

Design and construction having dwindled because of the depression, Mr. Shubin was engaged from 1932 to 1934 in bridge maintenance work, in charge of the county's forty-five major bridges in that period. He again resumed design work for Allegheny County in 1934, becoming bridge design engineer in 1936 in the department, later known as the County Department of Works, and was head of bridge and structural design until his death.

In 1934 Mr. Shubin had charge of the design of extensive improvements in North Park, Allegheny County, consisting of a dam and spillway for a large artificial lake, a boathouse, a swimming pool, and a bathhouse.

Mr. Shubin supervised the design of new structures at the County Airport in 1936, including a hangar which had the largest automatically operated doors theretofore constructed, with a clear opening of forty by one hundred and fifty feet. Later, he was responsible for the design of the Highland Park Bridge over the Allegheny River, the Dookers Hollow Bridge over a deep ravine, the clover leaf at the intersection of state route No. 51 and the Lebanon Church Road, and the elevated roadways and bridges of the Water Street and Duquesne Way improvements along the banks of the Monongahela and Allegheny rivers in downtown Pittsburgh.

Two other structures which he conceived remained to be constructed after his death—the Babcock Boulevard and the Nelson Run Road traffic interchanges on the new McKnight Road, under construction. At his death, plans were practically completed also for the four million dollar Dravosburg Bridge over the Monongahela River, and others were well under way for the Rankin Bridge over the same stream, both included in the county's thirty-four million dollar bridge, road, and airport program.

In addition to his other duties, Mr. Shubin frequently appeared on behalf of the county in public utility commission hearings.

Mr. Shubin was for twenty-three years a member of the Rodef Shalom Congregation. He was a man of high ideals and integrity and enjoyed the respect of his many friends, associates, and acquaintances. His character is clearly shown by the following quotation summarizing his record, from a paper he prepared shortly before his death:

> "I thank God for having lived in this great land of freedom and opportunity. By his Grace he has inscribed me in the book of life as a builder of monuments for a future world, which I hope will be a world of peace and tranquility, and with human dignity."

He was a registered professional engineer in Pennsylvania and a charter member of the Pittsburgh Chapter of the Pennsylvania Society of Professional Engineers.

In 1916 Mr. Shubin was married to Anna Shubin, to whom two children were born, Murray J. and Allan M. Mrs. Shubin and their two sons survive.

Mr. Shubin was elected an Associate Member of the American Society of Civil Engineers on June 7, 1926, and a Member on June 27, 1932.

ALFRED VARLEY SIMS, M. ASCE[1]

DIED JANUARY 21, 1944

Alfred Varley Sims was born in Port Hope, Ont., Canada, on September 21, 1864. His father, Alfred William Sims, an American civil engineer, was on a professional mission, accompanied by his wife Adelaide (Sowden) Sims.

Some years later the family returned to the United States and settled temporarily in Orbisonia, Pa., where young Alfred attended a private school. As is the usual lot of the civil engineer, Mr. Sims was forced by his professional activities to move from place to place, so that the record of his son's early education is somewhat vague.

There is no doubt that his father's vocation influenced Alfred in the profession he was to embrace, and inured him to the almost nomadic existence that he maintained throughout the greater part of his life. However, his father evidently took a hand in his son's technical education; at seventeen the young man began his engineering career in the field, making surveys for the Rock Hill Iron and Coal Company and filling various subordinate positions with the Pennsylvania Railroad Company from 1881 to 1885.

That these five years of actual engineering work were profitable is shown by the fact that in 1886 he entered the University of Pennsylvania in Philadelphia, as a junior, and was graduated two years later with the class of 1888. It is indicative of his character that during his university days he was active in extracurricular pursuits; he played football and baseball in both his junior and senior years, and was a member of various committees concerned with the social affairs of his class. He was popular with his associates, being ready and willing to do his share in any common enterprise. These traits he bore throughout his life and, by them, earned quickly the respect and affection of his fellows in many parts of the world.

For the seven years following graduation his energies were directed to various railroad construction jobs. He was chief engineer for the New England Terminal Company (1888–1889), the Atlantic and Danville Railway Company (1898–1890), the Danville and East Tennessee Railroad Company, and the Atlantic Coast and North Western Railroad Company (1890–1892).

During his association with the Atlantic Coast and North Western Railroad Company he met Ruth Hairston to whom he was married on June 10, 1891. She was of an old Virginia family, and there were murmured protests to her marriage to a "damn Yankee." They were soon quieted when it became apparent that Mr. Sims was in complete sympathy with the problems and viewpoint of the South, and embraced their Democratic faith wholeheartedly.

Mrs. Sims had inherited considerable plantation acreage, in Virginia and North Carolina, which was to afford Mr. Sims an intense and constructive interest in farming that continued through his lifetime.

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[1] Memoir prepared by Alden Arthur Knipe, Esq., New York, N.Y.

During the year 1892, Mr. Sims was chief engineer, assigned to a confidential survey of Death Valley, Calif., undertaken by the Utah, Nevada and California Railroad Company. This assignment nearly ended his career. When his desert guide admitted they were lost, it took his skilful combination of keen observation, animal lore, instinct, and practical knowledge of engineering to bring them back to civilization before their water and supplies were exhausted. Because of the secret nature of the expedition Mrs. Sims, who accompanied her husband as far as a desert border hotel, had to acquiesce in the mild fiction that her husband was in the desert for his health, and to receive the heartily expressed sympathy of newly made friends.

During the year 1894 Mr. and Mrs. Sims were in the ancestral home at Berry Hill, Pittsylvania County, Virginia, where Mr. Sims' inquiring mind immediately centered on the farming and social problems of the region. It was tobacco country where the tenants lived a hand-to-mouth existence, and the solution of their problems demanded a radical change in methods. Rehabilitation of the land, diversity of crops, and a more modern system of cultivation were the essential changes Mr. Sims inaugurated. He devised new ways of curing, handling, and storing tobacco, while hampered by the resistance of his tenants who "had always done it that way" and saw no reason to change even though their returns grew less year by year. However, he made headway against their dyed-in-the-wool prejudices, and his sincere desire to promote the welfare of the farmers, in time, dissolved the fogs of misunderstanding that existed between landlord and tenant and that had grown into an antagonistic tradition throughout the region. The "sharecropper" policy Mr. Sims found bad for the land, demoralizing for the individual, and unprofitable for all concerned. Throughout the rest of his life he combated these conditions with tireless energy.

In 1895 Mr. Sims was called to the chair of engineering at the State University of Iowa at Iowa City, where, until 1904, he devoted his attention to teaching. By no means was he the typical professor, a title he disliked, because to his thinking the formalities incident to a pedagogical status placed a barrier between himself and his students. It was his feeling that he and his pupils were as one in the investigation of the engineering problems presented in the curriculum. Of necessity his classes were scheduled, but there was never any limit set for discussion and consultation. Mr. Sims was always available to his scholars, who were quick to embrace these opportunities for informal instruction. In spite of his dislike for the rigidity inherent in the life of a university professor Mr. Sims found a great deal of pleasure in the years spent in Iowa.

During the nine years at the university he did considerable research in cement, concrete, and sand that resulted in the inventions of mechanisms and methods for uniform testing of cement, and the determination of laws of forces and resistances in granular substances. He harnessed them for punching clean holes in metal, building flat arches of sand without cement, and constructing paper and sand columns that exceeded concrete in strength. He was the author of the following pamphlets: "The Engineer and the Death

Rate," "Discussion on Concrete," and "Computation of Structural Stresses."

After nine years of teaching, mitigated by vacation trips to the farms in Virginia, Mr. Sims abandoned the title of professor and returned to active engineering work. In 1904 he was appointed president, chief engineer, and general manager of the Cuba Eastern Railroad Company, the Northwestern Cuba Railroad Company, the Cuba Eastern Terminal Company, and eight other allied enterprises. He was stationed in Guantanamo, Cuba, where for four years he struggled with a hostile Cuban government, a revolutionary junta, and the constant, rather dubious, intrigues of rival railroad competition. After an extremely busy and anxious time, he was on the way to success when the panic of 1907–1908 closed the Knickerbocker Trust Company in New York, N. Y., which was the financial support of the extensive enterprise. It was a discouraging experience because Mr. Sims was already building new railroad lines which would have been profitable and expanding the other affiliated organizations into money-making undertakings. The companies, however, were forced into liquidation, and Mr. Sims returned to the United States, rather worn in health and keenly disappointed in the realization that his hard work had gone for nothing through no fault of his management in Cuba, but because of a lax banking system in New York.

During the rest of his life it was characteristic of him to cherish the remembrance of his years in Cuba. Shortly after returning to the United States he received a handsome watch with the inscription, "To Mr. A .V. Sims from the boys of The Cuba Eastern in recognition of many a square deal, 1908." He retired for a time to Virginia to regain his health and to carry out many plans for the betterment of his tenants and the improvement of his lands.

For a year, 1913–1914, he was in London, England, promoting a hardwood project in Cuba. World War I, however, put an end to these negotiations and he returned to New York. Here he established himself as a consulting engineer and a representative of The James Boyd Company in the sale of four-wheel-drive trucks to the allied armies.

During this period he became aware that the valves obtainable for reciprocating pumps left much to be desired, both functionally and materially. Characteristically, he set to work to invent a better valve; during lulls in his engineering consultations, he produced the "Sims Inclined Port Rotating Pump Valve" which incorporates many features that solve most of the problems inherent in such a mechanism. He was owner and president of the Sims Pump Valve Company of New York. For the remainder of his life, Mr. Sims, in partnership with his son Alfred, devoted much of his time to the development and refinement of this invention. Its wide distribution and general acceptance as power plant equipment in both the marine and stationary fields attest to a correct harnessing of nature's laws into a mechanism for the benefit of man.

The need for air raid shelters during World War II resulted in the request that Mr. Sims make further experiments with granular substances. His early patent of 1905 on sand columns, arches, and floors was supplemented by another that was not issued until after his death. His failing health prevented his following through with his son on their application for the construction of bombproof shelters. However, he had the gratification of seeing the interest of military and civil engineers in the model he constructed in 1941. United States patents were issued to Alfred Varley Sims covering the following: Columns, arches, and floors of sand; ticket holders; processes and apparatus for manufacturing articles from paper pulp; safety sea suit; pump valves (two individual patents); hay drying and the like; bilge pump mechanisms; mixing fluids of different temperatures and densities; and granular supports.

Mr. Sims was a large man, six feet two inches tall, and broad in proportion with a big man's wide, tolerant outlook upon life. He was forever inventing gadgets for the fun of it, or delving into obscure problems to satisfy an insatiable curiosity—to find out what made the physical "wheels go round." His hobby was playing with nature's laws and harnessing them for the use of mankind; his recreation was his family and its interests. He had an innate fondness for horses, dogs, and all nature's children. He lent a ready and sympathetic ear to the everyday troubles of his associates giving advice when it was asked, but tempering its import with a fine sense of humor. His friends, and there were hosts of them, deeply regret his passing; but they are consoled by the fact that he lived a long, useful, and successful life that was inspired by high ethical standards and was rewarded by the gratification that comes with a multitude of tasks well done.

In addition to membership in this Society Mr. Sims belonged to the honor society of Sigma Xi and he was a member of the Episcopalian Church. He is survived by a son, Alfred W., and three daughters, Elsie H. (Mrs. F. Q. Rickards), Ruth H. (Mrs. George Schaeffer), and Adelaide Varley (Mrs. Paul M. Zorn).

Mr. Sims was elected a Member of the American Society of Civil Engineers on March 4, 1896.

WALTER LYNES SMITH, M. ASCE[1]

Died January 29, 1947

Walter Lynes Smith was born at Middleburg in Schoharie County, N. Y., on July 15, 1876. He was graduated from the public school in Chicopee, Mass., in June, 1896, and from Union College in Schenectady, N. Y., on June 27, 1900, with the degree of Bachelor of Science in Civil Engineering.

After graduation Mr. Smith entered the employ of the American Bridge Company as draftsman and, after one year, he left to work for the New York, New Haven and Hartford Railroad Company in the same capacity.

In January, 1906, he joined the Toledo-Massillon Bridge Company as assistant chief engineer and in September, 1908, was made contracting engineer of the Pennsylvania Bridge Company.

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[1] Memoir prepared by Hurieosco Austill, M. ASCE.

His next employment, beginning in September, 1909, was with the Pennsylvania Railroad Company in the office of the engineer of bridges and buildings on the lines west of Pittsburgh, Pa., and, in May, 1919, he was appointed assistant engineer of bridges and buildings. In the latter capacity he was in special charge of design and construction of two lift bridges (about $1,400,000) and of a bridge over the Maumee River at Toledo, Ohio (about $400,000).

While working for the Pennsylvania Railroad Company Mr. Smith was associated with that eminent bridge engineer, J. C. Bland,[2] who was serving as consulting engineer for the Terminal Railroad Association of St. Louis, Mo. There was important work to be done on the terminal's two bridges over the Mississippi River, and on March 1, 1920, Mr. Smith was appointed bridge engineer of the Terminal Railroad Association of St. Louis, which position he retained with distinction until his voluntary retirement on August 30, 1941.

He was particularly fond of music and enjoyed his retirement by devoting his time to the higher things of life and especially to the study of foreign languages. His rather sudden death occurred after a collapse. He was a bachelor and had no near kin.

Mr. Smith was elected an Associate Member of the American Society of Civil Engineers on February 4, 1914, and a Member on May 8, 1922. He became a Life Member in January, 1947.

RUSSELL ELSTNER SNOWDEN, M. ASCE[1]

Died December 5, 1946

Russell Elstner Snowden was born on December 22, 1880, at Snowden, Currituck County, N. C. He was the son of Milton Hume Snowden and Rosa (Halstead) Snowden, of Snowden. He traced his ancestry to Thomas Snowden, whose name appears in colonial records in 1682 as clerk of court of Albemarle County. This county has since been subdivided into Currituck, Camden, Pasquotank, Perquimans, Chowan, and Gates counties, occupying the extreme northeastern corner of the state. Albemarle disappeared as a county, but the name persists in Albemarle Sound, which washed its southern shores.

There are a Snowden crest and a coat of arms, which in all probability were honors borne by more remote ancestors. Some of the Snowdens dwelt in Yorkshire, England, and many families from Yorkshire were early settlers in America.

Russell attended private school in Snowden, until he was fourteen years old; and for the next four years he attended Elizabeth City Academy in Elizabeth City, N. C. In 1898 he entered North Carolina State College (then known as North Carolina Agricultural and Mechanical College) at

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[2] For memoir, see *Transactions*, ASCE, Vol. 95, 1931, p. 1453.

[1] Memoir prepared by O. B. Bestor, M. ASCE.

Raleigh. He not only stood well in his classes, but was active and proficient in military affairs, becoming the highest student officer with the rank of major of a battalion. He was graduated on May 28, 1902, with the degree of Bachelor of Engineering.

During the succeeding eight years Mr. Snowden engaged in railroad engineering. From November, 1902, to February, 1903, he was rodman on the construction of the Oxford and Coast Line Railway, a branch of the Seaboard Air Line Railway. From March, 1903, to November, 1905, he was, successively, rodman and transitman on preliminary and location surveys, then assistant resident engineer on heavy railroad construction, for the Coal and Coke Railway on the Clover Fork Residency in Lewis and Braxton counties, West Virginia. The work included a tunnel about 1,000 feet long through unstable rock and shale requiring timber lining. It was driven from both ends, the drifts meeting near the center of the tunnel. Mr. Snowden repeatedly checked the alinement and the stationing, using vertical and slope measurements to check those made horizontally. When the headings met, the closure was practically perfect, both in alinement and measurement. The approach cuts to the portals were made through material that became sliding when wet. Unusual foresight was shown by removing threatening material outside the normal cross sections, and establishing an effective system of ditching to prevent future slides.

Upon completion of the work in West Virginia, Mr. Snowden returned to North Carolina, and from February to July, 1906, engaged in preliminary surveys as field engineer for the Raleigh and Charleston Railroad Company. From July, 1906, to September, 1907, he was assistant engineer with the Southern Railway Company—for nine months in charge of a locating party revising the main line between Washington, D. C., and Atlanta, Ga., and for the succeeding six months on twelve miles of heavy construction, using ten steam shovels, on the revision and double tracking of the main line from Lynchburg to Danville, Va.

During the last quarter of 1907 he was special resident engineer for the Norfolk Southern Railroad Company, on construction from Washington, N. C., to Raleigh. Near the end of the following year he was for about four months field engineer for the North and South Carolina Railroad Company, making preliminary and location surveys. From April, 1909, to April, 1910, he performed similar work as topographer (actually transitman) and as transitman (actually chief of party) for the Seaboard Air Line Railway Company. From April to August, 1910, he was assistant locating engineer for the Coal and Coke Railway, making a survey to extend that railway from Elkins, W. Va., crossing the Morgantown and Kingwood Railroad at Rowlesburg, W. Va., to connect with the New York Central Railroad at Cheat Haven, Pa. From September, 1910, to the end of the year, he was assistant to the chief engineer of the Atlanta and Northeastern Railway Company on a location northward from Atlanta to an ultimate crossing of the Blue Ridge Mountains.

Mr. Snowden then turned from railroad to highway engineering. From February to December, 1911, he was highway engineer, District No. 3, Covington County, Mississippi.. However, malaria caused him to resign; and he returned to his home state where for the next twelve months he was chief engineer and general manager of the Reynolds Farms, near Winston-Salem, N. C. From February, 1913, to November, 1914, he was highway engineer in Craven County, and part of this time also highway engineer, Goldsboro Improvement, Wayne County. Then he spent two years in general practice, including drainage and per diem work for the North Carolina State Highway Commission.

In May, 1917, Mr. Snowden entered the employ of the reorganized North Carolina State Highway Commission, remaining until his retirement in July, 1931. During the first four years of this period he was division engineer in complete charge of all highway location and construction in the Eastern Division and later in the Fourth Division. In April, 1921, he became district engineer of the Second District, with headquarters at Kinston, embracing ten counties in the middle of the easternmost section of the state.

After his retirement, Mr. Snowden maintained a consulting office at his home in Snowden, specializing in land drainage and topographic, municipal, railway, and industrial surveys, including estimates, designs, specifications, and supervision. He was also the head of the Works Progress Administration work in Currituck County.

Mr. Snowden's most comprehensive and responsible work was during the period when he was district engineer for the State Highway Commission, particularly from 1921 to 1926, when the state carried out its great highway building program, beginning with a $50,000,000 bond issue and augmented by later bond issues. He was required to organize his office, field, and maintenance forces; to determine the location of the new highways, which, under the State Highway Act of 1921, were to be laid to connect all principal towns and county seats; to insure that the construction contracts were executed according to the specifications; and to make monthly estimates of the work done, which were forwarded to Raleigh for payment of the contractors.

The location problems were often serious: All sections wanted roads and the routings determined were of intense interest to everyone; factional disputes arose, merchants insisted that roads should pass their business houses, and landowners objected to having their fields split wide open. Mr. Snowden met these problems with tact and firmness and, of the many appeals carried to the head office in Raleigh, very few came from his district. He had expressly desired that his district should not include his home county, so that he might be entirely unbiased in his decisions.

Much of his district lay in the tidewater region of the state; the flatness of the terrain required careful study of methods of drainage, and drainage became a specialty with him. The high ground in the "pocosins" (swamp lands) which was sought for the road locations might be less than a foot above the general ground level: Mr. Snowden did not resort to accurate leveling to determine this, but found the high ground unerringly through his knowledge of the timber growths in the pocosins, a knowledge probably obtained early in life, and supplemented by thorough study.

"Currituck," as he was called by two or three of his early friends, was soft spoken and reserved, and did not invite familarity; yet he was a genial and entertaining companion, with a keen, dry humor, and a fund of droll stories. He was courageous and determined, and would not change his decisions even in minor matters. He had the reputation of making his word stick with contractors. His rugged honesty, sound judgment, and loyalty to his associates resulted in the establishment of an exceptionally faithful and efficient organization in his district. He inculcated a fine spirit of helpfulness to travelers in trouble, and auto trouble was frequent in those days of "detours" when the state was getting its roads out of the mud.

Mr. Snowden was married on October 24, 1906, at Snowden, to Mary Morris Stevens, who survives him. His one child, Rosa Elizabeth, died in 1916, when nine and one-half years old. There are no living brothers or sisters. One brother, Capt. Basil S. Snowden, was killed in World War I.

Mr. Snowden was a member of Masonic Hall Lodge No. 53, and a Past Master of that lodge. He belonged to the Providence Baptist Church, and taught the Men's Bible Class for several years. He was a charter member of the North Carolina Society of Engineers, which he assisted in founding in 1917, and later became an honorary member of that organization.

Mr. Snowden was elected an Associate Member of the American Society of Civil Engineers on June 18, 1918, and a Member on August 28, 1922.

WILLIAM FRANKLIN STROUSE, M. ASCE[1]

Died December 5, 1945

William Franklin Strouse, eldest son of Joseph and Anna (Krebs) Strouse, was born in State College, Pa., on December 19, 1864.

His is the typical success story of the American boy, born of poor but thrifty parents, who made good in his chosen profession. He was a farm boy and received his early education in the rural district school. A descendant of the early settlers who came to the United States with William Penn, his family had for generations been tillers of the soil. Thus, it came as a surprise, and caused no little anxiety to his parents, when young "Frank" expressed a desire to leave the land to become a civil engineer. He received little encouragement from his mother who was the dominant figure in the home. Indeed, had it not been for the kindly interest displayed by his teachers in the rural school, in addition to his own strong and indomitable courage, he might not have been able to realize his ambition. He was studious, conscientious, and untiring in his efforts to achieve the goal which he had set for himself, and no obstacle, financial or otherwise, could deter him. He worked early and late to earn the money needed to pay his expenses through

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[1] Memoir prepared by W. P. Irvin, Assoc. M. ASCE.

"prep" school and college, and many a night he burned the midnight oil over intricate figures and involved engineering problems after a long and tiring day of hard manual labor. All this he accomplished with an added handicap, the use of only one eye; an injury at birth had deprived him of the sight of his right eye.

Mr. Strouse was graduated from Pennsylvania State College at State College, in June, 1887, with the degree of Bachelor of Science. From July, 1887, to September, 1889, he was employed in the office of the Engineer of Construction at Philipsburg, Pa., and as transitman on surveys for the Pennsylvania Railroad Company. In September, 1889, he severed his connection with the Pennsylvania Railroad Company and accepted a position as draftsman with the Maryland Central Railway Company in Baltimore, Md., later working on surveys for the Baltimore Belt Railroad. During this period he continued his studies, and in June, 1890, he returned to Pennsylvania State College to receive the degree of Civil Engineer.

On his return to Baltimore, he was promoted to chief draftsman for the Maryland Construction Company, builders of the Baltimore Belt Railroad, and remained with them until 1893, when he transferred to the office of W. T. Manning, chief engineer of the Baltimore and Ohio Railroad Company, as chief draftsman. From March, 1897, to June, 1903, in addition to his duties as head of the drafting office, he was almost constantly exercising general or detail supervision over odd pieces of construction work or surveys, the most important being the remodeling of the passenger station and the construction of new baggage room and train sheds at Baltimore; construction of new freight and passenger stations at Clarksburg, W. Va., and Connellsville, Pa.; bulk yard and stations at Fairmont, W. Va.; surveys for proposed improvements of the Washington (D. C.) branch; and terminal improvements at Washington, D. C.

In June, 1903, he was appointed assistant engineer of the Washington Terminal Company in direct charge of all work under the jurisdiction of the Engineering Department of the Baltimore and Ohio Railroad Company, which included the construction of the Union Station, train sheds, powerhouses, engine houses, and shops—in short, all the work north of Massachusetts Avenue in Washington, D. C. Mr. Strouse wrote a detailed description of this project,[2] the largest of its kind ever built up to that time. The Union Station was the first building constructed under the McMillan Commission's plan for a more beautiful national capitol. It replaced the maze of unrestricted railroads which cut across the mall and ran down city streets, each to its own shabby terminal. The concourse alone is larger than those of the New York Central Railroad and Pennsylvania Railroad terminals in New York, N. Y., combined. It is a great tribute to Mr. Strouse's innate sense of justice, his patience, and deep understanding of human nature that, in a period of great labor unrest, he was able to complete this project, costing twenty-five million dollars and occupying a space of more than twenty-five acres, without a single strike or lawsuit.

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[2] "The Reconstruction of the Passenger Terminals at Washington, D. C.," by W. F. Strouse, *Transactions*, ASCE, Vol. LXXI, March, 1911, p. 11.

When the Union Station was completed in 1909, Mr. Strouse was confronted with the question of whether to remain in the employ of the Baltimore and Ohio Railroad Company, or to accept one of the very excellent offers he had received from other construction companies during this period. Since, in each instance, he would have been required to make drastic changes in his domestic affairs, he finally decided to continue his work with the Baltimore and Ohio Railroad, and thus remain in his beloved Roland Park, Md., where he had established his home.

Subsequent to the Washington Terminal improvements, from December, 1909, to October, 1918, Mr. Strouse was engaged in the construction of the Baltimore and Ohio Railroad bridge in the vicinity of the Gay Street Station in Baltimore. He was also in charge of the Curtis Bay Coal Pier construction in the early stages of that project. As assistant engineer, he had charge of all construction between Philadelphia, Pa., and Cumberland, Md., which included second track work between Sir John's Run and Great Cacapon, and Green Spring and Patterson's Creek, in West Virginia. During World War I, he was in charge of Baltimore and Ohio track improvements on the Washington, Baltimore and Annapolis Railroad between Annapolis Junction and Odenton in Maryland in connection with the costruction of Camp Meade.

In 1918, on the doctor's orders to "take things a little easier," Mr. Strouse resigned from the Baltimore and Ohio Railroad Company, and shortly thereafter accepted the position of Civilian Supervising Engineer with the United States Army, in charge of cantonment construction at Fort Howard, Md. This work came to a close with the end of the war in 1919.

His last professional activity was in the Engineering Department of the Public Service Commission of Maryland, where he served from April, 1919—first, in the capacity of valuation engineer, and later, as chief engineer—until his retirement in October, 1931.

The outstanding work of Mr. Strouse while associated with the Public Service Commission of Maryland was to make valuations of practically all the public utilities of the state, totaling millions of dollars. He also made some valuations of railroad properties, estimates of the cost of several ferry projects, and numerous investigations of service matters throughout the state. These investigations succeeded in clarifying a number of conflicting claims as to service and related matters.

Mr. Strouse was an able engineer, noted for his unimpeachable integrity, for his adherence to his beliefs, and for the very careful and painstaking way in which he did every job on which he was engaged. As an executive he was strict, but just, taking his work seriously and giving unstintingly of his time and effort. He expected those working under him to be equally conscientious, and he was rarely disappointed. His knowledge and experience he shared without reservation. Quiet and unassuming, he was never one to seek publicity, but believed credit should be given where credit was due. A man of rare judgment, tempered with good common sense, he instinctively inspired confidence, and, therefore, made deep and lasting friendships among his associates.

In addition to this Society, of which Mr. Strouse was a Life Member, he was also a life member of the American Railway Bridge and Building Association, and was an active participant in the work thereof, serving on various committees and as chairman of the Committee on the Elimination of Grade Crossings. Several of his articles on safety measures appeared from time to time in its *Proceedings.* In 1921 he served with distinction as president of the association. He was also a charter member and organizer of the Engineers' Club of Baltimore and in 1911, its president.

Mr. Strouse ardently supported his alma mater, and for many years was president of the Baltimore Chapter of the Penn State Alumni Association. During World War I, he kept "open house" for Penn State men in the armed services who were stationed in and around Baltimore. He belonged to the Phi Kappa Sigma fraternity.

His interests were many and diversified. As a young man in Baltimore, he was active in the work of the Second English Lutheran Church and through its Sunday school became interested in a group of teen-age boys. He encouraged them to fit themselves for specialized work and assisted them in finding suitable jobs. This interest was sustained throughout his later life, and through his efforts he made it possible for a number of young men to receive college educations. It was, therefore, a fitting tribute to his memory that so many of "his boys" attended his funeral.

Mr. Strouse was a great student, and over the years acquired a large technical library, which he later presented to the Engineers' Club of Baltimore. He was also intensely interested in the work of civic organizations, and was a loyal supporter of various charities.

After his retirement from the Public Service Commission of Maryland, he devoted a great deal of time to tracing his genealogy, and spent many pleasant hours driving from place to place with his wife, the former Harriet E. Irvin of Clearfield County, Pennsylvania, searching for records and collecting information for a book which he planned to write. However, this pleasant interlude in their lives came to an abrupt end in 1933, when Mrs. Strouse was stricken with paralysis and remained an invalid until her death in October, 1945. It was characteristic of Mr. Strouse to give up his various activities and devote his entire time to caring for her. With him, "first things came first," and once a decision was made he never faltered. It seemed rather significant, therefore, that two months to the day, after her death, his last work accomplished, he too passed away. A devoted husband for a period of fifty-four years, he was also a devoted father. He is survived by two daughters, Miriam (Mrs. M. J. Keller) and Edna (Mrs. E. C. Romaine); and two granddaughters.

Mr. Strouse was elected a Member of the American Society of Civil Engineers on March 1, 1905.

EDWARD GRAY TABER, M. ASCE[1]

Died February 19, 1946

Edward Gray Taber, the son of Marcus William and Olive Collins (Ashley) Taber was born in Freetown, Mass., on July 14, 1855. He was a descendant of Philip Taber, an Englishman from Essex, England, who landed at Plymouth, Mass., in 1630. The Tabers, in his father's time, were largely a seagoing family, and his father, Marcus Taber, spent the greater part of his active life at sea, mostly as a master on whaling ships.

Shortly after Edward's birth his parents moved to New Bedford, Mass., and it was there he received his early education, being graduated from high school in 1873. He completed a three-year course at the Massachusetts Institute of Technology at Boston, in 1877. Poor health, however, caused him to discontinue his schooling and kept him at home until 1880, when he entered the service of the Northern Pacific Railway Company which was then extending its line of railroad to the Pacific coast.

He enjoyed a long and active life in the almost continuous practice of his profession from 1880, when he began work with the Northern Pacific Railway Company, until June 30, 1941, when he retired. His engagements, principally with western railways, were as follows:

1880—rodman and levelman with the Northern Pacific Railway Company in Montana.

1881–1885—assistant engineer on location and construction with the Northern Pacific Railway Company, mostly in Montana.

1886–1887—assistant engineer on location and construction with the Northern Pacific Railway branch lines in Washington.

1888–1889—resident engineer in charge of location and construction for the Seattle, Lake Shore and Eastern Railway Company, built forty miles west from Spokane, Wash.

1890–1891—with the Union Pacific Railroad Company on the exploration and location of branch lines in western Washington.

1892—resident engineer in charge of construction for the Great Northern Railway Company in eastern Washington. (During this period he made explorations for the final location from Spokane to the Columbia River.)

1893–1898—principal assistant engineer on location and construction with the Nelson and Fort Sheppard Railway Company, the Red Mountain Railway Company, and extensions of the Spokane Falls and Northern Railway Company.

1899—assistant engineer with the Oregon Railroad and Navigation Company, constructing docks at Portland, Ore.; locating and resident engineer with the Snake River Valley Railroad Company; and chief engineer for the Moscow and Eastern Railway Company.

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[1] Memoir prepared by Oscar S. Bowen, M. ASCE.

1900—assistant engineer with the Northern Pacific Railway Company, engaged in extending branch lines.

1901–1903—assistant engineer for the Oregon Short Line Railroad Company on location and construction of branch lines.

1905–June 30, 1941—chief engineer with the Spokane International Railway Company, and during this period he also held the following assignments:

1907—chief engineer with the Spokane Valley Land and Water Company;
1908—chief engineer with the E. B. C. Railway Company; and
1910—chief engineer with the Flathead Valley Railroad Company, and the Pasadena Park Water Company.

Mr. Taber had established a splendid home in Spokane, and after his retirement spent most of his time in the care of his home in which he had always taken a great interest. He spent a good part of his life out of doors and was a great lover of nature. During his boyhood and early manhood in New Bedford, his out-of-doors sports were boating and hunting, and during his younger days in the West he hunted buffalo and other big game in Montana and for many years kept a fancy riding horse as a hobby. In his later years his hobby shifted to gardening. He was also an authority on birds and bird life and always saw to it that he had plenty of them about his home. His greatest pleasure and recreation in the latter part of his life, however, were in the care of his flower garden at his Spokane home. This was of such beauty that it was well known in Spokane.

When Mr. Taber retired, he was eighty-six years of age and in fairly good health. He remained quite active until about a year before his death, attending to his flowers and providing for the birds he attracted to his home.

He was a member of the Associated Engineers of Spokane; he was very active in local Society affairs. He also was recognized in a wider sphere of professional interest when he served as Director of the Society for the term 1926–1928, representing District 12. Until his death he was, both in age and time of residence, the oldest engineer in Spokane. He was of quiet disposition—a man of great integrity and exemplary habits. During his long residence in Spokane he acquired a wide acquaintance and many friends, and was held in great respect by engineers and others with whom he came in contact. His passing marked the end of a long and useful life.

Mr. Taber was first married in 1893 to Mrs. Marie Christine Bowen who died in 1923. In 1927 he was married to Mrs. Anna Brown who died in 1936. He is survived by a niece, Mrs. J. M. Duthie of Forsyth, Mont.; and by two cousins, James H. Winslow and B. S. Winslow, M.D., of New Bedford.

Mr. Taber was elected a Member of the American Society of Civil Engineers on September 2, 1914. He became a Life Member in January, 1939.

CARROLL ROSE THOMPSON, M. ASCE[1]

DIED FEBRUARY 21, 1946

Carroll Rose Thompson was born on October 6, 1885, in Philadelphia, Pa. His parents were George C. and Adele (Thomas) Thompson. He was a graduate of the old Philadelphia Central Manual Training School. To the best interests of his native city he devoted the major part of his life.

In his early career, from August, 1902, until May, 1908, he was employed as draftsman by the American Bridge Company at its Pencoyd (Pa.) plant, where he was active in plant drafting and checking, under the supervision of the assistant engineer, and where he was in charge of detailing structural steel. From July, 1908, to March, 1909, he was engaged by the Pennsylvania Steel Company at its Steelton (Pa.) plant, again as draftsman and checker.

Mr. Thompson entered the services of the Department of Wharves, Docks and Ferries of Philadelphia, in April, 1909, less than two years after the creation of that municipal agency, and continued in that department until his death, almost thirty-seven years later. He began as a draftsman and, in time, became chief engineer and one of the outstanding anthorities in the United States on waterfront construction and regulation, particularly on matters concerning riparian rights.

In January, 1910, with less than one year of service, Mr. Thompson was promoted to chief draftsman of the new department, which was then busily engaged in an intense program to provide the Port of Philadelphia with a group of modern municipal piers. As chief draftsman, reporting to the harbor engineer, he was in charge of the design and preparation of contract plans for piers, bulkheads, and all other harbor structures. Among the port facilities which he supervised during this period were 3,700 feet of Delaware Avenue bulkhead, costing $225,000; piers Nos. 38 and 40 of the Southwark group, costing (equipped) $1,315,305 each; pier No. 78 south of the Moyamensing group, costing (equipped) $3,400,000; and pier No. 9 north on Cherry Street, costing $1,300,000.

He was also in charge of the comprehensive planning for the development of the Southwark group of four piers and of the Moyamensing group of ten piers, including warehouse sites, freight yards, marginal ways, approaches, and similar construction. In addition, he reviewed the examination and checking of designs of numerous privately-owned harbor structures submitted to the Department of Wharves, Docks and Ferries for approval as to stability and location in accordance with city, state, and federal requirements.

From December, 1916, to December, 1918, a period during World War I, he served as first assistant to the harbor engineer, and was in charge of the engineer's office. In January, 1919, he became principal assistant engineer, and directed the design, planning, and preparation of contract plans and specifications for the construction, equipment, and maintenance of all city harbor structures. These improvements included two designs for Penn Treaty Pier No. 57 north, costing $80,000; substructures of piers Nos. 82 and 84 south of the Moyamensing group, costing about $1,500,000 each; pier No. 30 south of the Southwark group, costing $908,400; 1,150 feet of bulkhead along the Schuylkill River, costing $343,400; and substructure of pier No. 80 south, costing $1,300,000.

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[1] Memoir prepared by Edwin R. Cox, Director, Dept. of Wharves, Docks and Ferries, Philadelphia, Pa.

Mr. Thompson was appointed assistant director of the Department of Wharves, Docks and Ferries on January 16, 1920, and continued in that capacity until August 20, 1923, when he was named chief engineer. He continued in the latter position until his death. As chief engineer, he completed the plans and supervised the construction of the present group of municipal piers which the city authorities of Philadelphia cite as unequaled throughout the world.

For years, Mr. Thompson served as consulting engineer, without pay, for the Navigation Commission for the Delaware River and Its Navigable Tributaries. He was an examiner for the Pennsylvania State Board of Engineers and was a member of numerous engineering organizations, including the Engineers Club of Philadelphia, the American Association of Port Authorities, and the Society of American Military Engineers. He was also a member of the F. and A. M. and the Artisans.

Carroll R. Thompson died on February 21, 1946, at his home, 618 Leverington Avenue, Philadelphia. He had been ill for about a year, and had been confined to his home for six months.

Following his death, the writer, in a public statement declared:

> "The death of Mr. Thompson removes a most loyal and efficient official from the service of the city. His passing is a decided loss to our community and to the engineering profession and, particularly, to the Port of Philadelphia, to the interests of which he gave his most sincere efforts."

In Harrisburg, Pa., in 1908 Mr. Thompson was married to Florence Waldeck. He is survived by his widow; two brothers, Harry H. and Frank; and three sisters, Esther (Mrs. Charles Deychert), Irene (Mrs. Ralph Priest), and Virginia (Mrs. Sevill Schofield).

Mr. Thompson was elected an Associate Member of the American Society of Civil Engineers on March 12, 1918, and a Member on July 6, 1920.

CLARENCE SYDNEY TIMANUS, M. ASCE[1]

Died March 12, 1946

Clarence Sydney Timanus, the son of Elmer and Belle (Culley) Timanus, was born in Kansas City, Mo., on March 20, 1892.

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[1] Memoir prepared by Chester A. Smith, M. ASCE.

He attended the public schools of Kansas City, including Central High School and University High School. He enrolled in the Massachusetts Institute of Technology at Cambridge, in September, 1914, and was graduated in 1918. He also received the degree of Bachelor of Science from Harvard University, at Cambridge, in 1918. Prior to graduation, in April, 1918, he enlisted in the United States Navy, and was assigned for training in a Naval Aviation Detachment, stationed at Hingham, Mass.

On December 9, 1918, he entered the employ of Burns and McDonnell Engineering Company, consulting engineers of Kansas City, as a junior engineer. From 1920 to 1924, he served as chief draftsman and office engineer, and later he was classified as associate engineer in charge of the design of major engineering projects for the firm. In 1930 he became a partner in the firm and continued in this capacity until his death.

Because of his technical training, the reading of many technical publications, and association with a large variety of engineering projects, he had a broad knowledge of engineering subjects. His specialties were hydraulic, sanitary, and structural engineering designs.

Among the two hundred and ninety engineering assignments over which Mr. Timanus had personal direction was the twelve-million-gallon water softening and iron removal plant for Springfield, Ill. Later, when the new supply was developed, he had direct charge of the surveys, design, and general supervision of construction of the Lake Springfield dam, water treatment plant, pumping station, and municipal electric power plant—at a total cost of more than $6,000,000. He also had charge of the rehabilitation and enlargement of the $3,200,000 Cincinnati (Ohio) water purification plant.

In 1941 and 1942, he was assistant director of the construction of Camp Crowder, Missouri; and, in 1942 and 1943, he was director of the construction of the Smoky Hill Airfield at Salina, Kans. The runways at the Smoky Hill Airfield were originally designed for 300-ft width and 5,000-ft length, but a change order came through to increase these dimensions to 500 ft by 10,000 ft. This extension of runways was planned for the first B-29 bomber training center.

All his engineering associates, friends, clients, and contractors held Mr. Timanus in the highest regard, not only for his engineering ability but also for his eminent fairness in decisions—never taking advantage of mere technicalities to create an injustice to any person or client.

Mr. Timanus became a member of the Oak Park Christian Church of Kansas City in 1921. He participated very actively in the church organization—as a deacon, an elder, president of the Men's Bible Class, and vice-president of the official board of the church. His outstanding ability in organizing engineering projects was useful in his church work. The minister stated he knew everything would be ready for starting services when Mr. Timanus was in the city, as he was always there early and saw that all details were arranged. He was a lover of good music, and was a regular subscriber to the Kansas City Philharmonic Orchestra. At home he spent hours at the piano, solely for his own amusement and relaxation. He was a member of the

American Water Works Association, American Concrete Institute, and Engineers' Club of Kansas City.

On June 10, 1920, Mr. Timanus was married to Lella Glayd Saunders at Kansas City. He is survived by his widow and a daughter, Marjorie.

Mr. Timanus was elected an Associate Member of the American Society of Civil Engineers on August 31, 1925, and a Member on May 12, 1930.

LEE TREADWELL, M. ASCE[1]

DIED MAY 15, 1946

Lee Treadwell was born in Hill County, Texas, on May 18, 1864, the son of Steven Clement Treadwell, of Tennessee, and Missouri (Hodges)Treadwell, of Georgia. In 1865 the family moved to Arkansas where he was graduated from Arkansas Industrial University at Fayetteville, in 1888, with the degree of Civil Engineer. Mr. Treadwell was president of his senior class and became instructor of field practice in surveying at the university during his senior year and held the position for two months thereafter.

From November, 1888, to October, 1896, he was with J. A. L. Waddell, consulting engineer of Kansas City, Mo., for whom he became principal assistant engineer. In almost all his work, from 1891 to 1896, he was "in responsible charge." His work covered designs and foundations of six large bridges, one of which was the longest swing drawspan in the world, and surveys and foundations borings of five bridges. He worked on the elevated railroad in Chicago, Ill.; the train shed at Sioux City, Iowa; and other structures.

In 1895, he was with Sooysmith and Company; in 1896, with the Phoenix Bridge Company in Philadelphia, Pa., engaged on the Wissahickon Bridge; and, in 1898, again with Sooysmith and Company, in New York, N. Y. He was also employed on cofferdam and foundation work in Ottawa, Ont., Canada, and in 1899, he constructed a dam at Tariffville, Conn.

From November, 1899, to January, 1905, he was superintendent and engineer in general charge of construction for John Peirce and Company, contractor for the stone dry dock at the United States Navy Yard at Portsmouth, N. H.

It is not often that Navy Department records are made of acts of personal bravery of civilians who have not been members of the United States armed forces, or directly employed by the government. From the official navy yard record of the dry dock construction work is quoted the following:

> "At one time during the course of the construction he distinguished himself in a heroic manner. The cofferdam built across the entrance to the dock developed a serious leak which threatened its destruction. Mr. Tread-

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[1] Memoir prepared by L. F. Bellinger, M. ASCE.

well ordered all workmen off the cofferdam and personally went on the dam and with a large quantity of oakum succeeded in checking the leak so the cofferdam was saved."

He obtained patent No. 693465 dated February 18, 1902, for "Machines for Delivering Comminuted Solids," at constant rate and in measured quantities, as for concrete.

In 1904, he was consulting engineer for the Phoenix Bridge Company on the foundations of the Quebec Bridge, the superstructure of which suffered two wrecks. In January, 1904, he was consulted by the Imperial Drydock Company at St. John, N. B., Canada, and prepared an estimate of cost for the John Peirce Company.

From 1905 to 1907, he was superintendent for the Foundation Company, contractors for the caisson and slips at Chalmette, La. The caisson was named "Lee Treadwell."

In the period, from June, 1907, to July, 1913, he was one of the four founders of the Union Bridge and Construction Company of Kansas City, and held the positions of vice-president and chief engineer. Among the numerous items of work of this company that were handled personally by Mr. Treadwell were the Atchafalaya River Bridge in Louisiana, in 1907–1908; the Broadway Bridge in Portland, Ore., in 1911; and the Iowa Central Railroad Bridge at Keithsburg, Ill.

After 1913, he undertook only one job—that of inspection engineer for Fuller and Maitland of New York, on the water works of Kansas City.

In September, 1933, he moved to Atlanta, Ga., with his eldest daughter, Althea (Mrs. Dale Allen).

It is seldom that letters of reference to the candidates for transfer of membership in the Society are such warm, personal letters of esteem, as are preserved among Mr. Treadwell's papers. These references were written by seven of the most prominent engineers of that time (1893)—namely, the late William H. Burr,[2] M. ASCE, of Columbia University, in New York City; John Sterling Deans,[3] M. ASCE, chief engineer of the Phoenix Bridge Company; W. Kiersted, M. ASCE; C. C. Schneider,[4] Past-President, ASCE, chief engineer of the American Bridge Company; Charles Sooysmith, M. ASCE, contractor and soft foundation engineer; J. A. L. Waddell, Hon. M., ASCE, consulting bridge engineer; and Paul Wolfel,[5] M. ASCE.

He was married on December 22, 1896, at Kalamazoo, Mich., to Althea Laura Fletcher. He is survived by his two daughters, Althea Virginia (Mrs. Dale Allen) and Laura (Mrs. Robert Galbraith); three grandsons; one granddaughter; and one great-granddaughter.

Mr. Treadwell was elected a Junior of the American Society of Civil Engineers on November 5, 1890; an Associate Member on October 4, 1893; and a Member on December 2, 1896. He became a Life Member in January, 1928.

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[2] For memoir, see *Transactions*, ASCE, Vol. 100, 1935, p. 1617.

[3] For memoir, *ibid.*, Vol. LXXXIII, 1919–1920, p. 2187.

[4] For memoir, *ibid.*, Vol. LXXXI, 1917, p. 1665.

[5] For memoir, *ibid.*, Vol. LXXXIV, p. 940.

WOLFGANG GUSTAV TRIEST, M. ASCE[1]

Died September 21, 1946

Wolfgang Gustav Triest was born in Oppeln, Germany, in 1863, the son of Felix and Clara Triest. He was graduated from the Royal Technical College in Berlin in 1888 and in the same year moved to the United States.

Mr. Triest started his business career in March, 1888, when he joined the engineering department of the Passaic Steel Company in Paterson, N. J. From 1890 until 1895, he was the principal assistant of the late A. P. Boller,[2] M. ASCE, assisting in preparing plans and supervising construction for the Fifteenth Street viaduct in New York, N. Y., and also for a number of railroad bridges. This was followed by his partnership in the firm of James R. F. Kelley and Company, Contracting Engineers in New York, that lasted until May, 1897. The work that Mr. Triest helped to supervise for this company included the high service pumping station at 181st Street, in New York City.

For two years, from 1897 until 1899, he was engaged in private practice as a contracting engineer, with headquarters in New York City. The work executed during that period consisted of a number of private and government contracts for structural steel buildings which included the preparation of plans and the supervision of construction. He built several fireproof buildings in the Brooklyn Navy Yard.

On January 1, 1900, Mr. Triest and the late Frederick Snare, Affiliate, ASCE, formed a company then known as Snare and Triest, Incorporated, with Mr. Triest as vice-president. The firm later became known as the Snare and Triest Company. He continued with the corporation until 1919, at which time he left to form his own company, known as the Triest Contracting Corporation.

While Mr. Triest was vice-president of the Snare and Triest Company, he supervised the activities of the company within the United States. Some of the notable work done under his supervision was the building of a large coaling plant for the United States Navy at East Lemoine, Frenchmen's Bay, Maine, from designs prepared by his company; five of the Chelsea piers for the Cunard Line and other steamship companies; a section of the Lexington Avenue Subway, in which his company was associated with Arthur McMullen; the rebuilding of many miles of the New York elevated railroad for the Interborough Rapid Transit Company; the superstructure for the piers for the Hell Gate Bridge; the Cherry Street pier in Philadelphia, Pa., and the superstructure of piers thirty eight and forty in Philadelphia, both of which were built for the Department of Wharves and Docks of that city; the reservoir for the City of Perth Amboy, N. J.; and the Manhattan approach to the Blackwell's Island Bridge for the City of New York.

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[1] Memoir prepared by A. W. Buttenheim, Pres., Frederick Snare Corp., New York, N. Y.

[2] For memoir, see *Transactions*, ASCE, Vol. LXXXV, 1922, p. 1653.

During World War I he did a great deal of work for the United States Navy, including the construction of a fitting-out pier at League Island, in Philadelphia, and of many buildings at Lake Denmark and Iona Island, N. Y. He supervised the building of ten shipways at Hog Island, on the Delaware River near Philadelphia, and also the construction of the quartermaster terminal in Philadelphia and the Raritan ordnance depot at Perth Amboy, both for the United States Army.

After forming the Triest Contracting Corporation, he successfully completed several sections of the Independent Subway System for the City of New York and a section of subway in Camden, N. J., for the Delaware River Joint Commission. He also completed the Bay Way (N. J.) approaches to the Goethals Bridge, and an aqueduct for the Borough of Brooklyn. Additional work done by Mr. Triest's own company included the enormous doors of the Lakehurst (N. J.) Hangar, which he built for the United States Navy, and which are considered to be some of the largest doors ever constructed. He also erected a hangar for the United States Navy at Cape May, N. J.

In recent years Mr. Triest was not active in the field of construction. However, he was a regular visitor at the Engineers Club, of which he was a life member, and in this manner kept in close contact with his many friends in the construction industry.

In 1894 he was married to Lucy Wood, who died in 1899. In November, 1901, he was married to Lillie Macdonald, who, with their two sons, Willard G. and Carl G., survives him.

Mr. Triest's activities in engineering and construction work were carried on during a very interesting period. He started at a time when a capable engineer with small capital could carry on successfully and without undue competition. From the time he signed his first contract until he retired, he and his associates completed over 1,100 different contracts. That he was able to carry on so many and such diversified types of construction, and that he successfully completed every contract awarded to his company, is a record of which his family and associates can be very proud.

It is interesting that his death on September 21, 1946, occurred within a few hours of, and at the same age as, his old friend and former associate, Frederick Snare.

Mr. Triest was elected a Junior of the American Society of Civil Engineers on September 3, 1890; an Affiliate on March 6, 1900; and a Member on January 31, 1905. He became a Life Member in January, 1934.

HERMAN VAN DER VEEN, M. ASCE[1]

Died October 27, 1946

Herman van der Veen was born in Vollenhove, Holland, on July 1, 1878, and was awarded the degree of Civil Engineer at the University of Delft, Holland, in June, 1903.

He was a civil engineer of wide experience on hydraulic matters, his speciality being river improvements.

Having started his career as an engineer with the Triangulation Bureau of the Dutch government, Mr. van der Veen was an assistant to the professor in geodesy at the University of Delft, before leaving his native country in 1904 to fill the appointment of engineer with the Chung-ting fu in China. There he became engineer in charge of the Whangpoo Conservancy Board at Shanghi, China, in 1906.

In 1911 he left for the Netherlands East Indies where he held the position of acting engineer in chief of the Surabay Harbor and, afterward, chief of the Padang Harbor Works.

Mr. van der Veen returned to China in 1914 as adviser to the Conservancy Bureau of China, and later to the ministries of the interior and communications. In November, 1929, he was repatriated and returned to Holland. His task, however, was not yet finished and he continued his activities as government engineer in chief on the improvement of the river, the Meuse, and afterward was in charge of the Rotterdam waterway, from which position he resigned on August 1, 1943.

From this summary it can be seen that the greater part of his career was spent in China and, indeed, it was in this country that he developed his full energies. . It was certainly not an easy task to be an adviser to the government in this gigantic country where powerful rivers required drastic improvements to prevent fatal inundations, causing devastation and famine in wide areas. Being a prominent member and a technical adviser of many committees on this subject, he had a sound knowledge of the problem. Two of the committees with which he was associated were the Famine Relief Bureau of the Technical Committee of the Yangtze River and of the Yung-ting ho Commission.

Many reports containing valuable data and advices were prepared by him. Among his technical publications, "The Improvement of the Chihli River System" (1928) and "The Improvement of the Yellow River" (1924) are his important work.

Obviously most of the projects which he recommended required huge capital investments and could not be completed. Notwithstanding, all his efforts were directed toward the river improvement problem in China which will require many years of work and the utmost exertion to solve. Mr. van der

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[1] Memoir prepared by O. C. A. Van Lidth de Jeude, The Hague, Netherlands.

Veen, however, was a pioneer who has laid the foundations upon which others can continue the task.

His long experience in China made him what is called "an old China-hand," and after his repatriation he kept in contact with this fascinating country, where his lifework lies. The Chinese government recognized his services by conferring the order of "Golden Rice Blade" upon him.

This pioneering work suited his vital personality and his energetic character. He was popular with his friends because of his straightforwardness and cordiality and was a welcome guest in the Shanghai Club and in the Peking Club.

While in Tientsin, where he made his home for several years, he was married to Adriana Alida ten Nave on June 23, 1920. Mrs. van der Veen survives him.

Mr. van der Veen was elected a Member of the American Society of Civil Engineers on November 9, 1920.

LOUIS JOSEPH VOORHIES, M. ASCE[1]

Died August 25, 1946

Louis Joseph Voorhies, the son of Martin J. and Amelie (Bienvenu) Voorhies was born in Saint Martinville, La., on August 29, 1886. He was a distinguished member of a family which contributed much to the social, educational, and economic development of Louisiana. He received his early education in private schools in his home community and, in June, 1906, received the degree of Bachelor of Science in Civil Engineering from Louisiana State University at Baton Rouge. In college, Mr. Voorhies was not only an outstanding student, receiving the faculty medal for attaining the highest scholastic record in his graduating class, but also an athlete, being the regular catcher on the university baseball team for two years. He was a member of Tau Beta Pi, Kappa Sigma, and Phi Kappa Phi fraternities. In 1928, in recognition of his professional accomplishments, Louisiana State University conferred upon him the degree of Civil Engineer.

Following graduation, Mr. Voorhies was employed by the Southern Pacific Railroad Company from August, 1906, to December, 1911, for the first two years as rodman and instrumentman and for the remaining three years as resident engineer in charge of construction. From December, 1911, to June, 1912, he was employed by the Galveston, Houston and Henderson Railroad Company, appraising right-of-way and terminal grounds between Galveston and Houston, Tex. Between June, 1912, and September, 1913, he practiced engineering in Crowley, La., designing, locating, and constructing drainage and irrigation canals in southwest Louisiana.

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[1] Memoir prepared by a Committee of the Louisiana Section consisting of S. Steve Carnegie, Jun. ASCE, Chairman, Frederick Fischer Pillet, M. ASCE, and John Henry Bateman.

He was city engineer of Lafayette, La., from September, 1913, to May, 1915, during which period he designed street pavements, storm sewers, and other street facilities for the city. For a year beginning in May, 1915, he held the position of senior civil engineer with the Interstate Commerce Commission on the federal valuation of railways. Since he was unable to spend much time with his family, he decided to enter the commercial field and moved back to Lafayette, where he engaged in private practice. In February, 1917, he became district engineer in Louisiana for the J. B. McCrary Engineering Corporation of Atlanta, Ga., which had been specializing in municipal engineering for many years.

When the United States entered World War I, James C. Nagle,[2] M. ASCE, dean of engineering at the Agricultural and Mechanical College of Texas at College Station, urged Mr. Voorhies to join the faculty as associate professor of railway and highway engineering. Mr. Voorhies took a leave of absence from the J. B. McCrary Engineering Corporation and taught engineering from September, 1917, to June, 1918, during which time three thousand young men were undergoing intensive engineering and military training at this college.

He returned to the J. B. McCrary Engineering Corporation in June, 1918, as district manager for Louisiana and adjoining states with headquarters in Lafayette. During the following eight years, he developed an extensive engineering practice—designing and supervising the construction of municipal public work projects involving the expenditure of many million dollars.

In December, 1926, Mr. Voorhies decided to go into business for himself and opened an office in Baton Rouge. From 1926 to 1946 he served as consulting engineer for a large number of municipalities designing and supervising the construction of water distribution systems, water treatment plants, sanitary and storm sewer systems, sewage treatment plants, power plants, gas distribution systems, municipal buildings, swimming pools, pavements, parks, and other municipal improvements.

Mr. Voorhies possessed great natural ability and an unusual capacity for work. He continued his extensive and thorough studies of engineering developments throughout his professional career. His ability to deal with people in all walks of life was outstanding and, with his extensive knowledge and experience, enabled him to rise to the top of his chosen career in public works engineering.

His principal interests were his family, his work, and his friends. His great personal charm, his ready wit, his keen understanding of the economic development of the United States, his state, and his community, together with his unusual aptitude for expressing his thoughts in clear, concise language made visits in his home a delightful privilege. He will long be remembered by his friends and associates for his great ability, his outstanding achievements, his high standards of professional and personal conduct, and his wish to be of service to others.

The willingness of several of his employees to remain with him for a number of years reflects his personal character. One employee, Frederick

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[2] For memoir, see *Transactions*, ASCE, Vol. 93, 1929, p. 1878.

E. Smith, was associated with him for more than eighteen years, during which time Mr. Voorhies trained and guided Mr. Smith, with the intention of having someone to continue with the high professional standards that he himself had followed during his career. In October, 1945, they formed a partnership which continued until Mr. Voorhies' death.

In 1913, Mr. Voorhies was married to Hilda Labbe of Saint Martinville. Besides his widow, he is survived by one son, Louis Joseph, Jr.; three daughters, Corinne (Mrs. Wayne Amos), Hilda (Mrs. Roy Nystrom), and Mary Nell (Mrs. D. T. Waller); and nine grandchildren.

Mr. Voorhies was elected a Member of the American Society of Civil Engineers on August 14, 1939.

CHARLES EDWARD WADDELL, M. ASCE[1]

Died April 20, 1945

Charles Edward Waddell, the son of Francis Nash and Anne Ivey (Miller) Waddell, was born on May 1, 1877, on Mooresfield Plantation, near Hillsboro in Orange County, North Carolina. He received his early education at the Bingham Military Academy, in Asheville, N. C., from which he was graduated in 1894, and in the shops of the General Electric Company.

In 1883 Mr. Waddell's family moved to Asheville, where he received his first professional appointment in 1891, when, on October 30, he became the superintendent of the Asheville fire alarm system. He entered the shops of the General Electric Company in Boston, Mass., in 1894 as a student. This was followed by a period of field work for this company in Bangor, Me., and in Asheville. In 1897 he became superintendent of the Asheville and Biltmore Street Railway Company, supervising installation of lines, powerhouse, and equipment; in 1900, as engineer in charge of tracks for the General Electric Company, he rebuilt a greater part of the system in Asheville.

Mr. Waddell became chief engineer to the late George W. Vanderbilt in 1901, and designed and built all the engineering works on the Biltmore estate, including three substations, the largest electrical heating plant yet attempted, a complete refrigeration plant, and other works. He lived on the estate, which became his home for the rest of his life.

In 1903 Mr. Waddell entered, actively, the field of consulting engineering practice, and became one of the leading engineers in the southeast for hydroelectric and steam generated plants, hydraulic power resources, and water supply. An impressive list of works constructed under Mr. Waddell's supervision include, in addition to the development of the Vanderbilt estate, the North Carolina Electric Power Company's system with Weaver and Marshall hydroelectric plants and the Elk Mountain Steam Plant (now Western Division of the Carolina Power and Light Company); the Sunburst Arch Dam and the forty-million-gallon filter plant for the Champion Fibre Company in Canton, N. C.; the Bee Tree Dam and water supply system in the City of Asheville; and the water supply system for the American Enka Corporation in Enka, N. C.; besides a number of notable wooden, earthen, gravity concrete, arch or multiple arch, dams.

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[1] Memoir prepared by G. H. Maurice and D. M. Williams, Members, ASCE.

As consulting engineer, in 1913, Mr. Waddell served the United Electric Securities Company in Boston; in 1915 he made waterway studies for the Southern Railway Company on the main line south of Washington, D. C.; and in 1917 he made similar studies on the Queen and Crescent Railroad between Cincinnati, Ohio, and Harriman, Tenn.

During World War I he was a consulting engineer for the Power Section Council of National Defense for power resources in the southern states' war emergency and also Director of Conservation for the United States Fuel Administration in the State of North Carolina. He was engineer to the Quartermaster Department of the United States Army for construction on general hospitals Nos. 12 and 19.

In recognition of his work in developing hydroelectric power in North Carolina, the degree of Doctor of Science was conferred upon him by North Carolina State College of Agriculture and Engineering at Raleigh in 1925.

Dr. Waddell was a member and chairman of the North Carolina State Board of Engineering Examiners, from 1921 to 1926, of which he was one of the originators; a member of North Carolina Ship and Water Transportation Commission, from 1923 to 1924; commissioner of Biltmore Forest, from 1923 to 1927; consulting engineer with the North Carolina Corporation Commission in readjustment of utility rates, from 1932 to 1934; consulting engineer with the Tennessee Valley Authority, from 1936 to 1938; and consulting engineer for the City of Asheville for two terms, from 1925 to 1927 and from 1940 to 1941.

In 1927 Dr. Waddell's activities took him to South America. He was a consulting engineer for the Departmente de Antioquia in Colombia, and for the City of Medillin in Colombia, where he worked on the hydroelectric plants.

Dr. Waddell was a born leader and teacher who gave himself unsparingly to all civic and educational undertakings. To young engineers in his employ and to others who sought his advice, he was never too busy to impart to them the benefit of his vast store of knowledge and experience.

One of North Carolina's outstanding financiers recently stated: "Do you know what made Waddell great? It was his love for things beautiful." He would never compromise with quality, nor would he follow through with a project unless he was convinced that it was sound.

In his early years, while employed by the Biltmore Railway Company in Asheville, he received a blow on the head from a trolley. This injury was forgotten for years until, in 1913, after constant pain, it was discovered that the skull had been fractured and that a piece was now pressing on the brain.

Three major operations followed during the next fifteen years. After the last one, he suffered a partial paralysis of the legs, from which he never fully recovered. Although in almost constant pain, he maintained his characteristically cheerful outlook on life. His civic activities were many: He was president of the Pen and Plate Club in Asheville in 1916; president of the Biltmore Hospital, from 1920 to 1925; president of the Civitan Club in Asheville in 1923; governor of the Biltmore Forest Country Club from 1923 to 1933; and a member of the Cosmos Club in Washington, D. C.

Dr. Waddell was one of the men who organized the North Carolina Section of the Society on October 24, 1923, at Durham, N. C., and was elected, and served as President from 1923 to 1925. He retained membership in the Section after the western counties of the state were transferred to the Tennessee Valley Section, and was made an Honorary Life Member of the North Carolina Section on January 30, 1943.

Besides membership in this Society, he was a Fellow of the American Institute of Electrical Engineers; a member of the American Society of Mechanical Engineers; and a member of the North Carolina Society of Engineers, of which he was president in 1928, and an honorary member in 1941. He was the author of numerous technical publications and papers.[2, 3, 4, 5]

On April 19, 1904, he married Eleanor Sheppard Belknap of Louisville, Ky., who survives him. A son, Charles E. Waddell, Jr., and a daughter, Eleanor Belknap (Mrs. George M. Stephens), also survive. Funeral services were held April 22, 1945, at All Souls Episcopal Church in Biltmore, of which Dr. Waddell had been a member of many years. Interment was in Riverside Cemetery.

An editorial in the *Asheville Citizen* closed with the following eulogy:

> "Charles Waddell lived and breathed his profession. He loved difficult problems in hydraulics and delighted to dwell in the realm of mathematics. He was a good and useful citizen, generous, socially-conscious and always a striver for the best causes in the community he loved and served. To a full life he gave more than is asked of most men, courageously and ruggedly, untiringly and to the very last."

Dr. Waddell was elected a Member of the American Society of Civil Engineers on March 1, 1910.

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[2] "Southern Appalachian Streams," by Charles E. Waddell, *Journal of the Franklin Institute.*, Vol. CLXIV, July–December, 1907, p. 161.

[3] "The Preservation of the Southern Appalachian Streams; A Forest Problem," by Charles E. Waddell, *Transactions*, A.I.E.E., Vol. 24, 1905, p. 889.

[4] "Notes on the Electrical Heating Plant of the Biltmore Estate," by Charles E. Waddell, *ibid.*, A.I.E.E., Vol. 27, Pt. I, p. 651.

[5] "Hydraulic-Fill Core Control," by Charles E. Waddell, *Engineering News-Record*, Vol. 105, 1930, p. 958.

RAYMOND FOWLER WALTER, M. ASCE[1]

Died June 30, 1940

Raymond Fowler Walter's parents, John Huffman and Susan (Garlock) Walter, established their home in Iowa but moved to Chicago, Ill., where their son, Raymond, was born on October 31, 1873. He was named Arthur Raymond Walter; but, as a youth, he dropped the name Arthur and added the middle name of Fowler, and he was thereafter known as Raymond Fowler Walter. His father was a printer and publisher, who, attracted by the stories of gold mining in Colorado, loaded his belongings and with his wife and son drove to Cripple Creek (Colo.), the center of the gold mining activity, where the family lived for a short time and then, like many miners, found their reward in agriculture.

Probably influenced by the establishment of the Greeley Colony in Weld County, of which the Town of Greeley (Colo.) became the county seat in 1886, the family finally settled at near-by Fort Collins, where young Raymond attended grade and high schools and continued at Colorado State College of Agriculture and Mechanical Arts. At first, he was an agricultural student, but, later, he studied engineering and was awarded the degree of Bachelor of Science in Civil Engineering in 1893.

He was thus ready to take advantage of the unprecedented interest and demand for irrigation development throughout the West, particularly in Colorado, where financing was easy and irrigation bonds were in demand. This condition unfortunately encouraged many wildcat projects, resulting in financial failures and much human suffering.

Mr. Walter's services as a young engineer were in demand for surveying and designing several undertakings in northeastern Colorado, but his knowledge and canny analysis prevented the mistakes too common in those times. He enlarged and completed Terry Lake and designed and constructed the Independence Canal for the Big Horn Mining and Irrigation Company, as well as the Big Creek Reservoir in Wyoming. In 1895 he became the junior partner in the firm of Baker and Walter in Greeley. This firm designed and built reservoir No. 6 for the North Poudre Irrigation Company. He also was elected county surveyor of Weld County, and was mostly engaged on the construction of roads and bridges for three years. From 1899 to 1901 he served as city engineer of Greeley, being responsible for the design and construction of water and sewer extensions.

Thereafter, Mr. Walter designed and built the Seven Lakes Reservoir at Loveland, Colo., and the Fossil Creek Reservoir at Fort Collins for irrigation. Meanwhile he designed and installed a sewer system for Eaton, Colo., and designed and supervised the construction of many irrigation works, including the Union Reservoir at Longmont, Colo.; McClellan and Law Reservoir at Windsor, Colo.; and the Clark Lakes Reservoir and canals at Wellington,

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[1] Memoir prepared by John C. Page, M. ASCE.

Colo.—among others. He served also as deputy state engineer in charge of river and canal flow records and at the same time designed Jackson Lake and Riverside reservoirs near Greeley.

Finally, in 1903, ten years after graduation, he was appointed assistant engineer in the United States Reclamation Service, and was assigned to making surveys and plans for the Belle Fourche Reclamation Project in South Dakota. At this time he was married to Lillian Leon Phillips of Fort Collins. In 1904 he was designated project manager and supervised the construction of Orman Dam and the canals and drains on the Belle Fourche project.

After completion of this project, Mr. Walter progressed to the position of supervising engineer at Denver, in charge of all projects of the Bureau of Reclamation in Colorado, Wyoming, Nebraska, and Utah. He held this assignment from 1909 to 1915, when, as senior engineer at El Paso, Tex., he supervised the Rio Grande Project, in New Mexico and Texas. After a year in Texas he returned to Denver as assistant chief of construction and assistant chief engineer. During this time he concentrated on administrative duties as well as on strictly engineering matters; and it was due to his careful scrutiny of all operations that he earned the title, within the organization, of the "watchdog of the treasury" because of his insistence on economy and efficiency. He guarded federal funds more zealously than his own.

Finally, on May 1, 1925, he was appointed chief engineer, with supervision of all construction activities of the Bureau of Reclamation. To increase efficiency and to promote the highest type of design and construction, he helped organize and man the design and construction organization which is recognized all over the world for outstanding ability and for the creation of new methods and improvements on great irrigation enterprises. During his period of service, Mr. Walter was instrumental in building, successively, "the highest dam in the world" on five separate occasions. These dams included, of course, Boulder, Shasta, and Grand Coulee, for which he was privileged to sign the construction contracts on behalf of the government. Can an engineer ask for finer memorials?

In addition to the industry and conscientious attention he brought to his constructive activities, he recognized his duty as a family man and a member of society. This was demonstrated by his care and education of his two children, Dorothy L. (Mrs. F. B. Cook) and Donald S., the latter following in his father's footsteps as a responsible engineer for the Bureau of Reclamation. Mr. Walter's widow and two children survive.

He joined the Presbyterian Church and the F. and A. M., becoming a member of the Shrine at Deadwood, S. Dak. Although lack of time curtailed his activity, both these organizations knew of his loyalty, sincere interest, and support.

He was by nature a friendly and courteous individual, even to the point of hurting himself to perform properly his administrative duties; and as a protection he shielded his feelings by a shell of gruffness which fooled no one who knew him well. It did protect him from imposition by individuals having only a superficial acquaintance.

Death by heart disease ended the career of this lifelong and outstanding public servant who left a memorable record of engineering and constructive accomplishment through a busy life. The value of his service is well described in the statement by the Hon. Harold L. Ickes, then Secretary of the Interior, who was particularly well qualified to evaluate the services rendered by Mr. Walter during his thirty-seven-year employment in this department:

> "The death of Chief Engineer Walter after a lifetime of service to the Bureau of Reclamation and the Department of the Interior is a severe blow. For 15 years Mr. Walter had been in active charge of what is generally recognized as the greatest engineering office in the world—the Denver engineering headquarters of the Bureau of Reclamation. His services were outstanding. His unfailing loyalty to his organization, to his Bureau and his department, and to the United States Government has provided an example for the thousands of young men working under him. His unexpected death is a shock to the whole Department."

Mr. Walter was elected a Member of the American Society of Civil Engineers on October 5, 1909.

WILLIAM MacINTIRE WHITE, M. ASCE[1]

Died April 25, 1946

William MacIntire White was born in Philadelphia, Pa., on January 28, 1873, the son of Thomas Earle White and Emmaline (Dunot) White. He was graduated from the University of Pennsylvania at Philadelphia with the degree of Bachelor of Science in Engineering in 1892, and received the degree of Civil Engineer in 1893.

In the fall of 1893 he began his engineering work as second assistant in the district surveyor's office in the City of Philadelphia. In March, 1894, he became a draftsman at the Pencoyd Iron Works, and after two years was made an assistant in the engineering department. He remained with the Pencoyd Iron Works until 1899, when his company recommended him to the Union Pacific Railroad Company. He was employed in the Omaha (Nebr.) office of the Union Pacific as assistant engineer in the bridge department for a year and a half, and also worked for the Baltimore and Ohio Railroad Company for several months as assistant engineer in the bridge department.

Mr. White returned to the American Bridge Company in December, 1900, where he remained as assistant engineer, estimating and designing at Pencoyd, in Pittsburgh, Pa., and in New York, N. Y., until 1906. During this period he had charge of the work on the reconstruction of the Poughkeepsie Bridge in 1905, inspecting the bridge and designing changes to strengthen it.

In 1906 he was granted a leave of absence to go to Peru. The Peruvian railroads were expecting to install heavier rolling equipment and had asked the American Bridge Company to recommend an engineer to inspect the bridges to see if they could stand the increased loads. He inspected the sixty-five bridges of the Ferrocarril Central del Peru and the Ferrocarril del Sur Peru, and submitted a report recommending the necessary repairs and renewals. When this tour of duty was completed, he returned to the American Bridge Company for six months before being sent to China in June, 1907. In China, where he remained a year and a half, he was resident engineer for the United States Steel Products Export Company with headquarters in Shanghai; he designed the Canton River Bridge and other bridges; and he also sold various U. S. Steel products.

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[1] Memoir prepared by William G. Grove, M. ASCE.

When he was in China, Mr. White traveled quite extensively, sometimes for business reasons and sometimes to see the country. He made a trip to the great wall where he visited the chief engineer of the railroad who had been a Chinese student at the University of Pennsylvania. Chinese art interested him and he brought home some lovely pictures and embroideries. He got along well with the Chinese businessmen and admired the Chinese people very much.

While in Peru, Mr. White had an unusual experience. One day when he was inspecting a bridge over a deep canyon, his Mexican helper, stationed at the mouth of a tunnel to warn him if a train should be coming, fell asleep while he was out on the bridge. Mr. White looked up just in time to see the train coming out of the tunnel and on to the bridge, and was forced to drop down and hang on to the ties as the train thundered over his head.

Returning to the United States in 1909, he was located in the Chicago (Ill.) office of the American Bridge Company and later in the New York office of the chief engineer as engineer, estimating, designing, and handling special contracts. In 1913 he was transferred to the contracting department and was a contract manager in the New York office until his retirement in 1938, after forty-three years of continuous service.

Mr. White lived in Fanwood, N. J., from 1909 until his death in 1946, and had been active in the civic affairs of that town. He served on the board of health for several years and was the chairman of the building committee of the board of education when the township high school was built. He was the president of the civic association and active on the defense council during World War II.

In 1913 he was married to Katherine Brown of Plainfield, N. J. They had two children. Mr. White is survived by his widow; a son, William M., Jr.; a daughter, Katherine (Mrs. E. R. Beecher); and two grandchildren, William M. White, III, and Katherine M. Beecher.

Throughout his long business career he had the happy faculty of making lasting friendships and his many friends will remember him for his high ideals of living.

Mr. White was elected an Associate Member of the American Society of Civil Engineers on January 2, 1901, and a Member on March 2, 1909. He became a Life Member in January, 1936.

RALPH WHITMAN, M. ASCE[1]

Died February 3, 1946

From the date of his commission as Ensign until he reached the age limit for active service, when he was retired as Rear Admiral, Ralph Whitman was on active duty as an officer of the Civil Engineer Corps of the United States Navy. This covered a period of nearly thirty-seven years and was the principal part of his professional life. It is one of the longest tours of active duty by any member of the Civil Engineer Corps since its inception in the 1860's.

Ralph Whitman was born in Boston, Mass., on April 7, 1880, a son of Kilborn and Ella M. (Wightman) Whitman, and a direct descendant of Edward Winslow, the first governor of the Massachusetts Bay Colony. He was a member of the John Alden family and also of the Briggs family, prominent shipbuilders of sailing vessels from early Colonial days.

Mr. Whitman was educated in the public schools of Boston, and in 1901 he was graduated from the Massachusetts Institute of Technology in Boston with the degree of Bachelor of Science in Civil Engineering. His education was further continued later in life when, as Commander Whitman, he was assigned to the senior class at the United States Naval War College, in Newport, R. I., composed of commanders, captains, and rear admirals. Here he became familiar with the broader problems in naval operations, logistics, and policy which were to be of value to him and to the Navy in later years. In 1923 he was granted a certificate of graduation on completing the course.

On being graduated from the Massachusetts Institute of Technology in 1901, Mr. Whitman was engaged in field and drafting room service for the bridge department of the City of Boston and continued with this department until 1905. From 1905 to 1907, he was employed by the Isthmian Canal Commission on preliminary estimates and studies for the Panama Canal.

As a result of his success in a competitive examination, he was commissioned an Ensign in the Civil Engineer Corps of the U. S. Navy on August 12, 1907. His ability, together with his satisfactory performance of duties, led to the successive promotions to Lieutenant (junior and senior grades), Lieutenant Commander, Commander, and Captain. Finally, on March 22, 1939, he was commissioned Rear Admiral, Civil Engineer Corps, U. S. Navy, having been selected by a board of senior officers as the best qualified on a list of eligibles. He was the senior officer on the active list under the chief of civil engineers for a period of more than five years, and he continued with this rank after retirement from active duty.

His naval career began, when, as Ensign, he was assigned to the public works department of the navy yard in Philadelphia, Pa., from 1907 to 1910, and until 1911, to the Bureau of Yards and Docks of the Navy Department.

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[1] Memoir prepared by R. E. Bakenhus, M. ASCE.

As Lieutenant, j.g., he was the public works officer of the United States Naval Station, Guantanamo, Cuba, in independent charge of the construction work from 1911 to 1913. In 1913 he became public works officer of the United States Naval Academy at Annapolis, Md., and had charge of new construction, and of the maintenance of buildings and grounds until 1917. While at Annapolis he was promoted to Lieutenant, senior grade.

In April, 1917, as Lieutenant, he was detailed to duty as aide on the staff of the United States Military Governor of Santo Domingo, Dominican Republic, and, from 1919 to 1920, he was a member of the Dominican Claims Commission. During this tour of duty in Santo Domingo he was promoted to the rank of Lieutenant Commander.

From May, 1920, to May, 1922, Lieutenant-Commander Whitman was public works officer of the United States Naval Ordnance Plant at South Charleston, W. Va., when he had charge of the construction of the naval armor plant, largely by day labor. While stationed at South Charleston he was promoted to Commander. His course at the U. S. Naval War College covered the year 1922–1923. After its completion he was assigned, appropriately, for two years to administrative duty in the Bureau of Yards and Docks. This tour of duty involved the public works of the Navy as a whole, and their relations to other branches of the naval service.

Subsequently, Commander Whitman was public works officer in responsible charge of new construction and maintenance at one of the largest navy yards, namely—Norfolk, Va., where he remained for nearly four years. In 1929 Commander Whitman was ordered to important duty as public works officer of the navy yard at Mare Island, Calif., and also of the 12th Naval District, comprising northern California, Utah, Colorado, and Nevada. While on this duty, he was promoted to Captain. In 1934 Captain Whitman was again assigned to administrative duty in the Bureau of Yards and Docks. This particular assignment continued until January, 1939. He was then ordered to duty as public works officer of the naval operating base and 5th Naval District at Hampton Roads, Va., and, while on this duty, was promoted to Rear Admiral.

He became a member of the Hepburn Board, created in 1938 by an act of Congress as a naval board for the purpose of investigating and reporting on the need for further naval bases, including those for naval aircraft, submarines, destroyers, and mines. This board was, in reality, created so that the United States might be in a more secure position in the event of a second world war. The recommendations of this board were largely the basis for the important naval shore station development that was finally carried out. Admiral Whitman was the engineer member of the board, his training at the Naval War College being of particular benefit.

Upon detachment from this duty in June, 1939, Admiral Whitman was assigned as public works officer of the 3d Naval District, with headquarters in New York, N. Y., and covering northern New Jersey and New York State. He served for some months in Washington, D. C., as a member of the War Production Board. Admiral Whitman was retired on reaching the statutory age limit of sixty-four years and was detached from all duty on April 30, 1944.

The engineering projects which Admiral Whitman dealt with in his naval career covered an unusual variety, including such items as dredging, waterfront improvements, dry docks, industrial plants at navy yards and the naval armor plant, shipbuilding ways, piers, wharves, housing of all kinds, power plant construction and operation, and railroads and highways in industrial plants, as well as the operation of railway and truck transportation systems and many other incidental types.

Outstanding characteristics of Admiral Whitman were his painstaking ability as an engineer, his unusual devotion to duty, and his unshakable loyalty to his high principles and to the naval service, as well as to his family and friends. His integrity was exemplified in all his undertakings—he was the "Captain of his Soul."

Admiral Whitman served in World War I and was awarded a special letter of commendation with Silver Star, a Victory Medal with West Indies Clasp, and the United States Marine Corps Commemorative Expeditionary ribbon for services in Santo Domingo. For his service in World War II he was awarded a number of commendatory ribbons; posthumously, he was awarded the American Defense Service Medal and the Victory Medal for World War II.

The order retiring him from active duty contained the following statement:

> "The Secretary of the Navy regrets your retirement from the active list of the Navy and takes this occasion to extend to you his heartiest congratulations and appreciation for your long and distinguished service to our Nation. During the time which you have so faithfully and efficiently served, you have witnessed many advancements in the morale, strength and efficiency of the Navy; and you have the satisfaction of knowing that you have contributed to the accomplishment of these results. May I wish for you continued success and many years of health and happiness."

He was married to Frances Guyon Seabrook of Westminster, Md., on December 12, 1916. His wife and daughter, Frances Guyon Whitman, survive him. A sister, (Mrs. William Hanna, and a brother, Kilborn Whitman, Jr., also survive him.

He was a member of the Metropolitan Section of the Society; of the Boston Society of Civil Engineers; the American Association for the Advancement of Science; the Society of American Military Engineers; the American Society of Naval Engineers; the United States Naval Institute; and the Newcomen Society of London, England (American Section).

Admiral Whitman was elected an Associate Member of the American Society of Civil Engineers on November 6, 1907, and a Member on June 24, 1914.

WALTER FRANK WHITTEMORE, M. ASCE[1]

Died October 27, 1944

Walter Frank Whittemore, the son of Walter D. and Philomena Antoinette (O'Gier) Whittemore, was born in Camden, Me., on June 12, 1858. His father was a sea captain, and that may have accounted for young Walter's attraction to the life at sea, which resulted in his signing up as a seaman on a sailing vessel out of New York, N. Y., bound for foreign ports. He was, successively, promoted to second mate and mate, and by the time he reached the age of twenty one, he was master of his own ship and licensed by the United States Government as a sailing master authorized to command a steamship "on any ocean."

As a career, he evidently did not find a sailor's life to his liking, and decided on engineering as the profession of his choice. He was matriculated in New York University, in New York City, and was graduated in 1883 with the degrees of Bachelor of Science and Civil Engineer. On being graduated he was appointed instructor at New York University, and at the same time attended postgraduate courses which led to the degree of Master of Science, awarded him in 1886. In the meantime, in 1884, he had been appointed assistant professor, a position which he held until 1887.

While teaching at New York University, he spent his spare time in the employ of the late Charles B. Brush,[2] M. ASCE, with offices in Hoboken, N. J. He held the position of assistant engineer until 1889, when he was promoted to principal assistant engineer. In 1896 he became a member of the firm, known as Charles B. Brush and Company, and in 1915, upon the death of Mr. Brush, he continued as successor in the capacity of consulting engineer, maintaining offices for that purpose until his death.

While specializing in harbor improvements, Mr. Whittemore was also retained on a great variety of engineering projects, including water works, sewage disposal, and railroad and highway bridges. He designed the piers and buildings of the Hoboken Ferry Company at Barclay Street, in New York City, which were built in 1887 and 1897, and those of the various steamship companies, including the Fletcher Iron Works, for almost the full length of Hoboken, north of the Lackawanna Railroad Terminal on the Hudson River front. Outstanding among these are the piers and buildings of the Holland-American and North German Lloyd Steamship Lines, including a sea wall nine hundred feet long, and three piers one thousand feet long, surmounted by two-story steel pier sheds.

He was appointed a member of the State Highway Commission by the governor of New Jersey in 1920, and served for three years in that capacity.

He joined the New Jersey National Guard in 1897 and was duly commissioned a Captain, then Major, and finally a Lieutenant Colonel, the latter in the Engineering Corps. Rifle shooting was practically his only hobby. In 1900 he won the Individual National Rifle Championship at Sea Girt, N. J. In later years he liked to say that he made good use of that skill, keeping a twenty-two caliber rifle beside his breakfast table with which he accounted for many a rabbit and woodchuck attempting to raid the garden near his house.

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[1] Memoir prepared by John L. Vogel, M. ASCE.

[2] For memoir, see *Transactions*, ASCE, Vol. LXXXIV, 1921, p. 818.

His life at sea evidently endowed him with a rugged constitution; he deplored the softness of the students who could not come to class in the famous blizzard of 1888, whereas, he seemed to find no difficulty in getting to Washington Square in New York City from Brooklyn, only to find an empty classroom. In similar vein, he often told of his father's instructions prior to his going to sea: "Never get in a fight. If you have to hit a man, be sure you are in the right, and then hit him once, so hard that it will end the argument instantly." Mr. Whittemore exemplified this type of person; he was not pugnacious, but he certainly was virile.

Although courteous and kind to all he met, he did not make friends readily; but those whom he did honor with his friendship remained faithful through life. In an era when an "educated" engineer was frowned upon, he maintained that the engineer should be a gentleman of culture, in addition to possessing an extensive technical training.

After his retirement, he enjoyed the active management of his farm, and rode his saddle horse almost daily, from early spring to late fall. The day after Election Day, however, always found Colonel and Mrs. Whittemore installed in a midtown hotel in New York City, which was a more convenient base for attending the theater, concerts, the opera, libraries, museums, and lectures, and for visiting his close friends.

In 1885 he was married to Alice Jayne in Hoboken. He was devoted to his wife, and was deeply affected by her death in 1939. He left no children of his own. A brother, Joseph O. Whittemore, M. ASCE, survives.

His was a long life replete with accomplishment in many fields of endeavor, including public service. Colonel Whitemore was a member of the Marine Society of the City of New York and served as its president, and was a trustee of Sailors' Snug Harbor, a home for seamen. He was also a member of the New York State Chamber of Commerce.

Colonel Whittemore was elected a Junior of the American Society of Civil Engineers on March 6, 1889; an Associate Member on April 6, 1892; and a Member on October 31, 1905.

WILLIAM HOOK WOODBURY, M. ASCE[1]

DIED JULY 5, 1946

William Hook Woodbury was born on August 22, 1876, in Saint Clair, Mo. He received his Bachelor of Arts degree from the University of Kansas at

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[1] Memoir prepared by Wayne A. Clark, M. ASCE.

Lawrence, in 1903, and completed the three-year course required for the degree of Bachelor of Science in Engineering.

Immediately following graduation he began what proved to be an exceptionally busy, varied, and continuous career in his profession. He worked in the west and in the south of the United States, and in parts of Canada—always in connection with railroad activities.

His civil engineering career covered a broad field in which he served as rodman, draftsman, assistant engineer on maintenance of way, resident engineer on construction, inspector, bridge erector, and reconnaissance and location engineer. After 1916, he also engaged in railroad valuation work. He held various railroad engineering positions with the Kansas City Southern Railway Company; the Atchison, Topeka and Santa Fe Railway Company; the Northern Pacific Railway Company; the Great Northern Railway Company; the Soo Line; and the Canadian Pacific Railroad Company.

In 1916 Mr. Woodbury joined the Duluth and Iron Range Railroad Company as valuation engineer and continued with its successor, the Duluth, Missabe and Iron Range Railway Company. In this position he handled the railroad's many problems and controversies with the government that arose in an effort to determine its so-called "excess earnings." He continued in that capacity until December, 1933, at which time he became engineer of maintenance of the same railroad company, holding the latter position until he retired on October 1, 1942, because of physical disability. At this time he moved to Kansas City, Mo., his former home.

Mr. Woodbury became a member of the Society in 1913 and was a regular attendant at the meetings of the Duluth Section during his residence in that city. In 1918 he was active in organizing the Duluth Engineers Club and in 1920 was its president. He also represented that organization on the American Engineering Council and was active in its work. In addition M. Woodbury was a member of the American Railway Engineering Association, serving on its committees from time to time.

Being particularly interested in the development of public improvements along sound engineering lines, he was an active member of the Duluth Chamber of Commerce, and a member and director of the Duluth Automobile Club. He was unassuming, friendly, and exceptionally energetic.

His religious and social affiliations included the Pilgrim Congregational Church and the F. and A. M.

On July 8, 1908, in Kansas City, Kans., he was married to Della Mae Hamilton, who survives him. Two sisters, Amy and Blanche Woodbury, and a brother, Richard, also survive.

Mr. Woodbury was elected a Member of the American Society of Civil Engineers on December 31, 1913.

LOUIS YAGER, M. ASCE[1]

Died November 22, 1946

Louis Yager, son of Frederick R. and Mary (Eberlein) Yager, was born in Germantown, Wis., on July 12, 1877. He was graduated from the University of Minnesota in Minneapolis, with a degree in civil engineering in 1900. While at the university he was a member of the honorary society of Sigma Xi.

When he was graduated from the university, Louis Yager entered the service of the Northern Pacific Railway Company; and, from that time until his death, was continuously in the employ of that company, with the exception of the years 1919 and 1920, when he was Chief Maintenance of Way Engineer of the United States Railroad Administration in Washington, D. C.

He served the Northern Pacific Railway Company in the following capacities: Rodman and inspector from 1900 to 1901; assistant engineer on construction from 1901 to 1902; supervisor of bridges and buildings in Minneapolis from 1902 to 1907; assistant engineer on construction from 1907 to 1910; division engineer in St. Paul, Minn., from 1910 to 1917; engineer on maintanence of way on the lines east of Paradise, Mont., from 1917 to 1919; engineer on maintenance of way in St. Paul from 1920 to 1922; and assistant chief engineer of the system, from 1922.

Among the important work on which he was in charge as construction engineer was the rebuilding of the St. Louis Bay bridges, drawspans for Minnesota and Wisconsin channels, between Duluth, Minn., and Superior, Wis., in order to handle modern rolling stock safely, and provide sufficient openings for lake traffic; and the construction of line from Glendive to Sidney in Montana, a distance of fifty-five miles.

In 1934–1935 he was chairman of the directing committee of the Twin City Terminal Study for coordination of terminal facilities of the various railroads entering St. Paul and Minneapolis, as well as a similar study of the Head of the Lakes Terminals, at Duluth and Superior.

During his service with the United States Railroad Administration in Washington, D. C., he was chairman of a special maintenance committee appointed to study the effects of time and use on the various elements that make up track structure and that are involved in the cost of maintenance of way and structures. The formula developed for each element of track structure, such as ties, rail, ballast, etc., has become known as the "Yager Formula," and is in general use among railroads today.

Mr. Yager's great interest in the affairs of the American Railway Engineering Association is evidenced by his assignments and by the offices he held: Committee III—Ties, member from 1913 to 1920, inclusive; Committee IV—Rail, member from 1922 until his death, serving as vice-chairman in 1937 and 1938; Committee XXI—Economics of Railway Operation, member from 1921 to 1922, inclusive; Committee on Stresses in Railroad Track, member from 1937 to 1940, inclusive, serving as vice-chairman in 1940; Committee on Research Administration, vice-chairman from 1941 to 1943, inclusive; and director from 1923 to 1926, inclusive. He was elected second vice-president in 1927 and served as first vice-president in 1928, and as president in 1929. While he was president he represented the association as delegate to the World Engineering Congress in Japan. In addition to his interest in the American Railway Engineering Association, he was a member of this Society and also of the American Association for the Advancement of Science.

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[1] Memoir prepared by Bernard Blum, M. ASCE.

Mr. Yager attended the House of Hope Presbyterian Church and was a member of its Board of Deacons. He belonged to the Palestine Lodge of the A. F. and A. M. at Duluth, and the Scottish Rite and Shrine bodies at St. Paul.

On August 14, 1902, he was married to Hester Elizabeth Whiteley. He is survived by his widow and a sister, Mrs. Mary Y. Riphenburg of Wilton, Wis.

Mr. Yager was elected a Member of the American Society of Civil Engineers on September 9, 1919.

LAWRENCE RICHARD YOUNG, M. ASCE[1]

Died August 24, 1946

Lawrence Richard Young was born in South Colton, N. Y., on December 26, 1897, the son of Oscar and Flossa (Powers) Young. He was graduated from Clarkson College in Potsdam, N. Y., in 1922 with the degree of Bachelor of Science in Civil Engineering.

Soon after graduation he became associated with the New York Power Corporation, where he worked on a number of smaller hydroelectric power plants under William P. Creager, M. ASCE. This early experience seems to have set him on his course, for the remainder of his professional life, with but few interruptions, was in the field of water resources development and control.

A short period as designer on the Beauharnois project with the late W. S. Lee, M. ASCE, at Charlotte, N. C., was followed by more than ten years of work with the United States Engineer Department. For this government department Mr. Young was engaged on the Passamaquoddy project, the Muskingum (Ohio) Flood Control, the Los Angeles (Calif.) Flood Control, and in related work in the offices at Washington, D. C.; Seattle, Wash.; Kansas City, Mo.; and Little Rock, Ark. Much of his time at the Little Rock office was devoted to the planning of Norfolk Dam.

Mr. Young left the Little Rock office of the U. S. Engineer Department in 1941 to join the Aluminum Company of America in Pittsburgh, Pa., where he made preliminary layouts of a dam for the Fontana site. He went west again as an engineer for the Morrison-Knudsen Company in Boise, Idaho, for whom he estimated and designed the construction plant for the Norfolk Dam,

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[1] Memoir prepared by W. G. Huber, Dist. Mgr., International Eng. Co., Inc., Denver, Colo.

Dale Hollow Dam, and numerous other jobs. A brief trip to South America in the spring of 1945 completed his wartime assignments.

The International Engineering Company, Incorporated, in Denver, Colo., with Mr. Young as vice-president and general manager brought him back to water power development work. This time he collaborated with John L. Savage, Hon. M. ASCE, consultant, on the general design of Bhakra Dam for Punjab Province, India. On this project he had an opportunity to exercise his talent for organizing an engineering force and guiding the arrangement of a large development.

He was less patient with details but indefatigable in his striving for the most effective and economical combination of the major features of a job. Great as was his love of the out of doors, particularly of fishing, his work always was given first claim on his time. This devotion to duty, his sense of justice, and his generosity will long be remembered by his many friends and acquaintances in the engineering and construction fields throughout the United States.

In 1927, Mr. Young was married to Florence J. Quady of Rome, N. Y., who survives him.

Mr. Young was elected an Associate Member of the American Society of Civil Engineers on June 6, 1927, and a Member on September 9, 1935.

GEORGE WILLIAM BORDEN, Assoc. M. ASCE[1]

DIED JUNE 11, 1946

George William Borden was born on August 21, 1881, in Kansas City, Mo. He was one of two brothers, the son of James Thomas and Sarah R. (Felan) Borden. While he was still a child, the family moved to Terre Haute, Ind., where he received his elementary school education. Later his parents established their residence in Paris, Ill., and it was there that he was graduated from high school.

At this period, Mr. Borden evidenced a trait of character which was to remain with him for the rest of his life. His main interest was in people; he delighted in passing on to others, particularly younger people, any knowledge or information which he had gathered. It was not surprising, therefore, that for two years succeeding the completion of his high school course he remained at the school to teach mathematics. However, his personal ambition lay in the field of civil engineering, and he determined that he would make it his chosen profession.

Accordingly, in the fall of 1903, he entered the University of Illinois at Urbana, in the School of Civil Engineering. As has been the case with so many young men, he was forced by circumstances to make his own way, and he temporarily abandoned his college career in October, 1905. He worked for a year as chainman and rodman for the Chicago and Northwestern Railway Company, after which he returned to the university for a short period.

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[1] Memoir prepared by a Committee of the Sacramento Section consisting of Henderson E. McGee and Roy J. Sykes, Assoc. Members, ASCE.

Recognizing the tremendous possibilities for young engineers in the Pacific Northwest, Mr. Borden left the university in April, 1907, and went to the State of Washington where he accepted a job as instrumentman and draftsman with the Oregon-Washington Railroad and Navigation Company. Another opportunity to teach could not be resisted; in October, 1907, he became a member of the faculty of the Lyle (Wash.) High School where he remained for more than a year.

About this time the State of Washington was experiencing a rapid development in its road and highway systems. Beginning with his election as county engineer of Klickitat County, Washington, in January, 1909, and carrying through until 1919, Mr. Borden was intimately connected with the development of roads and highways in that state. He served two two-year terms as county engineer of Klickitat County, and the remainder of the time, with the exception of a short six-month period in private practice. For the State of Washington he was engaged in planning, design, construction, and maintenance of road systems.

In 1919 he was appointed assistant state highway engineer of the State of Nevada, and, from 1921 to 1927, he was state highway engineer for that state. During these periods a great many of the important highway systems in Nevada were planned, and construction work was initiated on several.

In 1927 Mr. Borden became sales engineer for the Santa Cruz Cement Company in San Francisco, Calif., and, from 1929 through 1932, he was assistant manager of sales in the asphalt department of the Shell Oil Company. In 1931, while on this latter assignment, he worked with the Argentine Government, spending six months in that country furnishing expert advice on the use of asphalts in road building. From 1933 to 1941, he was, successively, park engineer and associate engineer with the National Park Service, engaged on the development of roads in the national parks of the West.

In 1941, just before the commencement of World War II, during the period when the national defense program was being undertaken, Mr. Borden felt that he should make his engineering experience available to this program. Accordingly, he became a member of the staff of the United States Engineer Office in Sacramento, Calif., where he served until his death. During this latter engagement all the years of experience in road building were channeled into the development of military airfields and other military installations; his contribution to the war effort, through this service, was of considerable consequence and importance.

Mr. Borden was a member of the F. and A.M., and while at the University of Illinois was affiliated with the Triangle fraternity. He also belonged to the Society of American Military Engineers.

In his work, he was steady and meticulous, and he delighted in having placed under his direct supervision young men whom he could father and nurture in the highest ethical principles of the profession. As a result, there are many men in the states of Washington, Nevada, and California who owe their earliest practical professional training to the guidance given them by Mr. Borden.

Mr. Borden was justly proud of his family. In October, 1909, he was married to Catherine Knox of Lucas, Wash. During the years of their married life Mrs. Borden was a constant inspiration both to him and to his many friends and associates. His widow and two sons, Thomas Knox and Donald Spencer, survive him.

Mr. Borden was elected an Associate Member of the American Society of Civil Engineers on November 26, 1918.

LYNN MOORE BURROWS, Assoc. M. ASCE[1]

DIED FEBRUARY 10, 1944

Lynn Moore Burrows was born in Saginaw, Mich., on June 17, 1884. He was the son of Lorenzo and Julia Louise (Moore) Burrows. His family had recently moved to Saginaw, but they returned to Albion, Orleans County, N. Y., shortly thereafter, where they lived the remainder of their lives.

The Burrows family, of pioneer stock, migrated from Connecticut to western New York in 1824. Their activities in the political, financial, commercial, and civic life had much to do with the development of western New York. This section became a prosperous region having varied interests that contributed to progress and expanding growth. Mr. Burrows' life and affairs were greatly influenced by these activities and by the progress of the region.

Mr. Burrows received his early education in private schools, and later attended Rensselaer Polytechnic Institute, Troy, N. Y., from which he was graduated in 1907 with the degree of Civil Engineer. Immediately thereafter he took charge of sewer construction for the Village of Medina, N. Y. At that time, the State of New York had just begun to enlarge the Erie Canal into what became known as the New York State Barge Canal. This project was to employ many new methods of construction which brought out the ingenuity and talents of the engineering profession. Mr. Burrows started work during the year following his graduation as assistant engineer in charge of construction of one of the large sections of the Barge Canal. This had a broadening influence on his engineering life, which was later very valuable to him. Since he was small of stature, this interesting out-of-door work strengthened him physically as well as mentally and he often referred to the unusually beneficial effects of such work. During this period, he was in close contact with many men and many problems connected with this large project, as it was then considered. By nature he was a little reticent, and not blustering. He knew how to make friends and to get along with his compeers, as well as those in lesser positions, and this brought him very successful results.

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[1] Memoir prepared by Schuyler Hazard, M. ASCE.

Likewise, after the years on the Barge Canal project that so broadened Mr. Burrows' outlook in engineering matters, he applied his unusually potent talents to the solution of problems of development and opening of large areas of rich virgin lands for intensive agricultural treatment and use. Although he was not interested directly in this engineering problem, he took note of its progress. These large areas amounted, in the aggregate, to a little less than fifteen square miles. They were the remains of prehistoric swamps, and were rather flat and level tracts of large and continuous sections. They were generally referred to as muck lands. To prepare these areas for cultivation required the application of well-balanced engineering talent, since drainage of the land produced many problems.

Mr. Burrows was particularly tactful in straightening out ownership of the land in this general area. The region concerned reached from the north state line of Pennsylvania to the shore of Lake Ontario and from the Niagara frontier on the Niagara River eastward to the Genesee Valley near Rochester, N. Y. In early days, it was referred to as "The Genesee Country." In general, the region had been surveyed in the latter part of the eighteenth century, following out "The Hundred-Thousand Acre Tract," "The Connecticut Tract," and similar grants. The original notes of these different portions had not been properly preserved in many cases, and some had been kept with meticulous care. Many of them, when placed together and studied, made it possible to map the entire territory. By deep analysis and the diligent application of the true spirit and resourcefulness of the engineer, it was possible to bring this land into proper juxtaposition, but strict application and painstaking tramping over the areas were necessary. Although Mr. Burrows did not have direct responsibility, he was very much intrigued, and gave his ideas and counsel which aided materially in finding the right answer.

Mr. Burrows spent considerable time and effort in land surveying, and engineering and drainage, from April, 1909, to December, 1924. He was greatly interested in the design and construction of, and the surveying for, water distribution, sewer systems, and drainage—to which he gave his close attention for fifteen years. During this period he worked in partnership with others, but the various projects with which he was concerned were completed largely by his own efforts.

Mr. Burrows' forefathers, from pioneer stock, early foresaw the possible future value of the region in which they, and later he, lived, and had given an impetus to its development. One result of this background was his interest in the spanning, by means of bridges, of the Niagara River Gorge at the frontier along the International Boundary Line between the United States and Canada. Such bridges were primarily for railroad traffic, but were designed for combination uses to include vehicular and pedestrian traffic as well. As greater demands were made, these facilities had to be increased and revised, and Mr. Burrows was given charge of designing the improvement of the approaches to the Niagara Railway Arch Bridge at Niagara Falls, N. Y. He was consulting engineer and in charge of construction for such improvement, including the consequent changes in affected water and sewer systems—

water and sewer systems being his specialities. About this time he became a director of the Niagara Falls International Bridge Company at Niagara Falls, but still resided at Albion. Also, during this period, greater demands were made on Mr. Burrows, but he had considerable versatility and aptitude; and, when called upon, he undertook the work of designing and supervising the construction of the sewer extension and disposal of factory wastes for the Village of Albion. Concurrently, he made surveys for subdivisions of property for city planning in the general region referred to as western New York.

In addition to his duties connected with the office of director, he was chosen as president of the Niagara Falls International Bridge Company and filled that position with honor and success, although part of that time illness interfered somewhat with his activities.

Mr. Burrows was a likable man, and had many friends in and out of the engineering profession. He was quiet and unassuming. An ardent yachtsman, his love for yachting was a great help in maintaining his health and spirits, and he took delight in annual cruises in his yachts. In earlier days he enjoyed sailing craft, and later motor cruisers. He always took pleasure in having his friends accompany him on cruises (including a trip along the Canadian shore, with suitable stops en route, and many shorter trips across Lake Ontario and along the shore of the lake east and west of his home port of "Oak Orchard-on-the-Lake," where he had his summer quarters for rest and recreation).

Mr. Burrows was a member of the Rensselaer Society of Engineers; the Officers Reserve Corps (Captain of Engineers); the University Club of Rochester; the Oak Orchard Yacht Club ("Commodore"); the Rochester Yacht Club; the Town Club of Albion; Renovation Lodge of Benevolent and Protective Order of Elks at Albion; American Red Cross (treasurer of the local chapter); the Board of Commissioners of Mt. Albion Cemetery (treasurer); Albion Chapter of International Rotary; and the Board of Directors of Arnold Gregory Memorial Hospital at Albion. He was a Republican and belonged to the Christ Episcopal Church at Albion.

On July 24, 1907, Mr. Burrows was married to Beatrice Signor of Albion. He is survived by his widow.

Mr. Burrows was elected an Associate Member of the American Society of Civil Engineers on December 15, 1924.

WILLIAM ROBERTS CONARD, Assoc. M. ASCE[1]

Died April 30, 1946

William Roberts Conard, the son of William and Julia Ann (Powell) Conard, was born in Burlington, N. J., on May 19, 1872. His father was the great-

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[1] Memoir prepared by Howard T. Critchlow, M. ASCE, and Wilfred G. Conard, Asst. Canal Supervisor, New Jersey Dept. of Conservation, Trenton, N. J.

great-great-grandson of Thomas Kunders (who later became known as Dennis Conard) and his wife, Elin, who settled in Pennsylvania in July, 1683. A member of the Society of Friends, Thomes Kunders came from Crefeld, on the lower Rhine in Germany, near the Netherlands border. Before sailing for America, Thomes Kunders, or Conard, obtained from Lenart Arets a warrant for five hundred acres of lands that were formerly purchased from William Penn. These lands lie in the northern part of the City of Philadelphia (Pa.), later known as Germantown. The seven children of Thomes and Elin and their offspring have spread the family of Conards, Cunards, Connards, Conrads, and Conreds throughout Chester, Montgomery, and Philadelphia counties in Pennsylvania and the bordering states of Delaware and New Jersey.

William Roberts Conard was the youngest son of William, who was employed by A. H. McNeal and Company in the manufacture of cast-iron pipes in Burlington. He was educated in the public schools and at Trenton Business College in Trenton, N. J., and worked in various clerical capacities while still a young man. During his employment with the Underwood Ink Company in Brooklyn, N. Y., he was married to Carabelle Topping, a descendant of Robert Humphrey Nichols, a sailing master in the American Navy during the War of 1812.

After his marriage, he moved to Burlington, where he built up a business in inspection and testing engineering. Throughout the United States Mr. Conard was also widely known in the water works field and, as a consultant, was engaged by many communities in the eastern part of the United States, as well as by some communities in Mexico and South America. During World War I, he was with Hazen, Whipple and Fuller, constructing the pumping station and supply main to Camp Dix, New Jersey, and also at Mays Landing, N. J., where he developed the water and sewage systems for the Bethlehem Loading Company.

At the end of the World War I, he entered into partnership with John Stewart Buzby, who had previously been his principal assistant. Conard and Buzby expanded their business; it finally covered the inspection of cast-iron pipes and fittings imported into the United States from Europe with inspectors on both continents. They also branched out into the construction field, dealing principally with the installation of hydraulic equipment, with work involving mass concrete, and with bridge building.

Of an inventive turn of mind, Mr. Conard designed and developed many items used in the water works field. He was interested in the General Pipe Cleaning Company of Philadelphia, which engaged in a business of hydraulic pipe line cleaning. Always progressive, and always one to avail himself of the latest developments and modern trends, it is remembered that he was the owner of the first shaft and gear driven bicycle in town, a Pierce Arrow. When the automobile appeared, again he was owner of one of the first in town.

Mr. Conard was very fond of gunning and, when time permitted, spent many hours in the field. In later years this was supplanted by golf at the Burlington County Club where he was a life member. During World War II, he found time to take his turn at an aeroplane observation post. He devoted much time and energy to civic betterment; he advocated city zoning; he was a member of the Young America Fire Company of Burlington; and he actively worked for improved fire protection. All his life he was a Republican in politics and a member of the Union League of Philadelphia. He was a member and trustee of Broad Street Methodist Episcopal Church of Burlington as well as a member of, and officer in, numerous fraternal organizations. In Burlington, he had been postmaster, a member and president of the board of education, and secretary of the Burlington Postwar Planning Board. In the State of New Jersey, he served on the Water Policy and Supply Council of the Department of Conservation, and was a licensed professional engineer.

Mr. Conard was a man of sterling character, reticent and dignified, and in conversation he impressed his interlocutor with complete and instinctive confidence in his ability and integrity.

Despite his many interests, Mr. Conard was very active in the American Water Works Association and was one of the founders of its New Jersey section of which he served as chairman in 1937. He was a member of the New England Water Works Association, the American Society for Testing Materials, and the Engineers Club of Philadelphia.

He is survived by his widow; two sons, Wilfred George and Robert Powell Conard; and two daughters, Mrs. Carabelle Mitchell and Esther Laurie Conard.

Mr. Conard was elected an Affiliate of the American Society of Civil Engineers on January 3, 1906, and an Associate Member on July 7, 1915. He became a Life Member in January, 1941.

CLARENCE DEXTER CONWAY, Assoc. M. ASCE[1]

DIED JUNE 19, 1946

Clarence Dexter Conway was born on March 31, 1889, in Millersburg, Ky. His mother, Lida B. Johnson, and his father, William B. Conway, came from prominent Kentucky families. He received his early education in Paris, Ky. In 1905, he was graduated from Millersburg Military Institute, and in the same year entered the University of Kentucky at Lexington, to study civil engineering. In 1906 he came to California and entered civil engineering work, completing his engineering studies by correspondence courses and practical experience.

Mr. Conway's professional career began with construction work for the Western Pacific Railroad Company in the Feather River Canyon, extending from Oroville eastward to the California state line, from 1906 to 1910. On this project he was instrumentman and tunnel inspector part of the time. Following this position he was with the Southern Pacific Railroad Company in the division engineer's office on the construction of the Natron Cutoff in Oregon.

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[1] Memoir prepared by a Committee of the Sacramento Section consisting of Martin C. Polk, M. ASCE, and Fred T. Robson, Assoc. M. ASCE.

In 1911 and 1912 he was in charge of subdividing and laying out an irrigation system for a large tract of land consisting of about twelve thousand acres in the Sacramento (Calif.) Valley, known as the Los Molinos Land Colony. The next year he was in charge of maintenance and operation of the irrigation system for the same project; until March, 1916, he was occupied in intensive land development and irrigation engineering on various projects in the State of California. He then returned and became manager for the Los Molinos Land Company and chief engineer of the Coneland Water Company which irrigated the colony lands. Thus, Mr. Conway had complete charge of mainnance and operations of the irrigation system and management of the land company. He remained in this work until his death. During the time of his management of the land company he also specialized in, and had a large practice in, land development and irrigation engineering.

The Sacramento River and its problems of flood control, navigation, and irrigation especially interested Mr. Conway. He knew every foot of its length through Tehama County, California, and the damage that was being done by floods. Continually striving to have this menace curbed, he was untiring in his efforts to get help from federal and state sources and used moving pictures of the large floods on the Sacramento River to further the interest of flood control work. He was consulted in an advisory capacity by the state engineer's office and by the United States Army Engineers in charge of flood work on the Sacramento River, and was a member of the flood control committee of the chamber of commerce of the State of California.

During his thirty-five years as a resident of Tehama County, Mr. Conway was always a leader of any movement that might better the citizens of the county, either mentally or materially. For fifteen years he was a member of the board of the Corning (Calif.) Union High School District, as well as being a very active member of the Los Molinos Mosquito Abatement District, the Tehama County Planning Commission, and the Tehama County Probation Board. His efforts, in fact, were untiring in every way for the benefit of the fellow citizens of his community.

In 1917 he was married to Della E. Rowe of Oroville. He is survived by his widow, and two sons, Richard D. and William E. Conway.

Mr. Conway was elected a Junior of the American Society of Civil Engineers on November 28, 1916, and an Associate Member on January 19, 1920.

GWYNNE WALLACE ELLIS, Assoc. M. ASCE[1]

Died October 9, 1945

Gwynne Wallace Ellis, the son of Augustus Wallace Ellis and Katherine (Axline) Ellis, was born in Colfax, Wash., on October 23, 1882.

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[1] Memoir prepared by Chester A. Smith, M. ASCE.

In 1883 his parents moved to Pratt, Kans., where Mr. Ellis spent his boyhood days and attended the grade and high schools.

He received the degree of Bachelor of Science in Civil Engineering from the University of Kansas at Lawrence, in 1908.

During his senior year at the university, Mr. Ellis, in conjunction with another senior engineering student, prepared the plans and specifications for the sanitary sewer system for his home town. Upon his graduation from the university, he was appointed city engineer of Pratt and supervised the construction of the sewer system.

In November, 1909, he entered the employ of the Burns and McDonnell Engineering Company in Kansas City, Mo. He served as resident engineer in the supervision of construction of the sanitary sewers for Washington, Minneapolis, Osage City, and Harper in Kansas and the water works system for Sapulpa, Okla. He resigned from Burns and McDonnell Engineering Company in September, 1913, to engage in private consulting practice. In 1914 Mr. Ellis was appointed city manager of Osage City.

During World War I, in 1917, Mr. Ellis was commissioned as Captain in the United States Army Engineering Corps and, after finishing officers' training camp, was assigned to the 110th Engineering Regiment. Upon his discharge from the army, he again engaged in private engineering in Pratt, where for about four years he did considerable work on the design and construction of sewers, water systems, and paving, and also on power plant construction for several cities in the southern and western sections of Kansas.

In 1921, he moved to California and purchased a home in Glendale. There he entered the contracting business as G. W. Ellis, Contractor. Street and highway paving were the principal types of work required in his contracts. With the entry of the United States in World War II, Mr. Ellis placed his equipment for use on war projects, and maintained a repair yard for equipment from near-by localities as well as from the war zones.

Mr. Ellis was highly respected among engineers, contractors, material salesmen, and others with whom he was in contact for his honesty, integrity, and straightforward decisions. In his early engineering experience as supervisor of construction, he demanded careful work and adherence to the plans and specifications, and carried the same principles into his contracting work.

Although he led a very busy life with his engineering regardless of the type on which he was engaged, he spent his leisure hours reading good books and technical magazines. Also, although not active in outdoor sports, he enjoyed attending baseball, football, and other manly sports events.

On June 19, 1912, he was married to Reba Bauman of Osage City. He is survived by his widow, and his son, Ralph.

Mr. Ellis was elected an Associate Member of the American Society of Civil Engineers on September 3, 1913.

DAVID FERGUSON, Assoc. M. ASCE[1]

Died December 9, 1945

David Ferguson was born in Inverness, Scotland, on July 18, 1892. He was the son of John and Mary (Groat) Ferguson. When he was four years of age, his family settled in the vicinity of Boston, Mass., where he received his education. He attended grade school and high school at Cliftondale and Methuen in Massachusetts. He received his engineering education at Tufts College in Medford, Mass., and was graduated in 1916 with the degree of Bachelor of Science in Civil Engineering.

Following his graduation from college, from August, 1916, to August, 1917, he was employed by the War Department on survey work for the Mississippi River Commission at Memphis, Tenn.

During World War I, from October, 1917, to May, 1919, Mr. Ferguson served in the armed forces of the United States. In 1918, he saw service in France with Battery E, 319th Field Artillery of the Eighty-Second Division. His unit took part in the Saint-Mihiel and Meuse-Argonne offensives.

After the war he was with the United States Engineer Office at Memphis, as chief of survey party, from 1919 to 1923. In this capacity he surveyed the shore lines of the Mississippi River from Cairo, Ill., to the mouth of the White River in Arkansas.

From May, 1923, to November, 1927, Mr. Ferguson was employed by several steel fabricators: The Ingalls Iron Works; the Tennessee Coal, Iron and Railroad Company; the Virginia Bridge and Iron Company (Birmingham, Ala.); and the Richmond Structural Steel Company (Richmond, Va.). In November, 1927, he entered the service of the Chesapeake and Ohio Railway as a structural designer and remained with that company until June, 1930.

Mr. Ferguson was a structural engineer with the Treasury Department (Supervising Architect's Office and Procurement Division), and the Federal Works Agency in Washington, D. C., from 1930 to 1941. He was responsible for the preparation of structural drawings for various types of federal buildings throughout the United States. The court house and federal building at Salt Lake City, Utah, and the gold depository at Fort Knox, Ky., are examples of important structures which he helped to design.

In February, 1941, Mr. Ferguson returned to the War Department, as chief of the structural unit, Airport Section, Office of the Chief of Engineers. In addition to the usual airport construction, he was responsible for the review of plans and studies for airplane hangar construction submitted to the Office of the Chief of Engineers. In May, 1942, Mr. Ferguson was appointed assistant chief of the Fortifications and Protective Design Section, Engineering and Development Division, Office of the Chief of Engineers, and worked in this position until his death in December, 1945. The work included planning, tests, and designs of all kinds of protective structures, such as land and seacoast fortifications, submarine mine installations, and fixed antiaircraft defenses. The sound manner in which he attacked all engineering problems won him the respect of fellow engineers.

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[1] Memoir prepared by Christian Beck, M. ASCE, and Walter G. Hiner, Structural Engr., Office, Chf. of Engrs., War Dept., Washington, D. C.

Mr. Ferguson had been active in the work of the Baptist Church at Clarenden and Manassas, in Virginia. He was a member of the F. and A. M., Lewis Ginter Lodge No. 317 of Richmond. David Ferguson was a man of sterling character and high principles. Although of slight stature, he was energetic and fearless, and a man firm in his convictions. A characteristic sense of humor and friendly interest in others will long be remembered by his friends and associates.

Mr. Ferguson was married on May 14, 1918, in Long Island City, N. Y., to Hazel (Todd) Merrill. He is survived by his widow; one daughter, Evelyn; his father, John Ferguson; three sisters, Anna (Mrs. R. R. Gibson), Emma (Mrs. J. A. Henderson), and Margaret (Mrs. C. E. Wilkinson); and a brother, Daniel Ferguson. Funeral services were held at the Fort Meyer Chapel and burial was in the National Memorial Cemetery, Arlington, Va., on December 12, 1945.

Mr. Ferguson was elected an Associate Member of the American Society of Civil Engineers on March 12, 1923.

CHARLES NEEDHAM FORREST, Assoc. M. ASCE[1]

Died December 2, 1946

Charles Needham Forrest was born on September 30, 1873, in Baltimore, Md. He was the son of Andrew Jackson and Emily (Dorsey) Forrest.

In Baltimore, he attended the public schools and the Maryland Institute, and began his professional career when he entered the employ of the Baltimore and Ohio Railroad Company, in the Test Department. In this department he became, successively, assistant in the chemical and physical laboratory and principal assistant chemist.

In 1896 he was employed by the Southern Railway Company as chemist in the Test Department in Washington, D. C. During this year he took advantage of the evening courses that were being held at Columbian University (later George Washington University) in Washington, D. C., where he specialized in chemistry under the very eminent Charles Monroe.

In 1897 he became chemist and inspector for the Long Island Railroad Company in Long Island City, N. Y., remaining there until 1904. During this period he was active on the important projects of the Port Washington extension of the railroad and on laboratory and mill inspections. Later he supervised the chemical and physical laboratory work for the Atlantic Avenue

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[1] Memoir prepared by J. Strother Miller, Asphalt Technologist, Rahway, N. J.

Improvement project in Brooklyn, N. Y., and, for the last three years, served virtually as engineer of tests and assistant purchasing agent.

In 1904 Mr. Forrest was associated with Clifford Richardson as chief chemist in the New York Testing Laboratory at Long Island City. When the New York Testing Laboratory was moved to Maurer, N. J., in 1907, he was in full charge of the design and control of street and highway paving, and materials entering this field, particularly asphalt. In addition, he supervised the operations of the largest firm of asphalt paving contractors at that time, the Barber Asphalt Paving Company. When the laboratory was taken over by the Barber Asphalt Company, he continued with that corporation until his retirement, under its pension plan, in 1941. For these many years he served as manager of the Technical Bureau, entirely supervising paving operations, and plant operation, in both asphalt and oil refining. In 1927 he made a world tour in the interest of paving operations in various countries.

During the early years of improved highway paving, Mr. Forrest lectured on this subject. At that time he also took an active part in the meetings of the Society, even before the Highway Division was formed. Throughout the period of great specialization in asphalt and its many applications, Mr. Forrest served as an expert in many court cases including some very interesting patent suits.

In addition to the Society, he was a member of the American Society for Testing Materials (A.S.T.M.), the American Chemical Society, the Society for Chemical Industry, Franklin Institute (Philadelphia, Pa.), the Association of Asphalt Paving Technologists, the Society of Municipal Engineers, and the American Association for the Advancement of Science (a fellow).

He was most active in the A.S.T.M., serving on Committee E-1, Methods of Testing; Committee E-8, Nomenclature and Definitions; Committee D-1, Paint, Varnish, Lacquer and Related Products; Committee D-4, Road and Paving Materials (past-chairman); Committee D-2, Petroleum Products and Lubricants; and Committee D-8, Bituminous Waterproofing and Roofing Materials (past vice-chairman). He also served a term on the executive committee (later the board of directors) of A.S.T.M.

These activities are indicative of the leading part he had in standardization and research work for road and paving materials, waterproofing compounds, roofing, and petroleum products. He wrote many technical reports and papers presented before meetings of the A.S.T.M. and other societies. Mr. Forrest also served as chairman of the Advisory Research Committee of the Asphalt Roofing Industry Bureau in New York, N. Y., and continued in this capacity for several years after retirement from active corporate work.

His placid and pleasant personality, his wit, and his humor won for him many friends—in and out of his profession—at home and abroad. Many times when an impasse developed at a committee meeting, the situation was saved by his wit and humor, along with his ability to explain the technical necessities of the situation. Only his breadth of experience in his chosen fields could have solved many such situations.

He is survived by his widow, the former Cora Linthicum, and their two sons, Arthur Linthicum and Charles Dorsey.

Mr. Forrest was elected an Associate Member of the American Society of Civil Engineers on April 6, 1909.

JOHN HARRISON FOSS, Assoc. M. ASCE[1]

Died January 5, 1946

The life and career of John Harrison Foss well exemplify the widely varied influences and types of engineering, the rapid development, and yet the secure economic and technical controls of that most intriguing outpost of America—Hawaii.

Born in Loleta, Humboldt County, Calif., on January 7, 1879, the young and vigorous life of the West was a part of his heritage. The Foss family is of Norwegian extraction, their original American members being shipowners and traders. They became permanent residents of Maine in about 1700, although they may have engaged in American trade earlier. James C. Foss, the father of John Harrison Foss, came to California from Saco, Me., during the gold rush of the early 1850's. Lavina Dickson was born near Digby on the Bay of Fundy in Nova Scotia, Canada. Her family were of British and French Huguenot extraction and had been Tory refugees from New York during the Revolutionary War. She came to California to visit a brother, and there she met and married James C. Foss. This pioneering ancestry, and the frontier life in this early and remote California settlement contributed to the dependability and rugged independence of mind and character notable in their son John.

His career in engineering began with his matriculation at Leland Stanford Junior University, Stanford University, Calif., as a member of the class of 1903. In those early days of Stanford, the influence of that remarkable group of young men, the late C. D. Marx,[2] Hon. M. and Past-President ASCE, and the late C. B. Wing and J. C. L. Fish, Members, ASCE, who were the civil engineering faculty, was far more direct and potent a force in the formation of professional and personal ideals and character than can be the case with huge schools and mass education. The training fell on fertile ground. Since Mr. Foss was gifted with an acute and penetrating mind, an amazing patience, and a capacity for hard and thorough work, no economic or engineering problem ever appeared insoluble to him. It was merely a call for long hours of extra work, frequently late at night. The sound grasp and the amazingly detailed mastery of highly complex problems that he so frequently displayed were the results of his added efforts.

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[1] Memoir prepared by Joel B. Cox, Assoc. M. ASCE.

[2] For memoir, see *Transactions*, ASCE, Vol. 105, 1940, p. 1785.

Immediately following his graduation from Stanford in 1903, at the age of twenty four, Mr. Foss made his first contact with what was to prove the major work of his life—the construction and development of the collection system for mountain water from the slopes of Haleakala on east Maui, to irrigate the fertile isthmus between east and west Maui in Hawaii. Again it was a time of stimulating contacts, and of rapid growth and development. Annexation of Hawaii by the United States had brought renewed growth to the sugar industry, and one of Hawaii's greatest men, H. P. Baldwin, was at the height of his career. He had initiated the use of east Maui water for irrigation twenty-five years before, and now the full utilization and development was to begin. The late M. M. O'Shaughnessy,[3] M. ASCE, later better known as engineer of the City of San Francisco (Calif.), was employed to build the Koolau Ditch—an aqueduct with a capacity of seventy million gallons per day, extending the system of collection to a distance of thirty miles from its use. To bring this water to the Maui Agricultural Company's fields, the new Hamakua Ditch was built simultaneously.

Employed on the construction of the Hamakua Ditch as a young engineer, Mr. Foss rose in the short time of fourteen months to the position of engineer in charge, completing twelve miles of tunnels, siphons, and open ditch around the rugged flanks of ten-thousand-foot volcano. With the completion of the new Hamakua Ditch in 1905, Mr. Foss moved to the plantation proper as a civil engineer. He remodeled and extended the irrigation system; rebuilt the railroad to transport the greater volume of cane to the mill; built reservoirs, distributing ditches, flumes, siphons, weirs and regulating works, and housing for plantation employees; and constructed the other myriad structures necessary for so complex an enterprise.

After this introduction to the full scope of sugar plantation engineering, Mr. Foss was drawn back to Stanford for a period of twelve years as assistant professor of civil engineering. His clarity of thought, warm and human approach to his students, and his endless patience made him a most valuable teacher, particularly in drafting room courses in which he specialized.

Even during this academic period he was closely associated with the growth of Maui. He returned for a sabbatical year in 1912 to plan and largely build the Honolula Ditch, an eight-mile aqueduct in West Maui. He was previously married to a Maui girl, Irene V. Crook, in 1909. Two other members of his family, a brother, J. C. Foss, also a civil engineer, and a sister who was married to David T. Fleming, a Maui pineapple and cattle ranch manager, made their homes on Maui. Thus, it was as a member of the close-knit island community, rather than as a stranger, that he returned in 1919 to head the civil engineering staff of the group of corporations owning and operating the majority of Maui's plantations (both sugar and pineapple), ranches, and ports.

From this time until his death in 1946, Mr. Foss' professional life was a summary of the progress of his community. During this quarter of a century, the production of sugar increased nearly fifty per cent, a great pineapple industry rose and became established, and the community prospered and matured. His hand was everywhere. From the architecture of its churches to his latest improvement for the handling and shipping of sugar in bulk in specially designed ships, it was his designing, ability, and sound economic judgment which insured success.

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[3] **For memoir, see *Transactions*, ASCE, Vol. 100, 1935, p. 1710.**

His four children grew to maturity on Maui—John Harrison Foss, Jr; Capt. William Crook Foss, who was killed in action on Okinawa in May, 1945; Francis Dickson Foss; and Nancy Virginia Foss. Mrs. Foss died in October, 1931, and in June, 1940, Mr. Foss was married to Helen E. Van Keuren who survives him.

The wide range of his civic interests and his sense of public duty are reflected in the long list of memberships in community enterprises listed in "Who's Who in Engineering."

The Wailoa Ditch, with ten miles of concrete lined tunnels and a capacity of one hundred and eighty million gallons per day, completed in 1923, was his most important work from a technical standpoint. Not only is it the greatest in capacity of Hawaiian aqueducts but, in its use of automatic gates, high concrete flumes, and self-supporting steel pipe bridges, it is a milestone in engineering progress in the islands.

The development of hydroelectric power as a by-product of an irrigation system was Mr. Foss' contribution to the economy of the plantation. He was first to develop accurately a statistical measurement of the duty of water in sugar cane irrigation and to determine its economics closely. As a consultant, his services in the reconstruction and completion of the Alexander Dam in 1932 contributed largely to the successful methods of internal drainage which made possible the use of highly colloidal material in hydraulic-fill construction. His exploration of underground water conditions in the basalts of east Maui was an adventure in economic geology whose importance is only beginning to be realized. His latest work was the design of the bulk sugar storage and loading plant at the harbor of Kahului, Maui. This plant has storage room for forty thousand tons of sugar per day and loads four hundred tons per hour into ships.

Throughout this period of achievement, Mr. Foss' personal qualities were a guide and an inspiration to the entire engineering profession in Hawaii. His complete integrity, his careful and painstaking accuracy and dependability, and his patient and comprehensive study and mastery of complex problems were an inspiration. His generous gifts of time and interest in the technical problems of others, and his patient and warm-hearted understanding of personal problems made him deeply beloved. His passing will leave a gap which cannot be filled, but his work will endure and his well-loved island home will profit by his building for generations to come.

Mr. Foss was elected a Junior of the American Society of Civil Engineers on December 1, 1903, and an Associate Member on October 4, 1910. He became a Life Member in January, 1945.

ABRAM NUTE GEORGE, Assoc. M. ASCE[1]

Died May 6, 1946

Abram Nute George, the son of Andrew and Grace (Nute) George, was born in Hastings, Nebr., on May 27, 1890. In 1898, the George family moved to California and settled in San Diego. He was graduated from San Diego High School in June, 1908, and in 1915 completed a civil engineering course through the International Correspondence School.

Between May, 1910, and February, 1912, he worked as chainman and instrumentman for the San Diego County Highway Commission. When the California State Highway Department was organized in 1912, Mr. George started on February 15 of that year as transitman; his name appears on the first payroll of District VII. He soon advanced to chief of party and held this position from the spring of 1914 until September 28, 1917, when he entered the U. S. Army Air Force during World War I.

Upon graduation from the U. S. School of Military Aeronautics at Berkeley, Calif., he was commissioned a Second Lieutenant in the U. S. Air Service and proceeded to France. While in France he taught aerial gunnery and was a pursuit pilot at the front until the end of the war.

On April 7, 1919, Mr. George returned to the Division of Highways of the State of California and held the positions of chief of party, resident engineer, and office engineer until the fall of 1927. At that time he resigned from state service to become chief engineer for the B. W. Kuhn Corporation, Contractors. However, the urge of public service was strong and after a year he returned to the Division of Highways.

Between September, 1928, and November, 1931, he held the position of resident engineer and was in charge of many large and important construction projects. In 1933, his outstanding ability was further recognized by his promotion to acting district construction engineer, and on June 15, 1934, he was advanced to district construction engineer, which position he held until his death. As district construction engineer he supervised the construction of the Arroyo Seco Parkway, the first freeway built in the Los Angeles (Calif.) metropolitan area; the section of the Ramona Parkway in the City of Los Angeles; and many other noteworthy projects.

Mr. George was held in the highest esteem, both as an engineer and as a man of strong character and inflexible principles. These outstanding qualities won for him the respect and admiration of his associates and aided him materially in establishing and maintaining the reputation of the Division of Highways on its present high plane.

In 1924, Mr. George was married to Josephine K. Hirschler. He is survived by his widow; two sons, Abram N. George, Jr., and Jack Alan George; and a sister, Mrs. Emma Sherburn.

Mr. George was elected an Associate Member of the American Society of Civil Engineers on January 19, 1920.

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[1] Memoir prepared by S. V. Cortelyou, M. ASCE.

SUMNER GOWEN, Assoc. M. ASCE[1]

Died February 18, 1947

Sumner Gowen was born in Boston, Mass., on June 30, 1874. He was the son of Franklin Augustus Gowen and Mary Jane (Horne) Gowen and the oldest of four children. His mother came from a Maine family. The Gowen family moved to Wakefield, Mass., where his education was started and where he was graduated from high school. In the Massachusetts Institute of Technology at Boston, he took the civil engineering course and received the degree of Bachelor of Science in Civil Engineering in the class of 1897.

In August of that year, he was employed in the drafting room of the Phoenix Bridge Company at Phoenixville, Pa., and remained with that company for forty-nine and one-half years, until his death in February, 1947 —a record as unusual as it was remarkable.

In a short time after starting in the drafting room, he became familiar with detailing procedure and shop practice and showed such an interest in structural design that he was transferred to the designing department. At that time, the Phoenix Bridge Company principally produced railroad bridges, its contracts including designing, detailing, fabricating, and erecting girder and truss spans, drawbridges, turntables, cantilevers, etc. Railroad companies then generally did not have designing departments and there were relatively few consulting engineers. Bridge fabricating companies furnished designs for the majority of the contracts they executed.

Mr. Gowen's talents were particularly adapted to work of this type as he had a complete and lightning-like grasp of the mathematical phase of design, and also possessed an innate sense of the functional fitness of details and of assemblages. As a result of this happy combination of faculties, any design he turned out was adequate, economical, and most suitable.

Soon after he joined the Phoenix Bridge Company, that firm built the 3,000-ton Cornwall Bridge over the Saint Lawrence River in Canada—including a viaduct, three 368-foot through pin truss spans, one 240-foot drawspan, and an 841-foot cantilever. During the following years, Mr. Gowen contributed largely to designs which were required for bridges all over the United States and for others exported to Central and South America, to China, to Japan, and so on. In 1903 work involving unprecedented problems was begun on the first Quebec cantilever bridge over the Saint Lawrence River—a bridge with an 1,800-foot main span and two 500-foot anchor arms. In 1904 the 8,000-ton Cambridge Bridge, with its eleven plate girder arch spans, was built in Boston.

During his lifetime, Mr. Gowen saw many changes in steel construction and participated in advances in design. In the first two decades of the century, many bridges of various types were required for railroads—changes for line extensions and renewals for heavier rolling equipment. For example, there were numerous orders from railroads such as the Southern Pacific Lines and the Southern Railway System requiring stress sheets. At first, in design and construction practice, plate girders were used for relatively short spans, and trusses were pin connected. Gradually, plate girders were used for longer and longer spans, and in the shorter truss spans riveted joints began to be used. Finally the pin-connected truss became almost obsolete. Where viaducts with 30-foot steel towers and 60-foot spans—or a similar arrangement—had been common practiee, high masonry piers and long span plate girders were preferred. Plate girder and truss swing spans gave way in general to the rolling lift, the trunnion, and later to the vertical lift types of movable bridges. Mr. Gowen's work for the Phoenix Bridge Company was in the front line of this march of progress.

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[1] Memoir prepared by Roberts S. Foulds, Contracting Engr., The Phoenix Bridge Co., Phoenixville, Pa.

One of his attractive designs in 1915 was the 665-ton, 600-foot highway cantilever bridge for the Palisades Interstate Park Commission with an outline like the braced arch for the Popolopen Creek crossing in New York near Bear Mountain Bridge.

With the improvement of highways, accelerated by the automobile, contracts for highway bridge work replaced railroad bridge contracts to a considerable extent. Railroads and state highway departments began to do much of their own designing. For most of the larger projects, consulting engineers were retained. About 1920, the Phoenix Bridge Company began to contract for steel for buildings, and Mr. Gowen made many building designs. To design had been added cost estimating and, as time went on, Mr. Gowen's work included less designing and more estimating until in later years estimating took the major part of his time. In estimating, just as in designing, he could turn out a prodigious amount of work—all carefully done.

Until the last few years, golf was his principal recreation. Earlier he had played a good deal of tennis and had side interests such as photography and literature. An omniverous reader, he had a particular mastery of English history and literature from the time of Elizabeth to George IV. Over the years, his associates could always count on getting help from him and many found contact with his keen mind delightful. He was clear headed and kind of heart.

On May 12, 1909, he was married to Keturah Kreamer Reeves of Phoenixville. His only brother had died at the age of twenty one and his sisters, Jessie (Mrs. Ernest E. Tyzzer) and Ruth (Mrs. Harry B. Fiske), died in recent years. He is survived by his widow and three nephews, Gerald and Franklin Tyzzer and Benjamin Fiske.

Mr. Gowen was elected a Junior of the American Society of Civil Engineers on May 2, 1899, and an Associate Member on June 4, 1902.

JOHN GREER, Assoc. M. ASCE[1]

DIED MAY 23, 1946

John Greer, the son of William and Mary (Greenfield) Greer, was born at Belfast, Ireland, on February 5, 1885. He received his education at Queens University in Belfast, and was graduated with the degrees of Bachelor of Arts and Bachelor of Engineering in 1907 and 1911, respectively.

His professional career began immediately after graduation, and, from 1911 to 1913, he held the position of surveyor with the Irish Land Commission. In 1909, he became an associate member of the British Institute of Civil Engineers.

He emigrated to Canada and from March, 1913, to December, 1914, he was an instrumentman for the Canadian Northern Railway, stationed at Vancouver Island, B. C., Canada. From January to December, 1915, he was an instrumentman at Portland, Ore., on various surveys, including a toll line survey for the Pacific Telegraph and Telephone Company on the new Columbia Highway in Oregon.

During World War I, from January, 1916, to November, 1918, he was with the Canadian Expeditionary Force in France, including one year on railway construction and bridge repair with the First Canadian Railway Troops.

After the war, from March to June, 1919, he was employed as draftsman by the Foundation Company of New York, N. Y., on details in new power plant construction at Sidney, Nova Scotia, Canada. Between July, 1919, and December, 1921, he was employed by the Grand Trunk Lines for one year as assistant engineer on a valuation survey, and for one and a half years as assistant engineer in full charge of maintenance of way and structures of its Portland Division in New England.

In July, 1922, he entered the service of the Maine Central Railroad Company in the capacity of resident engineer, reporting to the engineer of construction. As resident engineer he was responsible for laying out and supervising the construction of various structures, including bridges, roundhouses, shops, and trackwork. He resigned in May, 1926, and for the next six months was employed by the Technical Advisory Corporation of New York, as assistant engineer in charge of making studies and reports for the elimination of grade crossings at Elmira, N. Y.

His next employment, during 1927, was with Allen N. Spooner and Sons of New York, as superintendent of construction, building piers and retaining walls on New York City's waterfront.

The following year he was engaged by the Town of Belleville, N. J., as assistant engineer in connection with a special study it was making of its sanitary sewer system.

Completing this study he entered the service of the Erie Railroad Company as designing draftsman in the office of the engineer of structures. He was engaged principally on design of railroad and highway bridges in connection with grade crossing eliminations. In 1945, he was promoted to designer, which position he held at the time of his death.

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[1] Memoir prepared by W. R. Swatosh, M. ASCE.

Mr. Greer's work was always characterized by accuracy and good judgment. He was a gentleman of very likable personality and made many lasting friendships.

He was married in Belfast on October 9, 1916, to Katherine Gemmel who, with two daughters, Elizabeth Barbara (Mrs. Philip Tremain Johnson) and Mary Clarke, survives him.

Mr. Greer was elected an Associate Member of the American Society of Civil Engineers on February 25, 1924.

ISAAC HAUSMAN, Assoc. M. ASCE[1]

DIED OCTOBER 29, 1946

Isaac Hausman, the son of Dr. Jacob L. and Jennie (Burna) Hausman, was born in Russia on February 5, 1889. His parents, coming to the United States when he was about three years old, settled in Kansas.

He completed his early education there and obtained part of his engineering training at the University of Kansas in Lawrence, finishing his college work at the Massachusetts Institute of Technology in Boston, in 1911, when he was graduated with the degree of Bachelor of Science in Civil Engineering.

During his college years and after his graduation, Mr. Hausman spent some time on field work with the Union Pacific Railroad Company; with C. M. Bolton and Company in Garrison, Mont.; with Tunnel Contractors in Garrison, Mont., and Millwood, N. Y.; and with Post and McCord and George A. Just and Company, both in New York. In 1913 he moved to Toledo, Ohio, to work as structural engineer, designer, and detailer for the Toledo Bridge and Crane Company.

The following year Isaac Hausman started his own business, The Hausman Steel Company in Toledo, engaging in structural engineering and supplying fireproof building materials. Thousands of schoolhouses, offices, hotels, and apartment houses were subsequently handled by his firm; the emphasis was gradually placed on fireproof construction and, especially, on reinforced concrete structures. He acquired the patent rights of the Ambursen Form System from the Blaw-Knox Company and subcontracted for the steel form work on reinforced concrete buildings throughout the eastern United States. The jobs were scattered from Maine to Wisconsin, and from Texas to Florida, including such work as the eighteen million dollar charity hospital in New Orleans, La. His firm specialized in rigid frame construction and other indeterminate forms of concrete work, developing some very interesting and novel designs.

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[1] Memoir prepared by Raymond C. Reese, Assoc. M. ASCE.

Mr. Hausman was a member of the American Concrete Institute, the Alumni Association of the Massachusetts Institute of Technology, and the Toledo Rotary Club; he was a registered professional engineer in the State of Ohio. He was always very active in the affairs of the Concrete Reinforcing Steel Institute, particularly the Concrete Form Section, in standardizing sizes, improving techniques, raising ethical standards, and any other work that would fall to this section.

Besides his professional and business activities, one of his chief interests was golf, a game which he learned late in life, after the pressure of developing a business permitted some relaxation. For a number of years he was a prominent member of the Glengary Country Club in Holland, Ohio, where he was once presented with a plaque commemorating his guidance in reconstructing the club.

Although trained in the cold science of engineering, Isaac Hausman's interests were very largely in human beings. He was vitally interested in all his employees, their families, home life, successes, and difficulties, and was similarly interested in the personal lives and successes of his customers and suppliers—with anyone with whom he came in contact. His memory of intimate details in the lives of casual acquaintances was phenomenal.

On March 10, 1915, in Toledo, Mr. Hausman was married to Clara Staadecker. He is survived by his widow; three sons, Frederick I., James S., and William L. Hausman; his mother; and four brothers, Myer R., Benjamin F., Lewis W., and Nathan Hausman.

Mr. Hausman was elected an Associate Member of the American Society of Civil Engineers on May 28, 1923.

WILLIAM ELLIS ROW IRWIN, Assoc. M. ASCE[1]

Died March 25, 1946

William Ellis Row Irwin, the son of Franklin Fisher and Minnum (Row) Irwin, was born in Philipsburg, Pa., on July 15, 1896. He was graduated from Philipsburg High School and received the degree of Bachelor of Science, in 1922, from the School of Civil Engineering at the University of Michigan in Ann Arbor. Later he studied finance and economics at the Wharton School of the University of Pennsylvania in Philadelphia. Prior to his graduation he served as an Ensign in the United States Navy during World War I, seeing action in European waters and in Germany and England; and, as a member of Col. Edward M. House's staff, he attended the Peace Conference in Versailles, France.

Mr. Irwin's first assignment after graduation was with the Pennsylvania Department of Highways as inspector on concrete and asphalt road construction. In the early part of 1923, he entered the service of Day and Zimmerman as instrumentman in charge of a field party on construction of the Holtwood Power Plant at Holtwood, Pa.

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[1] Memoir prepared by Leo M. Odom, M. ASCE, and Byron P. Lyons, Dept. of Public Works, State of Louisiana, Baton Rouge, La.

In December, 1923, he was employed by Gibbs and Hill, who were working on the electrification of the Virginia Railway. His first position with this firm was as instrumentman at Narrows, Va., on transmission line surveys, estimates, and inspection during construction of the Narrows Power Plant. He advanced rapidly with Gibbs and Hill, his next assignment being chief of party in charge of construction of shops and other facilities at Mullens, W. Va. He then became office engineer at Roanoke, Va., and later assistant engineer.

In 1926 he severed his connection with Gibbs and Hill and returned to the employ of Day and Zimmerman as assistant engineer at West Chester, Pa., in charge of the drafting room. In this capacity he prepared data for the construction of the Conowingo transmission line. From 1928 until 1933 he was field engineer for the firm in charge of construction of the New Eastern State Penitentiary at Graterford, Pa.

In 1933 Mr. Irwin entered the service of E. I. du Pont de Nemours and Company as assistant construction superintendent. His first assignment was at Deepwater, N. J., on construction of units of the Dye Works. In 1936 this company started construction of a plant to manufacture tetraethyl lead for the Ethyl Corporation at Baton Rouge, La. Mr. Irwin was selected as superintendent of construction and, under his supervision, the plant was built and put into operation. In 1941 he was called to the main office of E. I. du Pont de Nemours and Company at Wilmington, Del., and was made district superintendent of general construction. In this capacity he supervised part of the large war construction program of the company throughout the United States. In June, 1943, he devoted his entire time to the construction of the atomic bomb project at Oak Ridge, Tenn., and, although he remained in Wilmington, he supervised all construction at Oak Ridge until this plant went into operation late in 1943. Early in 1944, he was assigned to special work at the atomic bomb project in Pasco, Wash. In May, 1944, upon completion of the war construction program, Mr. Irwin returned to Baton Rouge to become works engineer in the organic chemicals department, in charge of the operation of several departments at the ethyl plant.

In the latter part of 1945, all interests of E. I. du Pont de Nemours and Company were closed out in Baton Rouge and the operation of the plant manufacturing tetraethyl lead was taken over by the Ethyl Corporation. Mr. Irwin was appointed assistant resident manager by the Ethyl Corporation and held this position until his death. He was stricken while attending to his duties at the ethyl plant and died March 25, 1946. He was buried at Media, Pa.

He was a man respected by all who knew him and was known as "Bill Irwin" to men connected with the construction and operation of the Baton Rouge plant. He always had time to listen to any man who thought he had a complaint, regardless of what position the man held.

William Irwin was married to Virginia Kinnier on July 15, 1918, at Charleston, S. C. His widow and son, William K., survive him.

He was a member of the American Legion, the University of Michigan Club of Philadelphia, and the Phi Mu Alpha fraternity.

Mr. Irwin was elected an Associate Member of the American Society of Civil Engineers on January 14, 1929.

OSCAR WILLIAM LANZENDORF, Assoc. M. ASCE[1]

DIED APRIL 13, 1945

Oscar William Lanzendorf, the son of Ernest William and Bertha (Hanisch) Lanzendorf, was born in San Francisco, Calif., on July 5, 1888.

After receiving his preparatory education at the California School of Mechanical Arts in San Francisco, he was employed by the William Little Estate Company, contractors and builders, in San Francisco, from 1907 to 1908, as a draftsman, estimator, and bookkeeper. He then entered the University of California in Berkeley, where he completed a course in the College of Civil Engineering and was graduated with the degree of Bachelor of Science in 1913.

From 1913 until 1917 he was employed by the California Highway Commission, first as draftsman and later as resident engineer on highway planning, construction, and bridge design.

The United States Army called in 1917, and he enlisted as a private in the Corps of Engineers. He served in Forty-Third Engineers and Twentieth Engineers in France, attaining the rank of First Lieutenant.

When he returned to civilian life after World War I, he was again employed by the California Highway Commission from 1919 until 1921, engaged in general engineering in the office of Division IV in San Francisco.

In 1921 he was employed by Sutter County, California, as assistant county engineer and in 1925 became county engineer, working on the many phases of road and bridge design and construction until 1942. During World War II, he was assistant post engineer at Camp Beale, California.

On December 24, 1927, Mr. Lanzendorf was married to Julie White. He is survived by his widow.

Mr. Lanzendorf was elected an Associate Member of the American Society of Civil Engineers on January 19, 1920.

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[1] Memoir prepared by a Committee of the Sacramento Section consisting of Harold F. Percival, Chairman, Joel B. Hodges and Martin H. Blote, Assoc. Members, ASCE.

LESLIE LOU LEVEQUE, Assoc. M. ASCE[1]

DIED MAY 5, 1946

Leslie Lou LeVeque was born on November 9, 1892, in Porie, Mich. He was the son of Lou J. and Anna B. (Coltes) LeVeque. His primary and preparatory schooling were completed in Marquette, Mich., and in 1916 he was graduated from the University of Michigan at Ann Arbor, with the degree of Bachelor of Science in Civil Engineering.

Immediately following his graduation and until the beginning of World War I, Mr. LeVeque was associated with A. Bentley and Sons in Toledo, Ohio, as structural engineer. With the declaration of war in 1917, he enlisted in the United States Army and received his commission as a Second Lieutenant at Fort Sheridan, Ill. Promoted to First Lieutenant in the field, he saw action with the First Gas Regiment (Thirtieth Engineers), Company E, in the Meuse-Argonne and St.-Mihiel offensive and was decorated three times.

Upon his return from the war he was married to Elsa Will, M.D., from Rochester, N. Y., on May 1, 1920.

Prior to the organization of his own construction company, Mr. LeVeque was associated with H. J. Speaker, General Contractors, in Toledo as engineer and superintendent of construction; he left that company to do some research at Ohio State University in Columbus where he taught mathematics in the Engineering Department.

In early spring of 1921 he left the university and opened a Columbus Office for the Watts-Survier Company, General Contractors, in Toledo. He had full charge of all operations in this area and under his management this office soon had more contracts than did the main office.

In the fall of 1921 he resigned from Watts-Survier Company and organized The L. L. LeVeque Company in Columbus; and, as president of the company, he directed the construction of schools, hospitals, dormitories, industrial plants, federal buildings, apartment buildings, theaters, and warehouses throughout the United States and Canada. These structures varied from $300,000 warehouses to $7,500,000 apartment dwellings. Among the principals with whom Mr. LeVeque contracted were the Great Atlantic and Pacific Tea Company, the Ohio Bell Telephone Company, Ohio State University, the Ralston Steel Car Company, as well as city, state, and federal governments.

As early as 1935 Mr. LeVeque envisioned the plans of a model village that would not merely "house" families, but that would also afford the comforts and social enjoyments generally attained by only the most successful businessmen. In Columbus, on the scenic banks of the Olentangy River, on a sixty-six acre tract, lies the fullfilment of that vision. Olentangy Village is a beautifully landscaped plan of apartment dwellings, capturing the spirit of old Williamsburg (Va.). Here, a living memorial is perpetuated in the community spirit of the 1,150 Olentagny residents, whose public pride is engendered by their own Olentangy club rooms, victory gardens, swimming pool, tennis and badminton courts, horseshoe courts, ball diamonds, picnic grounds, playgrounds, and fishing and boating facilities.

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[1] Memoir prepared by Myron T. Jones and Howard E. Bonham, Members, ASCE, and Sanders A. Frye, Business Mgr., Otterbein College, Westerville, Ohio.

In 1940 a family recreation center was designed and constructed in Olentangy Village. The Olentangy Village Bowling Center was a pioneering venture. Skeet, rifle, and archery ranges, as well as golf driving mats and table tennis were included with the thirty-two bowling lanes. The pleasant atmosphere, in addition to the strict policy forbidding the selling of any alcoholic beverage, has made Olentangy a community for the young people in Columbus.

In 1942 Mr. LeVeque directed his energies from the field of construction to the field of manufacturing. With record-breaking speed and precision, the Wolab Corporation made delivery of vital tools, jigs, and fixtures for military aricraft. No undertaking seemed too great—no obstacle was too big. As an industrial engineer, Mr. LeVeque exemplified the spirit of the "Army-Navy E" to which award he led his employees.

Born from his intolerance of waste, a fully mechanical, antomatic, pin setting machine for tenpins in bowling was developed by Mr. LeVeque at the close of the war. Steps for the reconversion of the Wolab factories were already being taken when the tragedy of his plane crash took the lives of Mr. and Mrs. LeVeque.

Mr. LeVeque organized and was president of many realty companies, residential hotels (including the Parc Vendome in New York, N. Y.), apartment buildings, manufacturing companies, and office buildings (including the LeVeque-Lincoln Tower in Columbus).

His professional contributions and achievements in the fields of civil and industrial engineering and finance reflect the intenseness with which Mr. LeVeque lived every hour of his career. Mr. LeVeque was modest in self-appraisment, daring in his conceptions, and courageous in the execution of his plans which made his life one that was full of action. He was a man of his time, reflecting the tempo of the era in which he lived. He was not one to seek the limelight—he preferred to have appraisal of his fellow citizens based on his accomplishments. His efficiency, pioneering foresight and vision, and the tenacity with which he saw every undertaking completed enabled him to translate his ideas into practical action and made him one of the outstanding figures of his time.

All those who knew him will remember him not as much in the light of his personal accomplishments but more as the man he was. This fact stands out above all others regardless of the association, whether it was one of business or one of personal friendship. His generosity and thoughtfulness, his genuine smile, his good humor, his love of life and work, his honesty, his efficiency, his modesty, and sense of fair play live in the memories of those who knew him.

Financier, engineer, sportsman, and patron of the arts, Mr. LeVeque lived a full life. His untimely death has deprived his community of an esteemed citizen and a great leader, but his spirit and the good of his accomplishments will remain as a perpetual memorial to him and his ideals. He is survived by a son, Frederick Will, and two daughters, Betty Marx and Patricia Lou.

Mr. LeVeque was elected an Associate Member of the American Society of Civil Engineers on August 28, 1922.

JAMES EVERETT MACKIE, Assoc. M. ASCE[1]

Died February 21, 1946

James Everett Mackie, the son of William G. and Caroline (Overman) Mackie, was born on March 18, 1898, at Duluth, Minn. He received his early education in Green Bay, Wis., and was employed in that city at the beginning of World War I.

During that war he served as private, corporal, and sergeant in Battery E, 121st Headquarters Field Artillery, 57th Brigade, 32d Division, American Expeditionary Forces. Shortly after receiving his discharge, he entered the University of Wisconsin at Madison, from which he was graduated in 1923 with the degree of Bachelor of Science in Civil Engineering. In 1929 he was awarded the degree of Civil Engineer by the University of Wisconsin.

During summers and holidays of his college years Mr. Mackie obtained practical experience with construction firms, and in various capacities with the Wisconsin State Highway Commission. He subsequently moved west to Long Beach, Calif., where he was employed in the city building inspection department from 1924 until 1927, having been appointed chief building inspector in December, 1925.

During this period he became interested in the pioneering efforts, started in 1922, by a group of western building officials to formulate a "Uniform Building Code." Progress on this project had reached the third preliminary draft stage in 1926 when Mr. Mackie was elected secretary of the Pacific Coast Building Officials Conference. For several years afterward he was largely responsible for the preparation and editing of the various printed editions of the "Uniform Building Code" of the conference.

Perhaps the outstanding accomplishment of his career was the guidance of the early editions of this code through the troubled waters of politics, engineering principles, taxpayers' pulling and materials producers' hauling, and the practical possibilities of enforcement. Truly it was a task well done.

From July, 1927, to April, 1929, Mr. Mackie was employed as full-time secretary of the conference. In April, 1929, he left that position to become structural engineer for the National Lumber Manufacturers' Association.

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[1] Memoir prepared by Arthur C. Horner, Assoc. M. ASCE.

During the many months he spent lecturing on wood as a structural material at colleges and technical schools throughout the United States, and consulting with engineers, architects, and wood users—generally on a wide range of problems pertaining to the use of wood—he had the opportunity not only to put to use his considerable engineering ability but also to make a host of friends throughout the profession. At his death, on February 21, 1946, Mr. Mackie was western manager of the National Lumber Manufacturers Association, with offices in San Francisco, Calif.

His interest in public welfare, as attested by the many hours of gratuitous work spent on building codes with city councils and civic organizations, as well as on technical and lay committees, was fittingly recognized by his election as an honorary member of the Pacific Coast Building Officials Conference.

On October 25, 1923, Mr. Mackie was married to Gene Langson Plumb. He is survived by his widow and one daughter, Travis.

Mr. Mackie was elected a Junior of the American Society of Civil Engineers on November 26, 1923, and an Associate Member on June 9, 1930.

LEROY MONROE MILNER, SR., Assoc. M. ASCE[1]

DIED MAY 7, 1946

LeRoy Monroe Milner, Sr., was born in Birmingham, Ala., on March 8, 1890. He was the son of John A. and Sally Milner, early settlers of that city.

From September, 1909, to February, 1913, Mr. Milner was a mining engineer with the Tennessee Coal, Iron, and Railroad Company in Birmingham. For the next four years, until 1917, he was associated with the Milner and Browne Engineering Company of Birmingham, engaged in a general engineering practice.

In 1917 he returned to the Tennessee Coal, Iron, and Railroad Company, remaining in its engineering department until 1922. The following year he became a partner in the Milner Engineering Company of Sheffield (Ala.), Florence (Ala.), and Tuscumbia (Ala.). This organization played an important part in the development of the Muscle Shoals district at Wilson Dam in Alabama.

From September, 1928, until his death, Mr. Milner was an inspector in the Memphis District of the United States Engineer Department. His various activities included construction of levees and inspection of revetments, maintenance and improvement work on the tributaries of the Mississippi River, and emergency flood protection measures on the tributary rivers.

In 1912 he was married to Myrtle Lucy Rutherford of Birmingham. He is survived by his widow in Memphis, Tenn., and two sons, both engineers, LeRoy Milner, Jr., of Sheffield, and Robert Milner of Memphis.

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[1] Memoir prepared by Robert Milner, Mech. Engr., Buckeye Cotton Oil Co., Memphis, Tenn.

Mr. Milner belonged to the St. Johns Methodist Church of Memphis, the Memphis Engineers Club, and the Rotary Club of Newport, Ark.; and he was a member of the Society of American Military Engineers.

He was highly respected by his many friends throughout the midsouth for his strong character, professional loyalty, honesty, and perseverance.

Mr. Milner was elected an Associate Member of the American Society of Civil Engineers on May 13, 1940.

RALPH NOBLE PRIEST, Assoc. M. ASCE[1]

Died May 14, 1946

Ralph Noble Priest was born in Roxborough, Pa., on August 9, 1882. His parents were Andrew Priest and Sarah Noble, descendants of the pioneer German stock that has been so prominently identified with the growth of eastern Pennsylvania.

After attending Philadelphia (Pa.) High School at night, for about three years, he entered the drafting room of the Pencoyd Iron Works, at the age of eighteen, where he worked for a short time. From 1900 to 1908, he was with Amos W. Barnes, an engineer in Philadelphia.

Studying diligently under the direction and encouragement of Mr. Barnes, he showed early natural ability and capacity for hard work. He attended classses at Drexel Institute of Technology, in Philadelphia, and then completed a course of study with the International Correspondence Schools, receiving their certificate in civil engineering. During this period Mr. Priest designed structural work for an incinerating plant in Atlantic City, N. J.; for the Lardners Point Pumping Station in Philadelphia; and for an addition to the Coplay Cement Company plant, in Coplay, Pa. In addition, he was in charge of a survey for an electric freight line in Delaware—about one hundred miles long.

Mr. Priest was in charge of the structural steel design and erection of the original Forrest Theater in Philadelphia. From 1908 to 1910, he was with George F. Pawling, M. ASCE, engineer and contractor, as designer and estimator. In 1911 he was again with Amos W. Barnes as general assistant engineer, looking after details of a lift bridge across the Chicago River.

Associations with M. Ward Easby and W. E. Wark and Company, as vice-president, followed, covering the years to 1916. This period included work on the design of the Mulberry Street viaduct in Scranton, Pa., and on heavy shoring for deep foundations for large city buildings.

From 1916 to 1941, Mr. Priest maintained his own office at 1627 Sansom Street, Philadelphia, as consulting engineer. He was retained by Morris-Wheeler and Company, Incorporated, as an independent engineer and structural steel detailer for more than ten years. This work covered theaters, apartment buildings, office buildings, and other structures. Work completed from his own office included the design of the MacAndrews and Forbes plant in Camden, N. J., and the Electric Storage Battery Company buildings and Royal Apartments, both in Philadelphia.

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[1] Memoir prepared by W. E. Belcher, M. ASCE.

His experience acquainted him with considerable wood truss and framing design work. This experience brought invitations for consultation with R. B. Okie and other prominent architects, and led to a special study of early American barn framing, church roof trusses of the early Pennsylvania type, and Pennsylvania Dutch architecture.

Some outstanding examples of Mr. Priest's work are: Roof trusses and timber framing of the Pennsbury Manor House on the Delaware River near Bristol, Pa. (R. B. Okie, architect); slow-burning wood framing for the Lang Paper Mills (Alfred Mackay, supervising engineer); trusses and framing for the barn of A. E. Pew in Elberson, Pa., and for the residence of John M. Cross in Bethlehem, Pa. (R. B. Okie, architect); truss and beam construction for McDowell Paper Mill in Manayunk, Pa., for the Bell residence in Devon, Pa., and for the Presbyterian Church at Mechanicsburg, Pa.

From 1941 to the spring of 1945, he was employed in the drafting room of the United Engineers and Constructors, Incorporated, as structural designer and checker on various industrial and public utility projects. Because of ill health, Mr. Priest was obliged to cease active engineering work, about one year before his death.

He had a quiet unassuming disposition, was always ready to assist someone else, and was a good citizen in the community in which he lived. Having had no college training, he was well educated, however, and had unusual engineering ability. His experience was quite extensive, and he was ready for any problem.

Mr. Priest was a registered professional engineer of the State of Pennsylvania and a member of the Roxborough Lodge of the F. and A. M. For sixteen years he was an active member of the Mendelssohn Club, starting under Dr. Gilchrist. He had a fine bass voice and enjoyed choral work. As an ardent student of nature, he was well versed in bird lore, and was a member of the Delaware Ornithological Society. He also took great pride in his home, garden, and flowers.

On October 25, 1917, he was married to Irene Thompson at Roxborough, where they made their home for many years, until recently, when they moved to Gwynedd Valley, Pa. He is survived by his widow and their son, Ralph N. Priest, Jr.

Mr. Priest was elected an Associate Member of the American Society of Civil Engineers on March 12, 1923.

ROBERT CHARLES RATCLIFFE, Assoc. M. ASCE[1]

Died August 16, 1945

Robert Charles Ratcliffe, the son of George McAfee Ratcliffe and Susie (Hetherwick) Ratcliffe, was born on April 12, 1903, in Natchitoches, La. He was the sixth in a family of seven children and was left an orphan at five years of age. His mother died when he was four years old and his father before he was six. In 1912, the family moved to Colorado under the guardianship of the oldest sister.

He was graduated from North Denver High School in 1920, and entered the University of Colorado at Boulder, in 1922. During this interval he worked for the local telephone company. He was graduated from the University of Colorado in 1927 with the degree of Bachelor of Science in Civil Engineering. During the summer vacations he had worked for the United States Bureau of Public Roads as rodman, chainman, and levelman on highway location and construction in Colorado.

After graduating from the university, he worked as draftsman and instrumentman on a real estate subdivision for the Redfeather Lakes Development Company. This was followed by eight years of work on highway inspection and construction. In March, 1928, he joined the Iowa State Highway Commission at Ames, as proportioning plant and concrete slab inspector, and later became bridge inspector and chief of party on highway construction and location; in 1931, he became bridge inspector for the Wyoming State Highway Department, a position that he held for two years; and finally he worked as bridge inspector for the Colorado State Highway Department for three years, becoming eligible for appointment as resident engineer.

Robert Ratcliffe was appointed chief of the Department of Public Works of the City of Grand Junction, Colo., in July, 1936. He was in charge of the water sewer and highway divisions, and was actively engaged in the design, construction, maintenance, and development of these public utilities. In 1940, he became resident engineer for the construction of the sewage disposal plant in Grand Junction.

At the approach of World War II, he turned his energy and skill to army camp construction, with particular reference to the sanitary problems involved —water supply and sewage disposal. Working for several contracting firms in Colorado he was assigned to the United States Army housing project at Fort Logan, the Denver ordinance plant, and the Camp Carson project. At the outbreak of the war, Robert Ratcliffe joined the staff of United States Army Engineers at Albuquerque, N. Mex., as engineer on specifications, and was soon rated associate civil and sanitary engineer. He either designed or passed on all designs for sanitary works in the area under the jurisdiction of the Albuquerque office, an area which comprised New Mexico, Arizona, southern Colorado, western Texas, and Oklahoma.

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[1] Memoir prepared by Roland B. Queneau, M. ASCE.

Early in 1943, he applied for a commission in the miltary government teams that the U. S. Army was organizing for use in conquered territory; the application was favorably considered, but he was turned down after failing to pass the physical examinations because of perforated eardrums. In January, 1945, he joined the United States Bureau of Reclamation as engineer in the Cheyenne (Wyo.) district, where he worked until his death on August 16, 1945.

Robert Ratcliffe was married to Charlotte West, a childhood friend, on November 29, 1933, in Denver. He is survived by his widow and a son, Peter Gordon Ratcliffe.

From early childhood, Robert Ratcliffe's character was earnest and serious. The tragedy in his childhood and, later, his partial deafness, caused by a delicate ear operation during his student days, merely increased his determination to acquire a thorough education and become a fully self-reliant and capable engineer. His personal integrity was always beyond question, he strove constantly for perfection in himself and expected others to do likewise; he had little patience with shoddy or dishonest work. His personality was open and generous, and he was devoted to his family and friends.

As an engineer, Robert Ratcliffe's contribution to the modern age consisted in work carefully planned with thorough technical skill and ample vision, and carried out under scrupulous supervision.

Mr. Ratcliffe was elected a Junior of the American Society of Civil Engineers on November 14, 1927, and an Associate Member on November 9, 1936.

CHARLES MORRISON SCUDDER, Assoc. M. ASCE[1]

DIED JULY 12, 1946

Charles Morrison Scudder was born in Marinette, Wis., on February 27, 1890, the son of Henry Townsend and Janet (Morrison) Scudder. He attended the public school at Marinette and was graduated, as valedictorian of his class, from the Marinette High School, in 1907. He received the degree of Bachelor of Science in Civil Engineering from the University of Wisconsin at Madison, in 1911, and the degree of Civil Engineer in 1912, after a year's postgraduate work in hydraulics.

For two years he was with the Knoxville Power Company in Alcoa, Tenn., in various capacities, and then served for three years as an instructor in surveying and mechanics at the University of Wisconsin. During World War I he served as Captain of Company C, First Battalion, Wisconsin Engineers, of the National Guard, and Company F, 107th Engineers, in the Thirty-Second (Red Arrow) Division.

Following the war he worked with the Allis-Chalmers Company at the West Allis (Wis.) plant on hydraulic power plant and other design work for several years. In 1928 he was with the First Wisconsin National Bank of Milwaukee (which was later a part of "Wisconsin Bankshares") and for ten years was employed by them in various capacities, where his engineering education and experience were of value. Much of this time he was in charge of the Gilson-Bolens plant at Port Washington, Wis., a plant in which the bank was interested.

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[1] Memoir prepared by Lewis M. Hammond, M. ASCE.

Mr. Scudder enjoyed this work very much, as it gave him an opportunity to apply his engineering training to practical business problems. In 1938 he went back to engineering, and for the last eight years was assistant engineer in charge of drafting and pressure vessels at the A. O. Smith Corporation in Milwaukee.

He had two hobbies, boats and music. The Wisconsin lakes, in summer, were a never-ending invitation and opportunity for him to enjoy his boats. They were mostly motorboats although he was also interested in sailing. His music he enjoyed everywhere. During his service in World War I, he organized, outfitted, and led the 107th Engineers' Band. He played almost every instrument well, and he and his piano accordion were the life of countless parties. He was always ready to give of his talents to entertain or amuse his friends. Mr. Scudder enjoyed singing, too. He took a keen interest in the National Association for the Preservation and Encouragement of Barber Shop Quartets and he helped organize and sang in one of the Milwaukee quartets.

He took an active interest in the affairs of the Local Section of the Society, the Wauwatosa (Wis.) Co-operative Club, and the American Legion. In the American Legion he was first affiliated with the Alonzo Cudworth Post in Milwaukee, later transferring to the Bernard Diedrich Post in Wauwatosa. At the time of his death he was writing a history of the 107th Engineers.

In 1916 Mr. Scudder was married to Eleanor Lee, of Oconto, Wis. He is survived by his widow; a daughter, Janet; a son, Charles; and a sister, Marjorie Scudder.

Because of his connections over a period of twenty-seven years with two of Milwaukee's largest industrial concerns, and its leading bank, Mr. Scudder had an unusually large circle of friends.

Mr. Scudder was elected a Junior of the American Society of Civil Engineers on May 28, 1912, and an Associate Member on October 2, 1922.

FELIX CHARLES STEHLE, Assoc. M. ASCE[1]

DIED AUGUST 14, 1946

Felix Charles Stehle was born on May 10, 1872, in New York, N. Y. His parents were Francis S. and Phillipina (Hausel) Stehle.

He received his early education in New York City schools, and was graduated from De La Salle Institute in 1888, after completing the academic course.

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[1] Memoir prepared by Glenn B. Woodruff, M. ASCE.

In 1892 he received the degree of Civil Engineer from Manhattan College, also in New York City.

His engineering career began with the position of structural draftsman with the Cornell Iron Works in Brooklyn, N. Y., and later with the Union Bridge Company at Athens, Pa. Upon the consolidation of the latter company with the American Bridge Company, he was transferred to Pencoyd, Pa., and later was made assistant plant engineer at Brooklyn.

In 1913 Mr. Stehle accepted the position of bridge inspector of the Lehigh Valley Railroad Company, with headquarters in Bethlehem, Pa., making his home in Towanda, Pa. At this time the railroad started a program of rebuilding most of its main line bridges. He continued in this work for the remainder of his active professional life.

In addition to his duties in charge of all bridge inspection, Mr. Stehle directed all field work on this reconstruction. The work proceeded with a remarkable freedom from accidents. No fatal or permanent disability cases occurred on any of this work, which was conducted without interruption of traffic. Mr. Stehle was retired from active service in 1941.

During his career he was most concerned with aiding the younger men with whom he came in contact. His knowledge of steel details, acquired in the drawing room and in the inspection of structures in service, made his advice invaluable.

In 1904 he was married in Towanda, to Mary Espy, who, with one daughter Katrina (Mrs. Edson Barnes), survives him. Another daughter, Adelaide (Mrs. Joseph Stevens) of New York City died on November 23, 1942. He was a member of the Roman Catholic Church and of the Knights of Columbus.

Mr. Stehle was elected an Associate Member of the American Society of Civil Engineers on June 3, 1903.

FRED WALTER STIEFEL, Assoc. M. ASCE[1]

DIED JULY 25, 1946

Fred Walter Stiefel was born in New York, N. Y., on September 12, 1892. He began his engineering work in July, 1910, and thereupon was engaged on city surveys, building construction, highway projects, and sewer work in New York City.

Although he started work at an early age, without the opportunity of attending engineering school, Mr. Stiefel studied during the evenings. He attended engineers' meetings and was able to learn through his own experience many engineering principles relating to the important projects with which he was connected. In his work as an engineer he showed confidence and originality.

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[1] Memoir prepared by James F. Sanborn, M. ASCE.

In October, 1911, he went to work as junior engineer for the Public Service Commission in the Office of Transit Construction (later the Transit Commission) in New York City. He supervised subway construction work, which was actively under way at that time. During this experience, he became acquainted with the late Robert Ridgway,[2] Past-President, ASCE, at that time chief engineer of the subway program. Mr. Stiefel greatly admired Mr. Ridgway, who also had succeeded in his engineering work without a formal education.

Mr. Stiefel left the employ of the Transit Commission in 1918 and worked for the Mason and Hanger Company for about four years, as engineer and assistant manager of construction on two contracts of the New York subway. For the same company he was later engaged on the construction of a two and one-half mile rock tunnel for a hydroelectric project in Vermont.

After 1925, Mr. Stiefel was employed by the Rosoff Subway Construction Company under the late Harry Sanford, M. ASCE, on the construction of a section of the Eighth Avenue Subway at St. Nicholas Avenue. On the death of Mr. Sanford in 1928, Mr. Stiefel was appointed chief engineer of the contracting company, a position he held until he resigned in 1942.

During this period he was also chief engineer of the Rosoff Tunnel Corporation and associated contracting companies, in charge of all engineering and construction contracts. The work performed by these companies included subways and sewer tunnels in New York City, a section of the Delaware Aqueduct tunnel (seventy-five thousand feet long), the "Third Lock" project for the Panama Canal, and other construction jobs.

Mr. Stiefel was always ready to try new devices on the work with which he was connected, and he designed many ingenious methods to reduce cost and promote safety. Some of his progressive ideas included the use of the following: Diesel locomotives in mining and tunnel construction; Diesel driven shovels under decking for subway work; welded steel pipe for by-passing gas mains; special equipment for reclaiming trolley tracks rapidly and cheaply; subway decking on underpinning piers for the support of elevated columns; the cofferdam method for supporting buildings along a subway excavation; and many others.

In 1944 Mr. Stiefel was awarded the Construction Engineering Prize of the Society for his paper, "Inclined Mine Shaft Sunk in the Adirondacks."[3] He also contributed papers to the American Institute of Mining and Metallurgical Engineers of which he was a member.

At his death in July, 1946, he was president of the Stiefel Construction Corporation in New York City.

Mr. Stiefel was elected an Associate Member of the American Society of Civil Engineers on August 4, 1924.

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[2] For memoir, see *Transactions*, ASCE, Vol. 106, 1941, p. 1527.

[3] *Civil Engineering*, April, 1944, p. 137.

DONALD RAYMOND BROWN, Jun. ASCE[1]

DIED JANUARY 17, 1946

Donald Raymond Brown was born in Watsonville, Calif., on July 17, 1915. He was the son of G. Raymond Brown and Myrtle Esther (Wolfe) Brown.

He attended school at Healdsburg, Calif., and the Santa Rosa District Junior College, at Santa Rosa, Calif. During that time he was employed, through vacation periods, by Marshall M. Wallace, the county surveyor. Mr. Brown continued his education at the Oregon State College, at Corvallis, and immediately upon receiving the degree of Bachelor of Science in Civil Engineering in June, 1939, he was employed by the County of Sonoma (California) on highway and bridge design. In September, 1939, his ability and interest in his profession won him an appointment to the position of deputy county surveyor.

While serving as deputy county surveyor, Mr. Brown designed and superintended construction of several bridges, including the Franquelin Bridge over Sonoma Creek, west of Sonoma, Calif.—a reinforced concrete structure consisting of two twenty-foot approach spans, two thirty-five-foot approach spans, and one seventy-foot main span.

In May, 1942, Mr. Brown enlisted in the United States Naval Reserve and entered the United States Naval Academy at Annapolis, Md., for officers' training, receiving his commission as Ensign on September 8, 1942. Upon volunteering for submarine duty, Ensign Brown was sent to Key West, Fla., and New London, Conn. Completing this training on June 26, 1943, Ensign Brown was assigned to duty on the submarine *Wahoo* in the Pacific Area. While on patrol duty in the Sea of Japan, the *Wahoo,* after completing her mission, attacked an enemy convoy with surface guns, was torpedoed amidship, and sank at once with loss of the entire crew.

Lieutenant Brown received his promotion on October 1, 1943, was reported missing on November 30, 1943, and continued in that status until declared as officially deceased by the Navy Department, as of January 7, 1946. He was awarded the Silver Star Medal, with citation; the Submarine Combat Insignia, with gold stars; and the Purple Heart Medal, with certificate. He is survived by his parents, who received his decorations.

He was an active member of the Order of DeMolay, having received the Order of Chevalier; the Masonic Order, Sotoyome Lodge No. 123, of Healdsburg; and the 20–30 Club of Healdsburg. He actively participated in such sports as golfing, skiing, hunting, fishing, and swimming, and was a qualified Red Cross campaign instructor and lifesaver.

Lieutenant Brown was elected a Junior of the American Society of Civil Engineers on October 16, 1939.

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[1] Memoir prepared by Marshall M. Wallace, County Surveyor & Road Commissioner, Santa Rosa, Calif.

JAMES CRISP AKERS SALTER, Jun. ASCE[1]

Died August 2, 1945

James Crisp Akers Salter, the son of Martha (Akers) Salter and James C. Salter, was born on November 15, 1919, in Montgomery, Ala. He was graduated from the Boys High School in Atlanta, Ga., in 1938; and for one year he attended Wheaton College, in Wheaton, Ill., later transferring to Clemson College in Clemson, S. C. In May, 1942, he received the degree of Bachelor of Science in Civil Engineering from Clemson and, at the same time, was commissioned a Second Lieutenant, Corps of Engineers, Army of the United States.

Lieutenant Salter immediately transferred to the Army Air Forces and thereupon was ordered to Maxwell Field at Montgomery on August 6, 1942, for pre-flight training. After subsequent pilot training at Dorr Field in Florida and at Cochran Field and Turner Field in Georgia, he was assigned to the 87th Bomb Squadron, 46th Bomb Group, at Will Rogers Field, Oklahoma, as pilot of an A–20 bomber.

Because of the acute need for capable pilots in the Pacific theater of operations, he was transferred to the 499th Bomb Squadron, 345th Bomb Group, and arrived at Port Moresby, Australia, on August 8, 1943. There he served as a B–25 pilot, attached to the 5th Air Force under Lt.-Gen. George C. Kenney. During Lieutenant Salter's assignment to this command he participated in the entire New Guinea campaign and the campaign of the Bismarck Archipelago, bombing Rabaul, New Britain, and the Admirality Islands. His squadron was then moved up to Hollandia, from which base he bombed Sorong, Halmahera, the Moluccas, and Nenado. By this time he had been promoted to Captain. He also participated in the invasion of the Philippines, before being returned to the United States because of combat fatigue.

During his assignment to the 499th Squadron from August, 1943, to February, 1945, Captain Salter flew about a thousand hours on fifty-eight combat missions; he, personally, was cited for bravery by General Kenney; he was awarded the Air Medal three times; and he became squadron leader of this "Bats Out O'Hell" Squadron. Captain Salter's combat flying, as shown on his officer's qualification card, was graded "excellent" or "superior" in every instance. In addition to the Air Medal with two clusters, he wore the Asiatic-Pacific ribbon with three bronze stars, representing the New Guinea, Bismarck Archipelago, and Philippine liberation campaigns.

Upon his return to the United States, Captain Salter was stationed at Williams Field, Arizona, where he served as a radar precision bomber pilot. His death, at the age of twenty five, occurred in an automobile accident while he was at that station. He was buried in Atlanta on August 7, 1945, with full military honors.

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[1] Memoir prepared by Harry B. Johnston, Jr., D.D.S., Atlanta, Ga.

Captain Salter's death, so early in life, prevented active participation in his profession, yet during his brief lifetime he displayed such skill and devotion to assigned duty as is seldom seen. Throughout his entire life, he exhibited all the rare traits of the fine Christian gentleman that he was.

He is survived by his mother and his grandmother, Mrs. F. M. Akers.

Captain Salter was elected a Junior of the American Society of Civil Engineers on September 9, 1942.

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